Short-range expansion and three-nucleon SRCs

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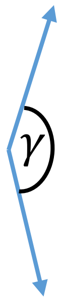
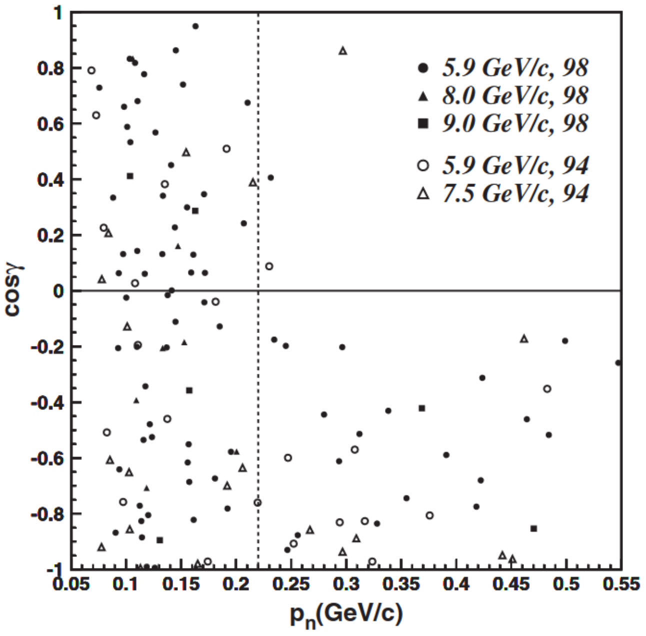
Short-Distance nuclear structure and PDFs ECT* workshop, July 18th, 2023

Nuclear short-range correlations

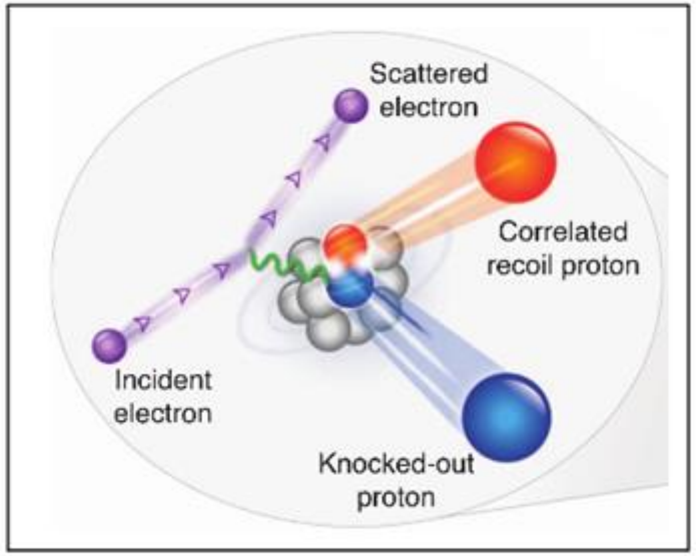
Studied experimentally using large momentum transfer quasi-elastic reactions

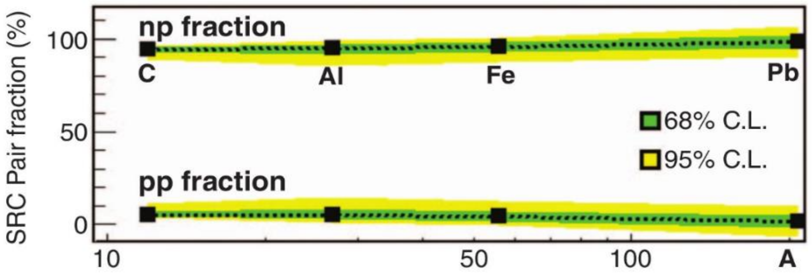


$^{12}\mathrm{C}(p, ppn)X$

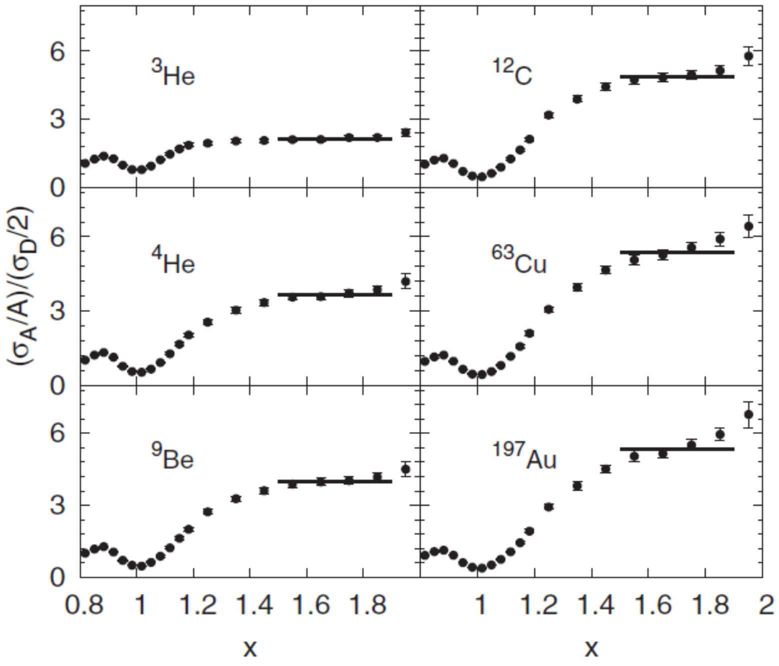


Piasetzky et al., PRL 97, 162504 (2006)





O. Hen et al., Science 346, 614 (2014)



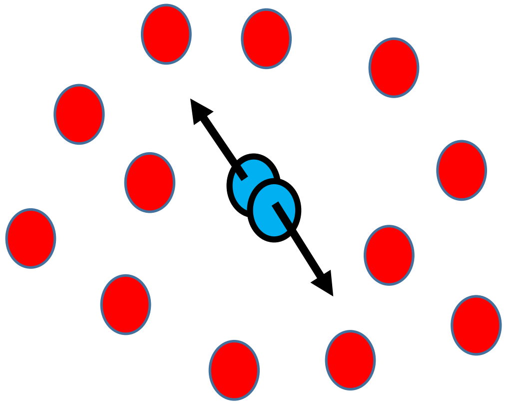
*Fomin et. al., PRL **108**, 092502 (2012)*

Nuclear short-range correlations

Studied experimentally using large momentum transfer quasi-elastic reactions

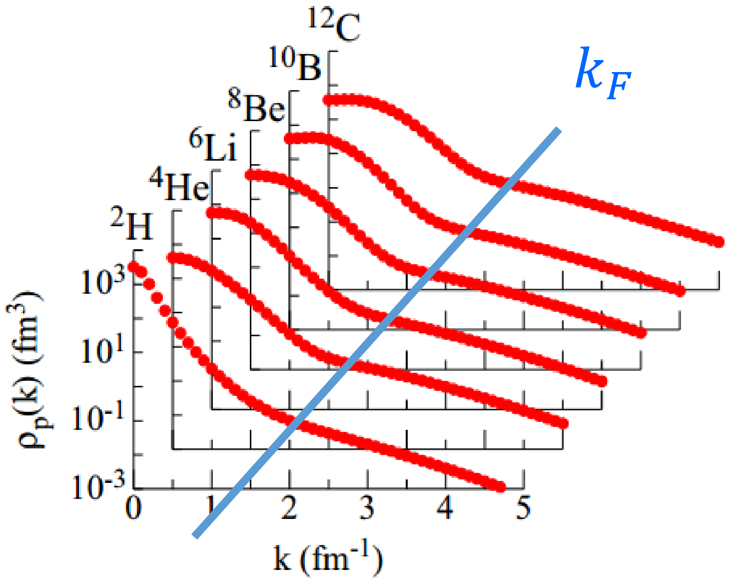
Main features:

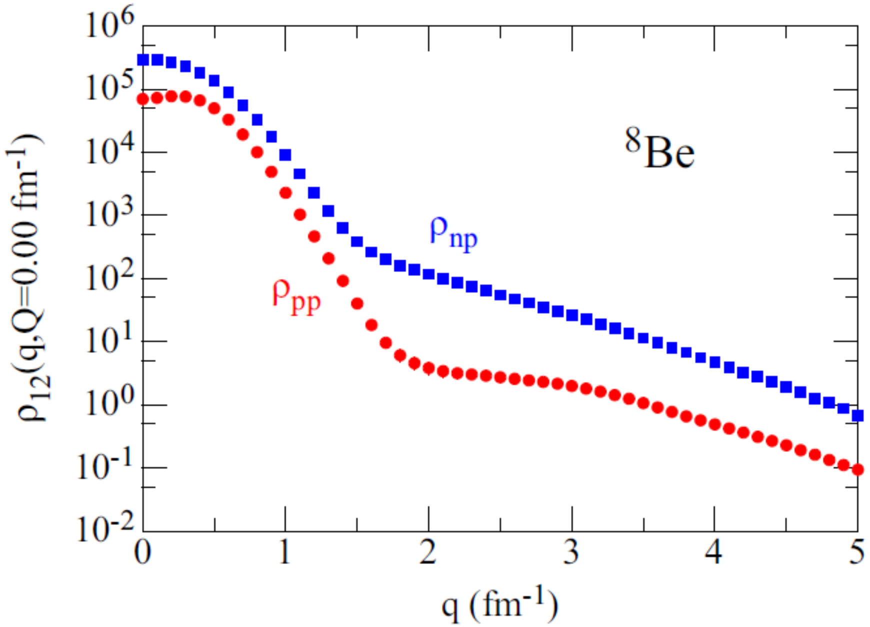
- High momentum particles with back-to-back configuration
- Universal behavior – “isolated pair”
- Neutron-proton dominance



Nuclear short-range correlations

Similar features are seen in ab-initio calculations:



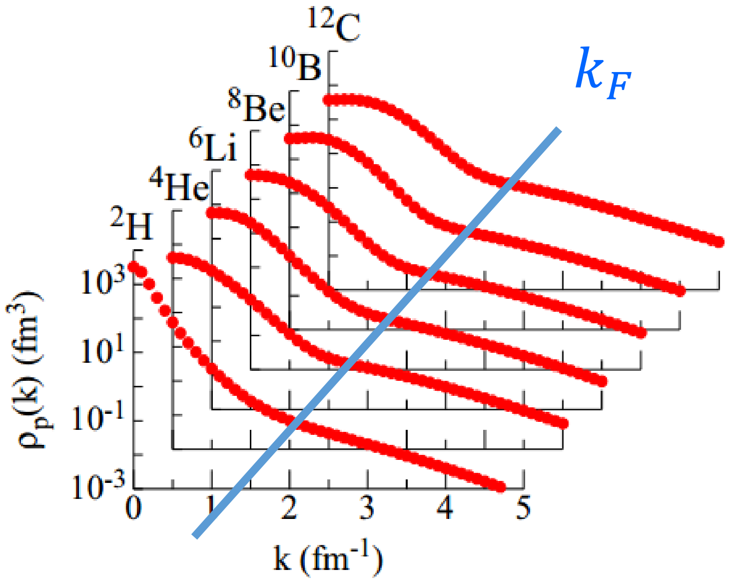


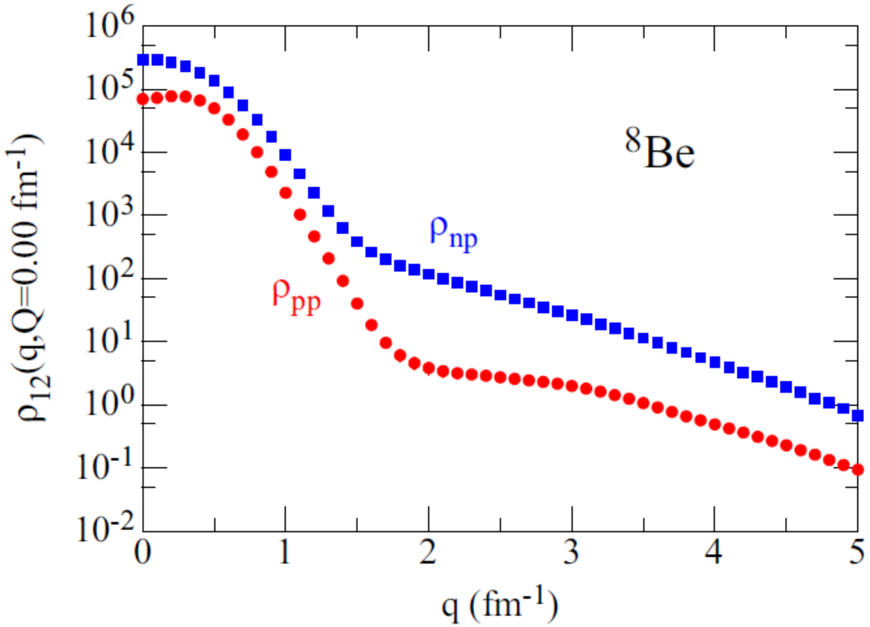
R.B Wiringa et. al., Phys. Rev. C 89, 024305 (2014)

Nuclear short-range correlations

Similar features are seen in ab-initio calculations:

- Limited to light nuclei (for “hard” NN interactions)
- Difficult to describe relevant reactions





R.B Wiringa et. al., Phys. Rev. C 89, 024305 (2014)

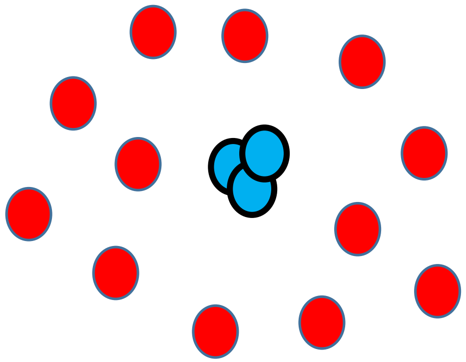
Nuclear short-range correlations

- How do we compare experiments to theory? Can we do that without solving the full many-body problem?
- Can we develop a systematic description?
- How can we use knowledge of SRCs for describing more general observables?

Outline

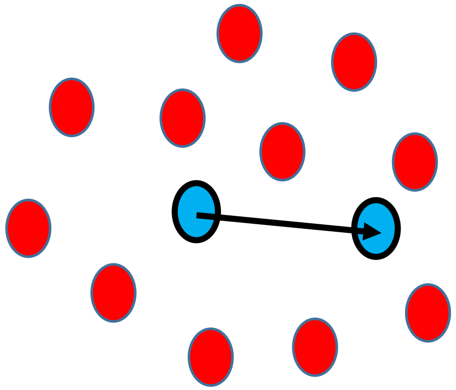
- The Generalized Contact Formalism (GCF)
  - Effective theory for describing SRC pairs
  - Comprehensive description of SRCs in nuclear structure and reactions

- Towards a systematic short-range expansion



Three-nucleon correlations

Systematic expansion for pairs



The Generalized Contact Formalism (GCF)

RW, B. Bazak, N. Barnea

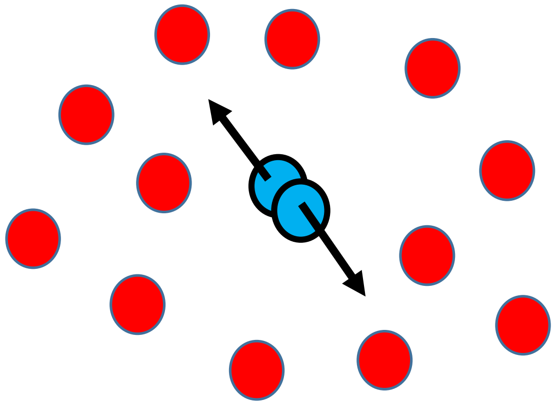
Generalized Contact Formalism

- Generalizing Tan's work for atomic systems

S. Tan, Ann. Phys. (N.Y.) 323, 2952 (2008); Ann. Phys. (N.Y.) 323, 2971 (2008); Ann. Phys. (N.Y.) 323, 2987 (2008)

- Starting point – Short-range factorization

$$\Psi(r_1, r_2, \ldots, r_A) \xrightarrow{r_{12}\to 0} \varphi(\boldsymbol{r}) \times A\left(\boldsymbol{R}, \{\boldsymbol{r}_k\}_{k\neq 1,2}\right)$$



Universal function
(but depends on the potential)

Nucleus-dependent function

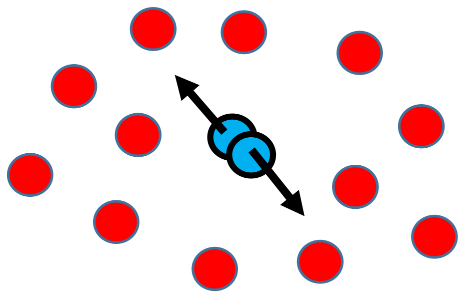
$\varphi(r) \equiv$ Zero-energy solution of the **two-body** Schrodinger Eq.

RW, B. Bazak, N. Barnea, PRC 92, 054311 (2015)

Generalized Contact Formalism

$$\Psi(r_1, r_2, \ldots, r_A) \xrightarrow{r_{12}\to 0} \varphi(\boldsymbol{r}) \times A(\boldsymbol{R}, \{\boldsymbol{r}_k\}_{k\neq 1,2})$$

universal function



For any **short-range** two-body operator $\hat{O}$

$$\langle \hat{O} \rangle = \langle \varphi | \hat{O}(r) | \varphi \rangle \, C \qquad C \propto \langle A | A \rangle$$

- Two-body dynamics
- Universal for all nuclei
- Simply calculated

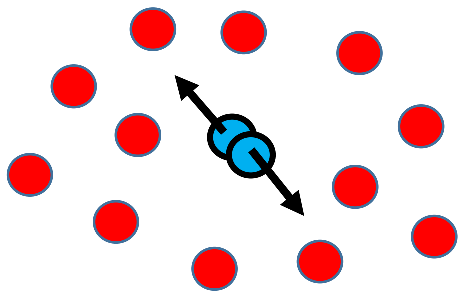
- The “contact”
- Number of correlated pairs
- Depends on the nucleus
- Independent of the operator

RW, B. Bazak, N. Barnea,
PRC 92, 054311 (2015)

Generalized Contact Formalism

$$\Psi(r_1, r_2, \dots, r_A) \xrightarrow{r_{12}\to 0} \varphi(\boldsymbol{r}) \times A\left(\boldsymbol{R}, \{\boldsymbol{r}_k\}_{k\neq 1,2}\right)$$

universal function



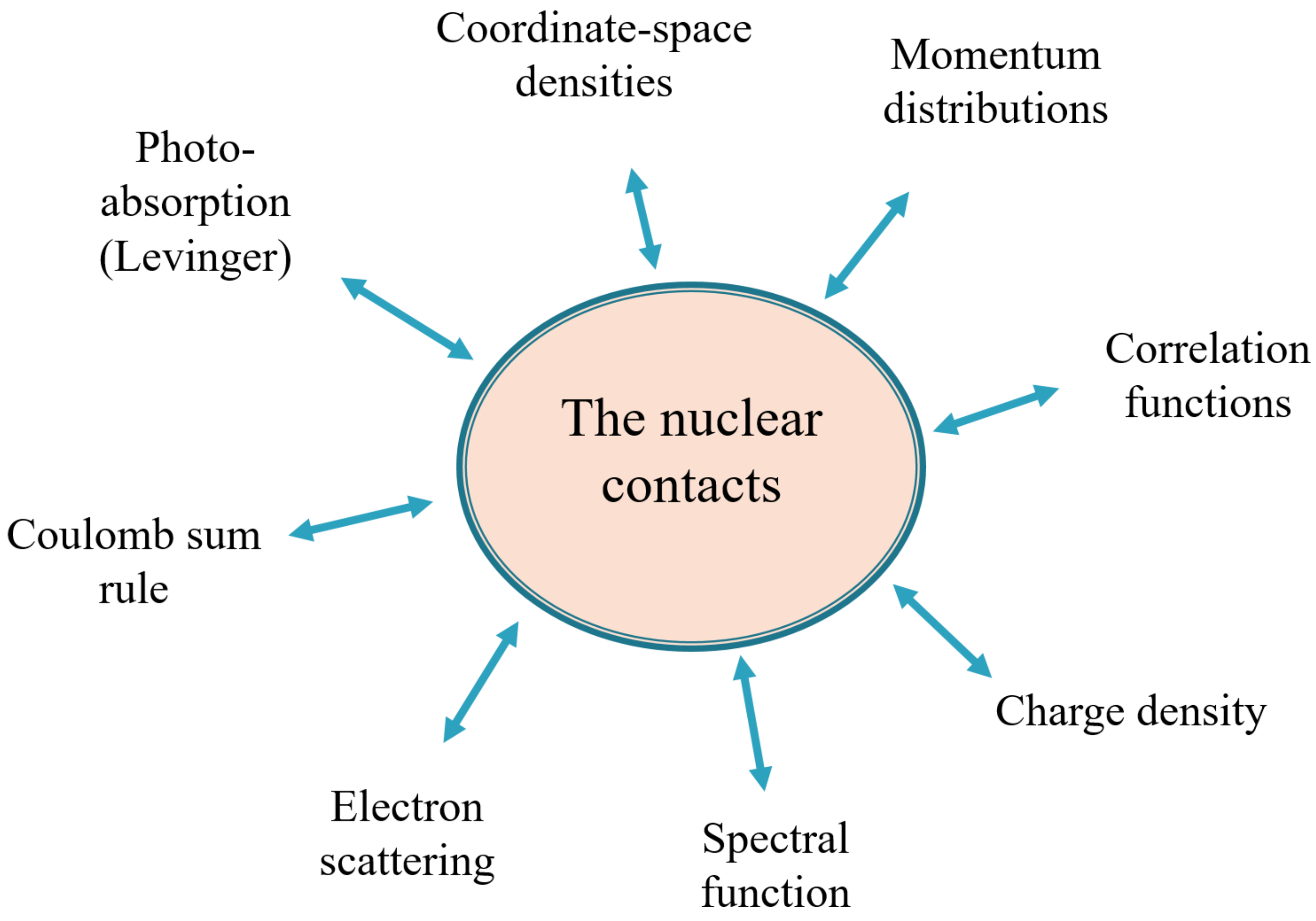
For any **short-range** two-body operator $\hat{O}$

$$\langle \hat{O} \rangle = \langle \varphi | \hat{O}(r) | \varphi \rangle \, C \qquad C \propto \langle A | A \rangle$$

This factorized form can be derived using:

- RG arguments — S. K. Bogner and D. Roscher, Phys. Rev. C 86, 064304 (2012). A. J. Tropiano, S. K. Bogner, and R. J. Furnstahl, Phys. Rev. C 104, 034311 (2021)
- Coupled Cluster expansion — S. Beck, RW, N. Barnea, Phys. Rev. C 107, 064306 (2023) S. Beck, RW, N. Barnea, arXiv:2305.17649 [nucl-th] (2023)

The nuclear contact relations



RW, B. Bazak, N. Barnea, PRC 92, 054311 (2015)

RW, B. Bazak, N. Barnea, PRL 114, 012501 (2015)

RW, R. Cruz-Torres, N. Barnea, E. Piasetzky and O. Hen, PLB 780, 211 (2018)

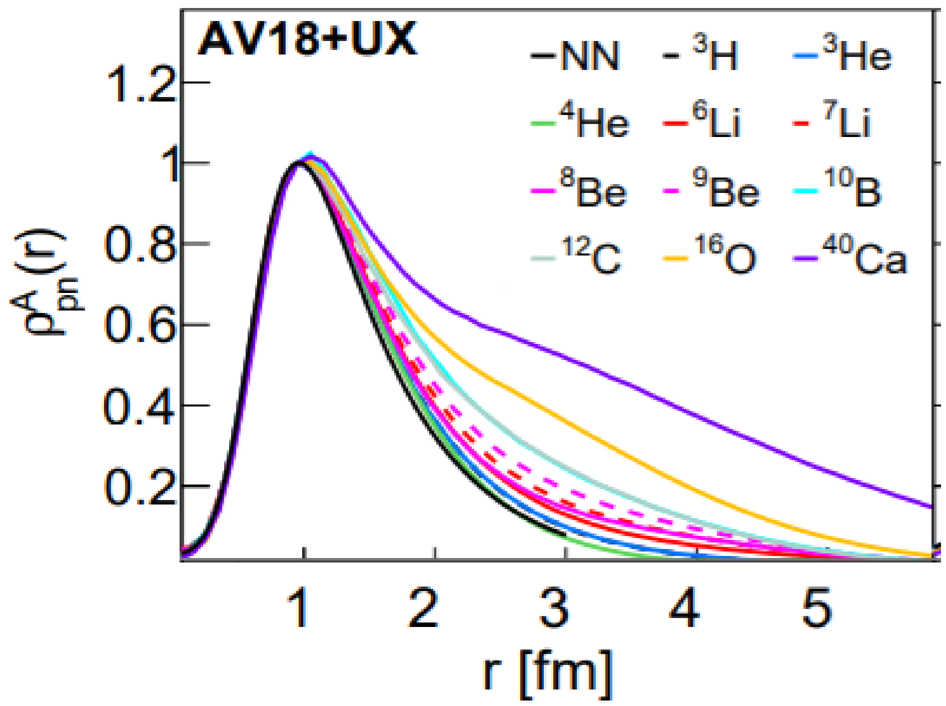
RW, I. Korover, E. Piasetzky, O. Hen and N. Barnea, PLB 791, 242 (2019)

R. Cruz-Torres, D. Lonardoni, RW, et al., Nature Physics (2020)

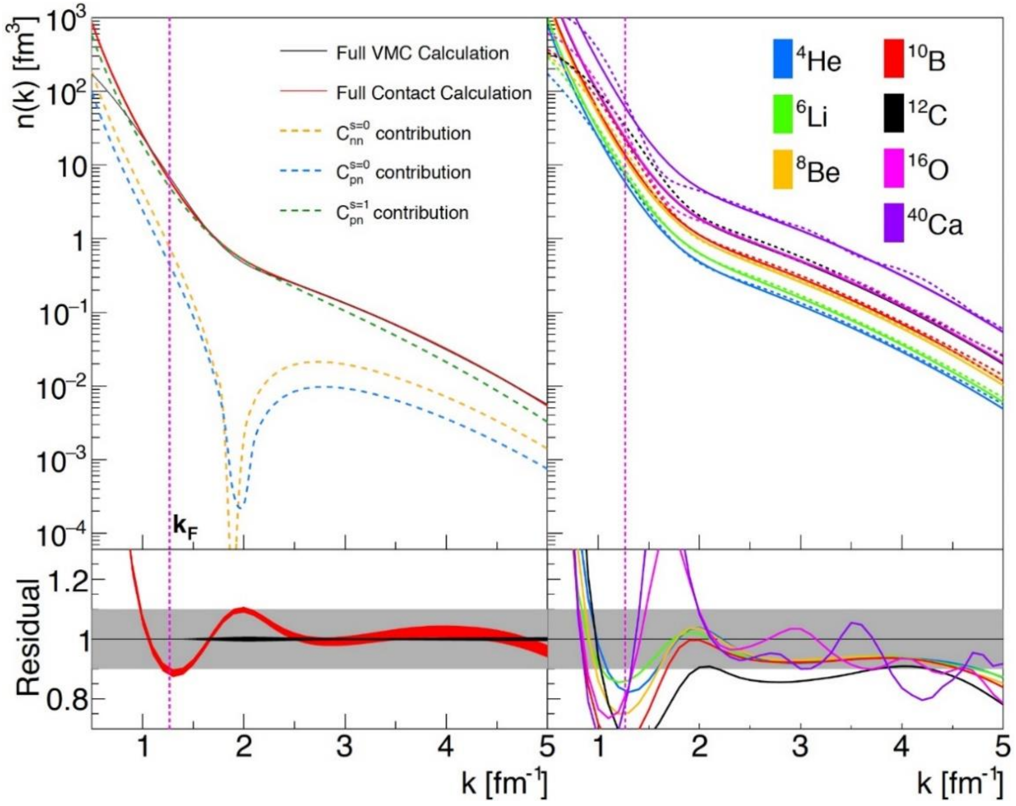
A. Schmidt, J.R. Pybus, RW, et al., Nature 578, 540 (2020)

The nuclear contact relations

Two-body density



Momentum distribution



$$\langle\hat{O}\rangle = C\langle\varphi|\hat{O}(r)|\varphi\rangle$$

R. Cruz-Torres, D. Lonardoni, RW, et al., Nature Physics (2020)

RW, R. Cruz-Torres, N. Barnea, E. Piasetzky and O. Hen, PLB 780, 211 (2018)

The nuclear contact relations

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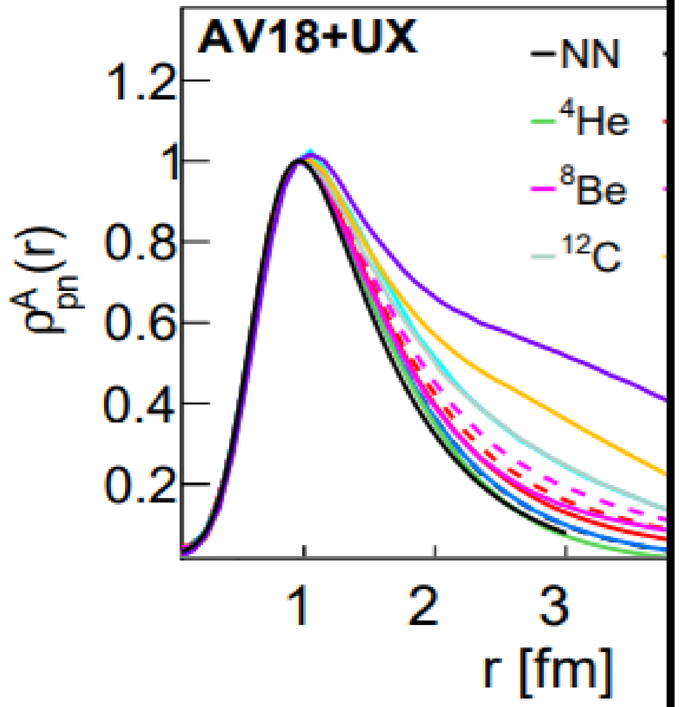
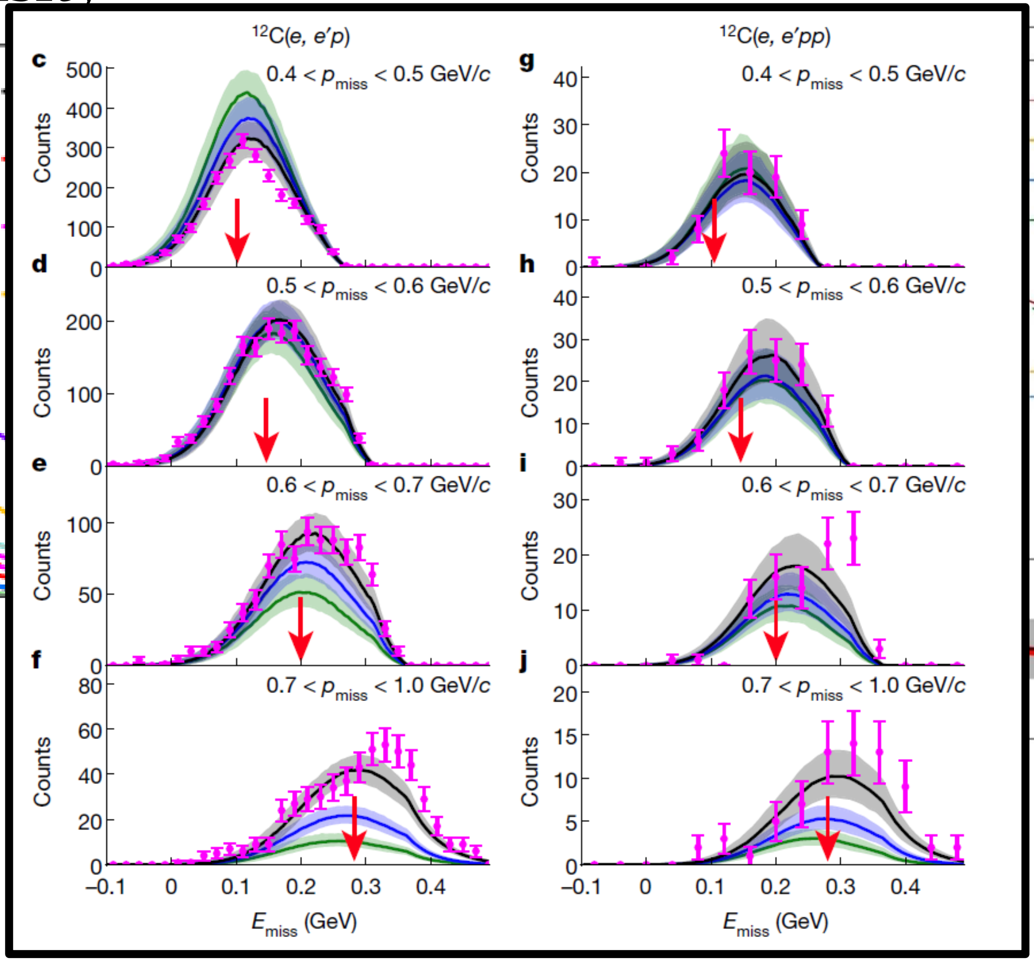
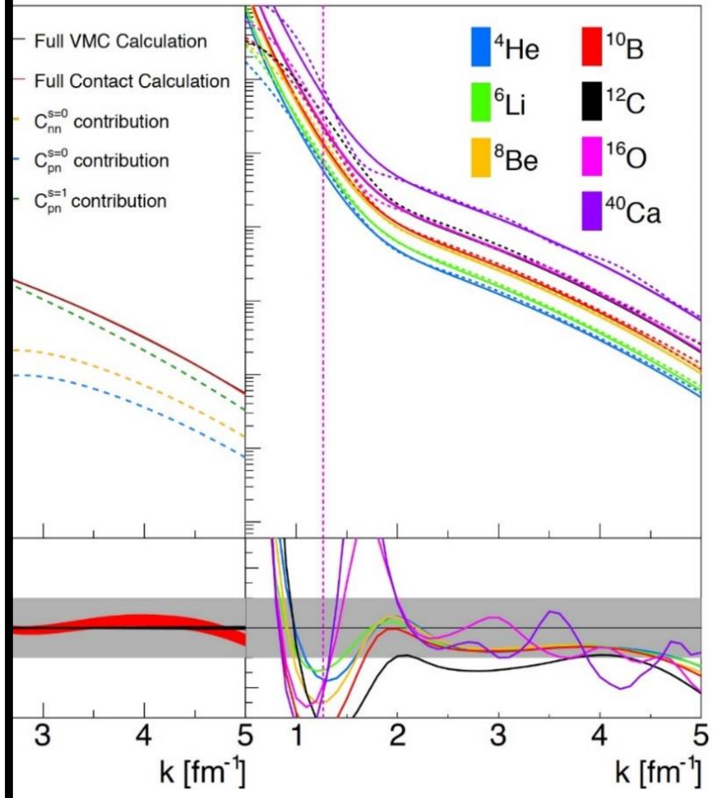


Electron scattering



Momentum distribution



A. Schmidt, J.R. Pybus, RW, E. P. Segarra, A. Hrnjic, A. Denniston, O. Hen, *et al. (CLAS collaboration), Nature 578, 540 (2020)*

Towards a systematic short-range expansion:

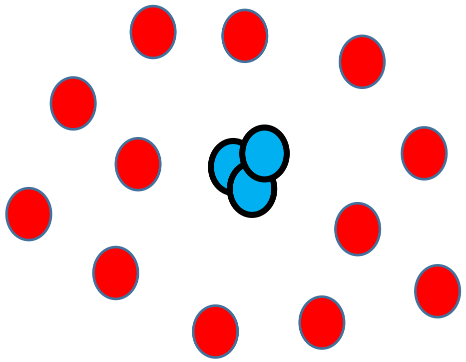
Corrections to the GCF

Corrections to the GCF

- GCF is based on the short-range factorization

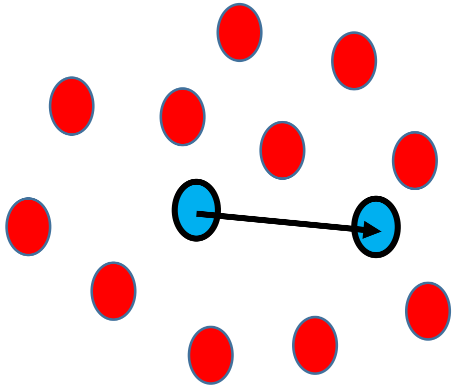
$$\Psi(r_1, r_2, \dots, r_A) \xrightarrow{r_{12}\to 0} \varphi(\boldsymbol{r}) \times A\left(\boldsymbol{R}, \{\boldsymbol{r}_k\}_{k\neq 1,2}\right)$$

- Possible corrections:



Three-nucleon correlations

Pairs at larger relative distance

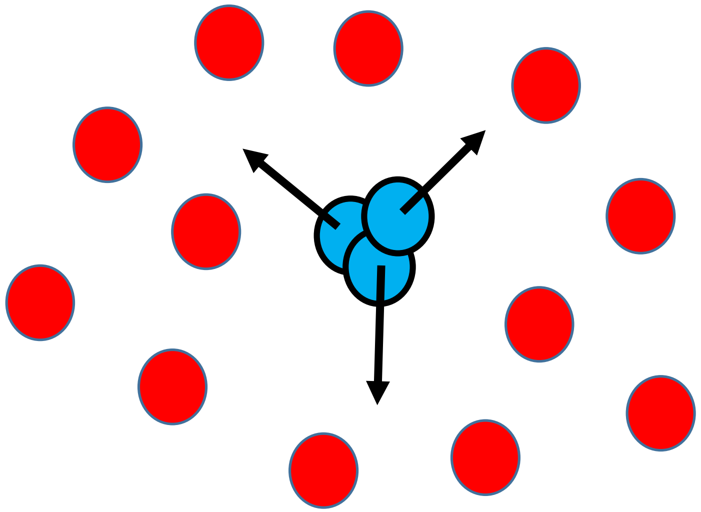


Three-body correlations

RW and S. Gandolfi, arXiv:2301.09605 [nucl-th] (2023) (accepted as a letter in PRC)

Three-body correlations

- There is no clear experimental signal of 3N SRCs
- No ab-initio calculations sensitive to 3N SRC features



- Factorization?

$$\Psi(r_1, r_2, \dots, r_A) \xrightarrow{r_{12}, r_{13}, r_{23} \to 0} \varphi(\boldsymbol{r}_{12}, \boldsymbol{r}_{13}) \times B\left(\boldsymbol{R}_{123}, \{\boldsymbol{r}_k\}_{k \neq 1,2,3}\right) \quad ?$$

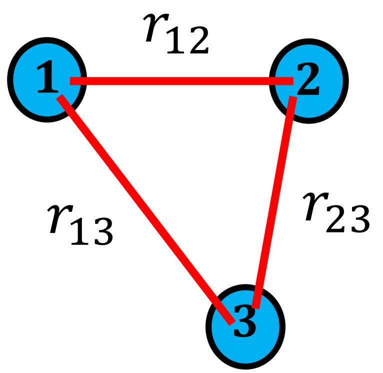
universal function

Three-body density

Ab-initio calculations – AFDMC (with Stefano Gandolfi):

$$\rho_3(r_{12}, r_{13}, r_{23}) \equiv \binom{A}{3} \langle\Psi|\delta(|\boldsymbol{r}_1 - \boldsymbol{r}_2| - r_{12})\delta(|\boldsymbol{r}_1 - \boldsymbol{r}_3| - r_{13})\delta(|\boldsymbol{r}_2 - \boldsymbol{r}_3| - r_{23})|\Psi\rangle$$

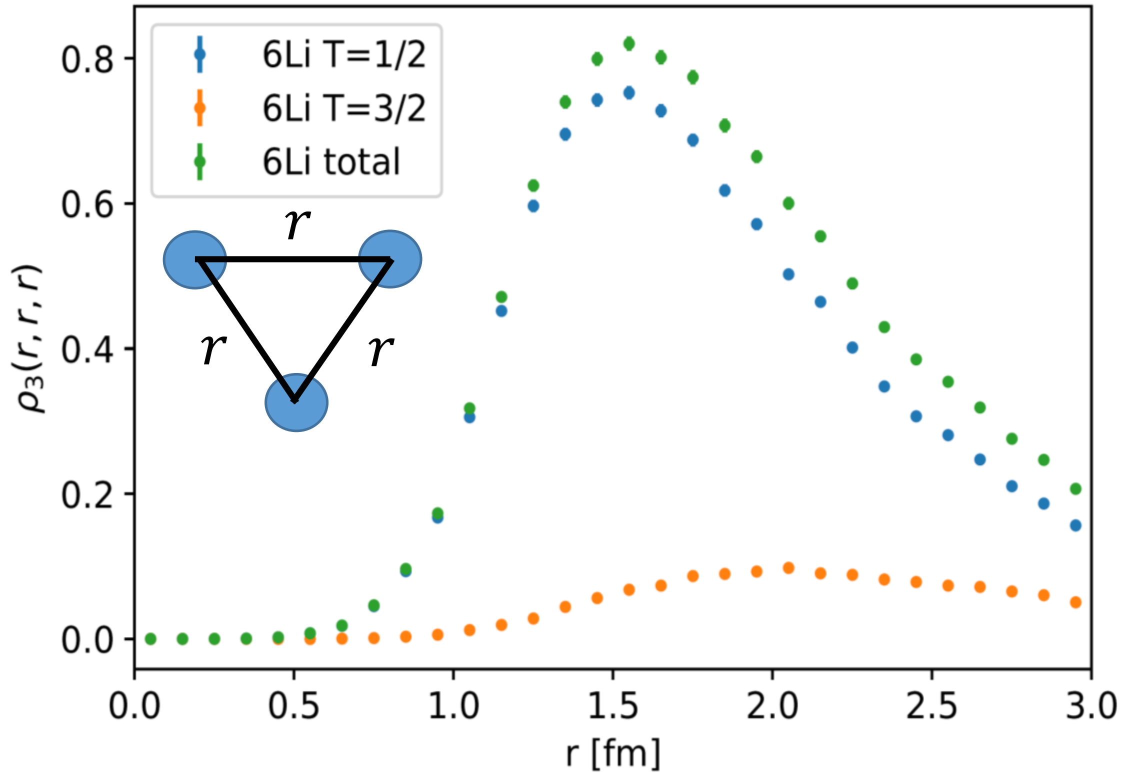
- Projections to $T = \frac{1}{2}$ and $T = \frac{3}{2}$
- N2LO($R = 1.0$ fm)E1 local chiral interaction
- Nuclei: ^{3}He, ^{4}He, ^{6}Li, , ^{16}O



Three-body density

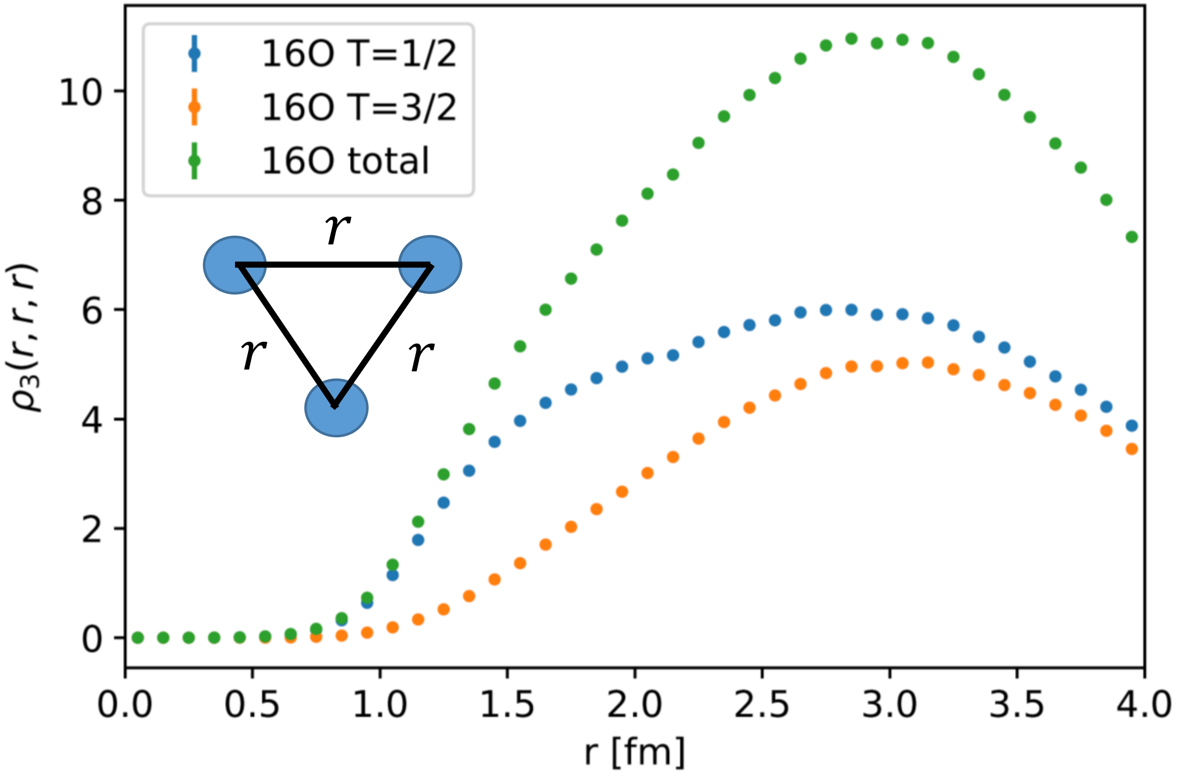
$T = 1/2 \quad \text{vs} \quad T = 3/2$

^{6}Li



Total number of triplets: $T = \frac{1}{2}: 16 \quad ; \quad T = \frac{3}{2}: 4$

^{16}O

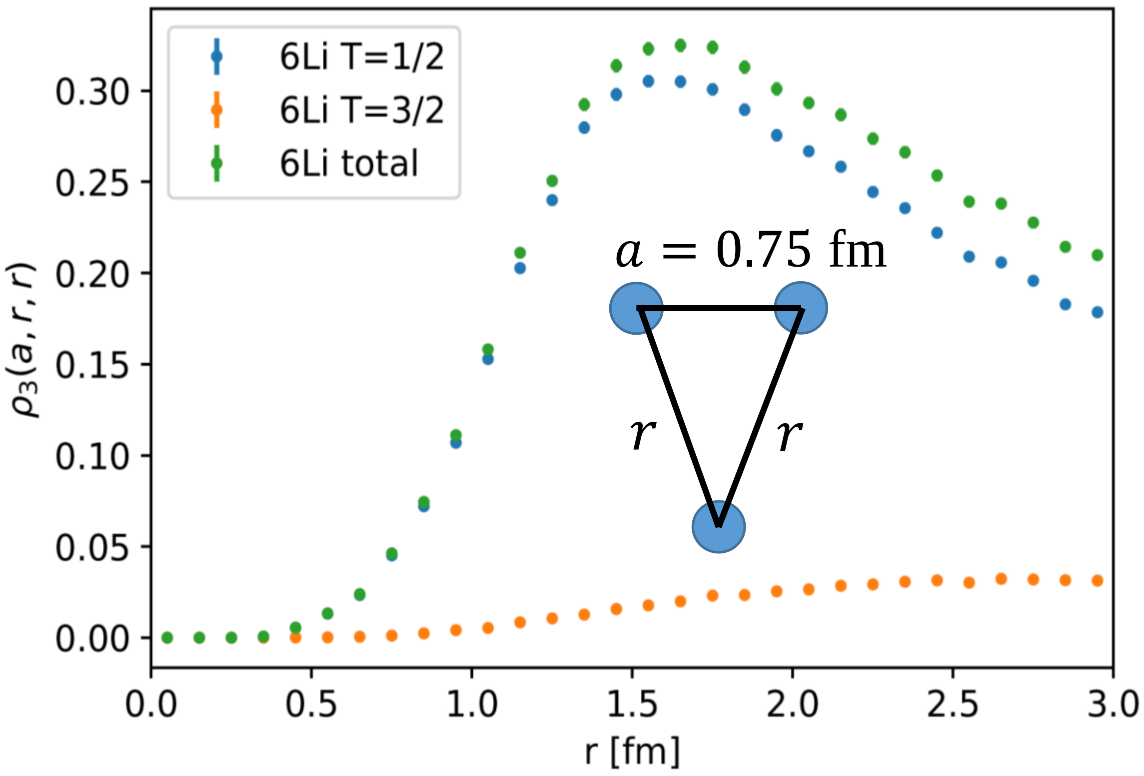


Total number of triplets: $T = \frac{1}{2}: 336 \quad ; \quad T = \frac{3}{2}: 224$

Three-body density

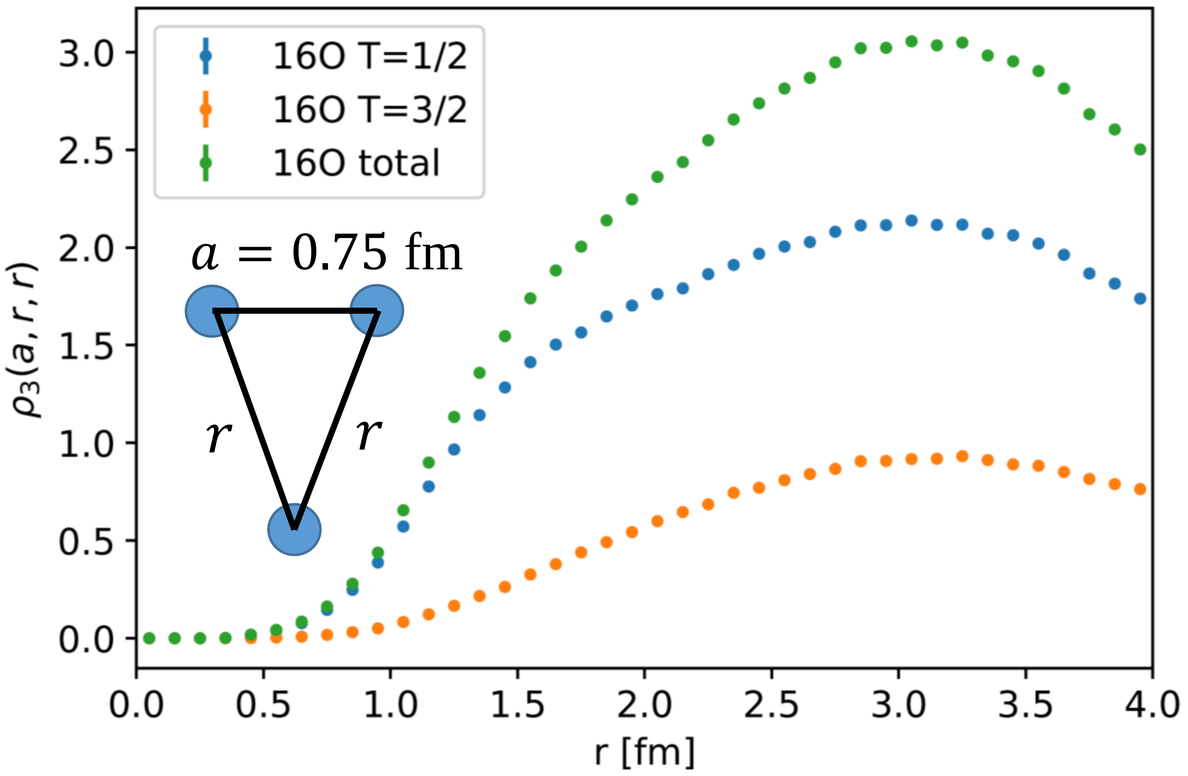
$T = 1/2 \quad \text{vs} \quad T = 3/2$

^{6}Li



Total number of triplets: $T = \frac{1}{2}: 16 \quad ; \quad T = \frac{3}{2}: 4$

^{16}O



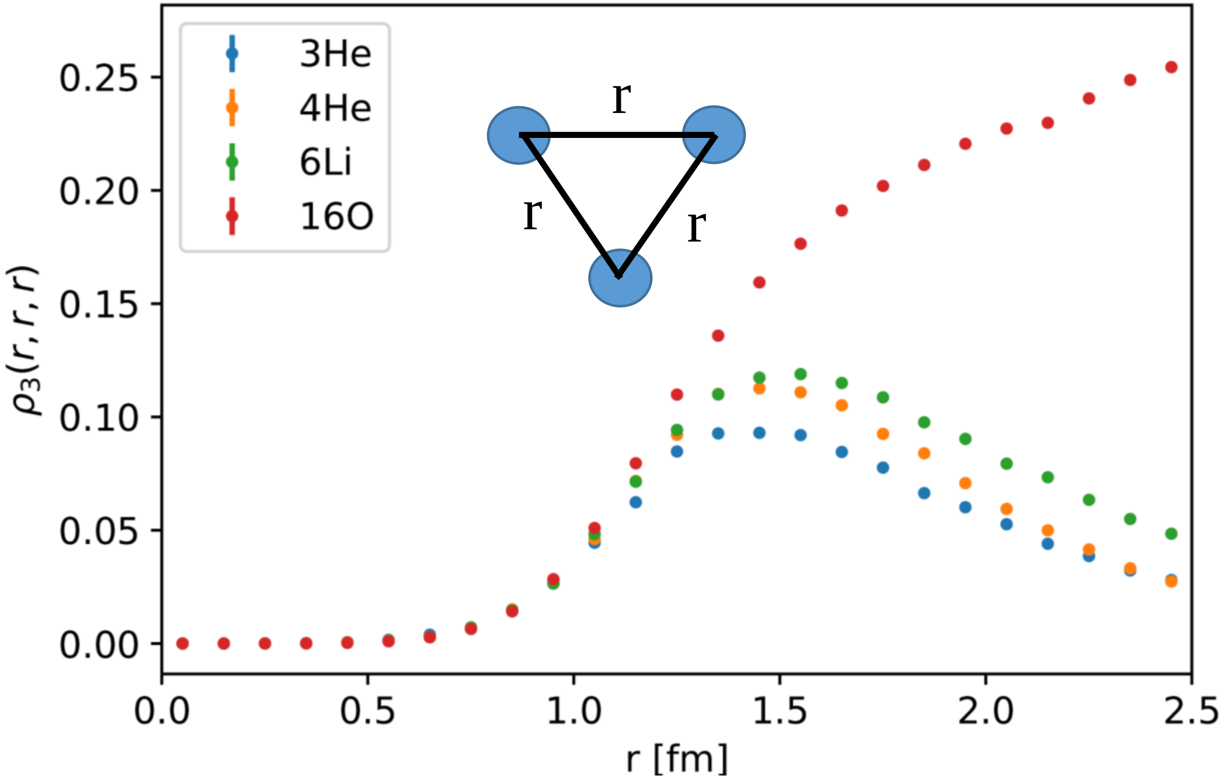
Total number of triplets: $T = \frac{1}{2}: 336 \quad ; \quad T = \frac{3}{2}: 224$

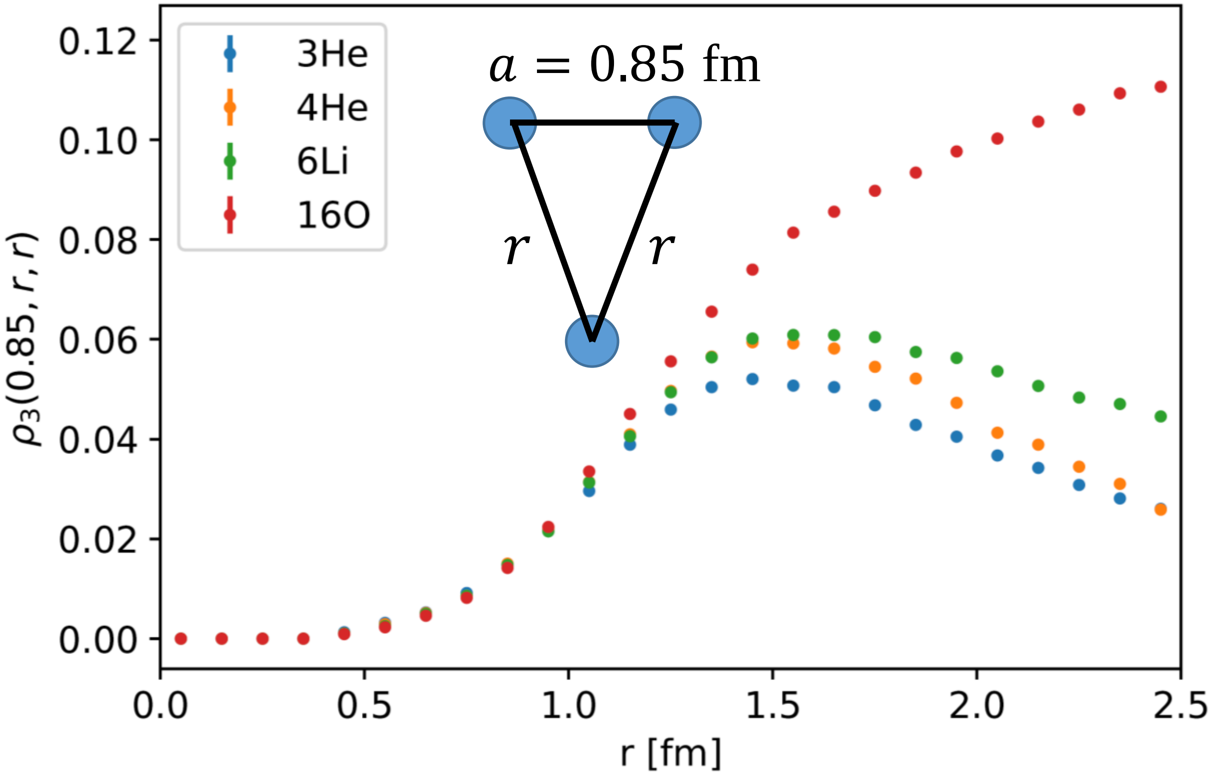
Three-body density

Same scaling factor for all geometries!

$T = \frac{1}{2}$ **universality:**

rescaled densities



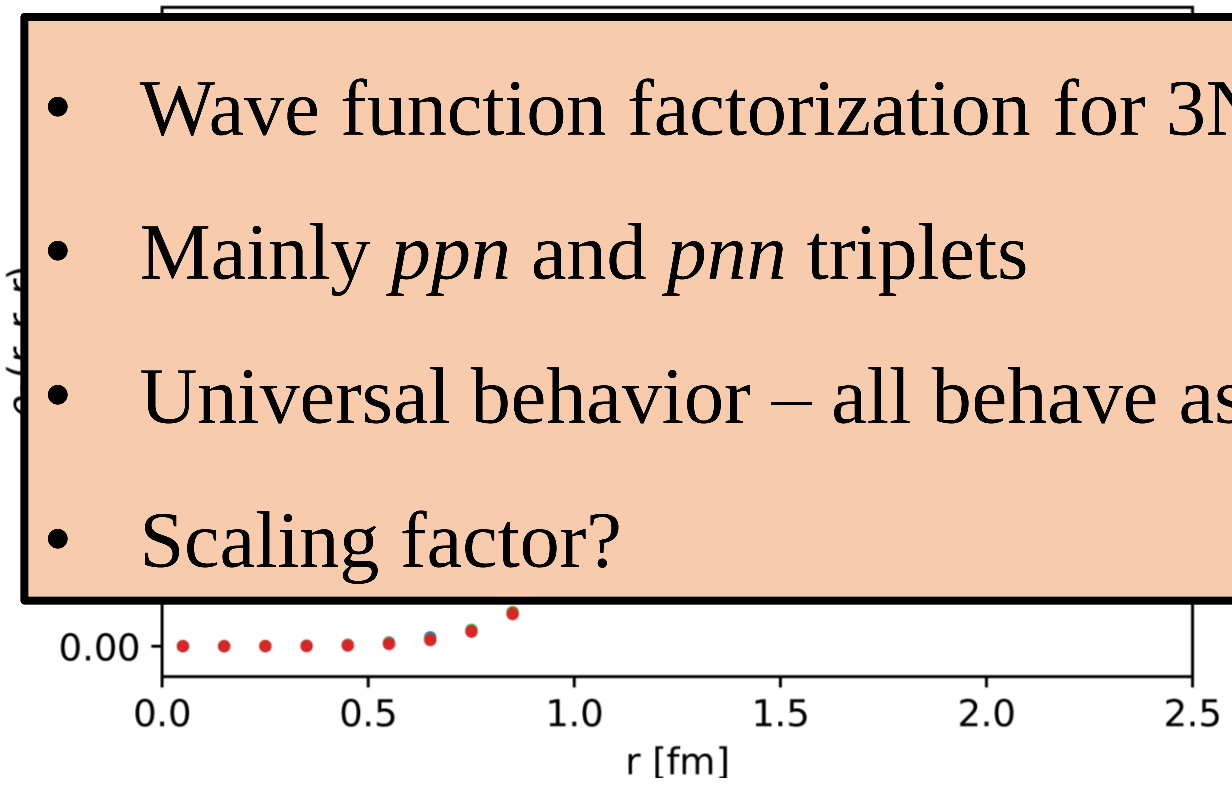


Three-body density

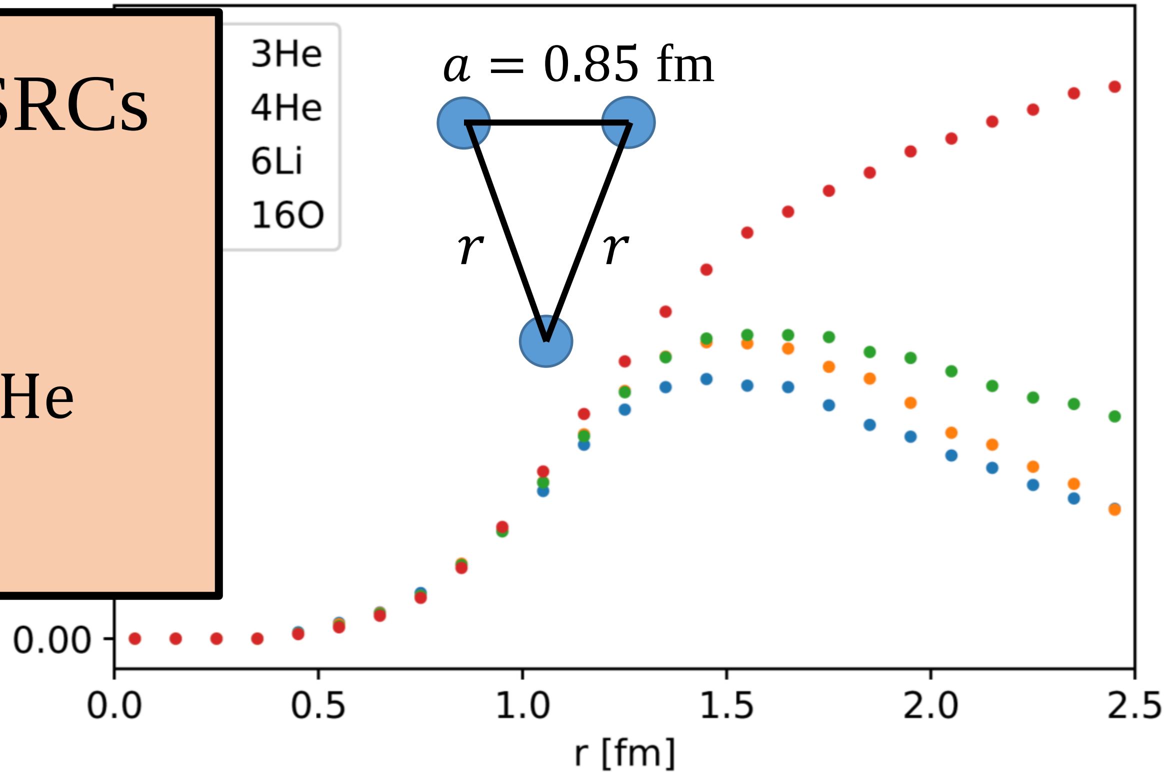
Same scaling factor for all geometries!

$$\boldsymbol{T = \frac{1}{2}}\ \textbf{universality:}$$

rescaled densities



- Wave function factorization for 3N SRCs
- Mainly *ppn* and *pnn* triplets
- Universal behavior – all behave as ^{3}He
- Scaling factor?



Three-body contact values ($T = 1/2$)

$$\frac{C({}^{4}\mathrm{He})}{C({}^{3}\mathrm{He})} \times \frac{3}{4} = 3.8 \pm 0.3$$

$$\frac{C({}^{6}\mathrm{Li})}{C({}^{3}\mathrm{He})} \times \frac{3}{6} = 3.1 \pm 0.3$$

$$\frac{C({}^{16}\mathrm{O})}{C({}^{3}\mathrm{He})} \times \frac{3}{16} = 4.2 \pm 0.5$$

Three-body contact values ($T = 1/2$)

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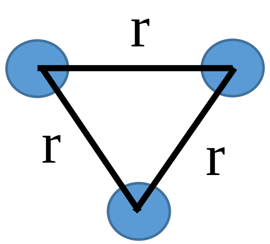
Can be compared to inclusive cross section ratios (in the appropriate kinematics)

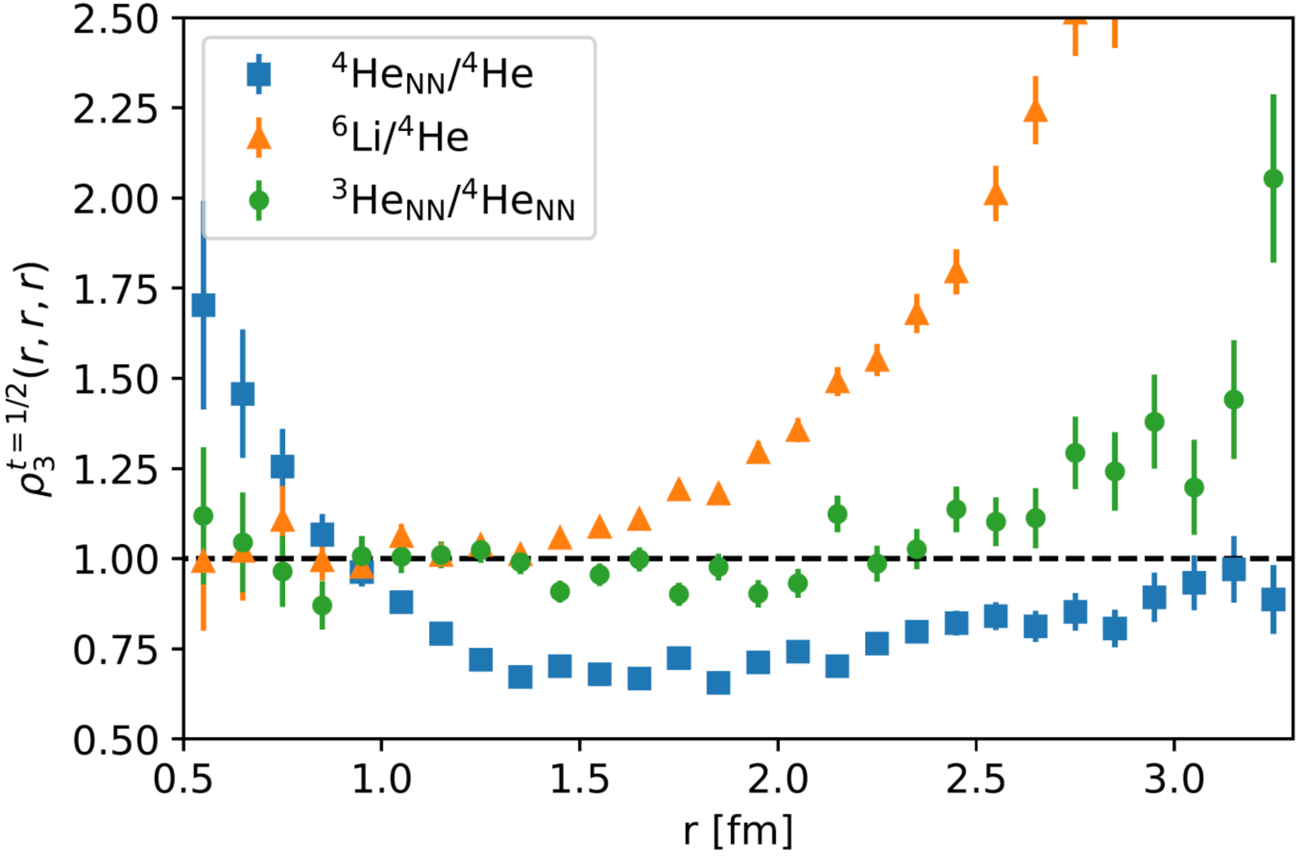
$$a_3(A) = \frac{3}{A} \frac{\sigma_{eA}}{(\sigma_{e\,^3\mathrm{He}} + \sigma_{e\,^3\mathrm{H}})/2}$$

For a symmetric nucleus A

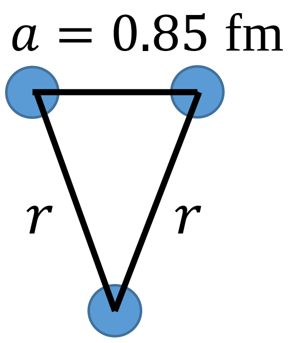
$$a_3(A) = \frac{3}{A} \frac{C(A)}{C(^3\mathrm{He})}$$

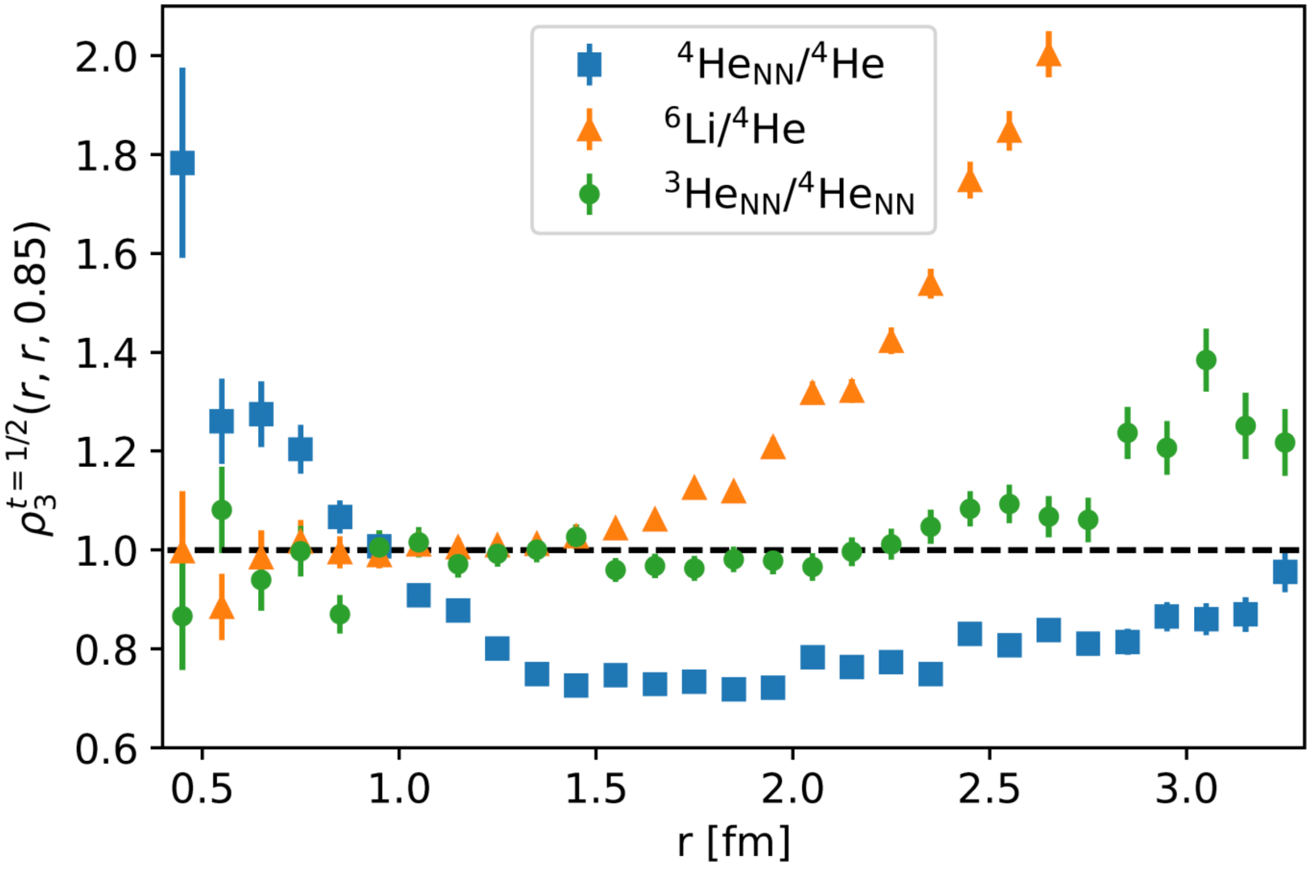
Sensitivity to three-body forces



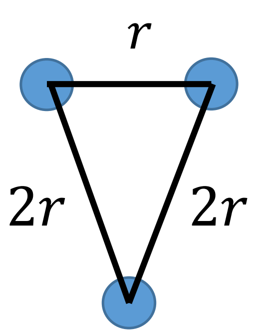


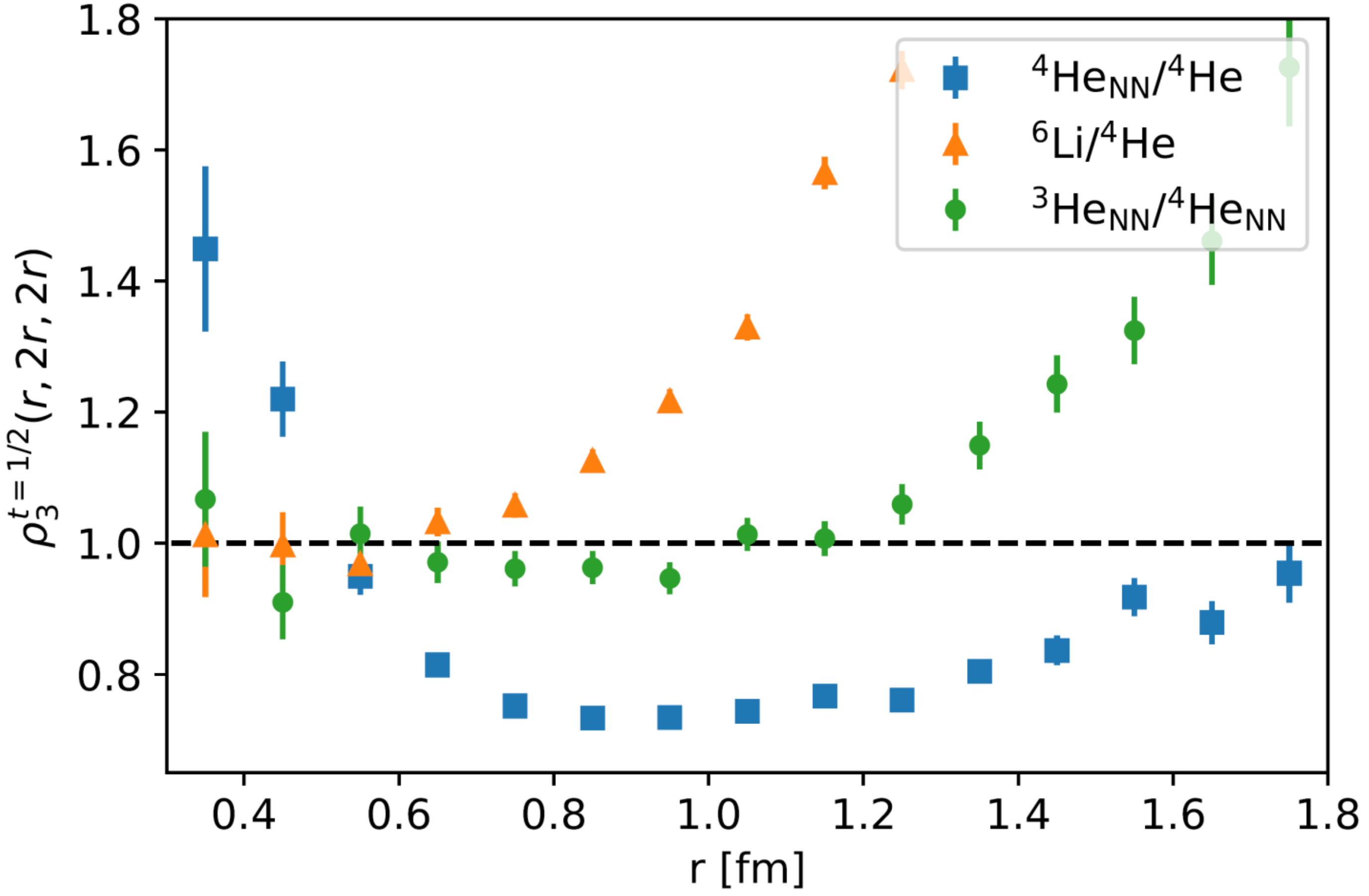
Sensitivity to three-body forces





Sensitivity to three-body forces





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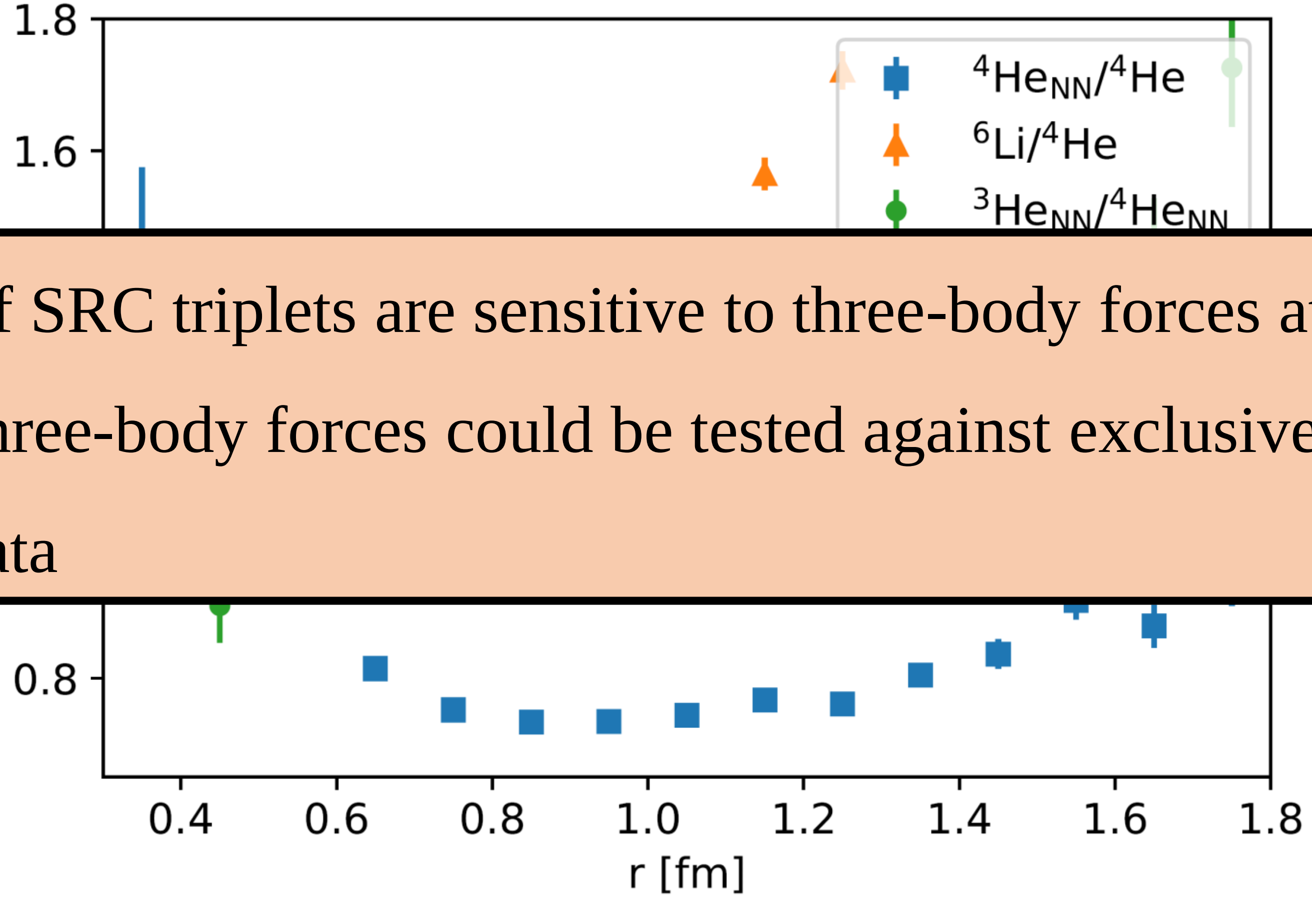
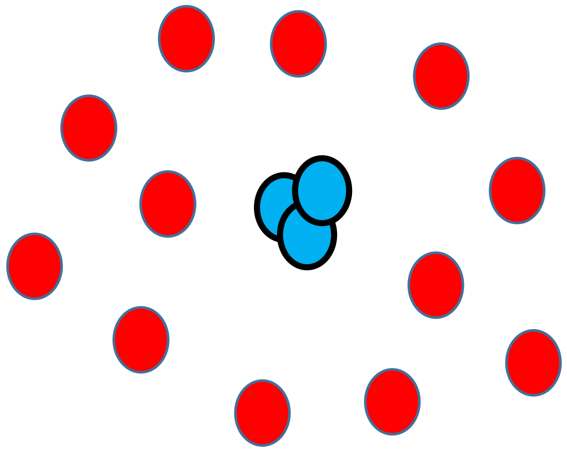


- Dynamics of SRC triplets are sensitive to three-body forces at short distances
- Models of three-body forces could be tested against exclusive electron scattering data

Three-body correlations

Future work:

- Dominant configurations
- Model dependence – Additional interactions
- Impact on momentum distributions
- Spectral function, electron scattering…

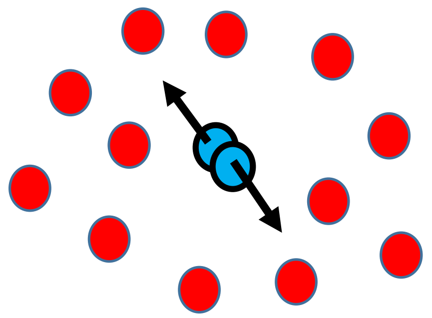


Subleading terms for SRC pairs:

Beyond factorization

RW, D. Lonardoni, S. Gandolfi, arXiv:2307.05910 [nucl-th] (2023)

Short-range expansion



- Factorization for short distances

$$\Psi(r_1, r_2, \dots, r_A) \xrightarrow{r_{12}\to 0} \varphi(r) \times A\left(\boldsymbol{R}, \{\boldsymbol{r}_k\}_{k\neq 1,2}\right)$$

- $\varphi(r) \equiv$ Zero-energy solution of the two-body Schrodinger Eq.

- The two-body system:

$$\left[-\frac{\hbar^2}{m}\nabla^2 + V(r)\right]\varphi^E(r) = E\varphi^E(r)$$

- For $r \to 0$: The energy becomes negligible

$$E \ll \frac{\hbar^2}{mr^2}$$

Short-range expansion

- For $r \to 0$:

$$\varphi^{E}(r) = \varphi^{E=0}(r)$$

- Taylor expansion around $E = 0$:

$$\varphi^{E}(\boldsymbol{r}) = \varphi^{E=0}(\boldsymbol{r}) + \left(\frac{d}{dE}\varphi^{E=0}(\boldsymbol{r})\right)E + \frac{1}{2!}\left(\frac{d^2}{dE^2}\varphi^{E=0}(\boldsymbol{r})\right)E^2 + \cdots$$

Short-range expansion

- For $r \to 0$:

$$\varphi^{E}(r) = \varphi^{E=0}(r)$$

- Taylor expansion around $E = 0$:

$$\varphi^{E}(\boldsymbol{r}) = \varphi^{E=0}(\boldsymbol{r}) + \left(\frac{d}{dE}\varphi^{E=0}(\boldsymbol{r})\right)E + \frac{1}{2!}\left(\frac{d^2}{dE^2}\varphi^{E=0}(\boldsymbol{r})\right)E^2 + \cdots$$

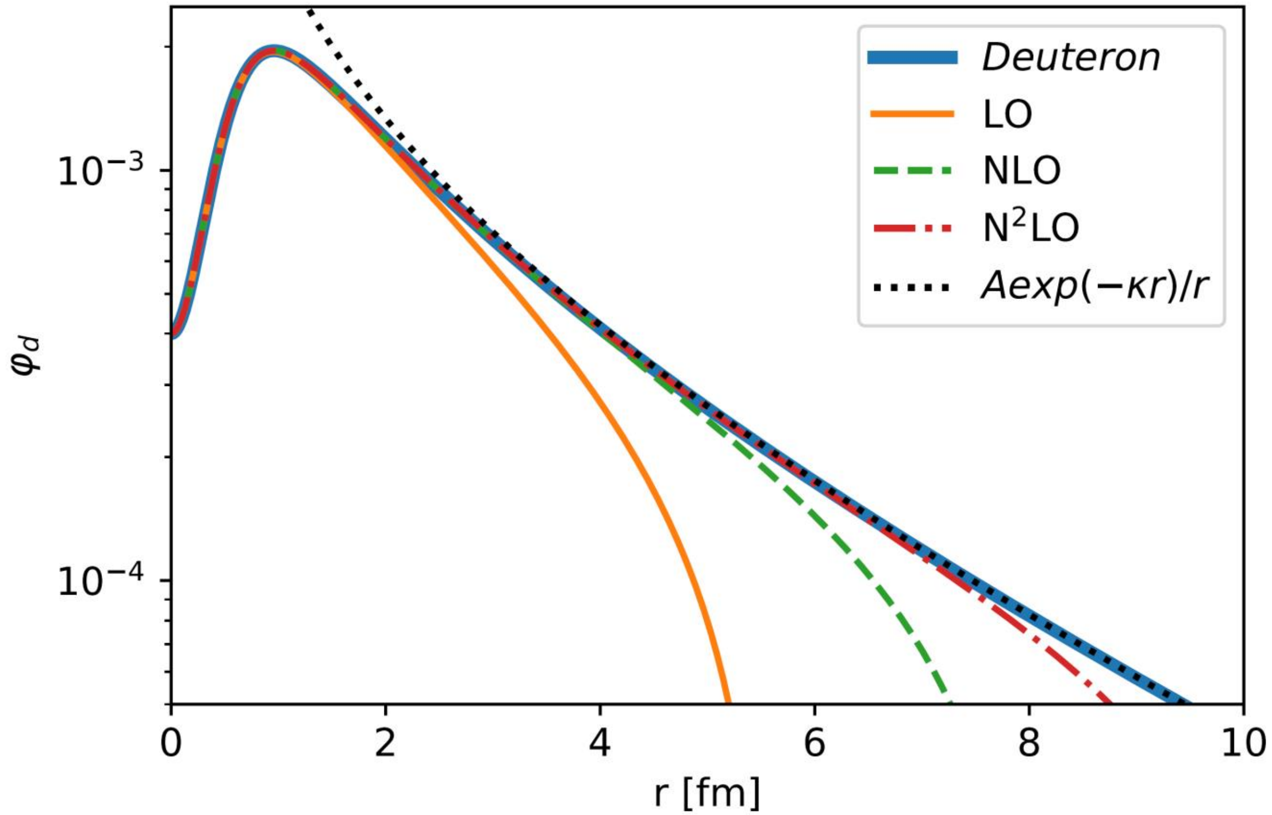
GCF
leading term

At short distances:
energy derivative is small

→ Short-range expansion

Short-range expansion

$$\varphi^{E}(\boldsymbol{r}) = \varphi^{E=0}(\boldsymbol{r}) + \left(\frac{d}{dE}\varphi^{E=0}(\boldsymbol{r})\right)E + \frac{1}{2!}\left(\frac{d^2}{dE^2}\varphi^{E=0}(\boldsymbol{r})\right)E^2 + \cdots$$



AV4'
Deuteron channel
Bound state

Short-range expansion

- The many-body case: Exact expansion

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_A) = \sum_{E,\alpha} \varphi_\alpha^E(\boldsymbol{r}_{12}) A_\alpha^E(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_A)$$

(α – quantum numbers)

Complete set of two-body functions

Short-range expansion

- The many-body case: Exact expansion

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_A) = \sum_{E,\alpha} \varphi_\alpha^E(\boldsymbol{r}_{12}) A_\alpha^E(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_A)$$

(α – quantum numbers)

- Taylor expansion around $E = 0$:

$$\varphi_\alpha^E(\boldsymbol{r}) = \varphi_\alpha^{E=0}(\boldsymbol{r}) + \left(\frac{d}{dE}\varphi_\alpha^{E=0}(\boldsymbol{r})\right)E + \frac{1}{2!}\left(\frac{d^2}{dE^2}\varphi_\alpha^{E=0}(\boldsymbol{r})\right)E^2 + \cdots$$

Short-range expansion

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$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \dots, \boldsymbol{r}_A) = \sum_\alpha \varphi_\alpha^{E=0}(\boldsymbol{r}_{12}) A_\alpha^{(0)} + \sum_\alpha \left(\frac{d}{dE}\varphi_\alpha^{E=0}(\boldsymbol{r})\right) A_\alpha^{(1)} + \sum_\alpha \left(\frac{d^2}{dE^2}\varphi_\alpha^{E=0}(\boldsymbol{r})\right) A_\alpha^{(2)} + \cdots$$

Short-range expansion

- The many-body case: Exact expansion

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \dots, \boldsymbol{r}_A) = \sum_{E,\alpha} \varphi_\alpha^E(\boldsymbol{r}_{12}) A_\alpha^E(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \dots, \boldsymbol{r}_A) \qquad (\alpha - \text{quantum numbers})$$

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$$\varphi_\alpha^E(\boldsymbol{r}) = \varphi_\alpha^{E=0}(\boldsymbol{r}) + \left(\frac{d}{dE}\varphi_\alpha^{E=0}(\boldsymbol{r})\right) E + \frac{1}{2!}\left(\frac{d^2}{dE^2}\varphi_\alpha^{E=0}(\boldsymbol{r})\right) E^2 + \cdots$$

GCF factorization

Next-order terms

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \dots, \boldsymbol{r}_A) = \sum_\alpha \varphi_\alpha^{E=0}(\boldsymbol{r}_{12}) A_\alpha^{(0)} + \sum_\alpha \left(\frac{d}{dE}\varphi_\alpha^{E=0}(\boldsymbol{r})\right) A_\alpha^{(1)} + \sum_\alpha \left(\frac{d^2}{dE^2}\varphi_\alpha^{E=0}(\boldsymbol{r})\right) A_\alpha^{(2)} + \cdots$$

Short-range expansion

- The many-body case:

$$\Psi(\boldsymbol{r}_1,\boldsymbol{r}_2,\ldots,\boldsymbol{r}_A) = \sum_\alpha \varphi_\alpha^{E=0}(\boldsymbol{r}_{12})A_\alpha^{(0)} + \sum_\alpha \left(\frac{d}{dE}\varphi_\alpha^{E=0}(\boldsymbol{r})\right)A_\alpha^{(1)} + \sum_\alpha \left(\frac{d^2}{dE^2}\varphi_\alpha^{E=0}(\boldsymbol{r})\right)A_\alpha^{(2)} + \cdots$$

- Two-body density:

$$\rho_2(r) = \sum_\alpha \left(|\varphi_\alpha^{E=0}(r)|^2 C_\alpha^{00} + \varphi_\alpha^{E=0}(r)\frac{d}{dE}\varphi_\alpha^{E=0}(r) C_\alpha^{01} + \cdots \right)$$

- Subleading contacts:

$$C_\alpha^{mn} \propto \langle A_\alpha^{(m)} | A_\alpha^{(n)} \rangle$$

Short-range expansion

$$\rho_2(r) = \sum_\alpha \left(|\varphi_\alpha^{E=0}(r)|^2 C_\alpha^{00} + \varphi_\alpha^{E=0}(r) \frac{d}{dE} \varphi_\alpha^{E=0}(r) C_\alpha^{01} + \cdots \right)$$

- Power counting is needed
- Two relevant parameters:
  - Number of energy derivatives
  - Orbital angular momentum ($s, p, d, \ldots$)
- Can be analyzed analytically for the two-body system

Neutron matter: two-body density

$$\rho_2(r) = \sum_\alpha \left(|\varphi_\alpha^{E=0}(r)|^2 C_\alpha^{00} + \varphi_\alpha^{E=0}(r) \frac{d}{dE} \varphi_\alpha^{E=0}(r) C_\alpha^{01} + \cdots \right)$$

- Neutron matter:

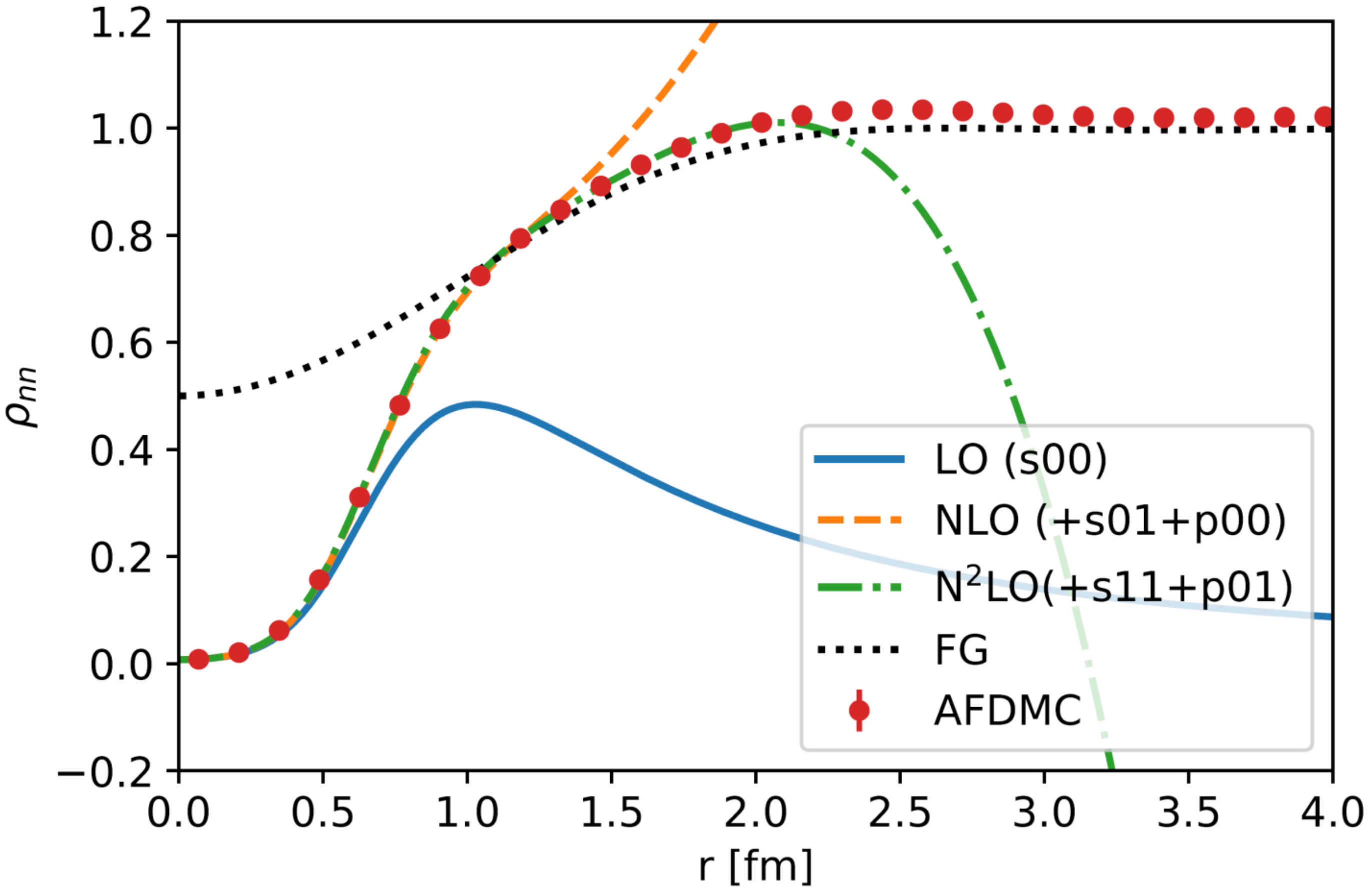
AFDMC by Diego Lonardoni & Stefano Gandolfi: AV4'+UIX_C $n = 0.16\ fm^{-3}$

5 fitting parameters at N^2LO

$(S + \ell = \text{Even})$

s-wave: $\ell = 0, S = 0, j = 0$

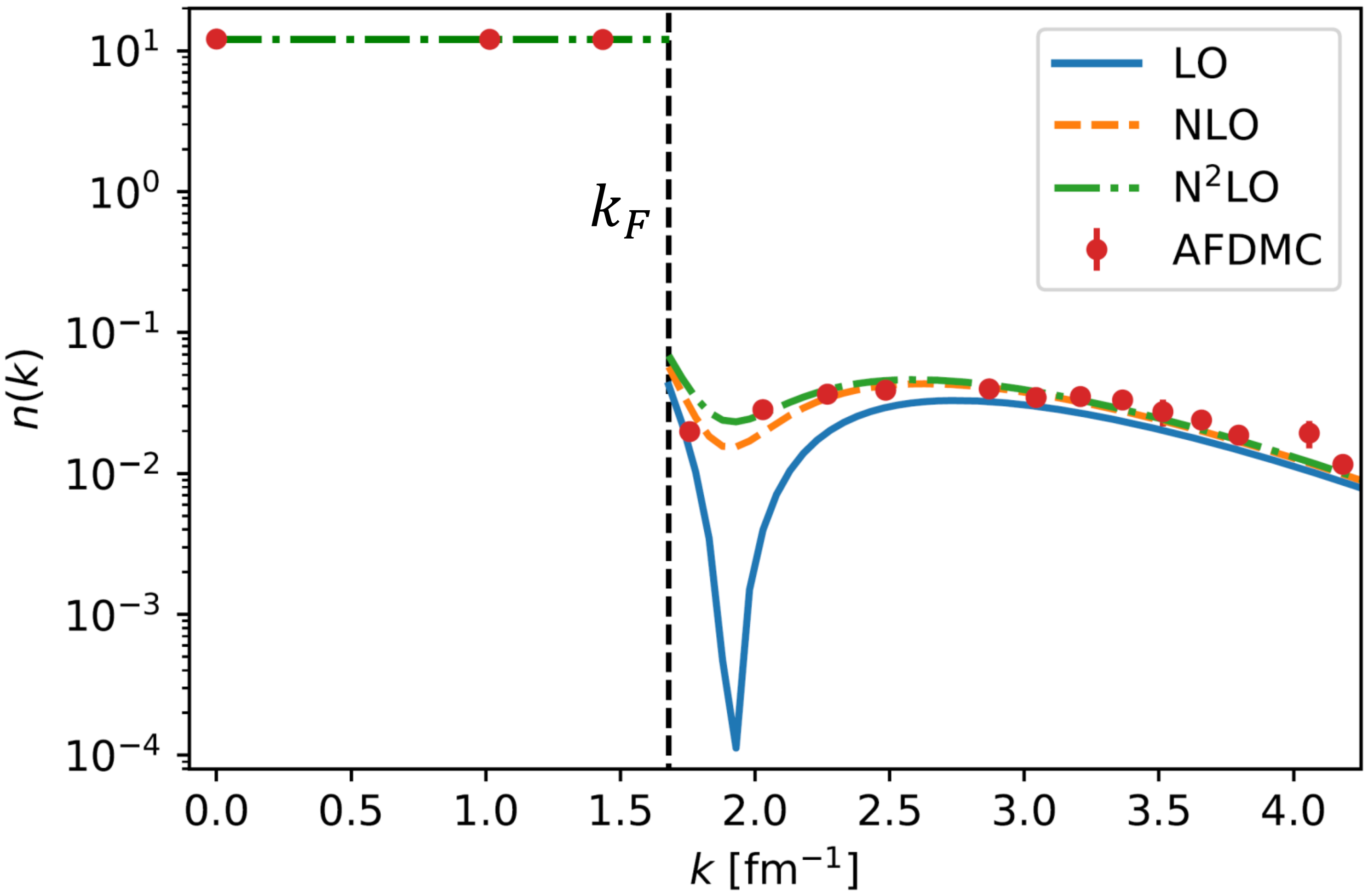
p-wave: $\ell = 1, S = 1, j = 0/1/2$



Neutron matter: one-body momentum distribution

Momentum distribution

No fitting parameters!



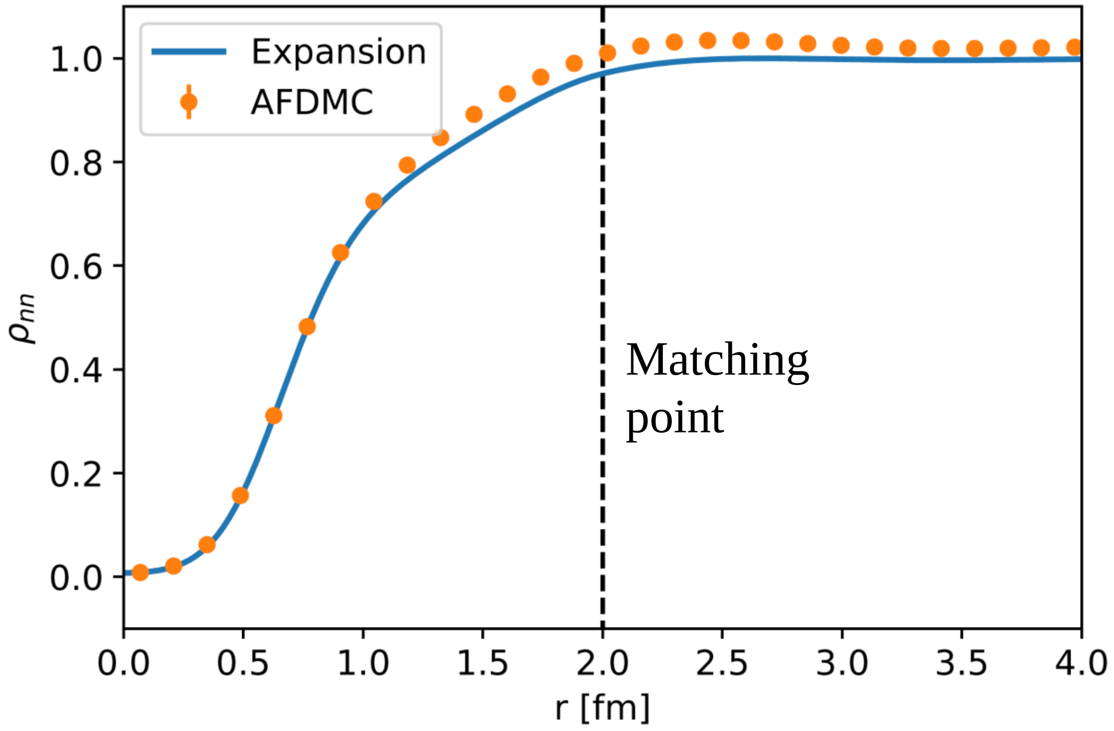
$(S + \ell = \text{Even})$

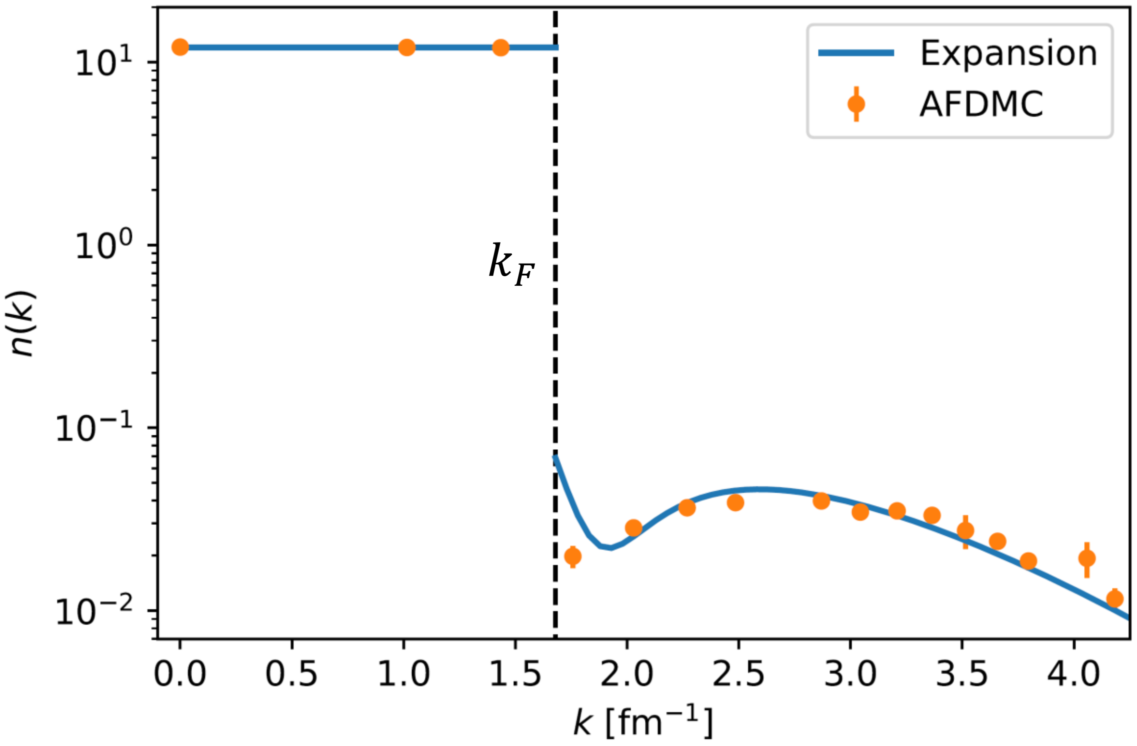
s-wave: $\ell = 0, S = 0, j = 0$

p-wave: $\ell = 1, S = 1, j = 0/1/2$

Matching to long-range model

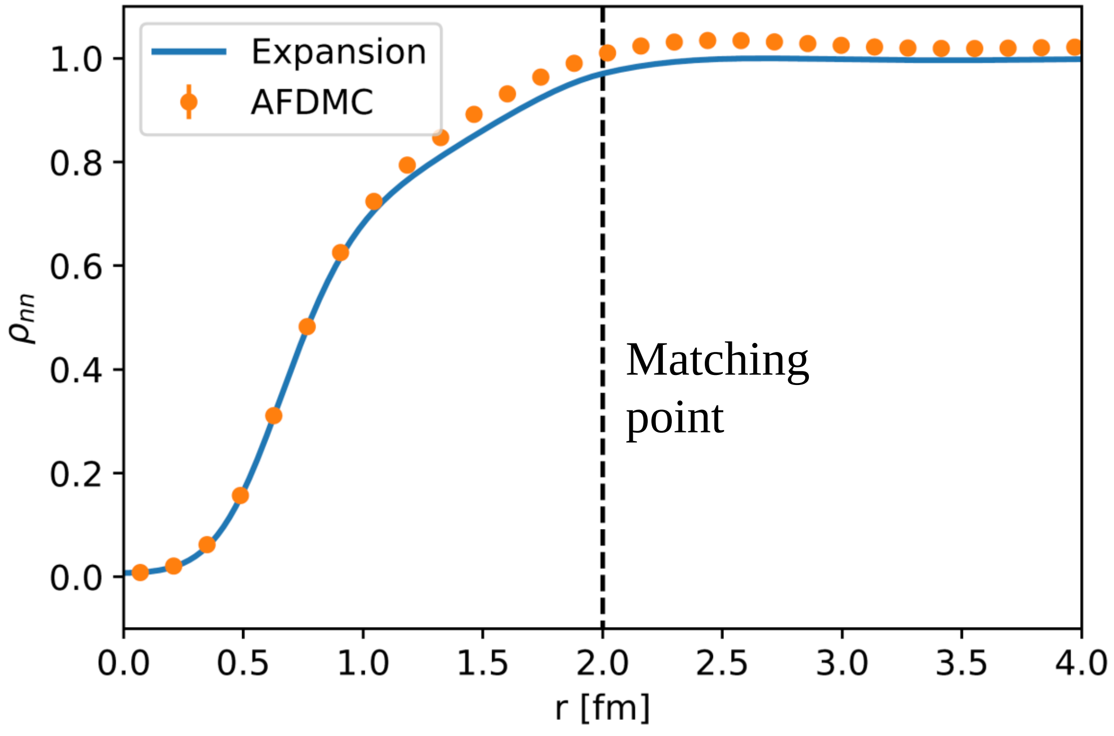
Fitting only the LO contact and matching to FG:

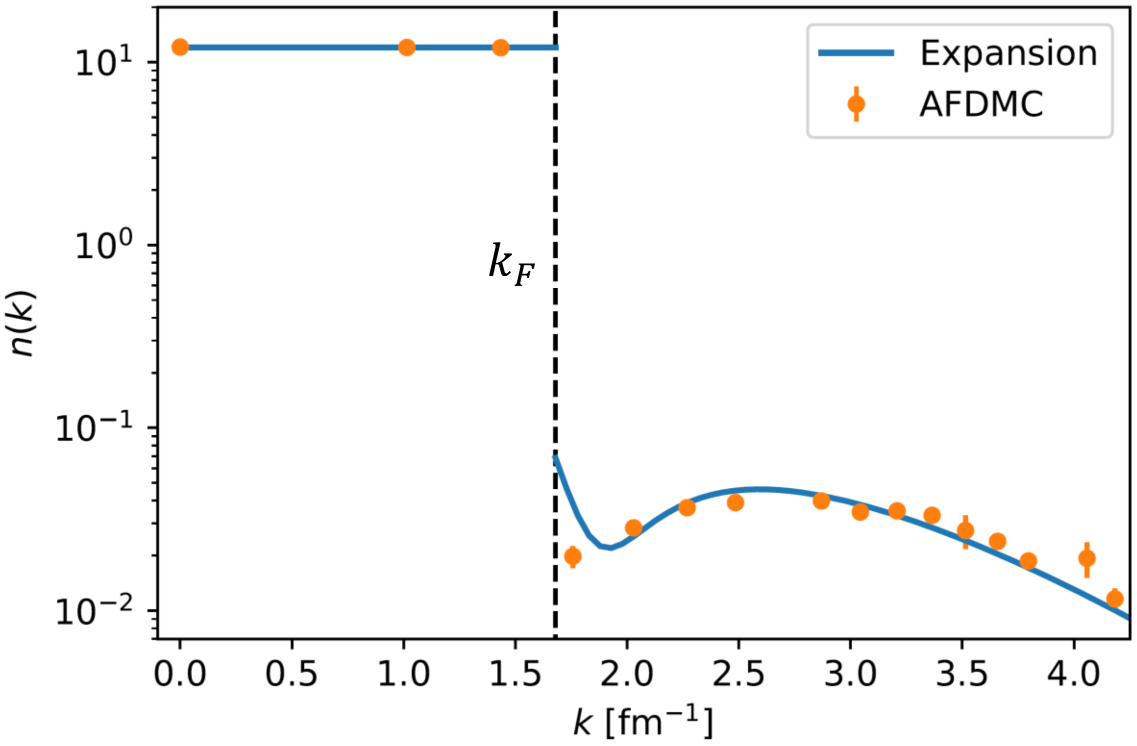




Matching to long-range model

Fitting only the LO contact and matching to FG:



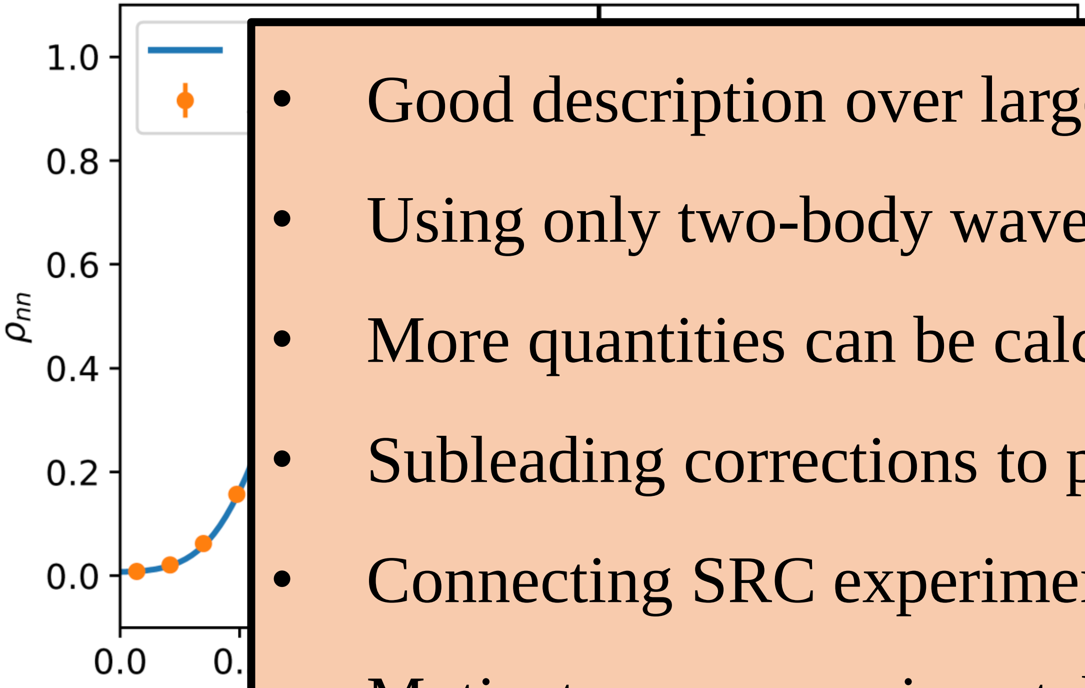


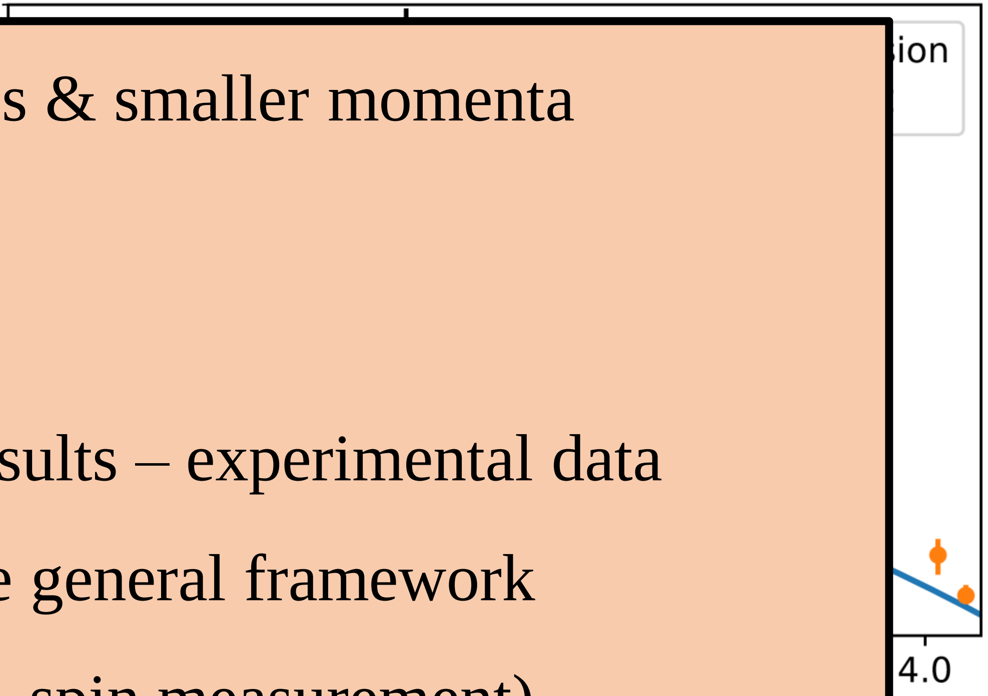
Obtained NN Potential Energy $\langle V_2 \rangle = -29.3$ MeV
Exact NN Potential Energy $\langle V_2 \rangle = -30.1$ MeV

Obtained Kinetic Energy $\langle T \rangle = 43.2$ MeV
Exact Kinetic Energy $\langle T \rangle = 43.3$ MeV

Matching to long-range model

Fitting only the LO contact and matching to FG:





- Good description over larger distances & smaller momenta
- Using only two-body wave-functions
- More quantities can be calculated!
- Subleading corrections to previous results – experimental data
- Connecting SRC experiments to more general framework
- Motivate new experimental data (e.g., spin measurement)

Obtained NN Potential Energy $\langle V_2 \rangle = -29.3$ MeV
Exact NN Potential Energy $\langle V_2 \rangle = -30.1$ MeV

Obtained Kinetic Energy $\langle T \rangle = 43.2$ MeV
Exact Kinetic Energy $\langle T \rangle = 43.3$ MeV

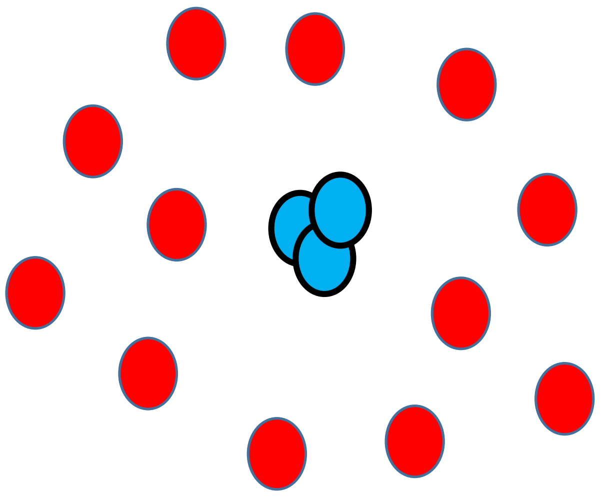
Summary

Summary

- Leading-order GCF provides **consistent and comprehensive description of short-range correlated pairs**

$$\Psi(r_1, r_2, \ldots, r_A) \xrightarrow{r_{12}\to 0} \varphi(\boldsymbol{r}) \times A\left(\boldsymbol{R}, \{\boldsymbol{r}_k\}_{k\neq 1,2}\right)$$

- **3N SRCs** – clear signal of correlated triplets
  - Wave function factorization
  - Single leading channel - $j^{\pi} = \frac{1}{2}^{+}, t = \frac{1}{2}$
  - Universal behavior of SRC triplets
  - Extracted scaling factors – 3N contact ratios
  - Sensitivity to three-body force

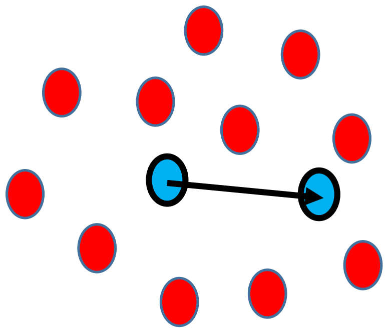


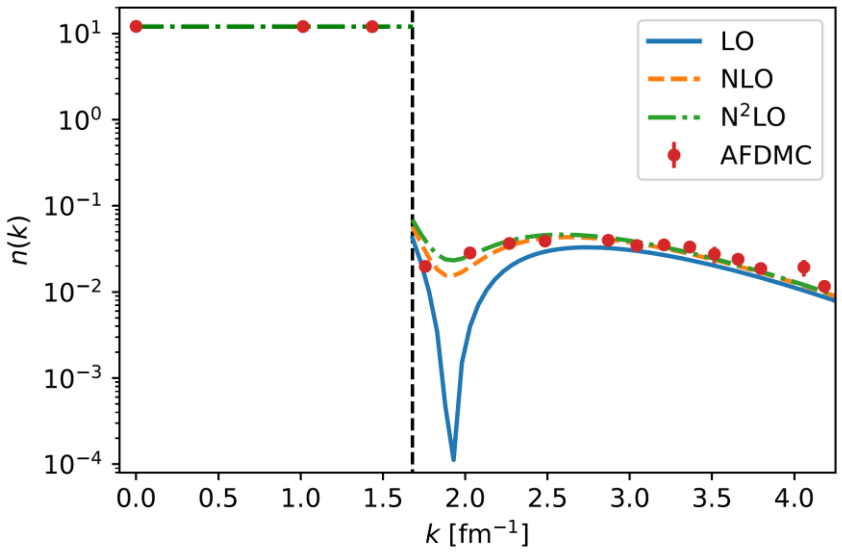
Summary

- **Short-range expansion -**

$$\rho_2(r) = \sum_\alpha \left(|\varphi_\alpha^{E=0}(r)|^2 C_\alpha^{00} + \varphi_\alpha^{E=0}(r) \frac{d}{dE} \varphi_\alpha^{E=0}(r) C_\alpha^{01} + \cdots \right)$$

  - Systematic expansion
  - Valid for larger distances / lower momenta
  - More observables can be described
  - Improved comparison with data
  - Motivates new experiments





BACKUP

Generalized Contact Formalism

$$\Psi \xrightarrow{r_{ij}\rightarrow 0} \sum_\alpha \varphi_{ij}^\alpha(\boldsymbol{r}_{ij}) A_{ij}^\alpha(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}) \quad ; \quad \boldsymbol{C}_{\boldsymbol{ij}}^{\boldsymbol{\alpha\beta}} \propto \langle A_{ij}^\alpha | A_{ij}^\beta \rangle$$

Channels $\alpha = (\ell_2 S_2) j_2 m_2$

Universal functions

The pair kind $ij \in \{pp, nn, pn\}$

3 matrices of Nuclear Contacts

Main channels:

The **deuteron** channel: $\ell_2 = 0{,}2\, ; s_2 = 1\, ; j_2 = 1\, ; t_2 = 0$

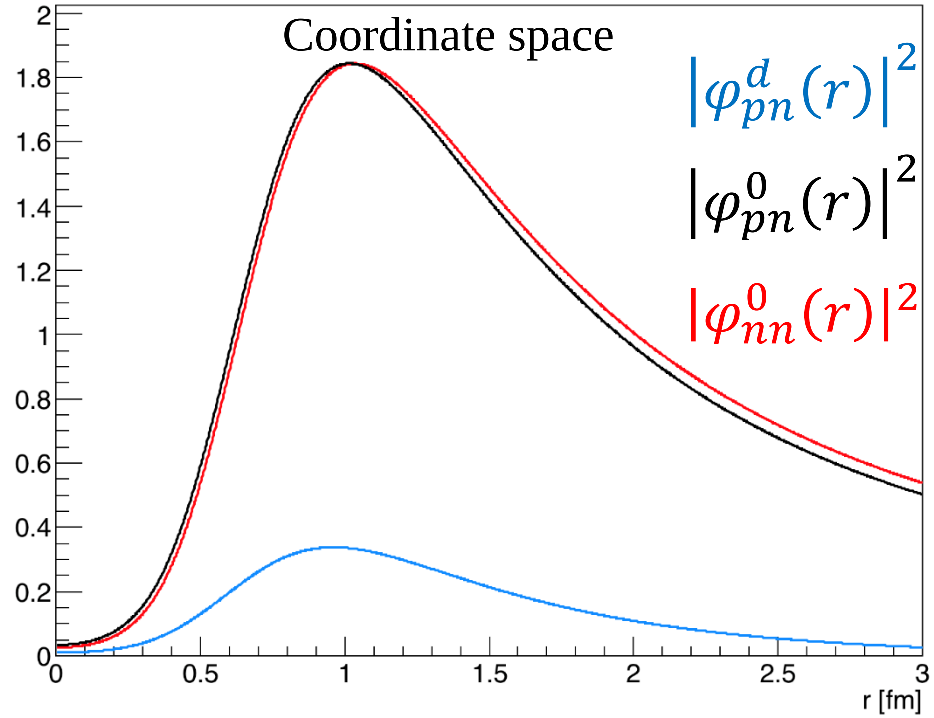
The **spin-zero** channel: $\ell_2 = 0\, ; s_2 = 0\, ; j_2 = 0\, ; t_2 = 1$

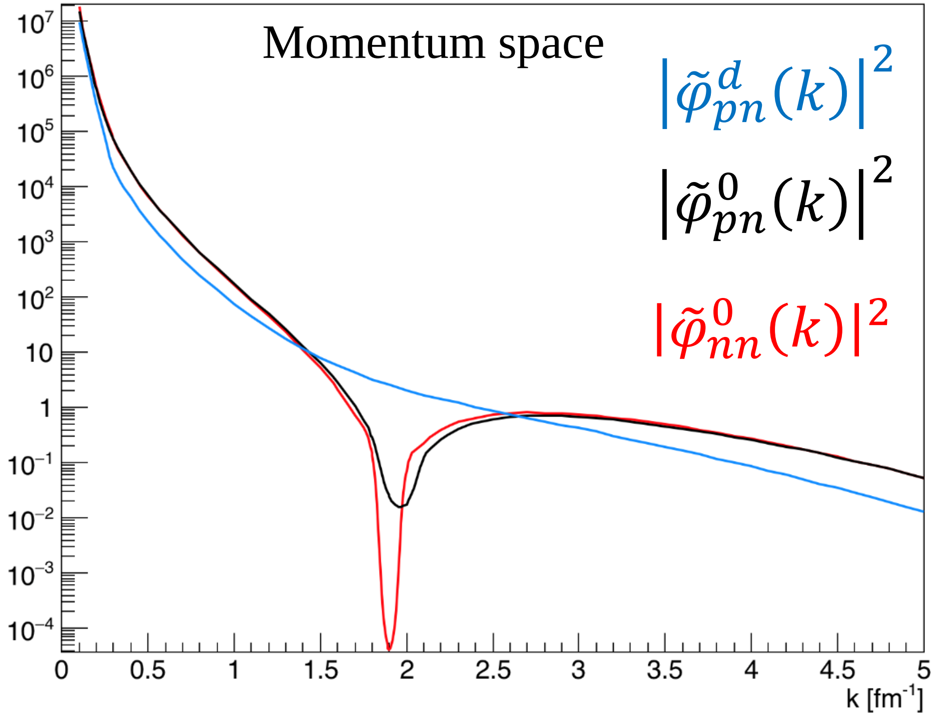
RW, B. Bazak, N. Barnea, PRC 92, 054311 (2015)

Generalized Contact Formalism

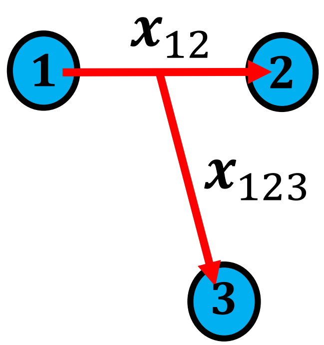
$$\Psi \xrightarrow{r_{ij}\to 0} \sum_{\alpha} \varphi_{ij}^{\alpha}(\boldsymbol{r}_{ij}) A_{ij}^{\alpha}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}) \quad ; \quad \boldsymbol{C_{ij}^{\alpha\beta}} \propto \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle$$

The universal functions using the AV18 potential





Three-body correlations



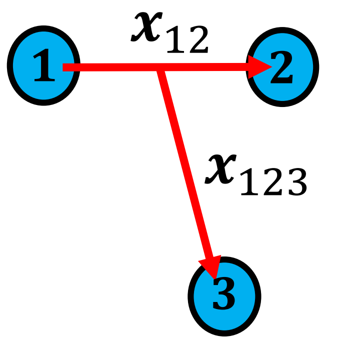
$$\Psi(r_1, r_2, \dots, r_A) \xrightarrow{r_{12}, r_{13}, r_{23} \to 0} \varphi(\boldsymbol{x}_{12}, \boldsymbol{x}_{123}) \times B\left(\boldsymbol{R}_{123}, \{\boldsymbol{r}_k\}_{k \neq 1,2,3}\right)$$

Three-body wave functions – Quantum numbers: π, j, m, t, t_z

- S-wave dominance at short distances $\ell = 0$ ⟹ $\pi = +$
- Spin $S = \frac{1}{2}, \frac{3}{2}$ + $\ell = 0$ ⟹ $j = \frac{1}{2}, \frac{3}{2}$
- Isospin $t = \frac{3}{2}$ (symmetric function) – suppressed due to Pauli blocking ⟹ $t = 1/2$
- Spin $S = \frac{3}{2}$ (symmetric function) – suppressed due to Pauli blocking ⟹ $j = 1/2$

Three-body correlations

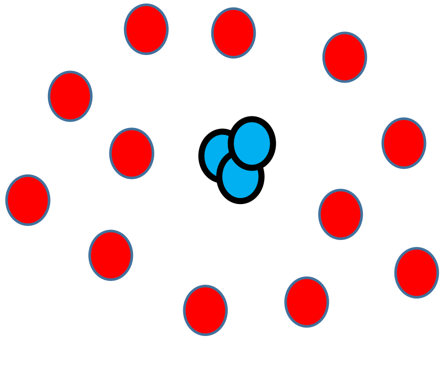
$$\Psi(r_1, r_2, \dots, r_A) \xrightarrow{r_{12}, r_{13}, r_{23} \to 0} \varphi(\boldsymbol{x}_{12}, \boldsymbol{x}_{123}) \times B\left(\boldsymbol{R}_{123}, \{\boldsymbol{r}_k\}_{k \neq 1,2,3}\right)$$



- A **single** leading channel:

$$j^\pi = \frac{1}{2}^+, t = \frac{1}{2}$$

- The same quantum numbers as ^{3}He and ^{3}H
- Therefore, **at short-distances** we expect:
  - $\boldsymbol{T = 1/2}$ **dominance** (over $T = 3/2$)
  - **Universality** - All nuclei should behave like ^{3}He



Three-body contact values ($T = 1/2$)

$$\frac{c(\,^4\mathrm{He})}{c(\,^3\mathrm{He})} \times \frac{3}{4} = 3.8 \pm 0.3$$

$$\frac{c(\,^6\mathrm{Li})}{c(\,^3\mathrm{He})} \times \frac{3}{6} = 3.1 \pm 0.3$$

$$\frac{c(^{16}\mathrm{O})}{c(\,^3\mathrm{He})} \times \frac{3}{16} = 4.2 \pm 0.5$$

Sargsian et al predicted [PRC 100, 044320 (2019)]:

$$a_3(A) = 1.12 \frac{a_2(A)^2}{a_2(\,^3\mathrm{He})^2}$$

$\Rightarrow$

$$a_3(\,^4\mathrm{He}) \approx 3.15$$

Three-body contact values ($T = 1/2$)

$$\frac{C(^{4}\mathrm{He})}{C(^{3}\mathrm{He})} \times \frac{3}{4} = 3.8 \pm 0.3$$

$$\frac{C(^{6}\mathrm{Li})}{C(^{3}\mathrm{He})} \times \frac{3}{6} = 3.1 \pm 0.3$$

$$\frac{C(^{16}\mathrm{O})}{C(^{3}\mathrm{He})} \times \frac{3}{16} = 4.2 \pm 0.5$$

Additional effects might be important:

- CM motion of the triplet in nucleus A
- Energy of the $A - 3$ system
- Contribution of $t = 3/2$ triplets (*e.g.*: ppp, nnn)

Detailed reaction calculations are needed

Short-range expansion

- The two-body system:

$$\left[-\frac{\hbar^2}{m}\nabla^2 + V(r)\right]\varphi^E(r) = E\varphi^E(r)$$

- Taylor expansion around $E = 0$:

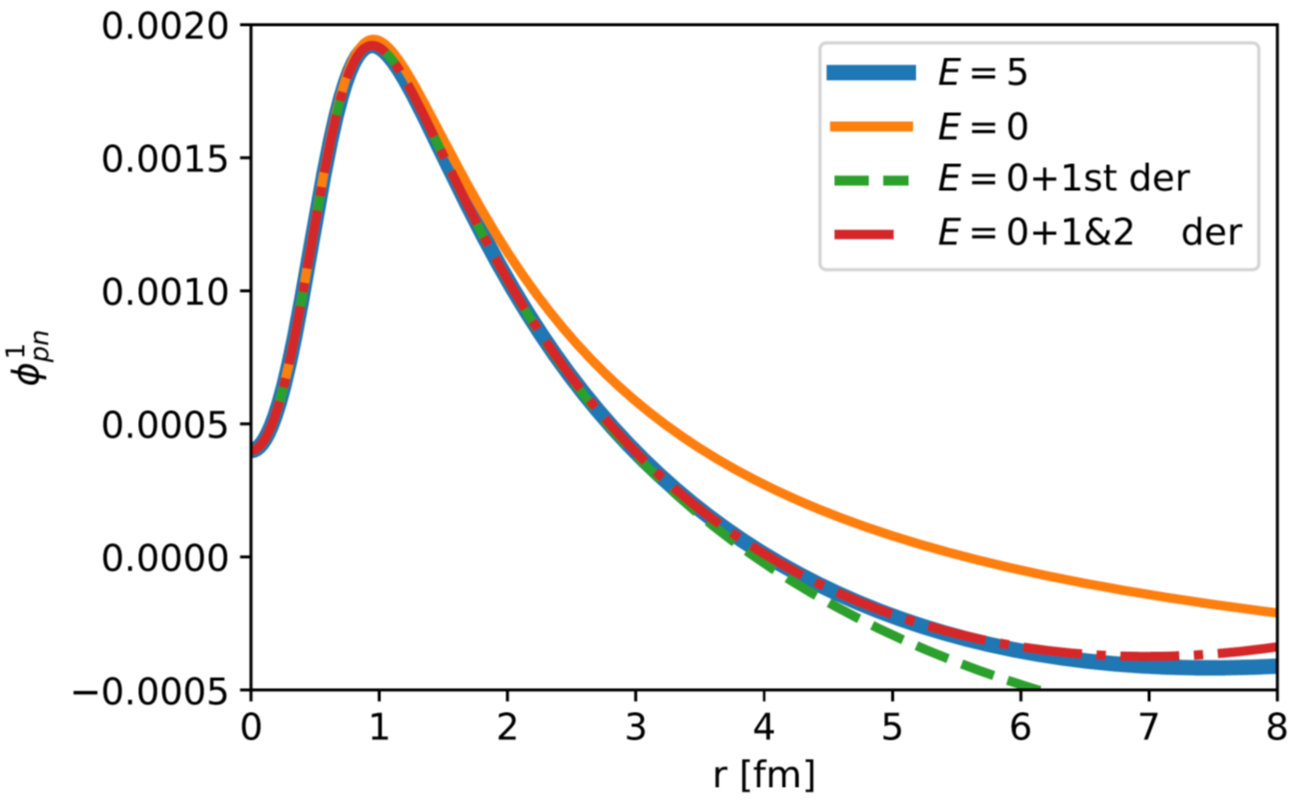
$$\varphi^E(\boldsymbol{r}) = \varphi^{E=0}(\boldsymbol{r}) + \left(\frac{d}{dE}\varphi^{E=0}(\boldsymbol{r})\right)E + \frac{1}{2!}\left(\frac{d^2}{dE^2}\varphi^{E=0}(\boldsymbol{r})\right)E^2 + \cdots$$

GCF
leading term

Next-order
terms

Short-range expansion

$$\varphi^{E}(\boldsymbol{r}) = \varphi^{E=0}(\boldsymbol{r}) + \left(\frac{d}{dE}\varphi^{E=0}(\boldsymbol{r})\right)E + \frac{1}{2!}\left(\frac{d^2}{dE^2}\varphi^{E=0}(\boldsymbol{r})\right)E^2 + \cdots$$



AV4'
Deuteron channel
Scattering state

Short-range expansion: Next order terms

The many-body case:

$$\Psi(\boldsymbol{r}_1,\boldsymbol{r}_2,\dots,\boldsymbol{r}_A) = \sum_\alpha \varphi_\alpha^{E=0}(\boldsymbol{r}_{12}) A_\alpha^{(0)} + \sum_\alpha \left(\frac{d}{dE}\varphi_\alpha^{E=0}(\boldsymbol{r})\right) A_\alpha^{(1)} + \sum_\alpha \left(\frac{d^2}{dE^2}\varphi_\alpha^{E=0}(\boldsymbol{r})\right) A_\alpha^{(2)} + \cdots$$

$$A_\alpha^{(0)}(\boldsymbol{R}_{12},\boldsymbol{r}_3,\dots,\boldsymbol{r}_A) = \sum_E A_\alpha^E(\boldsymbol{R}_{12},\boldsymbol{r}_3,\dots,\boldsymbol{r}_A)$$

$$A_\alpha^{(1)}(\boldsymbol{R}_{12},\boldsymbol{r}_3,\dots,\boldsymbol{r}_A) = \sum_E E A_\alpha^E(\boldsymbol{R}_{12},\boldsymbol{r}_3,\dots,\boldsymbol{r}_A)$$

$$A_\alpha^{(2)}(\boldsymbol{R}_{12},\boldsymbol{r}_3,\dots,\boldsymbol{r}_A) = \frac{1}{2!}\sum_E E^2 A_\alpha^E(\boldsymbol{R}_{12},\boldsymbol{r}_3,\dots,\boldsymbol{r}_A)$$

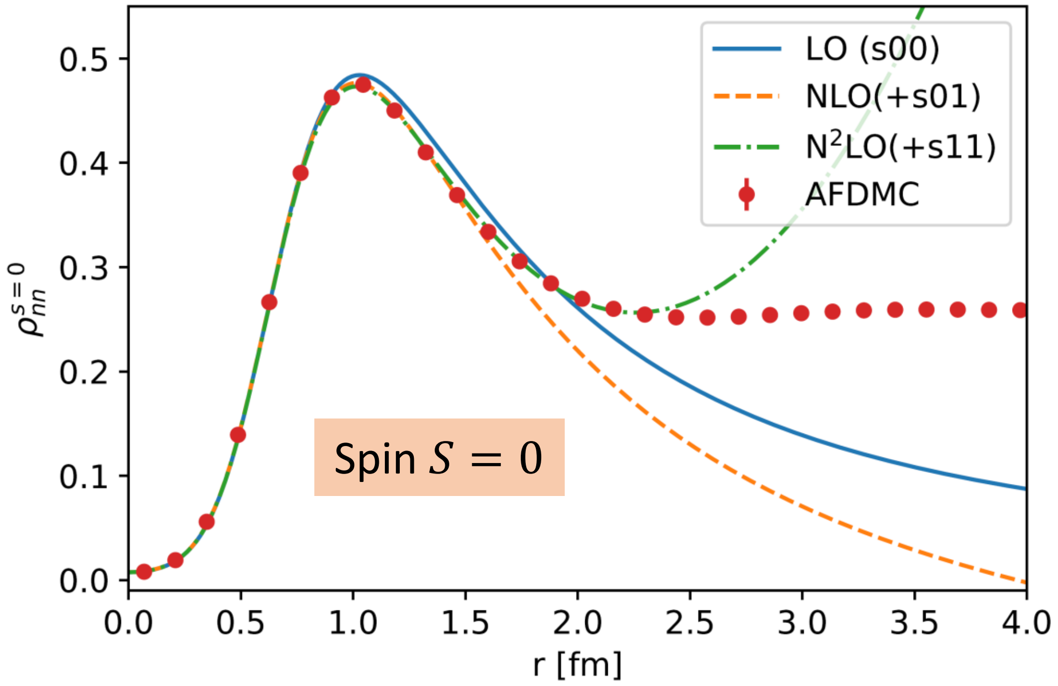
Short-range expansion

$$\rho_2(r) = \sum_\alpha \left(|\varphi_\alpha^{E=0}(r)|^2 C_\alpha^{00} + \varphi_\alpha^{E=0}(r) \frac{d}{dE} \varphi_\alpha^{E=0}(r) C_\alpha^{01} + \cdots \right)$$

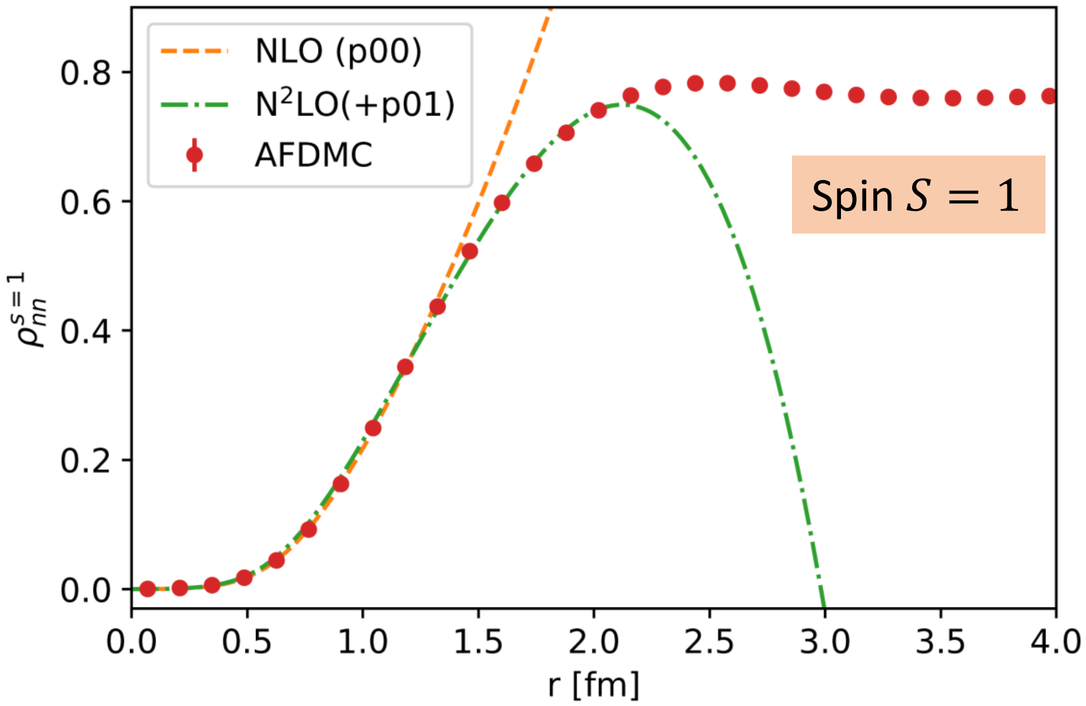
- Neutron matter:

  ($S + \ell =$ Even)

AFDMC by Diego Lonardoni & Stefano Gandolfi: AV4' $n = 0.16\ \mathrm{fm}^{-3}$



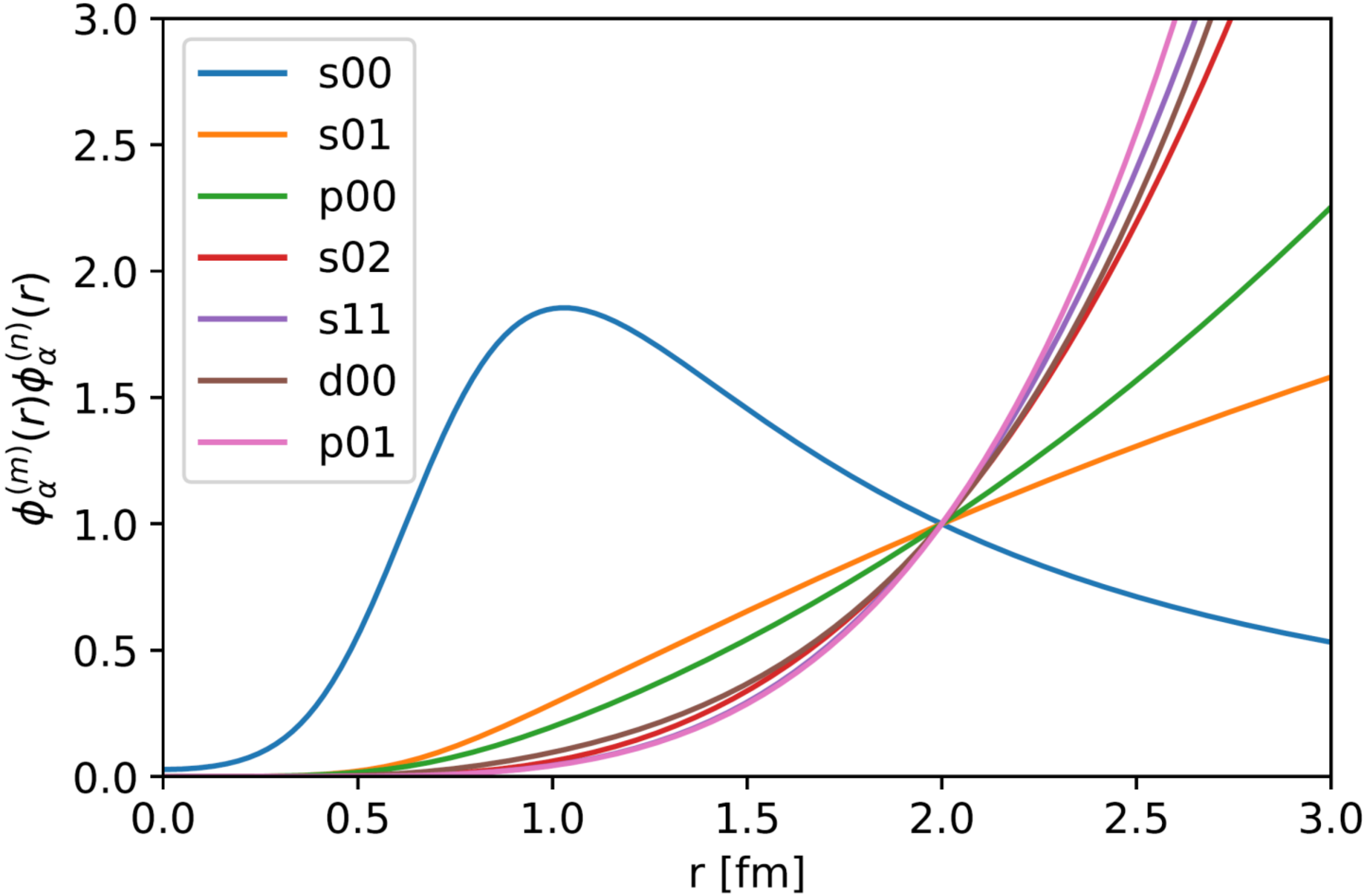
s-wave: $\ell = 0, S = 0, j = 0$



p-wave: $\ell = 1, S = 1, j = 0/1/2$

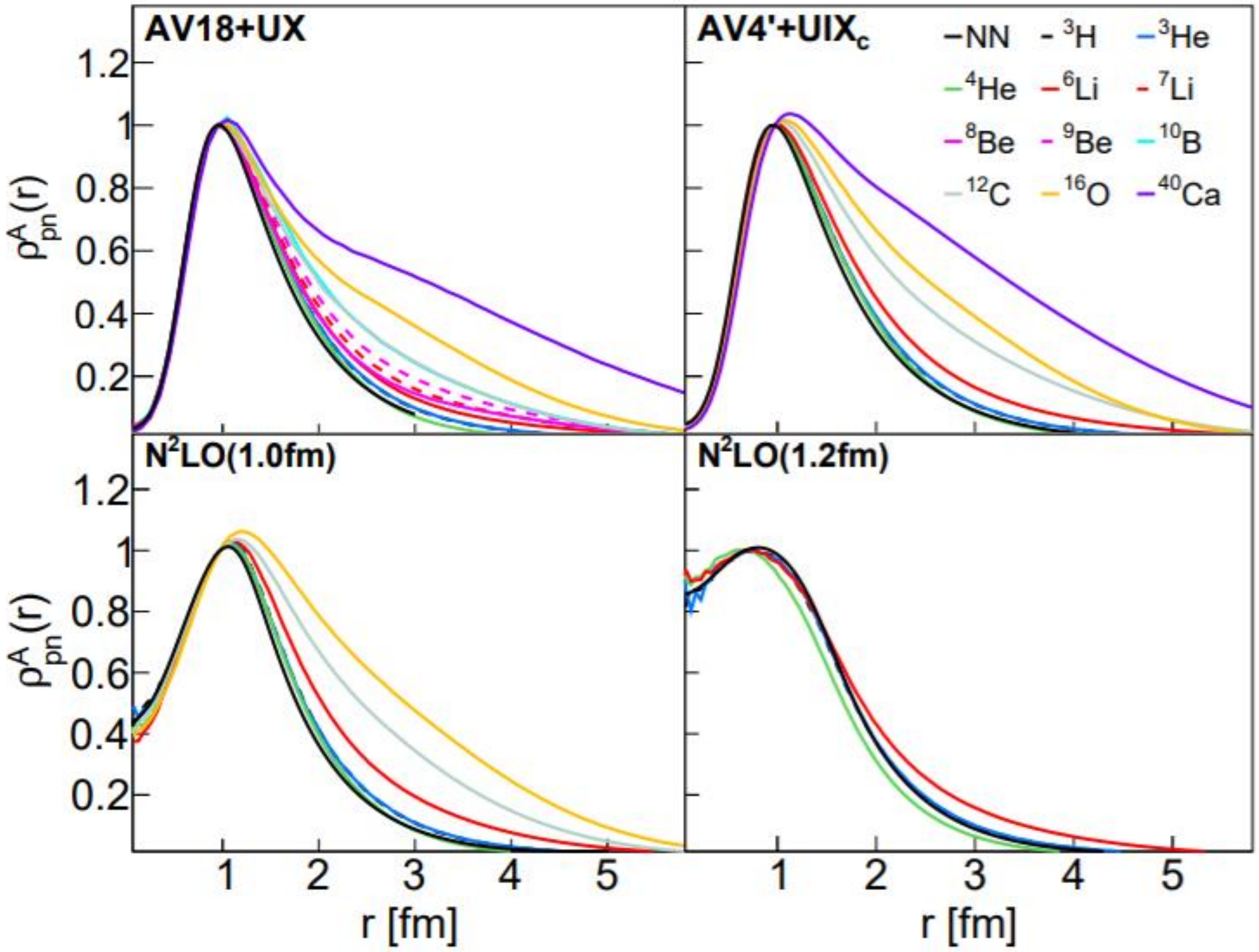
Short-range expansion

$$\rho_2(r) = \sum_\alpha \left(|\varphi_\alpha^{E=0}(r)|^2 C_\alpha^{00} + \varphi_\alpha^{E=0}(r) \frac{d}{dE} \varphi_\alpha^{E=0}(r) C_\alpha^{01} + \cdots \right)$$



Two-body density

$$\langle \hat{O} \rangle = \langle \varphi | \hat{O}(r) | \varphi \rangle \, C$$



Shows the validity of the factorization

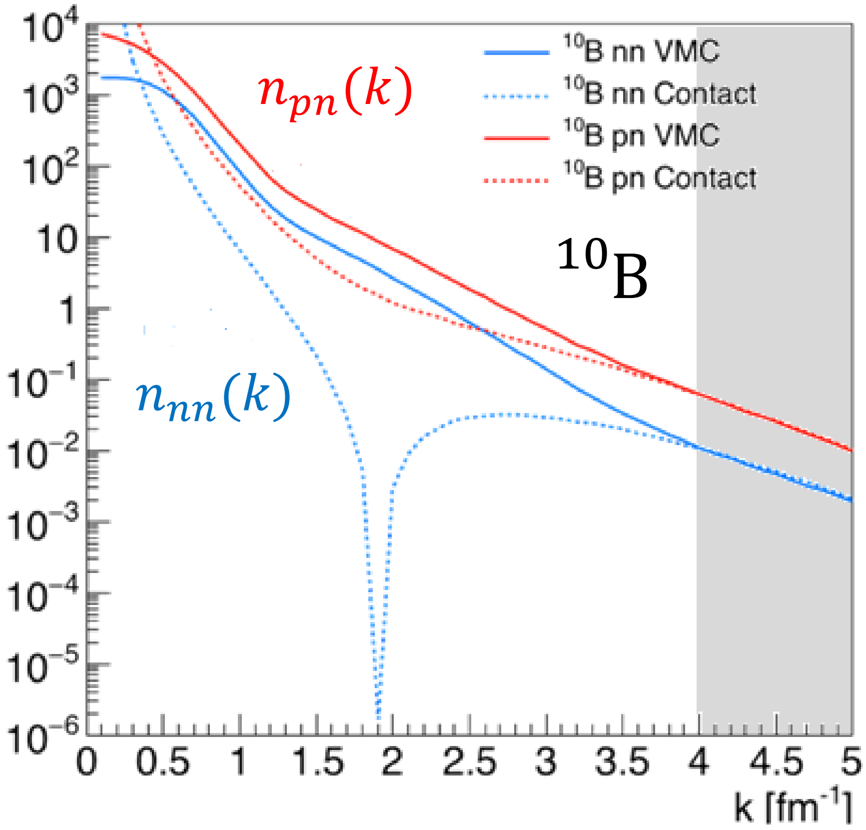
R. Cruz-Torres, D. Lonardoni, RW, et al., Nature Physics (2020)

Two-body momentum distribution

Relative momentum distribution

$$n_{pn}(k_{rel}) \xrightarrow[k\to\infty]{} C_{pn}^{d}\left|\varphi_{pn}^{d}(k_{rel})\right|^{2} + C_{pn}^{0}\left|\varphi_{pn}^{0}(k_{rel})\right|^{2}$$

$$n_{nn}(k_{rel}) \xrightarrow[k\to\infty]{} C_{nn}^{0}|\varphi_{nn}^{0}(k_{rel})|^{2}$$

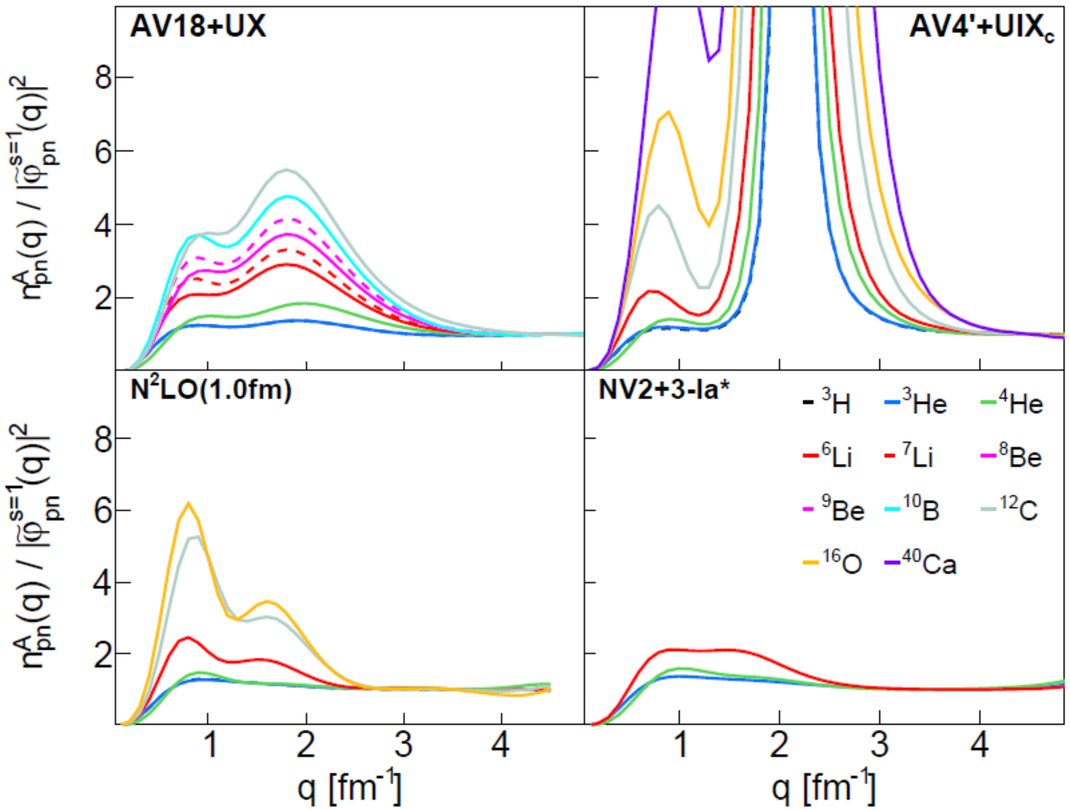


Two-body momentum distribution

Relative momentum distribution

$$n_{pn}(k_{rel}) \xrightarrow[k\to\infty]{} C_{pn}^{d}\left|\varphi_{pn}^{d}(k_{rel})\right|^2 + C_{pn}^{0}\left|\varphi_{pn}^{0}(k_{rel})\right|^2$$

$$n_{nn}(k_{rel}) \xrightarrow[k\to\infty]{} C_{nn}^{0}|\varphi_{nn}^{0}(k_{rel})|^2$$

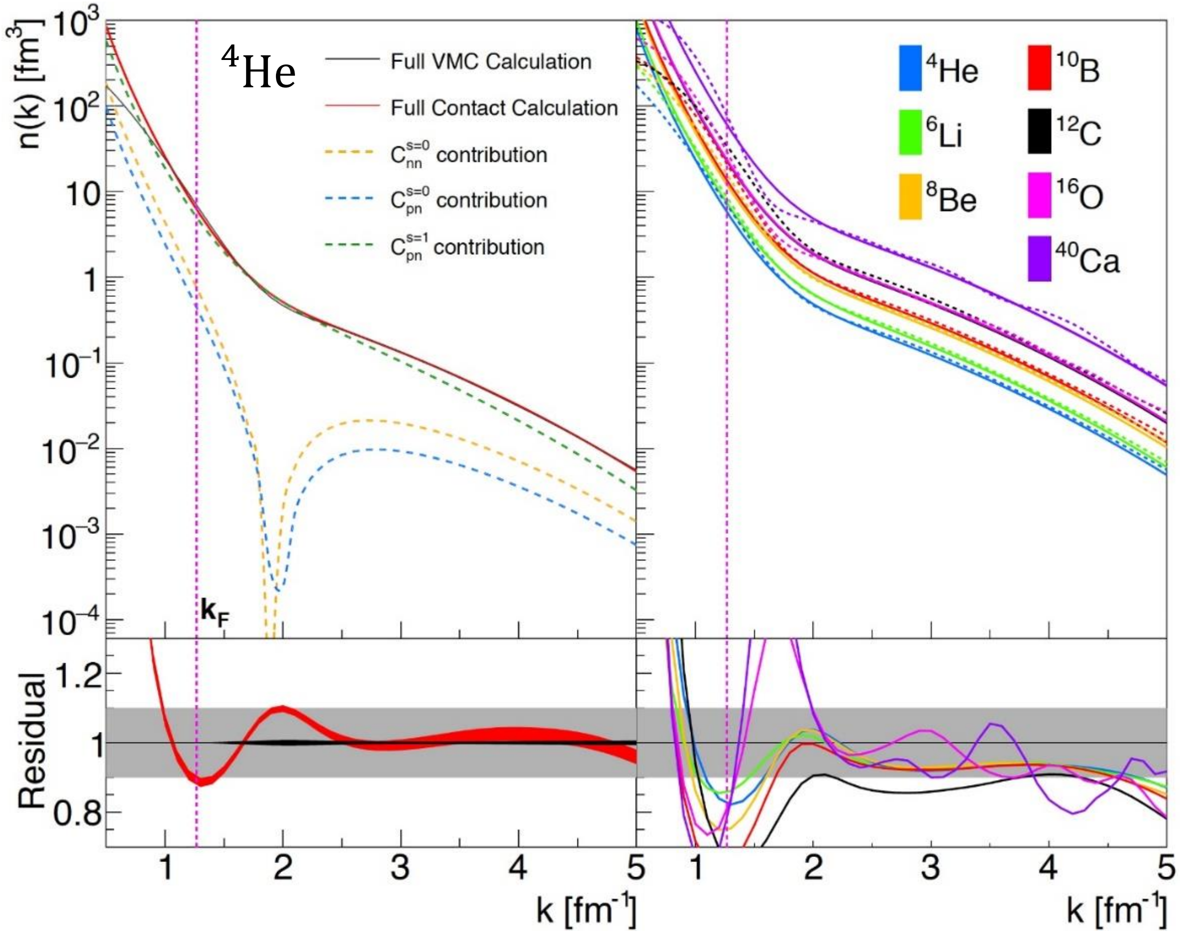


R. Cruz-Torres, D. Lonardoni, RW, et al., arXiv: 1907.03658 [nucl-th], Nature physics (2020)

One-body momentum distribution

$$n_p(k) \xrightarrow[k\to\infty]{} C_{pn}^{d}\left|\varphi_{pn}^{d}(k)\right|^2 + C_{pn}^{0}\left|\varphi_{pn}^{0}(k)\right|^2 + 2C_{pp}^{0}\left|\varphi_{pp}^{0}(k)\right|^2$$

$n_p(k)$



No fitting parameters!

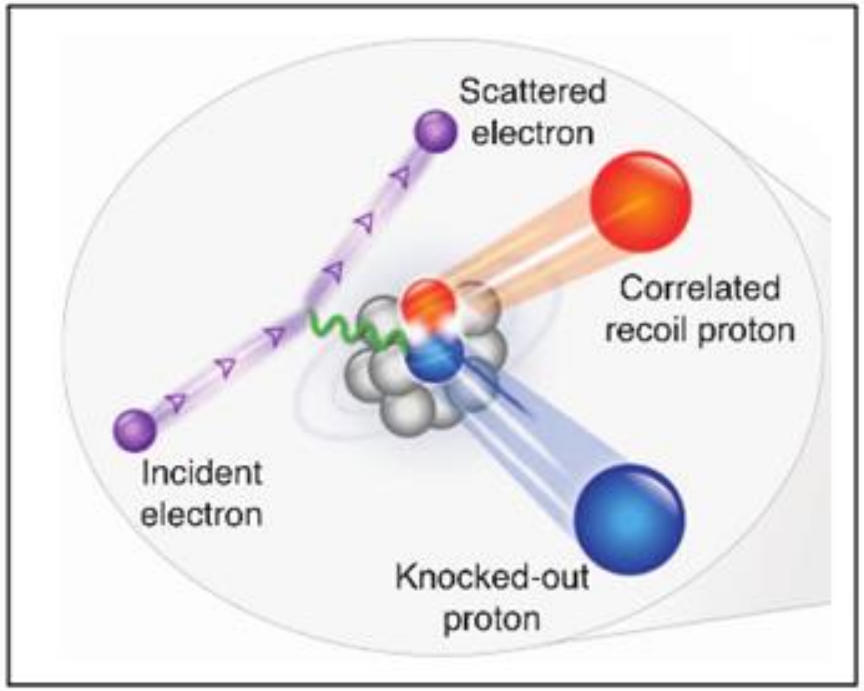
RW, R. Cruz-Torres, N. Barnea, E. Piasetzky and O. Hen, PLB 780, 211 (2018)

Electron-scattering experiments

- $A(e,e'N)$ and $A(e,e'NN)$ cross sections

- Described using the spectral function (the probability to find nucleon with momentum $\boldsymbol{p}_1$ and energy ϵ_1 in the nucleus)

- Using the GCF:



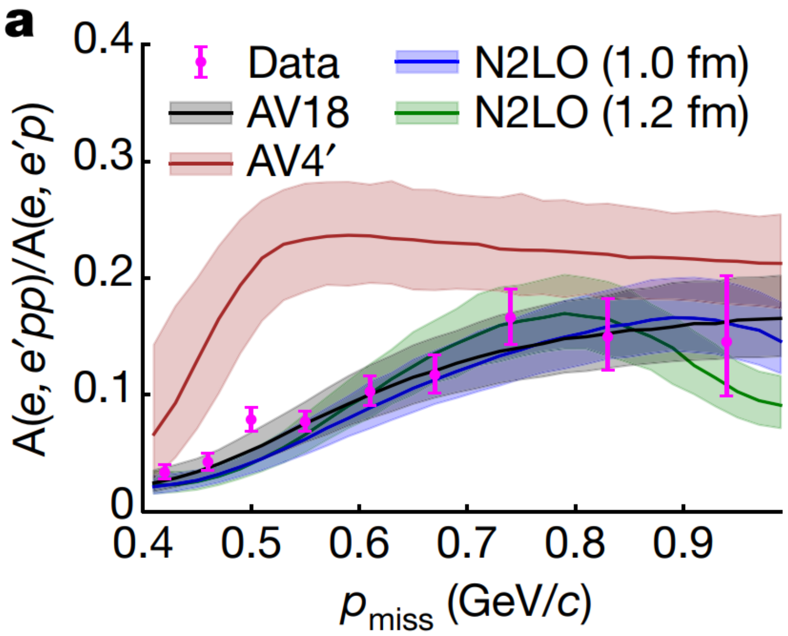
$$S^p(\boldsymbol{p_1},\epsilon_1) = C^1_{pn}S^1_{pn}(\boldsymbol{p_1},\epsilon_1) + C^0_{pn}S^0_{pn}(\boldsymbol{p_1},\epsilon_1) + 2C^0_{pp}S^0_{pp}(\boldsymbol{p_1},\epsilon_1)$$

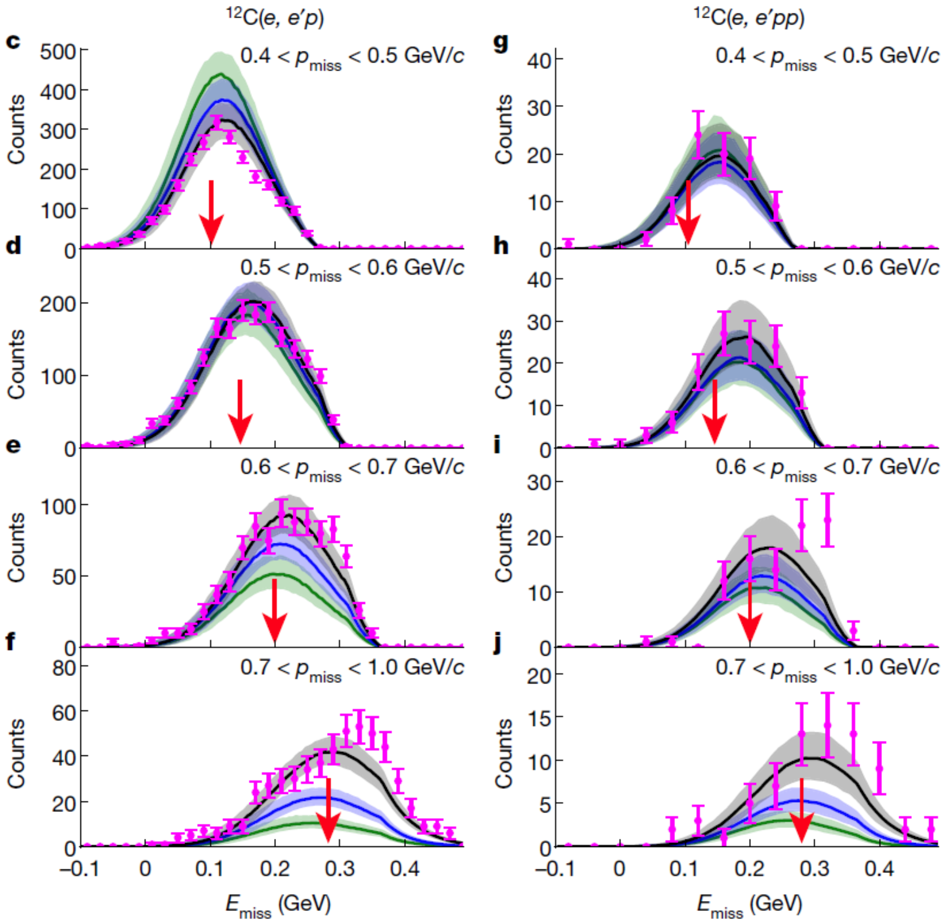
$(p_1 > k_F)$

RW, I. Korover, E. Piasetzky, O. Hen and N. Barnea, PLB 791, 242 (2019)

Electron-scattering experiments

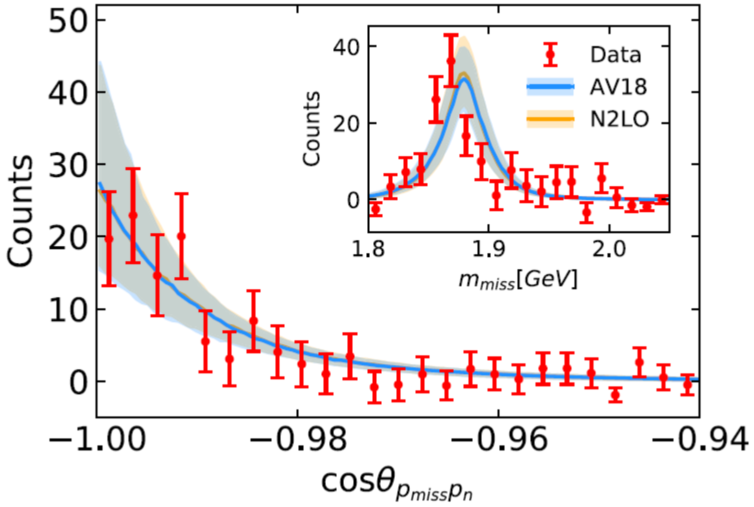
- Good description of experimental data:

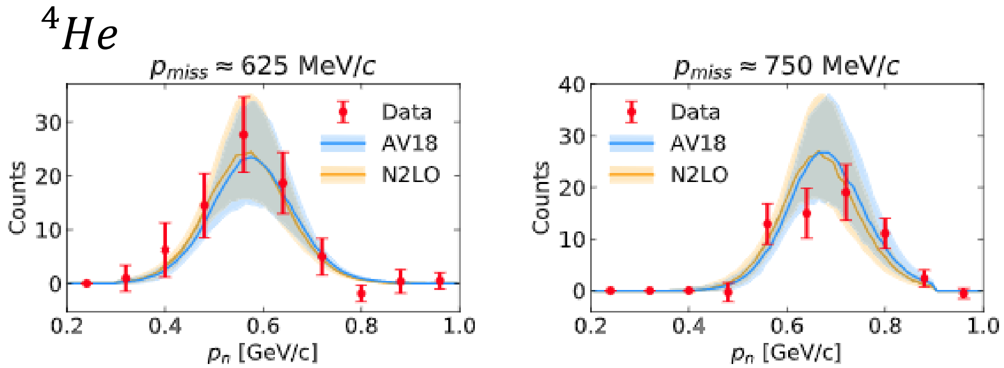




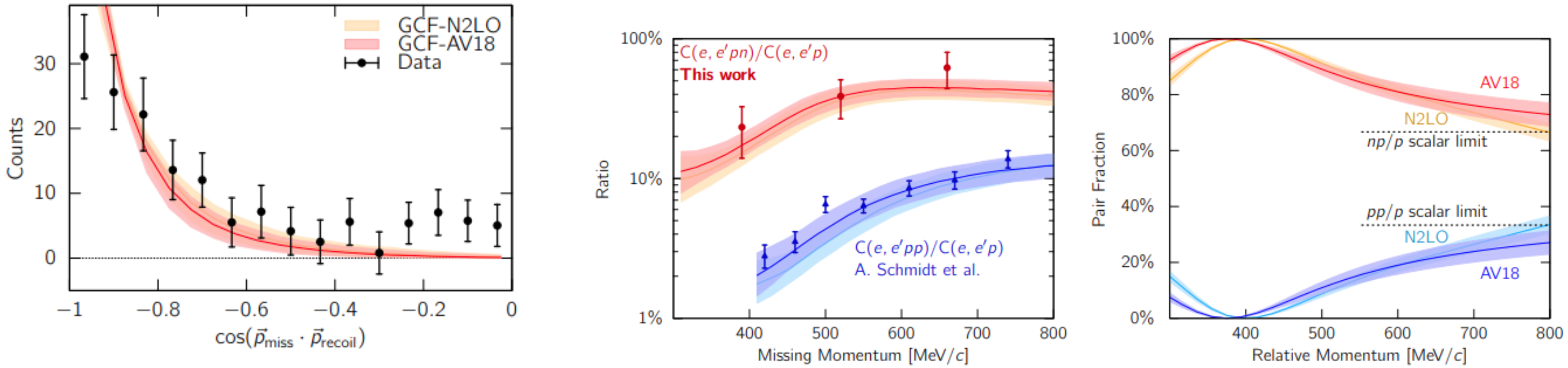
A. Schmidt, J.R. Pybus, RW, E. P. Segarra, A. Hrnjic, A. Denniston, O. Hen, *et al. (CLAS collaboration), Nature 578, 540 (2020)*

Electron-scattering experiments





J.R. Pybus et al., PLB 805, 135429 (2020)



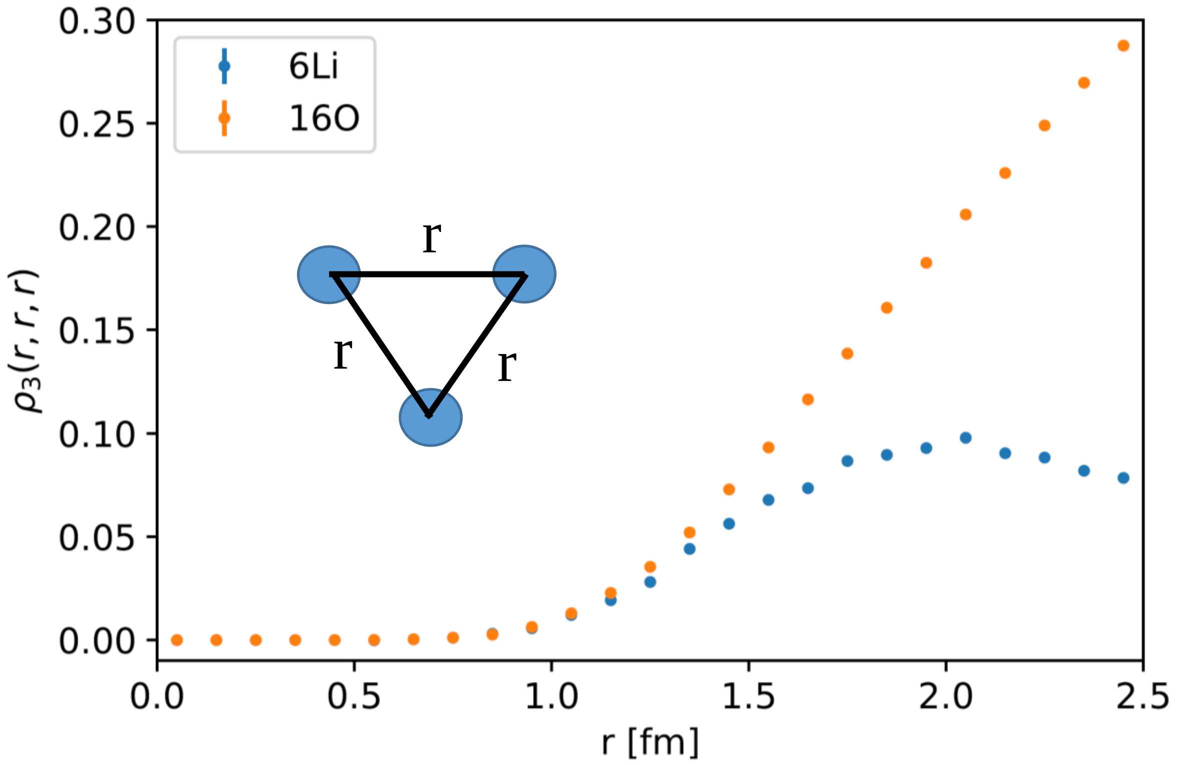
I. Korover et al., arXiv:2004.07304 (2020)

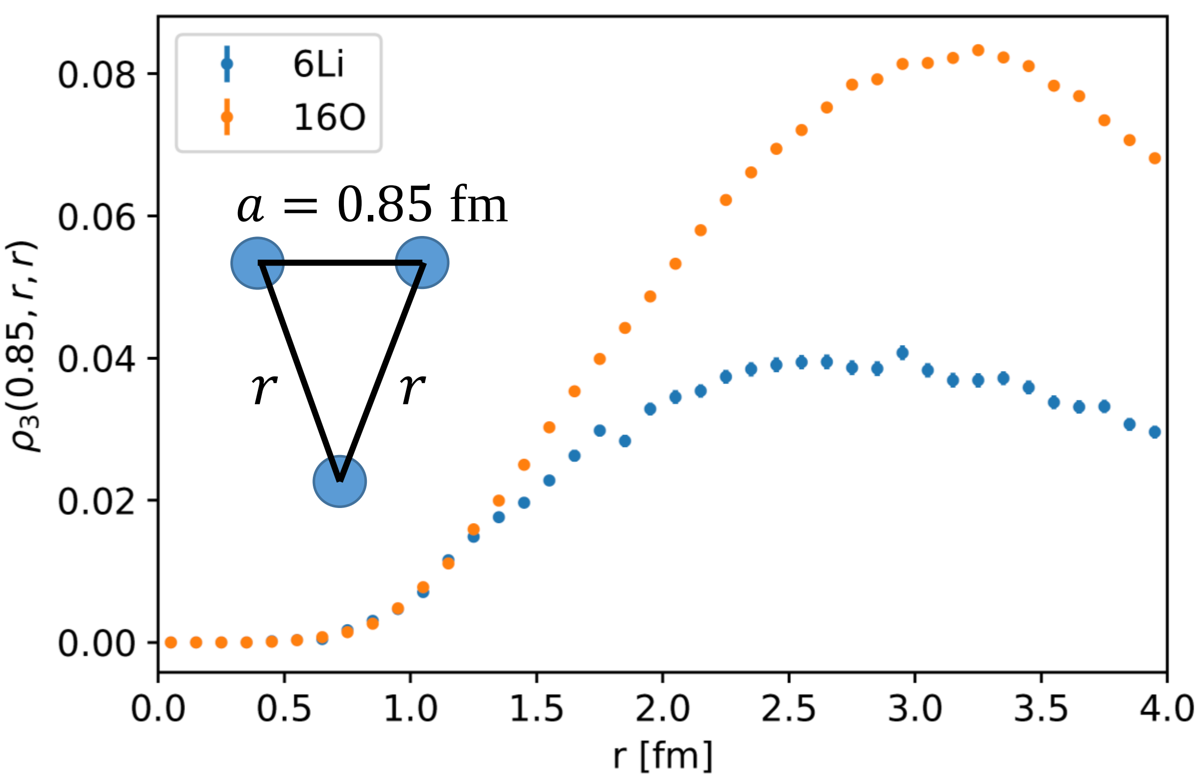
Three-body density

Same scaling factor for all geometries!

$$T = \frac{3}{2}\ \textbf{universality:}$$

rescaled densities





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