

# Measurement in topological superconducting systems

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Center for Quantum Information Physics*

## Theory Collaborators

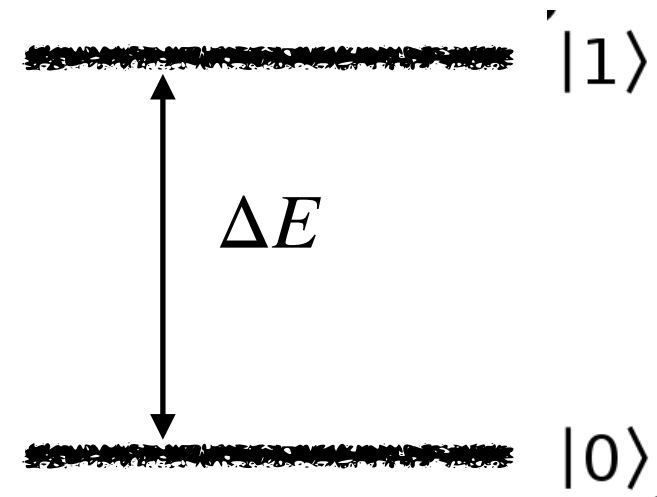
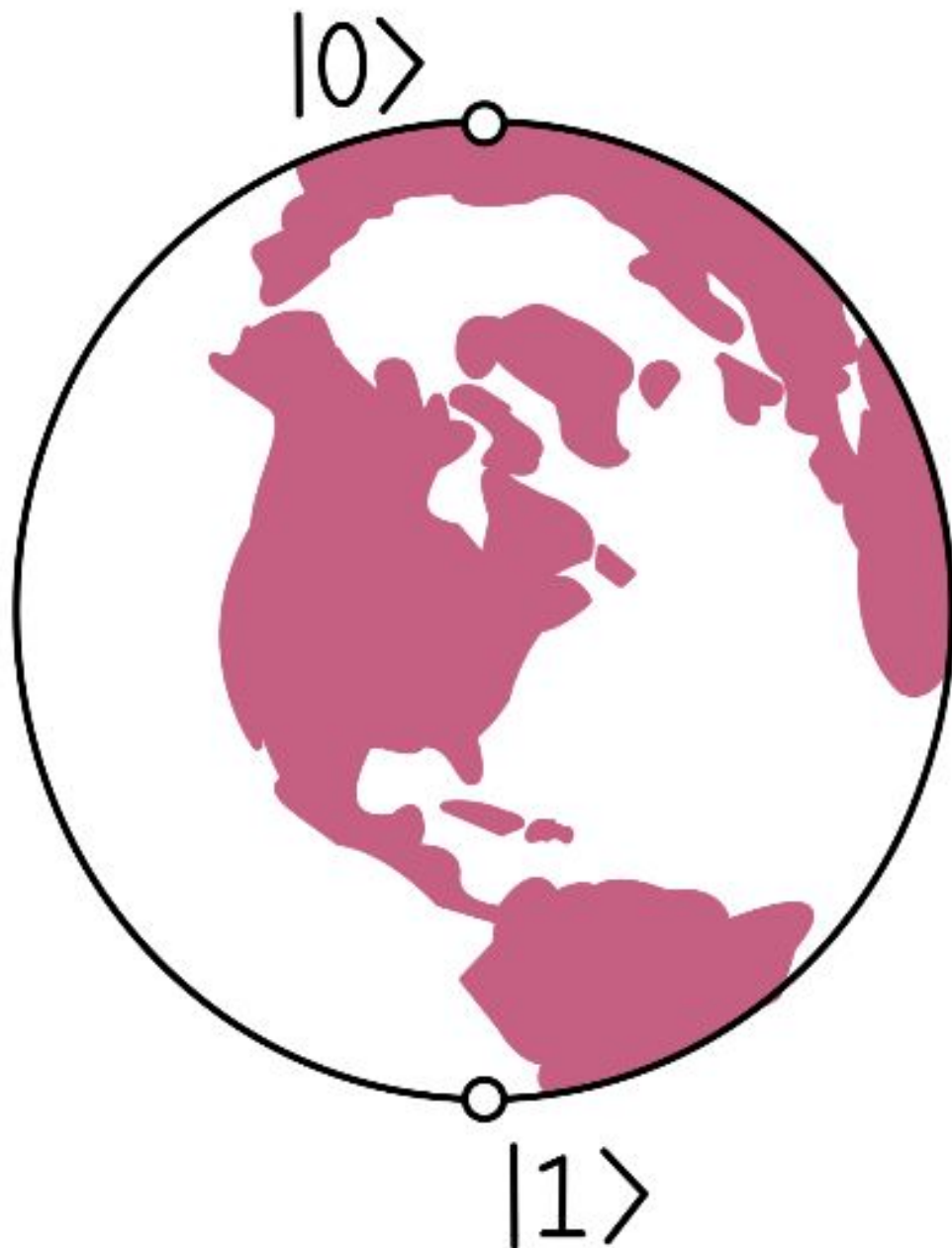
Igor Zutic      Alex Matos Abiague  
Tong Zhu        Baris Pekerten  
Jong Han        Enrico Rossi  
Leonid Glazman    Joseph Cuzzo

## Experimental Collaborators

Vladimir Manucharyan  
Eli Levenson-Falk  
Hugh Churchill  
Angel Kou  
Andrew Higginbotham



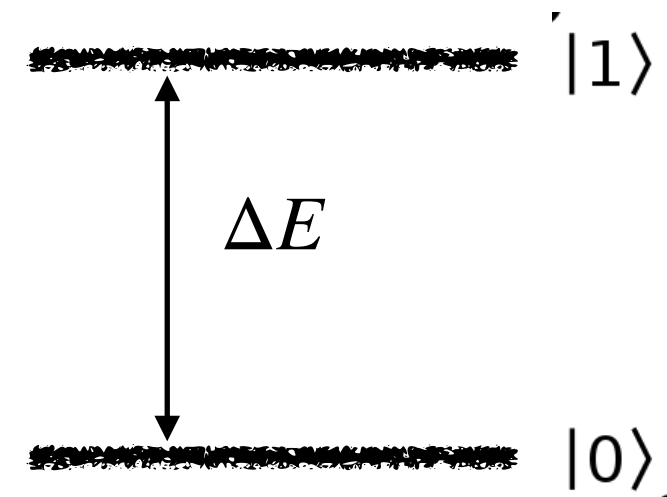
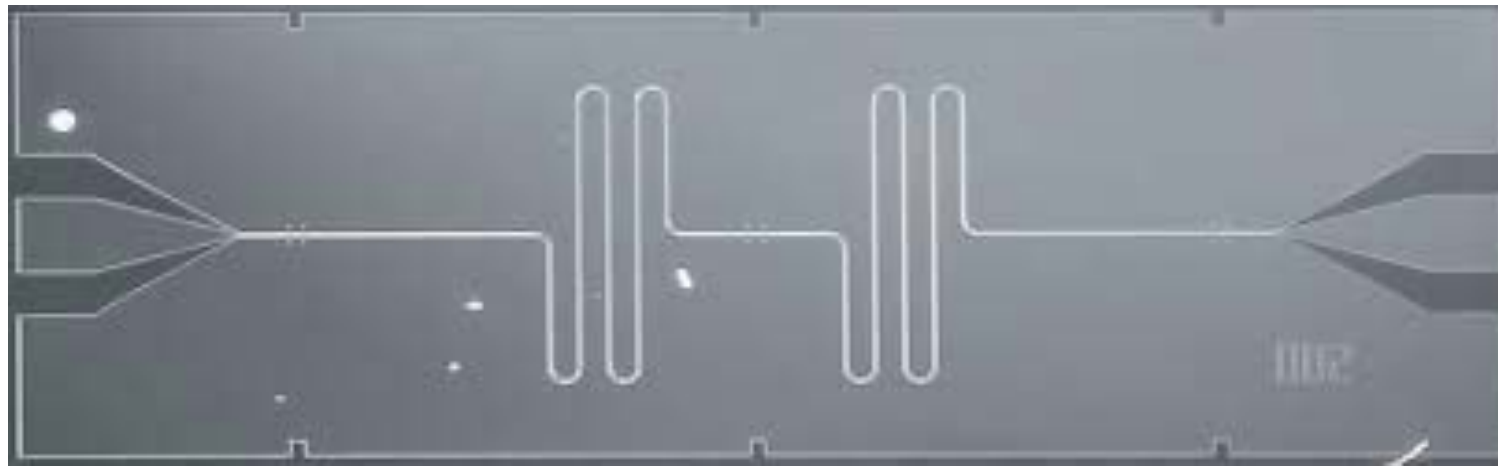
Two-level system governed by Schrodinger equation (to become quantum)



$$k_B T \ll \Delta E \quad (\text{Thermal})$$

$$\Gamma \ll \Delta E \quad (\text{Dissipation})$$

Two-level system governed by Schrodinger equation (to become quantum)

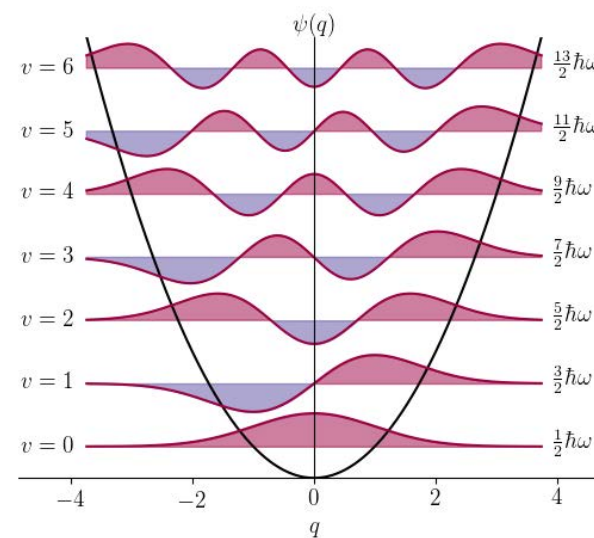


E.g. Superconducting resonator

$$\Delta E \approx 10\text{GHz} \approx 500\text{mK}$$

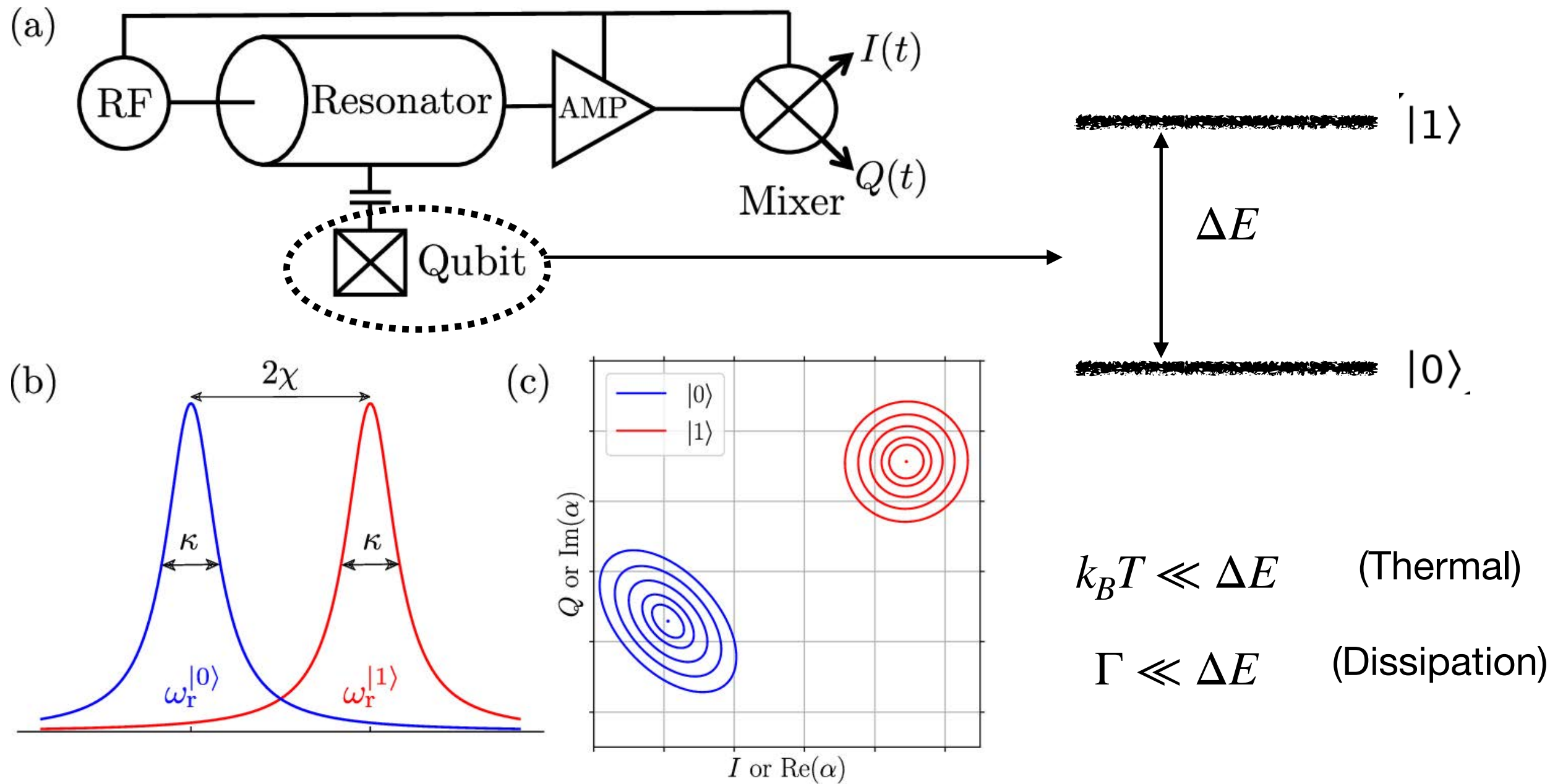
$$20\text{mK} \ll 500\text{mK}$$

$$\Gamma = 0 \ll \Delta E$$

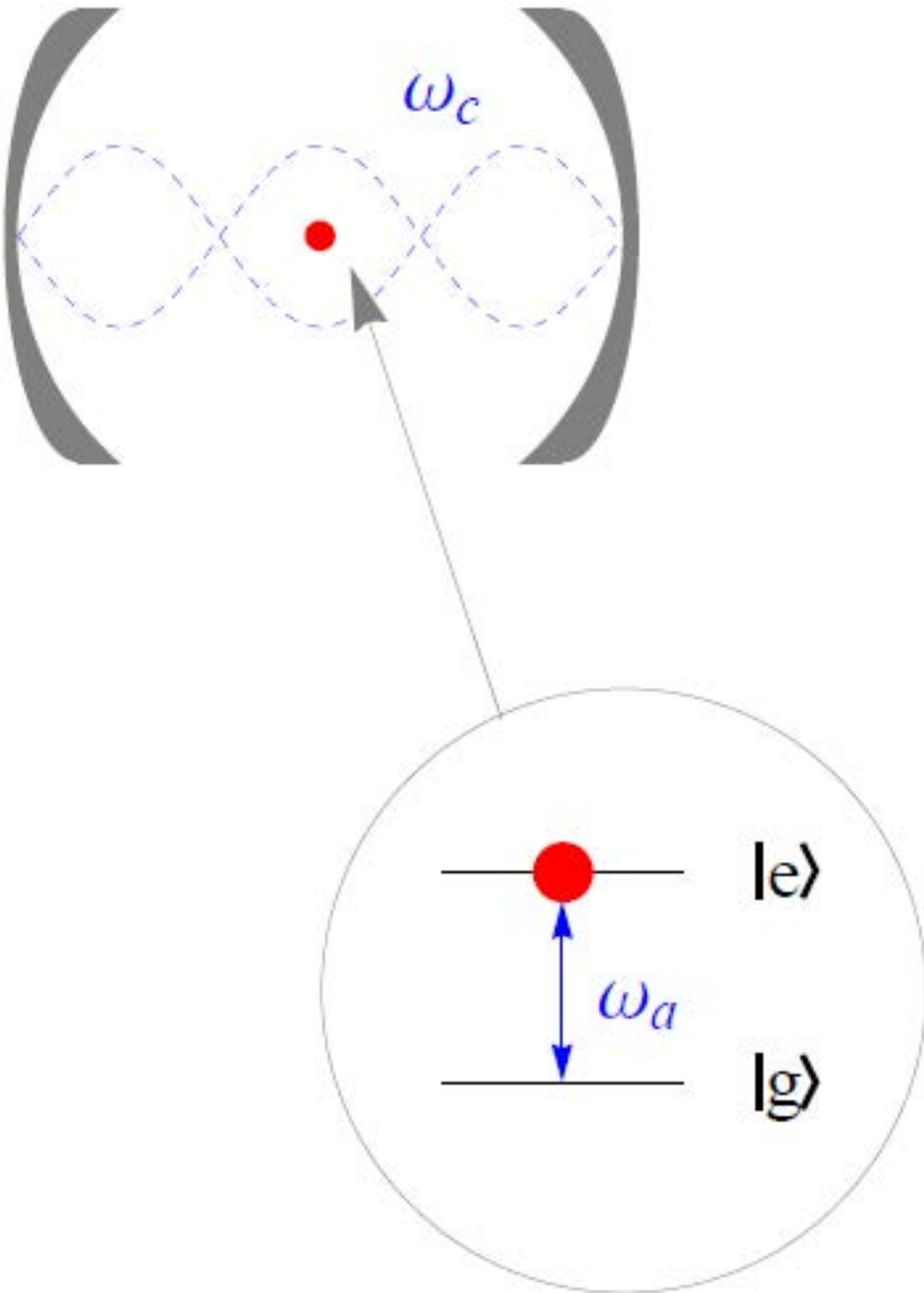


$$k_B T \ll \Delta E \quad (\text{Thermal})$$

$$\Gamma \ll \Delta E \quad (\text{Dissipation})$$







$$\hat{H} = \hat{H}_{\text{field}} + \hat{H}_{\text{atom}} + \hat{H}_{\text{int}}$$

$$\hat{H}_{\text{field}} = \hbar\omega_c \hat{a}^\dagger \hat{a}$$

$$\hat{H}_{\text{atom}} = \hbar\omega_a \frac{\hat{\sigma}_z}{2}$$

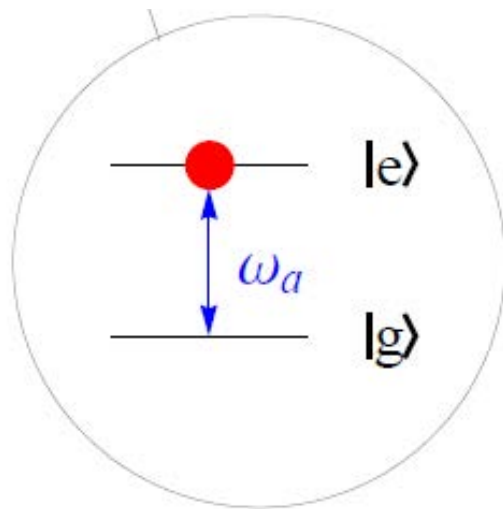
$$\hat{H}_{\text{int}} = \frac{\hbar\Omega}{2} \hat{E} \hat{S}$$

$$\hat{E} = E_{\text{ZPF}} (\hat{a} + \hat{a}^\dagger)$$

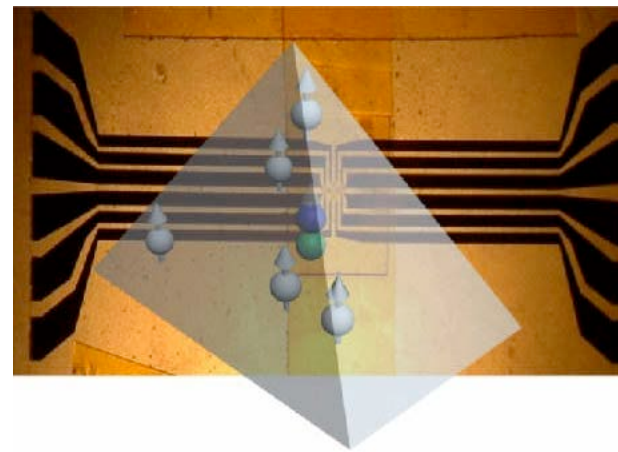
Resonator: Bosonic creation and annihilation operators

$$\hat{S} = \hat{\sigma}_+ + \hat{\sigma}_-$$

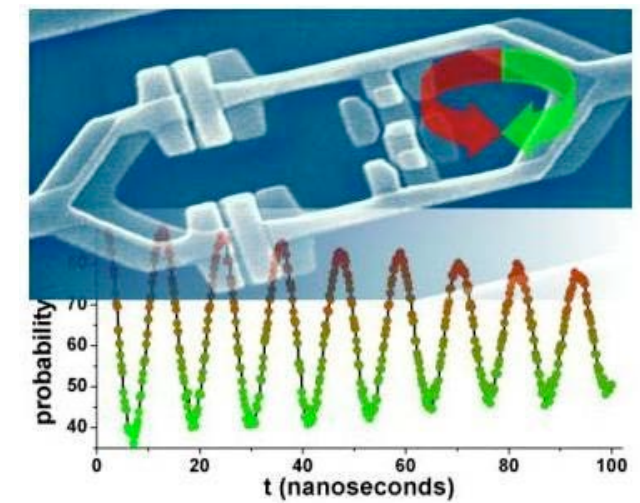
Qubit: Two-level atom is equivalent to a spin-half



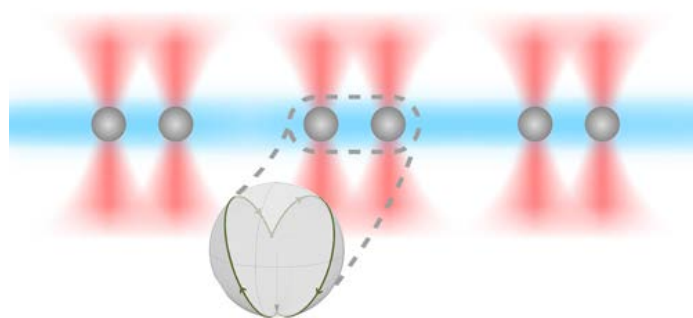
Two level qubits



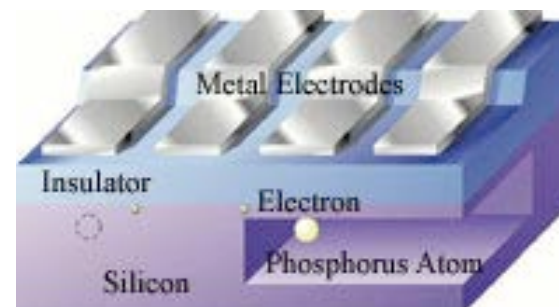
Diamond based



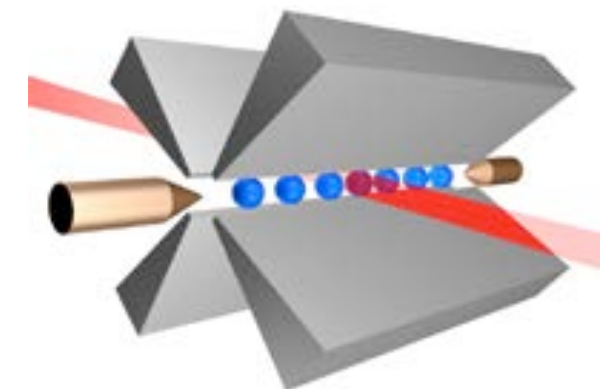
Superconducting



Neutral atoms


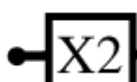



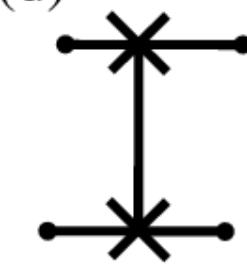

Silicon based


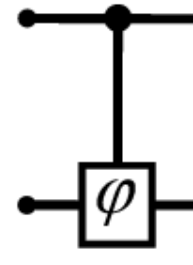


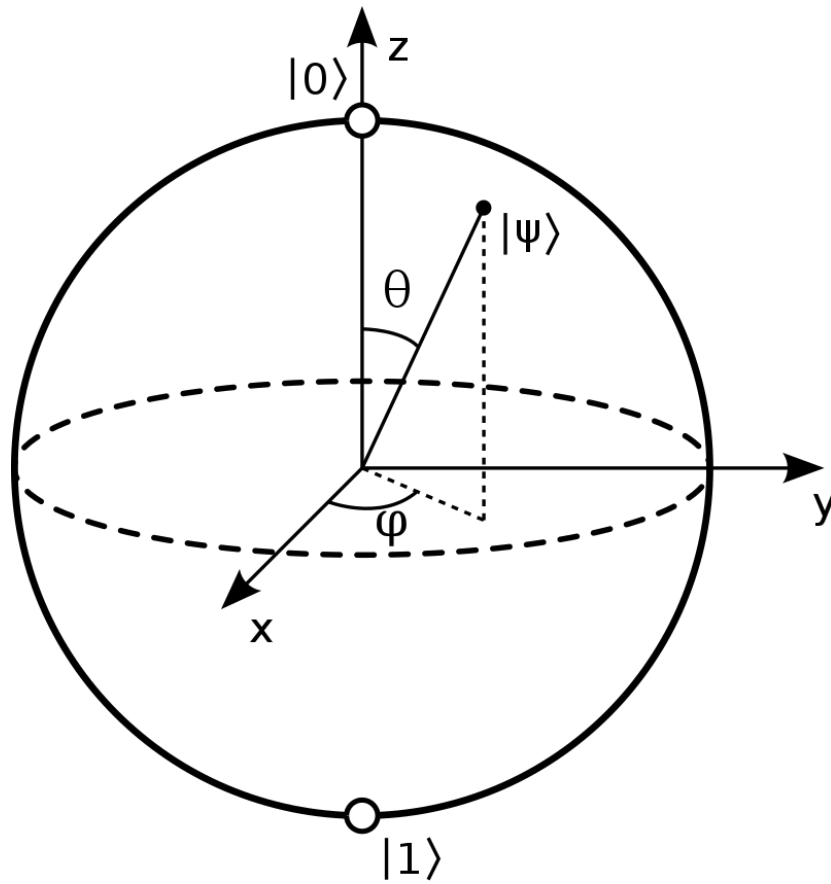
Ion trap

- Systems are evolved by unitary transformations, Measurement are not
  - **Quantum** operations are reversible
  - non-reversible operation is a **Measurement** of a qubit

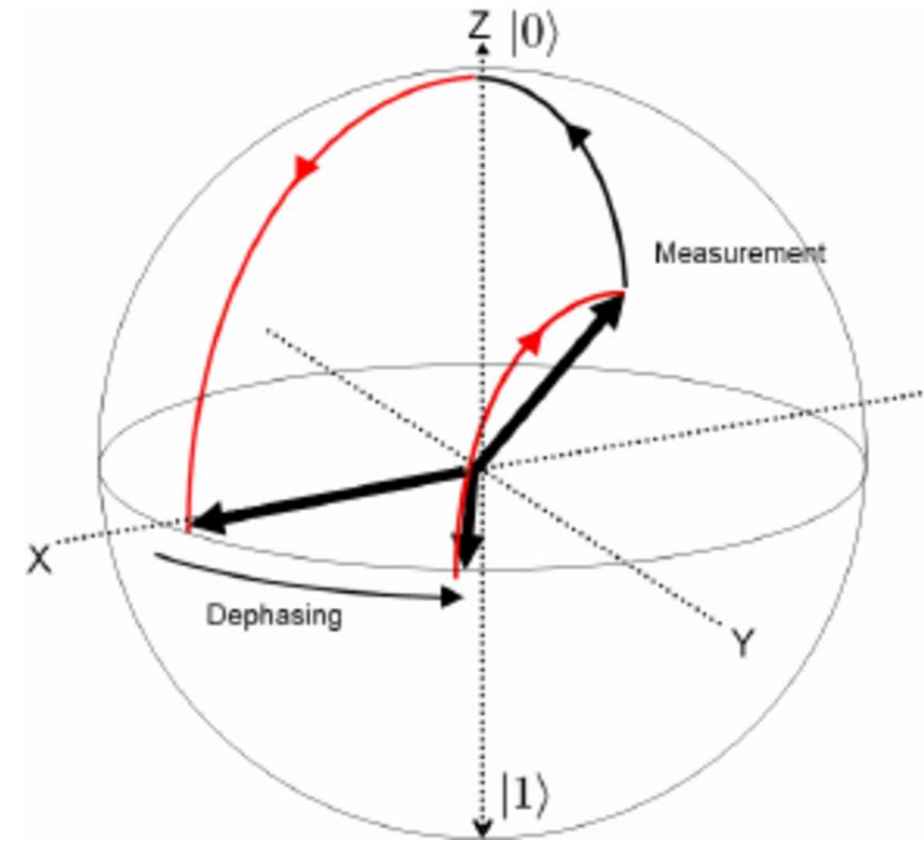
(a)  =  $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix};$     (b)  =  $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix};$     (c)  =  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix};$

(d)  =  $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$     (e)  =  $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix};$

(f)  =  $\begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix};$     (g)  =  $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\varphi} \end{pmatrix}$



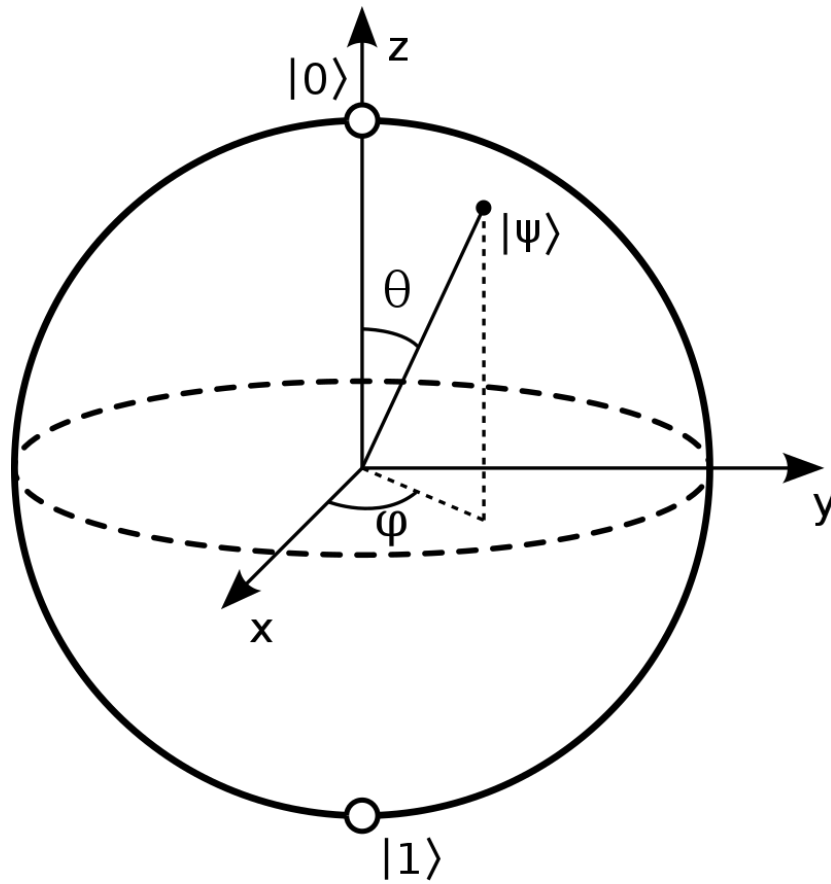
No noise



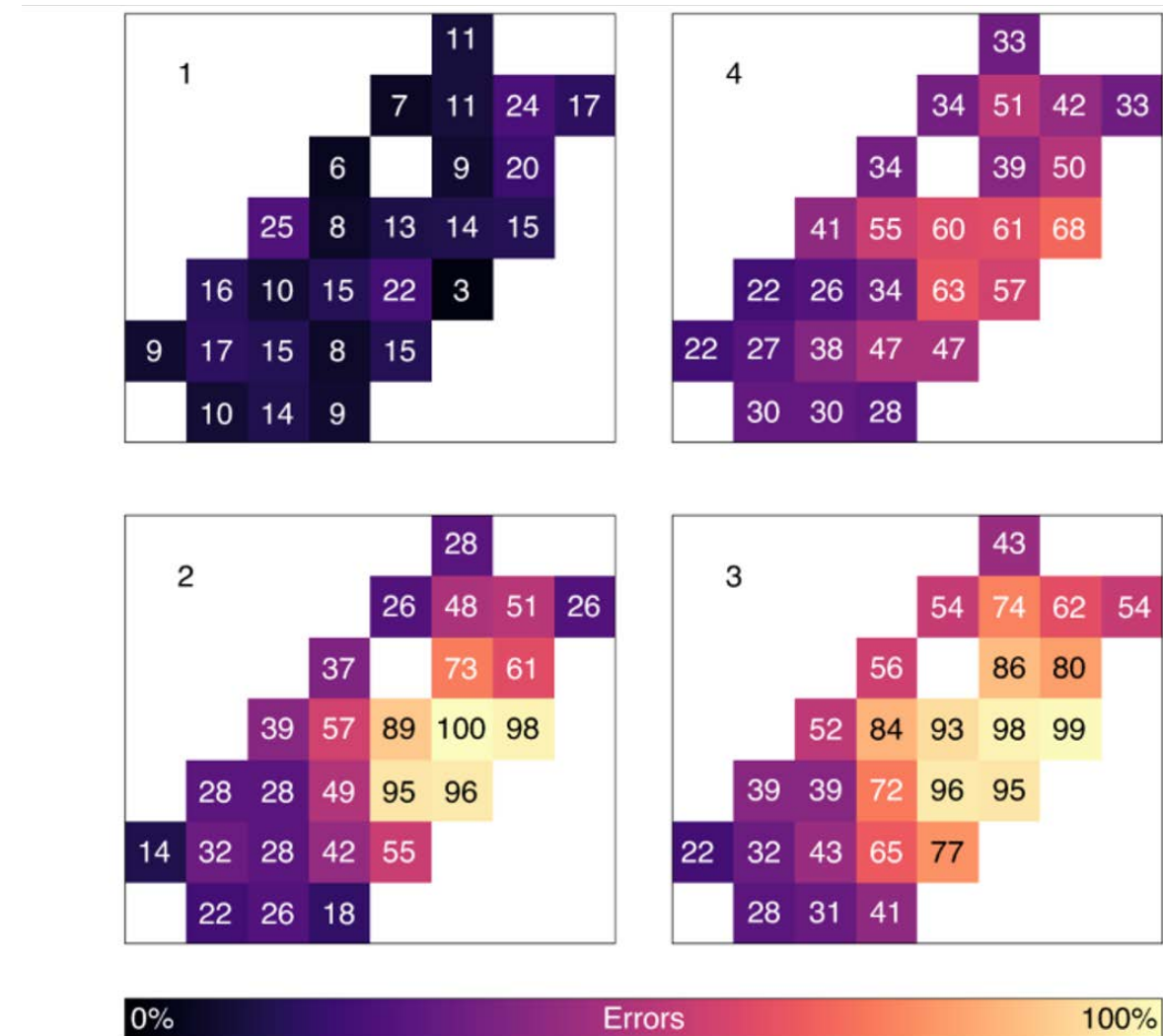
Decoherence

Simple two level systems have no intrinsic protection from relaxation or decoherence





No noise

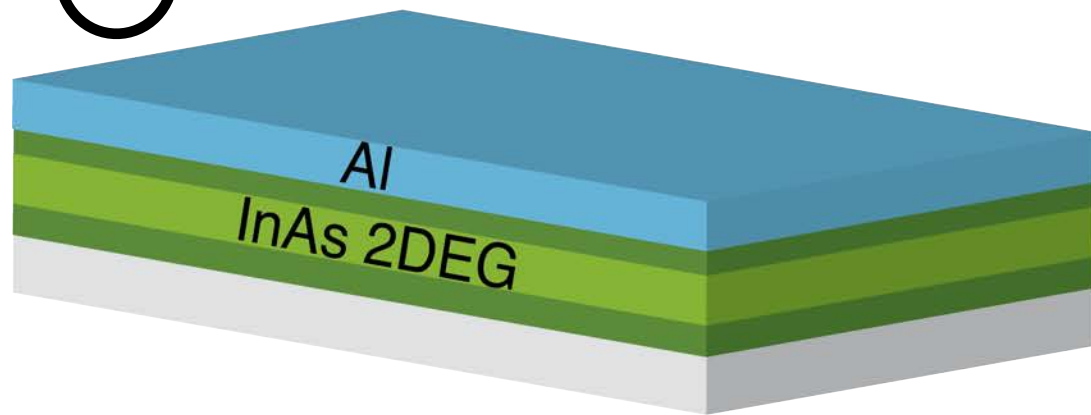


[McEwen et al., Nature Physics, 18, 107–111 \(2022\)](#)

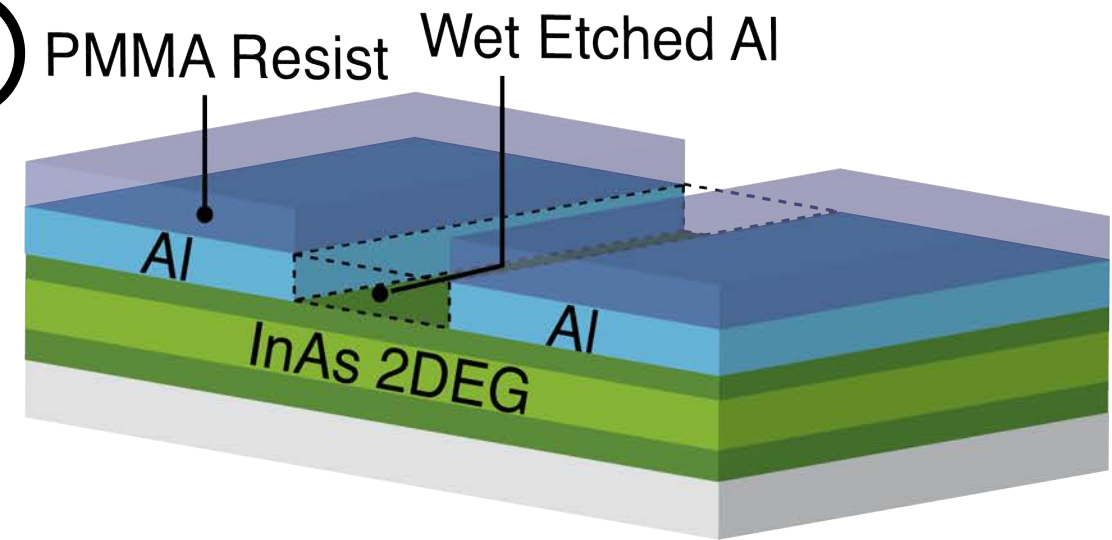
Title: Resolving catastrophic error bursts from cosmic rays in large arrays of superconducting qubits

Simple two level systems have no intrinsic protection from relaxation or decoherence

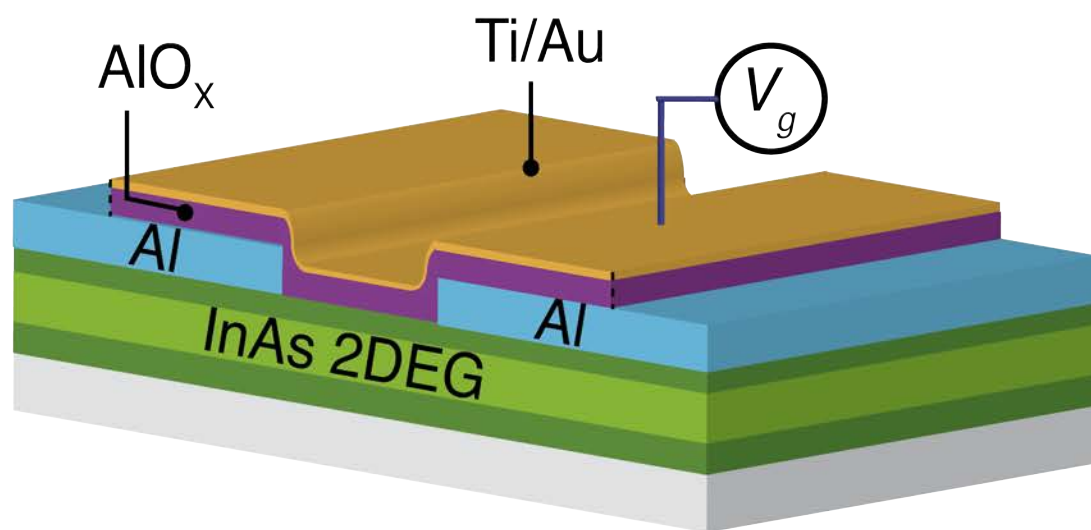
①



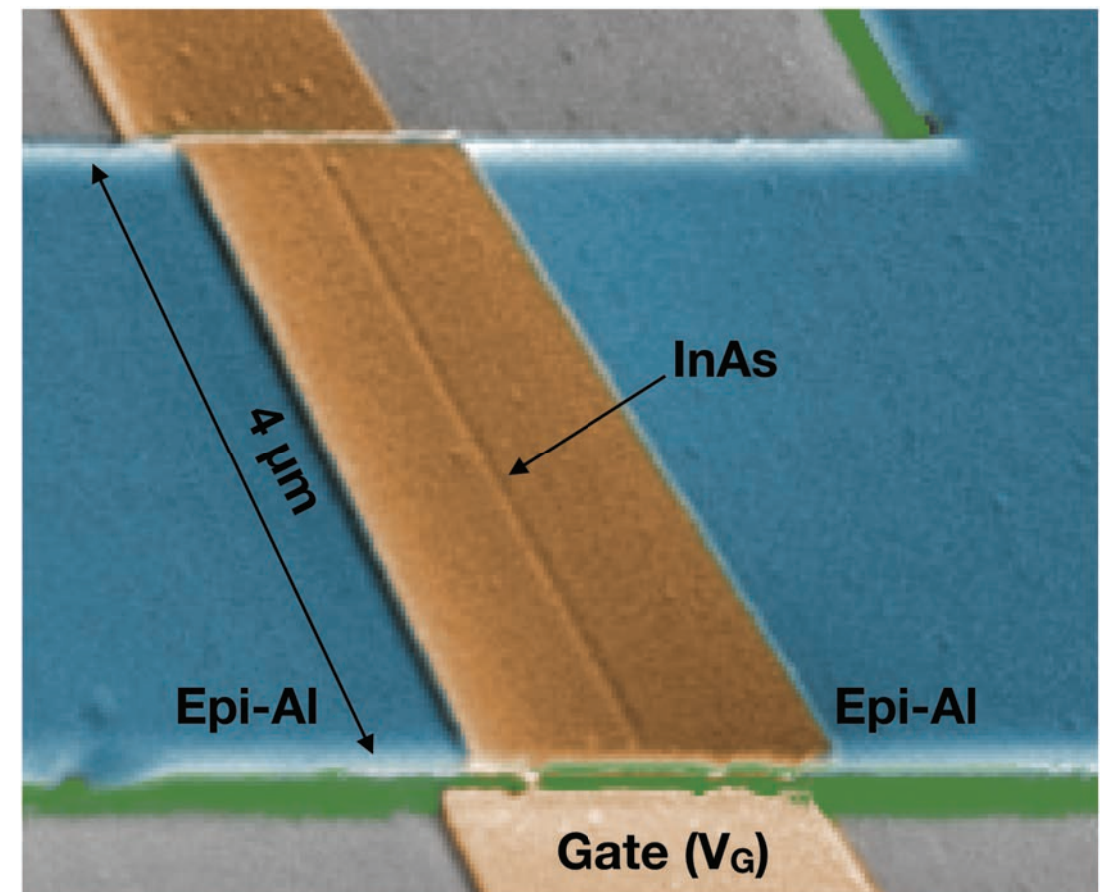
②



③

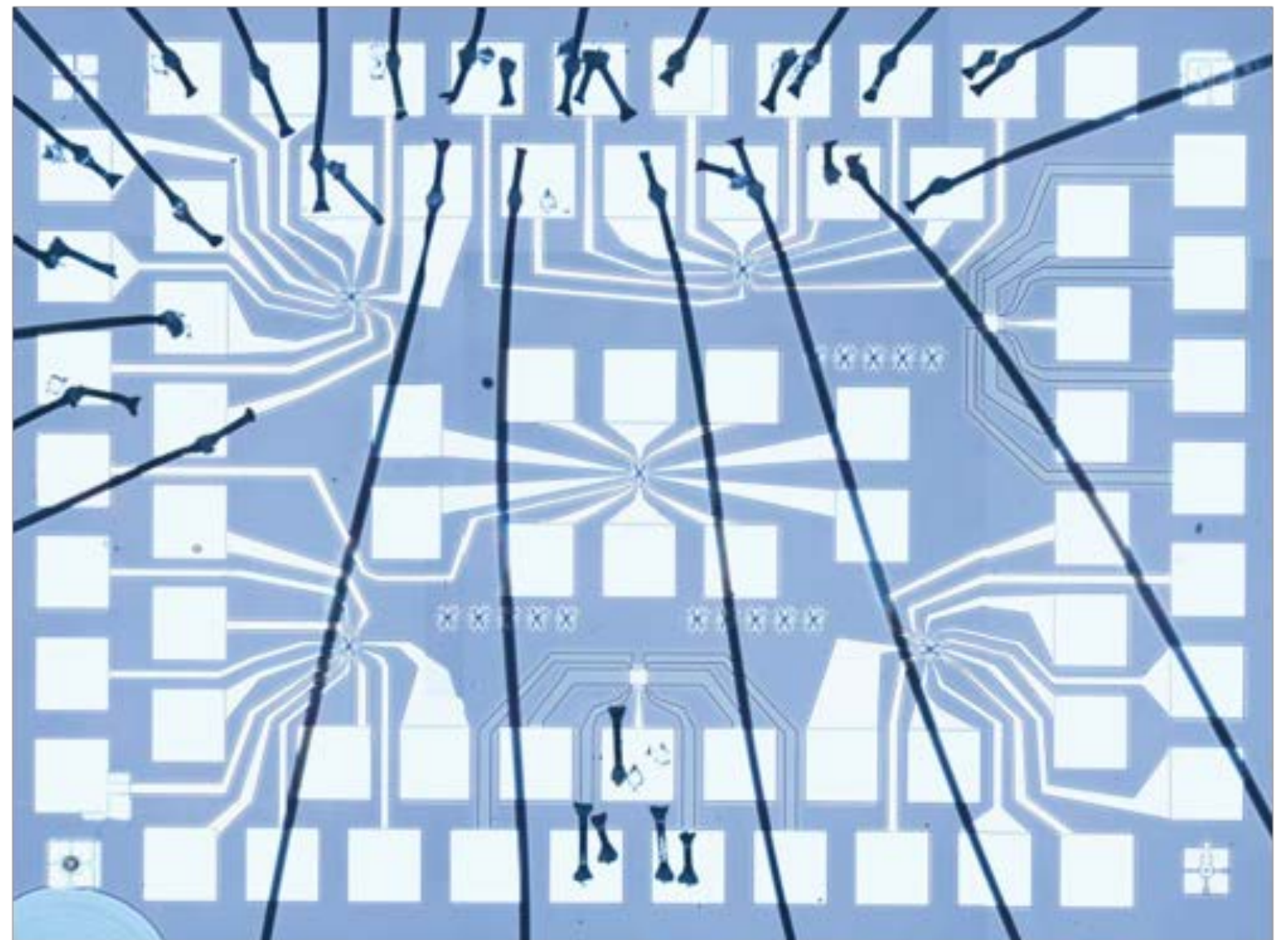
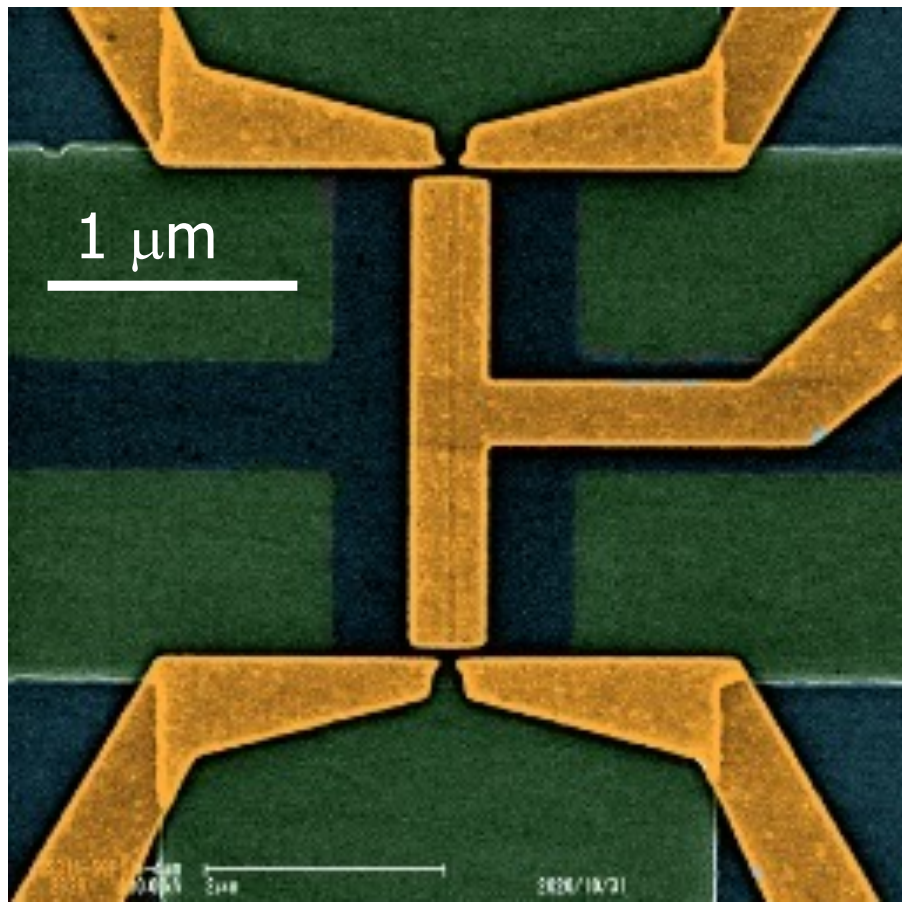
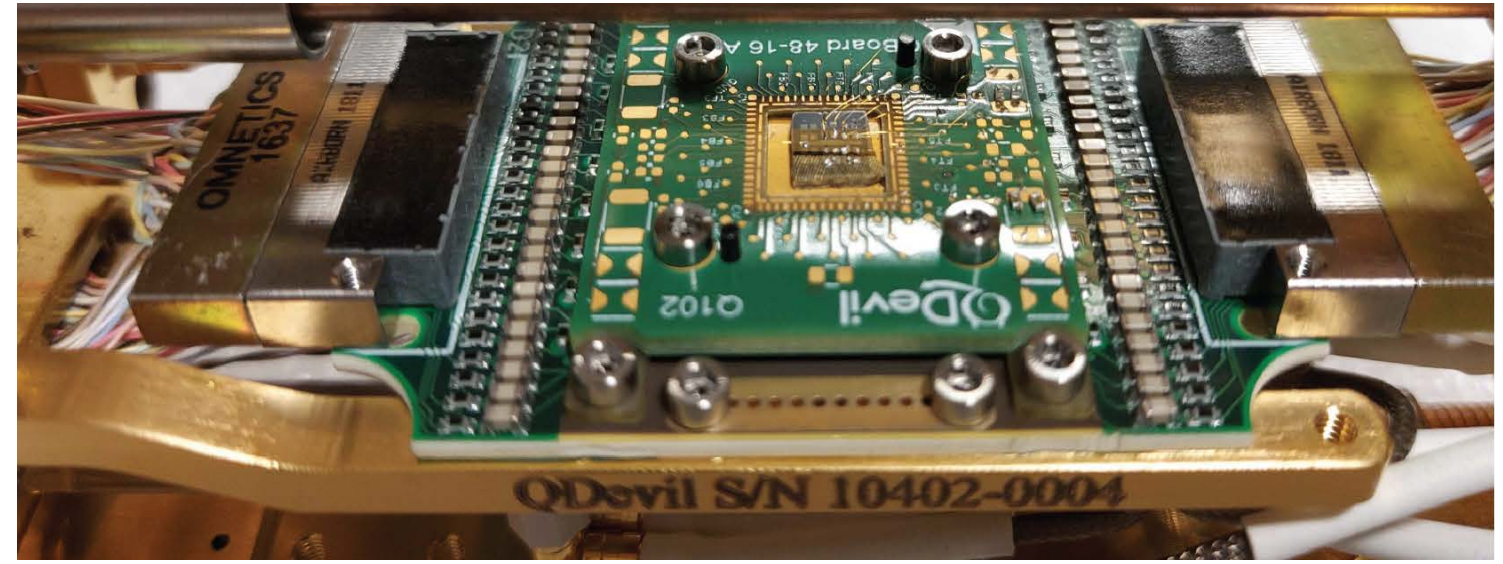
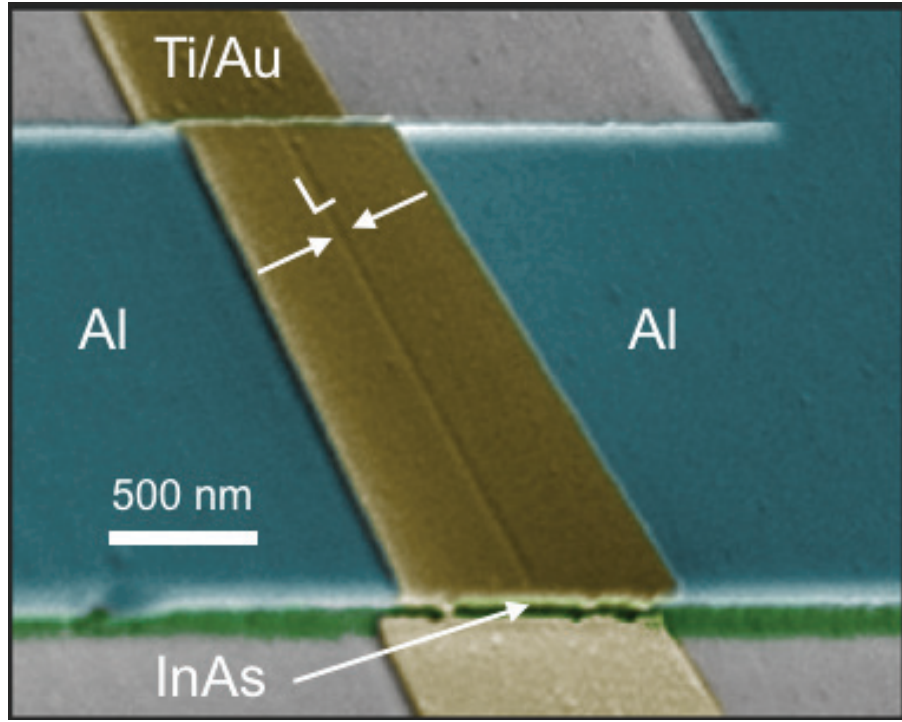


SEM Image (false colored)

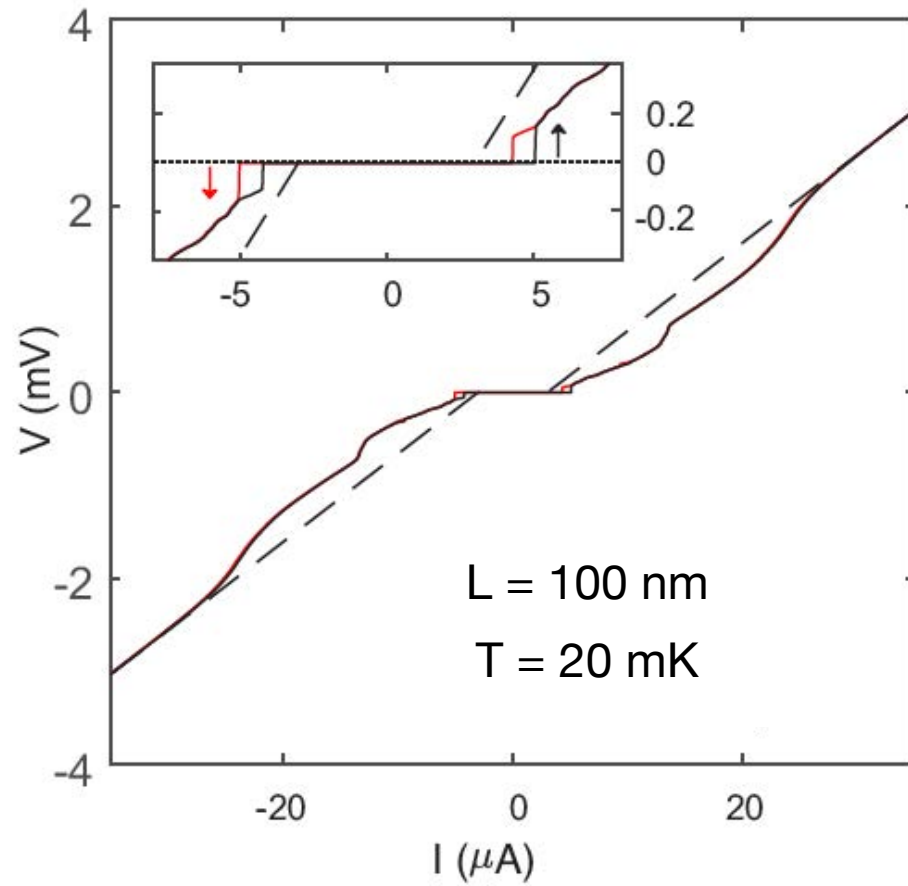




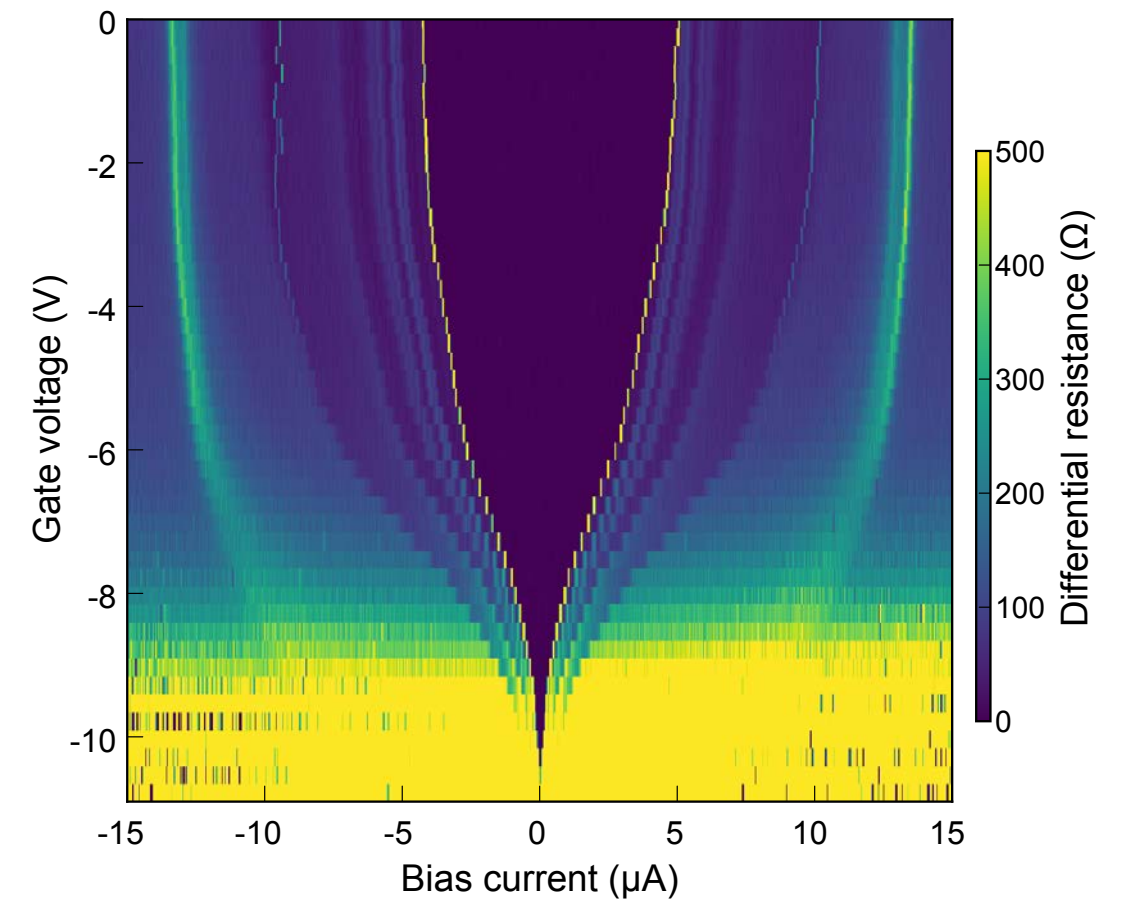
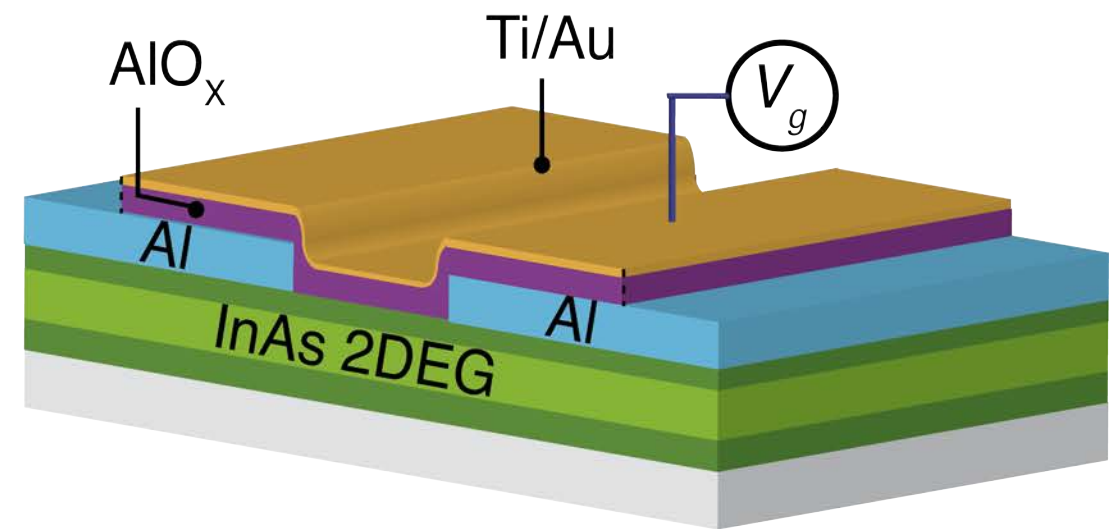
# Josephson Junction Devices



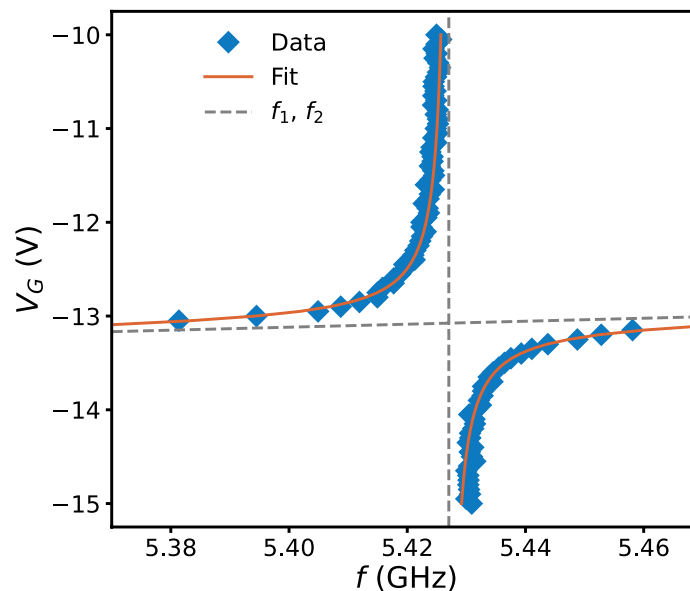
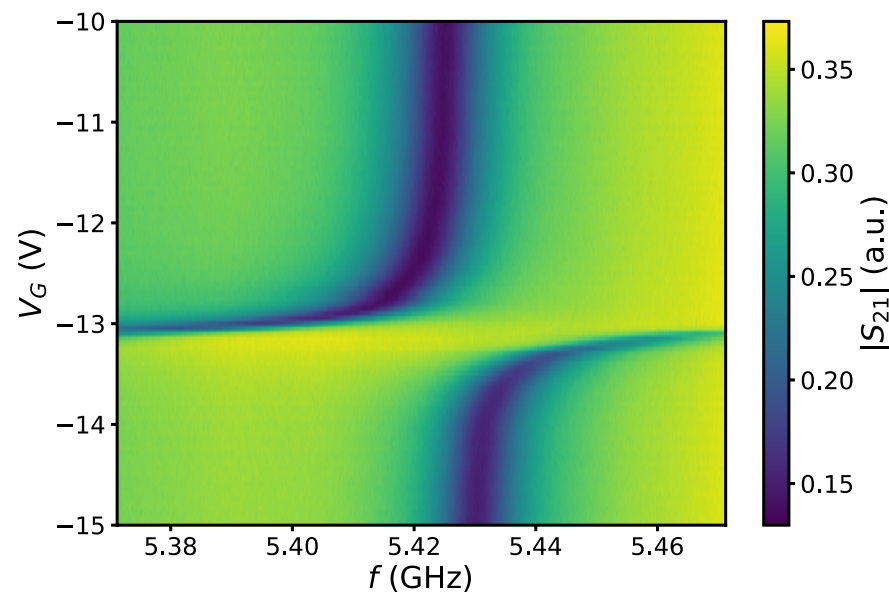
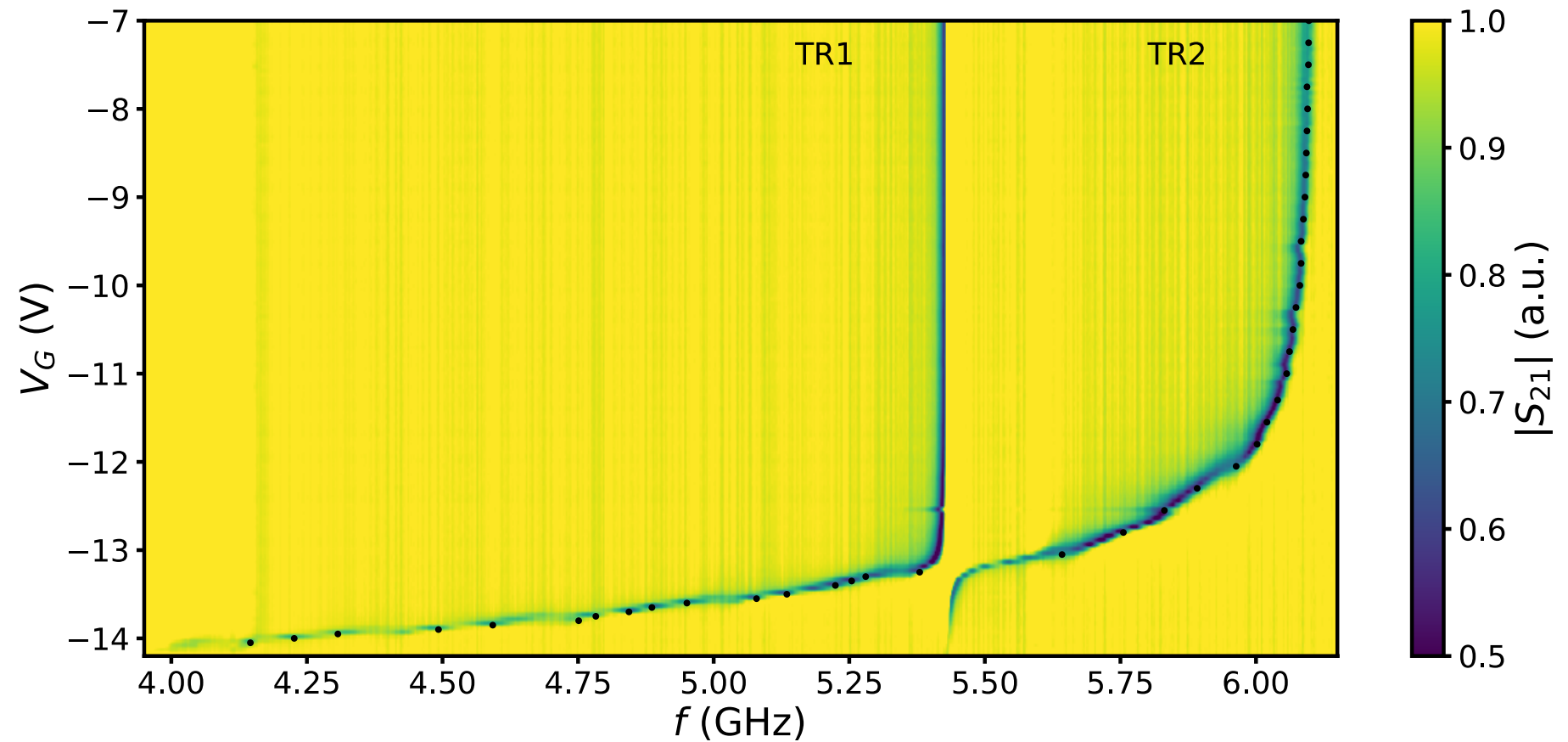
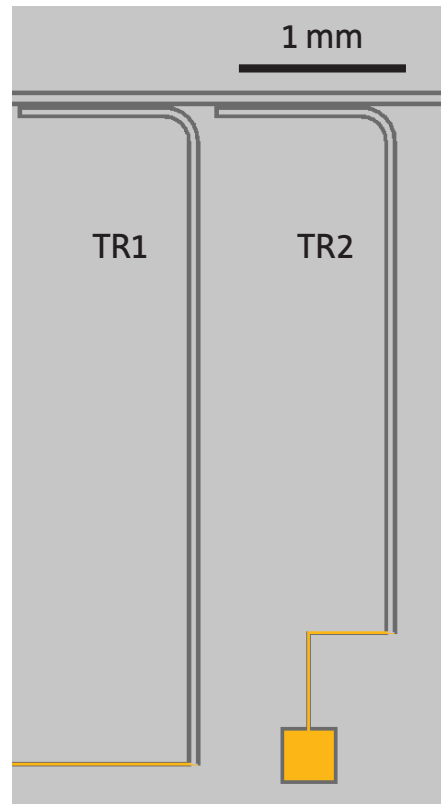




$$\omega \sim \frac{1}{\sqrt{(L_0 + L_J)C_0}}$$

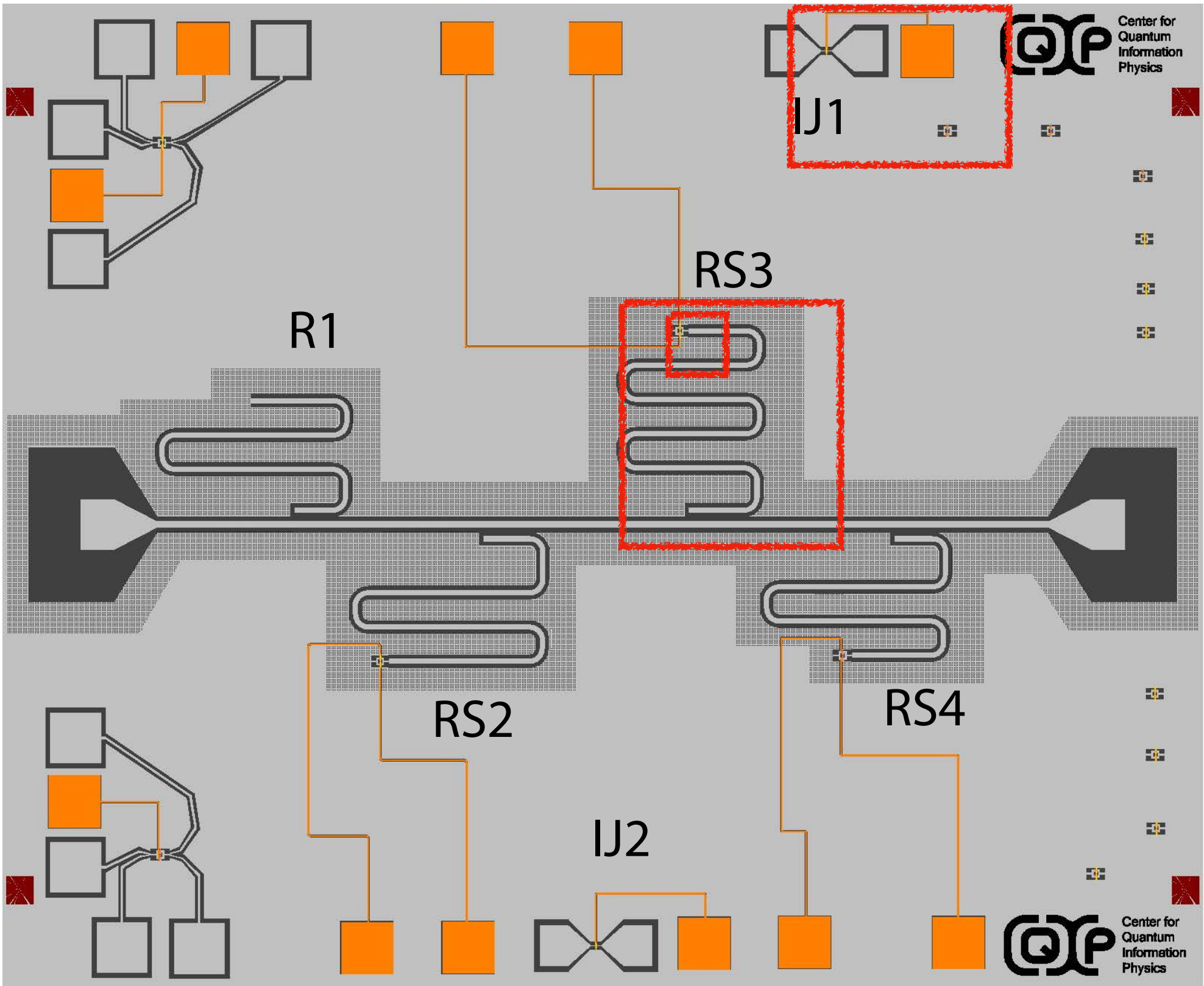


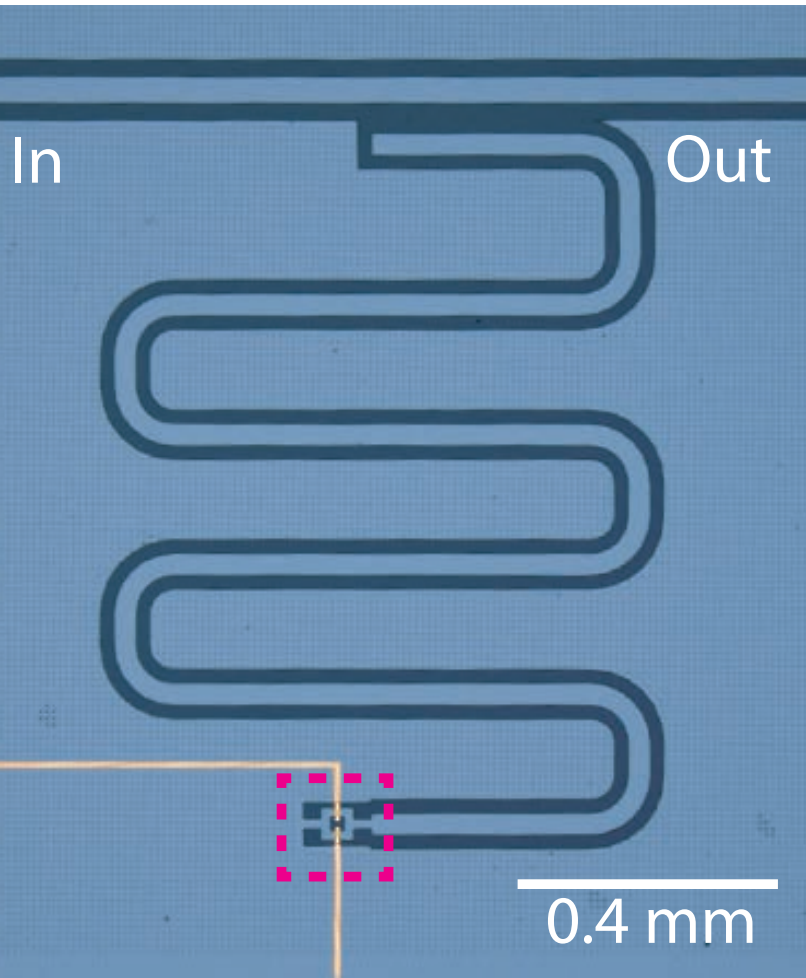




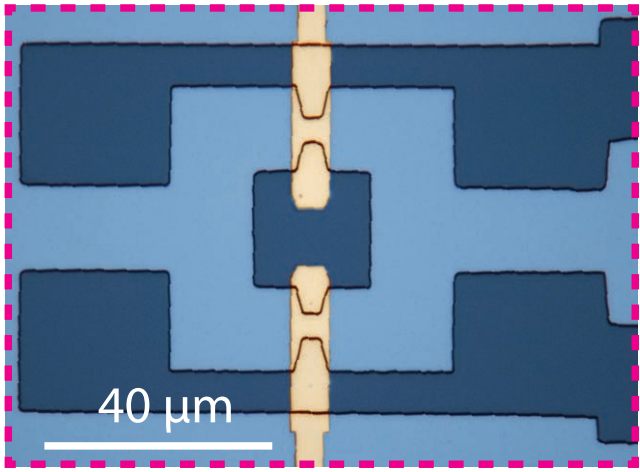
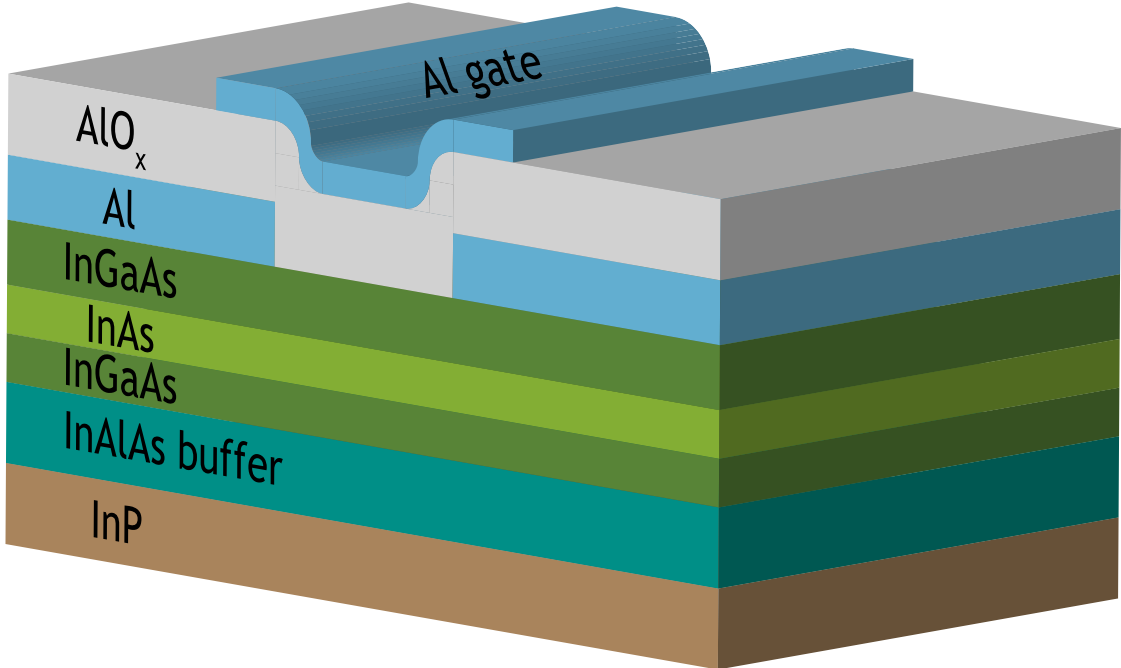
- Fit to two oscillators model with coupling strength  $g$ , find a coupling strength of  $g = 51.203$  MHz

$$f_{\pm} = \frac{1}{2}(f_1 + f_2) \pm \sqrt{g^2 + \frac{1}{2}(f_1 - f_2)^2}$$

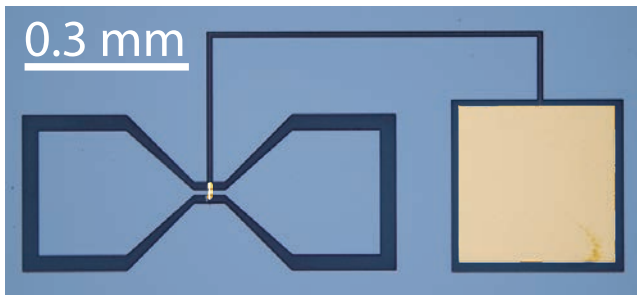




Resonator-SQUID



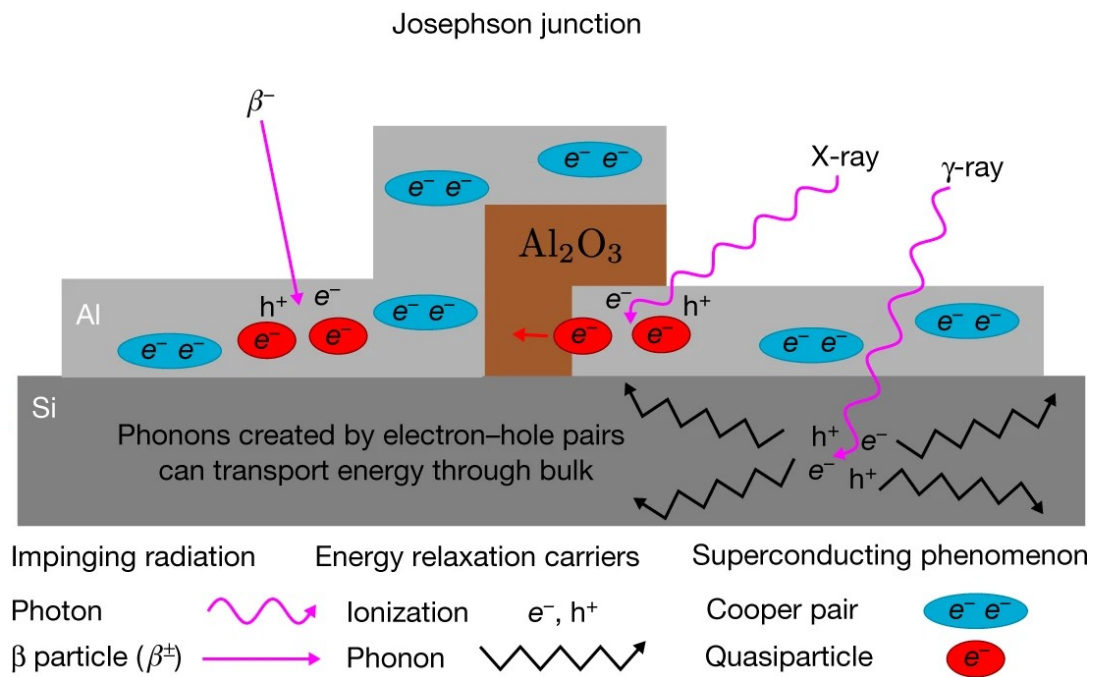
SQUID



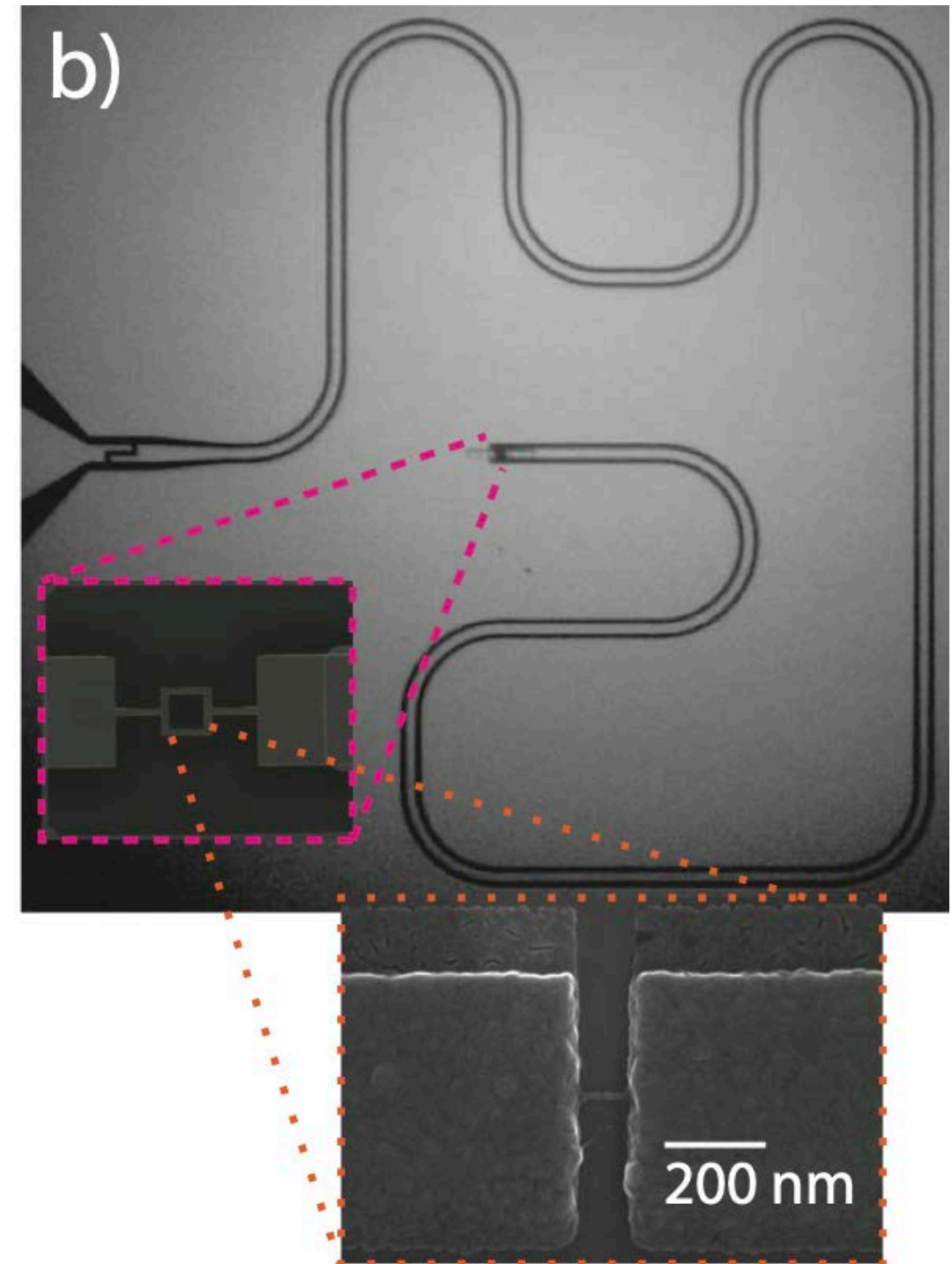
QP injector junction



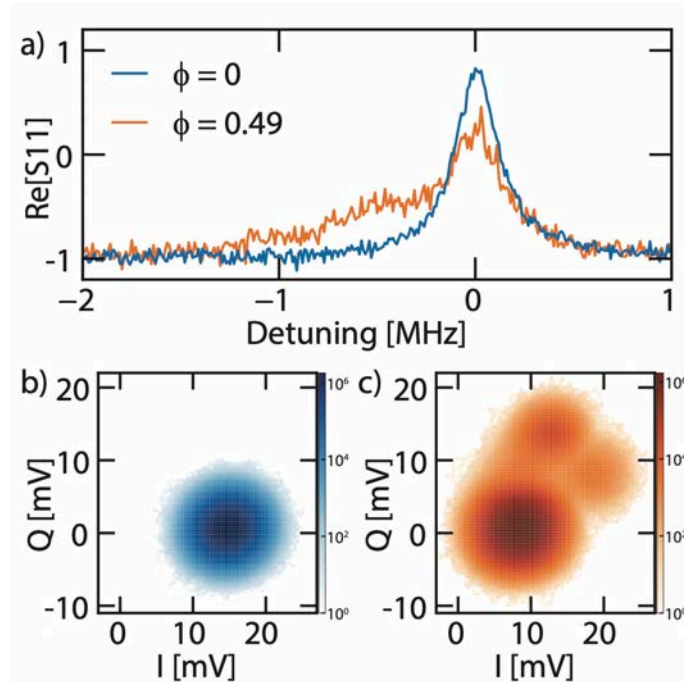
# Control Steps: Quasi Particle Poisoning



Resonator-SQUID



A. P. Vepsalainen et al., Nature (2020)



[https://topocondmat.org/w2\\_majorana/signatures.html](https://topocondmat.org/w2_majorana/signatures.html)

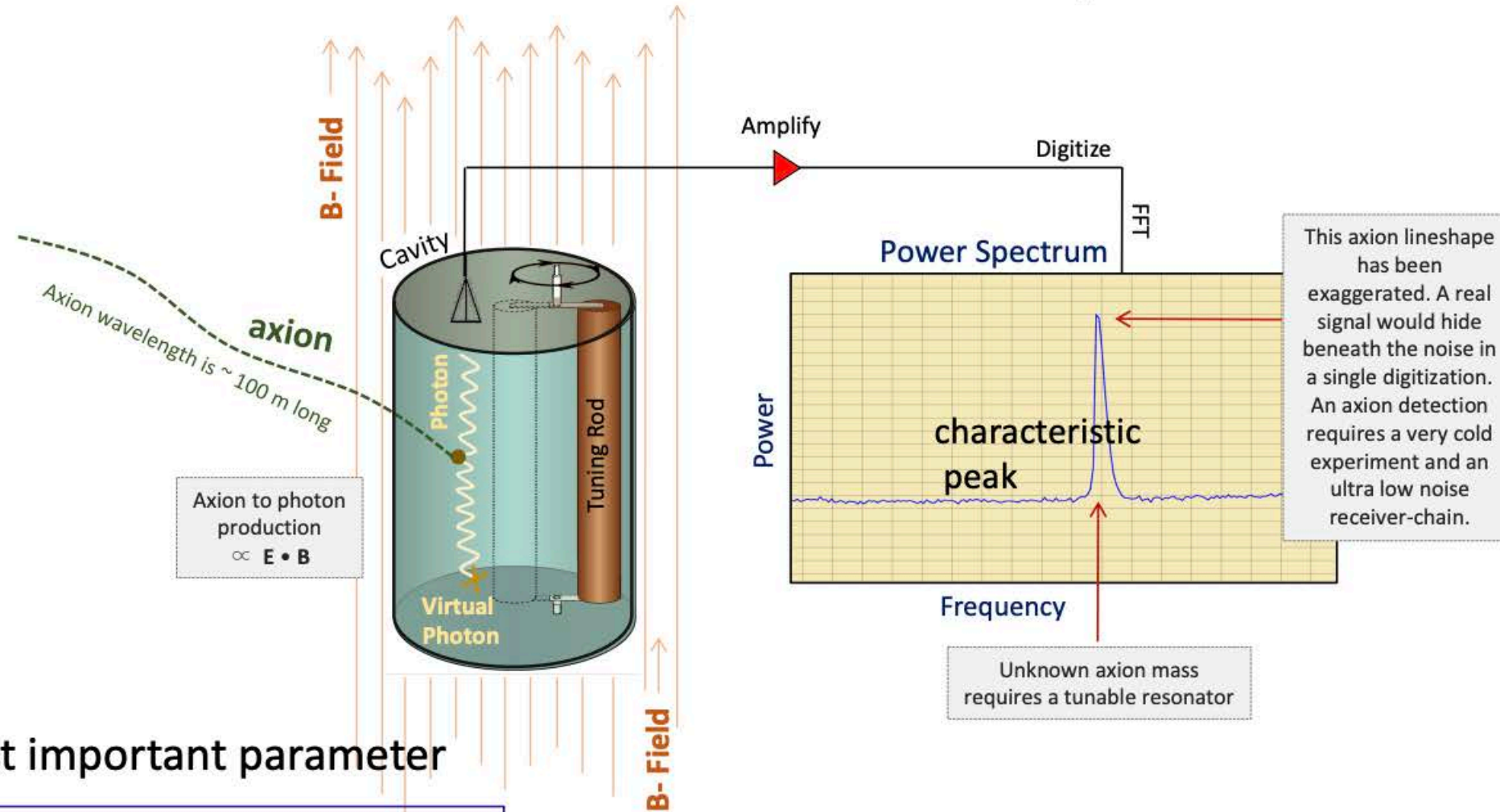
Hays, Springer Theses (2021)

J. Farmer et al., APL (2021)



# Axion dark matter radio

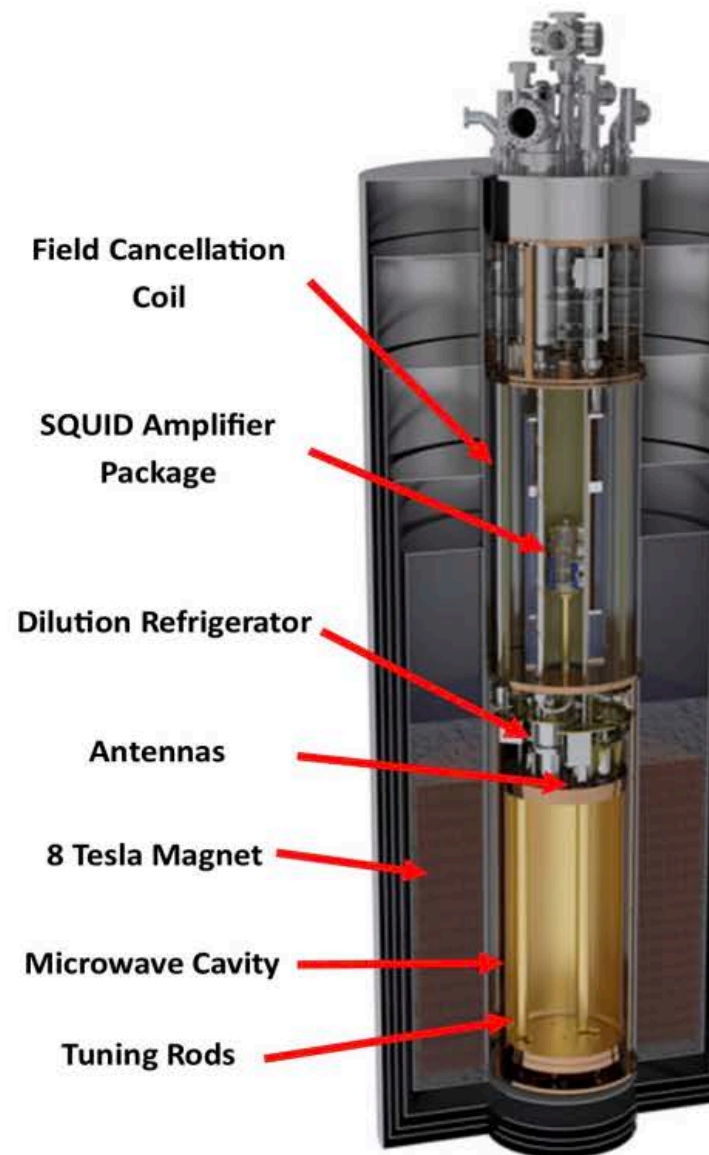
## The Axion Haloscope



Most important parameter

$$SNR \propto \frac{P_{out}}{k_B T_{system}} \sqrt{\frac{t}{b}} \propto \frac{g_{ay}^2 \rho_a f Q C_{mnp} B^2 V t^{\frac{1}{2}}}{b^{\frac{1}{2}} T_{system}}$$

# ADMX detector



**Field cancellation coil:** cancels the residual magnetic field around the SQUID electronics

**Superconducting QUantum Interference Device (SQUID) amplifiers:** amplifies the signal while being quantum noise limited

**Dilution refrigerator:** cools the insert to  $\sim 90\text{mK}$

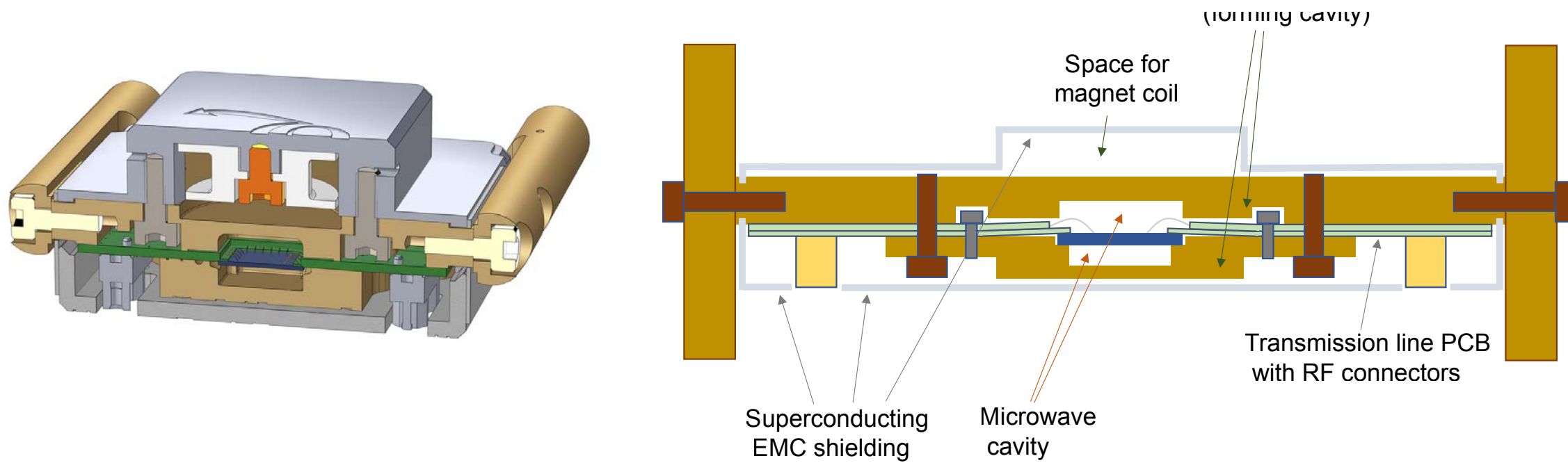
**Antennas:** pick up signal

**Magnet:** facilitates the axion conversion to photons, 8T

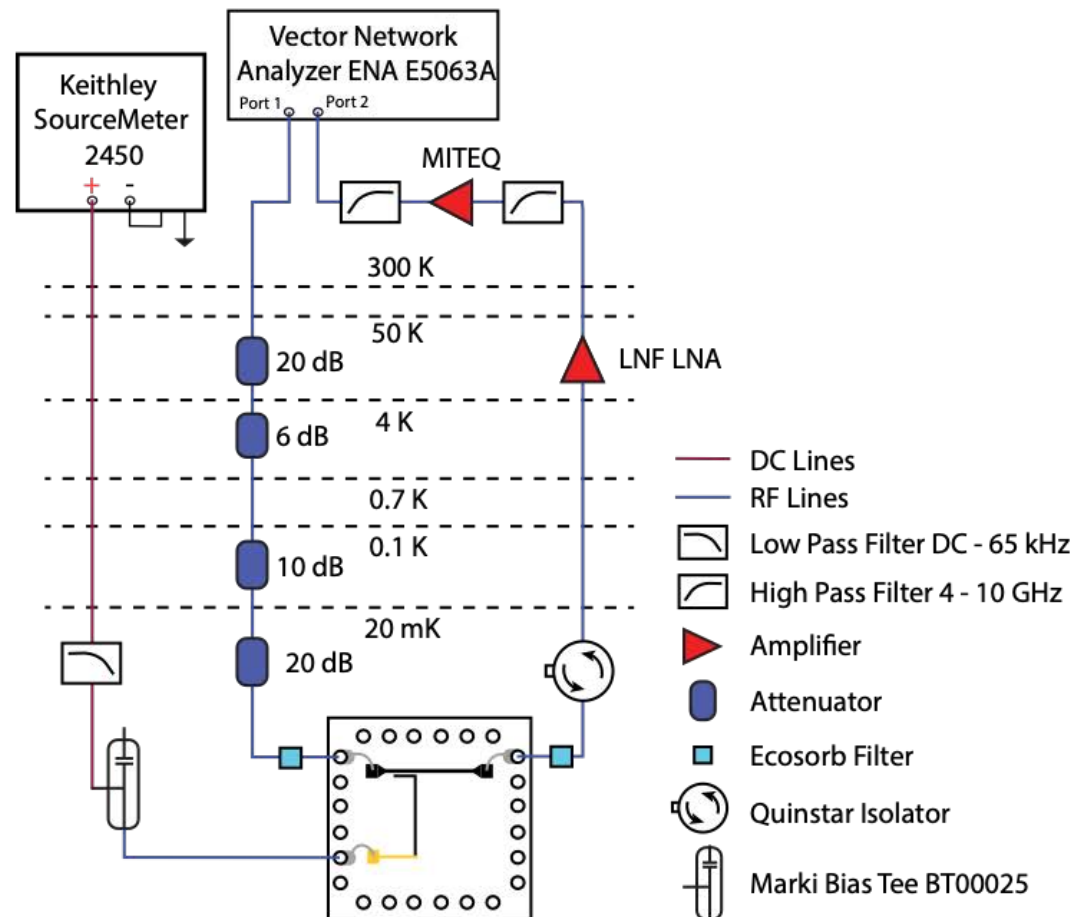
**Microwave Cavity:** converts axions into photons, tunable



# Control Steps: Quasi Particle Poisoning



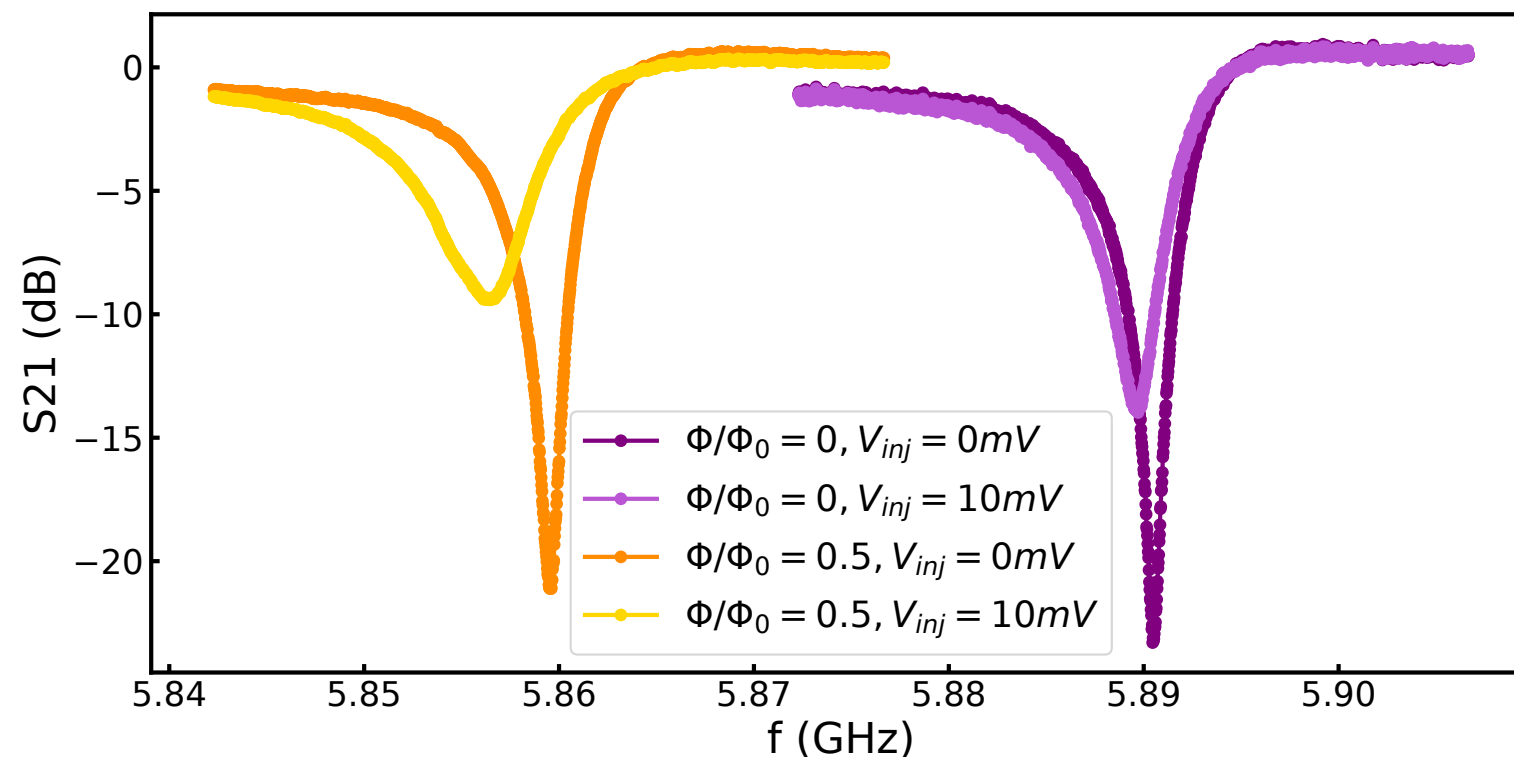
## 6-3-1 T Vector Dilution Fridge with qubit setup



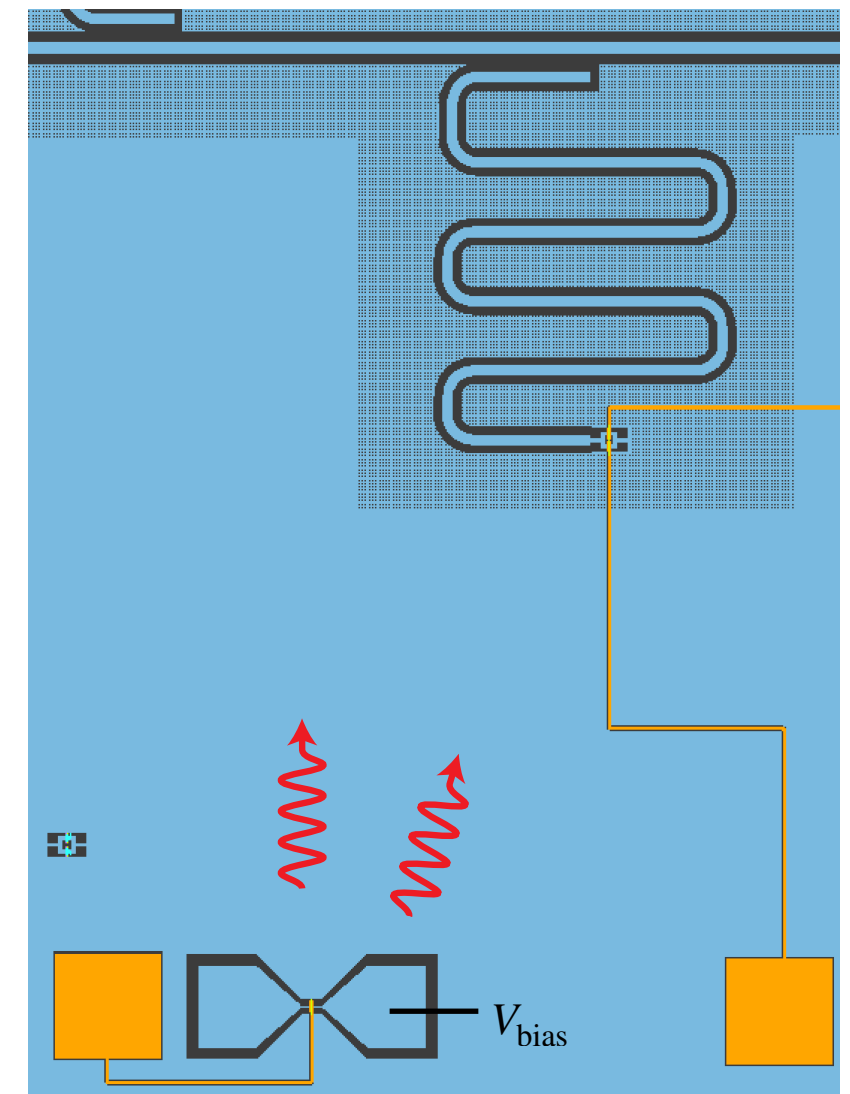
- Deepest trap ( $\tau = 1$ ) at  $\phi = 0.5$ :

$$\Delta_A = \Delta - \Delta \sqrt{1 - \tau \sin^2(\phi/2)} = 15.6 \text{ GHz}$$

- Applied clearing tone frequency = 18GHz



Resonator-SQUID

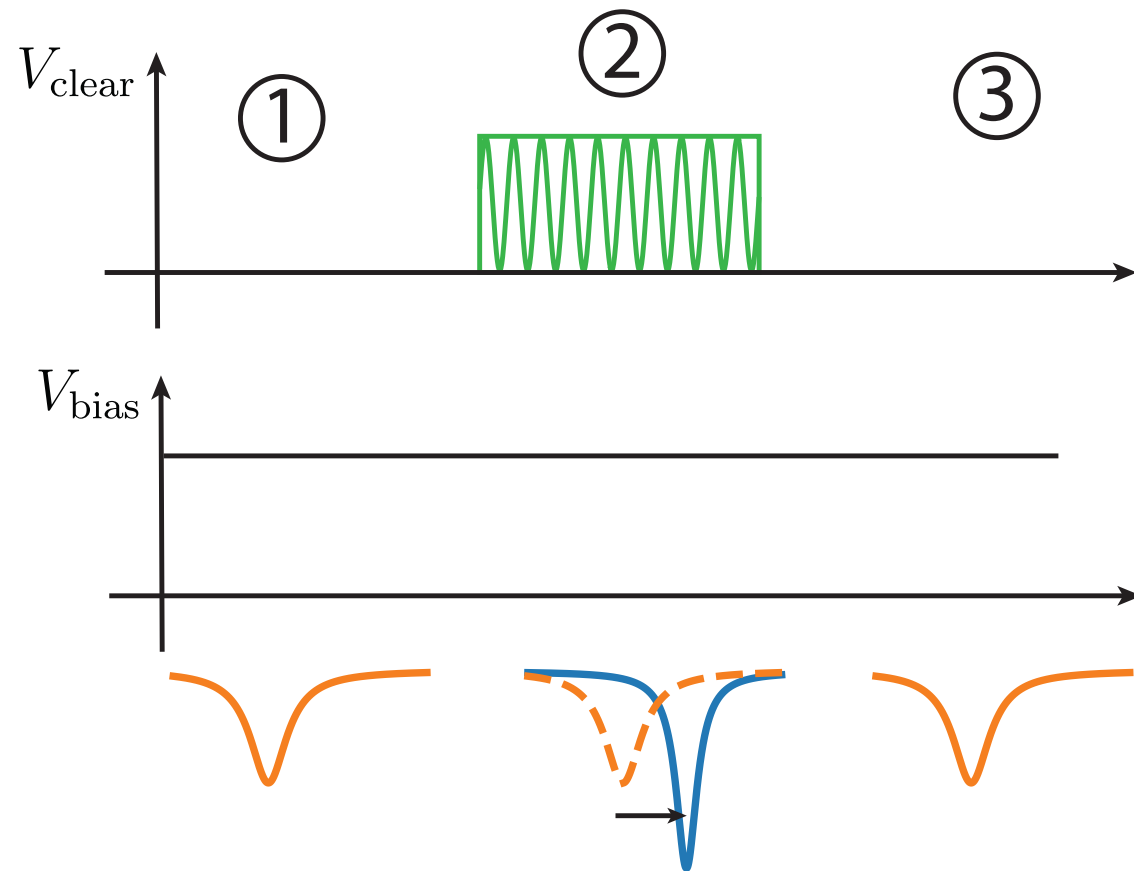


Topological gap ( $\sim \text{ns}$ )  $\ll$  Majorana qubit clock time  $\ll$  QPP time



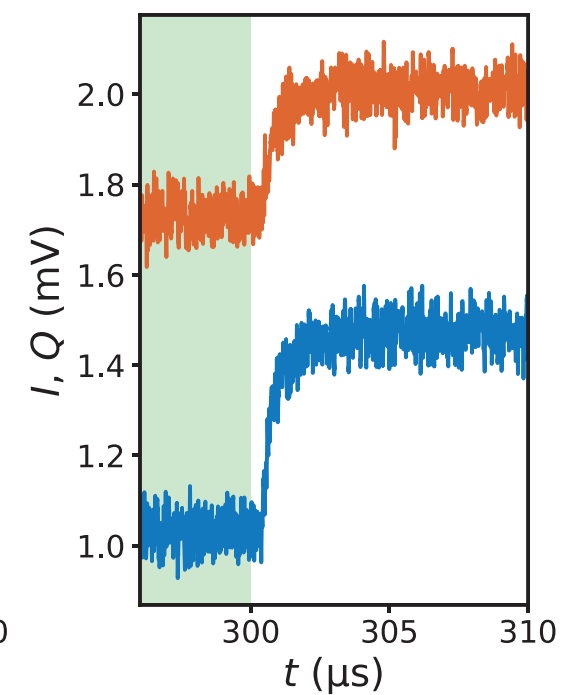
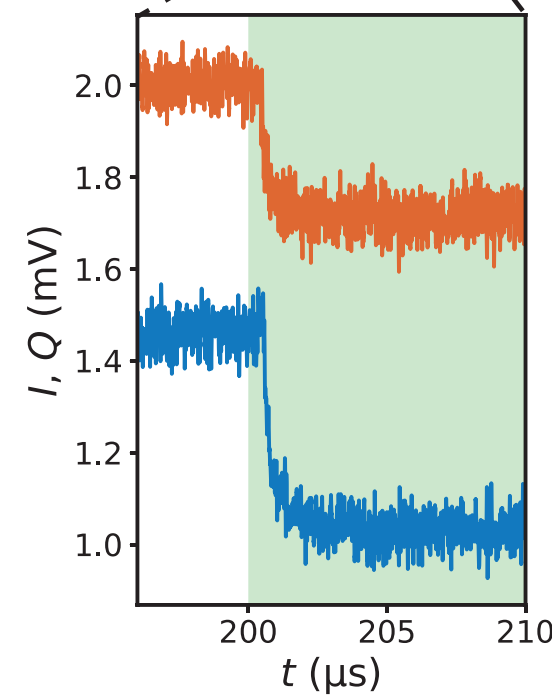
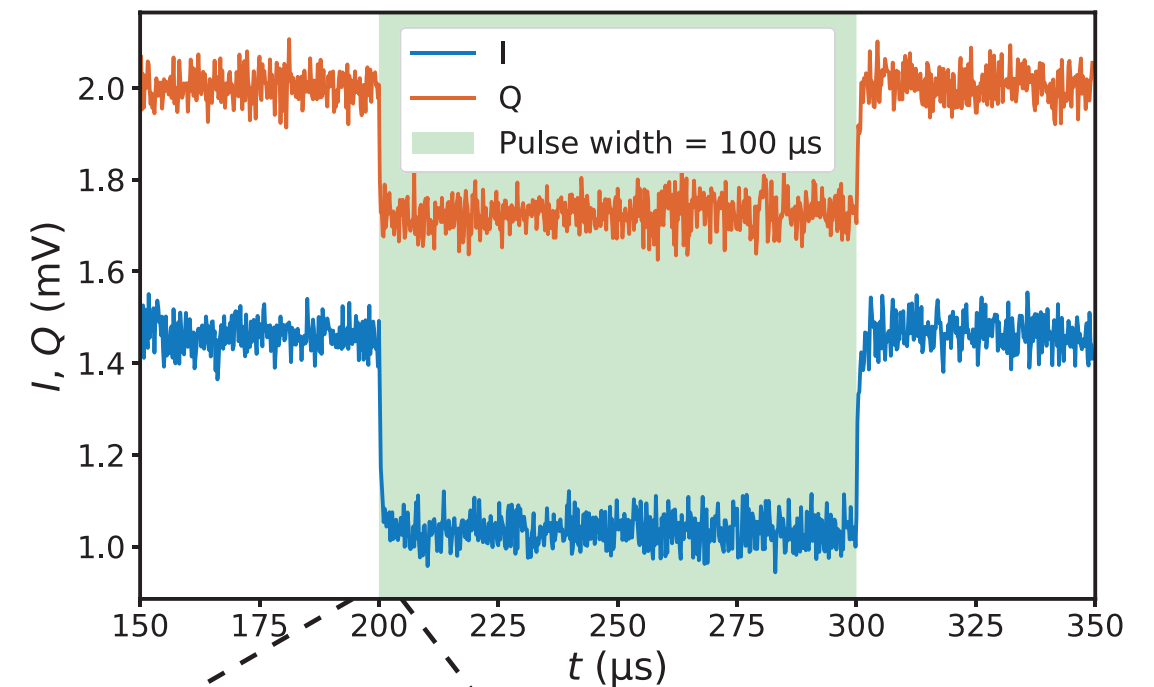
# Control Steps: Quasi Particle Poisoning

Applied clearing tone frequency = 18GHz

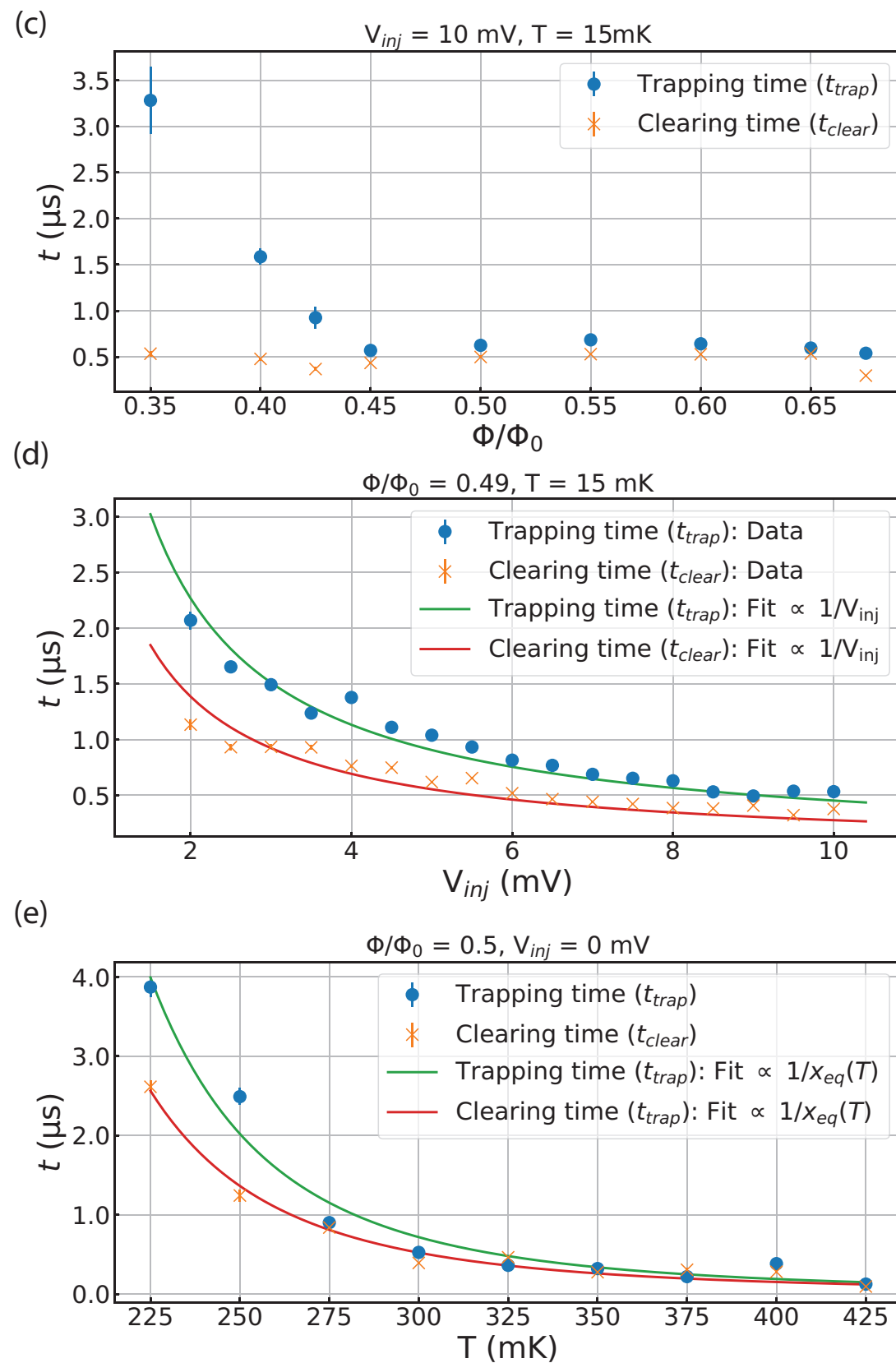
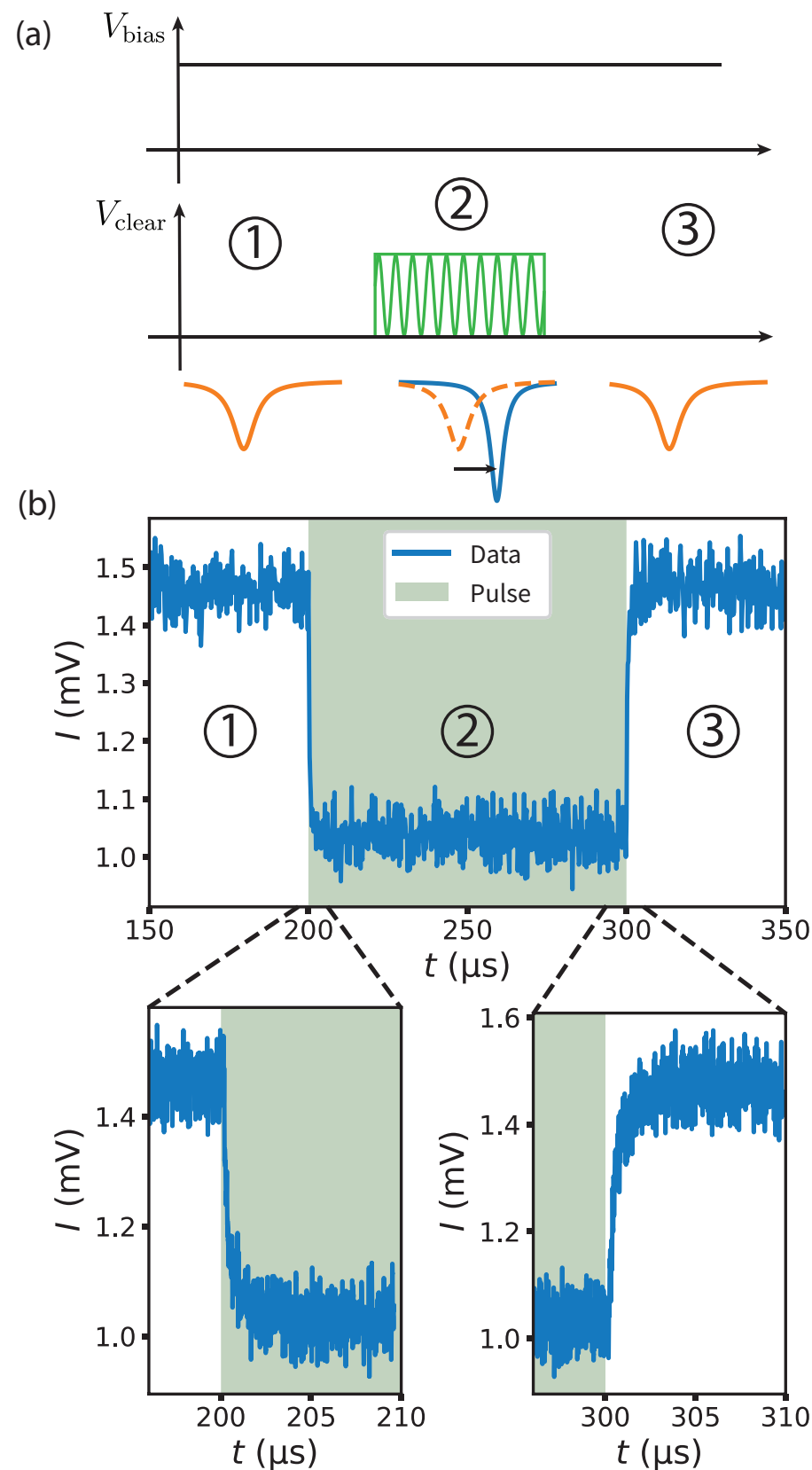


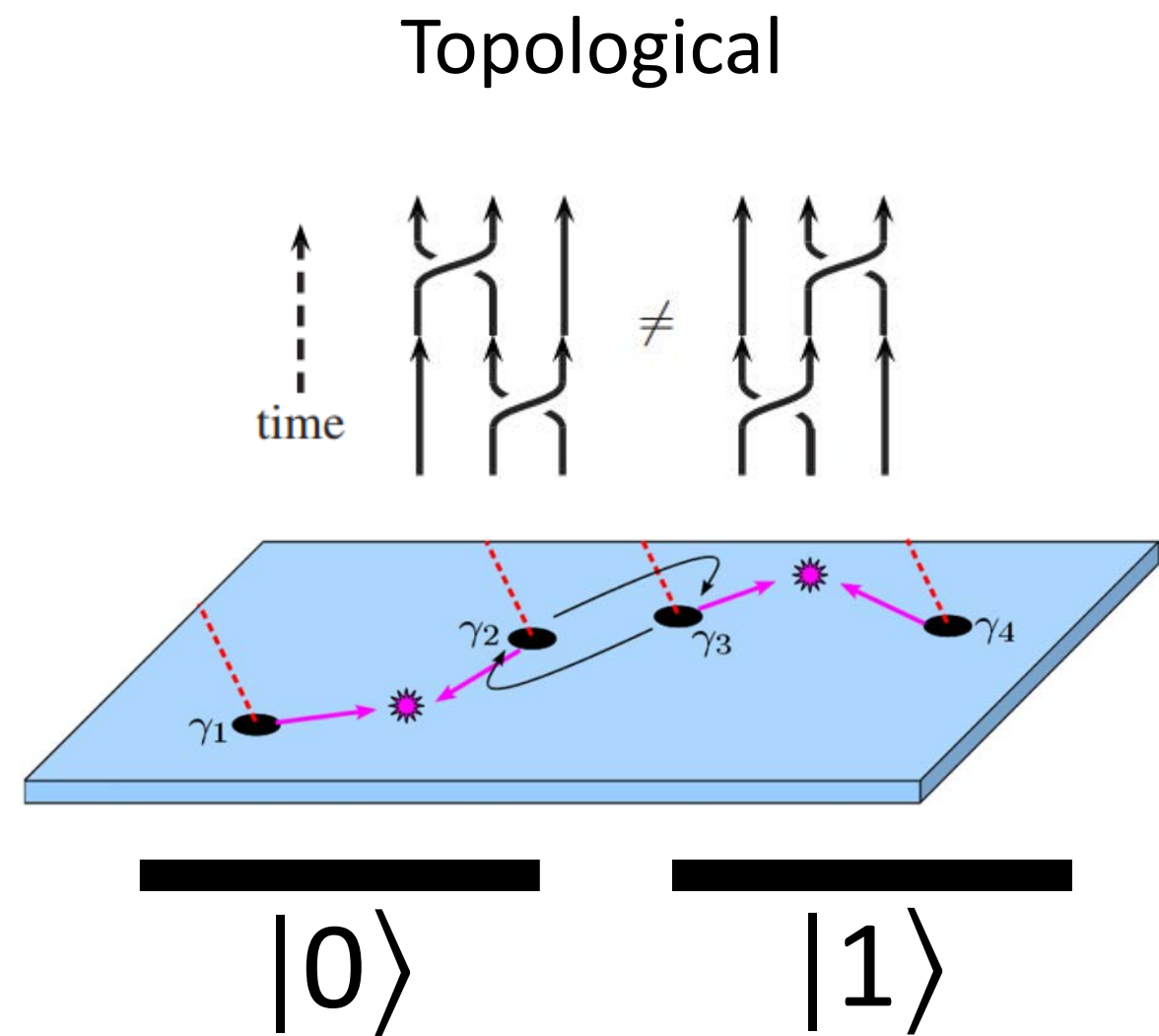
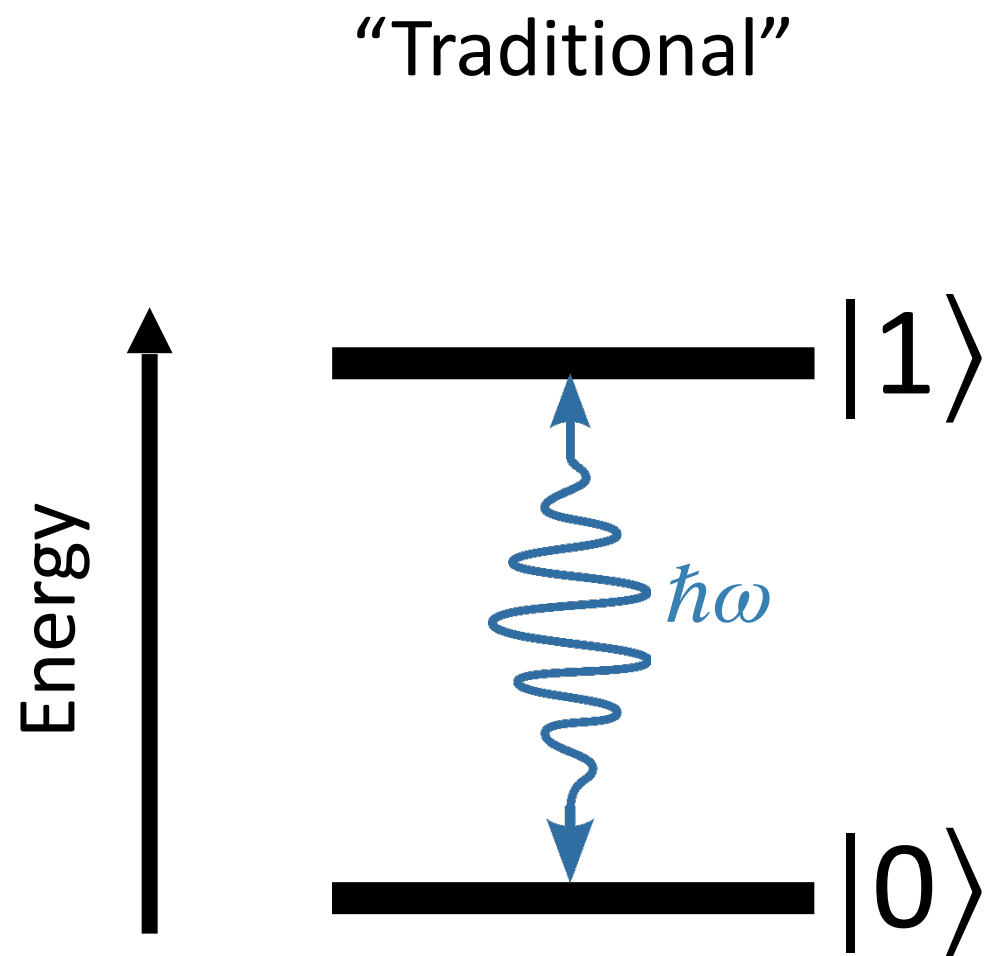
**Clearing time:** time it takes for QPs to clear out of the Andreev trap after pulse starts

**Trapping time:** time it takes QPs to fall into Andreev trap after pulse ends

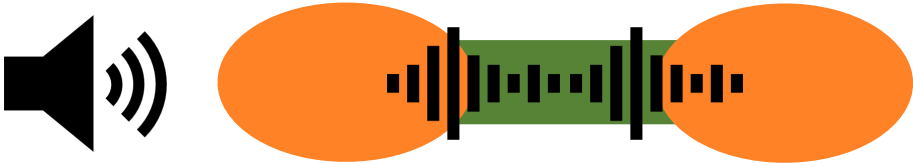
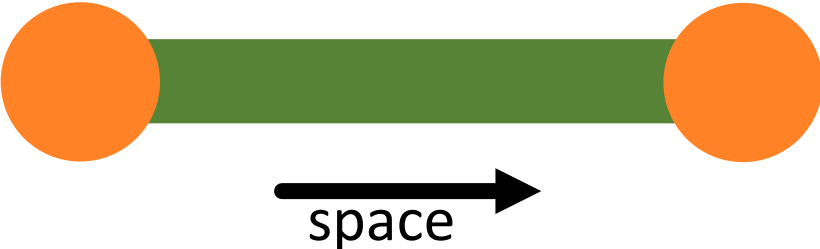


# Control Steps: Quasi Particle Poisoning

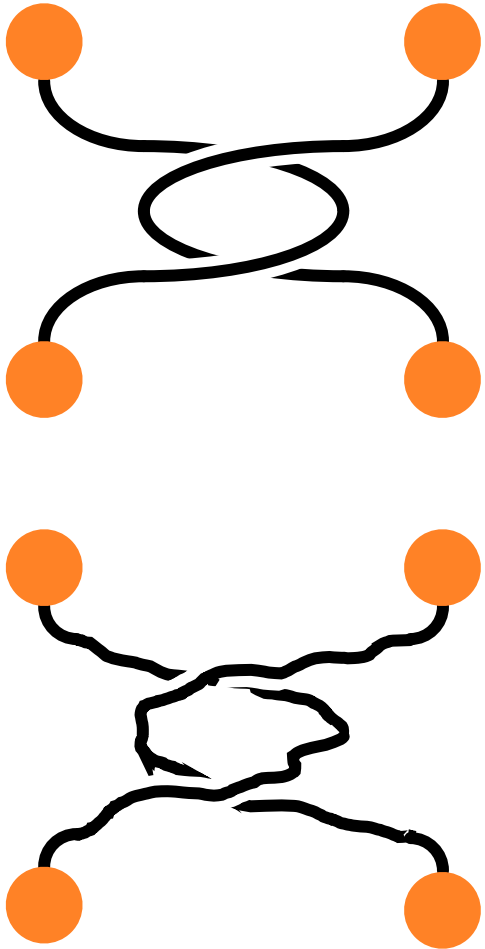
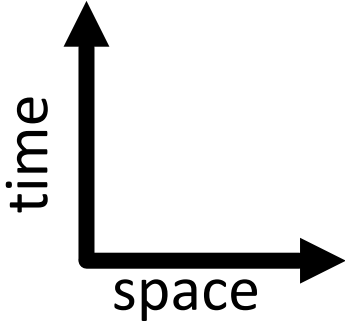




## Protected Encoding



## Protected Manipulation





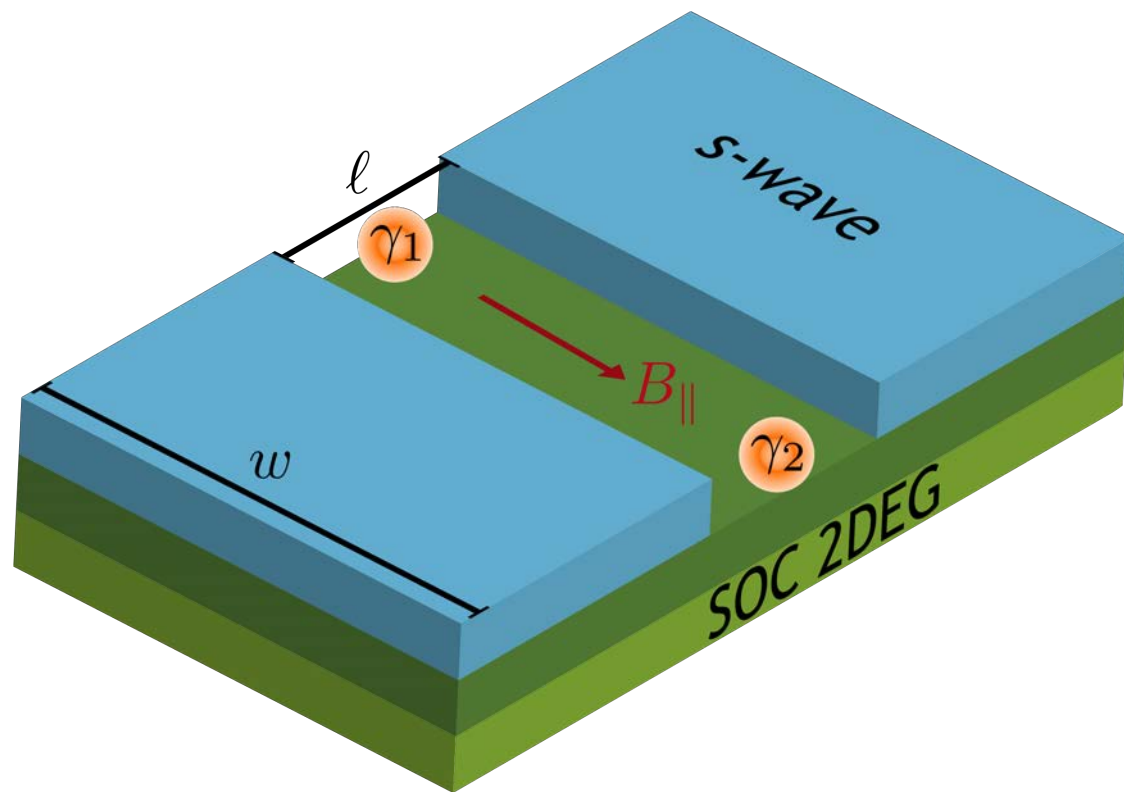
spin-orbit coupling

Zeeman effect

superconductivity

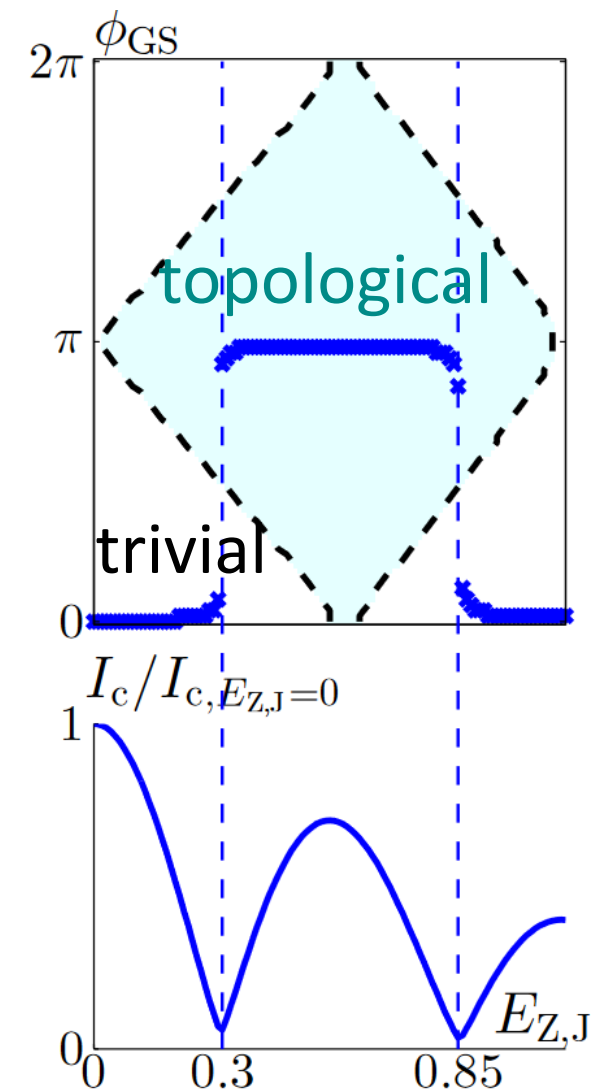
$$H = \left[ \frac{\mathbf{p}^2}{2m^*} - \mu + \frac{\alpha}{\hbar} (p_y \sigma_x - p_x \sigma_y) \right] \tau_z - \frac{g^* \mu_B}{2} \mathbf{B} \cdot \boldsymbol{\sigma} + \Delta e^{i\varphi/2} \tau_+ + \Delta^* e^{-i\varphi/2} \tau_-$$

chemical potential



$$E_{SO} \gg E_Z > \sqrt{\Delta^2 + \mu^2}$$

phase



spin-orbit coupling

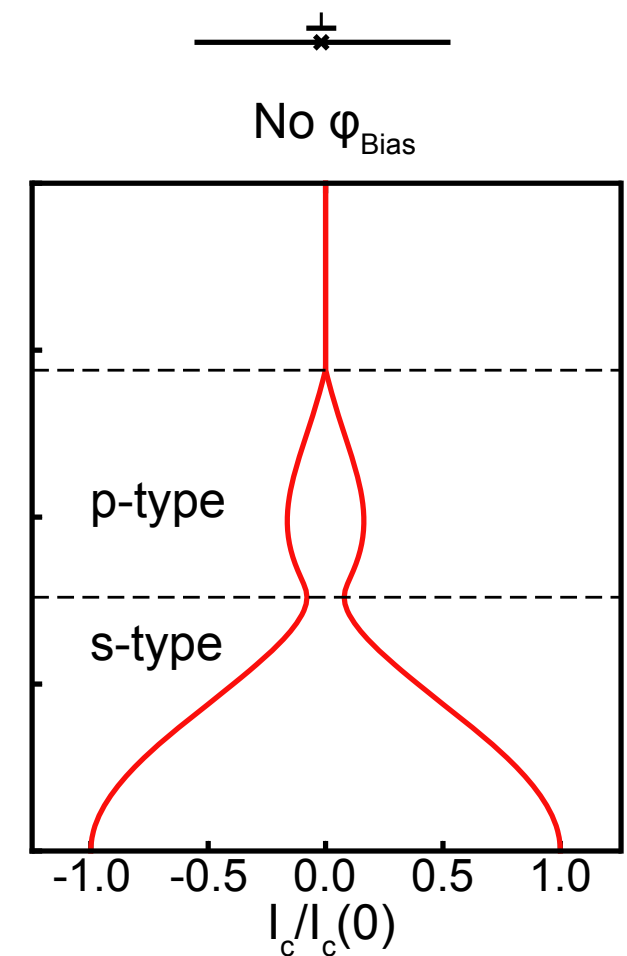
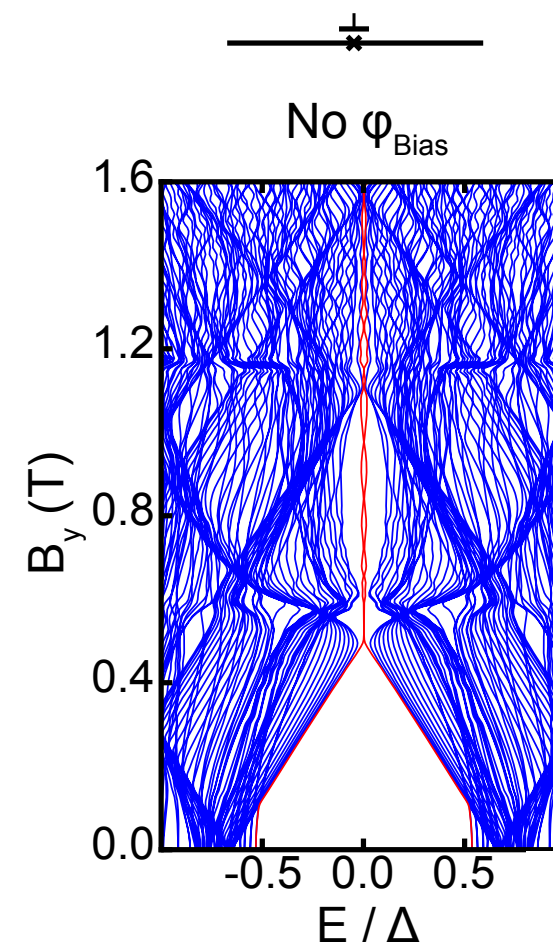
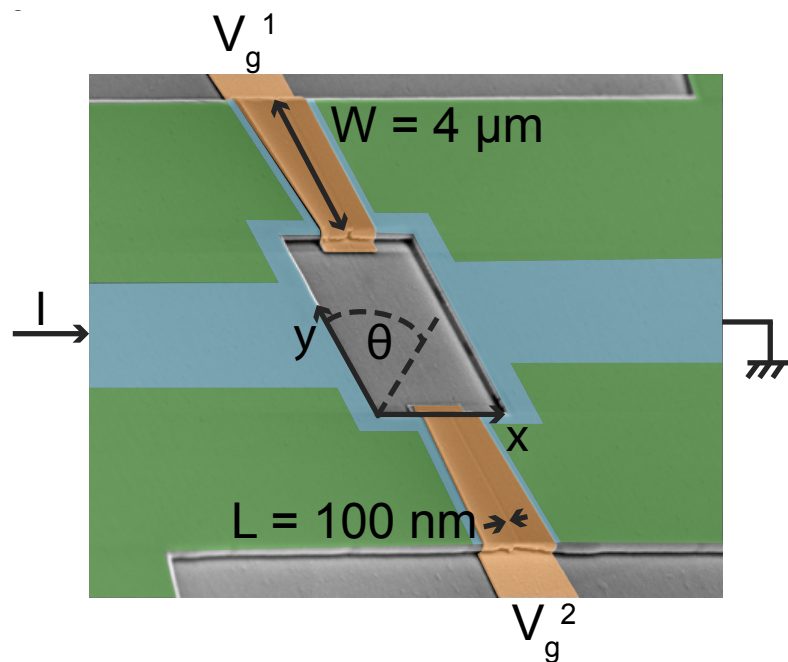
Zeeman effect

superconductivity

$$H = \left[ \frac{\mathbf{p}^2}{2m^*} - \mu + \frac{\alpha}{\hbar} (p_y \sigma_x - p_x \sigma_y) \right] \tau_z - \frac{g^* \mu_B}{2} \mathbf{B} \cdot \boldsymbol{\sigma} + \Delta e^{i\varphi/2} \tau_+ + \Delta^* e^{-i\varphi/2} \tau_-$$

chemical potential

phase



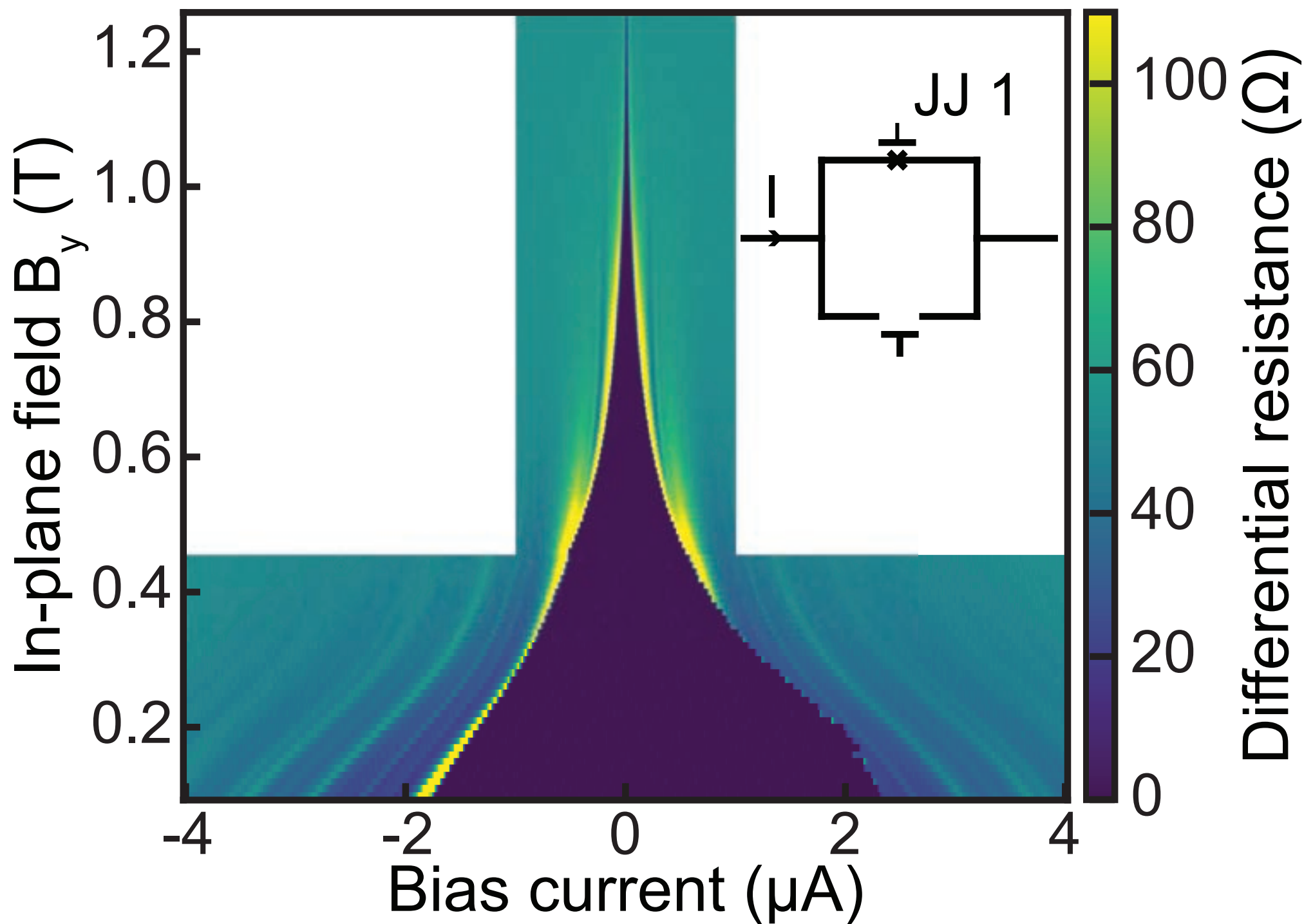
Simulations by Igor Zutic and Alex Matos-Abiague

Hell et al., PRL (2017).  
 Pientka et al., PRX (2017).  
 Tkachov, PRB (2019)  
 Scharf et al., PRB (2019)

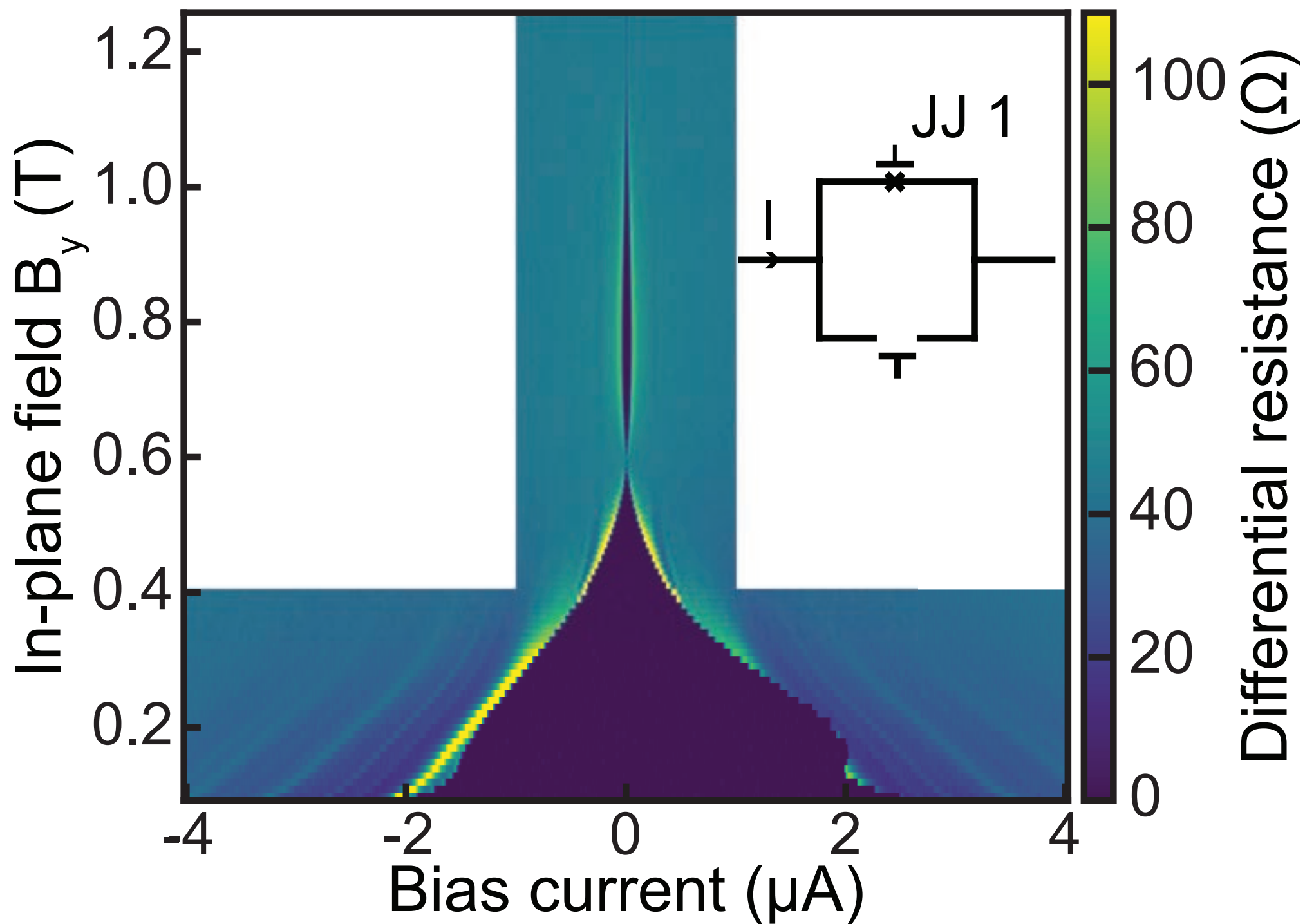
Setiawan, et al., PRB (2019)  
 Ren et al., Nature 2019  
 Fornieri et al., Nature 2019  
 Tong et al., PRL (2020)

Pakizer et al., PRR (2021)  
 Dartailh et al., PRL (2021)

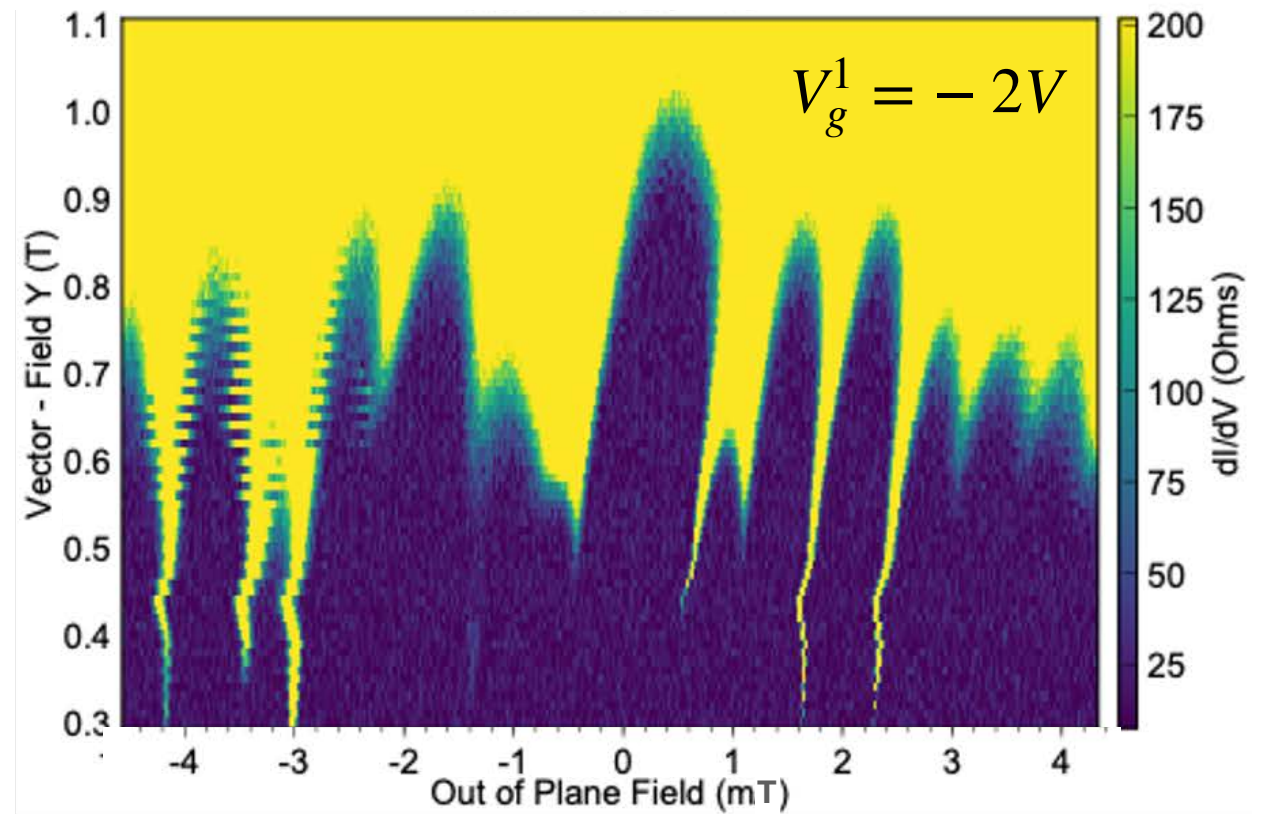
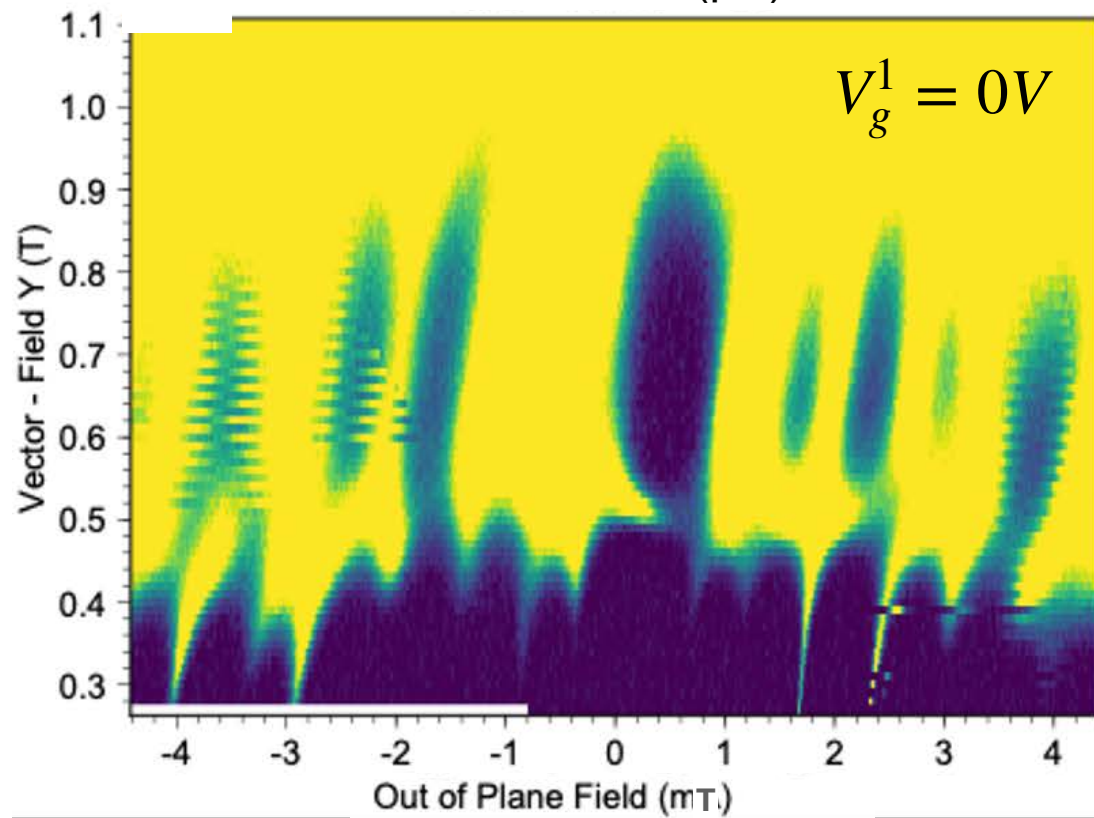
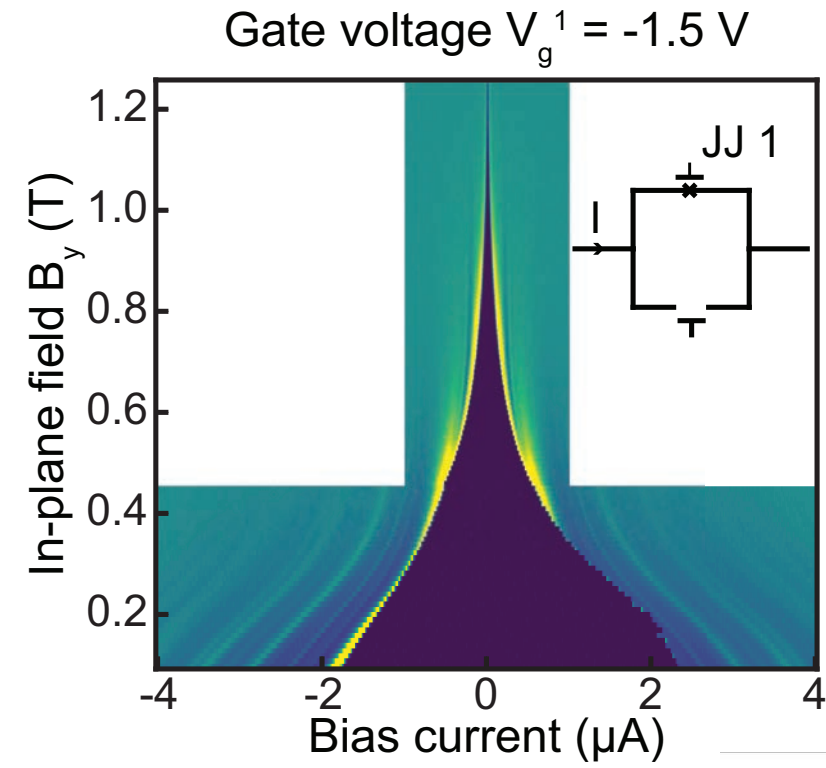
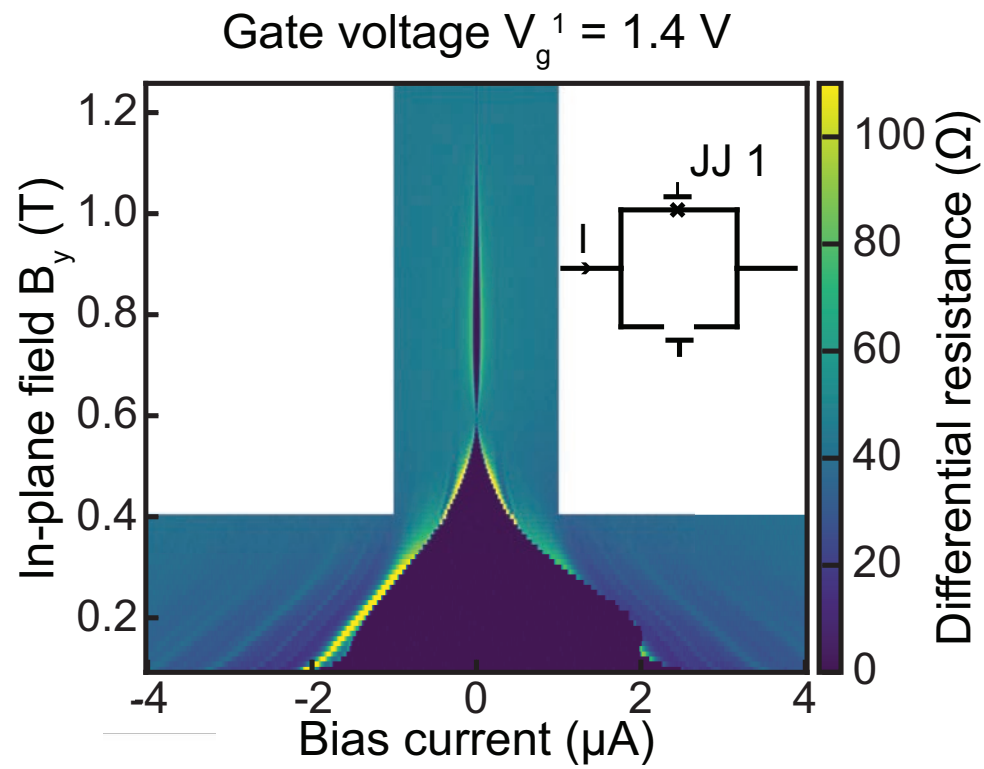
Gate voltage  $V_g^1 = -1.5 \text{ V}$

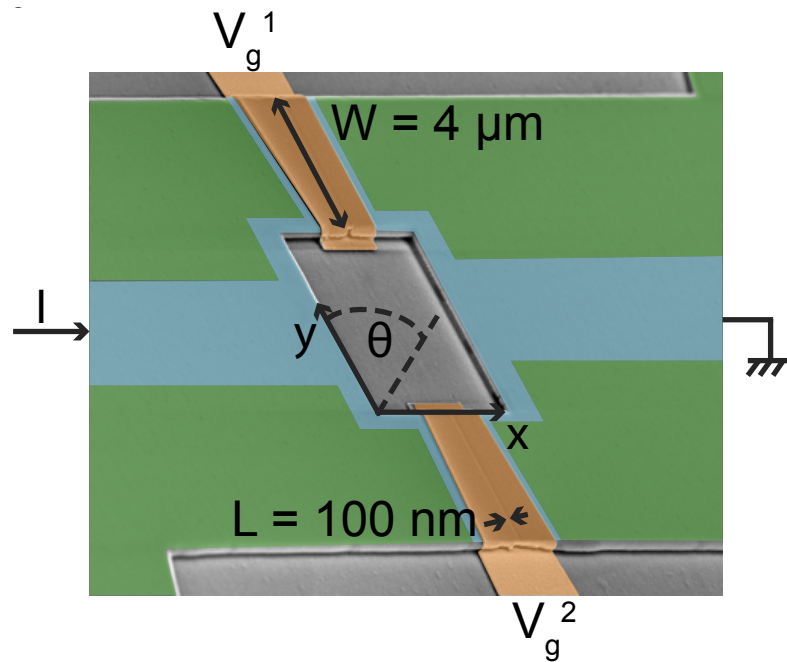


Gate voltage  $V_g^1 = 1.4 \text{ V}$





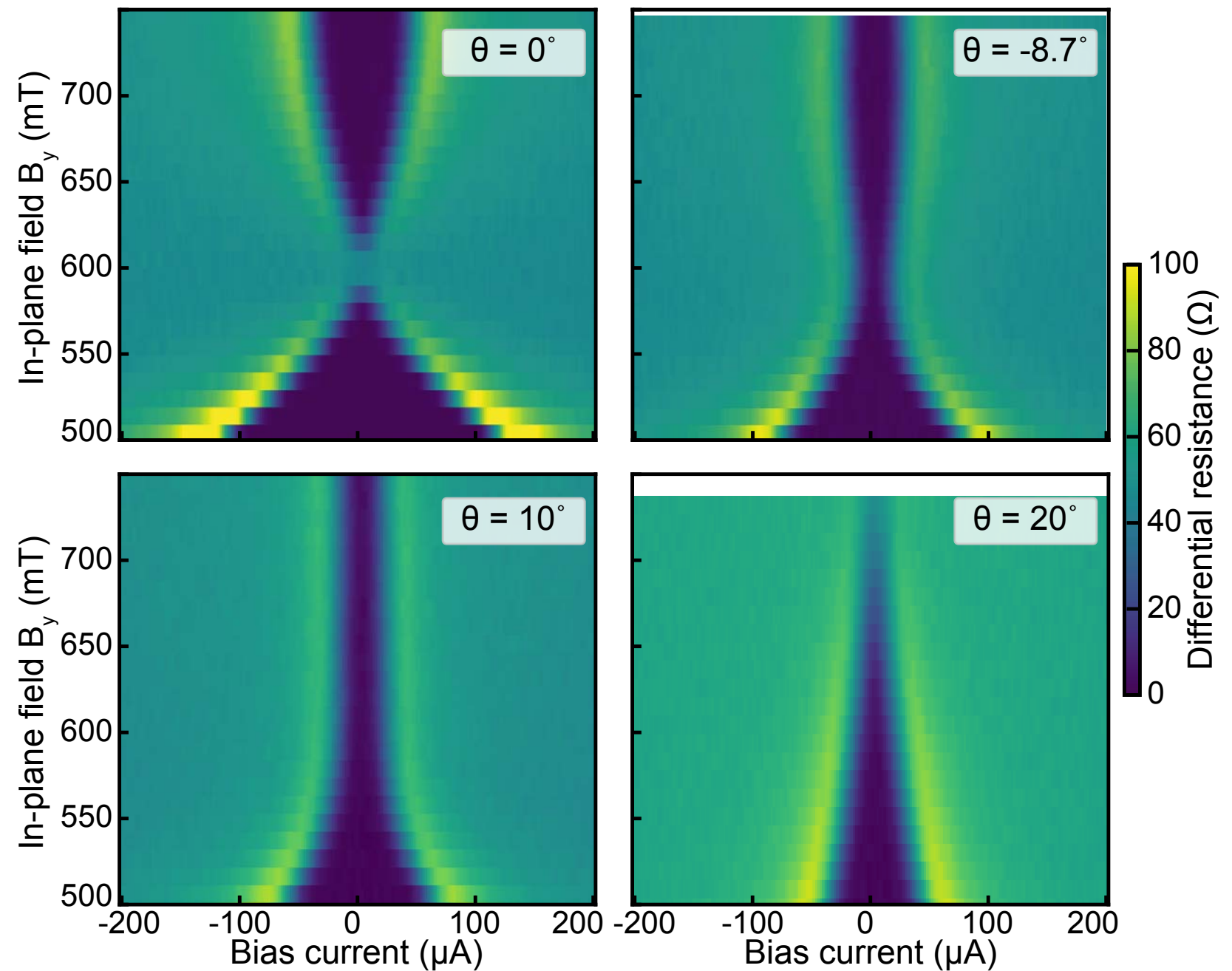


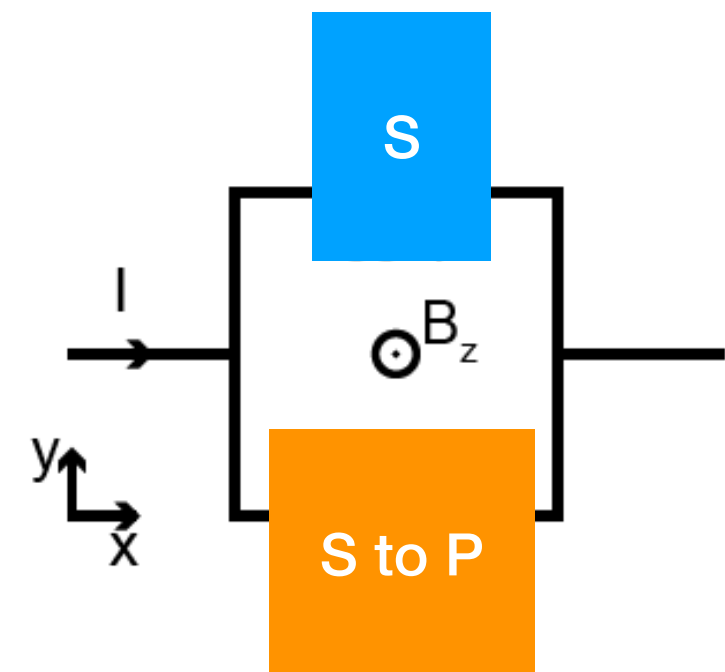
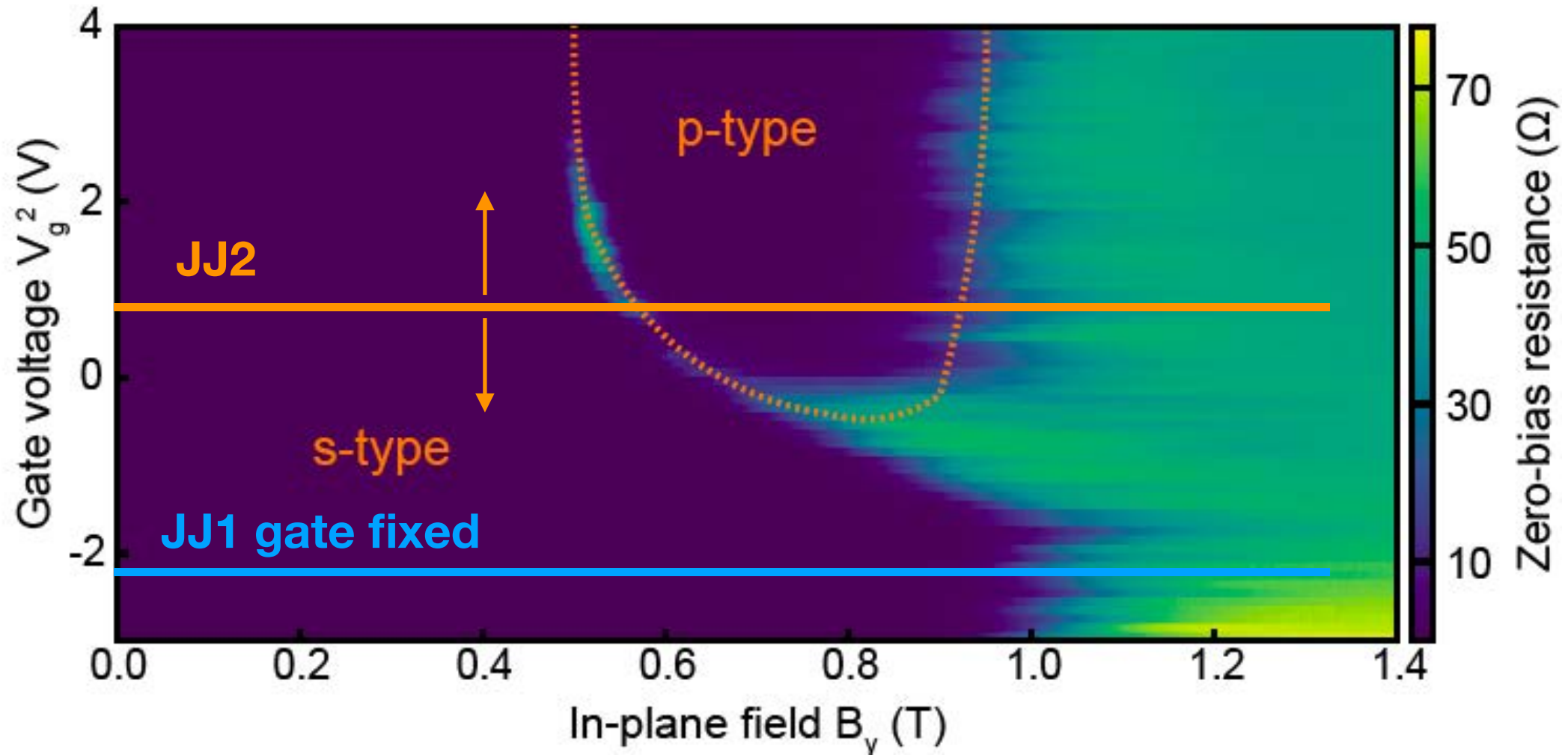


$$H \approx \frac{\alpha}{\hbar} (p_x \sigma_y)$$

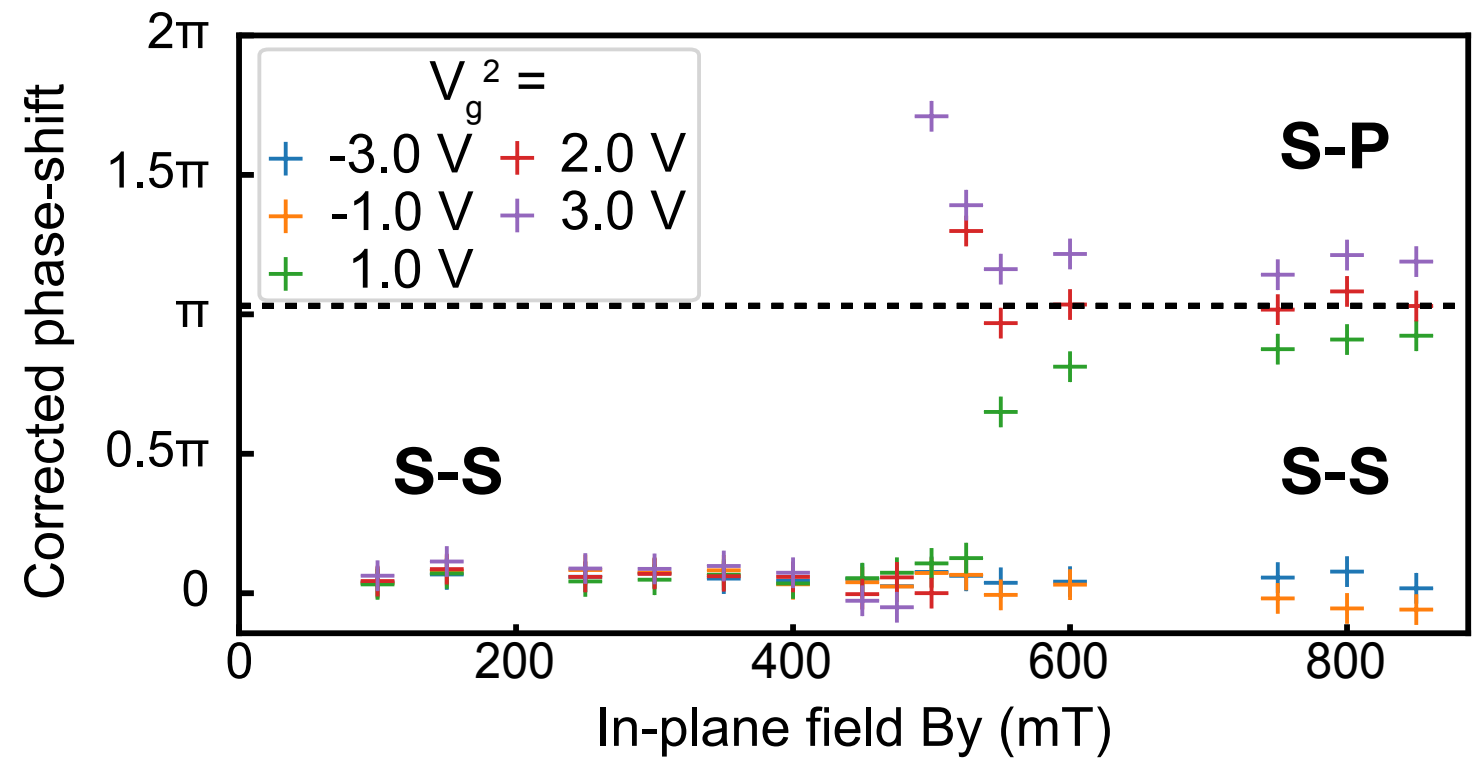
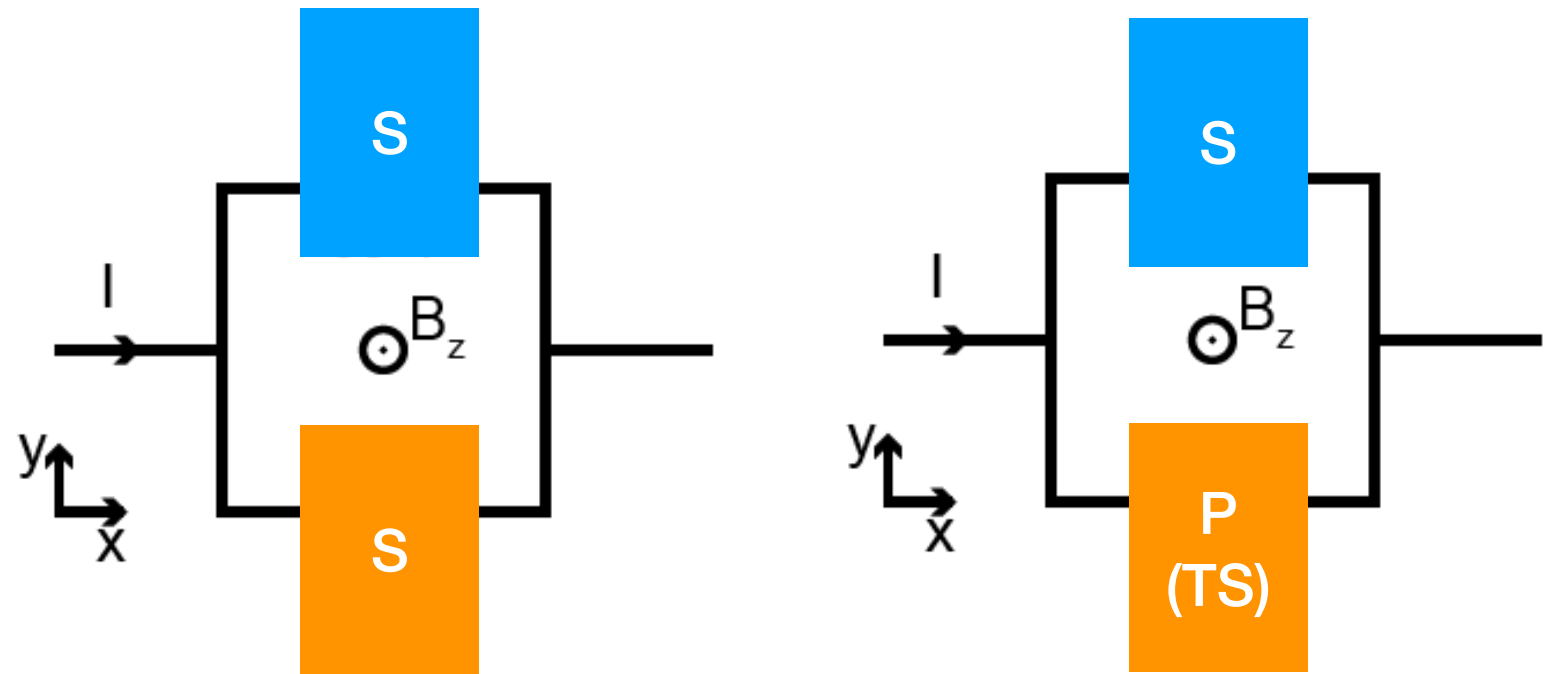
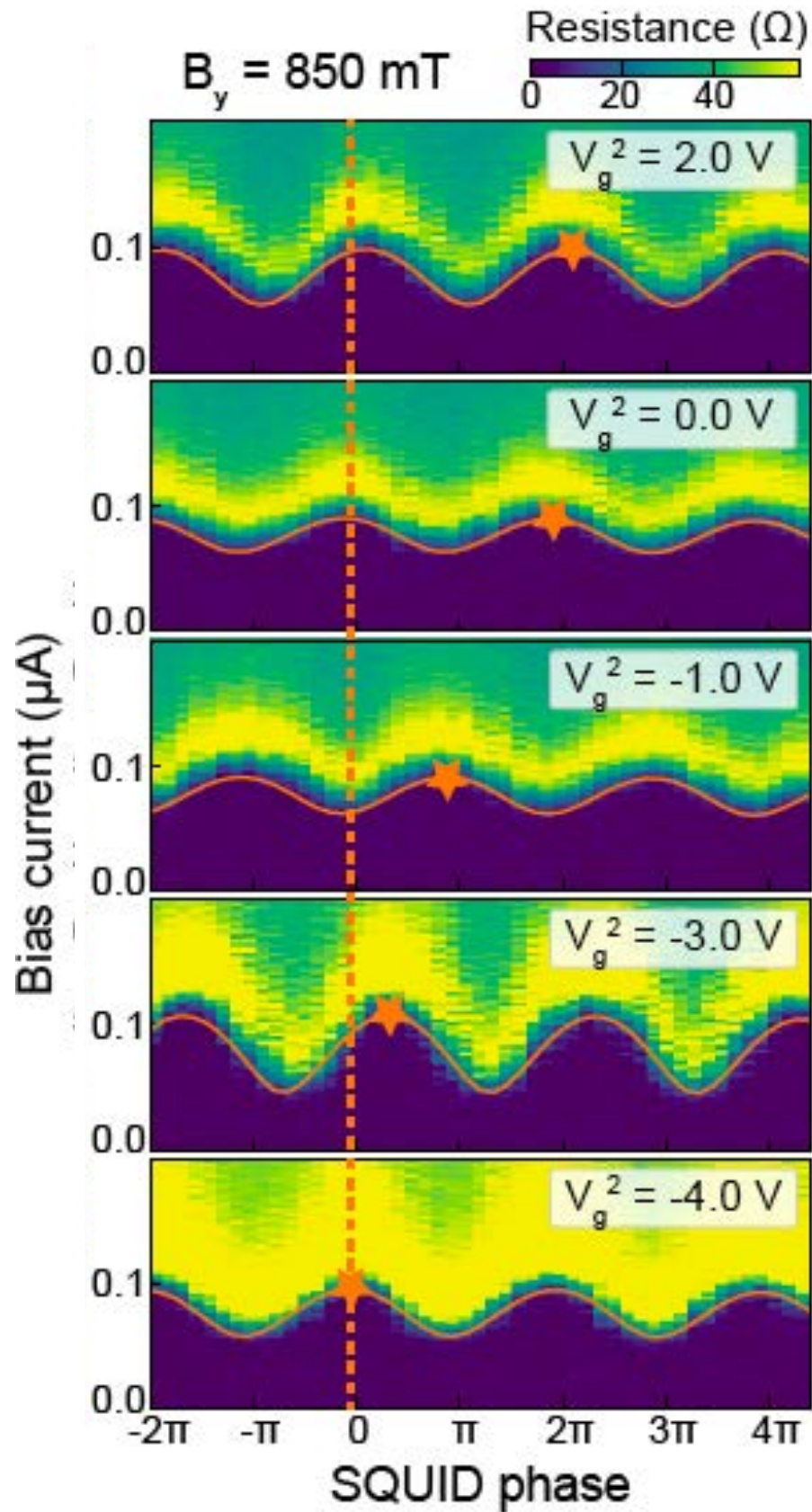
- Supercurrent is in x-direction

- Rashba SOC couples B to momentum *only if* B is oriented in y direction

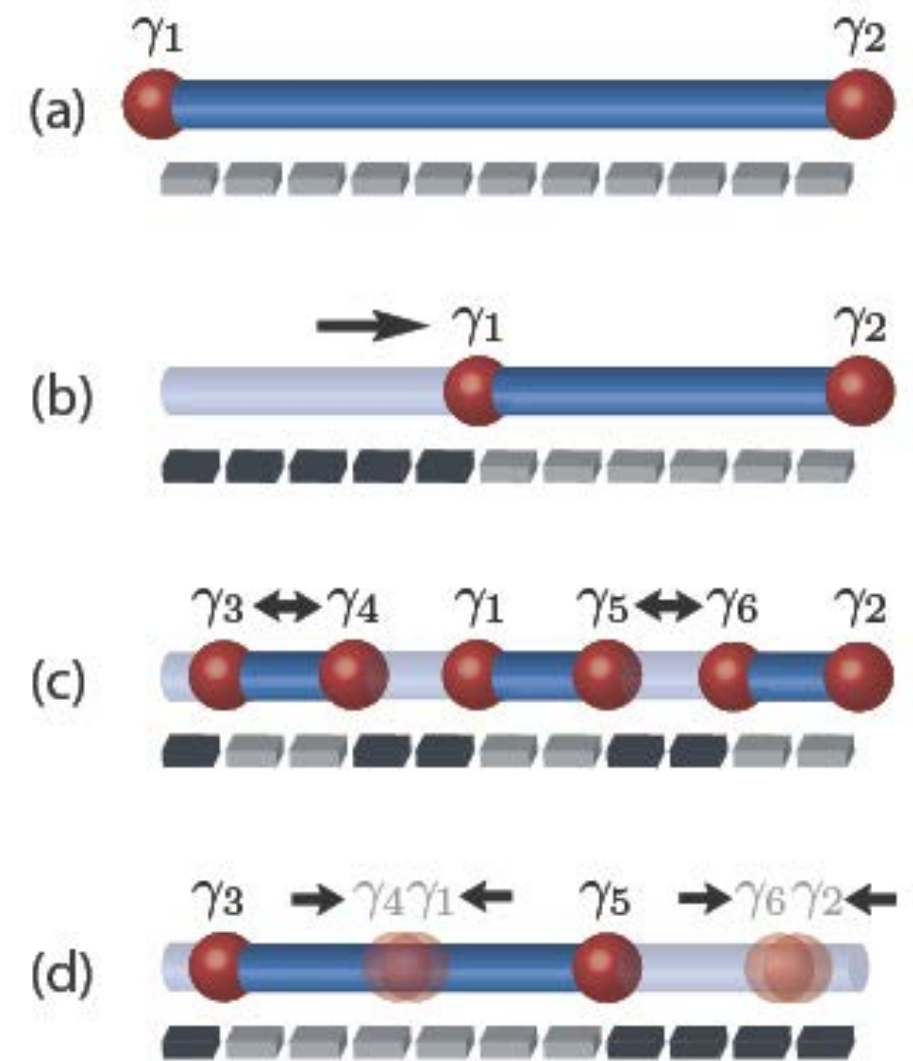
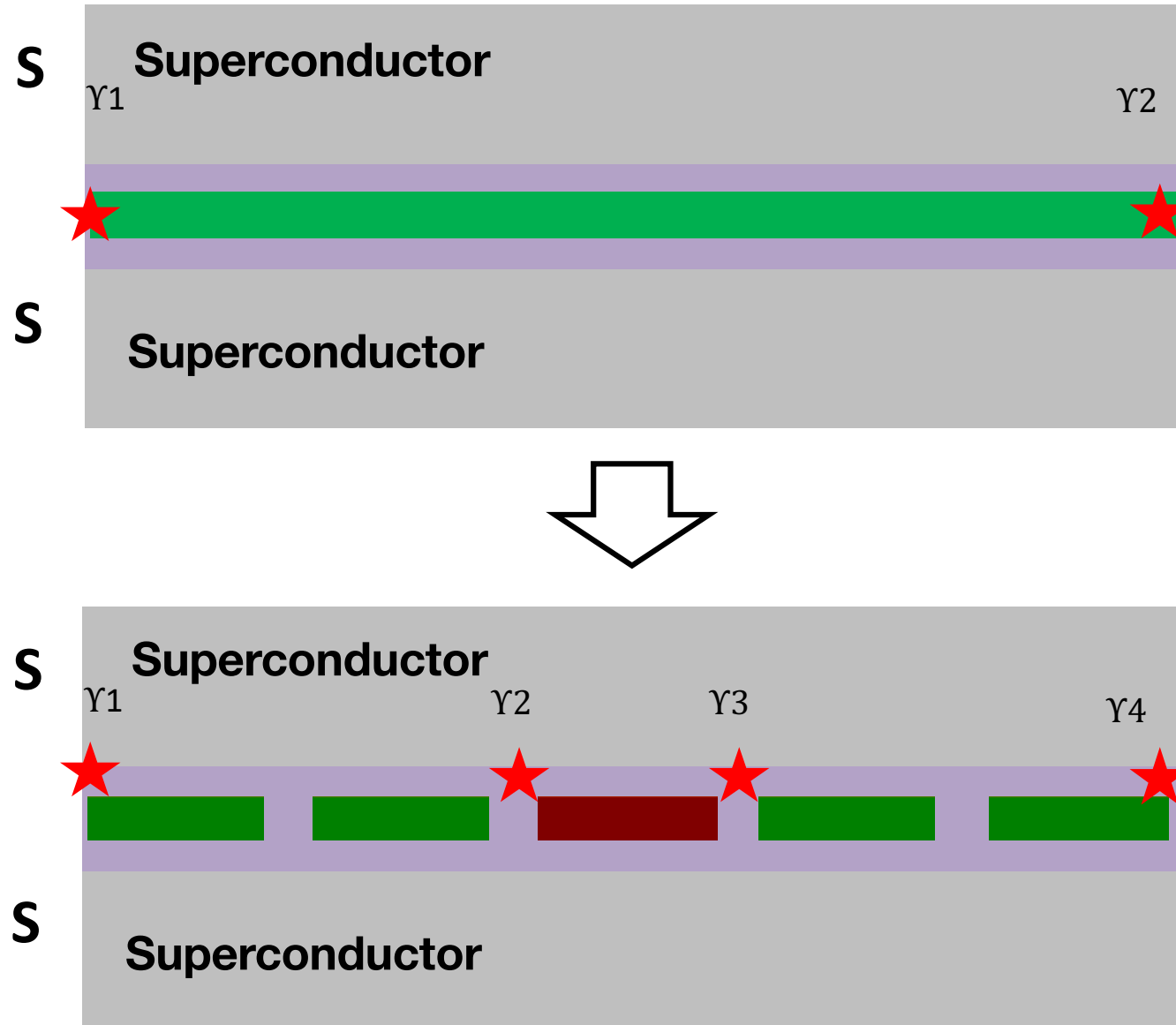








We showed single-gate JJs can make a phase transition. If this transition is topological then:



$$P_{ij} \Psi(\vec{x}_1 \dots \vec{x}_i \dots \vec{x}_j \dots \vec{x}_N) = e^{i\theta_{ij}} \Psi(\vec{x}_1 \dots \vec{x}_j \dots \vec{x}_i \dots \vec{x}_N)$$

The particles are **identical** since an overall phase is not observable.

But changing back amounts to nothing!

$$P_{ij}^2 \Psi(\vec{x}_1 \dots \vec{x}_i \dots \vec{x}_j \dots \vec{x}_N) = \Psi(\vec{x}_1 \dots \vec{x}_i \dots \vec{x}_j \dots \vec{x}_N)$$

$$P_{ij}^2 = 1 \quad \Rightarrow \quad e^{i\theta} = \pm 1$$

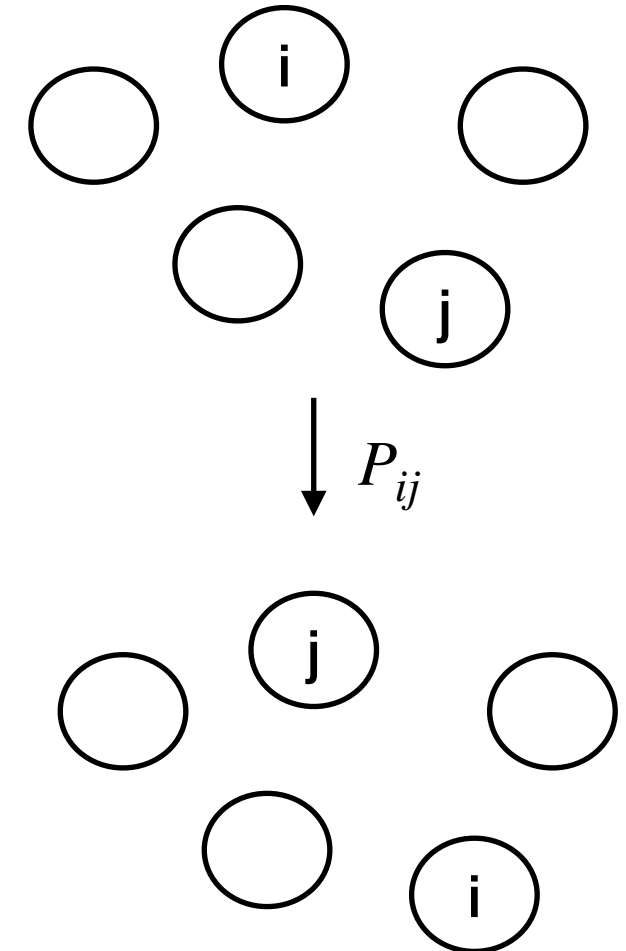
So there are only two alternatives:

$$P_{ij} = 1 \quad ; \quad \theta_{ij} = 0$$

**Boson**

$$P_{ij} = -1 \quad ; \quad \theta_{ij} = \pi$$

**Fermion**





$$P_{ij} \Psi(\vec{x}_1 \dots \vec{x}_i \dots \vec{x}_j \dots \vec{x}_N) = e^{i\theta_{ij}} \Psi(\vec{x}_1 \dots \vec{x}_j \dots \vec{x}_i \dots \vec{x}_N)$$

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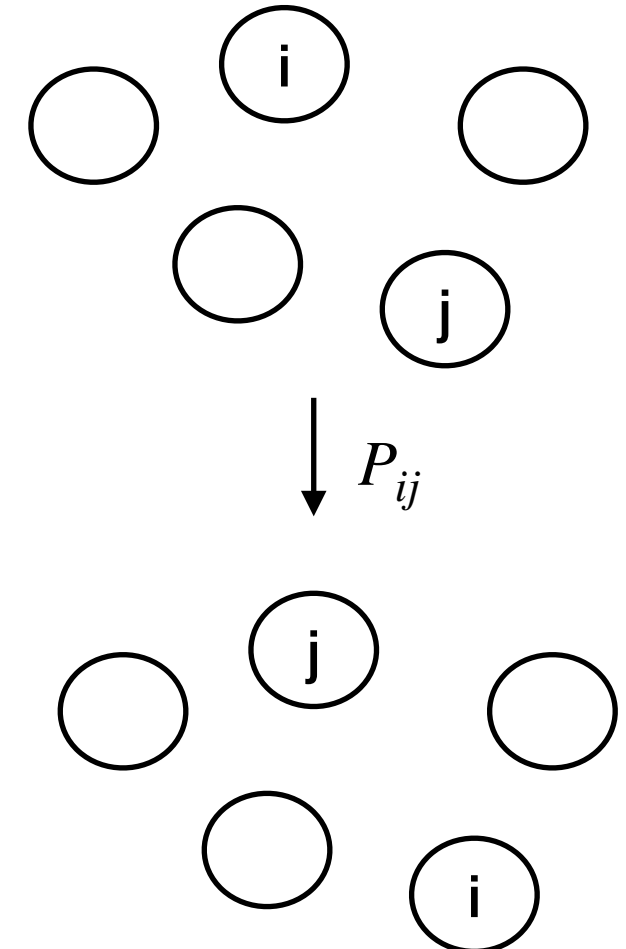
$$P_{ij} = 1 \quad ; \quad \theta_{ij} = 0 \quad \text{Boson}$$

$$P_{ij} = -1 \quad ; \quad \theta_{ij} = \pi \quad \text{Fermion}$$

- A quasiparticle emergent in 2-D quantum systems with **topological** nature.
- Unlike **fermions** and **bosons**, under particle exchange they obey statistics as:

$|\psi_1 \psi_2\rangle = e^{i\theta} |\psi_2 \psi_1\rangle$

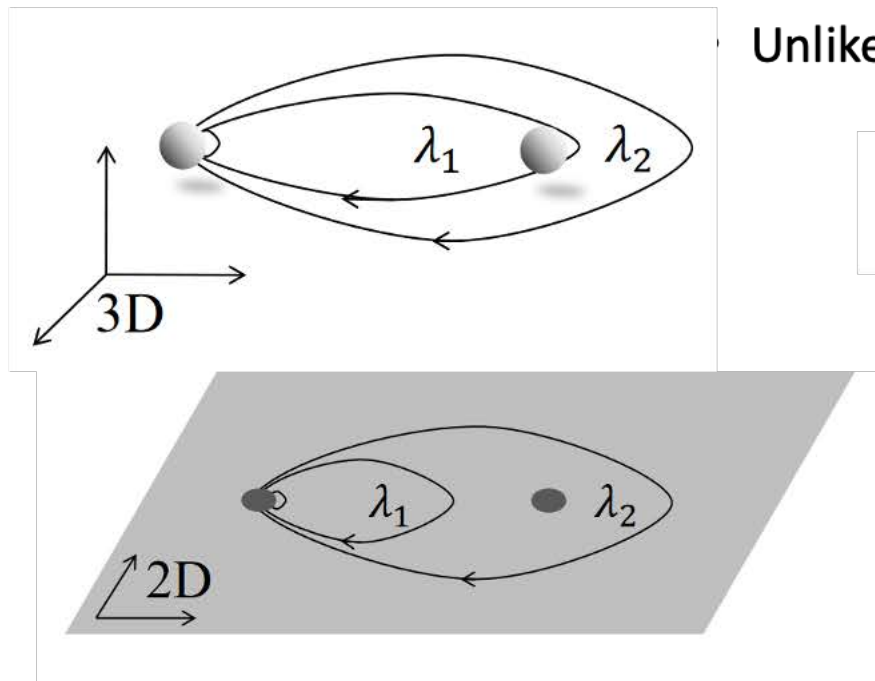
 where  $e^{i\theta}$  can be any value between -1 and +1.



## Anyons

- A quasiparticle emergent in 2-D quantum systems with **topological** nature. Unlike **fermions** and **bosons**, under particle exchange they obey statistics as:

$$|\psi_1 \psi_2\rangle = e^{i\theta} |\psi_2 \psi_1\rangle \text{ where } e^{i\theta} \text{ can be any value between -1 and +1.}$$



- For **non-Abelian** anyons,
  - $e^{i\theta} \rightarrow$  rotation matrix  $U(\theta)$
  - $\rightarrow$  can be used to create universal gates

### Example:

The Fibonacci model contains **two particle types**: the vacuum with charge 0 and denoted by  $\mathbf{0}$ , and the non-trivial anyon with charge 1 and denoted by  $\tau$ . Explicitly, the fusion rules are:

$$\tau \otimes \tau = \mathbf{0} \oplus \tau \quad \mathbf{0} \otimes \mathbf{0} = \mathbf{0} \quad \tau \otimes \mathbf{0} = \tau \quad \mathbf{0} \otimes \tau = \tau$$

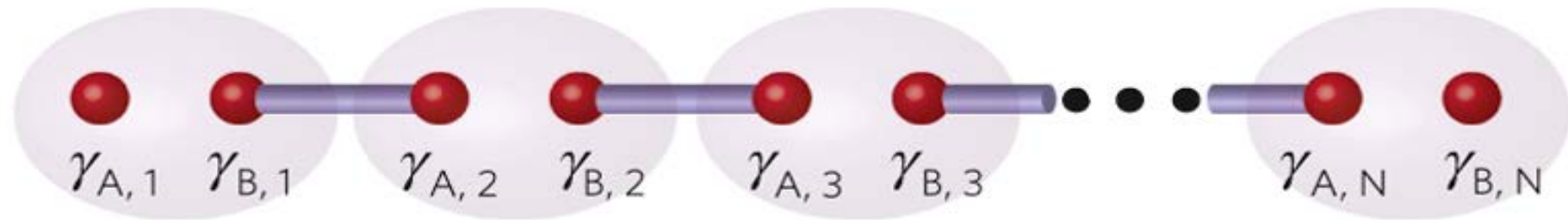
where  $\otimes$  denotes the fusion (merging) of two particles and  $\oplus$  denotes multiple possible outcomes.

- Majorana fermion (1940s): fermion that is its own antiparticle (maybe Neutrino?)

$$\gamma^\dagger = \gamma$$



Alexei Kitaev, 2000, 2001



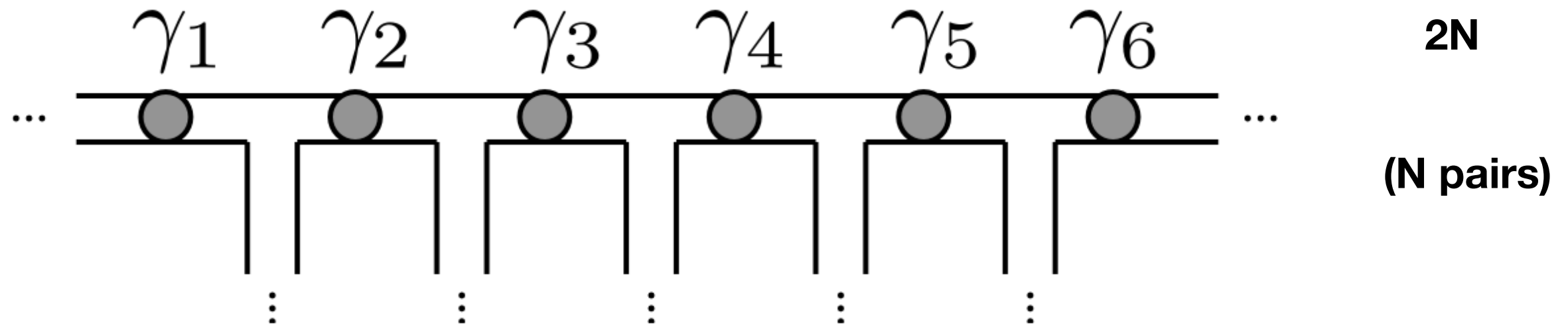
## 1D toy model

$$H_K = \sum_i \left( -\epsilon c_i^\dagger c_i - \frac{J}{2} (c_{i+1} - c_{i+1}^\dagger)(c_i + c_i^\dagger) \right) = \sum_i \left( -\epsilon c_i^\dagger c_i - \frac{J}{2} (c_{i+1}^\dagger c_i + c_i^\dagger c_i) + \frac{\Delta}{2} (c_{i+1} c_i + c_i^\dagger c_{i+1}^\dagger) \right)$$

**Kitaev predicted that a 1D chain under appropriate conditions can host delocalized Majorana modes**



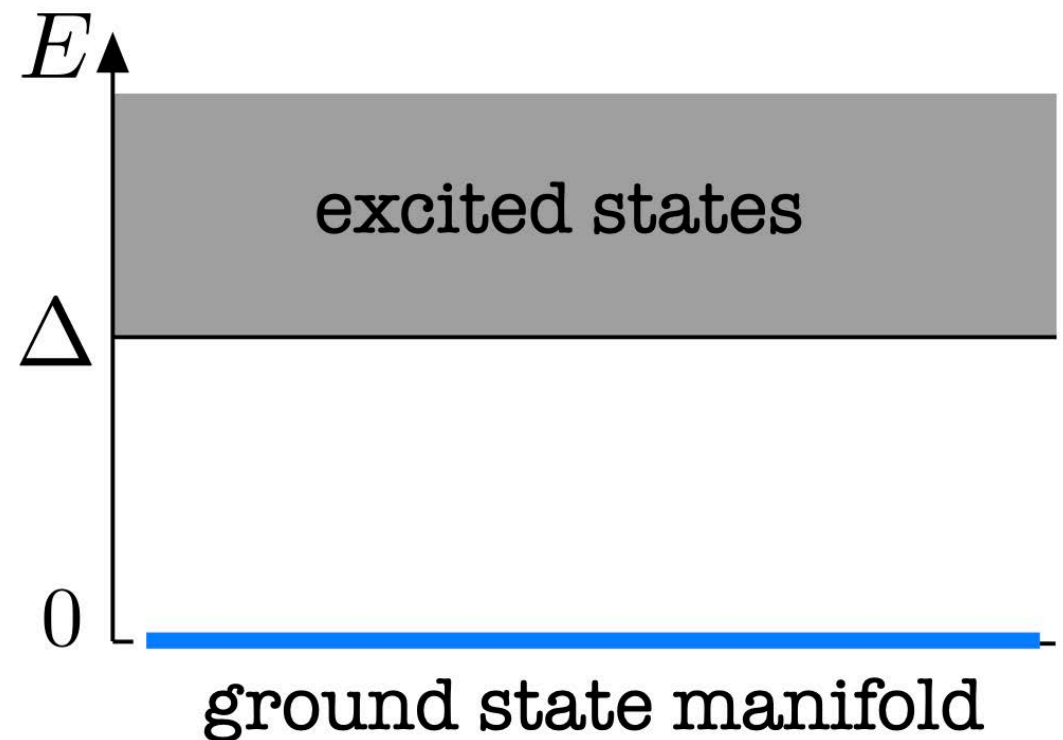




- The only thing that distinguishes the Majorana zero modes is their position in the network.
- They have no other “flavour” that would allow us to characterize them. They are identical to each other, just like all electrons are identical to each other.
- If we exchanged two Majoranas in space, the system after the exchange would look exactly the same as it looked before the exchange.

$$\gamma_i^\dagger = \gamma_i$$

= Degenerate zero energy-state manifold



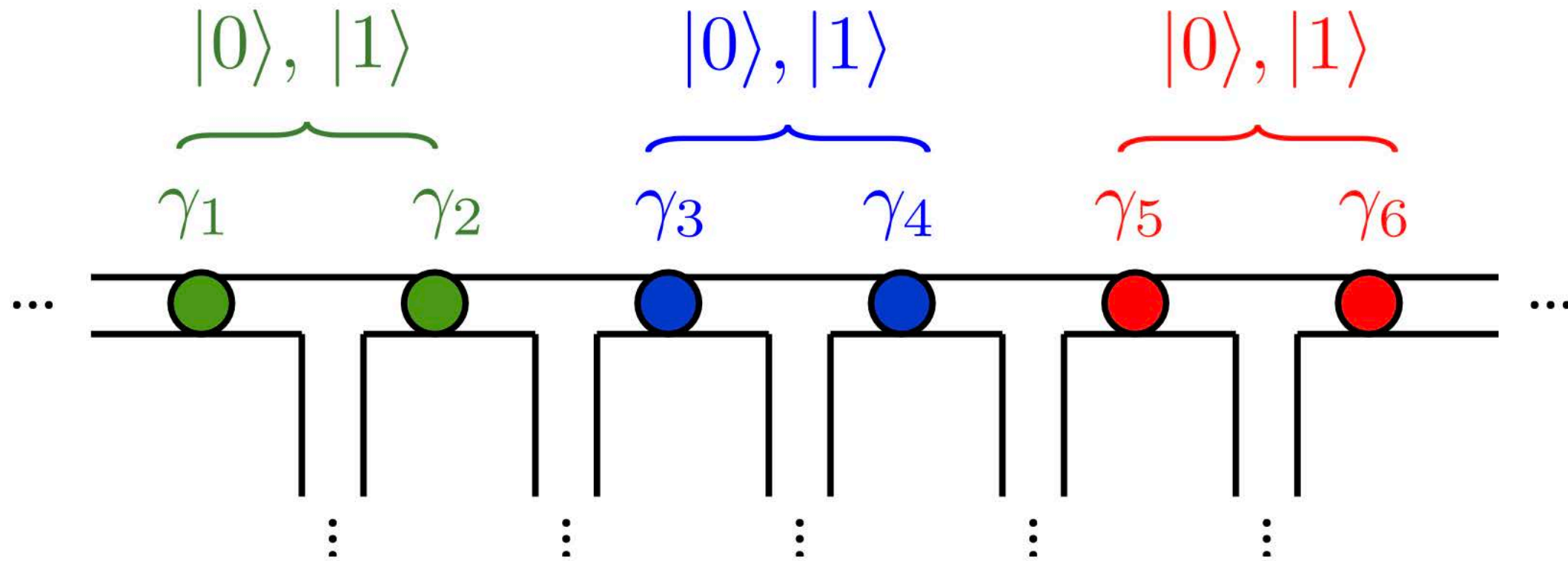
By construction, we can pair the Majoranas and form fermionic modes

$$c_n^\dagger = \frac{1}{2}(\gamma_{2n-1} + i\gamma_{2n}),$$

$$c_n = \frac{1}{2}(\gamma_{2n-1} - i\gamma_{2n}), \quad n=1, \dots, N$$

We have now a set of  $N$  fermionic modes with corresponding creation and annihilation operators. Every mode can be empty or it can be occupied by a fermion, giving us two possible degenerate quantum state:

$|0\rangle$     $|1\rangle$    for each pair of Majoranas



- 8 possible states, corresponding to all the possible combinations of the occupation numbers of the 3 fermionic modes

$2^N$  possible quantum states for  $N$  pairs of Majoranas



$|s_1\rangle$       if the fermionic mode is not occupied = 0      If occupied = 1

$|s_1, s_2, \dots, s_N\rangle$

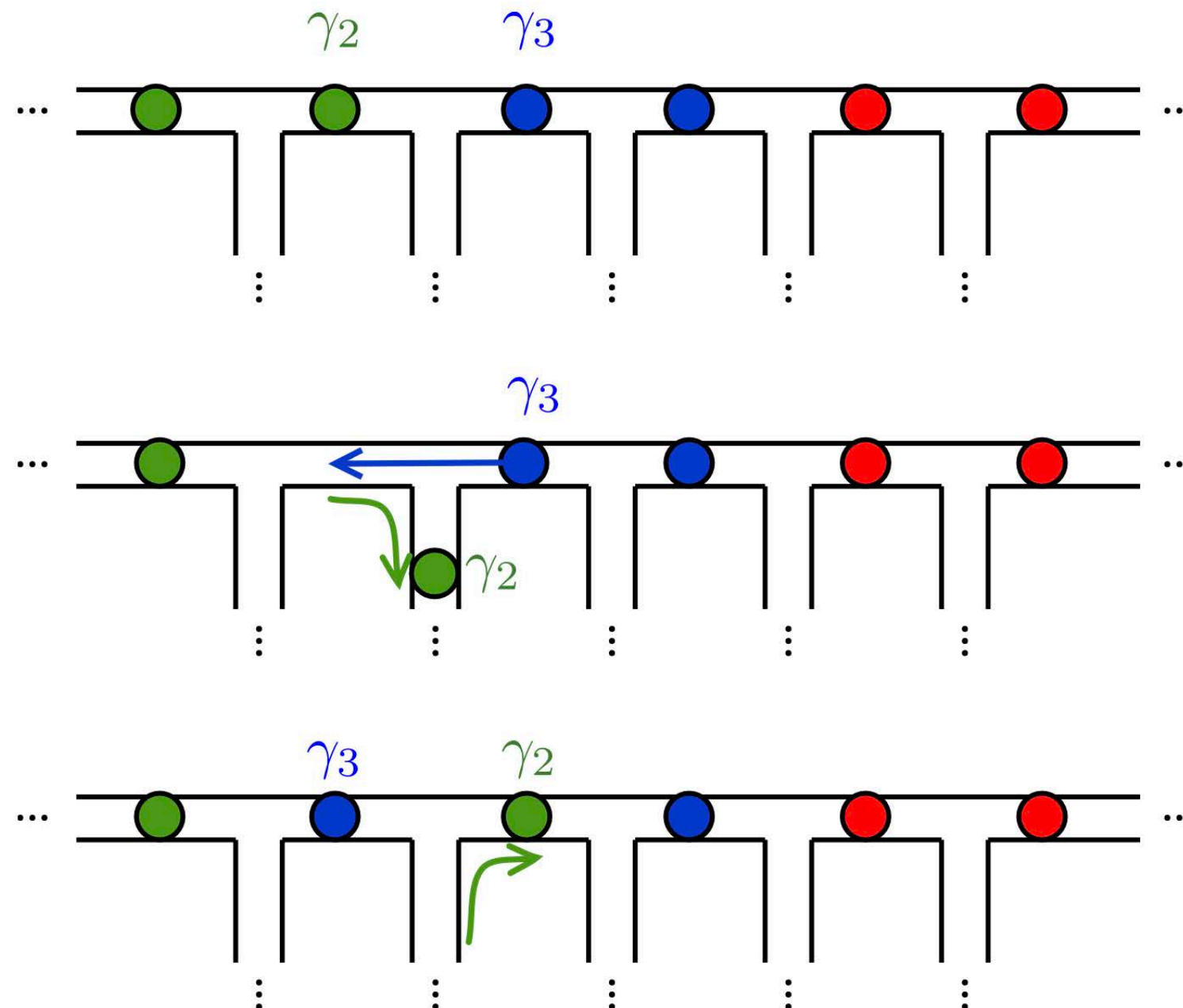
- These states are a *complete basis* for the Hilbert space of the set of Majorana modes
- These basis states are all eigenstates of the operators

$$P_n = 1 - 2c_n^\dagger c_n = i\gamma_{2n-1}\gamma_{2n}$$

**Fermion parity operator**  
for the pair of Majoranas  $2n-1$  and  $2n$

$$|\Psi\rangle = \sum_{s_n=0,1} \alpha_{s_1 s_2 \dots s_N} |s_1, s_2, \dots, s_N\rangle$$

We, experimentalist, are not only like to build such a network, but also to move the position of the domain walls and swap the positions of two Majoranas, for instance by performing the following trajectory:

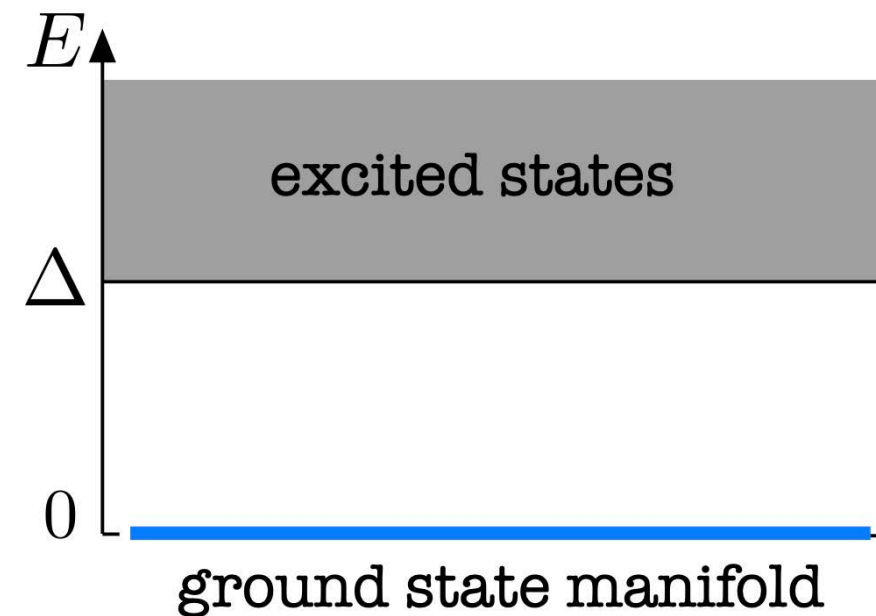


Adiabatic: During moving we never leave the ground state manifold with  $2^N$  states

$$|\Psi\rangle \rightarrow U |\Psi\rangle \quad 2^N \times 2^N \text{ unitary matrix}$$

- The adiabatic exchange of two Majoranas does not change the parity of the number of electrons in the system

$$[U, P_{\text{tot}}] = 0$$



$U$ : the exponential of  $i$  times a Hermitian operator is a unitary operator

$$U \equiv \exp(\beta \gamma_n \gamma_m) = \cos(\beta) + \gamma_n \gamma_m \sin(\beta) \quad \beta = \pm\pi/4$$



$$U = \exp\left(\pm \frac{\pi}{4} \gamma_n \gamma_m\right) = \frac{1}{\sqrt{2}}(1 \pm \gamma_n \gamma_m)$$

Consider **four Majoranas**:

$|00\rangle, |11\rangle, |01\rangle, |10\rangle$

$$c_1^\dagger = \frac{1}{2}(\gamma_1 + i\gamma_2)$$

First digit is the occupation number of the fermionic mode

$$c_2^\dagger = \frac{1}{2}(\gamma_3 + i\gamma_4)$$

Second digit is the occupation number of the fermionic mode

$$U_{12} = \exp\left(\frac{\pi}{4} \gamma_1 \gamma_2\right) \equiv \begin{pmatrix} e^{-i\pi/4} & 0 & 0 & 0 \\ 0 & e^{i\pi/4} & 0 & 0 \\ 0 & 0 & e^{-i\pi/4} & 0 \\ 0 & 0 & 0 & e^{i\pi/4} \end{pmatrix},$$

$$U_{23} = \exp\left(\frac{\pi}{4} \gamma_2 \gamma_3\right) \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i & 0 & 0 \\ -i & 1 & 0 & 0 \\ 0 & 0 & 1 & -i \\ 0 & 0 & -i & 1 \end{pmatrix}$$

- These matrices indeed act in a very non-trivial way on the wave function:

Initial state  $|00\rangle$

If we exchange Majoranas 2 and 3

$$|00\rangle \rightarrow U_{23} |00\rangle = \frac{1}{\sqrt{2}}(|00\rangle - i |11\rangle)$$

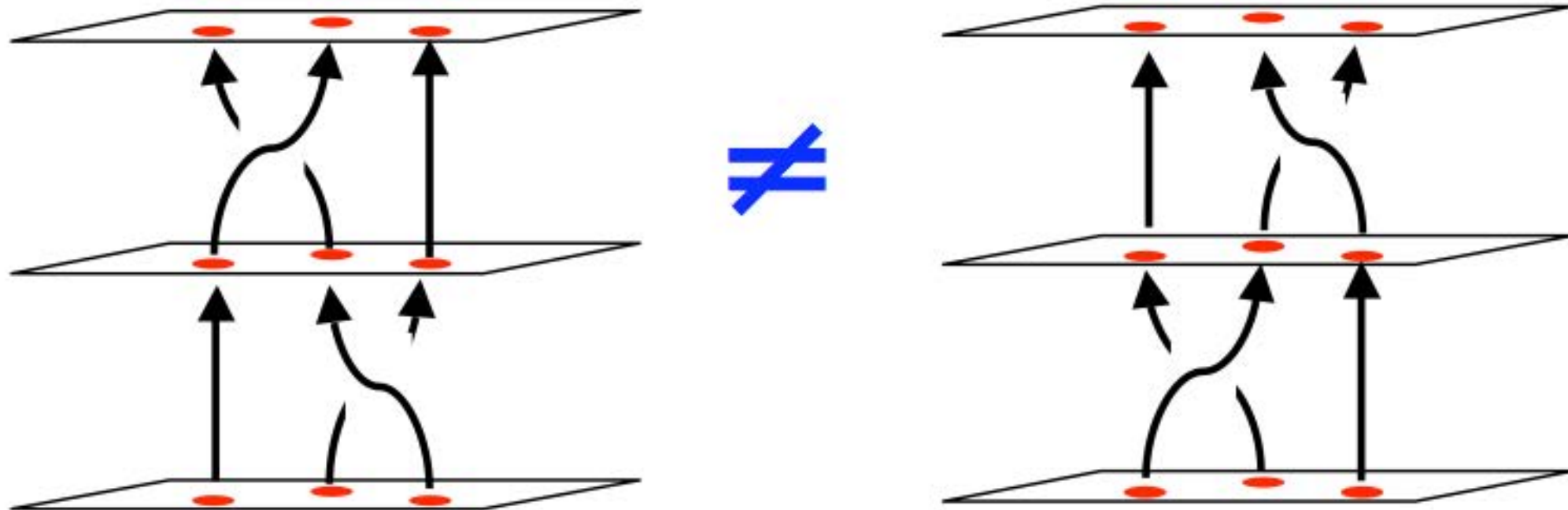
which is a superposition of states!

***Exchange two Majoranas on the wavefunction amounts to much more than just an overall phase, as it happens for bosons and fermions.***

- Let's try sequence of two exchanges which basically means multiplying the corresponding  $U$ :

$$U_{23}U_{12} \neq U_{12}U_{23}$$

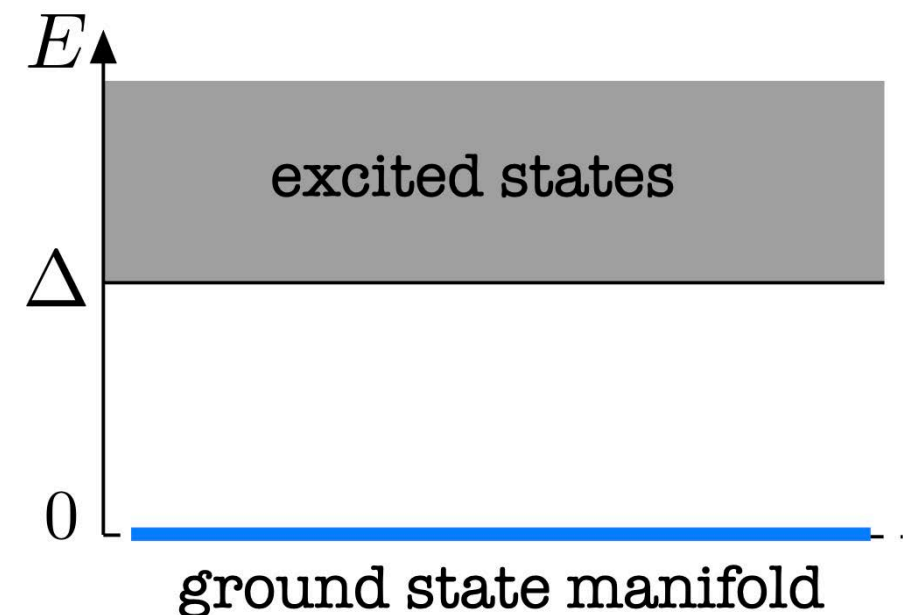
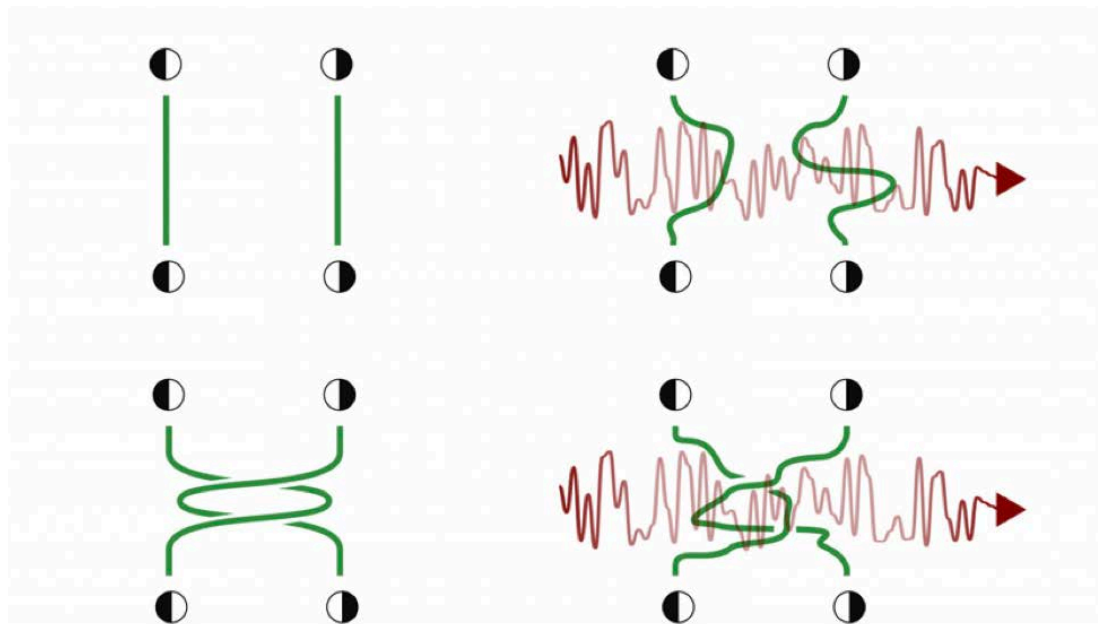
braiding



2d (space) + 1 (time)

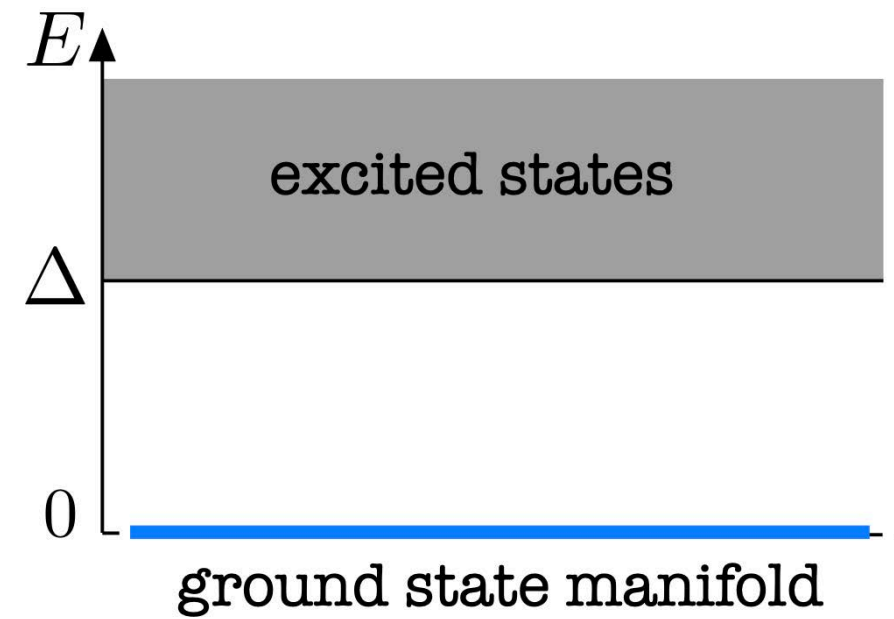
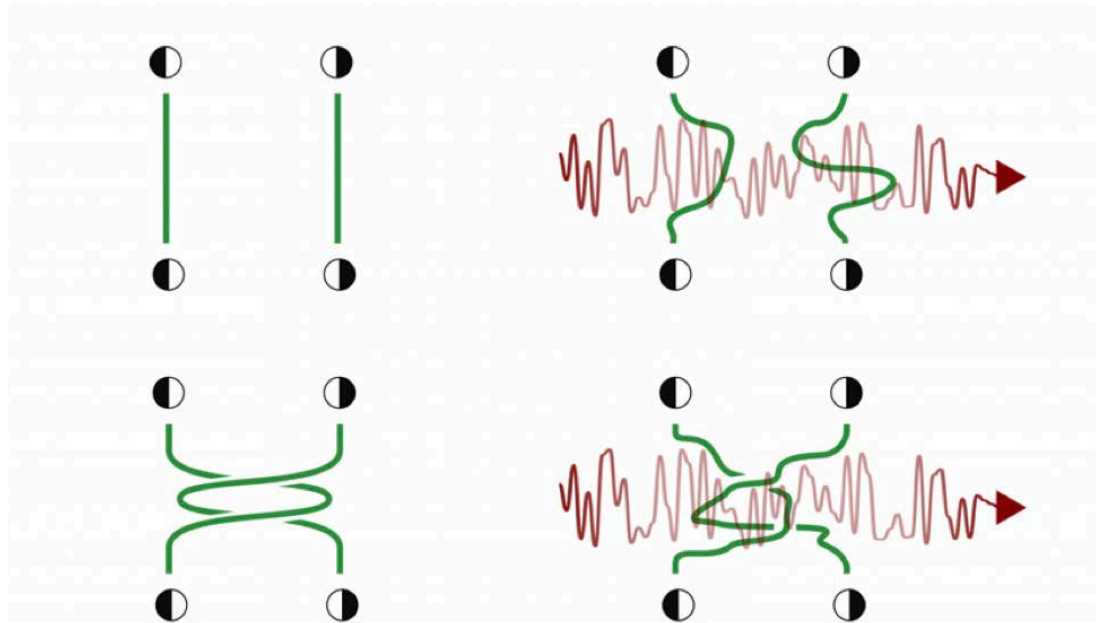


- You could say this is another way to obtain a given unitary operator acting on the wave function. In principle both the state of the register and the algorithms are ***topologically protected***.



- The state of the register is encoded in the fermion parity degrees of freedom, which are shared *non-locally* by the Majoranas.** This means that no local perturbation can change the state of the register and cause *decoherence* of the quantum state.
- The environment cannot access the information stored in the Majoranas, as long as they are kept far away from each other. The only exception is a change in fermion parity.

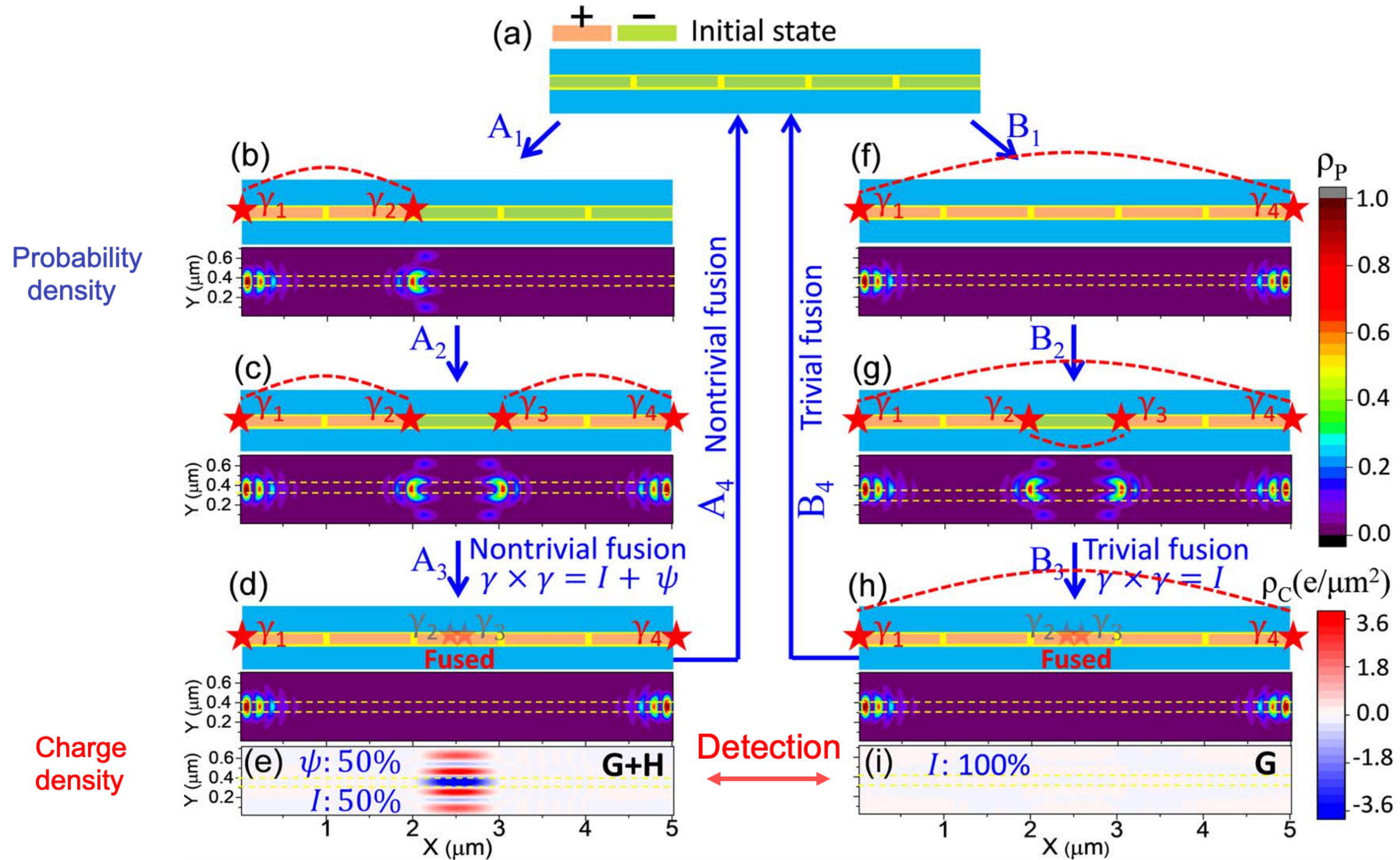
# How about "Measurement"?



One way is to measure exchange properties like fusion:

- need configuration, fast but adiabatic





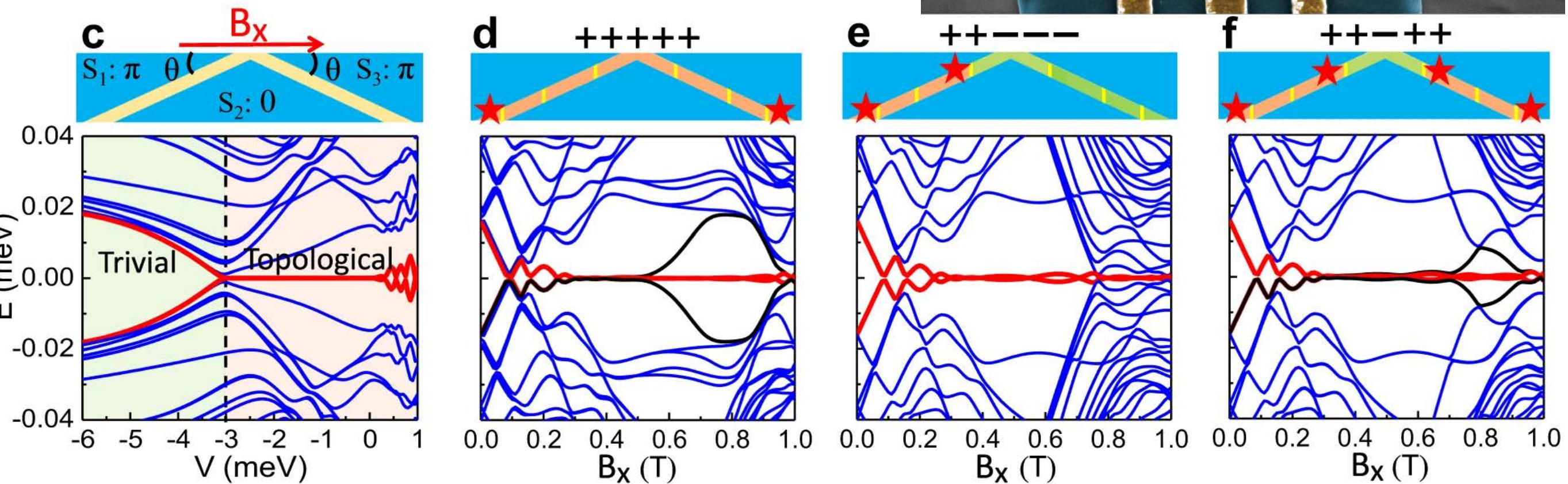
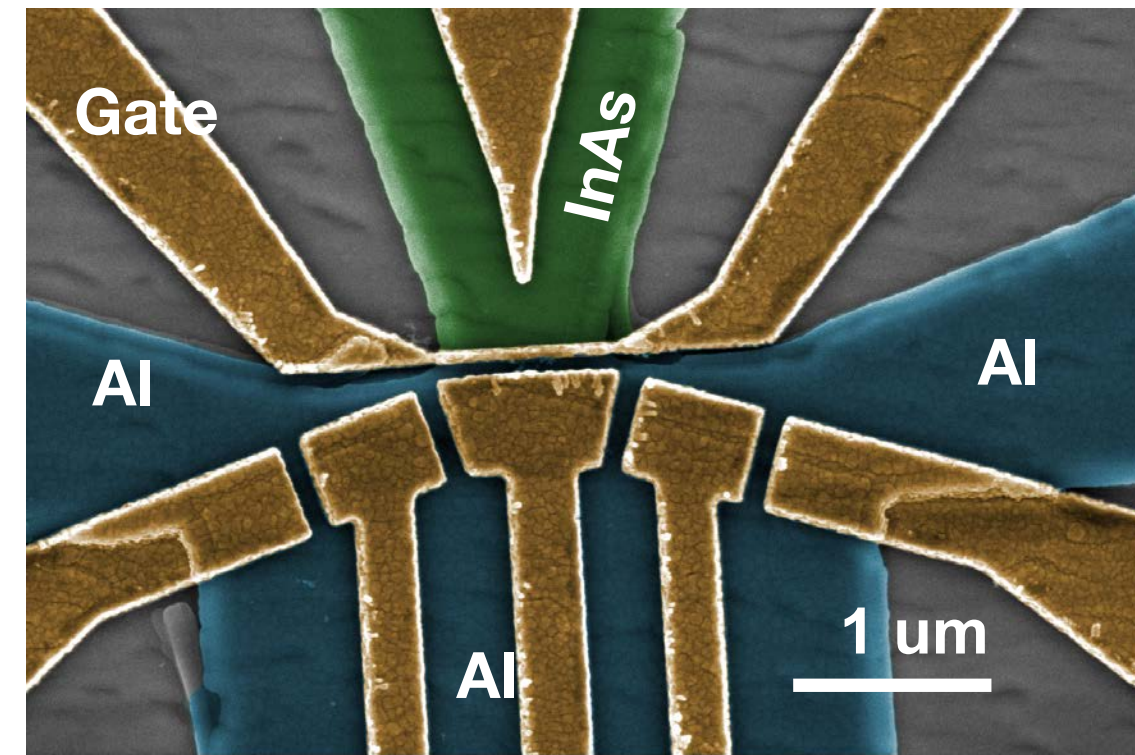
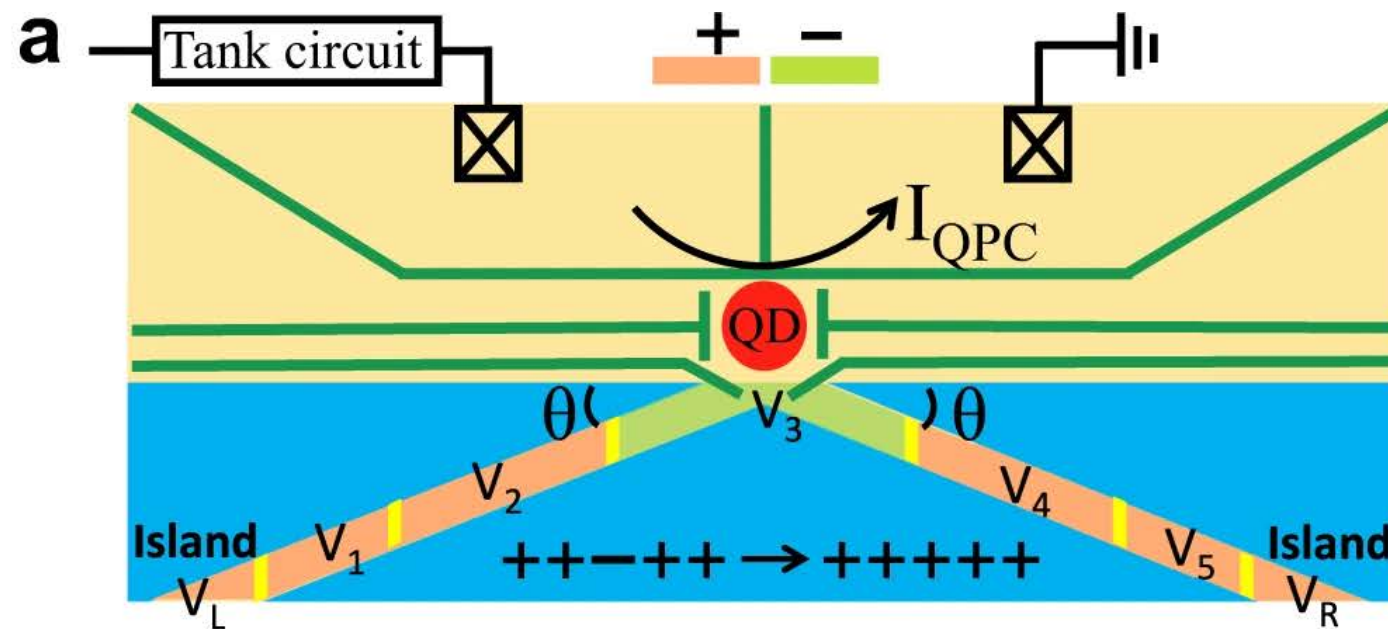
**Measurement is equivalent to sending “charge”**



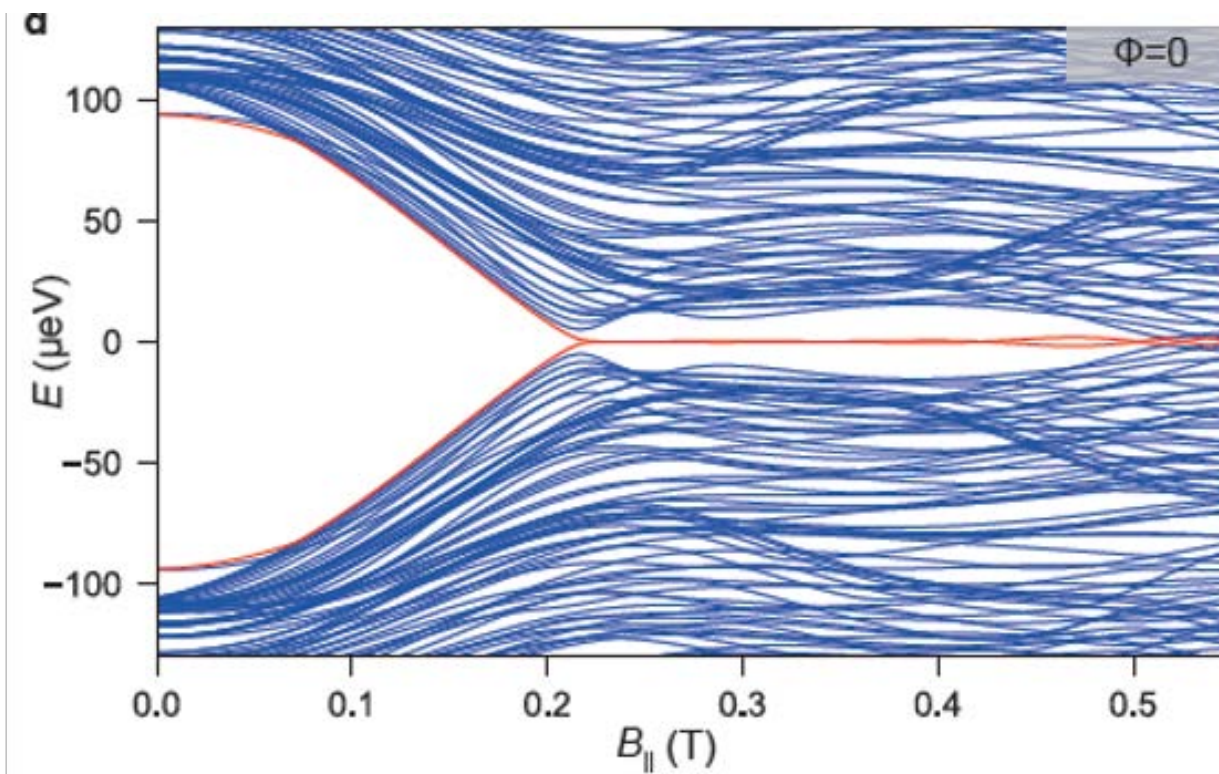
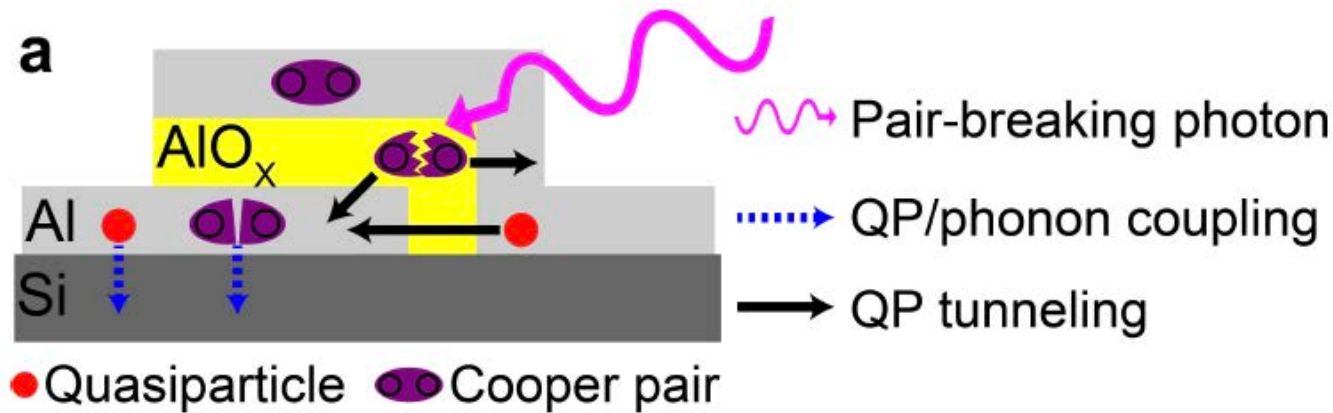




# One possible experimental approach: mini gate







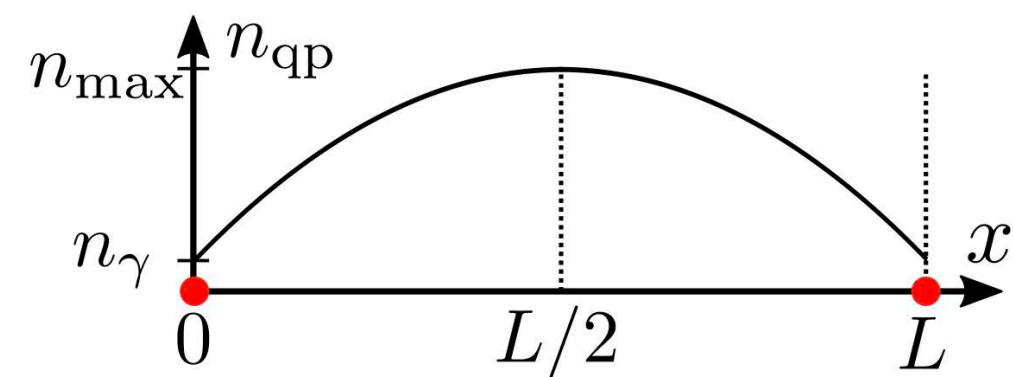
Quasi Particle Poisoning



Parity change

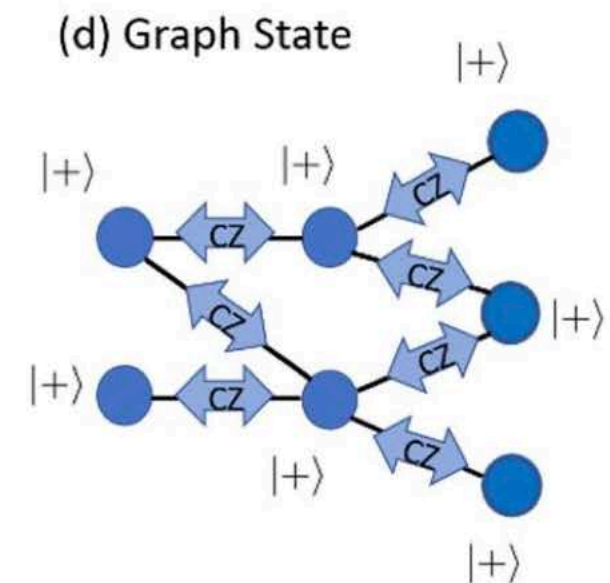
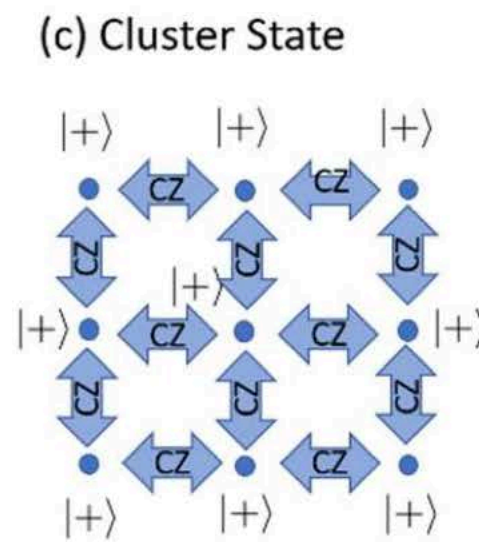
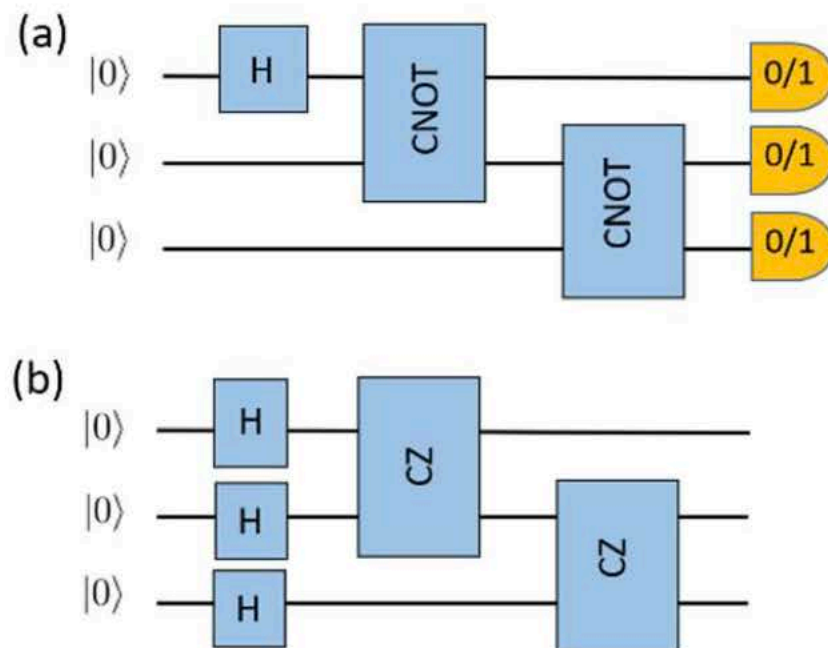


Upper time scale of Majorana qubits



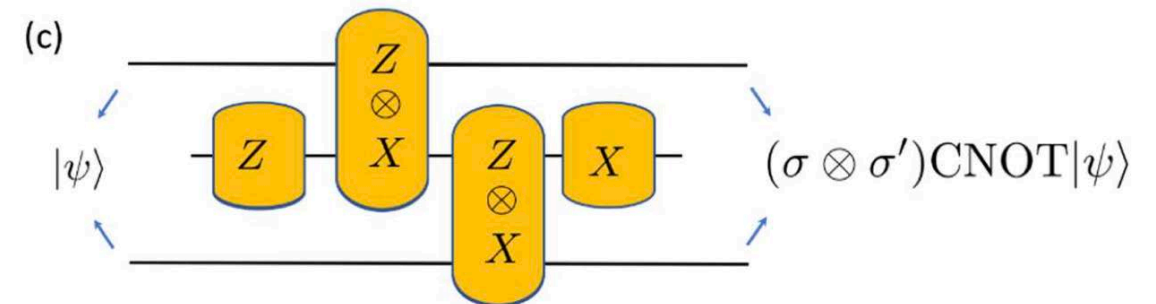
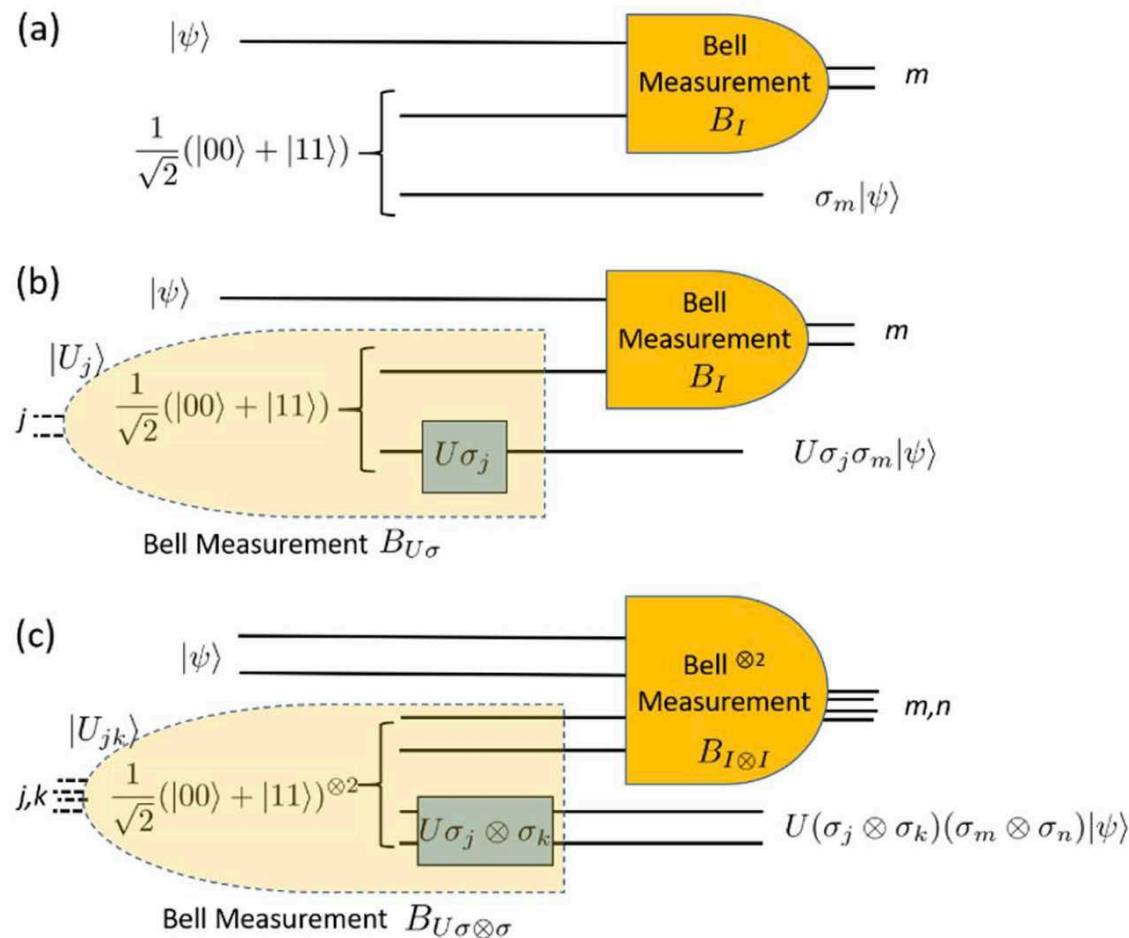
Topological gap ( $\sim\text{ns}$ )  $\ll$  Majorana qubit clock time  $\ll$  QPP time

- Entanglement is used as a resource
- Local measurements on qubits are used to drive the computation (one-way quantum computer of Raussendorf and Briegel, who introduced the so-called cluster)
- The randomness in the measurement outcomes can be dealt with by adapting future measurement axes so that computation is deterministic





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State transfer- based two-qubit CNOT gate

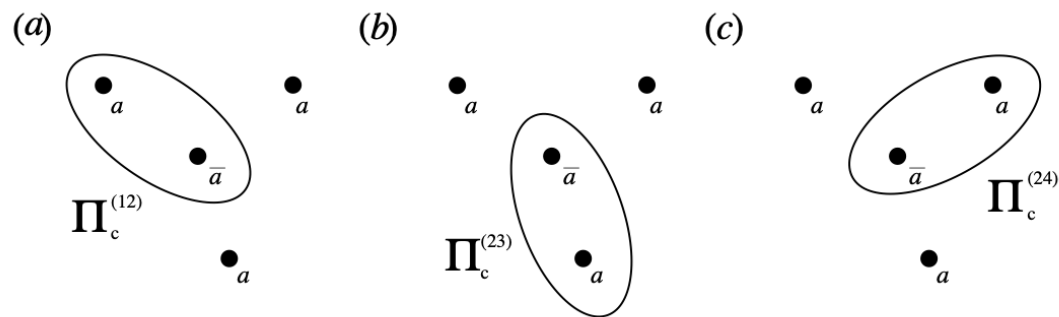
## Measurement-Only Topological Quantum Computation

Parsa Bonderson,<sup>1</sup> Michael Freedman,<sup>1</sup> and Chetan Nayak<sup>1,2</sup>

<sup>1</sup>Microsoft Research, Station Q, Elings Hall, University of California, Santa Barbara, CA 93106

<sup>2</sup>Department of Physics, University of California, Santa Barbara, CA 93106

(Dated: August 15, 2008)



$$\gamma \times \gamma = I + \psi$$

Starting from a maximally entangled anyon pair

+

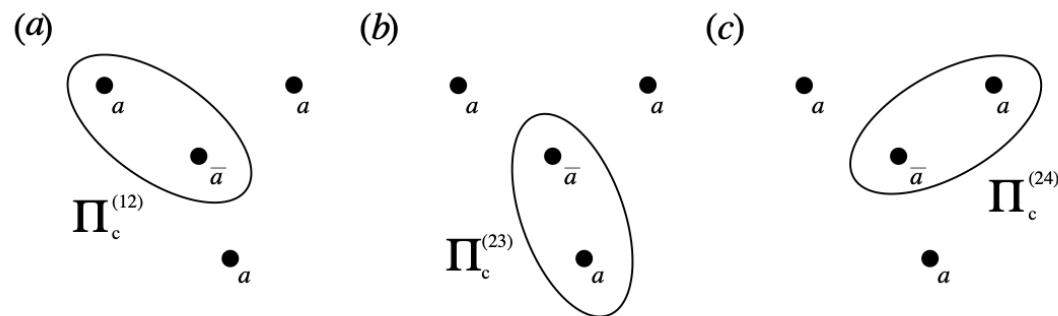
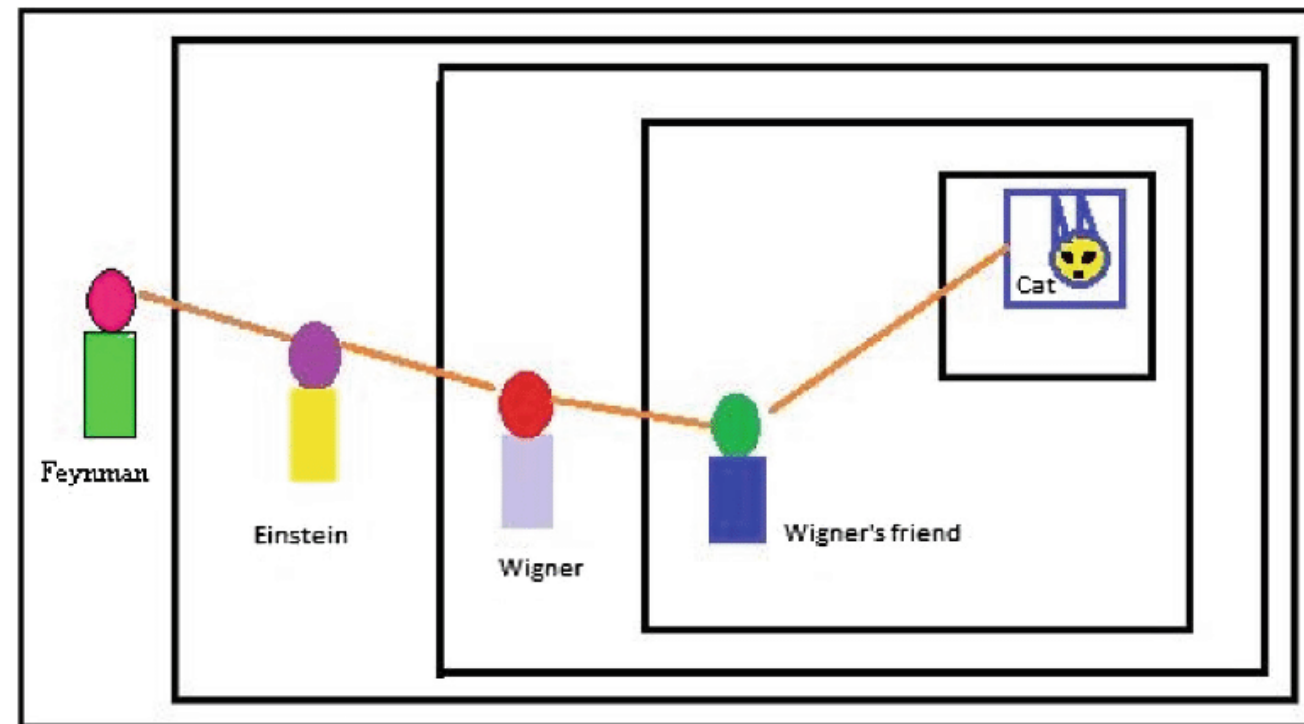
forced measurements

=

Entanglement resource is fully replenished and returned to its original location

=

Measurement-generated braiding transformations to be employed repeatedly, without exhausting the resources



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