Testing light-cone dynamics of deuteron and nuclear core using tensor polarized deuteron: $T_{20}=A_{zz}$,...

Mark Strikman, PSU

* talk is mostly based on the studied we did between 1976 and 1983 + (e,e') analysis of 1993

Tensor polarized Solid Target workshop Jlab, Jan 12, 2014

Outline

Emergence of light cone dominance at high energies Deuteron - LC - nonrelativistic correspondence Polarized deuteron ***** disentangling S- and D-wave in $e\vec{D} \rightarrow epn, \ \vec{D}(e, e')$ ***** Further tests of D spin structure: $\vec{e}\vec{D} \rightarrow epn$, * Collider eD: Tagging & tensor EMC effect

To resolve short-range structure of nuclei on the level of nucleon/hadronic constituents one needs processes which transfer to the nucleon constituents both energy and momentum larger than the scale of the NN short range correlations $q_0 \geq 1 GeV, \vec{q} \geq 1 GeV$

 \Rightarrow

Need to treat the processes in the relativistic domain. The price to pay is a need to treat the nucleus wave function using light-cone quantization - - One cannot use (at least in a simple way) nonrelativistic description of nuclei.

⇒ High energy process develops along the light cone.



Similar to the perturbative QCD the amplitudes of the processes are expressed through the wave functions on the light cone. *Note: in general no benefit for using LC for low energy processes.* LC quantization is uniquely selected in high energy processes if one tries to express cross section through elementary amplitudes near energy shell.

Consider the break up of the deuteron in the impulse approximation: $h+D\rightarrow X+N$, for $E_h\rightarrow\infty$



In quantum mechanical treatment energy in the $D \rightarrow NN$ vertex is not conserved. As a result

$$\Delta \equiv (s_{in} - s_f) \rightarrow 2 E_h (2\sqrt{m_N^2 + p_N^2 - m_D})_{\mid E_h \rightarrow \infty}$$

is infinite at high energies. Amplitude is far off energy shell.

In case of LC quantization along reaction axis

$$\Delta = (p_{\rm NN} + p_{\rm h})^2 - (p_{\rm D} + p_{\rm h})^2 = M_{\rm NN}^2 - M_{\rm D}^2 + (p_{\rm h})_+ (p_{\rm NN} - p_{\rm D})_- + (p_{\rm h})_- (p_{\rm NN} - p_{\rm D})_+$$
$$= M_{\rm NN}^2 - M_{\rm D}^2 + \frac{1}{2} (m_{\rm h}^2 / E_{\rm h}) (M_{\rm NN}^2 / M_{\rm D} - M_{\rm D}) \simeq M_{\rm NN}^2 - M_{\rm D}^2$$

Here M^2_{NN} is invariant mass squared of the two nucleon system

 Δ is fine and hence amplitude is close to the mass shell

Requirement of finite Δ uniquely fixes quantization axis for the high energy limit to be according to LC prescription

Onset of LC dominance in (e,e')

Consider example of high Q^2 (e,e') process at fixed large x >1 in the many nucleon approximation for the nucleus



The on-shell condition for the produced nucleon

$$(p^{\rm int} + q)^2 = m^2$$

$$q_{\pm} = q_0 \pm q_3$$

for any vector
$$a_{\mu} = (a_+, a_-, a_t)$$
,

Substituting

$$P_{+}^{A} = M_{A}^{2}/P_{-}^{A},$$

$$p_{+}^{\rm rec} = \frac{(m^{\rm rec})^{2} + p_{t}^{2}}{(1 - \alpha/A)P^{+}A_{-}},$$

and

$$(P^A - p^{\mathrm{rec}})_- = \frac{\alpha}{A}P^A_-,$$

we obtain

$$\tilde{m}^2 = \left(M_A^2 - \frac{(m^{\rm rec})^2 + p_t^2}{(A-\alpha)/A}\right)\frac{\alpha}{A} - p_t^2,$$

where m^{rec} is the mass of the recoiling (A-1) nucleus.

$$\tilde{m}^{2} + q_{+}p_{-}^{\text{int}} + q_{-}p_{+}^{\text{int}} + q^{2}$$

$$= \tilde{m}^{2} + q_{+}\frac{M_{A}}{A}\alpha + q_{-}\left(\frac{\tilde{m}^{2} + p_{t}^{2}}{\alpha(M_{A}/A)}\right) + q^{2} = m^{2}$$
Use the nucleus rest frame
$$P_{+}^{A} = P_{-}^{A} = M_{A}$$

$$\Rightarrow \frac{\partial \alpha}{\partial \tilde{m}^{2}} = -\left(\frac{1 + (q_{-}/\alpha)(M_{A}/A)}{(q_{+}M^{A}/A) - [q_{-}(\tilde{m}^{2} + p_{t}^{2})]/\alpha^{2}M_{A}/A}\right)$$

$$\longrightarrow 0 \text{ for large Q, fixed x, } \propto 1/q_{+}$$

In high energy limit the cross section depends only on the spectral function integrated over all variables but a - light-cone dominance, in particular no depend on the mass of the recoil system. Relevant quantity light-cone nucleon density matrix.

$$\frac{\sigma_{eA}(x,Q^2)}{\sigma_{eD}(x,Q^2)} = \frac{\rho_A(\alpha_{tn})}{\rho_D(\alpha_{tn})}$$

For intermediate Q^2 corrections can be treated by taking an average value of recoil mass. The two nucleon approximation for p_{-}^{rec} is

$$p_{-}^{rec} = m_{(A-2)*} + \frac{m^2 + p_t^2}{m(2-\alpha)} \qquad (*)$$

with Fermi motion of the pair leading to a spread of distribution over p_{-}^{rec} is but not to a significant change of $< p_{-}^{rec} >$.

"super"scaling of the (e,e') ratios in
$$\Omega_{t.n.}$$
 - Ω calculated using (*). Observed at Jlab.

For deuteron $m_{(A-2)}$ *=0

High energy processes are dominated by interactions near LCcross sections are simply expressed through LC wave functions

$$\int_{0}^{A} \alpha \rho_{\mathbf{A}}^{\mathbf{N}}(\alpha, k_{\perp}) \frac{\mathrm{d}\alpha}{\alpha} \mathrm{d}^{2} k_{\perp} = \int_{0}^{A} \rho_{\mathbf{A}}^{\mathbf{N}}(\alpha, k_{\perp}) \frac{\mathrm{d}\alpha}{\alpha} \mathrm{d}^{2} k_{\perp} \frac{\sum \alpha_{i}}{A} = A.$$

Example

$$F_{2A}(x,Q^2) = \sum_{\mathrm{N=p,n}} \int F_{2\mathrm{N}}(x/\alpha,Q^2) \rho_A^{\mathrm{N}}(\alpha,k_{\mathrm{t}}) \frac{\mathrm{d}\alpha}{\alpha} \mathrm{d}^2 k_{\mathrm{t}}.$$

Now we focus on the LC dynamics for two body case - more technical discussion

Decomposition over hadronic states could be useless if too many states are involved in the Fock representation

 $|D\rangle = |NN\rangle + |NN\pi\rangle + |\Delta\Delta\rangle + |NN\pi\pi\rangle + \dots$

Problem - we cannot use a guiding principle experience of the models of NN interactions based on the meson theory of nuclear forces - such models have a Landau pole close to mass shell and hence generate a lot of multi meson configurations. (On phenomenological level - problem with lack of enhancement of antiquarks in nuclei)

Instead, we can use the information on NN interactions at energies below few GeV and the chiral dynamics combined with the following general quantum mechanical principle - *relative magnitude of different components in the wave function should be similar to that in the NN scattering at the energy corresponding to off-shellness of the component.* Important simplification of the final states in NN interactions: direct pion production is suppressed for a wide range of energies due to chiral properties of the NN interactions:

$$\frac{\sigma(\mathrm{NN} \to \mathrm{NN}\pi)}{\sigma(\mathrm{NN} \to \mathrm{NN})} \simeq \frac{k_{\pi}^2}{16\pi^2 F_{\pi}^2}, \quad F_{\pi} = 94 \text{ MeV}$$

⇒ Main inelasticity for NN scattering for $T_P \leq I$ GeV is Δ -isobar production which is forbidden in the deuteron channel.

 $|\Delta \Delta>$ threshold is $k_N = \sqrt{m_\Delta^2 - m_N^2} \approx 800 \, MeV \, !!!$ Small parameter for inelastic effects in the deuteron WF, while relativistic effects are already significant as v/c ~l

For the nuclei where single Δ can be produced $k_N \approx 550 MeV$ Warning: Correspondence argument (WF \Leftrightarrow continuum) is not applicable for the cases when the probe interacts with rare configurations (EMC effect?) in the bound nucleons due to the presence of an additional scale

Light-cone Quantum mechanics of two nucleon system

Due to the presence of a small parameter (inelasticity of NN interactions) it makes sense to consider two nucleon approximation for the LC wave function of the deuteron.

Key point is presence of the unique matching between nonrelativistic and LC wave functions in this approximation. Proof is rather involved.

First step: include interactions which do not have two nucleon intermediate states into kernel V (like in nonrel. QM) to build a Lippman-Schwinger type (Weinberg type) equation.



The LC "energy denominator" is $1/(p_{n_+} - p_{f_+})$

Using explicit expression for the propagator in terms of the LC variables and using corresponding expressions for the two-body phase volume on LC we obtain:

$$T(\alpha_i, k_{it}, \alpha_f, k_{ft}) = V(\alpha_i, k_{it}, \alpha_f, k_{ft}) + \int V(\alpha_i, k_{it}, \alpha', k'_t) \frac{d\alpha'}{4\alpha'(1 - \alpha')} \frac{d^2k'_t}{(2\pi)^3}$$

$$\times \frac{T(\alpha', k'_t, \alpha_f, k_{ft})}{[(m^2 + {k'_t}^2)/\alpha'(1 - \alpha') - (m^2 + k_{ft}^2)/\alpha_f(1 - \alpha_f)]/2}$$

Second step: Impose condition that master equation should lead to the Lorentz invariance of the on-energy-shell amplitude of NN scattering

Introduce three-vector $\vec{k} = (k_3, k_t)$ with

$$\alpha = \frac{\sqrt{m^2 + k^2} + k_3}{2\sqrt{m^2 + k^2}}$$

Invariant mass of two nucleon system is

$$M_{NN}^2 = \frac{m^2 + k_t^2}{\alpha(1 - \alpha)} = 4m^2 + 4k^2$$

 $T(k_{\rm i}, k_{\rm f}, k_{\rm i3}, k_{\rm f3}) = V(k_{\rm i}, k_{\rm f}, k_{\rm i3}, k_{\rm f3}) \\ + \int V(k_{\rm i}, k', k_{\rm i3}, k'_{\rm 3}) \frac{\mathrm{d}^3 k'}{\sqrt{k'^2 + m^2}} \frac{1}{4(2\pi)^3} \frac{T(k', k_{\rm f}, k'_{\rm 3}, k_{\rm f3})}{k'^2 - k_{\rm f}^2}.$

On-mass-shell

$$T(k, k_3, k_{\rm f}, k_{\rm f3}) = T(k^2, k_{\rm f}^2, k_{\rm f})$$

$$V(k, k_3, k_{\rm f}, k_{\rm f3}) = V(k^2, k_{\rm f}^2, kk_{\rm f})$$

For rotational invariance of T it is sufficient that the same relation is satisfied for V off-mass-shell. The proof that this condition is also necessary is much more complicated (FS + Mankievich 91). At the same time it is obvious that it would be very difficult to satisfy the highly nonlinear equation for the on-shell amplitude if this condition were violated.

The proof uses methods of complex angular momentum plane and assumption that the amplitude is decreases sufficiently fast with momentum transfer (actually rather slow decrease was sufficient).

$$T(k,k_{\rm f}) = V(k,k_{\rm f}) + \int V(k,k') \frac{{\rm d}^3 k'}{4\sqrt{k'^2 + m^2}} \frac{1}{k'^2 - k_{\rm f}^2} \frac{1}{(2\pi)^3} T(k',k_{\rm f})$$

Very similar structure for the equation for the scattering amplitude in NR QM and for LC. If a NR potential leads to a good description of phase shifts the same is true for its LC analog. Hence simple approximate relation for LC and NR two nucleon wave function Spin zero /unpolarized case

rescale $\alpha \rightarrow 2\alpha$ so that $0 < \alpha < 2$ with $\alpha = 1$ corresponds to a nucleon at rest (more convenient when generalizing to A > 2)

$\begin{aligned} & \int \Psi_{NN}^2 \left(\frac{m^2 + k_t^2}{\alpha(2 - \alpha)} \right) \frac{d\alpha d^2 k_t}{\alpha(2 - \alpha)} = 1 & \int \phi^2(k) d^3 k = 1 \\ & \Psi_{NN}^2 \left(\frac{m^2 + k_t^2}{\alpha(2 - \alpha)} \right) = \frac{\phi^2(k)}{\sqrt{(m^2 + k^2)}} \end{aligned}$

Similarly for the spin I case we have two invariant vertices as in NR theory: $\psi^D_\mu \epsilon^D_\mu = \bar{u}(p_1) \left(\gamma_\mu G_1(M_{NN}^2) + (p_1 - p_2)_\mu G_2(M_{NN}^2)\right) u(-p_2) \epsilon^D_\mu$

hence there is a simple connection to the S- and D- wave NR WF of D

For two body system in two nucleon approximation the biggest difference between NR and virtual nucleon approximation and LC is in the relation of the wave function and the scattering amplitude

Let us illustrate this for the high energy deuteron break up $h(e) + D \rightarrow X + N$ in the impulse approximation with nucleon been in the deuteron fragmentation region - spectator contribution.

For any particle, b, in the final state in the target fragmentation region the light cone fractions are conserved under longitudinal boosts

$$\alpha_{\rm b}/2 = (E_{\rm b} + p_{\rm bZ})/(E_{\rm D} + p_{\rm DZ})$$

Hence in the rest frame

$$2 > \alpha_{\rm b} \equiv \left(\sqrt{m_{\rm b}^2 + p_{\rm b}^2} - p_{\rm bZ}\right) / M_{\rm D}$$



$$\frac{\mathrm{d}\sigma^{\mathrm{D}+\mathrm{h}\to\mathrm{N}+\cdots}}{(\mathrm{d}\alpha/\alpha)\mathrm{d}^{2}p_{\perp}} = \sigma^{\mathrm{h}\mathrm{N}}_{\mathrm{inel.}}[(2-\alpha)s_{NN}] \cdot \frac{[U^{2}(k)+W^{2}(k)]}{(2-\alpha)}\sqrt{k^{2}+m^{2}}$$
LC imp.approx.

 $\frac{\mathrm{d}\sigma^{\mathrm{D}+\mathrm{h}\to\mathrm{N}+\cdots}}{(\mathrm{d}\alpha/\alpha)\mathrm{d}^2 p_{\perp}} = \sigma^{\mathrm{hN}}_{\mathrm{inel.}}[(2-\alpha)s_{NN}] \cdot (2-\alpha)[U^2(p) + W^2(p)]\sqrt{p^2 + m^2}$ $| \mathsf{NR imp.approx.}$

LC nucleon: nonlinear relation between internal momentum k and observed momentum p (see next slide). Asymptotic behavior at $\alpha \rightarrow 2$ is determined by WF at $k \rightarrow \infty$. Similar to particle physics.

NR/Virtual nucleon: observed momentum is the same as in the WF, asymptotic at $\alpha \rightarrow 2, k_t=0$, is determined by WF at finite momentum 0.75 m, and has the same (2- α) dependence on α .



PHYSICS LETTERS

NEW DIRECT WAY OF CHECKING THE NUCLEAR CORE HYPOTHESIS IN INCLUSIVE HADRON SCATTERING OFF THE POLARIZED DEUTERON

L.L. FRANKFURT and M.I. STRIKMAN



The best way to look for the difference between LC and NR/Virtual nucleon seems to be scattering off the polarized deuteron

$$\frac{\mathrm{d}\sigma(\mathbf{e} + \mathbf{D}_{\Omega} \to \mathbf{e} + \mathbf{N} + \mathbf{X})}{(\mathrm{d}\alpha/\alpha) \,\mathrm{d}^{2}p_{\mathrm{t}}} / \frac{\mathrm{d}\sigma(\mathbf{e} + \mathbf{D} \to \mathbf{e} + \mathbf{N} + \mathbf{X})}{(\mathrm{d}\alpha/\alpha) \,\mathrm{d}^{2}p_{\mathrm{t}}}$$
$$= 1 + \left(\frac{3k_{i}k_{j}}{k^{2}} \,\Omega_{ij} - 1\right) \frac{\frac{1}{2}w^{2}(k) + \sqrt{2}u(k)w(k)}{u^{2}(k) + w^{2}(k)} \equiv P(\Omega, k)$$

 Ω is the spin density matrix of the deuteron, $\mathrm{Sp}\Omega=1$

Consider

$$R = T_{20} = \left[\frac{1}{2}(\sigma_{+} - \sigma_{-}) - \sigma_{0}\right] / \langle \sigma \rangle$$

$$R(p_{\rm s}) = \frac{3(k_{\rm t}^2/2 - k_z^2)}{k^2} \frac{u(k)w(k)\sqrt{2} + \frac{1}{2}w^2(k)}{u^2(k) + w^2(k)}$$

trivial angular dependence for fixed p

$$R^{\text{nonrel}}(p_{\text{s}}) = \frac{3(p_{\text{t}}^2/2 - p_z^2)}{p^2} \frac{u(p)w(p)\sqrt{2} + \frac{1}{2}w^2(p)}{u^2(p) + w^2(p)}$$





 p_s dependence of the(e,e'p) tensor polarization at $9=180^{\circ}$. Solid and dashed lines are PWIA predictions of the LC and VN methods, respectively. Marked curves include FSI.

High Q (e,e'), (e,e'N) processes

 $1 \text{ GeV}^2 \lesssim -q^2 \equiv Q^2$, $x = Q^2/2m_N q_0 > 1$, $1 \text{ GeV} > W - M_A > 50 - 100 \text{ MeV}$.

Two LC approaches

"the good current approach" FS81 "collinear approach" FS88 Good current approach follows logic of calculation of the form factor calculatio





$$\frac{1}{m_{\rm D}} W_{2\rm D}(\nu, q^2) = \frac{1}{4} \sum_{\rm N=p,n} \left(F_{1\rm N}^2(Q^2) + \frac{Q^2}{4m^2} F_{2\rm N}^2(Q^2) \right) \\ \times \int [u^2(k_{\rm i}) + w^2(k_{\rm i})] k_{\rm f} \, \frac{\sqrt{m^2 + k_{\rm i}^2}}{\sqrt{m^2 + k_{\rm f}^2}} \, \mathrm{d}\Omega_{\rm f},$$

$$k_{\rm i}^2 = \frac{m^2 + [k_{\rm ft} - (1 - \alpha/2)q_{\rm t}]^2}{\alpha(2 - \alpha)} - m^2.$$



SLAC data for the cross section for the D(e,e') reaction as a function of the minimum momentum of the nucleon in the deuteron wave function, kmin, allowed for the given values of Q^2 and W. Here Q^2_{el} is Q^2 for elastic scattering off a deuteron (or nucleon) at $9_e = 8^\circ$ and $\theta_e = 10^\circ$. The curves show calculations in the ``good current" approach with the Reid soft core wave function.

Serious problem: for exclusive final state an ad hoc choice of direction of qt leads to violation of rotational invariance for the NN final state. Happens for all WF except for the point-like vertex (which is the field theory situation for point-like particles). Can FSI restore rotational invariance? seems unlikely.

"Collinear approach": choose the quantization axis along the reaction axis. No problems with rotational invariance, but need to deal with bad and worst currents.

q_< 0 hence pair production is only due to instantaneous terms

The procedure is to use gauge invariance to express the contribution of the worst current through the good current

 $q_{-}\langle i|J_{+}(q)|R,p_{f}\rangle + q_{+}\langle i|J_{-}(q)|R,p_{f}\rangle = 0$

and deal with bad current correction like in QED.



ξt.n. -minimal LC fraction for the struck nucleon

 ξ t.n. ≥ I.2 - large D-wave contribution, strong tensor polarization effects



SLAC data compared with collinear approach for Reid (R) and Paris(P) wave functions. Open points is account of the FSI within NR approach by H.Arenhovel.

- Strong T₂₀ asymmetry for quasielastic (e,e')
 - **Αρρroximate** ξt.n. scaling of T₂₀
- On mass-shell fsi for T₂₀ to large extend cancels since it weakly depends on pn spin state.

Much larger effect than in the scaling limit





Same factor enters in the differential cross section of $\vec{e}\vec{D}
ightarrow epn$,

- **measure proton polarization**
- complementary way to test nonlinear relation between k and p

General case of polarized LC density matrix

$$\begin{split} \rho_{\Omega}^{\mathrm{D/N}}(\alpha, p_{\mathrm{t}}) &= \frac{2}{2-\alpha} \sum_{a,b} \Omega_{ab} \operatorname{Sp} \left(G_{1}(M_{\mathrm{NN}}^{2}) \varepsilon_{a} \gamma_{a} \right. \\ &+ (p_{1}-p_{2}, \varepsilon_{a}) G_{2}(M_{\mathrm{NN}}^{2}) (m+\hat{p}_{1}) (1-\gamma_{5}\hat{s}) (G_{1}(M_{\mathrm{NN}}^{2}) \hat{\varepsilon}_{b}^{*} \gamma_{b} \\ &+ (p_{1}-p_{2}, \varepsilon_{b}^{*}) G_{2}(M_{\mathrm{NN}}^{2}) (m-\hat{p}_{2}) \,, \end{split}$$

where G_1, G_2 are LC deuteron vertex functions

$$G_1(M_{\rm NN}^2) = (u(k) - w(k)/\sqrt{2})^{\frac{1}{4}}\sqrt{\varepsilon},$$

$$G_2(M_{\rm NN}^2) = -\sqrt{\varepsilon}/8k^2(u(k)(1 - m/\varepsilon) + w(k)\sqrt{\frac{1}{2}}(2 + m/\varepsilon))$$

Collider energies $\vec{e}\vec{D} \rightarrow ep + X$

If no EMC effect: $\frac{\mathrm{d}\sigma(\mathrm{e} + \mathrm{D}_{\Omega} \to \mathrm{e} + \mathrm{N} + \mathrm{X})}{(\mathrm{d}\alpha/\alpha) \,\mathrm{d}^{2}p_{\mathrm{t}}} / \frac{\mathrm{d}\sigma(\mathrm{e} + \mathrm{D} \to \mathrm{e} + \mathrm{N} + \mathrm{X})}{(\mathrm{d}\alpha/\alpha) \,\mathrm{d}^{2}p_{\mathrm{t}}}$ $= 1 + \left(\frac{3k_{i}k_{j}}{k^{2}} \,\Omega_{ij} - 1\right) \frac{\frac{1}{2}w^{2}(k) + \sqrt{2}u(k)w(k)}{u^{2}(k) + w^{2}(k)} \equiv P(\Omega, k)$

$$\frac{F_{2N}^{bound}(x/\alpha, Q^2, k)}{F_{2N}(x/\alpha, Q^2)} = \delta(x, Q^2, k)$$

expect:

- δ α nucleon off-energy shellness (virtuality)
- δ is different for S- and D waves since nucleon deformation since interaction of nucleon in a small configuration is reduced in different way for single pion and two pion potentials,..
- δ is different for the interaction with u and d quarks (may also differ for g₁ and F₂
 37

Conclusions

50

 $Q_{e1}^2 = 2, \theta = 8^\circ$

Light-cone approach allows to use a hidden small parameter of medium energy NN interactions small inelasticity.

M - M

 \implies Several qualitative differences from virtual nucleon approximation

