Calculations for Polarization Observables in D(e,e'p), and How to Enjoy Them Responsibly

SABINE JESCHONNEK (OSU),
BILL FORD (INDUSTRY),
J. WALLACE VAN ORDEN (JLAB, ODU)

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Outline

- relevance of D(e,e'p) reactions, basic observables
- a model calculation: results for unpolarized targets
  SJ & Van Orden, PRC 78, 014007 (2008) ;
  results for polarized targets SJ & Van Orden, PRC 80 054001 (2009);
  results for polarized ejectile protons SJ & Van Orden, PRC 81 014008 (2010)
- Theoretical error bands for calculations: SJ & Van Orden, PRC 95, 044001 (2017)

Beware: while many things (e.g. importance of FSIs) apply in general, the only way to know for sure is to **calculate an observable for the specific kinematics**

What may cause trouble?

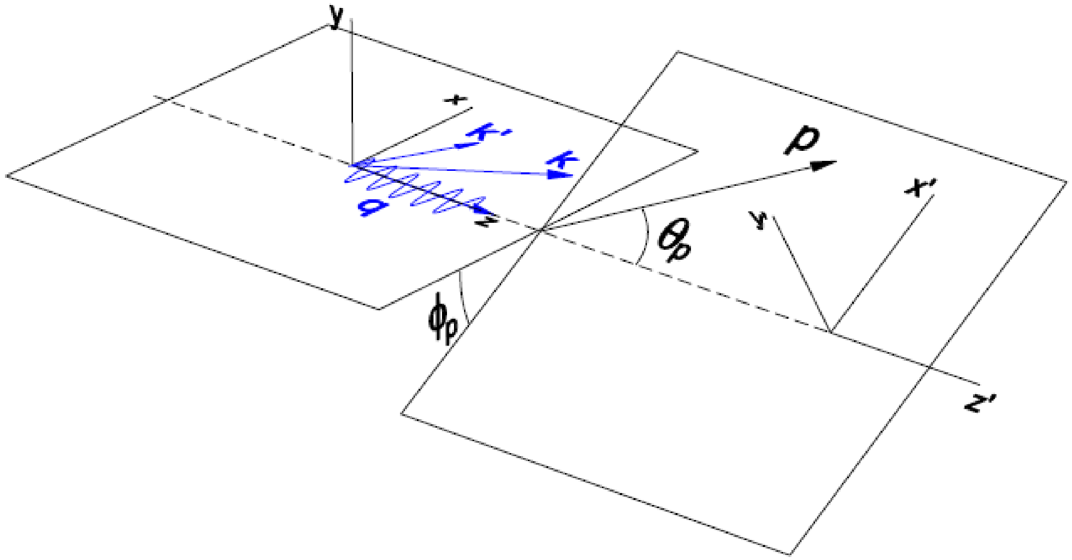
- Model building itself
  - E.g. no meson exchange current, no isobar states
  - Treatment of off-shell FSIs
  - Treatment of relativity
  - Off-shell effects in the current operator

- Input parameters
  - Nucleon form factors
  - Wave functions
  - NN interaction parametrizations

Differential Cross Section:

$$\left(\frac{d\sigma^5}{d\epsilon' d\Omega_e d\Omega_p}\right)_h = \frac{m_p\, m_n\, p_p}{8\pi^3\, M_d}\,\sigma_{Mott}\, f_{rec}^{-1}\Big[\Big(v_L R_L + v_T R_T$$

$$+ v_{TT} R_{TT}\cos 2\phi_p + v_{LT} R_{LT}\cos\phi_p\Big) + h v_{LT'} R_{LT'}\sin\phi_p\Big]$$



Details of the Calculation

- Relativistic deuteron w. f.: solution of Gross eqn.
- one-body e.m. current $\Gamma^\mu(q) = F_1(Q^2)\gamma^\mu + \frac{F_2(Q^2)}{2m} i\sigma^{\mu\nu} q_\nu$
- SAID parameterization of the NN scattering amplitude used
- All parts of the NN amplitude included
  - Central
  - Spin-orbit
  - Double spin-flip

SAID analysis for pn scattering up to 1.3GeV, see Arndt, Briscoe, Strakovsky, Workman,
Phys.Rev.C76:025209,2007

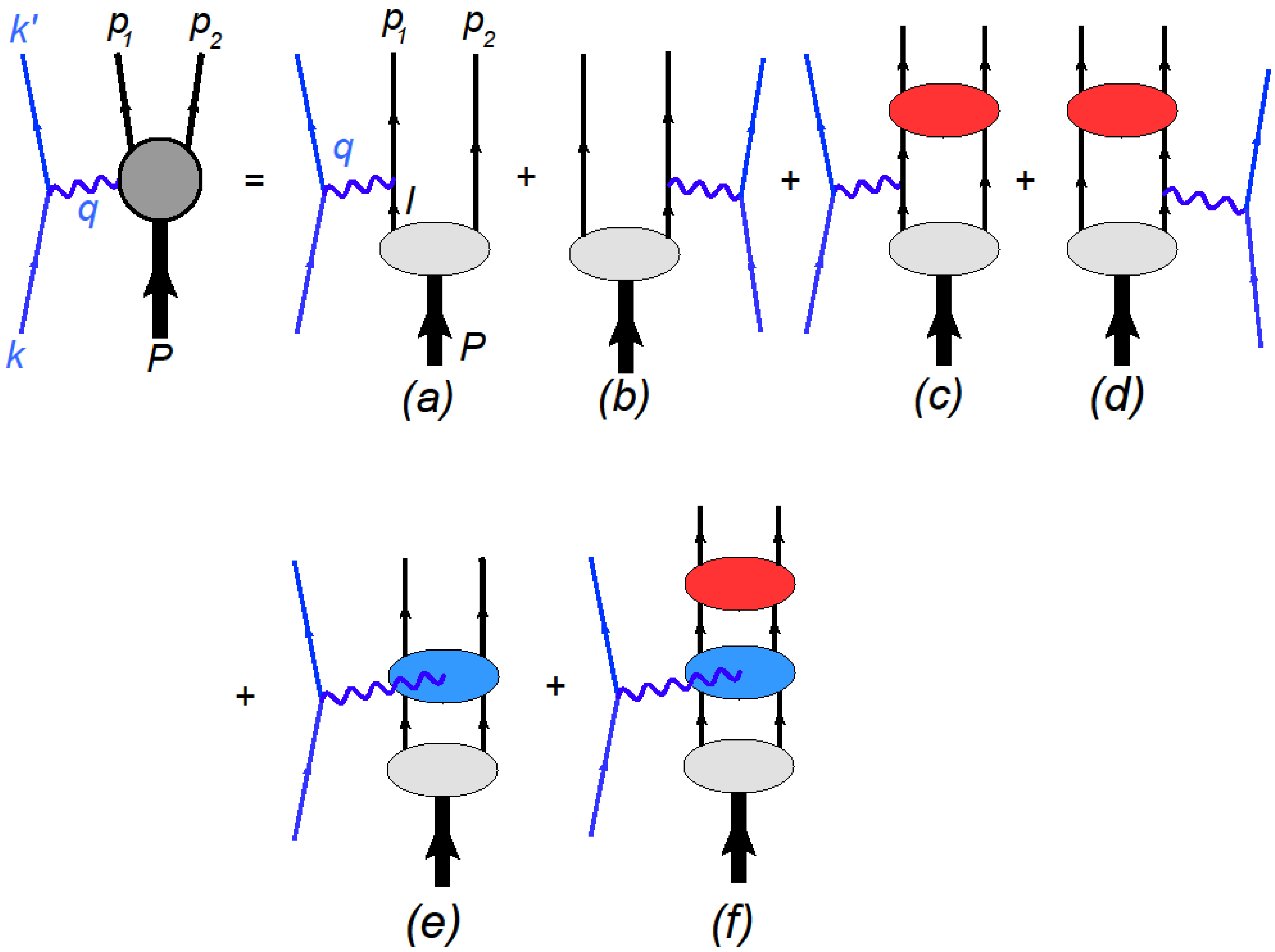
Nucleon-Nucleon Scattering Amplitude

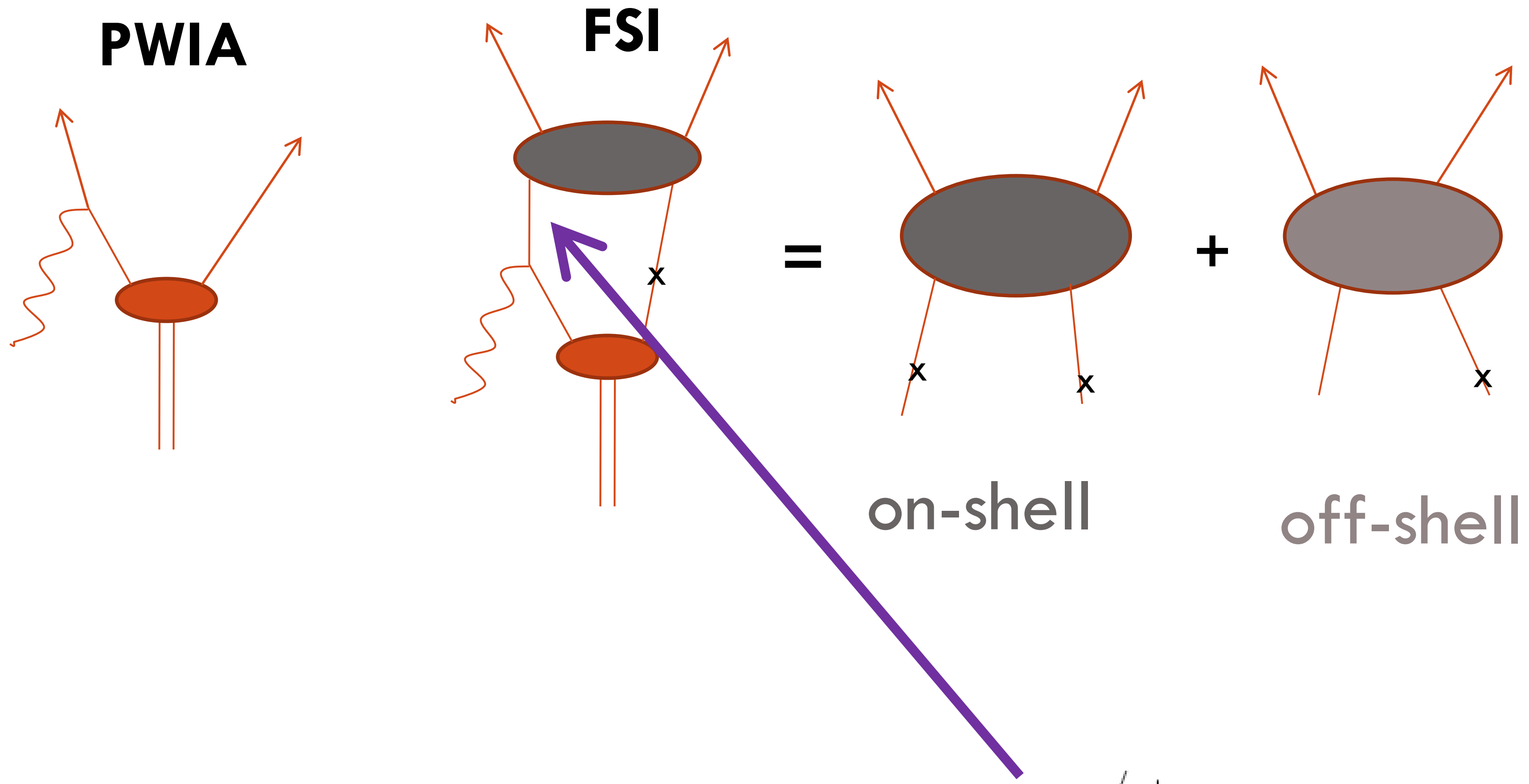
- SAID analysis for pn scattering up to 1.3GeV, see Arndt, Briscoe, Strakovsky, Workman, Phys.Rev.C76:025209,2007
- Saclay amplitudes:

$$M(\vec{k}', \vec{k}) = \frac{1}{2} \left[(a+b) + (a-b)\sigma_{1,n}\sigma_{2,n} + (c+d)\sigma_{1,m}\sigma_{2,m} + (c-d)\sigma_{1,l}\sigma_{2,l} + e(\sigma_{1,n} + \sigma_{2,n}) \right]$$

- Invariant amplitudes (McNeil, Ray, Wallace)

$$F = F_S + F_V\,\gamma_1{\cdot}\gamma_2 + F_T\,\sigma_1^{\mu\nu}\sigma_{2,\mu\nu} + F_P\,\gamma_1^5\gamma_2^5 + F_A\,\gamma_1^5\gamma_1^\mu\gamma_2^5\gamma_{2\mu}$$


k'
p1
p2
q
k
P
=
p1
p2
q
l
P
+
+
+
(a)
(b)
(c)
(d)
+
+
(e)
(f)



$$\frac{\not{p} + m}{p^2 - m^2 + i\varepsilon} =$$

off-shell FSI, **positive energy contribution**:
requires a dynamical model of the amplitude;
we estimate it with a simple prescription

$$\frac{m}{E_p} \sum_s \left(\frac{u(\vec{p}, s)\bar{u}(\vec{p}, s)}{p^0 - E_p + i\varepsilon} + \frac{v(-\vec{p}, s)\bar{v}(-\vec{p}, s)}{p^0 + E_p - i\varepsilon} \right)$$

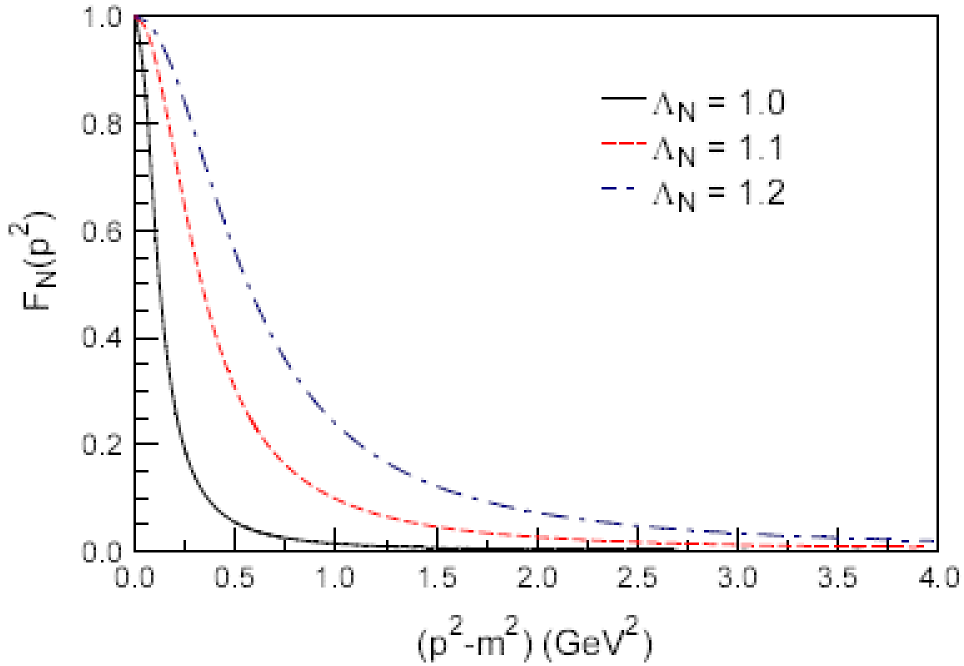
Positive Energy off-shell FSI prescription:

-retain the five on-shell invariants

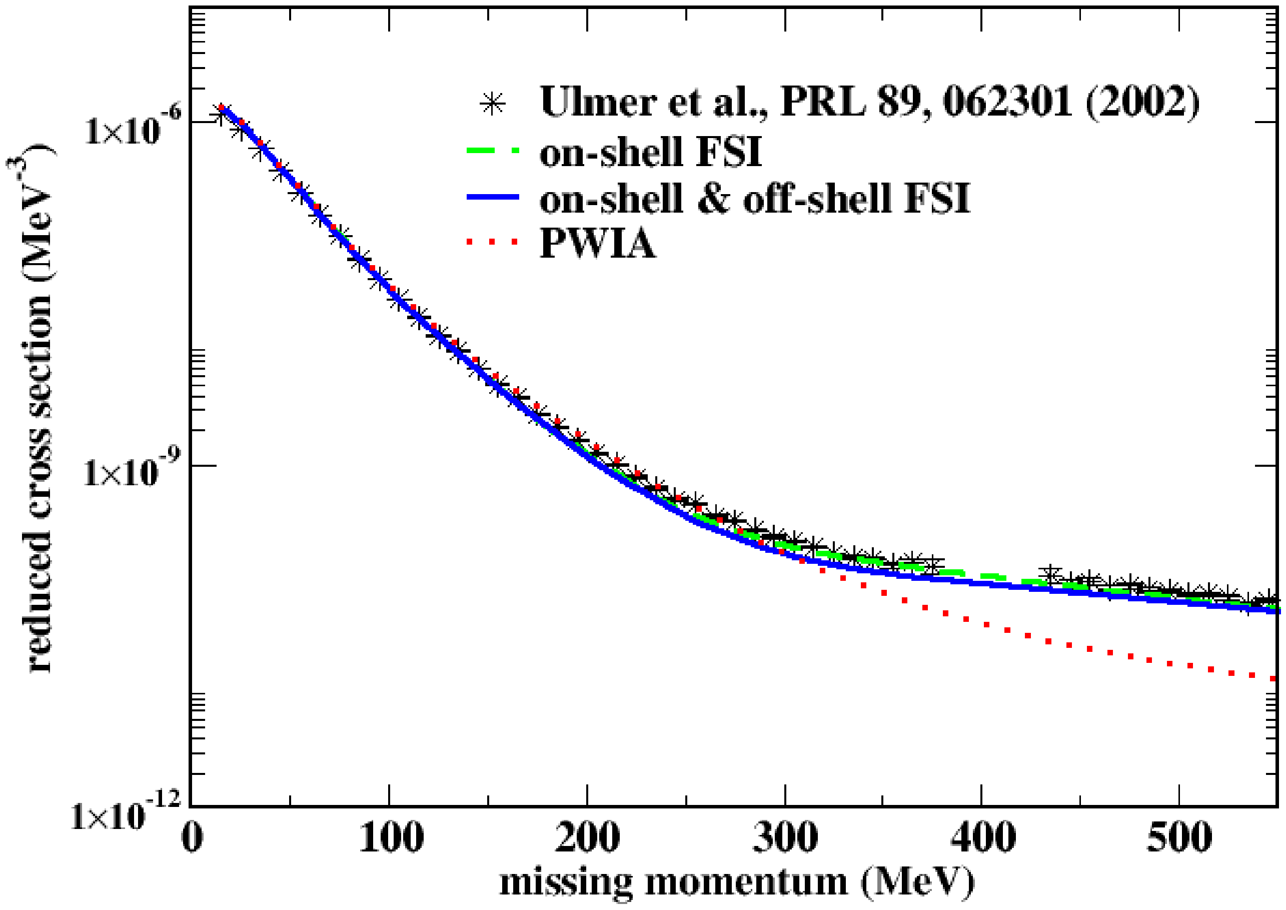
$$\mathcal{F}_i(s,t) \rightarrow \mathcal{F}_i(s,t,u) F_N(s+t+u-3m^2)$$

- use a form factor

$$F_N(p^2) = \frac{(\Lambda_N^2 - m^2)^2}{(p^2 - m^2)^2 + (\Lambda_N^2 - m^2)^2}$$



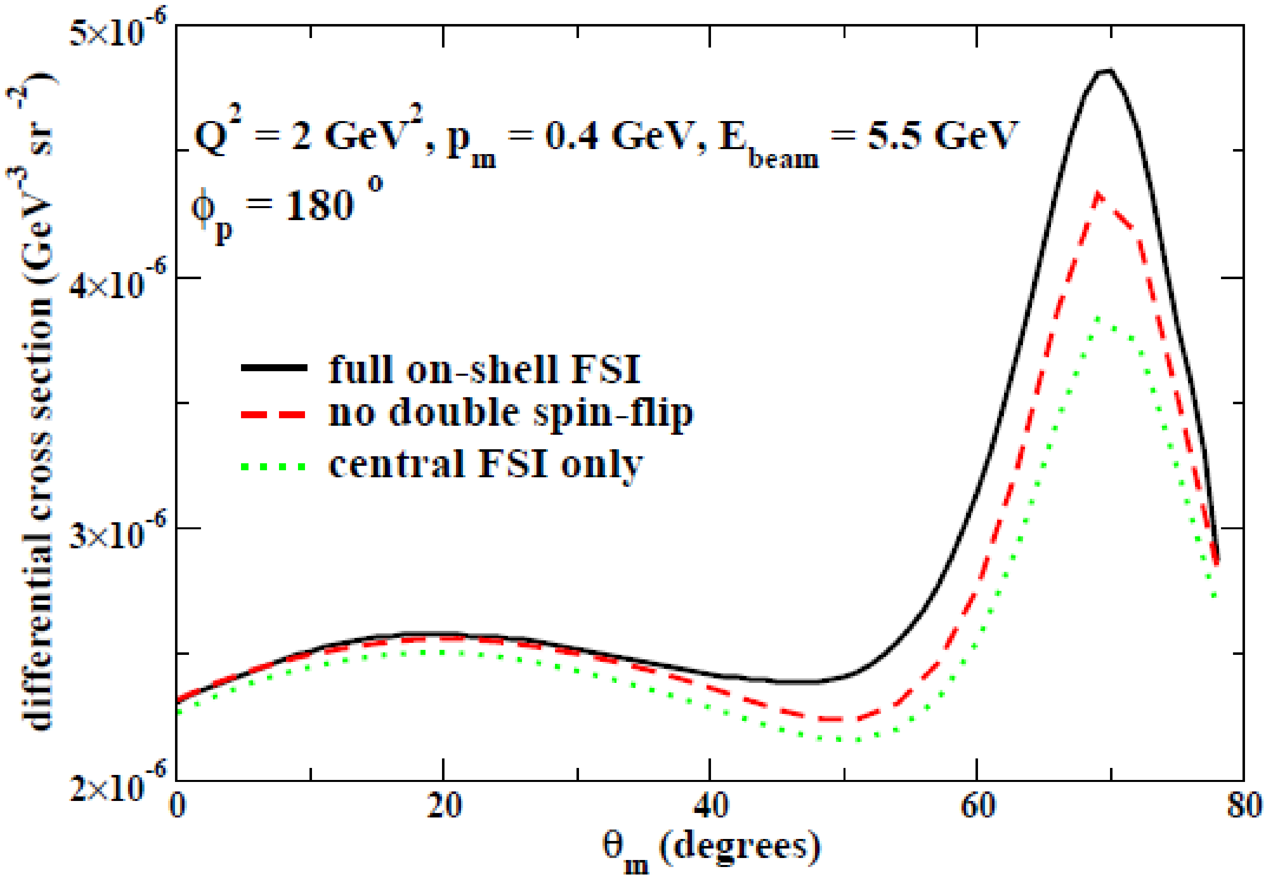
Diff. Cross Section Data from Hall A



data: Ulmer et al, PRL 89 (2002)

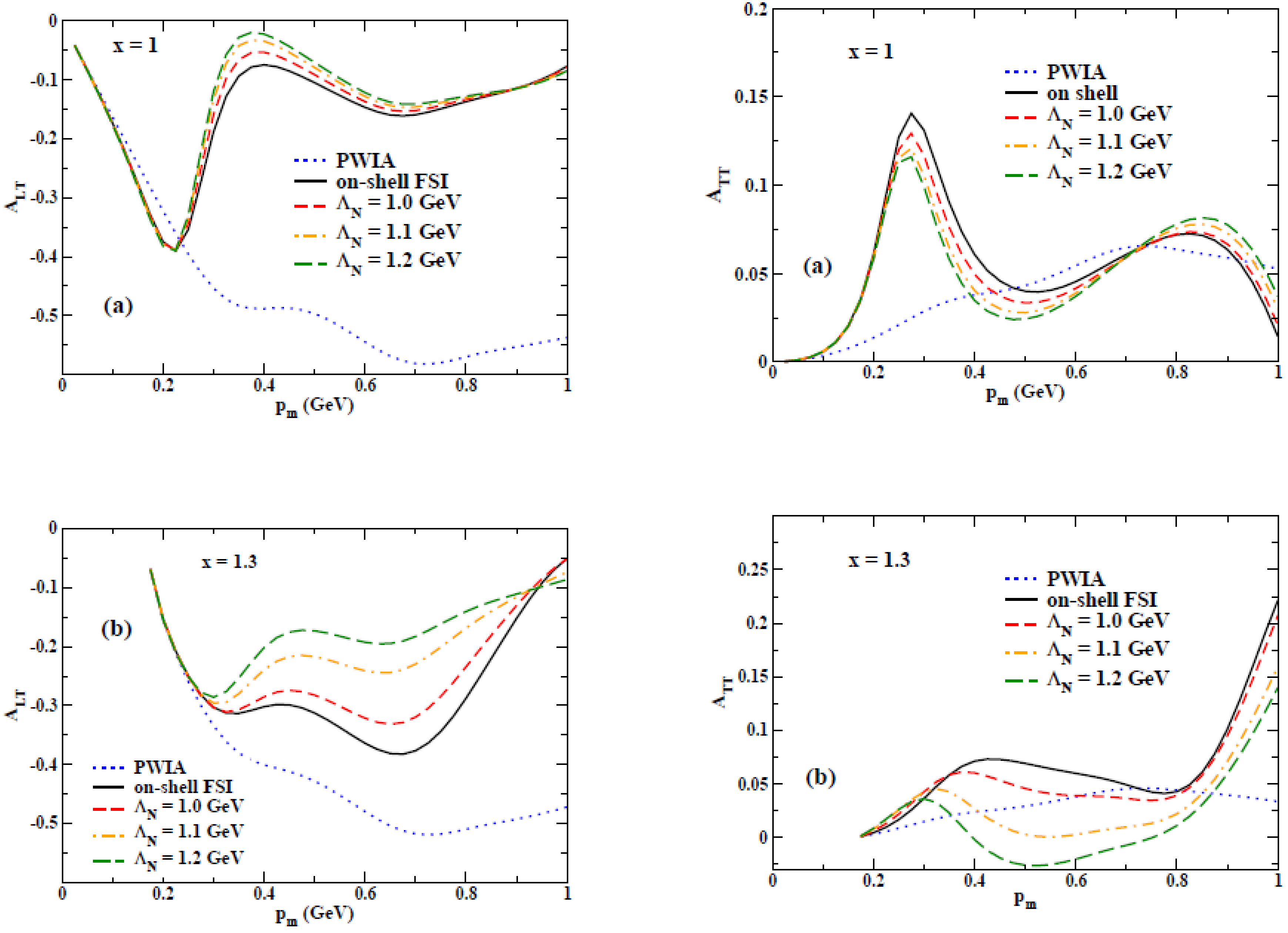
theory: SJ & JW Van Orden, Phys.Rev.C78:014007, 2008

Influence of the NN amplitude



Y-axis doesn't include zero, sorry!

Off-shell FSI Influence, Uncertainties



Summary: Unpolarized Targets

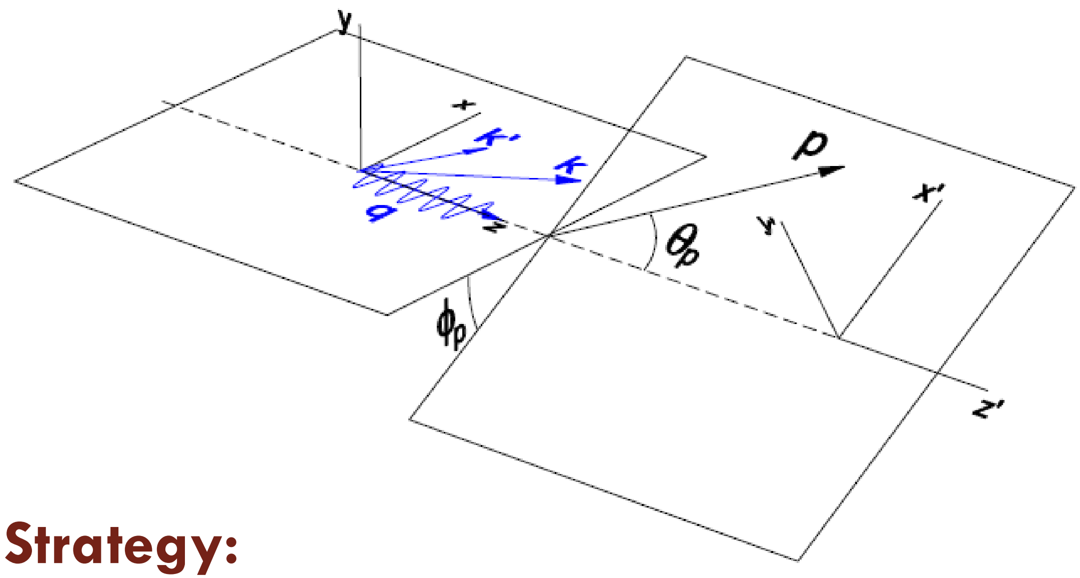
- Calculation with full NN scattering amplitude employed
  - Spin-dependent terms are important
  - Uncertainty introduced due to cut-off for off-shell FSI and prescription for positive energy off-shell FSI
  - NN amplitudes **not** available for all Jefferson Lab (Jlab) kinematics

Polarized Deuteron Targets

- deuteron has spin 1, M_J = -1, 0, +1
- deuteron can be **vector** polarized: $n_+ - n_-$

  or **tensor** polarized: $n_+ + n_- - 2n_0$
- **polarization axis**
  - theorist's choice: along the three-momentum transfer $\vec{q}$
  - experimentalist's choice: along the beam, along …

    SJ & Van Orden, PRC 80 054001 (2009)



Strategy:

1) define reduced responses in the **hadron plane**, this makes any Φ_p dependence **explicit**

2) use a **density matrix** to handle any type of deuteron polarization, e.g. T_{10} and T_{20}

3) **rotate** the density matrix to accommodate a **polarization axis** along the beam (or any other direction)

1) Define **reduced responses** in the **hadron plane**, this makes any **Φ_p dependence explicit**:

$$
\begin{aligned}
R_L(\overline{D}) &= \overline{R}_L^{(I)}(\overline{D}) \\
R_T(\overline{D}) &= \overline{R}_T^{(I)}(\overline{D}) \\
R_{TT}(\overline{D}) &= \overline{R}_{TT}^{(I)}(\overline{D}) \cos 2\phi_p + \overline{R}_{TT}^{(II)}(\overline{D}) \sin 2\phi_p \\
R_{LT}(\overline{D}) &= \overline{R}_{LT}^{(I)}(\overline{D}) \cos \phi_p + \overline{R}_{LT}^{(II)}(\overline{D}) \sin \phi_p \\
R_{LT'}(\overline{D}) &= \overline{R}_{LT'}^{(I)}(\overline{D}) \sin \phi_p + \overline{R}_{LT'}^{(II)}(\overline{D}) \cos \phi_p \\
R_{T'}(\overline{D}) &= \overline{R}_{T'}^{(II)}(\overline{D})
\end{aligned}
$$

$$
\begin{aligned}
\overline{R}_L^{(I)}(\overline{D}) &= \sum_i \overline{R}_L^{(I)}(\tau_i^{(I)})\overline{T}_i^{(I)} = \overline{w}_{00}(\overline{D}) \\
\overline{R}_T^{(I)}(\overline{D}) &= \sum_i \overline{R}_T^{(I)}(\tau_i^{(I)})\overline{T}_i^{(I)} = \overline{w}_{1,1}(\overline{D}) + \overline{w}_{-1,-1}(\overline{D}) \\
\overline{R}_{TT}^{(I)}(\overline{D}) &= \sum_i \overline{R}_{TT}^{(I)}(\tau_i^{(I)})\overline{T}_i^{(I)} = 2\Re(\overline{w}_{1,-1}(\overline{D})) \\
\overline{R}_{TT}^{(II)}(\overline{D}) &= \sum_i \overline{R}_{TT}^{(II)}(\tau_i^{(II)})\overline{T}_i^{(II)} = 2\Im(\overline{w}_{1,-1}(\overline{D})) \\
\overline{R}_{LT}^{(I)}(\overline{D}) &= \sum_i \overline{R}_{LT}^{(I)}(\tau_i^{(I)})\overline{T}_i^{(I)} = -2\Re(\overline{w}_{01}(\overline{D}) - \overline{w}_{0-1}(\overline{D})) \\
\overline{R}_{LT}^{(II)}(\overline{D}) &= \sum_i \overline{R}_{LT}^{(II)}(\tau_i^{(II)})\overline{T}_i^{(II)} = 2\Im(\overline{w}_{01}(\overline{D}) + \overline{w}_{0-1}(\overline{D})) \\
\overline{R}_{LT'}^{(I)}(\overline{D}) &= \sum_i \overline{R}_{LT'}^{(I)}(\tau_i^{(I)})\overline{T}_i^{(I)} = 2\Im(\overline{w}_{01}(\overline{D}) - \overline{w}_{0-1}(\overline{D})) \\
\overline{R}_{LT'}^{(II)}(\overline{D}) &= \sum_i \overline{R}_{LT'}^{(II)}(\tau_i^{(II)})\overline{T}_i^{(II)} = -2\Re(\overline{w}_{01}(\overline{D}) + \overline{w}_{0-1}(\overline{D})) \\
\overline{R}_{T'}^{(II)}(\overline{D}) &= \sum_i \overline{R}_{T'}^{(II)}(\tau_i^{(II)})\overline{T}_i^{(II)} = \overline{w}_{1,1}(\overline{D}) - \overline{w}_{-1,-1}(\overline{D}),
\end{aligned}
$$

The interference reduced responses are either real or imaginary parts of the hadronic tensor.

$$
\begin{aligned}
\overline{T}_i^{(I)} &\in \{U, \Im(\overline{T}_{11}), \overline{T}_{20}, \Re(\overline{T}_{21}), \Re(\overline{T}_{22})\} \\
\overline{T}_i^{(II)} &\in \{\overline{T}_{10}, \Re(\overline{T}_{11}), \Im(\overline{T}_{21}), \Im(\overline{T}_{22})\}
\end{aligned}
$$

2) Use a **density matrix** to handle any type of deuteron polarization, e.g. T_{10} and T_{20}

$$\rho = \frac{1}{3}\begin{pmatrix} 1+\sqrt{\frac{3}{2}}T_{10}+\frac{1}{\sqrt{2}}T_{20} & -\sqrt{\frac{3}{2}}(T_{11}^*+T_{21}^*) & \sqrt{3}\,T_{22}^* \\ -\sqrt{\frac{3}{2}}(T_{11}+T_{21}) & 1-\sqrt{2}\,T_{20} & -\sqrt{\frac{3}{2}}(T_{11}^*-T_{21}^*) \\ \sqrt{3}\,T_{22} & -\sqrt{\frac{3}{2}}(T_{11}-T_{21}) & 1-\sqrt{\frac{3}{2}}T_{10}+\frac{1}{\sqrt{2}}T_{20} \end{pmatrix}$$

T_{ij}: tensor polarization coefficients, experimental input

$$w_{\lambda'_\gamma,\lambda_\gamma}(D) = \sum_{s_1,s_2,\lambda_d,\lambda'_d} \langle \boldsymbol{p}_1 s_1; \boldsymbol{p}_2 s_2; (-)|\, J_{\lambda'_\gamma}\, |\boldsymbol{P}\lambda'_d\rangle^* \langle \boldsymbol{p}_1 s_1; \boldsymbol{p}_2 s_2; (-)|\, J_{\lambda_\gamma}\, |\boldsymbol{P}\lambda_d\rangle\, \rho_{\lambda_d\lambda'_d}$$

hadronic tensor, with the density matrix

3) **rotate** the density matrix

$$\overline{\rho}_{\lambda_d\lambda'_d} = \sum_{\Lambda\Lambda'} D^1_{\lambda_d\Lambda}(-\phi_p,\theta_{kq},0)D^1_{\lambda'_d\Lambda'}(-\phi_p,\theta_{kq},0)\tilde{\rho}^D_{\Lambda\Lambda'}$$

Target Polarization Observables (exclusive!)

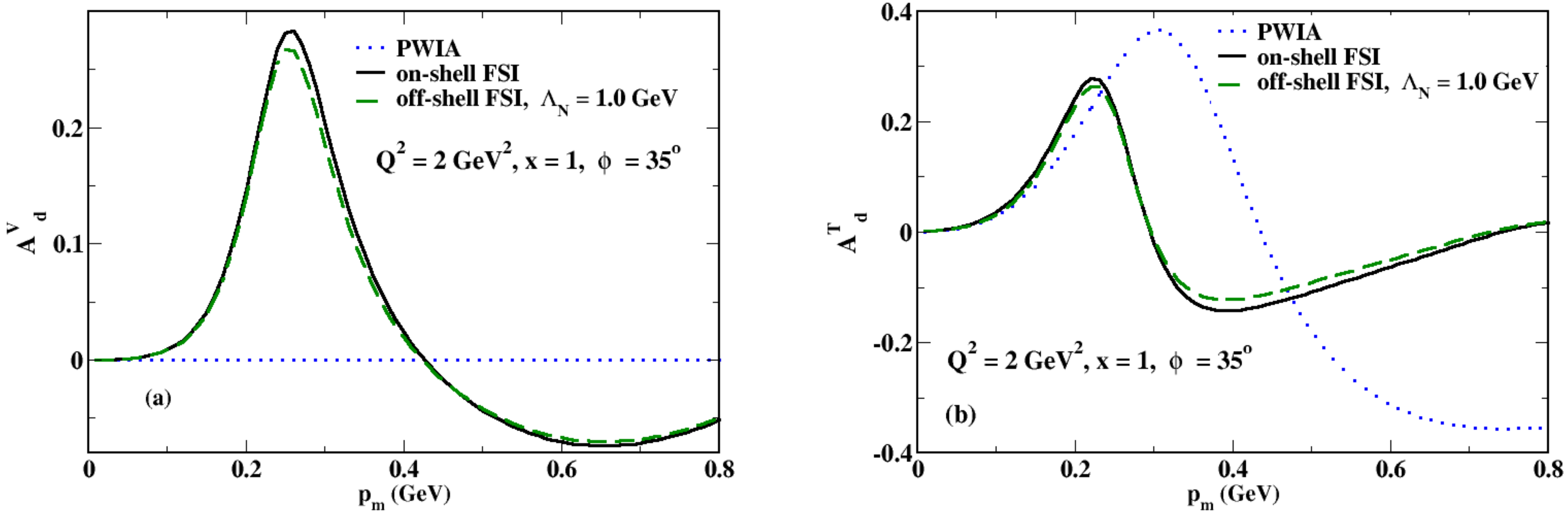
vector asym.: $$A_d^V = \frac{v_L R_L(\widetilde{T}_{10}) + v_T R_T(\widetilde{T}_{10}) + v_{TT} R_{TT}(\widetilde{T}_{10}) + v_{LT} R_{LT}(\widetilde{T}_{10})}{\widetilde{T}_{10}\Sigma}$$

tensor asym.: $$A_d^T = \frac{v_L R_L(\widetilde{T}_{20}) + v_T R_T(\widetilde{T}_{20}) + v_{TT} R_{TT}(\widetilde{T}_{20}) + v_{LT} R_{LT}(\widetilde{T}_{20})}{\widetilde{T}_{20}\Sigma}$$

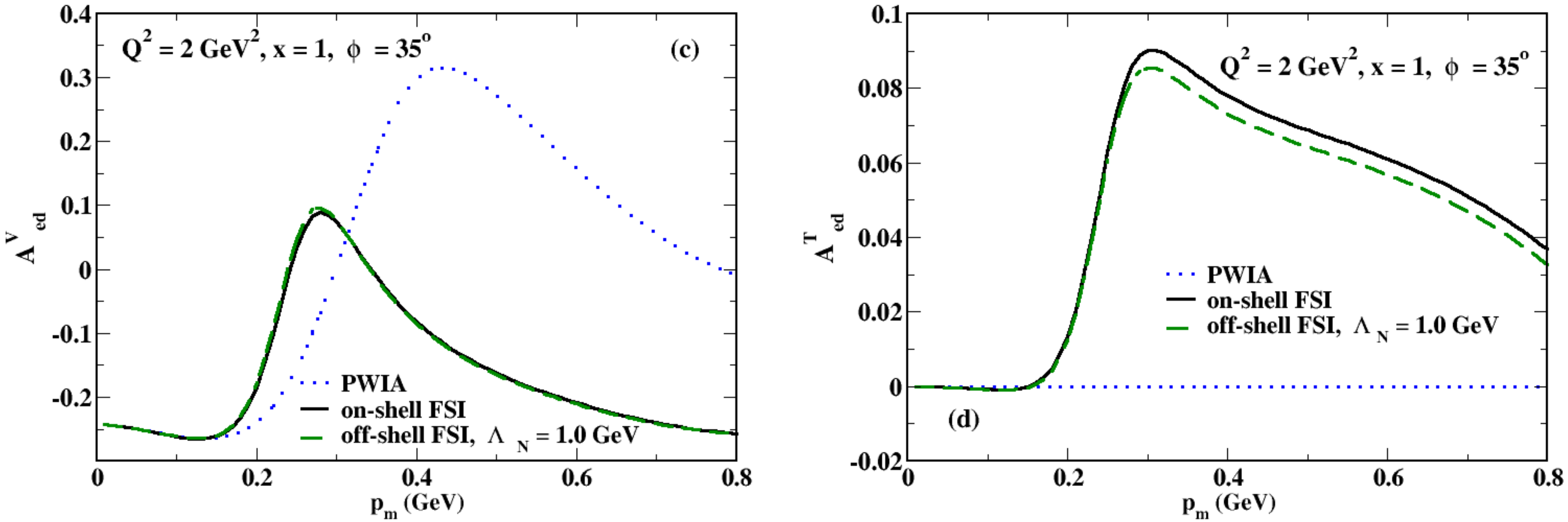
beam vector asym.: $$A_{ed}^V = \frac{v_{LT'} R_{LT'}(\widetilde{T}_{10}) + v_{T'} R_{T'}(\widetilde{T}_{10})}{\widetilde{T}_{10}\Sigma}$$

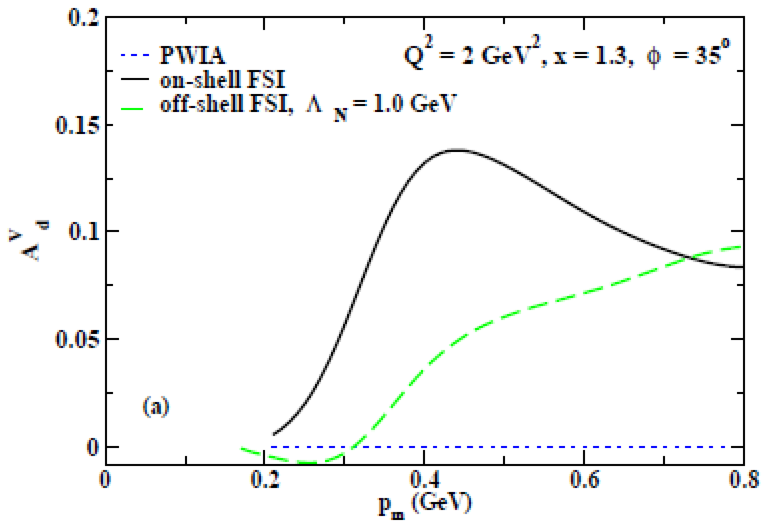
beam tensor asym.: $$A_{ed}^T = \frac{v_{LT'} R_{LT'}(\widetilde{T}_{20}) + v_{T'} R_{T'}(\widetilde{T}_{20})}{\widetilde{T}_{20}\Sigma}$$

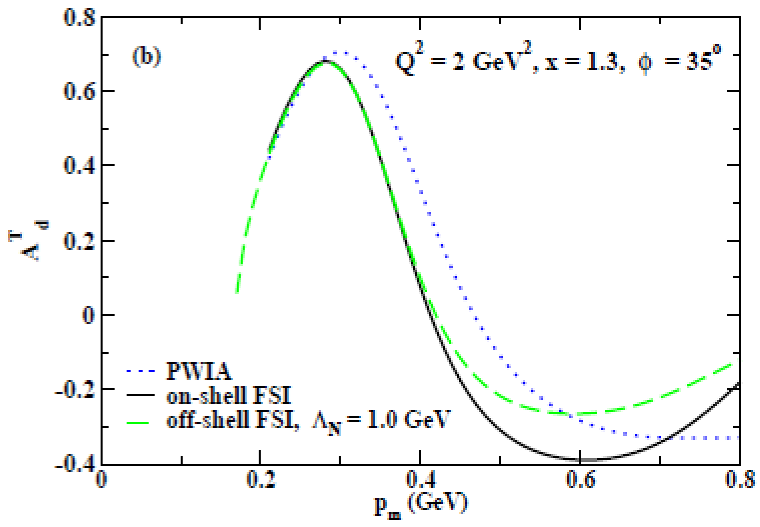
denominator, unpolarized: $$\Sigma = v_L R_L(U) + v_T R_T(U) + v_{TT} R_{TT}(U) + v_{LT} R_{LT}(U)$$



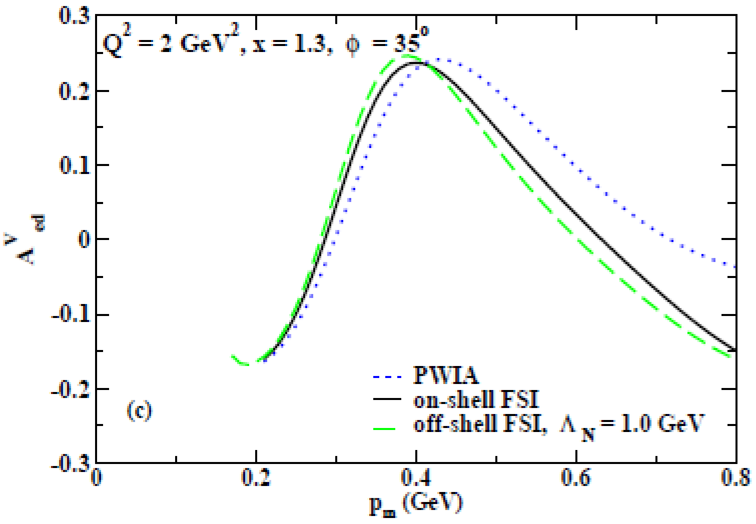
Momentum Distributions x = 1, Q^2 = 2 GeV^2

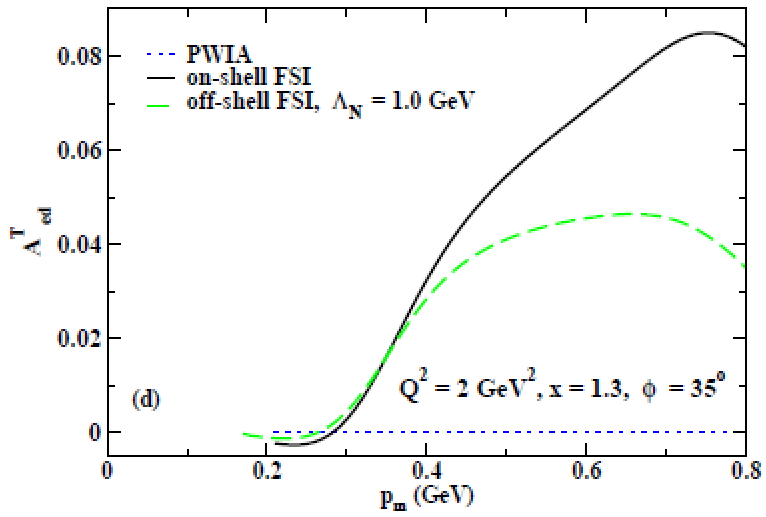


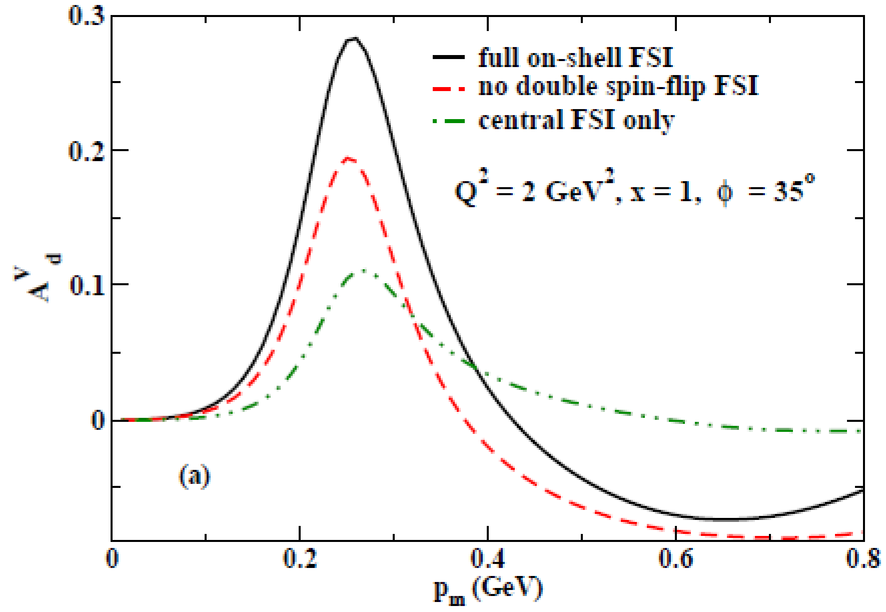


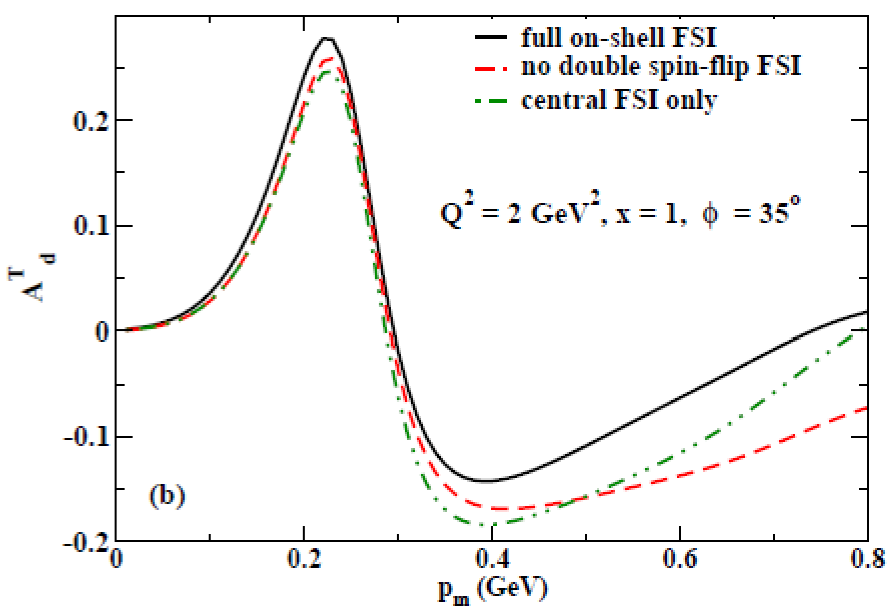


Momentum Distributions $x = 1.3$, $Q^2 = 2$ GeV^2

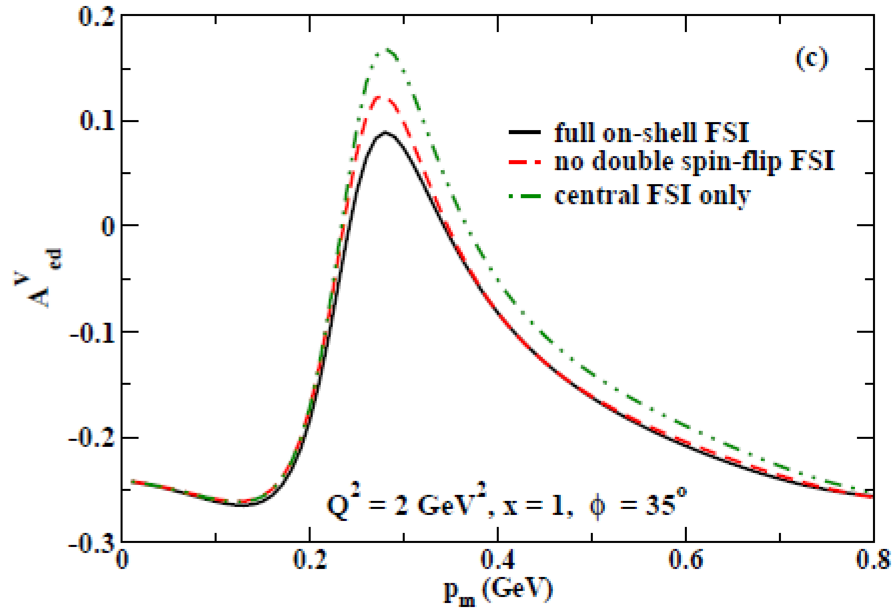


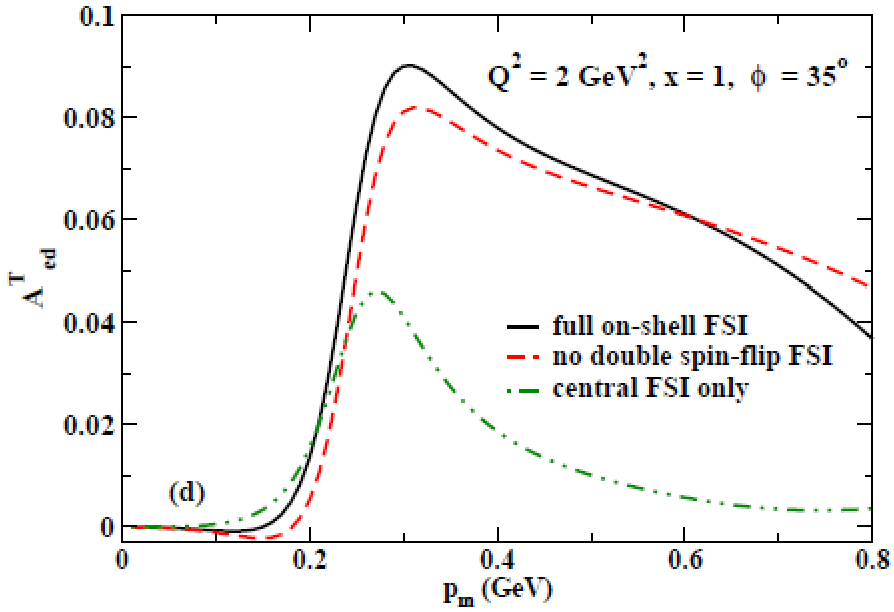






Role of Spin-Dependent FSIs: Single Spin Flip and Double Spin Flip



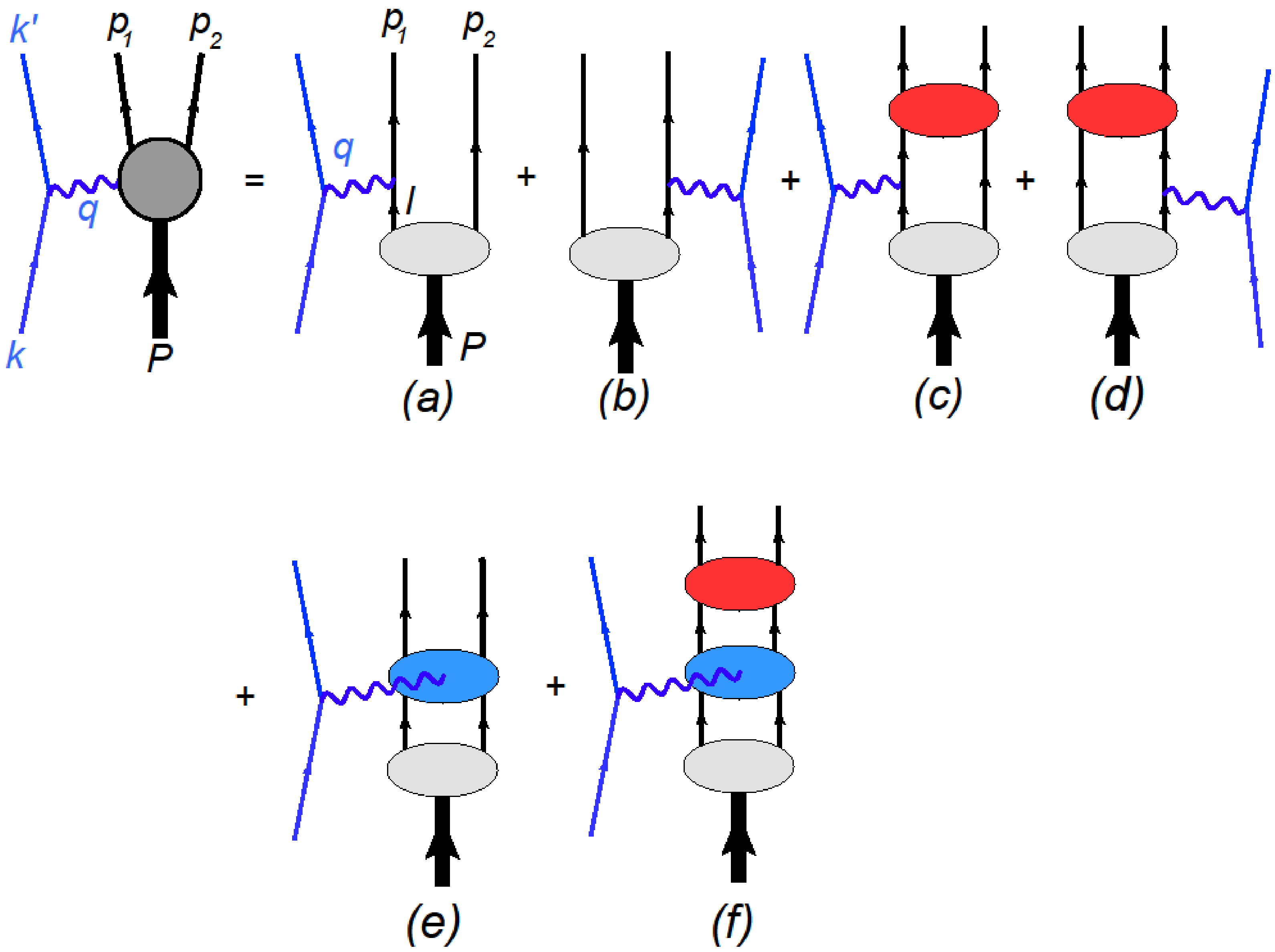


Summary: Polarized Target

- four asymmetries have been considered, two each are similar
- For our model, FSIs are hugely important, **just central FSIs are not enough – even in the quasi-elastic (x = 1) region**
- FSIs and ground state information are entangled
- Wishlist: measurement of A^{V}_{d} or A^{T}_{ed} at larger x

Role of Inputs into the Model

- NN scattering amplitude paramterizations
- Nucleon form factor parametrizations
- Deuteron wave functions


k'
p1
p2
q
k
P
=
q
l
P
+
+
+
(a)
(b)
(c)
(d)
+
+
(e)
(f)

Factorization – why A^T_d is so cool I

If we neglect p waves and only consider PWIA (graph (a)), then PRC 90, 064006 (2014) and PRC 95, 044001 (2017):

$$(A_d^T)_{factored} = \frac{n_+^{20}(\boldsymbol{p})T_{20} + \Re(n_+^{21}(\boldsymbol{p}))\Re(T_{21}) + \Re(n_+^{22}(\boldsymbol{p}))\Re(T_{22})}{n_+^{00}(p)\widetilde{T}_{20}}$$

$$= \frac{n_+^{20}(\boldsymbol{p})\frac{1}{4}(1+3\cos 2\theta_{kq}) + \Re(n_+^{21}(\boldsymbol{p}))\sqrt{\frac{3}{8}}\sin 2\theta_{kq} + \Re(n_+^{22}(\boldsymbol{p}))\sqrt{\frac{3}{32}}(1-\cos 2\theta_{kq})}{n_+^{00}(p)}$$

$$= -\sqrt{\frac{2\pi}{5}}\frac{2\Psi_1^2(p) - \Psi_3^2(p)}{\Psi_1^2(p) + \Psi_3^2(p)}\Xi(\theta_m, \phi, \theta_{kq})$$

Factorization – why A^T_d is so cool II

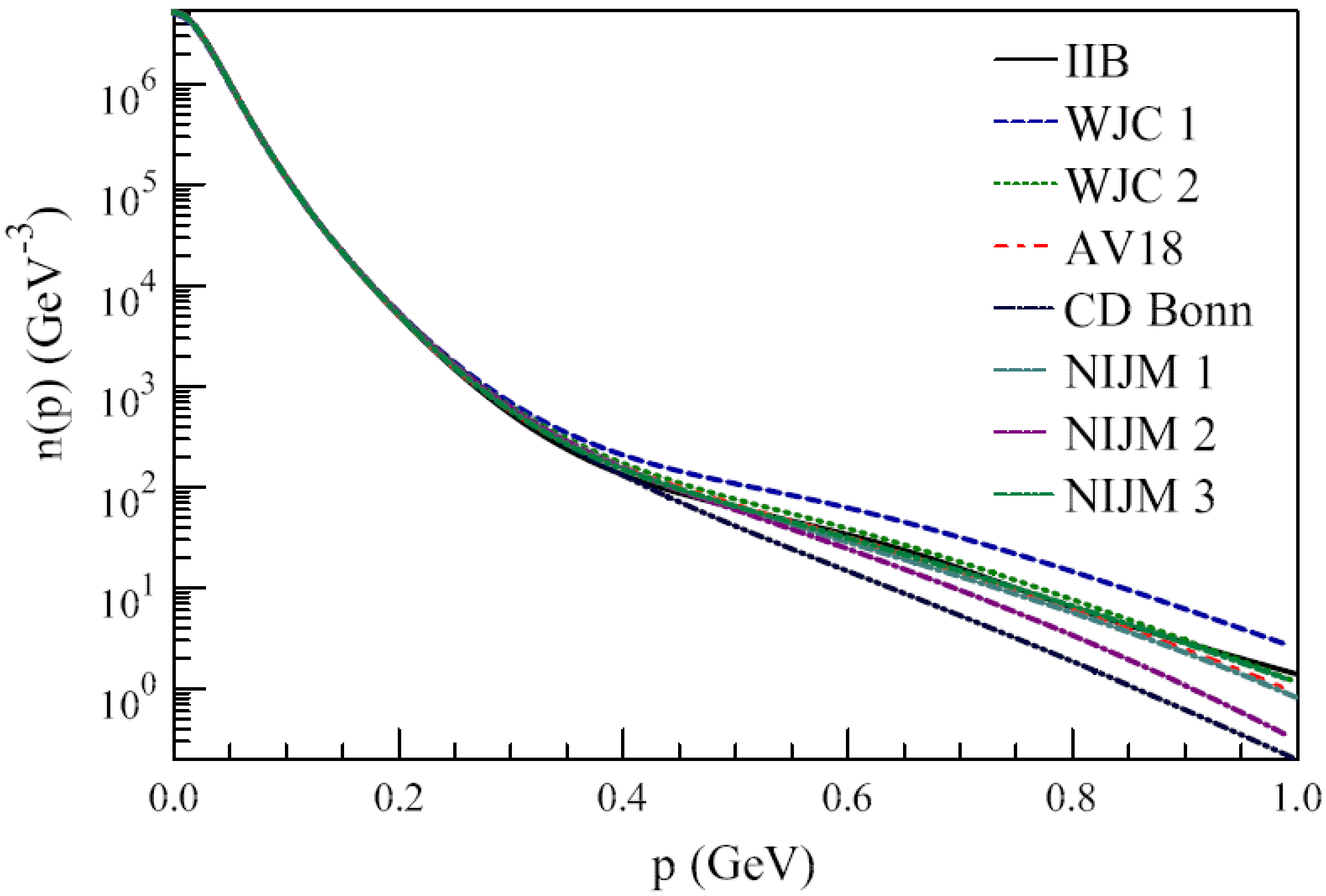
Define a reduced tensor asymmetry a^T_d

$$a_d^T = \frac{A_d^T}{\Xi(\theta_m, \phi, \theta_{kq})}$$

If we neglect p waves and only consider PWIA (graph (a)), then (PRC 95, 044001 (2017):

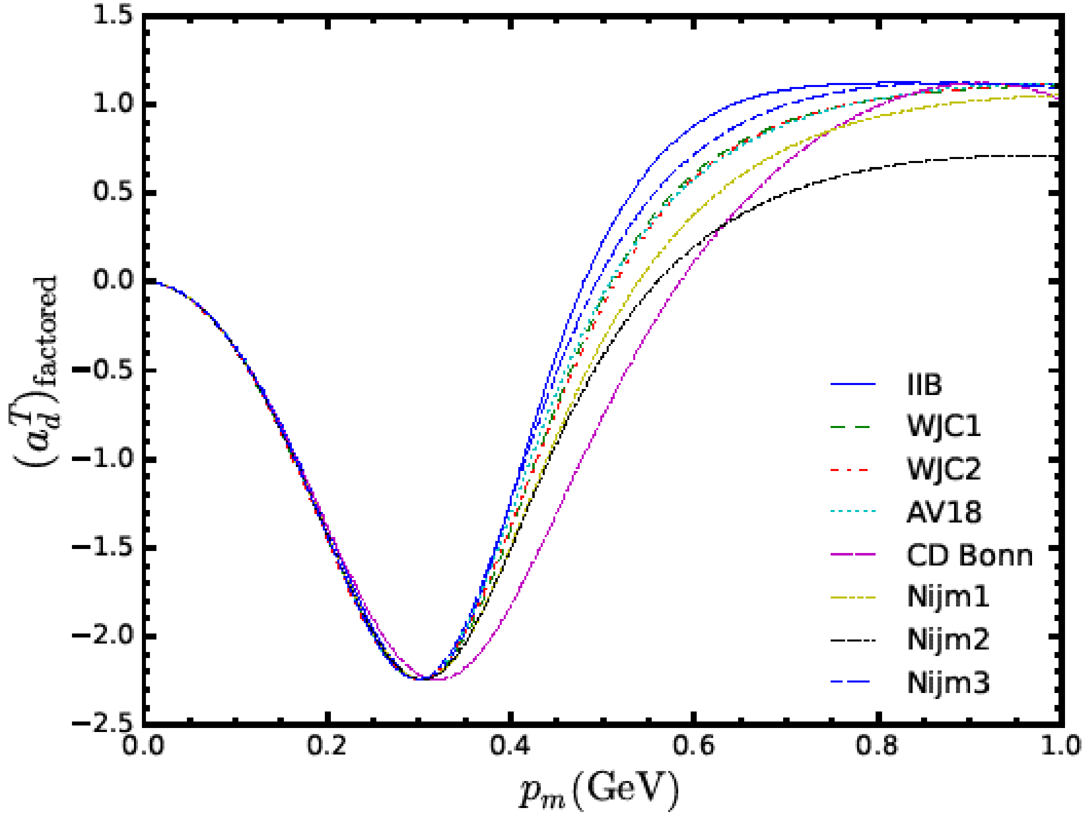
$$(a_d^T)_{factored} = -\sqrt{\frac{2\pi}{5}} \frac{w(p)(2\sqrt{2}u(p) + w(p))}{u^2(p) + w^2(p)}$$

Wave Functions

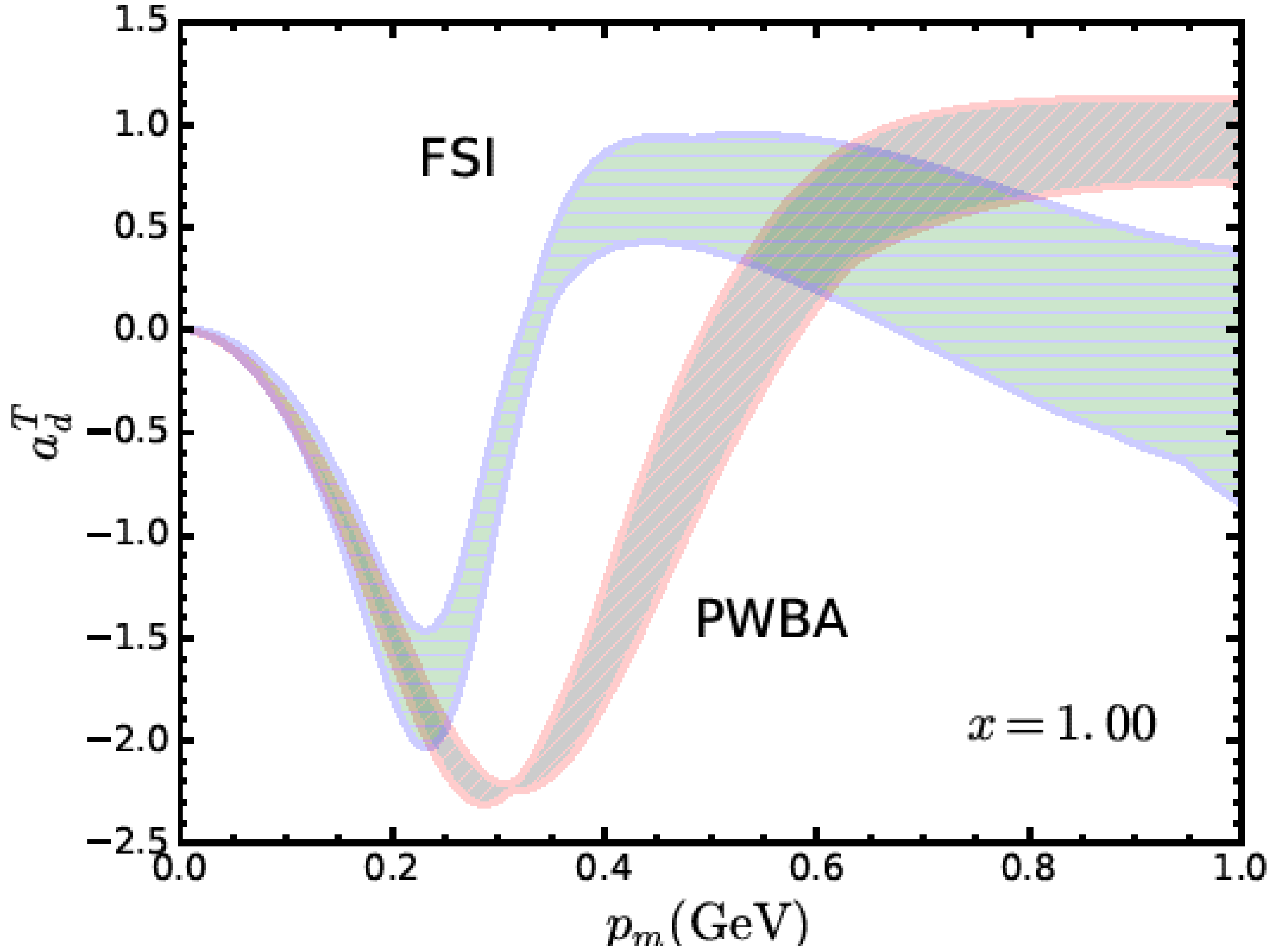


D-wave probabilities and a^T_d reduced

	s wave	d wave	triplet p wave	singlet p wave
IIB	94.82%	5.11%	0.06%	0.01%
WJC 1	92.33%	7.34%	0.11%	
WJC 2	93.60%	6.38%	0.01%	
AV18	94.24%	5.76%	0.00%	
CD Bonn	95.15%	4.85%	0.00%	
NIJM 1	94.25%	5.75%	0.00%	
NIJM 2	94.32%	5.68%	0.00%	
NIJM 3	94.35%	5.65%	0.00%	



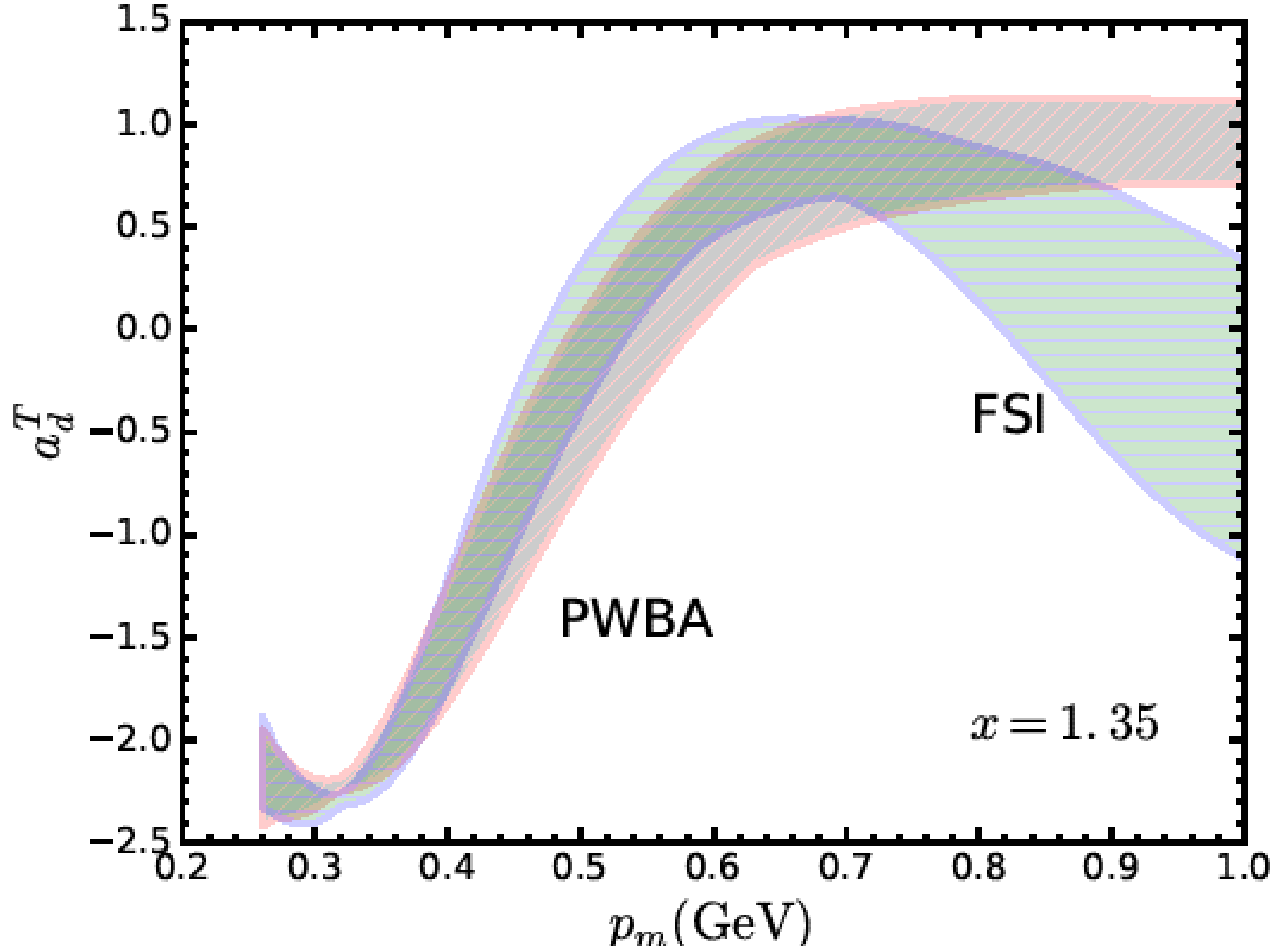
Calculations with FSI and in BA



Error bands include

- Nucleon Form Factor params (3)
- Wave functions (8)
- FSI with SAID NN and with Regge model parametrization (2)

Calculations with FSI and in BA



Summary

- In PWIA, neglecting p-waves, A^T_d factorizes, looks as if d-wave information is accessible
- Factorized a^T_d is NOT proportional to d wave content of the wave function
- theory calculations (should) come with a theory error bar/envelope – apart from actual model issues
- Calculations with FSI in Born approximation clearly break the factorization