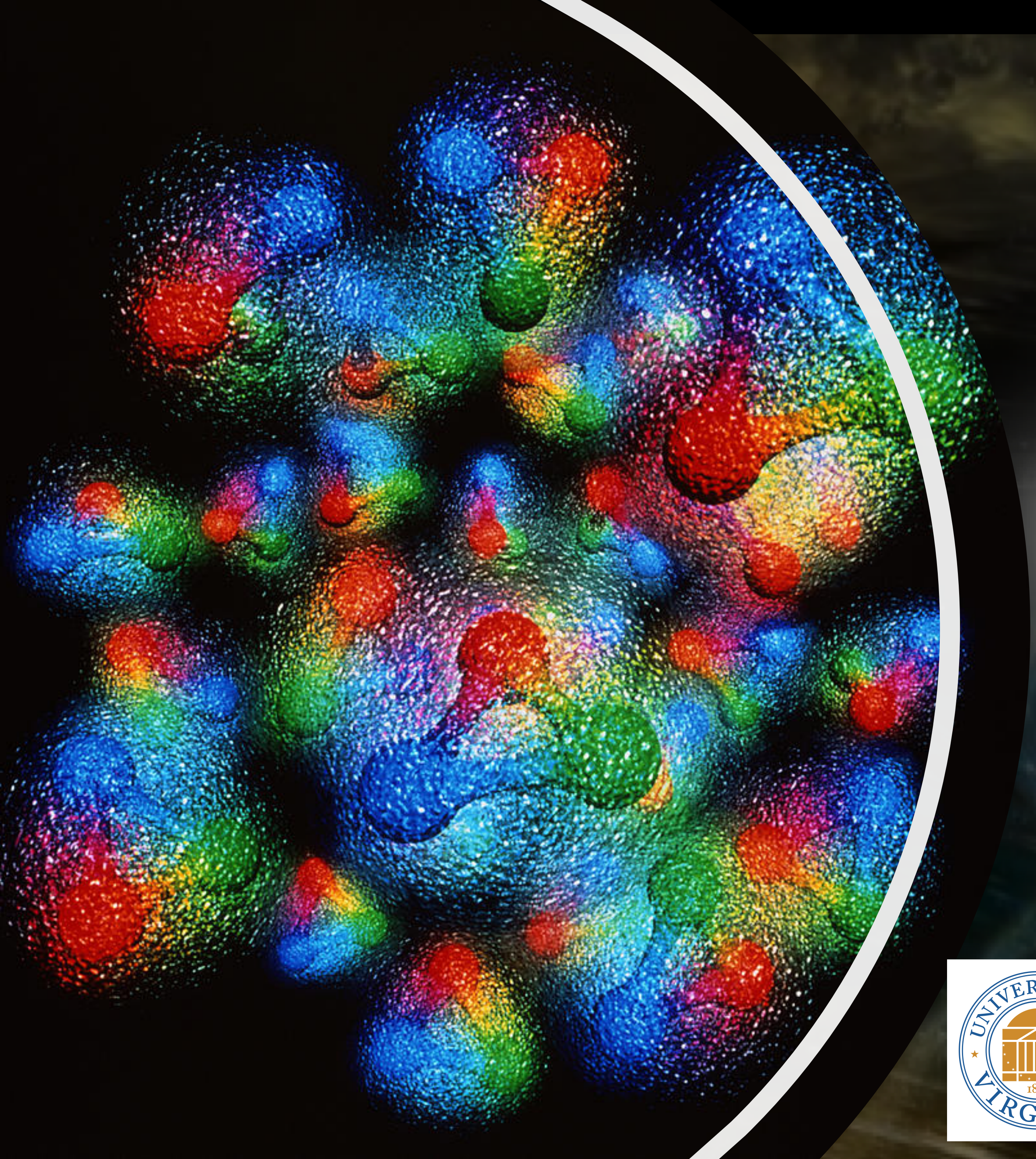

Deeply Virtual Exclusive Experiments on the Deuteron

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Motivation: to understand quarks and gluons angular momentum and the origin of the spin crisis

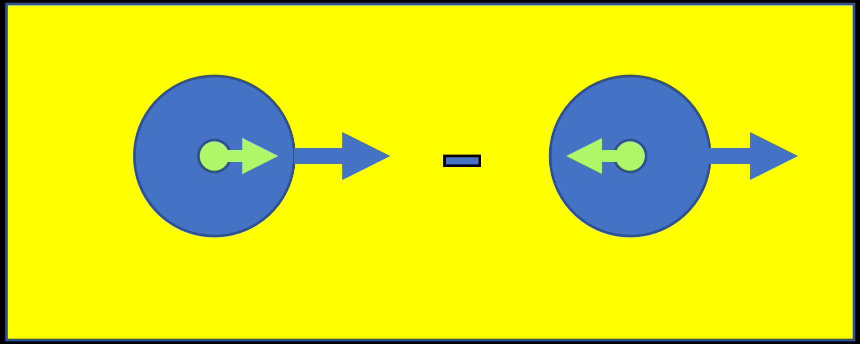
$$J_q + J_g = L_q + \frac{1}{2}\Sigma_q + J_g = \frac{1}{2}$$

This sum rule has longitudinal and transverse components, how do we access them through observables/quark and gluon distributions?

Fundamental role of deuteron through:

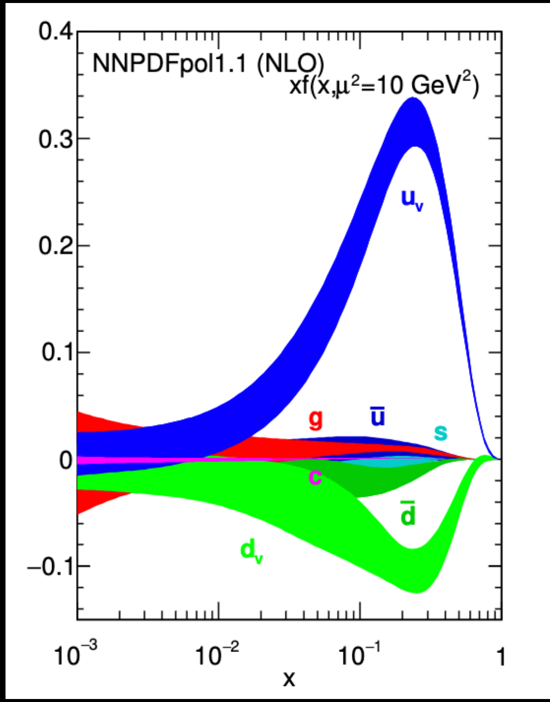
1. Extension of sum rule to spin 1 and interpretation of observables
2. Additional tensor-like observables

Longitudinal spin, S_z^q



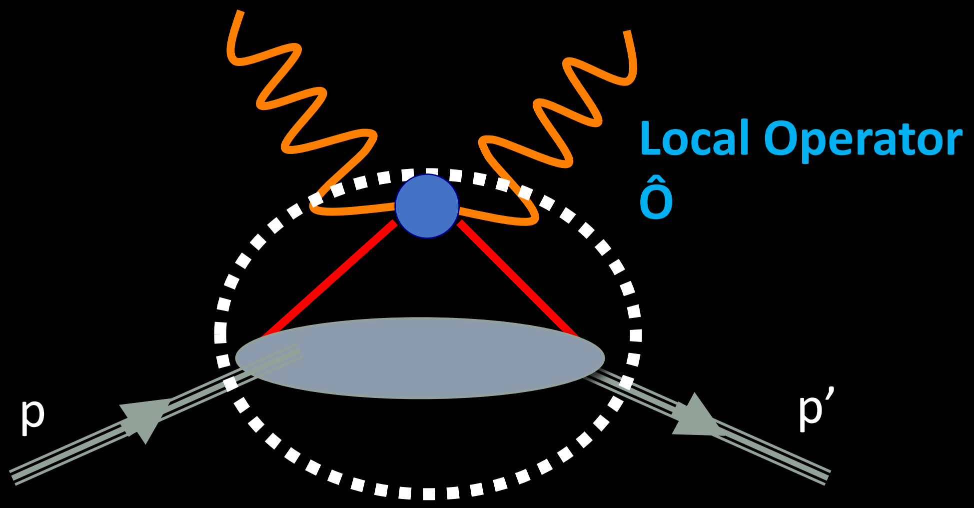
$$S_z^q = \frac{1}{2}\Delta\Sigma_q = \int_0^1 dx\, g_1^q(x)$$

g_1



Total angular momentum, J_z^q

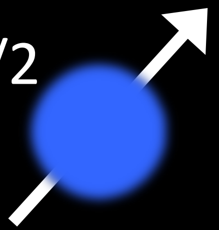
$$J_z^q = \int_0^1 dx\, x\left(H_q + E_q\right)$$



GPD Moments → QCD EMT Form Factors

X. Ji (1997)

S=1/2



(X. Ji, 1997)

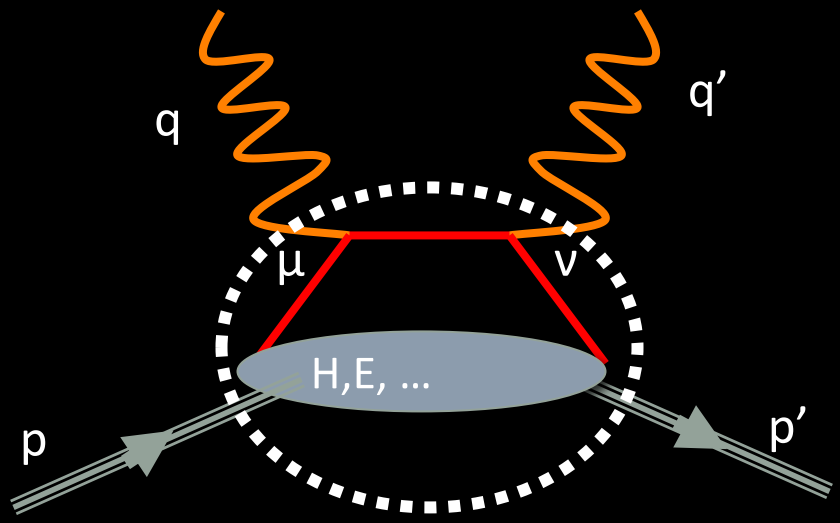
$$\langle p',\Lambda \mid T^{\mu\nu} \mid p,\Lambda\rangle = A(t)\bar{U}(p',\Lambda')[\gamma^\mu P^\nu + \gamma^\nu P^\mu]U(p,\Lambda) + B(t)\bar{U}(p',\Lambda') i\frac{\sigma^{\mu(\nu}\Delta^{\nu)}}{2M}U(p,\Lambda)$$
$$+C(t)[\Delta^2 g^{\mu\nu} - \Delta^{\mu\nu}]\bar{U}(p',\Lambda')U(p,\Lambda) + \widetilde{C}(t) g^{\mu\nu}\bar{U}(p',\Lambda')U(p,\Lambda)$$

off-forward

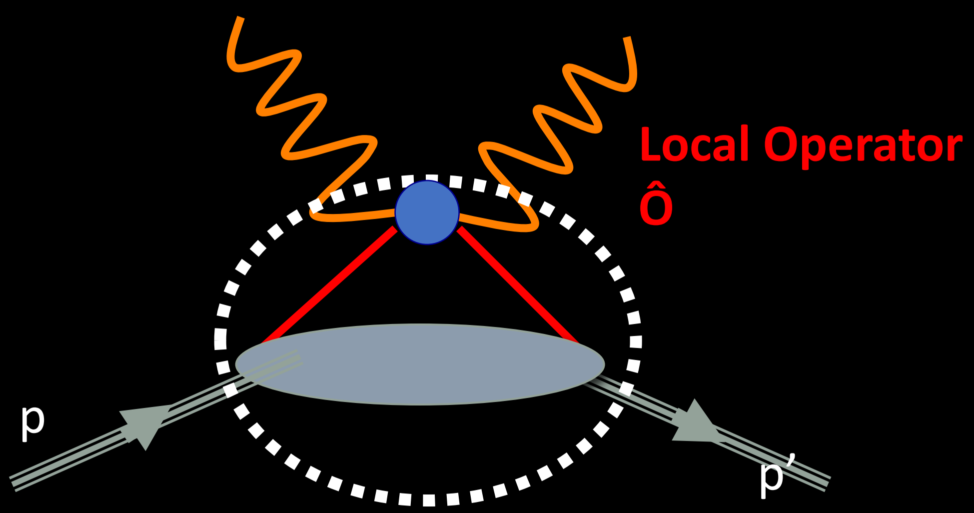
q and g not separately conserved

$$P = \frac{p+p'}{2}$$
$$\Delta = p' - p = q - q'$$
$$t = (p-p')^2 = \Delta^2$$

QCD Energy Momentum Tensor matrix elements as Generalized Parton Distributions Moments



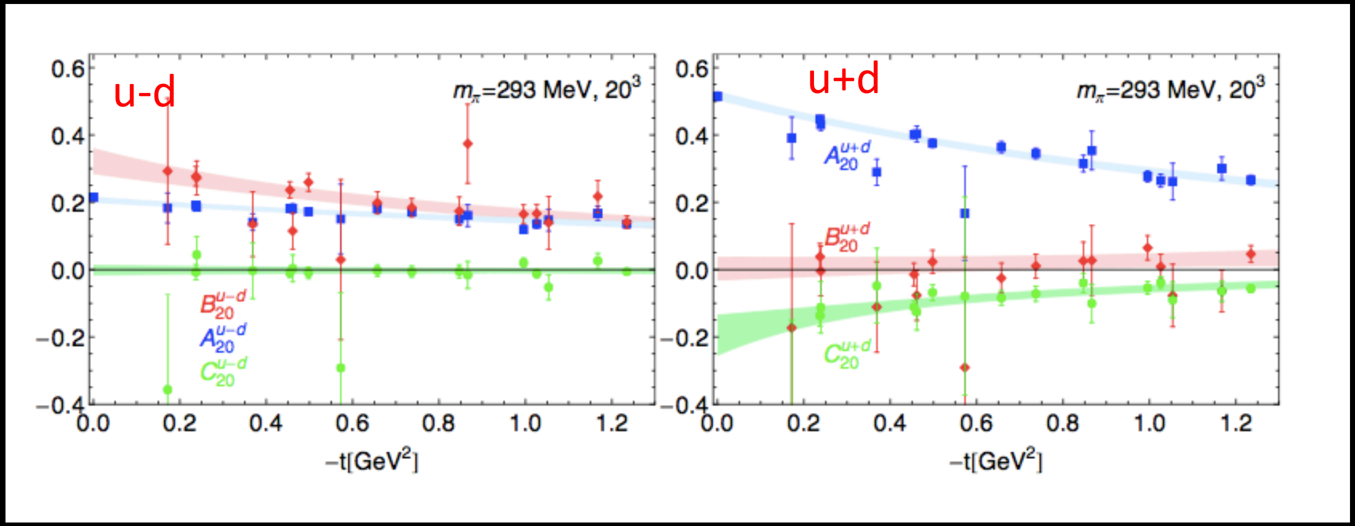
GPD



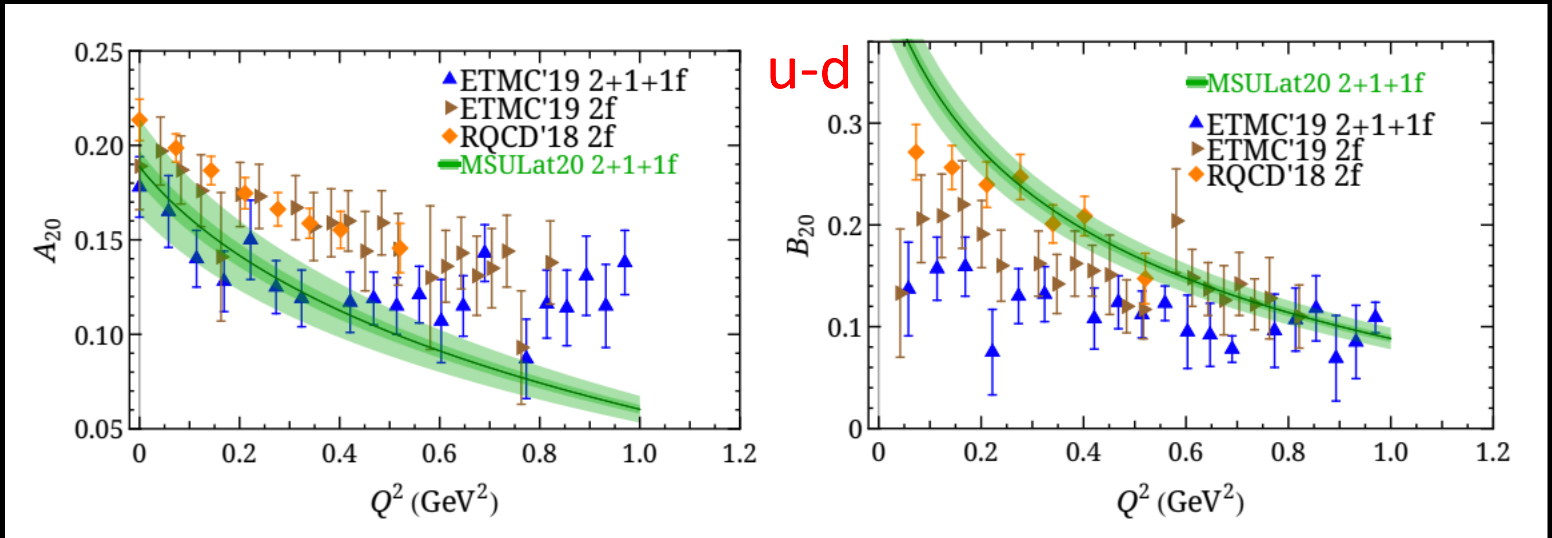
GPD Moments → EMT Form Factors

- Large momentum transfer $Q^2 >> M^2$ → "deep"
- Large Invariant Mass $W^2 >> M^2$ → equivalent to an "inelastic" process

J_z^q has not been measured reliably yet, but it is calculable in lattice QCD…

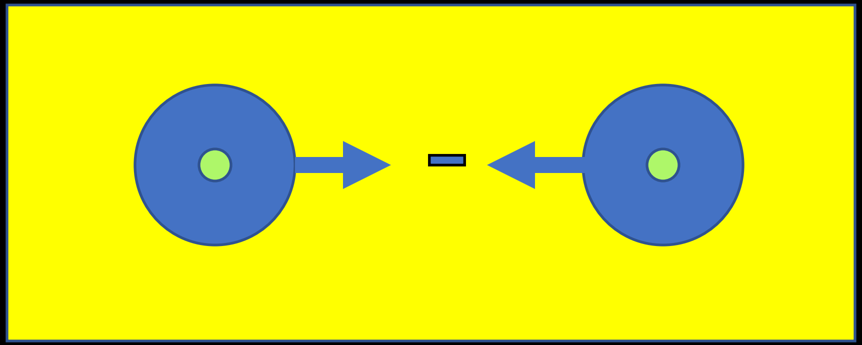


Ph. Haegler, JoP: **295** (2011) 012009

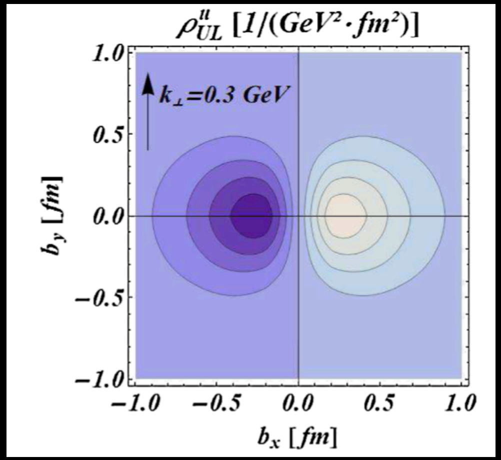


H-W Lin, *Phys.Rev.Lett.* 127 (2021)

Longitudinal OAM, L_z^q



F_{14}



Lorce, Pasquini, PRD (2013)

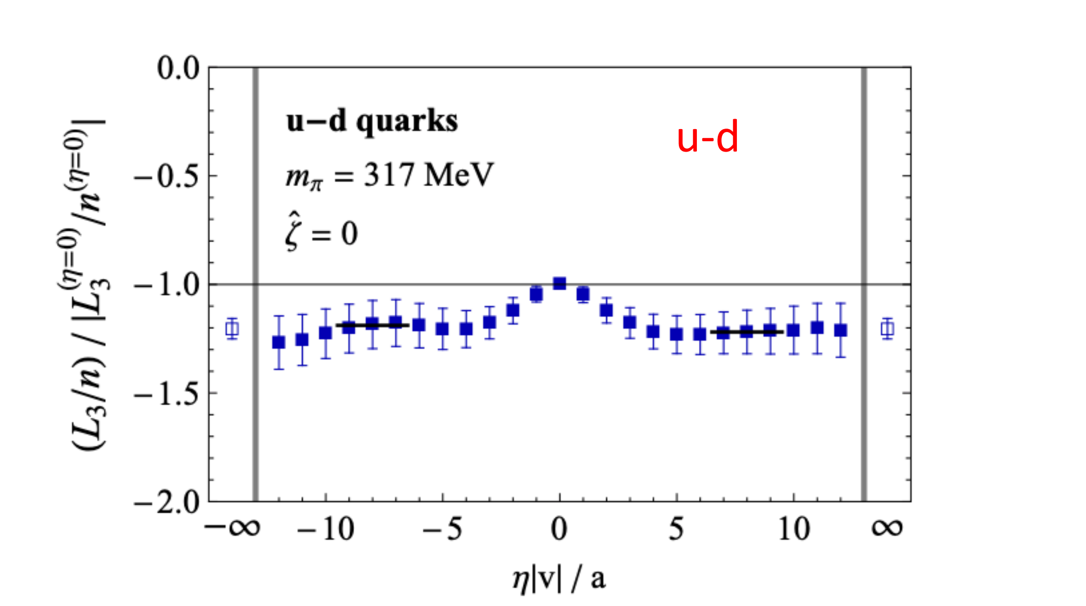
$$L_z^q = \int_0^1 dx \int d^2k_T \; k_T^2 F_{14}(x,0,0,k_T)$$

UL correlation Generalized TMD

QCD Correlation function: unintegrated and off-forward case

$$
\begin{aligned}
W^{\gamma^+}_{\Lambda'\Lambda} &= \frac{1}{2M}\overline{U}(p',\Lambda')\left[F_{11} + \frac{i\sigma^{i+}k^i}{P^+}F_{12} + \frac{i\sigma^{i+}\Delta^i}{P^+}F_{13} + \frac{i\sigma^{ij}k^i\Delta^j}{M^2}F_{14}\right]U(p,\Lambda) \\
&= \left[F_{11} + \frac{i\Lambda\epsilon^{ij}k^i\Delta^j}{M^2}F_{14}\right]\delta_{\Lambda'\Lambda} + \left[\frac{\Lambda\Delta^1 + i\Delta^2}{2M}(2F_{13} - F_{11}) + \frac{\Lambda k^1 + ik^2}{M}F_{12}\right]\delta_{-\Lambda'\Lambda}
\end{aligned}
$$

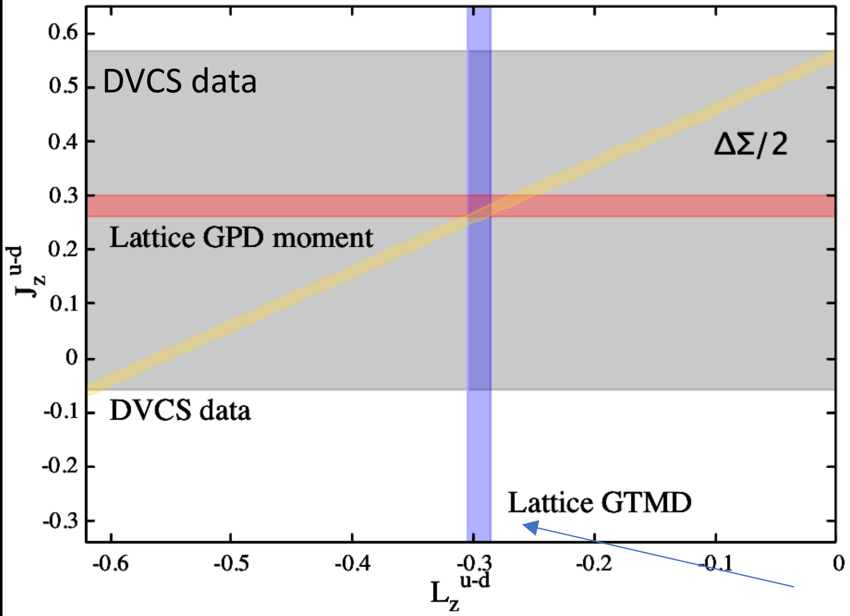
Calculable on lattice…



M. Engelhardt et al. *Phys.Rev. D* (2020)

Putting this all together: what we know from measurements and lattice

m_π = 320 MeV

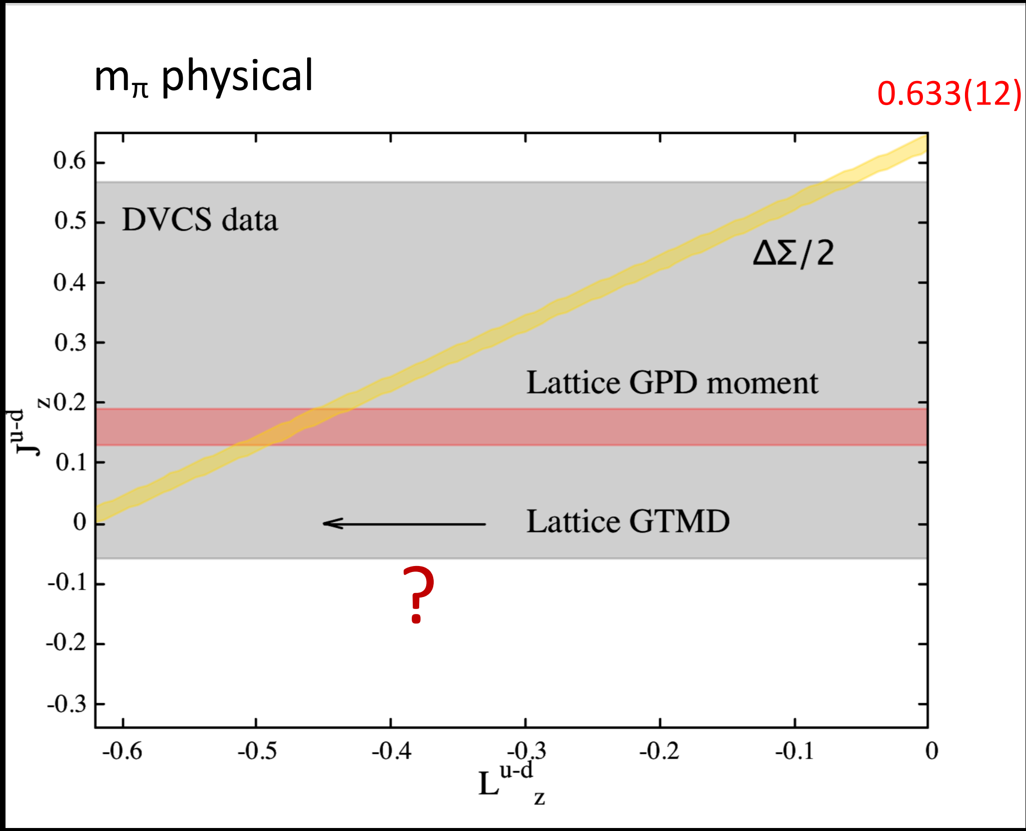


Mazouz et al.(Jlab Hall A) PRL (2007)

Engelhardt et al., Phys. Rev. D102, (2020)

q=u-d

$$J_q = L_q + \frac{1}{2}\Delta\Sigma_q$$


m_π physical
0.633(12)
DVCS data
$\Delta\Sigma/2$
Lattice GPD moment
Lattice GTMD
?
J_z^{u-d}
L_z^{u-d}
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0

Do we need to measure a GTMD to learn about OAM?

The other way that OAM is known

Polyakov Kiptily(2004), Hatta(2012)

$$\int_0^1 dx\, xG_2 = -\frac{1}{2}\int_0^1 dx\, x(H+E) + \frac{1}{2}\int_0^1 dx\, \tilde{H}$$

$$J_q = L_q + \frac{1}{2}\Delta\Sigma_q$$

A generalized integrated Wandzura Wilczek relation obtained using OPE for twist 2 and twist 3 operators for the off-forward matrix elements

While developing this approach we came across some interesting new aspects

- Advantage in going beyond the OPE k_T integrated structure to derive the different terms, using **nonlocal** matrix elements

- A dynamical picture emerges where we understand the working of final state interactions as being essential to generating quark angular momentum

- The interplay of partonic transverse momentum and twist three off-forward elements

A. Rajan, A. Courtoy, M. Engelhardt, S.L., PRD (2016)

A. Rajan, M. Engelhardt, S.L., PRD (2018)

A. Rajan, O. Alkassasbeh, M. Engelhardt, S.L., (2023)

Integral Relation

$$
\begin{aligned}
J_L \quad &= \quad L_L \quad + \quad S_L \\
\frac{1}{2}\int dx\, x(H+E) &= \int dx\, x(\widetilde{E}_{2T}+H+E) + \frac{1}{2}\int dx\, \widetilde{H} \\
&= -\int dx\, F_{14}^{(1)} + \frac{1}{2}\int dx\, \widetilde{H}
\end{aligned}
$$

- A. Rajan, A. Courtoy, M. Engelhardt, S.L., PRD (2016)
- A. Rajan, M. Engelhardt, S.L., PRD (2018)

*Twist 3 GPD notation from Meissner, Metz and Schlegel, JHEP(2009)

Quark Spin Orbit: $L \cdot S$: emerging role of chiral properties

$$\frac{1}{2}\int dx x \widetilde{H} + \frac{m_q}{2M}\kappa_T^q = \int dx x (2\widetilde{H}'_{2T} + E'_{2T} + \widetilde{H}) + \frac{1}{2} e_q$$

$J_z S_z$ $L_z S_z$ $S_z S_z$

- A. Rajan, A. Courtoy, M. Engelhardt, S.L., PRD (2016)
- A. Rajan, M. Engelhardt, S.L., PRD (2018)

*Twist 3 GPD notation from Meissner, Metz and Schlegel, JHEP(2009)

A closer look to $J_z S_z$

$$2(J_z S_z)_q \equiv 2[(J \cdot S)_q - (J_T \cdot S_T)_q] = \frac{1}{2}\int dx x\widetilde{H} + \frac{m_q}{2M}\kappa_T^q$$

$$\kappa_T^q = \int dx \left(E_T + 2\widetilde{H}_T\right) , \qquad e_q = \int dx\, H$$

Quark transverse anomalous magnetic moment
(M. Burkardt, PRD72 (2005)

Transverse Angular Momentum Sum Rule

O. Alkassasbeh, M. Engelhardt, SL and A. Rajan, (2022) soon on arXiv

$$\underbrace{\frac{1}{2}\int dx\, x(H+E) - \frac{1}{2}\int dx\, \mathcal{M}_T}_{J_T} = \underbrace{\int dx\, x\left(\widetilde{E}_{2T} + H + E + \frac{H_{2T}}{\xi}\right)}_{L_T} + \underbrace{\frac{1}{2}\int dx\, g_T - \frac{1}{2}\int dx\, x\mathcal{A}_T}_{S_T}$$

Twist 3 GPDs Physical Interpretation

GPD	$P_q P_p$	TMD	Ref. 1
$H^{\perp}$	UU	$f^{\perp}$	$2\widetilde{H}_{2T} + E_{2T}$
$\widetilde{H}_L^{\perp}$	LL	$g_L^{\perp}$	$2\widetilde{H}'_{2T} + E'_{2T}$
$H_L^{\perp}$	UL	$f_L^{\perp\,(*)}$	$\widetilde{E}_{2T} - \xi E_{2T}$
$\widetilde{H}^{\perp}$	LU	$g^{\perp\,(*)}$	$\widetilde{E}'_{2T} - \xi E'_{2T}$
$H_T^{(3)}$	UT	$f_T^{(*)}$	$H_{2T} + \tau \widetilde{H}_{2T}$
$\widetilde{H}_T^{(3)}$	LT	g'_T	$H'_{2T} + \tau \widetilde{H}'_{2T}$

1/Q correction to H

1/Q correction to $\widetilde{\mathrm{H}}$

NEW!! Orbital Angular Momentum L

NEW!! Spin Orbit correlation $\mathrm{L} \cdot \mathrm{S}$

NEW!! Transverse OAM $\mathrm{L_T}$

Transverse spin

(*) T-odd

[1] Meissner, Metz and Schlegel, JHEP(2009)

Energy Momentum tensor in deuteron

$$\langle p'|T^{\mu\nu}|p\rangle = -\frac{1}{2}P^\mu P^\nu(\epsilon'^*\epsilon)\mathcal{G}_1(t)$$
$$-\ \frac{1}{4}P^\mu P^\nu\frac{(\epsilon P)(\epsilon'^* P)}{M^2}\mathcal{G}_2(t) - \frac{1}{2}\left[\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right](\epsilon'^*\epsilon)$$
$$\times\ \mathcal{G}_3(t) - \frac{1}{4}\left[\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right]\frac{(\epsilon P)(\epsilon'^* P)}{M^2}\mathcal{G}_4(t)$$
$$+\ \frac{1}{4}\left[(\epsilon'^{*\mu}(\epsilon P) + \epsilon^\mu(\epsilon'^* P))P^\nu + \mu\leftrightarrow\nu\right]\mathcal{G}_5(t)$$
$$+\ \frac{1}{4}\left[(\epsilon'^{*\mu}(\epsilon P) - \epsilon^\mu(\epsilon'^* P))\Delta^\nu + \mu\leftrightarrow\nu\right]\mathcal{G}_6(t)$$
$$+\ \frac{1}{2}\left(\epsilon^{*\prime\mu}\epsilon^\nu + \epsilon'^{*\nu}\epsilon^\mu - \frac{1}{2}g^{\mu\nu}(\epsilon'^*\epsilon)\right)M^2\mathcal{G}_7(t)$$
$$+\ g^{\mu\nu}\left[(\epsilon'^*\epsilon)M^2\mathcal{G}_8(t) + (\epsilon P)(\epsilon^{*\prime}P)\mathcal{G}_9(t)\right]$$

DEUTERON

Non diagonal Hadronic Tensor for Deut.

$$\int\frac{d\kappa}{2\pi}e^{ix\kappa P.n}\langle p',\lambda'|\bar\psi(-\kappa n)\gamma.n\,\psi(\kappa n)|p,\lambda\rangle$$
$$= -(\epsilon'^*.\epsilon)H_1 + \frac{(\epsilon.n)(\epsilon'^*.P) + (\epsilon'^*.n)(\epsilon.P)}{P.n}H_2$$
$$-\frac{(\epsilon.P)(\epsilon'^*.P)}{2M^2}H_3 + \frac{(\epsilon.n)(\epsilon'^*.P) - (\epsilon'^*.n)(\epsilon.P)}{P.n}H_4$$
$$+\left\{4M^2\frac{(\epsilon.n)(\epsilon'^*.n)}{(P.n)^2} + \frac{1}{3}(\epsilon'^*.\epsilon)\right\}H_5 \qquad (7)$$

Berger, Cano Diehl, Pire, PRL(2001)

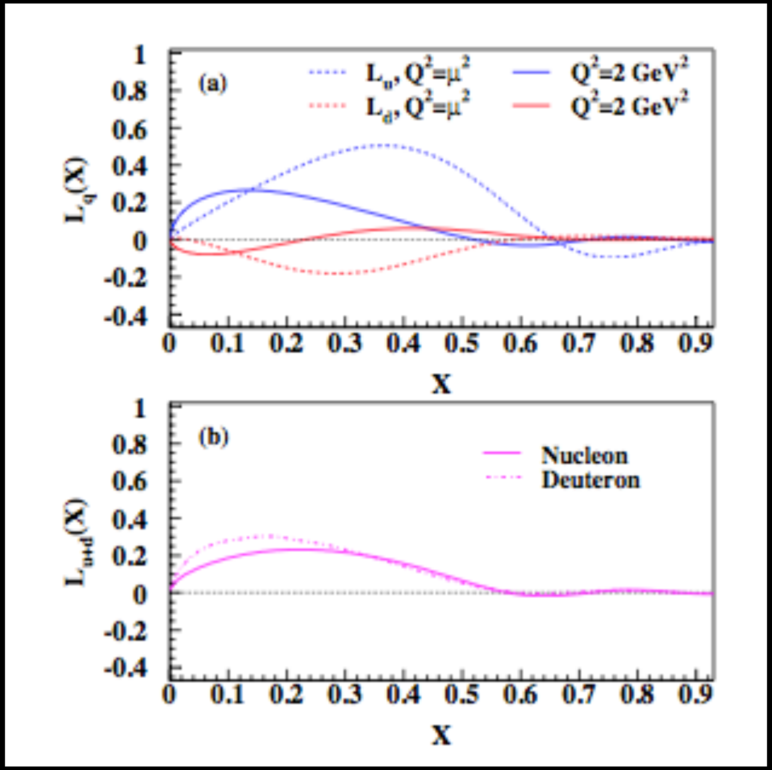
$$\langle p'| \int d^3x (\vec{x} \times \vec{T}^{0i}_{q,g})_z |p\rangle = G_{5,2}(0) \int d^3x \; p^0$$

$$G_{5,2}(0) = 2J^q_z$$

$$\begin{aligned}
\int dx x H_1(x,\xi,t) - \frac{1}{3}\int dx x H_5(x,\xi,t) &= G_{1,2}(t) + \xi^2 G_{3,2}(t) \\
\int dx x H_2(x,\xi,t) &= G_{5,2}(t) \\
\int dx x H_3(x,\xi,t) &= G_{2,2}(t) + \xi^2 G_{4,2}(t) \\
\int dx x H_4(x,\xi,t) &= \xi \; G_{6,2}(t) \\
\int dx x H_5(x,\xi,t) &= G_{7,2}(t)
\end{aligned}$$

J_z sum rule

$$J_q = \frac{1}{2} \int dx \, x \, H^q_2(x, 0, 0),$$



New sum rule for spin 1 system ➔deuteron

$$J_q = \frac{1}{2}\int dx\, x\left[H_q(x,0,0) + E_q(x,0,0)\right],$$

$$J_q = \frac{1}{2}\int dx\, x\, H_2^q(x,0,0),$$

$F_1+F_2= G_M$

G_M

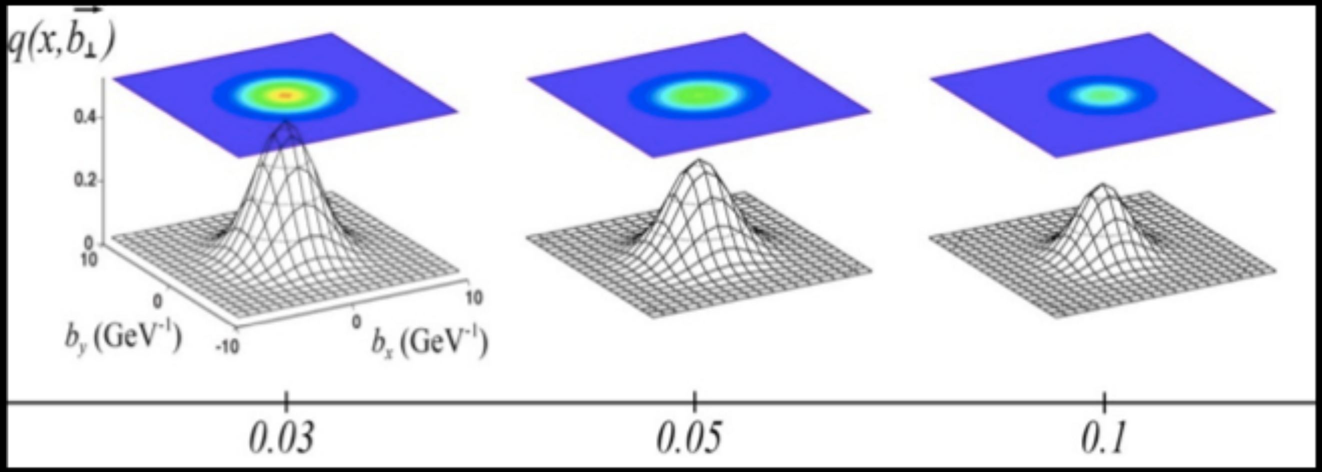
Summary of this part

- ✓ We obtain a clean connection to observables
- ✓ The sum rules, written in terms of twist-2 and twist-3 observables, are an intrinsic feature of (are derived directly from) the QCD EoM, and preserve Lorentz symmetry

Many open questions !!

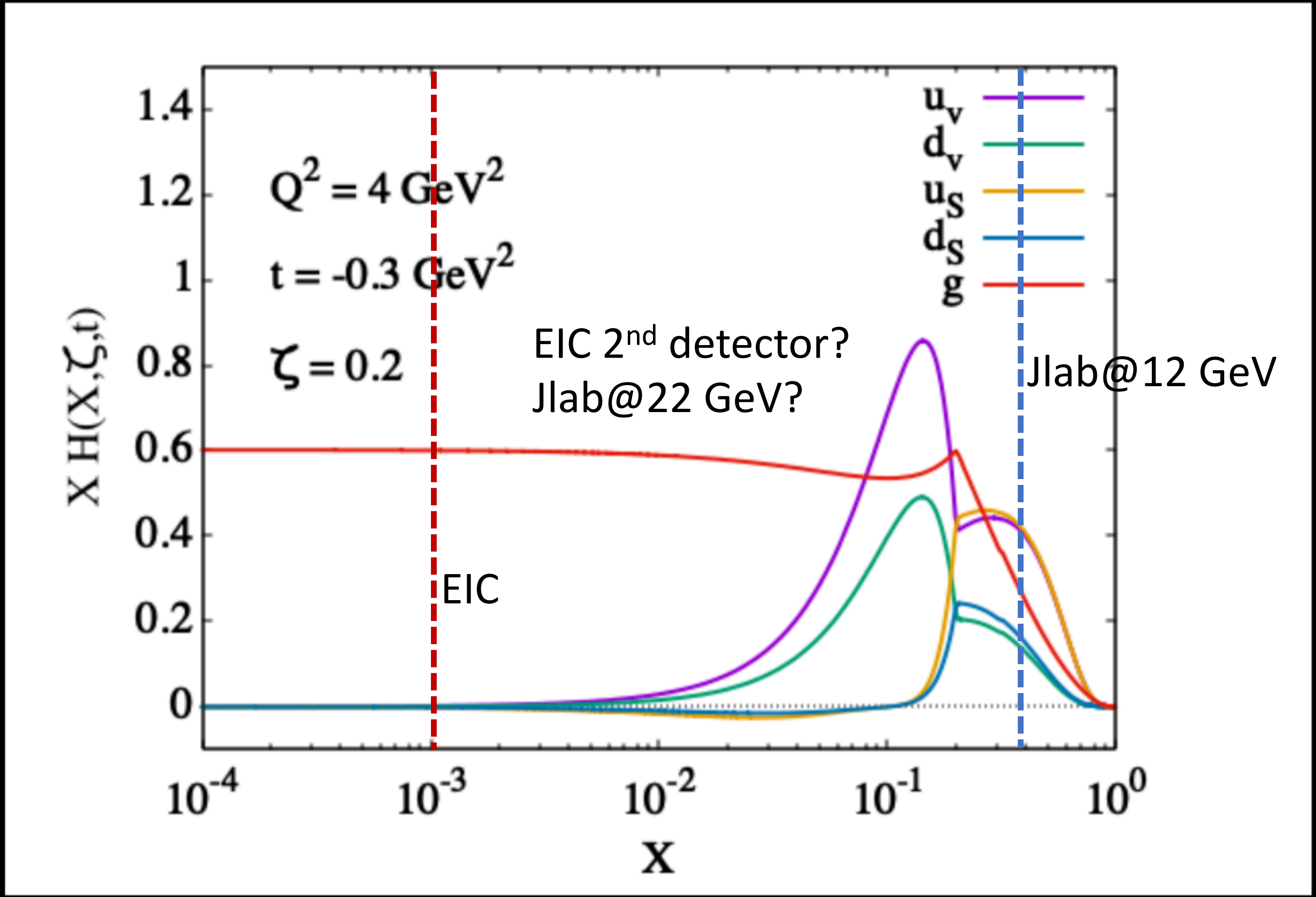
- Can we extend this to mass?
- What are the observables for the mass distribution?
- Tensor components?

Measuring All This



graph from M. Defurne

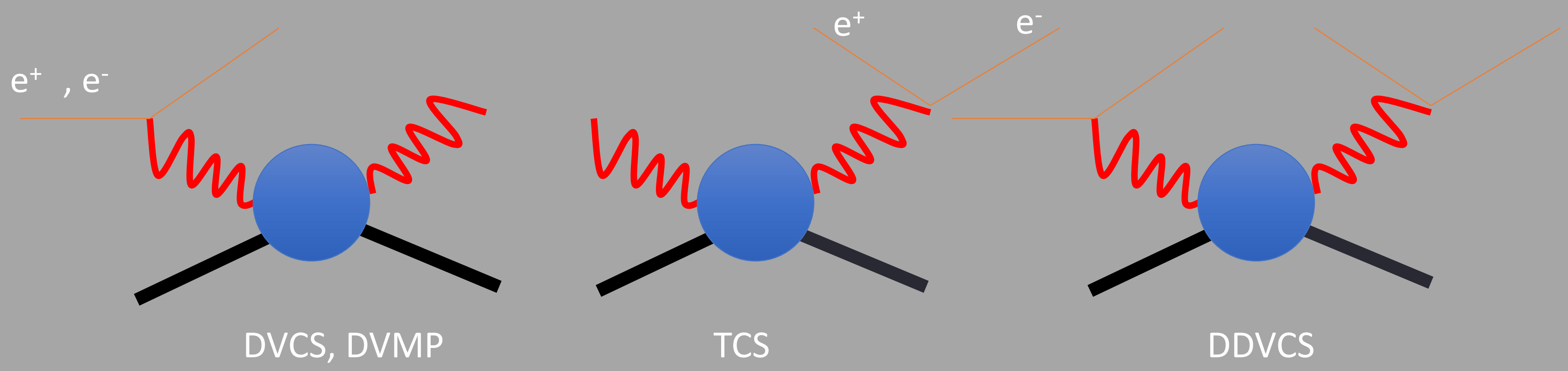
All GPDs

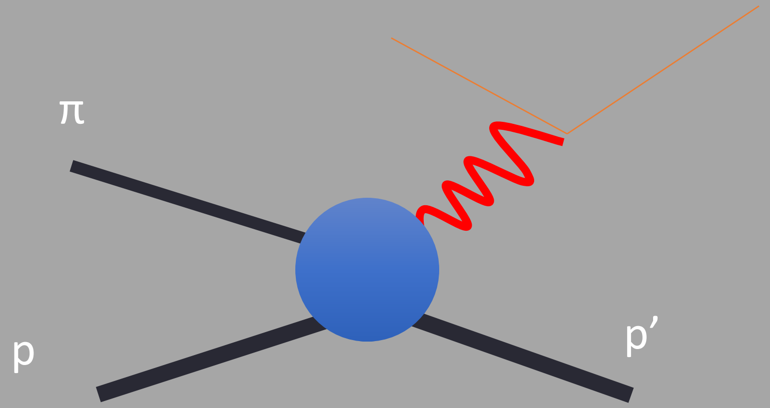


- B. Kriesten, P. Velie, E. Yeats, F. Y. Lopez and S. Liuti, *Phys. Rev D 105 (2022)*, arXiv:2101.01826

10/31/20

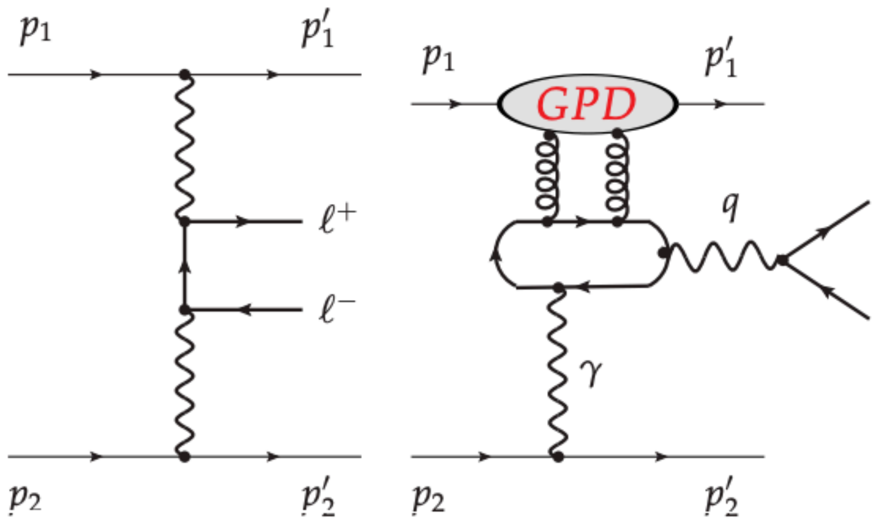
Harnessing/coordinating information from all channels





Exclusive pion induced DY (EDY)

Sawada et al., PRD93 (2016)



(a) BH (b) TCS with gluon GPDs

Based on…

arXiv:1903.05742

Extraction of Generalized Parton Distribution Observables from Deeply Vir… Electron Proton Scattering Experiments

Brandon Kriesten,* Simonetta Liuti,† Liliet Calero Diaz,‡ Dustin Keller,§ and Andrew Meyer¶

Department of Physics, University of Virginia, Ch…

Gary R. Goldstein

Department of Physics and Astronomy, Tufts Univ…

J. Osvaldo Gonzalez-Her…

INFN, Torino

(Dated: April 6, 201…

We provide the general expression of the cross section … troproduction from a spin 1/2 target using current param… function in a nucleon for different beam and target pola… accuracy. All contributions to the cross section including… Bethe-Heitler process, and their interference, are describe… work which is also relativistically … in a collider kinematic setting. C… components of the cross section … the electron scattering coinciden… tum, J_z, and on the orbital angu… extraction of J_z, given by the c… generalization of the Rosenbluth … the other, we single out for the … that are sensitive to L_z. The pr…

Theory of Deeply Virtual Compton Scattering off the Unpolarized Proton

Brandon Kriesten* and Simonetta Liuti†

Department of Physics, University of Virginia, Charlottesville, VA 22904, USA.

Using the helicity amplitudes formalism, we study deeply virtual exclusive electron photoproduction off an unpolarized nucleon target, $ep \to e'p'\gamma$, through a range of kinematic settings for the initial electron energy in the laboratory system, 6 GeV, 11 GeV and 24 GeV, which are ideal for studying the 3D quark structure of the nucleon. We use a reformulation of the cross section that brings to the forefront the defining features of the $ep \to e'p'\gamma$ process as a *coincidence scattering* experiment, where the observables are expressed as bilinear products of the independent helicity amplitudes which completely describe the process in terms of the electric, magnetic and axial currents for the nucleon. These different contributions to the cross section are checked against the Fourier harmonics-based formalism which has provided so far the underlying mathematical framework to study Deeply virtual Compton scattering and related experiments. Using two different sets of the …

Proton Compton Form Factors $\mathcal{H}$ and $\mathcal{E}$ from the Rosenbluth Separation Technique in Deeply Virtual Exclusive Photoproduction

Brandon Kriesten,* Simonetta Liuti,† and Andrew Meyer‡

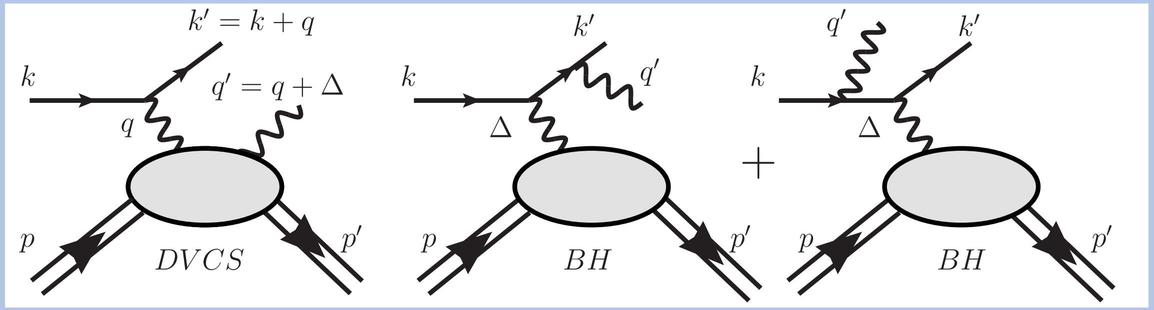
Department of Physics, University of Virginia, Charlottesville, VA 22904, USA.

We present an extraction from available Deeply Virtual Compton Scattering data on an unpolarized proton target, of the imaginary and real parts of the Compton Form Factors H, E, and, separately the combination, $H + E$. The latter is essential for the extraction of angular momentum. The extraction technique is a generalization of the Rosenbluth separation method which exploits the linear dependence in the kinematic coefficients of the reduced cross section to independently determine the Compton form factors as the slope and intercept.

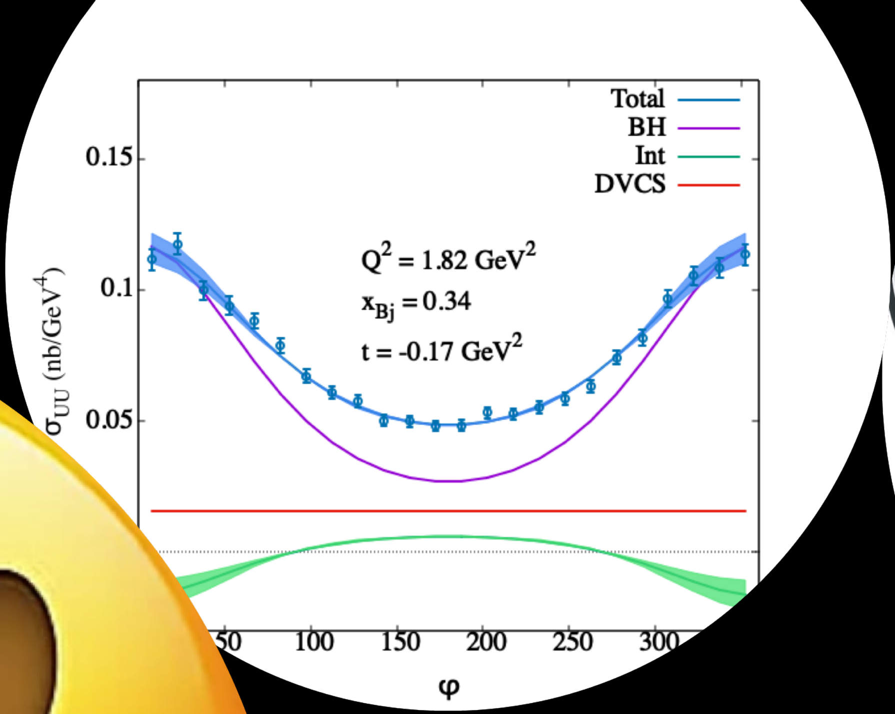
We need a robust framework for the cross section, where kinematic limits are under control (beyond "harmonics" model)

$$\frac{d^5\sigma}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}|T|^2,$$

$$T(k,p,k',q',p') = T_{DVCS}(k,p,k',q',p') + T_{BH}(k,p,k',q',p'),$$

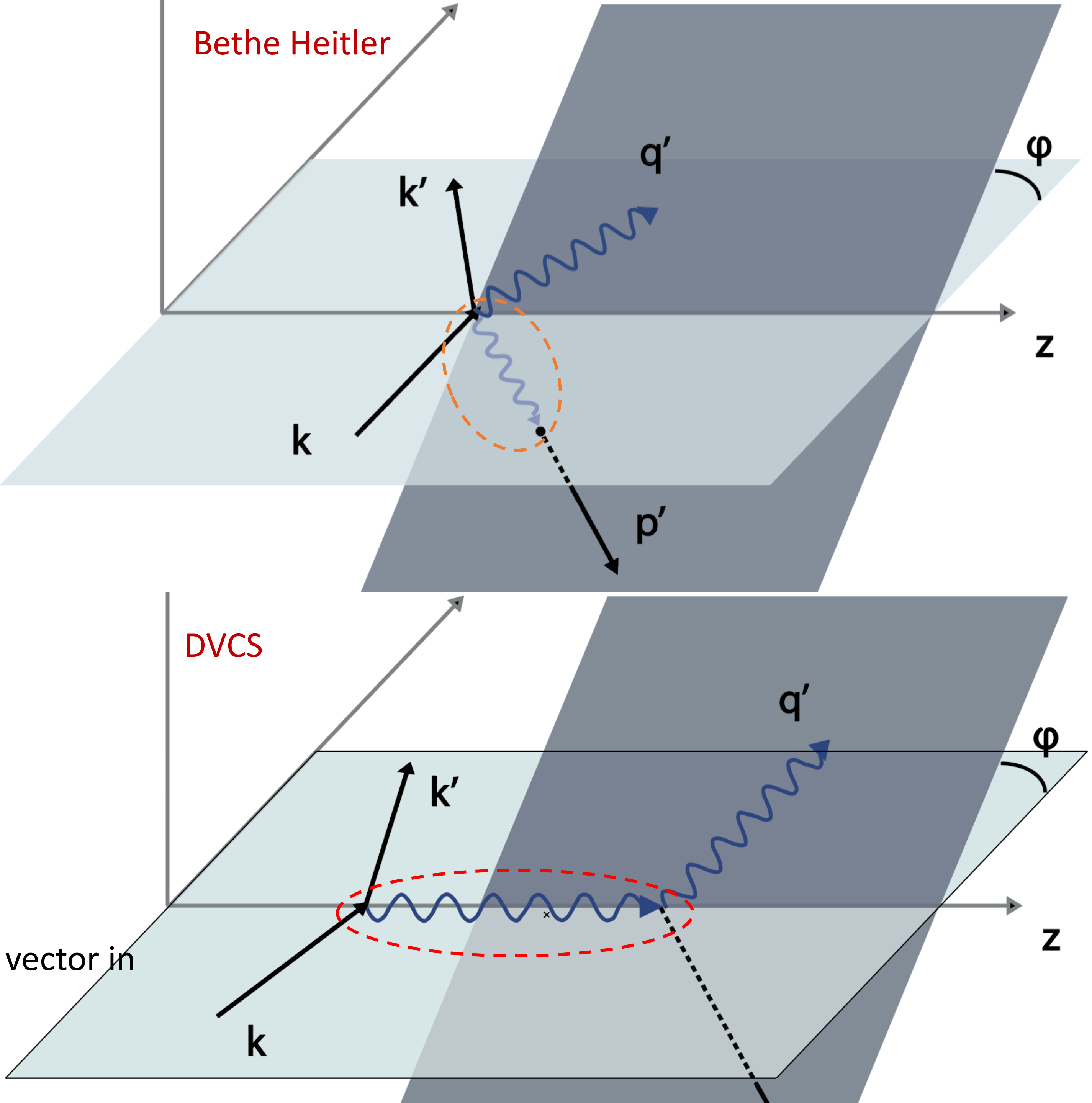


B. Kriesten et al, *Phys.Rev. D* 101 (2020)
B. Kriesten and S. Liuti, arXiv 2004.08890



… setting up the scene to enable QCD-based interpretations

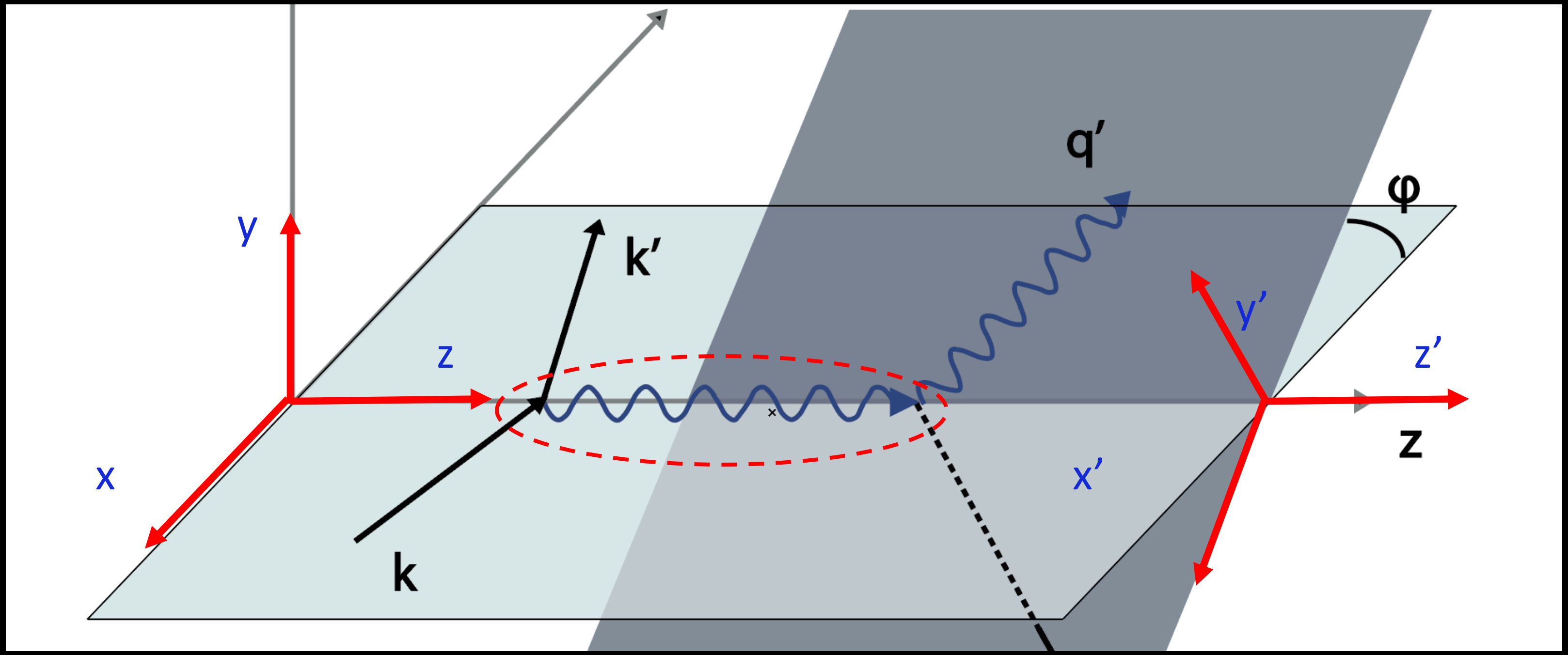
Demystification of harmonics formalism:



- In DVCS the virtual photon is along the z axis: φ dependence from usual rotation of polarization vector in helicity amp

To understand the cross section we need to understand the ϕ dependence

DVCS



The hadronic tensor is evaluated in the rotated frame

BH

$$\frac{d^5\sigma^{BH}_{unpol}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} \equiv \frac{\Gamma}{t}F^{BH}_{UU} = \frac{\Gamma}{t}\left[A(y,x_{Bj},t,Q^2,\phi)\left(F_1^2+\tau F_2^2\right) + B(y,x_{Bj},t,Q^2,\phi)\tau G_M^2(t)\right]$$

$$A = \frac{16\,M^2}{t(k\,q')(k'\,q')}\left[4\tau\left((k\,P)^2+(k'\,P)^2\right) - (\tau+1)\left((k\,\Delta)^2+(k'\,\Delta)^2\right)\right]$$

$$B = \frac{32\,M^2}{t(k\,q')(k'\,q')}\left[(k\,\Delta)^2+(k'\,\Delta)^2\right],$$

$$\epsilon_{BH} = \left(1+\frac{B}{A}(1+\tau)\right)^{-1}$$

…compared to ELASTIC SCATTERING

10/21/21

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{\epsilon (G_E^N)^2 + \tau (G_M^N)^2}{\epsilon (1+\tau)},$$

where $N = p$ for a proton and $N = n$ for a neutron, (
the recoil-corrected relativistic point-particle (Mott)
and τ, ϵ are dimensionless kinematic variables:

$$\tau = \frac{Q^2}{4m_N^2}, \quad \epsilon = \left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]^{-1},$$

J. Arrington, G. Cates, S. Riordan, Z. Ye, B. Wojsetowski, A. Puckett …

…compared to BKM, NPB (2001)

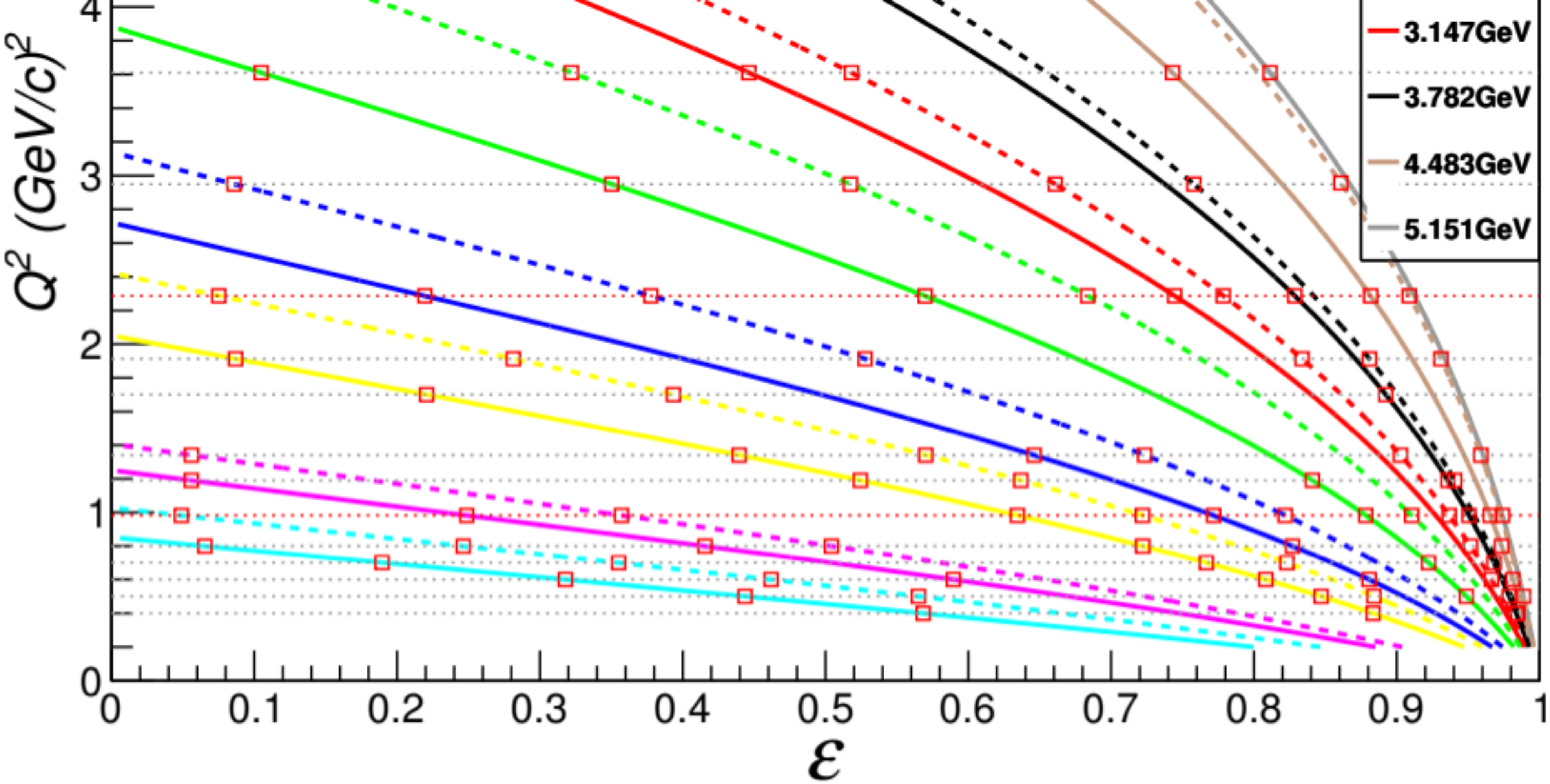
$$|\mathcal{T}_{\rm BH}|^2 = \frac{e^6}{x_{\rm B}^2 y^2 (1+\epsilon^2)^2 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \times \left\{ c_0^{\rm BH} + \sum_{n=1}^{2} c_n^{\rm BH} \cos(n\phi) + s_1^{\rm BH} \sin(\phi) \right\},$$

$$c^{\rm BH}_{0,\rm unp} = 8K^2\left\{(2+3\epsilon^2)\frac{Q^2}{\Delta^2}\left(F_1^2 - \frac{\Delta^2}{4M^2}F_2^2\right) + 2x_{\rm B}^2(F_1+F_2)^2\right\}$$
$$+ (2-y)^2\left\{(2+\epsilon^2)\left[\frac{4x_{\rm B}^2M^2}{\Delta^2}\left(1+\frac{\Delta^2}{Q^2}\right)^2 + 4(1-x_{\rm B})\left(1+x_{\rm B}\frac{\Delta^2}{Q^2}\right)\right]\left(F_1^2-\frac{\Delta^2}{4M^2}F_2^2\right) + 4x_{\rm B}^2\left[x_{\rm B}+\left(1-x_{\rm B}+\frac{\epsilon^2}{2}\right)\left(1-\frac{\Delta^2}{Q^2}\right)^2 - x_{\rm B}(1-2x_{\rm B})\frac{\Delta^4}{Q^4}\right](F_1+F_2)^2\right\}$$
$$+ 8\left(1+\epsilon^2\right)\left(1-y-\frac{\epsilon^2y^2}{4}\right) \times \left\{2\epsilon^2\left(1-\frac{\Delta^2}{4M^2}\right)\left(F_1^2-\frac{\Delta^2}{4M^2}F_2^2\right) - x_{\rm B}^2\left(1-\frac{\Delta^2}{Q^2}\right)^2(F_1+F_2)^2\right\},$$

A.V. Belitsky et al. / Nuclear Physics B 629 (2002) 323–392

$$c^{\rm BH}_{1,\rm unp} = 8K(2-y)\left\{\left(\frac{4x_{\rm B}^2M^2}{\Delta^2} - 2x_{\rm B} - \epsilon^2\right)\left(F_1^2-\frac{\Delta^2}{4M^2}F_2^2\right) + 2x_{\rm B}^2\left(1-(1-2x_{\rm B})\frac{\Delta^2}{Q^2}\right)(F_1+F_2)^2\right\},$$

$$c^{\rm BH}_{2,\rm unp} = 8x_{\rm B}^2K^2\left\{\frac{4M^2}{\Delta^2}\left(F_1^2-\frac{\Delta^2}{4M^2}F_2^2\right) + 2(F_1+F_2)^2\right\}.$$



igure 3.2: The E05-017 nominal kinematic coverage. The solid and dashed lines are constan

ttings.

M. Yurov, Ph.D. thesis

DVCS

$$\frac{d^5\sigma_{DVCS}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}\left|T_{DVCS}\right|^2$$

$$= \frac{\Gamma}{Q^2(1-\epsilon)}\Big\{\Big[F_{UU,T} + \epsilon F_{UU,L} + \epsilon\cos 2\phi F_{UU}^{\cos 2\phi}$$

$$+ \sqrt{\epsilon(\epsilon+1)}\left[\cos\phi F_{UU}^{\cos\phi} + \sin\phi F_{UU}^{\sin\phi}\right]$$

$$+ \lambda_e \sqrt{2\epsilon(1-\epsilon)}\sin\phi F_{LU}^{\sin\phi}\Big]$$

$$+ S_L \left[F_{UL} + \sqrt{\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi} + \epsilon\sin 2\phi F_{UL}^{\sin 2\phi}\right]$$

$$+ \lambda_e \sqrt{1-\epsilon^2}F_{LL} + 2\lambda_e\sqrt{\epsilon(1-\epsilon)}\cos\phi F_{LL}^{\cos\phi}\Big]$$

$$+ |S_T|\Big[\sin(\phi-\phi_S)\left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)}\right)$$

$$\epsilon\sin(\phi+\phi_S)F_{UT}^{\sin(\phi+\phi_S)} + \epsilon\sin(3\phi-\phi_S)F_{UT}^{\sin(3\phi-\phi_S)}$$

$$+\sqrt{2\epsilon(1+\epsilon)}\left(\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)}\right)\Big]$$

$$+ (\lambda_e\ S_L\ \Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S)F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\cos\phi_S F_{LT}^{\cos\phi_S}$$

$$+\sqrt{2\epsilon(1-\epsilon)}\cos(2\phi-\phi_S)F_{LT}^{\cos(2\phi-\phi_S)}\Big]\Big\}$$

twist two GPDs

twist three GPDs

U

L

T

Example: UNPOLARIZED target

$$F_{UU,T} = 4(1-\xi^2)\left\{\left|\mathcal{H}-\frac{\xi^2}{1-\xi^2}\mathcal{E}\right|^2+\left|\widetilde{\mathcal{H}}-\frac{\xi^2}{1-\xi^2}\widetilde{\mathcal{E}}\right|^2+\frac{t_o-t}{2M^2}\left(|\,\mathcal{E}\,|^2+\xi^2\,|\,\widetilde{\mathcal{E}}\,|^2\right)\right\}$$

$$F_{UU,L} = \frac{4(t_0-t)}{Q^2}x_{Bj}^2(1-\xi)^2\frac{t_0-t}{2M^2}\,2\left|\mathcal{H}^{\perp}+\widetilde{\mathcal{H}}_L^{\perp}-\xi\mathcal{H}_L^{\perp}-\xi\widetilde{\mathcal{H}}^{\perp}\right|^2$$

Example: UNPOLARIZED target

$$F_{UU}^{\cos\phi} = -\frac{2\sqrt{t_0-t}\,x_{Bj}(1-\xi)}{\sqrt{Q^2}}\,(1-\xi^2)\,\Re e\Big\{\left(\mathcal{H}^{\perp}+\widetilde{\mathcal{H}}_L^{\perp}\right)^*\left(\mathcal{H}-\frac{\xi^2}{1-\xi^2}\mathcal{E}\right)+\left(\mathcal{H}_T^{(3)}+\widetilde{\mathcal{H}}_T^{(3)}\right)^*\left(\mathcal{E}-\xi\widetilde{\mathcal{E}}\right)$$
$$-2\xi\left(\mathcal{H}_L^{\perp}+\widetilde{\mathcal{H}}_L^{\perp}\right)^*\left(\widetilde{\mathcal{H}}-\frac{\xi^2}{1-\xi^2}\widetilde{\mathcal{E}}\right)+\frac{\xi}{1-\xi^2}\left(\mathcal{H}_L^{\perp}+\widetilde{\mathcal{H}}_L^{\perp}\right)^*\left(\mathcal{E}-\xi\widetilde{\mathcal{E}}\right)+\frac{t_0-t}{16M^2}\left(\mathcal{H}_T^{\perp}+\widetilde{\mathcal{H}}_T^{\perp}\right)^*\left(\mathcal{E}+\xi\widetilde{\mathcal{E}}\right)\Big\}$$

$$F_{LU}^{\sin\phi} = -\frac{2\sqrt{t_0-t}\,x_{Bj}(1-\xi)}{\sqrt{Q^2}}\,(1-\xi^2)\,\Im m\Big\{\left(\mathcal{H}^{\perp}+\widetilde{\mathcal{H}}_L^{\perp}\right)^*\left(\mathcal{H}-\frac{\xi^2}{1-\xi^2}\mathcal{E}\right)+\left(\mathcal{H}_T^{(3)}+\widetilde{\mathcal{H}}_T^{(3)}\right)^*\left(\mathcal{E}-\xi\widetilde{\mathcal{E}}\right)$$
$$-\;2\xi\left(\mathcal{H}_L^{\perp}+\widetilde{\mathcal{H}}_L^{\perp}\right)^*\left(\widetilde{\mathcal{H}}-\frac{\xi^2}{1-\xi^2}\widetilde{\mathcal{E}}\right)+\frac{\xi}{1-\xi^2}\left(\mathcal{H}_L^{\perp}+\widetilde{\mathcal{H}}_L^{\perp}\right)^*\left(\mathcal{E}-\xi\widetilde{\mathcal{E}}\right)+\frac{t_0-t}{16M^2}\left(\mathcal{H}_T^{\perp}+\widetilde{\mathcal{H}}_T^{\perp}\right)^*\left(\mathcal{E}+\xi\widetilde{\mathcal{E}}\right)\Big\}$$

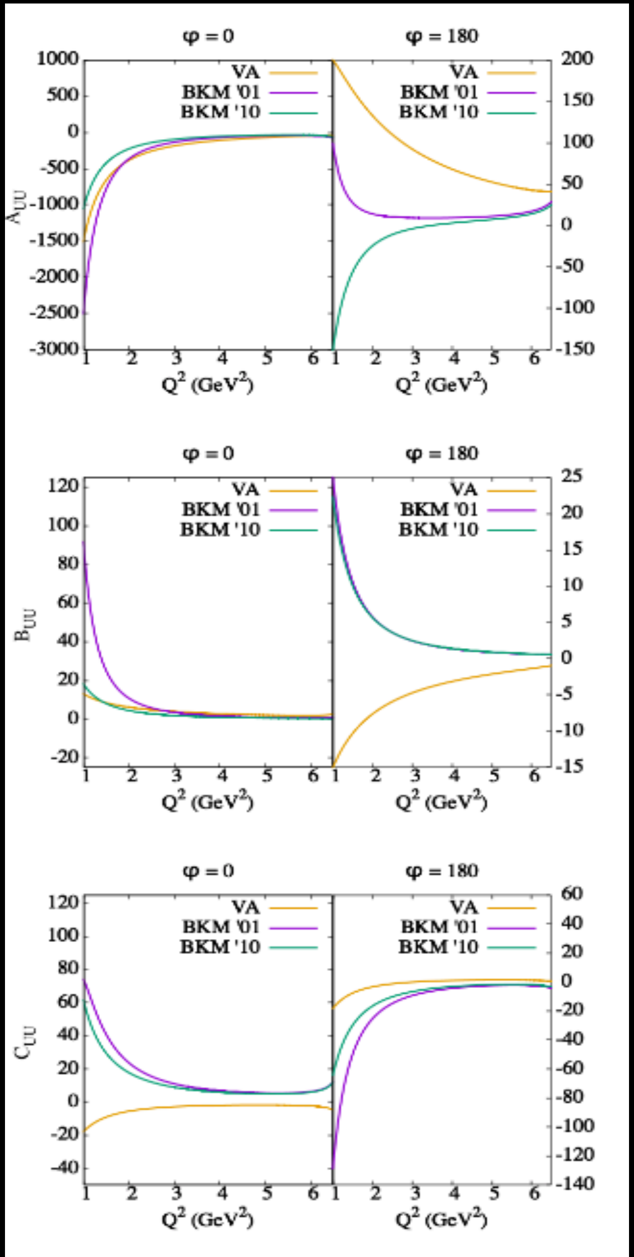
Example: UNPOLARIZED target

$$
\begin{aligned}
F_{UU}^{\cos 2\phi} = \; & -2\frac{\alpha_S}{2\pi}\sqrt{1-\xi^2}\frac{t_0-t}{4M^2}\,\Re\mathrm{e}\Bigg[\sqrt{1-\xi^2}\left(\widetilde{\mathcal{H}}_T^g + (1-\xi)\frac{\mathcal{E}_T^g+\widetilde{\mathcal{E}}_T^g}{2}\right)\left(\mathcal{H}+\widetilde{\mathcal{H}}-\frac{\xi^2}{1-\xi^2}(\mathcal{E}+\widetilde{\mathcal{E}})\right)^* \\
+ \; & \sqrt{1-\xi^2}\left(\widetilde{\mathcal{H}}_T^g + (1+\xi)\frac{\mathcal{E}_T^g-\widetilde{\mathcal{E}}_T^g}{2}\right)\left(\mathcal{H}-\widetilde{\mathcal{H}}-\frac{\xi^2}{1-\xi^2}(\mathcal{E}+\widetilde{\mathcal{E}})\right)^* \\
+ \; & \frac{\sqrt{t_0-t}}{2M}\left(\widetilde{\mathcal{H}}_T^g + (1+\xi)\frac{\mathcal{E}_T^g-\widetilde{\mathcal{E}}_T^g}{2}\right)\left(\mathcal{E}+\xi\widetilde{\mathcal{E}}\right)^* \\
- \; & \sqrt{1-\xi^2}\left(\mathcal{H}_T^g + \frac{t_0-t}{M^2}\widetilde{\mathcal{H}}_T^g - \frac{\xi^2}{1-\xi^2}\mathcal{E}_T^g + \frac{\xi}{1-\xi^2}\widetilde{\mathcal{E}}_T^g\right)\left(\mathcal{E}-\xi\widetilde{\mathcal{E}}\right)^*\Bigg]
\end{aligned}
$$

BH-DVCS interference

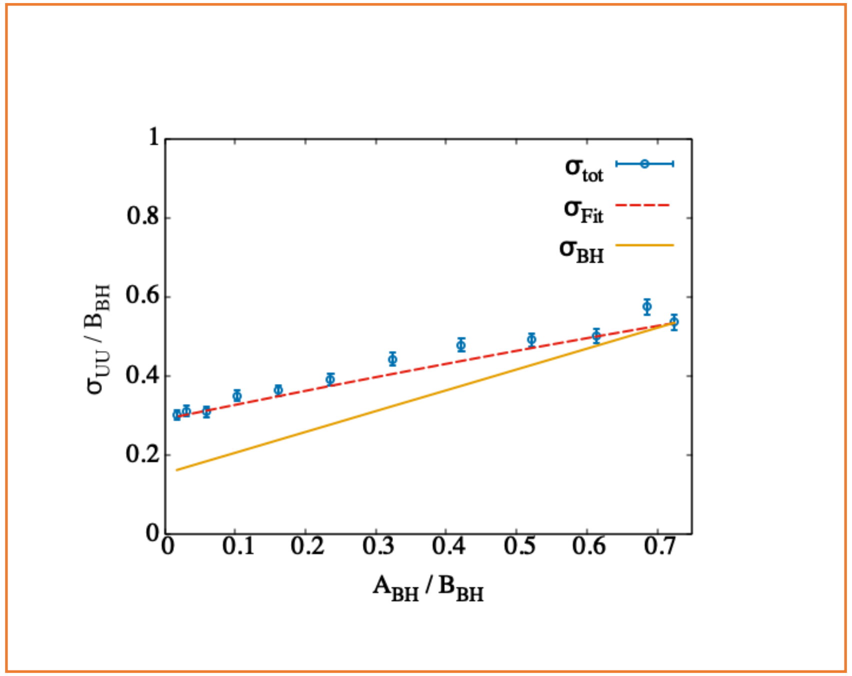
$$F_{UU}^{\mathcal{I},tw2} = A_{UU}^{\mathcal{I}} \Re e \left(F_1 \mathcal{H} + \tau F_2 \mathcal{E}\right) + B_{UU}^{\mathcal{I}} G_M \Re e \left(\mathcal{H} + \mathcal{E}\right) + C_{UU}^{\mathcal{I}} G_M \Re e \widetilde{\mathcal{H}}$$

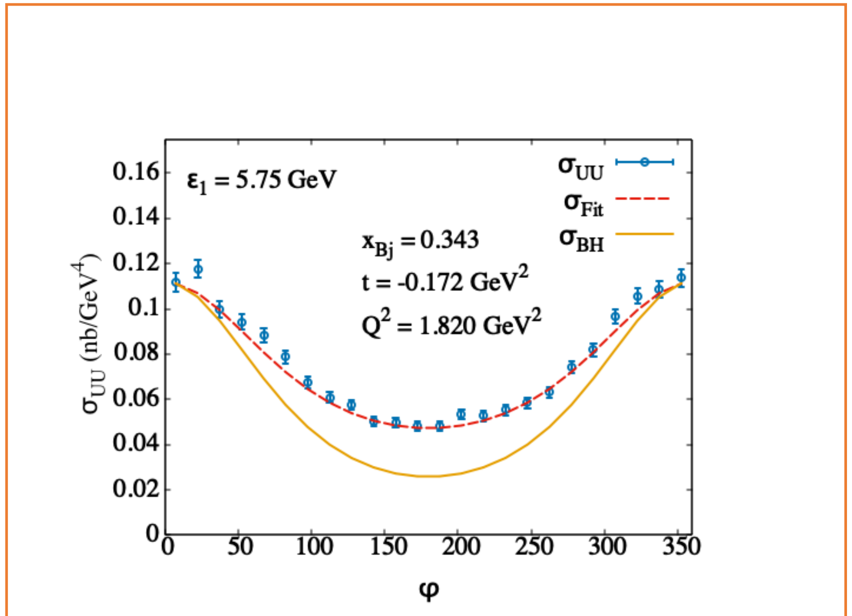
A_{UU}^{I} B_{UU}^{I} C_{UU}^{I} are φ dependent coefficients



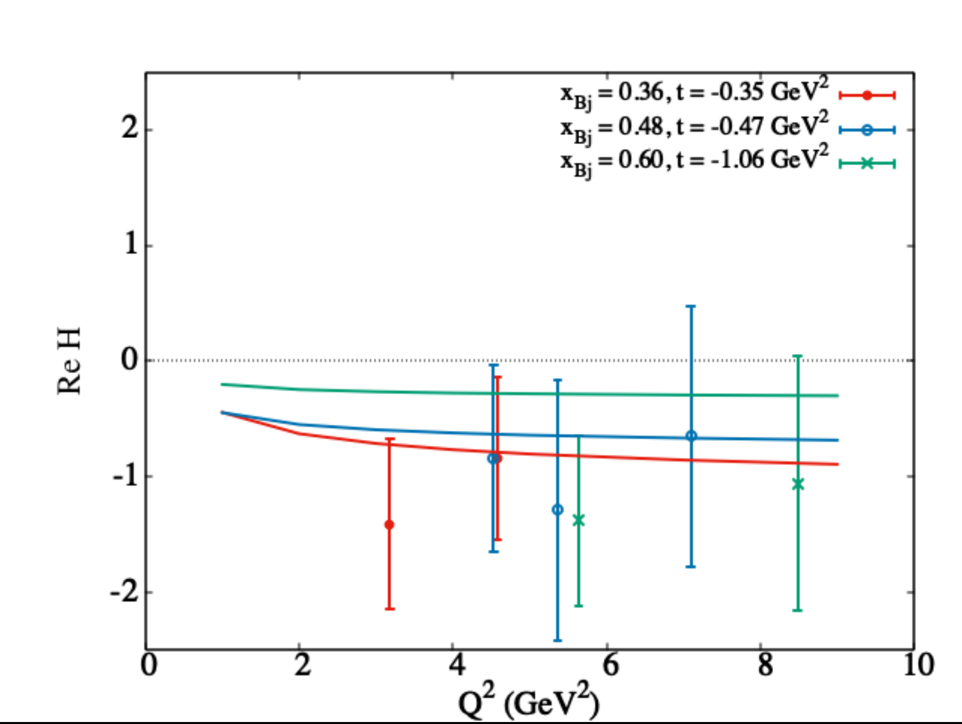
Streamlined description of cross section

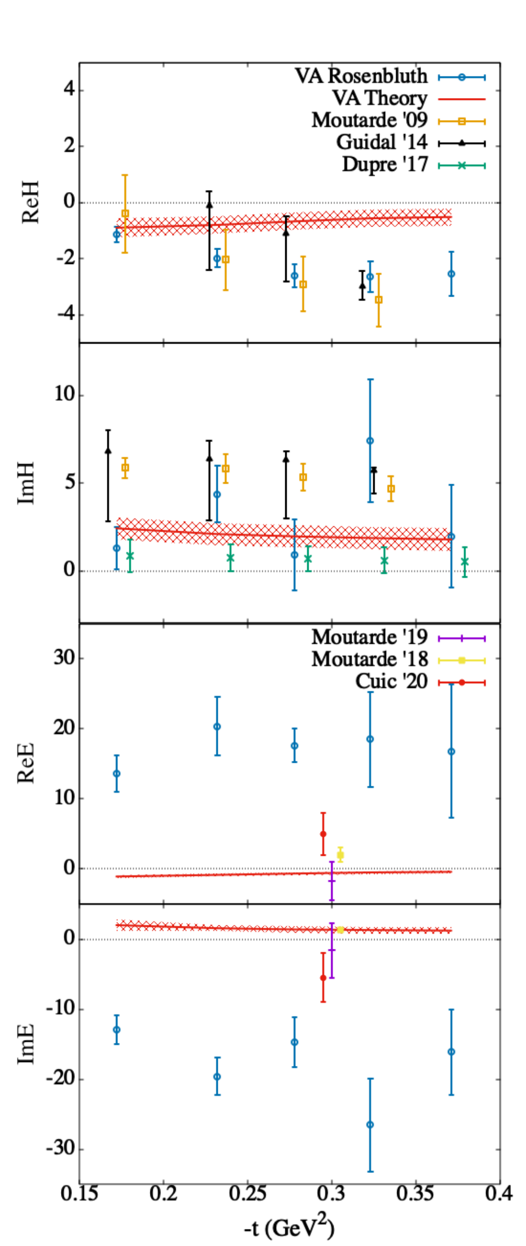
- Rosenbluth Separated BH-DVCS interference data





Compton Form Factor Extraction





Q^2 dependence

Deuteron Helicity Amplitudes

$$C_{\lambda'\lambda'_q,\lambda\lambda_q} = \sum_{\lambda_N,\lambda'_N} B_{\lambda'\lambda'_N,\lambda\lambda_N} \otimes A_{\lambda'_N\lambda'_q,\lambda_N\lambda_q},$$

$$H_2 = 2\left[(C_{++,++} + C_{+-,+-}) - \frac{1}{\sqrt{2D}}(C_{++,0+} + C_{+-,0-})\right]$$
$$= \int_0^{M_D/M} dz f^{++}(z)H_+(x/z,0,0) + f^{0+}(z)E_+(x/z,0,0),$$

Observable

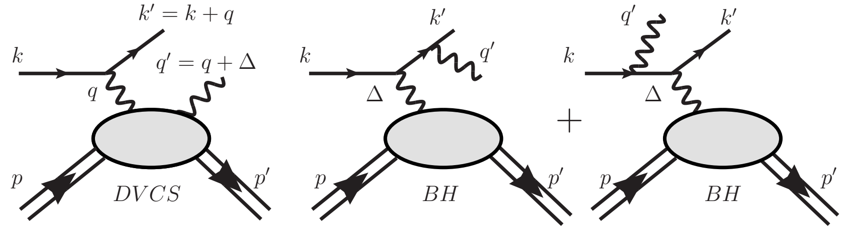
$$A_{UT} \sim \mathrm{Im}\{H_1^{*} H_5 + (H^{*}{}_1 + \frac{1}{6} H^{*}{}_5)(H_2 - H_4)\}$$

Tensor polarized GPD

We can access J !

We need a robust framework for DVES processes cross section, where kinematic limits are under control

To observe, evaluate and interpret GPDs and Wigner distributions at the subatomic level requires stepping up data analyses from the standard methods and developing new numerical/analytic/quantum computing methods



Bringing this further: ML based approach merging and organizing information from experiment and lattice with a faithful uncertainty representation (uncertainty quantification)

Conclusions

We have developed a comprehensive formalism for deeply virtual exclusive scattering experiments from the proton

This helicity amplitudes based formalism is ideal for defining the relation between polarization observables and generalized parton distributioms

Extension to the deuteron is on its way!

DVCS formalism

- B. Kriesten et al, *Phys.Rev. D* 101 (2020)
- B. Kriesten and S. Liuti, *Phys.Rev. D105 (2022),* arXiv 2004.08890
- B. Kriesten and S. Liuti, Phys. Lett. B829 (2022), arXiv:2011.04484

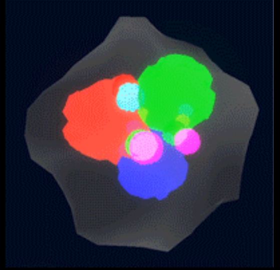
ML

- J. Grigsby, B. Kriesten, J. Hoskins, S. Liuti, P. Alonzi and M. Burkardt, Phys. Rev. D104 (2021)
- M Almaeen, J. Grigsby, J. Hoskins, B. Kriesten, Y. Li, H. W. Lin and S.~Liuti, ``Benchmarks for a Global Extraction of Information from Deeply Virtual Exclusive Scattering,'' [arXiv:2207.10766 [hep-ph]].

GPD Parametrization for global analysis

- B. Kriesten, P. Velie, E. Yeats, F. Y. Lopez and S. Liuti,

Phys. Rev D 105 (2022), arXiv:2101.01826



EXCLAIM – Collaboration

EXCLusives via Artificial Intelligence and Machine learning

- ✓ EXPERIMENT: M. Boer, T. Horn
- ✓ LATTICE QCD: M. Engelhardt, H-W Lin
- ✓ ML: G-W Chern, Y. Li, M. Almaeen, P. Alonzi, J. Hoskins,
- ✓ PHENOMENOLOGY: G. Goldstein, SL, M. Sievert, A. Courtoy, B. Kriesten