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JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Light-by-light sum rules: muon (g-2) and real light-by-light scattering

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[EPJ Web Conf. 274 (2022) 06007]

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Outline

1. General relations between amplitudes and structure functions for a spin-1 target
2. Application for the light-by-light scattering
3. Cottingham-like formula for e.m. isospin-breaking contribution to hadronic vacuum polarization using the dispersive representation of the LbL scattering amplitude
4. New physics from the real photon scattering at LHC. Constrains from LbL sum rules

Forward Compton tensor for the scattering off the deuteron

The complete gauge-invariant form of virtual forward Compton scattering off the deuteron of momentum p with a photon of momentum q is given by

$$\begin{aligned}
\mathcal{M}_{\mu\nu,\alpha\beta}(p,q) = {} & g_{\alpha\beta}\left[t_{\mu\nu}F_1 - \eta_\mu\eta_\nu F_2\right] + \mathrm{i}\kappa\epsilon_{\mu\nu\lambda\sigma}n^\lambda s^\sigma_{\alpha\beta}g_1 + \mathrm{i}\kappa^2\epsilon_{\mu\nu\lambda\sigma}n^\lambda(s^\sigma_{\alpha\beta} - s^\delta_{\alpha\beta}n_\delta\eta^\sigma)g_2 \\
& - t_{\mu\nu}\Sigma_{\alpha\beta}b_1 + \eta_\mu\eta_\nu\Sigma_{\alpha\beta}b_2 \\
& + (\kappa-1)\sqrt{\kappa}\left[\eta_\mu n_\beta t_{\alpha\nu} + \eta_\mu n_\alpha t_{\beta\nu} + \eta_\nu n_\beta t_{\alpha\mu} + \eta_\nu n_\alpha t_{\beta\mu} - 4\eta_\mu\eta_\nu n_\alpha n_\beta\right] b_3 \\
& + \frac{1}{2}\Bigg[\left(-t_{\mu\nu} + \frac{2Q^2}{\kappa\nu^2}\eta_\mu\eta_\nu\right)\left(\Sigma_{\alpha\beta} + \frac{2}{3}g_{\alpha\beta}\right) \\
& + (t_{\alpha\mu} - \eta_\mu n_\alpha)(t_{\beta\nu} - \eta_\nu n_\beta) + (t_{\alpha\nu} - \eta_\nu n_\alpha)(t_{\beta\mu} - \eta_\mu n_\beta)\Bigg]\Delta\,,
\end{aligned}$$

where $t_{\mu\nu} = g_{\mu\nu} - \dfrac{q_\mu q_\nu}{q^2},\ \Sigma_{\mu\nu} = \dfrac{1}{\kappa\nu^2}q_\mu q_\nu + \dfrac{1}{3}g_{\mu\nu},\ n_\mu = \dfrac{q_\mu}{\kappa\nu},\ \eta_\mu = \dfrac{1}{M}\left(p_\mu - \dfrac{M\nu}{q^2}q_\mu\right)$

$s^{\sigma\alpha\beta} = \mathrm{i}\epsilon^{\alpha\beta\sigma\delta}p_\delta/M \qquad \sqrt{\kappa} = |\vec{q}|/\nu$

Compton tensor in terms of polarized amplitudes

Following [Budnev et al., Nucl. Phys. B 34 (1971) 470-476], one can rewrite the Compton tensor in terms of 8 polarized amplitudes:

$$\mathcal{M}^{\mu'\nu',\mu\nu}(q,p) = R^{\mu\mu'}R^{\nu\nu'}\mathcal{M}_{TT} + \tfrac{1}{2}\left[R^{\mu\nu}R^{\mu'\nu'} + R^{\mu\nu'}R^{\mu'\nu} - R^{\mu\mu'}R^{\nu\nu'}\right]\mathcal{M}^{\tau}_{TT} + \left[R^{\mu\nu}R^{\mu'\nu'} - R^{\mu\nu'}R^{\mu'\nu}\right]\mathcal{M}^{a}_{TT}$$
$$+ R^{\mu\mu'}k_2^{\nu}k_2^{\nu'}\mathcal{M}_{TL} + k_1^{\mu}k_1^{\mu'}R^{\nu\nu'}\mathcal{M}_{LT} + k_1^{\mu}k_1^{\mu'}k_2^{\nu}k_2^{\nu'}\mathcal{M}_{LL}$$
$$- \left[R^{\mu\nu}k_1^{\mu'}k_2^{\nu'} + R^{\mu\nu'}k_1^{\mu'}k_2^{\nu} + (\mu\nu \leftrightarrow \mu'\nu')\right]\mathcal{M}^{\tau}_{TL} - \left[R^{\mu\nu}k_1^{\mu'}k_2^{\nu'} - R^{\mu\nu'}k_1^{\mu'}k_2^{\nu} + (\mu\nu \leftrightarrow \mu'\nu')\right]\mathcal{M}^{a}_{TL},$$

$$k_1^{\mu} \equiv \sqrt{\frac{-q^2}{X}}\left(p^{\mu} - \frac{p\cdot q}{q^2}q^{\mu}\right), \quad k_2^{\mu} \equiv \sqrt{\frac{M^2}{X}}\left(q^{\mu} - \frac{p\cdot q}{M^2}p^{\mu}\right), \quad X \equiv (p\cdot q)^2 - M^2q^2$$
$$R^{\alpha\beta} = R^{\beta\alpha} \equiv -g^{\alpha\beta} + \frac{1}{X}\left[(p\cdot q)(q^{\alpha}p^{\beta} + q^{\beta}p^{\alpha}) - q^2p^{\alpha}p^{\beta} - M^2q^{\alpha}q^{\beta}\right],$$

- Here all tensor structures are orthogonal to each other

Connection to the cross sections

Imaginary parts $W_Y = \mathrm{Im}\mathcal{M}_Y$ are related through the unitarity to the corresponding polarized photoabsorption cross sections (similarly to LbL)

$$W_{TT} = 2\sqrt{X}\sigma_{TT} = \sqrt{X}(\sigma_0 + \sigma_2) = \sqrt{X}(\sigma_{\parallel} + \sigma_{\perp})$$
$$W^{\tau}_{TT} = 2\sqrt{X}\tau_{TT} = 2\sqrt{X}(\sigma_{\parallel} - \sigma_{\perp})$$
$$W^{a}_{TT} = 2\sqrt{X}\tau^{a}_{TT} = \sqrt{X}(\sigma_0 - \sigma_2)$$
$$W_{TL} = 2\sqrt{X}\sigma_{TL} \qquad W_{LT} = 2\sqrt{X}\sigma_{LT} \qquad W_{LL} = 2\sqrt{X}\sigma_{LL}$$
$$W^{\tau}_{TL} = 2\sqrt{X}\tau_{TL} \qquad W^{a}_{TL} = 2\sqrt{X}\tau^{a}_{TL},$$

- all cross sections here are positive-definite
- for the fixed photon virtuality, the amplitudes can be written as dispersion relations in $\nu = q \cdot p$

$$\mathcal{M}_{\mathrm{even}}(\nu) = \frac{2}{\pi}\int_{\nu_{\mathrm{thr.}}}^{\infty} d\nu' \frac{\nu'}{\nu'^2 - \nu^2 - i0^+} W_{\mathrm{even}}(\nu'), \qquad \mathcal{M}_{\mathrm{odd}}(\nu) = \frac{2\nu}{\pi}\int_{\nu_{\mathrm{thr.}}}^{\infty} d\nu' \frac{1}{\nu'^2 - \nu^2 - i0^+} W_{\mathrm{odd}}(\nu'),$$

Relations between structure functions and Im parts

Direct relations

$$F_1 = \frac{W_{TL} + 2W_{TT}}{3}$$

$$F_2 = \frac{Q^2}{3(\nu^2 + Q^2)} \left(W_{TL} + 2W_{TT} + W_{LL} + 2W_{LT}\right)$$

$$g_1 = \frac{\nu}{\nu^2 + Q^2} \left(\sqrt{Q^2} W^a_{TL} - \nu W^a_{TT}\right)$$

$$g_2 = \frac{\nu^2}{\nu^2 + Q^2} \left(\frac{\nu}{\sqrt{Q^2}} W^a_{TL} + W^a_{TT}\right)$$

$$b_1 = W_{TL} - W_{TT}$$

$$b_2 = \frac{Q^2}{\nu^2 + Q^2} \left(W_{TL} - W_{TT} + W_{LL} - W_{LT}\right)$$

$$b_3 = \frac{\nu^2}{\sqrt{Q^2(\nu^2 + Q^2)}} W^\tau_{TL}$$

$$\Delta = W^\tau_{TT},$$

Inverse relations

$$W^a_{TL} = \frac{\sqrt{Q^2}}{\nu}(g_1 + g_2)$$

$$W^a_{TT} = \frac{Q^2}{\nu^2} g_2 - g_1$$

$$W^\tau_{TT} = \Delta$$

$$W^\tau_{TL} = \frac{\sqrt{Q^2(\nu^2 + Q^2)}}{\nu^2} b_3$$

$$W_{TT} = F_1 - \frac{b_1}{3}$$

$$W_{TL} = F_1 + \frac{2b_1}{3}$$

$$W_{LT} = -F_1 + \frac{b_1}{3} + \frac{\nu^2 + Q^2}{Q^2} \left(F_2 - \frac{b_2}{3}\right)$$

$$W_{LL} = -F_1 - \frac{2b_1}{3} + \frac{\nu^2 + Q^2}{Q^2} \left(F_2 + \frac{2b_2}{3}\right) .$$

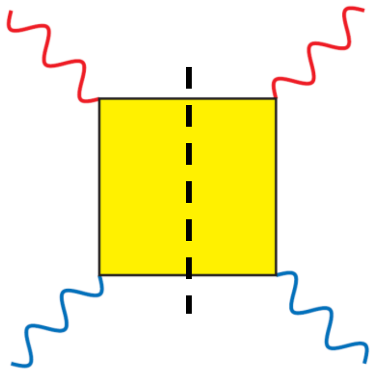
Compton scattering off the deuteron and LbL

It can be seen from the previous slides that the formalism for the Compton scattering off the deuteron and for the light-by-light scattering of virtual photons.

- Similar tensor parametrization, parity relations
- Same definitions of the absorptive parts
- Same dispersive representation of the amplitudes

The main difference is that the deuteron has electromagnetic moments, and the virtual photon does not.

Dispersion relation for traced LbL amplitude



Conventions:
[Budnev et al., Nucl. Phys. B 34 (1971) 470-476]
[Pascalutsa et al., Phys. Rev. D 85 (2012) 116001]

- Traced forward LbL amplitude in terms of the helicity amplitudes:

$$\mathcal{M} = \sum_{\lambda,\sigma=\pm1,0} (-1)^{\lambda+\sigma} \mathcal{M}_{\lambda\sigma\lambda\sigma} = 4\mathcal{M}_{TT} - 2\mathcal{M}_{LT} - 2\mathcal{M}_{TL} + \mathcal{M}_{LL}$$

$$\mathcal{M}_{TT} = \frac{1}{2}\left(\mathcal{M}_{++++} + \mathcal{M}_{----}\right), \quad \mathcal{M}_{LT} = \mathcal{M}_{0+0+}, \quad \mathcal{M}_{TL} = \mathcal{M}_{+0+0}, \quad \mathcal{M}_{LL} = \mathcal{M}_{0000}$$

- The imaginary part of the total traced forward LbL amplitude is given by

$$\mathrm{Im}\,\mathcal{M} = 2\sqrt{X}\sigma, \quad \sigma \equiv 4\sigma_{TT} - 2\sigma_{TL} - 2\sigma_{LT} + \sigma_{LL}$$

✓ The traced forward LbL amplitude satisfies once-subtracted dispersion relation:

$$\mathcal{M}(\nu, K^2, Q^2) = \mathcal{M}(\bar{\nu}, K^2, Q^2) + \frac{2(\nu^2 - \bar{\nu}^2)}{\pi} \int_{\nu_{\mathrm{thr.}}}^{\infty} d\nu' \frac{\nu'\, 2\sqrt{X'}\sigma(\nu', K^2, Q^2)}{(\nu'^2 - \bar{\nu}^2)(\nu'^2 - \nu^2)}$$

Dispersive representation for the subtraction function?

Apply the idea from [V.B et al., 2305.08814] written for the Compton scattering

- Look at the amplitude, unpolarized with respect to two photons with virtuality K^2

$$g^{\alpha\beta}\mathcal{M}_{\mu\alpha\nu\beta} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) M_1 + \frac{\hat{k}_\mu \hat{k}_\nu}{k^2} M_2, \quad \hat{k}_\mu = k_\mu - \frac{k\cdot q}{q^2} q_\mu .$$

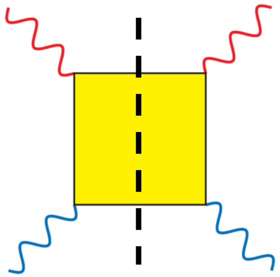
- Traced LbL amplitude is expressed via M_1 and M_2:

$$\mathcal{M} = -3M_1 - \frac{X}{q^2 k^2} M_2,$$

- Introduce longitudinal LbL amplitude. It is also expressed in terms of M_1 and M_2:

$$\mathcal{M}_L = M_{\mu\nu}\epsilon_L^\mu \epsilon_L^{*\nu} = -M_1 - M_2 \frac{X}{q^2 k^2}.$$

$$\mathrm{Im}\,\mathcal{M}_L = \mathrm{Im}\,(M_{0000} - M_{0+0+} - M_{0-0-}) = 2\sqrt{X}\,(\sigma_{LL} - 2\sigma_{LT}) \equiv 2\sqrt{X}\sigma_L .$$



- Consider the special kinematical point (Siegert limit): $\nu = KQ$.

$$\mathcal{M}_L(\nu = QK, Q^2, K^2) = -M_1(\nu = QK, Q^2, K^2) = \mathcal{M}(\nu = QK, Q^2, K^2)/3.$$

Dispersive representation for the subtraction function?

- Assume that the longitudinal LbL amplitude satisfies the unsubtracted sum rule

$$\mathcal{M}_L(\nu, K^2, Q^2) = \frac{4}{\pi} \int_{\nu_0}^{\infty} d\nu' \, \frac{\nu' \sqrt{X'} \sigma_L(\nu', K^2, Q^2)}{\nu'^{\,2} - \nu^2 - i0^+} \, ,$$

- Equaling the latter with the sum rule for the total traced LbL amplitude at $\nu = KQ$, we obtain the dispersive fully data-driven representation for the subtraction function!

$$\mathcal{M}(\bar{\nu}, K^2, Q^2) = \frac{4}{\pi} \int_{\nu_{\mathrm{thr.}}}^{\infty} d\nu' \frac{\nu'}{\sqrt{\nu'^{\,2} - (KQ)^2}} \left[3\sigma_L(\nu', K^2, Q^2) + \frac{\bar{\nu}^2 - (KQ)^2}{\nu'^{\,2} - \bar{\nu}^2} \sigma(\nu', K^2, Q^2) \right]$$

Dispersive representation for the subtraction function?

- The integral converges in QED! But to the wrong value…

$$\mathcal{M}^{\mathrm{QED}}(\bar{\nu}, K^2, Q^2) = \frac{4}{\pi} \int_{\nu_{\mathrm{thr.}}}^{\infty} d\nu' \frac{\nu'}{\sqrt{\nu'^2 - (KQ)^2}} \left[3\sigma_L^{\mathrm{QED}}(\nu', K^2, Q^2) + \frac{\bar{\nu}^2 - (KQ)^2}{\nu'^2 - \bar{\nu}^2} \sigma^{\mathrm{QED}}(\nu', K^2, Q^2) \right]$$

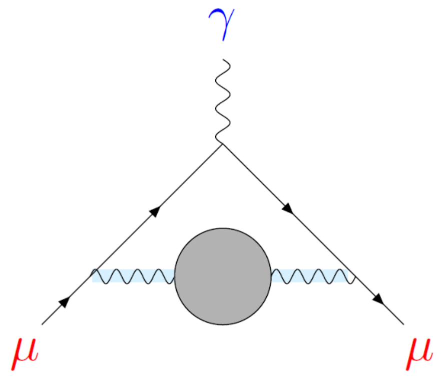
$$+3 \frac{32 \left(Q^4 + 4Q^2 - 2\sqrt{Q^2(Q^2+4)} \log\left(\frac{1}{2} \left(Q^2 + \sqrt{Q^2(Q^2+4)} + 2 \right) \right) \right)}{Q^2(Q^2+4)}$$

The residual corresponds to the value of the longitudinal LbL amplitude at infinite energy $\nu \to \infty$

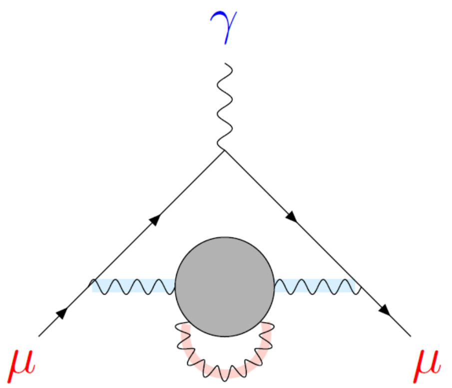
However, QED does not work at infinitely large energies…

Isospin-breaking corrections to HVP

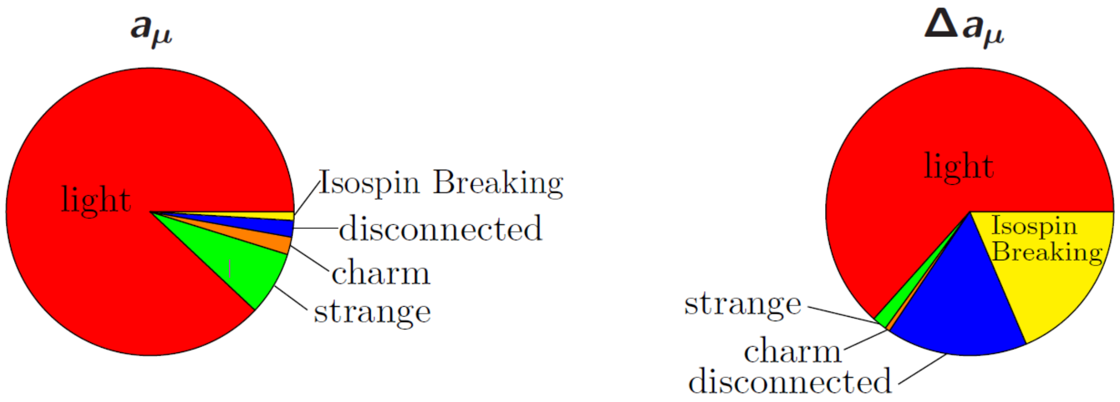
LO hadronic
vacuum polarization (HVP)



e.m. correction to HVP



Different contributions to HVP with relative uncertainties:

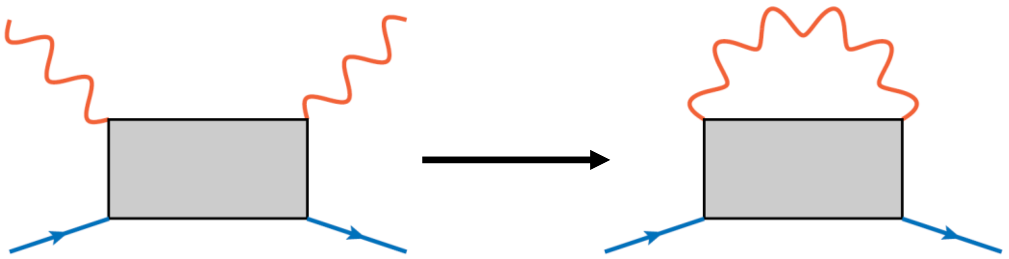


$\Delta a_\mu^{HVP} \sim 1\%$ from the lattice and dispersive estimations

- Nonequal quark masses, $m_u \neq m_d$
- Electromagnetic (radiative) corrections due to the different electric charges of the quarks

Main difficulty in the lattice is to control **finite size effects** arising from the power-law descend of QED correlators (massless photons)

Cottingham formula (for e.m. nucleon mass splitting)



[W. N. Cottingham, Ann. Phys. 25 (1963)]:

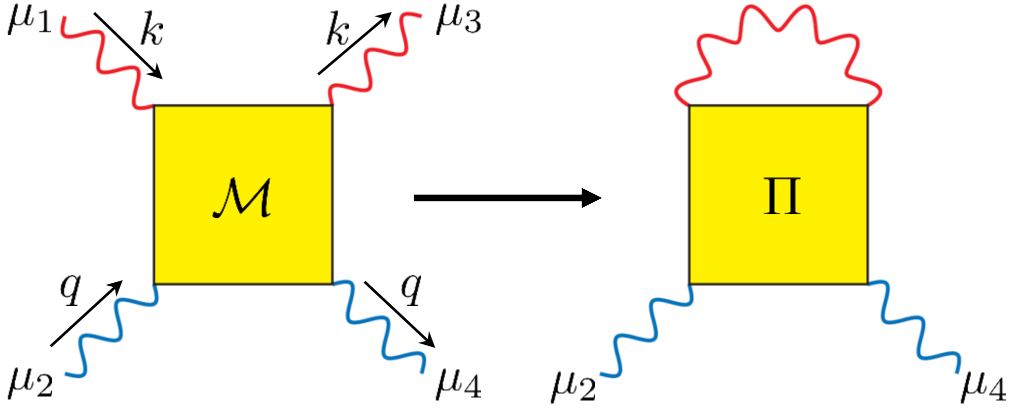
The virtual forward Compton amplitude was considered as an input to get a self-energy correction, which gives the e.m. mass splitting:

$$\delta M^{\gamma} = M_p - M_n = \frac{i}{2M}\frac{\alpha}{(2\pi)^3}\int d^4q\,\frac{T^{\mu}_{\mu}(p,q)}{q^2+i0^+}$$

with $T^{\mu}_{\mu} = -3T_1(\nu, Q^2) + \left(1+\frac{\nu^2}{Q^2}\right)T_2(\nu, Q^2)$ - the traced forward Compton amplitude

- Note the subtraction and counter term contributions! $\delta M^{\gamma} = \delta M^{\mathrm{el}} + \delta M^{\mathrm{inel}} + \delta M^{sub} + \delta M^{ct}$

Cottingham-like formula (for e.m. correction to HVP)



[JHEP 03 (2023) 194]

- The e.m. correction to the vacuum polarization is expressed through the traced LbL amplitude

$$\Pi_{4\mathrm{pt}}^{\mu_2\mu_4}(q^2,\Lambda) = -\frac{1}{2}\int\frac{d^4k}{(2\pi)^4}\left[\frac{-ig_{\mu_1\mu_3}}{k^2+i0^+}\right]_\Lambda \mathcal{M}^{\mu_1\mu_2\mu_3\mu_4}(k,q)$$

$$\Pi^{\mu\nu}(q) = \Pi(q^2)(q^2g^{\mu\nu} - q^\mu q^\nu)$$

- After the Wick rotation and angular integration, the formula for the traced vacuum polarization becomes:

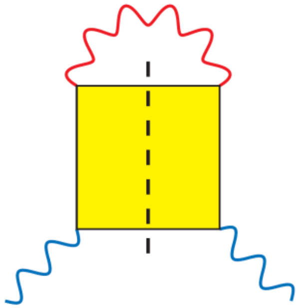
$$\Pi_{4\mathrm{pt}}(Q^2,\Lambda) = -\frac{1}{3Q^2(2\pi)^3}\int_0^\infty dK^2K^2\left[\frac{1}{K^2}\right]_\Lambda\int_0^1 dx\sqrt{1-x^2}\mathcal{M}(\nu,K^2,Q^2)$$

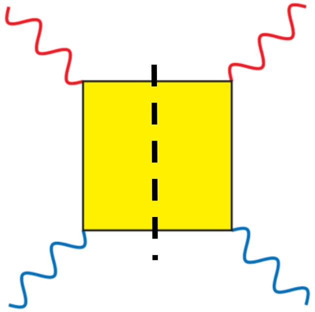
where $\mathcal{M} = g_{\mu_1\mu_3}g_{\mu_2\mu_4}\mathcal{M}^{\mu_1\mu_2\mu_3\mu_4}, \quad Q^2=-q^2>0, \quad K^2=-k^2>0, \quad \nu = k\cdot q$

Cottingham-like formula in dispersive representation

✓ Then the contribution to the vacuum polarization can be expressed via total LbL cross section as follows

$$\Pi_{4\mathrm{pt}}(Q^2,\Lambda) = -\frac{1}{3(2\pi)^3 Q^2}\int_0^\infty dK^2 K^2 \left[\frac{1}{K^2}\right]_\Lambda \Bigg[\frac{\pi}{4}\mathcal{M}(\bar{\nu},K^2,Q^2) + \int_{\nu_{\mathrm{thr.}}}^\infty d\nu \left(\frac{2}{\nu+\sqrt{X}} - \frac{\nu}{\nu^2-\bar{\nu}^2}\right)\sqrt{X}\sigma(\nu,K^2,Q^2)\Bigg]$$





This approach produces numerically stable results in QED and scalar QED.
(in comparison to the integration of LbL amplitude, which is evaluated directly via well-known libraries for one-loop integration, e.g. LoopTools, Collier, etc.)

Cottingham-like formula: advantages for the lattice

$$\Pi_{4\text{pt}}(Q^2,\Lambda) = -\frac{1}{3Q^2(2\pi)^3}\int_0^\infty dK^2 K^2 \left[\frac{1}{K^2}\right]_\Lambda \int_0^1 dx\sqrt{1-x^2}\mathcal{M}(\nu,K^2,Q^2)$$

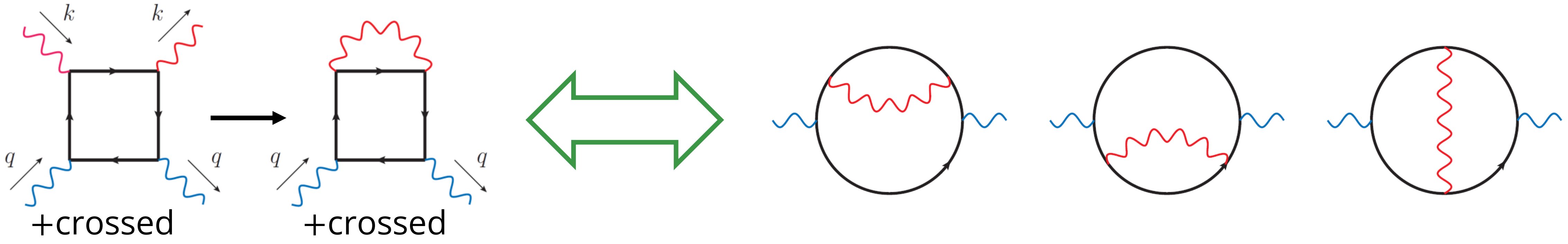
Forward doubly-virtual LbL amplitude has already been successfully calculated on a lattice [J. Green et al., PRL 115, 222003 (2015)]

$$\frac{1}{k^2} = \left(\frac{1}{k^2} - \frac{1}{k^2+\Lambda^2}\right) + \frac{1}{k^2+\Lambda^2}$$

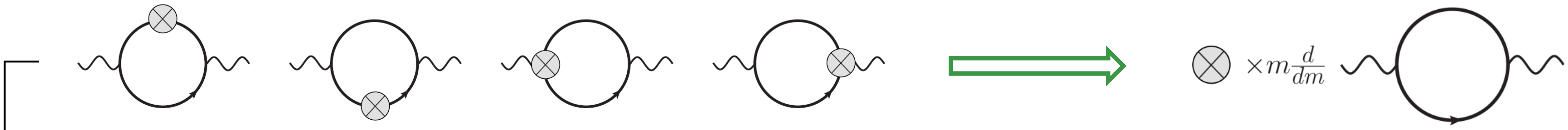
Ultraviolet-finite contribution, contains long-distance effects ($\pi^0\gamma$ and $\eta\gamma$ channels). It can be treated analogously to the HLbL on the lattice.

Can be treated entirely in lattice regularization (“massive” photon lives on the same lattice as the QCD fields)

- ✓ It was checked that the Cottingham-like formula provides the complete set of two-loop corrections to vacuum polarization in QED



- ✓ The subtraction at $Q^2 = 0$ is required and the counter terms should be added. The latter cannot be obtained from the one-loop LbL amplitude through the Cottingham formula, and are given by the following set of diagrams:

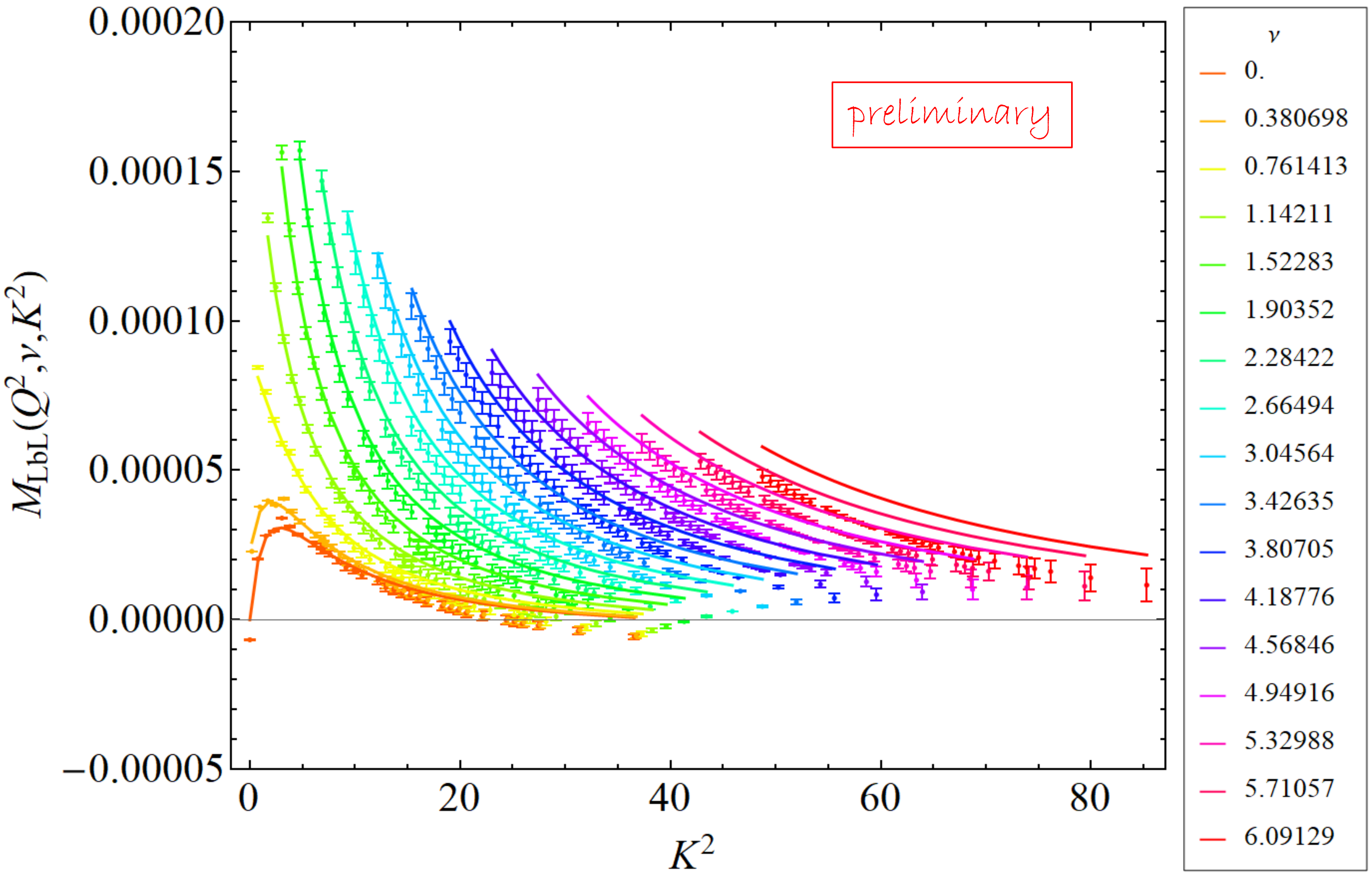


$$3\frac{\alpha}{2\pi}\left(\frac{1}{4}+\log\frac{\Lambda}{m}\right) m\frac{d}{dm}\overline{\Pi}_{2\mathrm{pt}}(Q^2,m), \quad \overline{\Pi}_{2\mathrm{pt}}(Q^2,m)\equiv \Pi_{2\mathrm{pt}}(Q^2,m)-\Pi_{2\mathrm{pt}}(0,m)$$

in Pauli-Villars regularization

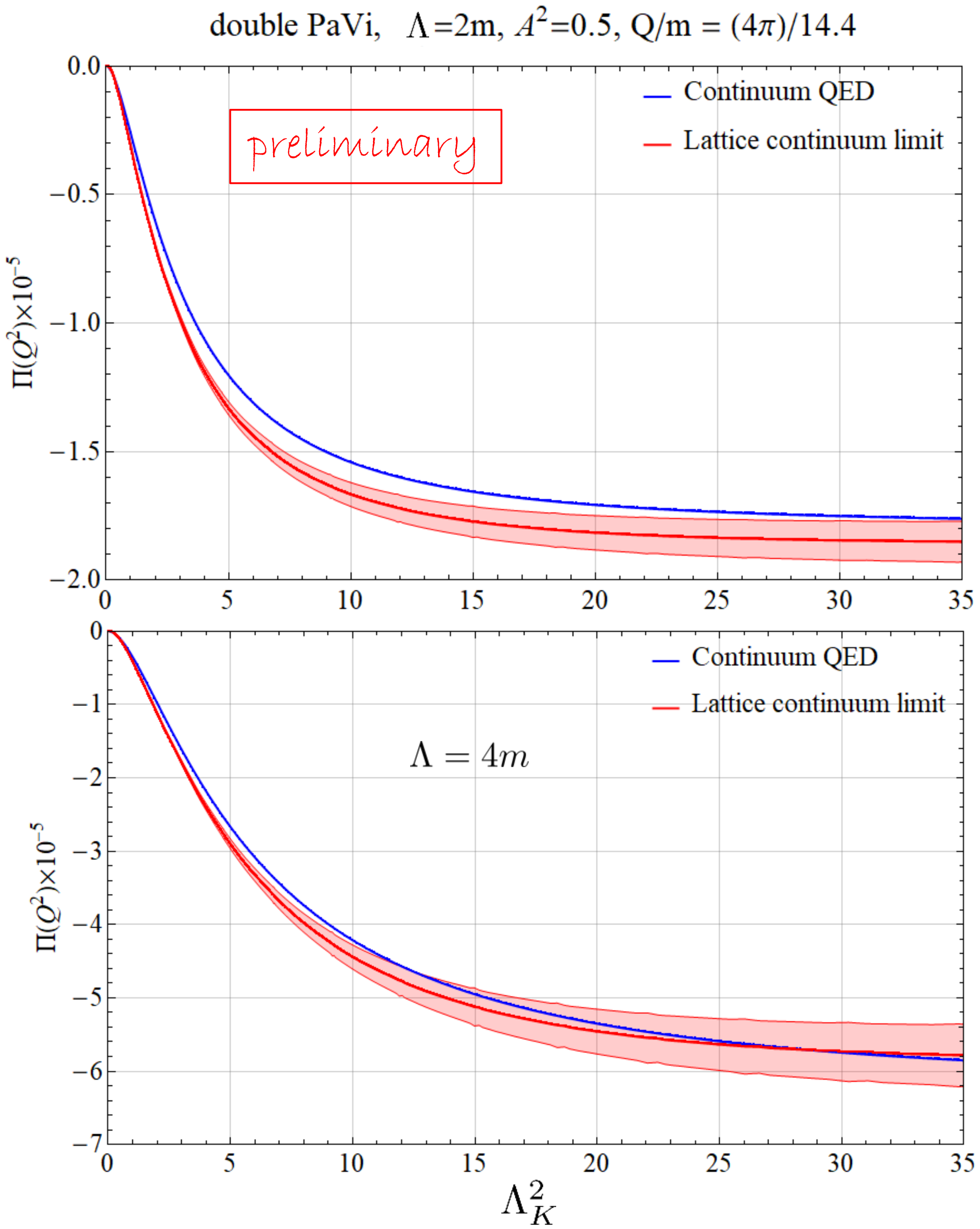
QED in continuum vs. on a lattice

Lattice data for LbL amplitude in QED (in continuum limit) vs. exact continuum calculation



Lattice data points are produced by Jeremy Green

QED in continuum vs. on a lattice



Comparison of the vacuum polarization, obtained via Cottingham formula, on a lattice and in continuum.

$$\Pi_{4\mathrm{pt}}(Q^2,\Lambda) = -\frac{1}{3Q^2(2\pi)^3}\int_0^{\Lambda_K} dK^2 K^2 \left[\frac{1}{K^2}\right]_\Lambda \int_0^1 dx\sqrt{1-x^2}\mathcal{M}(\nu,K^2,Q^2)$$

- Double Pauli-Villars regulator is used for faster convergence (to reduce the cut-off effects on a lattice)

$$\left[\frac{1}{K^2}\right]_\Lambda = \frac{\Lambda^4 A^2}{K^2(K^2+\Lambda^2)(K^2+A^2\Lambda^2)}$$

- Contribution to (g-2) at $\Lambda = 3m_\mu$

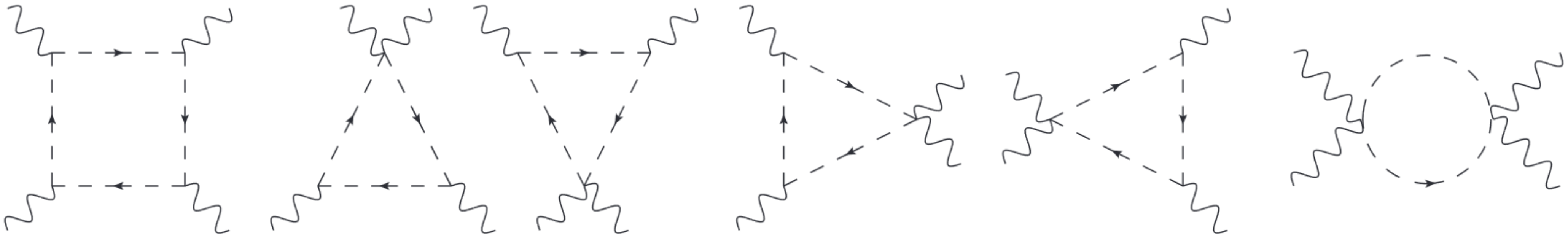
$$\Delta a_\mu^{\mathrm{cont.}} = -7.50\times 10^{-11} \quad \text{vs.} \quad \Delta a_\mu^{\mathrm{lat.}} \approx -7.5(4)\times 10^{-11}$$

Scalar QED

In order to compare the calculations in continuum with the full lattice calculation, the model for nonperturbative QCD is needed. One can consider meson exchanges and loops as an effective model for the long-range quark interactions.

- ➢ The Cottingham-like formula is needed to be checked for one-loop LbL scattering in sQED

- ✓ It was checked that the Cottingham-like formula provides the complete set of two-loop corrections to vacuum polarization in sQED



+crossed

Real LbL scattering

- Forward scattering: 3 independent helicity amplitudes.
Others are related via crossing and other symmetries

- Sum rules for the forward LbL helicity amplitudes:

$$M_{++++}(s) + M_{+-+-}(s) = \frac{2s^2}{\pi} \int_0^\infty ds' \frac{\sigma_0(s') + \sigma_2(s')}{s'^2 - s^2 - i0^+},$$

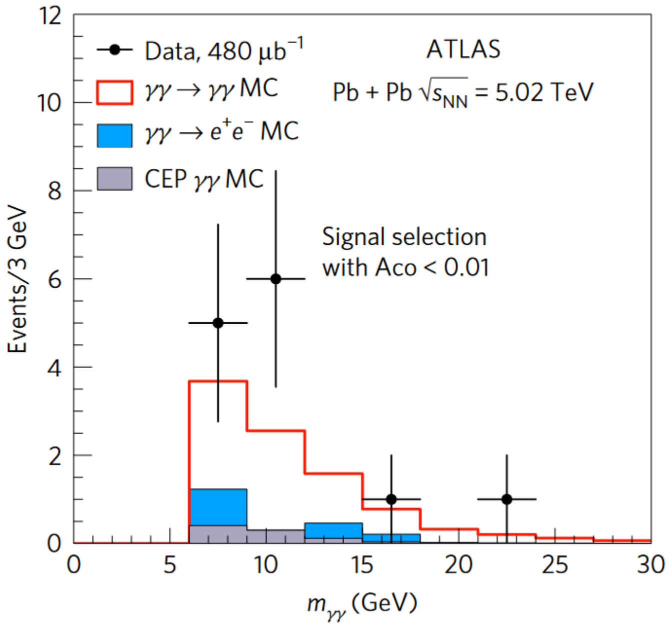
$$M_{++++}(s) - M_{+-+-}(s) = \frac{2s}{\pi} \int_0^\infty ds' \frac{s' [\sigma_0(s') - \sigma_2(s')]}{s'^2 - s^2 - i0^+},$$

$$M_{++--}(s) = \frac{2s^2}{\pi} \int_0^\infty ds' \frac{\sigma_\parallel(s') - \sigma_\perp(s')}{s'^2 - s^2 - i0^+}.$$

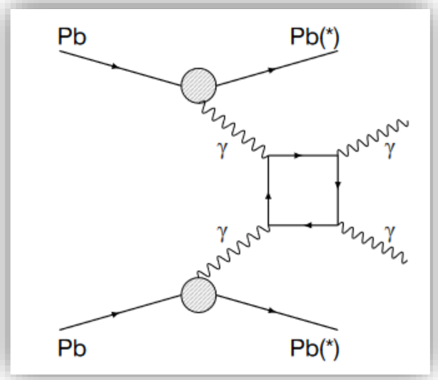
[Budnev et al., 1975]
[Pascalutsa and Vanderhaeghen, 2010]
[Pascalutsa, Pauk, Vanderhaeghen, 2012]

Light-by-light scattering at LHC: pilot experiments

[ATLAS collaboration, 2017]



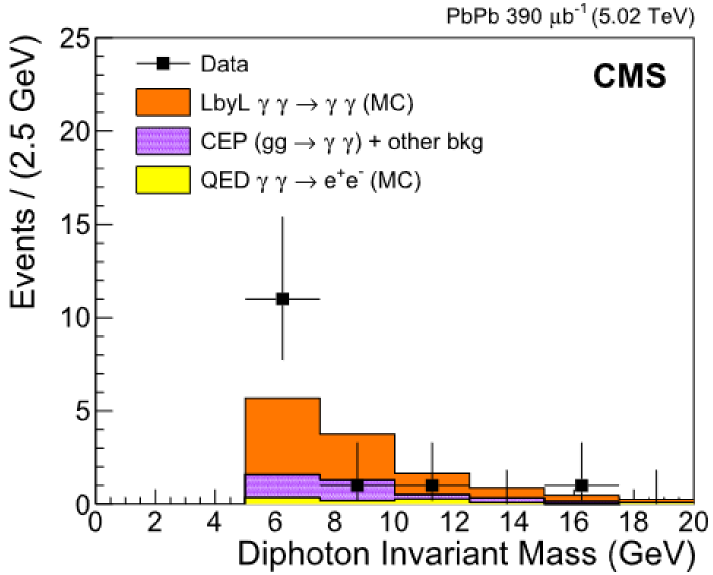
$$\sigma_{fid.} = 70 \pm 24(stat.) \pm 17(syst.)\ nb$$



$$\sigma^{excl}_{\gamma\gamma\to\gamma\gamma} = \sigma(AB \xrightarrow{\gamma\gamma} A\gamma\gamma B)$$
$$= \int d\omega_1 d\omega_2 \underbrace{\frac{f_{\gamma/A}(\omega_1)}{\omega_1}\frac{f_{\gamma/B}(\omega_2)}{\omega_2}} \sigma_{\gamma\gamma\to\gamma\gamma}\left(\sqrt{s_{\gamma\gamma}}\right)$$

Photon fluxes in equivalent photon approximation

[CMS collaboration, 2019]



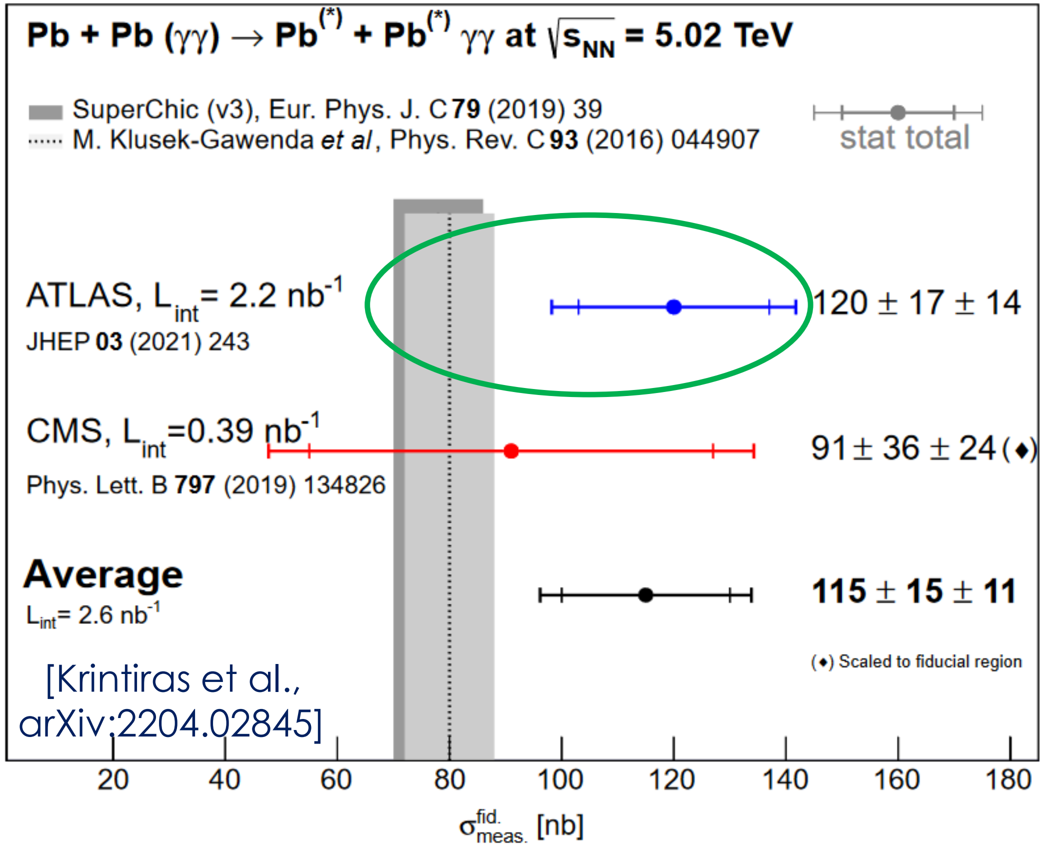
$$\sigma_{fid.} = 120 \pm 46(stat.) \pm 28(syst.) \pm 12(lumi.)\ nb$$

The unfolded data have not been provided.

Light-by-light scattering at LHC: full Run-2 dataset

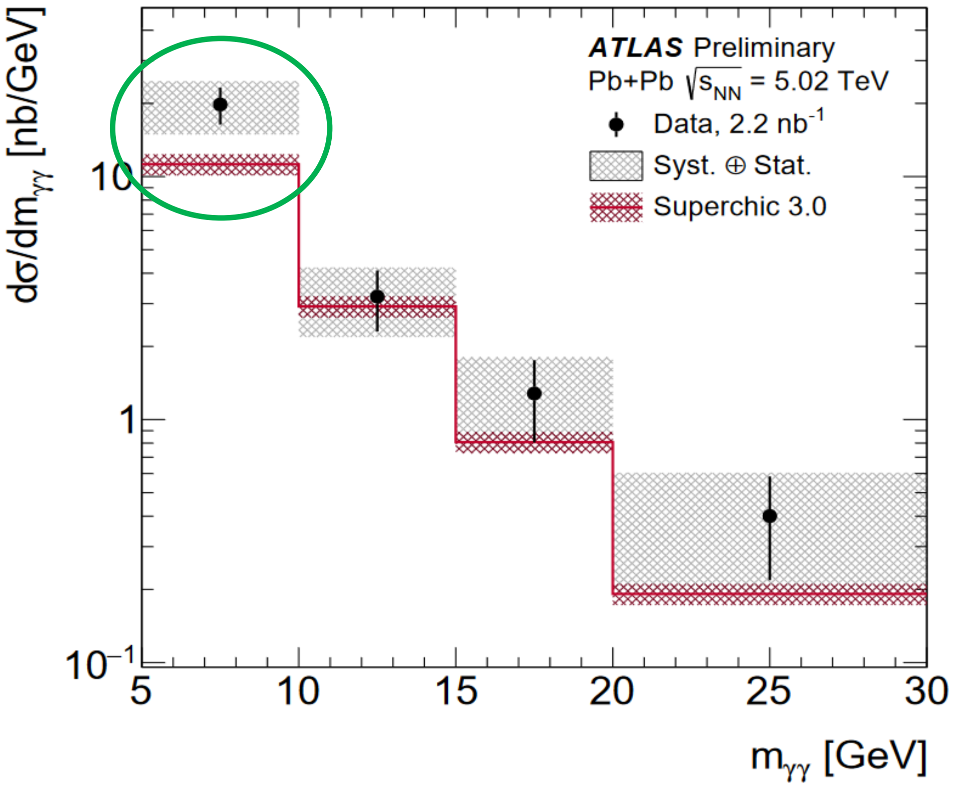
[ATLAS collaboration, 2021] • provide full Run-2 dataset with improved statistics and unfolded data

Total fiducial cross section



[Krintiras et al., arXiv:2204.02845]

Differential fiducial cross section vs. diphoton invariant mass (diphoton invariant mass spectrum)

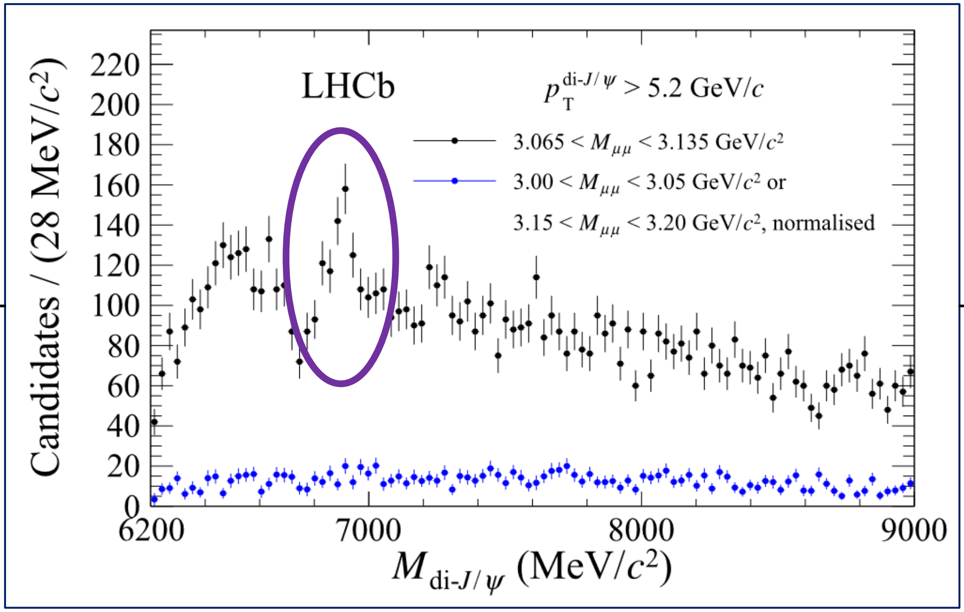


- Results show ~2σ discrepancy between experimental observations and theoretical predictions
- The excess is centered on the bin of 5-10 GeV of diphoton invariant mass

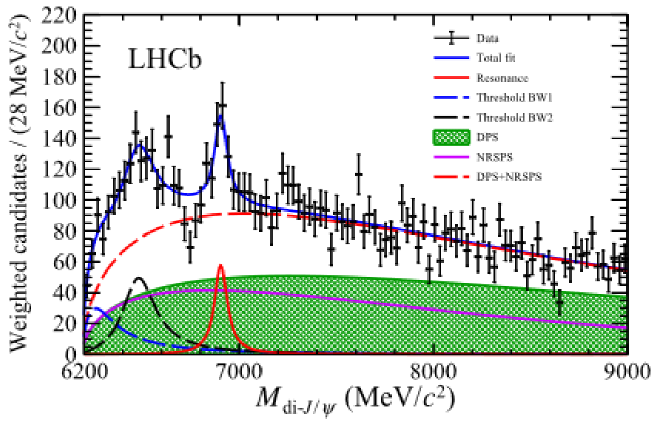
X(6900) at LHC

[LHCb collaboration, 2020]

Di-J/ψ mass spectrum



No-interference fitting scenario



$m[X(6900)] = 6905 \pm 11 \pm 7$ MeV$/c^2$,

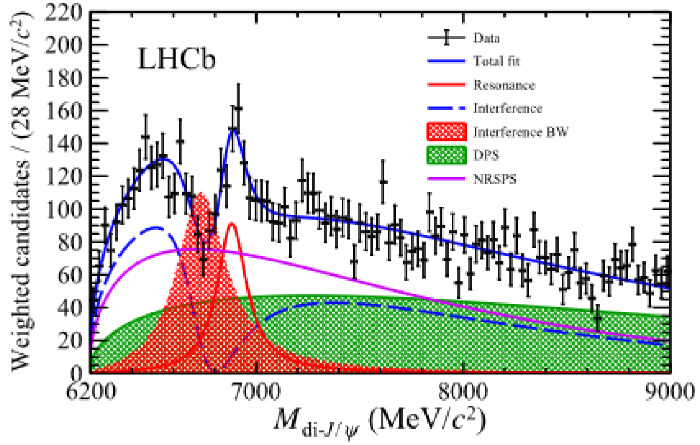
and

$\Gamma[X(6900)] = 80 \pm 19 \pm 33$ MeV,

This state is interpreted as possibly the lightest fully-charmed tetraquark state.

The following quantum numbers are considered for it in the literature on the tetraquark spectra:
$J^{PC} = 0^{++}, 0^{-+}, 1^{-+}, 2^{++}$.

Interference fitting scenario



$m[X(6900)] = 6886 \pm 11 \pm 11$ MeV$/c^2$

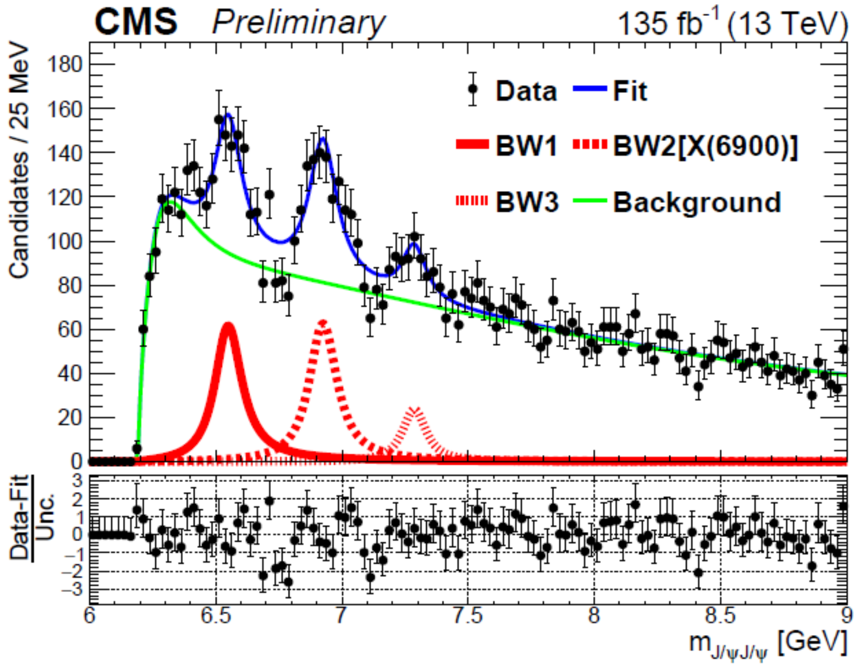
and

$\Gamma[X(6900)] = 168 \pm 33 \pm 69$ MeV.

New ATLAS and CMS data: X(6900), X(6600), X(7300), …

[CMS collaboration, 2022]

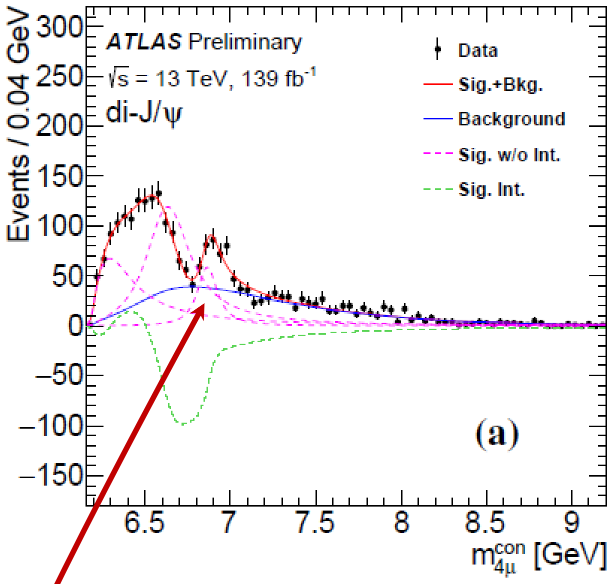
Di-J/ψ mass spectrum



	BW1	BW2	BW3
m	$6552 \pm 10 \pm 12$	$6927 \pm 9 \pm 5$	$7287 \pm 19 \pm 5$
Γ	$124 \pm 29 \pm 34$	$122 \pm 22 \pm 19$	$95 \pm 46 \pm 20$
	6.5σ	9.4σ	4.1σ

[ATLAS collaboration, 2022]

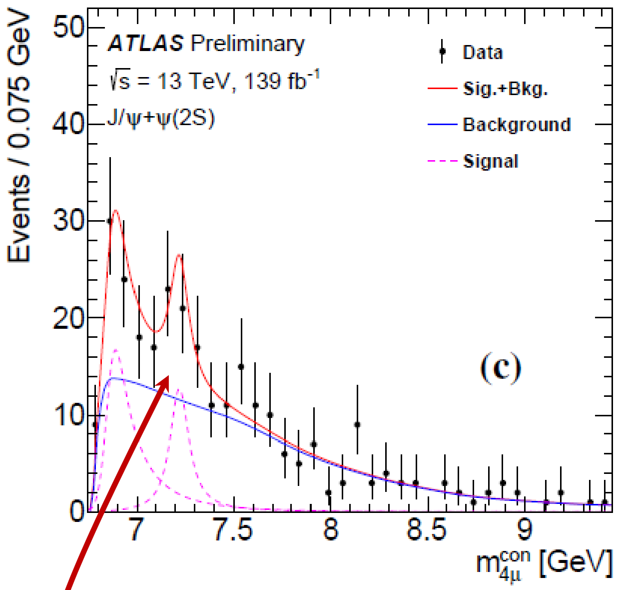
Di-J/ψ mass spectrum



m, GeV	Γ, GeV
$6.22 \pm 0.05^{+0.04}_{-0.05}$	$0.31 \pm 0.12^{+0.07}_{-0.08}$
$6.62 \pm 0.03^{+0.02}_{-0.01}$	$0.31 \pm 0.09^{+0.06}_{-0.11}$
$6.87 \pm 0.03^{+0.06}_{-0.01}$	$0.12 \pm 0.04^{+0.03}_{-0.01}$

10σ

J/ψ-$\psi(2S)$ mass spectrum



m, GeV	Γ, GeV
$7.22 \pm 0.03^{+0.02}_{-0.03}$	$0.10^{+0.13+0.06}_{-0.07-0.05}$
$6.78 \pm 0.36^{+0.35}_{-0.54}$	$0.39 \pm 0.11^{+0.11}_{-0.07}$

3.2σ

Fitting X(6900) into the light-by-light data

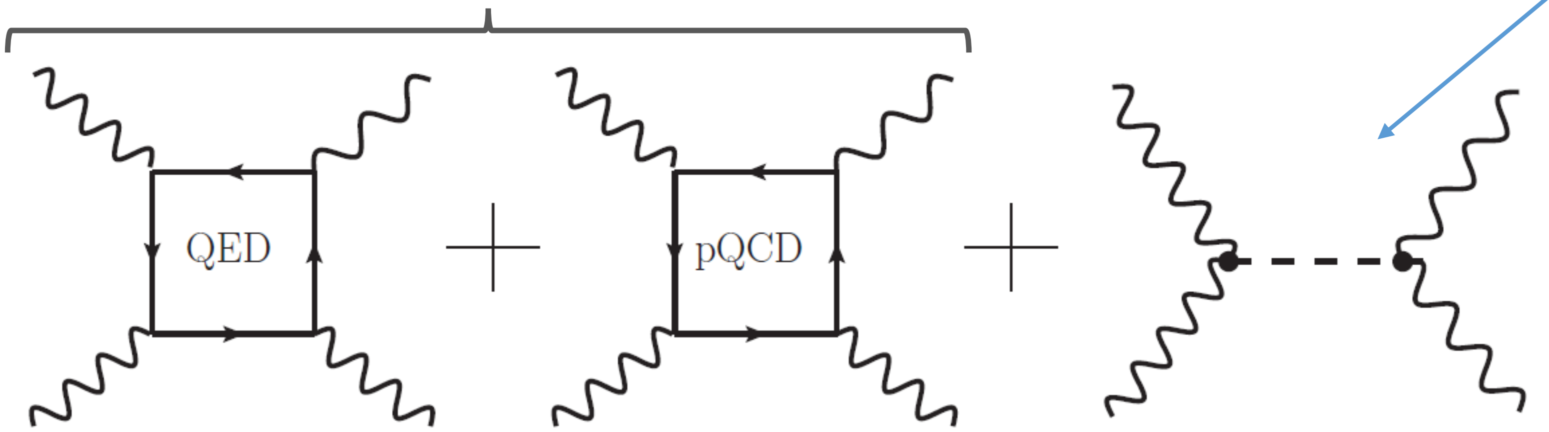
Since X(6900) decays into two J/ψ, then it would likely couple to two photons and hence contribute to the LbL scattering.

- That could explain the discrepancy in ATLAS data for the light-by-light scattering!
- One can extract the $X \to \gamma\gamma$ decay width of X(6900) exactly from the light-by-light data.

Basic SuperChic code
[superchic.hepforge.org]

Bottomonium resonances included in the model:

Meson	J^{PC}	M, [MeV]	Γ_{tot}, [MeV]	$\Gamma_{\gamma\gamma}/\Gamma_{tot}$ [%]
$\eta_b(1S)$	0^{-+}	9399.0	17.9	5.87×10^{-3}
$\eta_b(2S)$	0^{-+}	9999.0	8.34	5.86×10^{-3}
$\chi_{b0}(1P)$	0^{++}	9859.44	3.39	5.87×10^{-3}
$\chi_{b0}(2P)$	0^{++}	10232.5	3.54	5.41×10^{-3}



QED loops

pQCD loops

Bottomonium and X(6900) exchanges

"background"

"signal"

Constraints on meson exchange helicity amplitudes via forward LbL sum rules

The effective interaction Lagrangian term

$$\mathcal{L}_{X\gamma\gamma} = -g_{X\gamma\gamma}\phi_X F^{\mu\nu}F_{\mu\nu}$$

(for the pseudoscalar interaction: $F^{\mu\nu} \rightarrow \widetilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$)

produces the following tree-level helicity amplitudes:

Does it satisfy LbL sum rules?

$$M^P_{++++}(s,t,u) = -\frac{16\pi s^2\Gamma_{\gamma\gamma}}{m^3\,(s-m^2)},$$

$$M^P_{+++-}(s,t,u) = 0,$$

$$M^P_{++--}(s,t,u) = -P\frac{16\pi\Gamma_{\gamma\gamma}}{m}\left(\frac{s}{s-m^2} + \frac{t}{t-m^2} + \frac{u}{u-m^2}\right),$$

$$P = \pm 1$$

$$M^P_{++++}(s,t,u) = -\frac{16\pi s\,\Gamma_{\gamma\gamma}}{m\,(s-m^2)}.$$

Sum rules for the forward LbL helicity amplitudes:

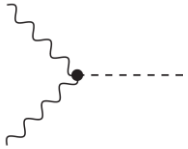
$$M_{++++}(s) + M_{+-+-}(s) = \frac{2s^2}{\pi}\int_0^\infty ds' \frac{\sigma_0(s') + \sigma_2(s')}{s'^2 - s^2 - i0^+},$$

$$M_{++++}(s) - M_{+-+-}(s) = \frac{2s}{\pi}\int_0^\infty ds' \frac{s'\,[\sigma_0(s') - \sigma_2(s')]}{s'^2 - s^2 - i0^+},$$

$$M_{++--}(s) = \frac{2s^2}{\pi}\int_0^\infty ds' \frac{\sigma_\parallel(s') - \sigma_\perp(s')}{s'^2 - s^2 - i0^+}.$$

[Budnev et al., 1975]
[Pascalutsa and Vanderhaeghen, 2010]
[Pascalutsa, Pauk, Vanderhaeghen, 2012]

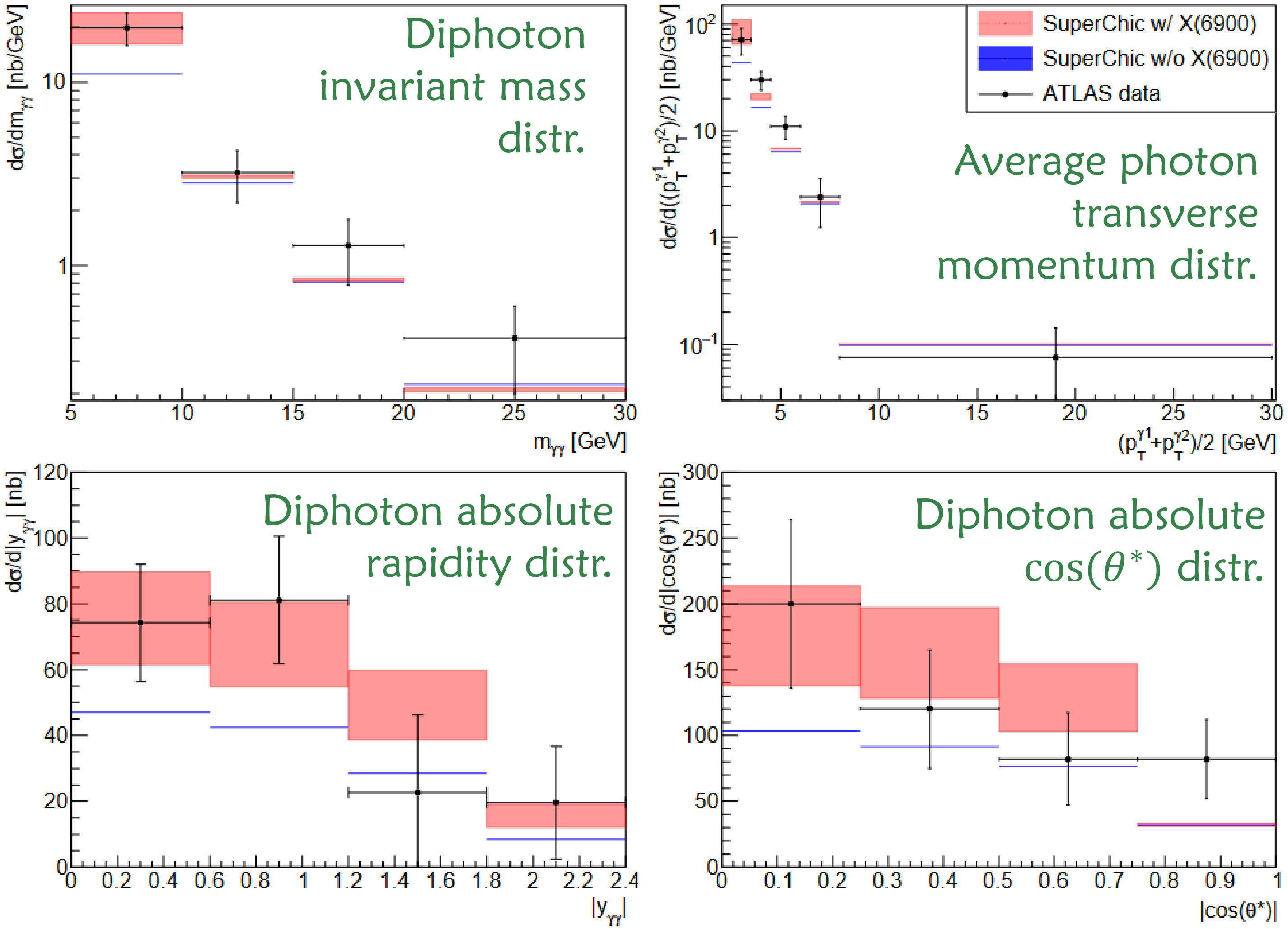
In case of meson exchanges the cross sections has the following form:



$$\sigma_0(s) = 16\pi^2\frac{\Gamma_{\gamma\gamma}}{m}\,\delta(s-m^2), \quad \sigma_2(s) = 0,$$

$$\begin{cases} \sigma_\parallel(s) = \sigma_0(s), & \sigma_\perp(s) = 0, \quad \text{for scalar}, \\ \sigma_\perp(s) = \sigma_0(s), & \sigma_\parallel(s) = 0, \quad \text{for pseudoscalar}. \end{cases}$$

Distributions of the observables:



- Here we used the mass and di-J/ψ decay width of X(6900), which has been measured by LHCb.
- The di-J/ψ decay width was treated as the total decay width of X(6900).

$X \to \gamma\gamma$ decay width:

Parameter	Interference	No-interference
m_X [MeV]	$6886 \pm 11 \pm 11$	$6905 \pm 11 \pm 7$
$\Gamma_{X\to J/\psi J/\psi}$ [MeV]	$168 \pm 33 \pm 69$	$80 \pm 19 \pm 33$
$\Gamma_{X\to\gamma\gamma}$ [keV]	67^{+15}_{-19}	45^{+11}_{-14}

Branching ratio:

$$B(X \to \gamma\gamma) = \begin{cases} 5.6^{+1.3}_{-1.6} \times 10^{-4}, & \text{No-int. sc,} \\ 4.0^{+0.9}_{-1.1} \times 10^{-4}, & \text{Int. sc..} \end{cases}$$

Summary

- The formalism for the forward Compton scattering off the deuteron is very similar to the one for virtual LbL scattering. However, the crucial difference is in the fact that the photon does not have electromagnetic moments.

- The calculation of e.m. isospin-breaking correction to HVP for muon (g-2) using Cottingham-like formula is suitable for lattice. It allows
  - separate long-range contributions from UV-divergent ones
  - avoid power-law effects

- The quasireal LbL scattering at the LHC can be used as a prospective filter for the fully-charm tetraquark quantum numbers. The LbL sum rules can be used to constrain the models of new physics.

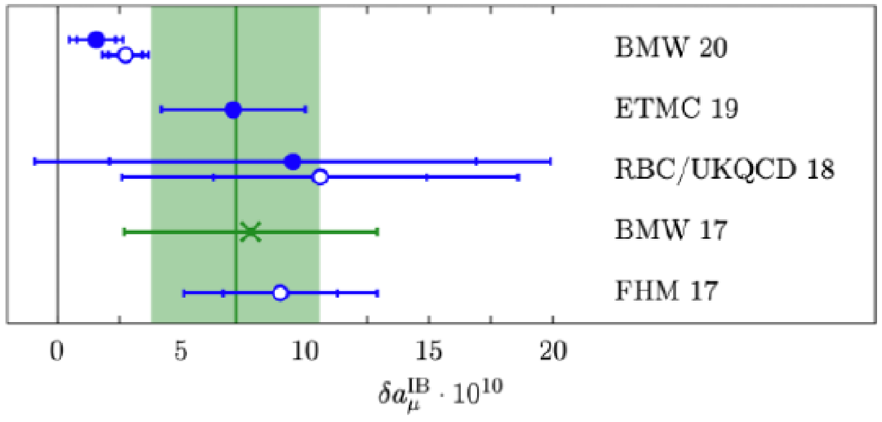
Thank you for attention!

Backup slides

ISOSPIN BREAKING CORRECTIONS

- LQCD estimates of e.m. IB have large uncertainty
  - Issue: finite-volume effects arising from the power-law descend of QED correlators (massless photons), …
- Cross checks between LQCD implementation are difficult
  - Issue: different implementations of isospin-symmetric LQCD & expansion points for IB effects
- No comparison with data-driven dispersive predictions possible
  - Issue: different conventions of the "physical point" in isospin-symmetric LQCD & counter terms associated with quark masses and strong coupling

H. Wittig, talk at Nov. '20 topical workshop.



Collaboration	$\delta a_\mu^{\mathrm{HVP,\,LO}} \times 10^{10}$	Comments
ETM-19 [12]	7.1 (2.9)	SIB+QED, perturbative method: QED = (V, S, S_T) in Fig. 42, SIB = M in Fig. 43. $M \ni$ scalar/pseudoscalar(PS) masses, where PS is for keeping maximal twist.
RBC/UKQCD-18 [11, 403]	9.5 (10.2)	SIB+QED, perturbative method: QED = (V, S, F) in Fig. 42, SIB = M in Fig. 43. F with no gluon between two quark-loops belongs to NNLO and is excluded.
FHM-17 [9]	9.5 (4.5)	Simulations with full-SIB for *ud*-conn: $m_d - m_u \neq 0$ while $\alpha = 0$.
BMW-17 [10]	7.8 (5.1)	(SIB + QED) using ChPT and dispersion: ρ–ω mix., FSR, $M_\pi^{\mathrm{ISLim}} \to M_{\pi^\pm}$, $\pi^0\gamma$, $\eta\gamma$.
CSSM/QCDSF/UKQCD Preliminary [437]	$\lesssim 1\% \times a_\mu^{\mathrm{HVP,\,LO}}$	Simulations with Full-QED for *ud*-conn: $\alpha \neq 0$ while $m_d - m_u = 0$. $M_\pi \sim 400$ MeV.

Cottingham-like formula: counterterms

OPERATOR PRODUCT EXPANSION (OPE) OF LBL

- Understand the UV divergence in $\Pi_{4\text{pt}}(q^2,\Lambda) = -\frac{1}{6q^2}\int \frac{d^4k}{(2\pi)^4}\left[\frac{-i}{k^2+i0^+}\right]_\Lambda \mathcal{M}(k,q)$
- Use OPE to determine large-k^2 behaviour of $\mathcal{M}(k,q)$ for fixed q:

$$\lim_{k^2\to\infty}\int \frac{d\Omega_k}{2\pi^2}\left\langle \int d^4x \int d^4y\, e^{ik(x-y)}\, V_\mu(x)V_\mu(y)V_\sigma(z)V_\lambda(0)\right\rangle = -\frac{6}{k^2} m\frac{\partial}{\partial m}\left\langle V_\sigma(z)V_\lambda(0)\right\rangle$$

$\longrightarrow$ purely determined by the mass-derivative of the vector two-point function

- Act on both sides with operator $-\frac{e^4}{2}\int \frac{d^4k}{(2\pi)^4}\left[\frac{1}{k^2}\right]_\Lambda \int d^4z\, H_{\lambda\sigma}(z)$, where $\overline{\Pi}(Q^2) = e^2\int_z H_{\lambda\sigma}(z)\,\langle V_\sigma^{\text{em}}(z)\, V_\lambda^{\text{em}}(0)\rangle$:

$$\lim_{\Lambda\to\infty}\overline{\Pi}_{4\text{pt}}(Q^2,\Lambda) = \frac{3e^2}{8\pi^2}\log\left(\frac{\Lambda}{\mu_{\text{IR}}}\right) m\frac{\partial}{\partial m}\overline{\Pi}_{e^2}(Q^2)$$

$\longrightarrow$ $\log\Lambda$ canceled precisely by the QED counter term

- Similar for QCD: counter terms are associated with the parameters of the theory (gauge coupling & quark masses)

Cottingham-like formula: counterterms

ISOSPIN-SYMMETRIC LQCD

- $(N_f + 1)$ bare parameters b_i^{lat} (quark masses and coupling) describing isospin-symmetric LQCD need to shift by δb_i^{lat} when implementing isospin breaking

- Requiring the theory with isospin breaking to reproduce suitable experimental observables:

physical hadron mass ⟶
$$M_h^{phys} = M_h^{iso} + M_{4pt,h}^{lat}(a;0) + \sum_{i=1}^{N_f+1} J_h^{lat(i)}\, \delta b_i^{lat} \qquad (h = 1, \ldots, N_f + 1)$$

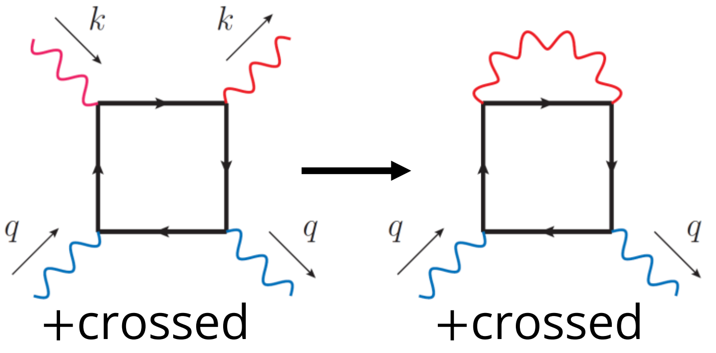
prediction in isospin-symmetric LQCD

$J_h^{lat(i)} = \partial M_h / \partial b_i^{lat} = \langle h \,|\, O_{lat}^{(i)} \,|\, h \rangle$ is the forward matrix element of the operator conjugate to parameter b_i^{lat}

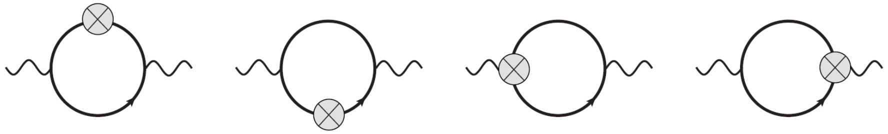
$O(e^2)$ e.m. contribution — to be calculated similarly to Π_{4pt}^{lat}

- Counter term will have the form: $\Pi_{ct}^{lat}(Q^2, a) = \sum_{i=1}^{N_f+1} \delta b_i^{lat} \frac{\partial}{\partial b_i^{lat}} \Pi_{e^2}(Q^2)$

QED example: analytic results



$$\Pi_{4\mathrm{pt}}(Q^2,\Lambda) = \frac{\alpha^2}{\pi^2}\left[-\frac{1}{2} + \frac{329}{1620}\frac{Q^2}{m_\ell^2} - \frac{2333}{75600}\left(\frac{Q^2}{m_\ell^2}\right)^2 + \frac{43579}{7938000}\left(\frac{Q^2}{m_\ell^2}\right)^3 + \mathcal{O}(Q^8)\right]$$
$$- \frac{\alpha^2}{\pi^2}\log\frac{\Lambda}{m_\ell}\left[-\frac{1}{2} + \frac{1}{5}\frac{Q^2}{m_\ell^2} - \frac{3}{70}\left(\frac{Q^2}{m_\ell^2}\right)^2 + \frac{1}{105}\left(\frac{Q^2}{m_\ell^2}\right)^3 + \mathcal{O}(Q^8)\right],$$



$$\overline{\Pi}_{\mathrm{ct}}(Q^2,\Lambda) = 6\,\frac{\alpha^2}{\pi^2}\left(\frac{1}{4} + \log\frac{\Lambda}{m_\ell}\right)\left[\frac{1}{6} - \frac{1}{\kappa^2} + \frac{4}{\kappa^3\sqrt{4+\kappa^2}}\operatorname{arctanh}\left(\frac{\kappa}{\sqrt{4+\kappa^2}}\right)\right]$$
$$= \frac{\alpha^2}{\pi^2}\left(\frac{1}{4} + \log\frac{\Lambda}{m_\ell}\right)\left[\frac{1}{5}\kappa^2 - \frac{3}{70}\kappa^4 + \frac{1}{105}\kappa^6 + \mathcal{O}(\kappa^8)\right], \quad \kappa = Q/m_\ell$$

Renormalized total result:

$$\Delta\overline{\Pi}(Q^2) \equiv \Pi_{4\mathrm{pt}}(Q^2,\Lambda) + \overline{\Pi}_{\mathrm{ct}}(Q^2,\Lambda) - \Pi_{4\mathrm{pt}}(0,\Lambda)$$
$$= \frac{\alpha^2}{\pi^2}\left[\frac{41}{162}\frac{Q^2}{m_\ell^2} - \frac{449}{10800}\left(\frac{Q^2}{m_\ell^2}\right)^2 + \frac{62479}{7938000}\left(\frac{Q^2}{m_\ell^2}\right)^3 + \mathcal{O}(Q^8)\right].$$

(in agreement with the well-known result, which can be obtained via the dispersive formula)

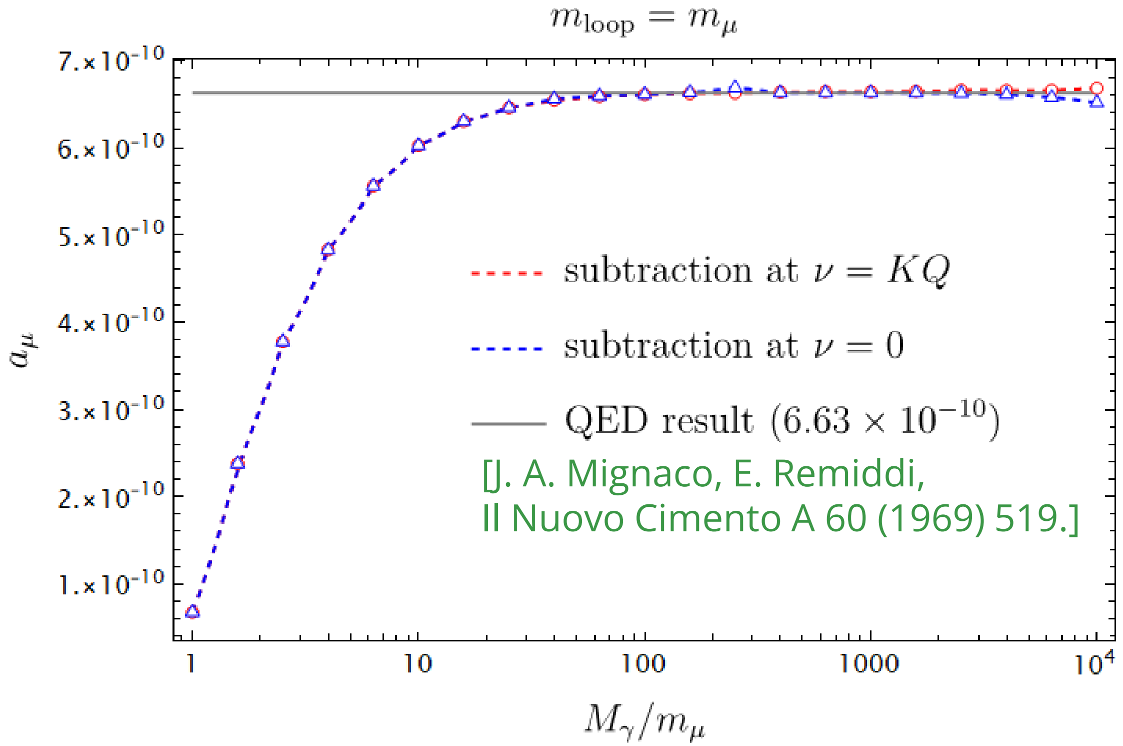
$$\Delta\overline{\Pi}(Q^2) = -\frac{Q^2}{\pi}\int_{4m_\ell^2}^{\infty}\frac{dt}{t(t+Q^2)}\mathrm{Im}\Pi(t).$$

QED example: contribution to (g-2)

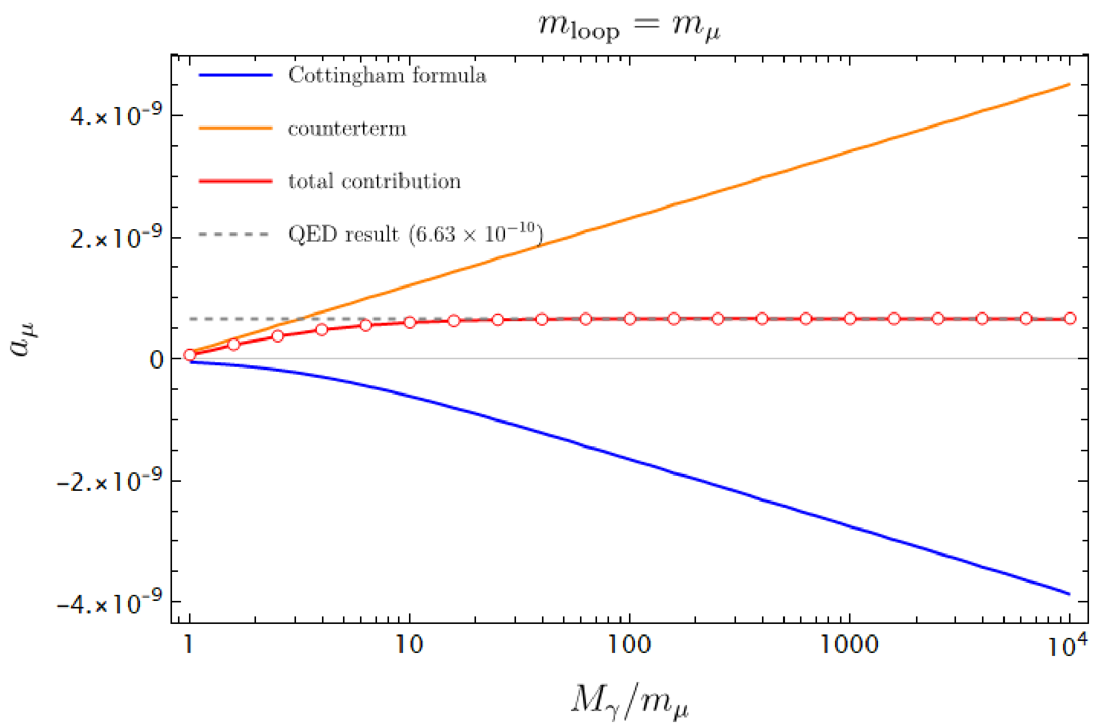
[F. Jegerlehner, Springer Tracts Mod. Phys. 274 (2017)]
[M. Davier, Nucl. Part. Phys. Proc. 287-288, 70 (2017)]

Dispersive approach to HVP: $a_\mu^{\rm VP} = \frac{\alpha}{\pi}\int_0^\infty dQ^2 \mathcal{K}(Q^2)\overline{\Pi}(Q^2) = -\frac{\alpha}{2\pi}\Pi(0) + \frac{\alpha}{\pi}\int_0^\infty dQ^2 \mathcal{K}(Q^2)\Pi(Q^2)$

$$\mathcal{K}(Q^2) = \frac{1}{2m_\mu}\frac{v-1}{2v(v+1)}, \qquad v = \sqrt{1+\frac{4m_\mu^2}{Q^2}}$$



[J. A. Mignaco, E. Remiddi, Il Nuovo Cimento A 60 (1969) 519.]



Scalar QED

preliminary

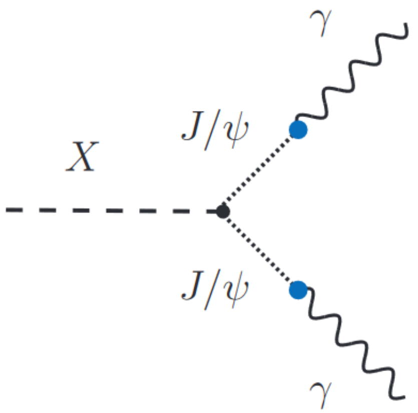
- ✓ The counter terms were also found in cut-off regularization, and the known answer for the two-loop vacuum polarization was reproduced.

  [J. Schwinger, "Particles, sources and fields", Vol.III] - provides $Im\Pi$, but has a serious typo in Eq.(5-4.132)
  [J. Bijnens, PRD 100, 014508 (2019)]

- Since the LbL amplitude in sQED has worse behavior with K then in QED, the additional regularization is needed. It can be double Pauli-Villars instead of a single one for QED, or one can insert form factors (e.g. VMD form factors) in each photon-meson vertex of the loop.

$\gamma\gamma$-decay width estimate

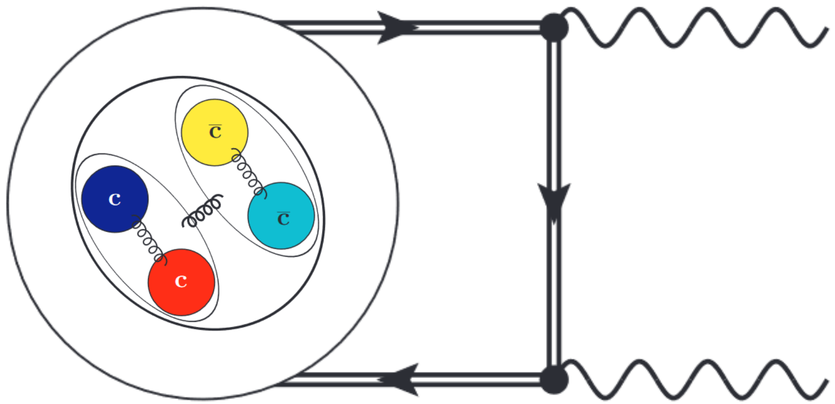
1. Vector-meson-dominance model: $\gamma\gamma$-decay via virtual J/ψ mesons.



	$B_{VMD}(X(6600) \to \gamma\gamma)$	$B_{VMD}(X(6900) \to \gamma\gamma)$	$B_{VMD}(X(7300) \to \gamma\gamma)$
scalar	$(4.12 \pm 0.54) \times 10^{-6}$	$(2.77 \pm 0.36) \times 10^{-6}$	$(2.19 \pm 0.28) \times 10^{-6}$
pseudoscalar	$(15.7 \pm 2.2) \times 10^{-6}$	$(6.07 \pm 0.80) \times 10^{-6}$	$(3.73 \pm 0.49) \times 10^{-6}$

$\Gamma_{X\to\gamma\gamma} \lesssim$ 1keV

2. Radiative decay of diquark-antidiquark state
– gives bigger $\gamma\gamma$-decay width!



$\Gamma_{X\to\gamma\gamma} \sim$ 10keV

* According to the works on the spectrum of fully-charm tetraquark excitations, the results obtained for diquark-antidiquark structure are in the best agreement with the experimentally observed resonances.

[Debastiani, 2019]