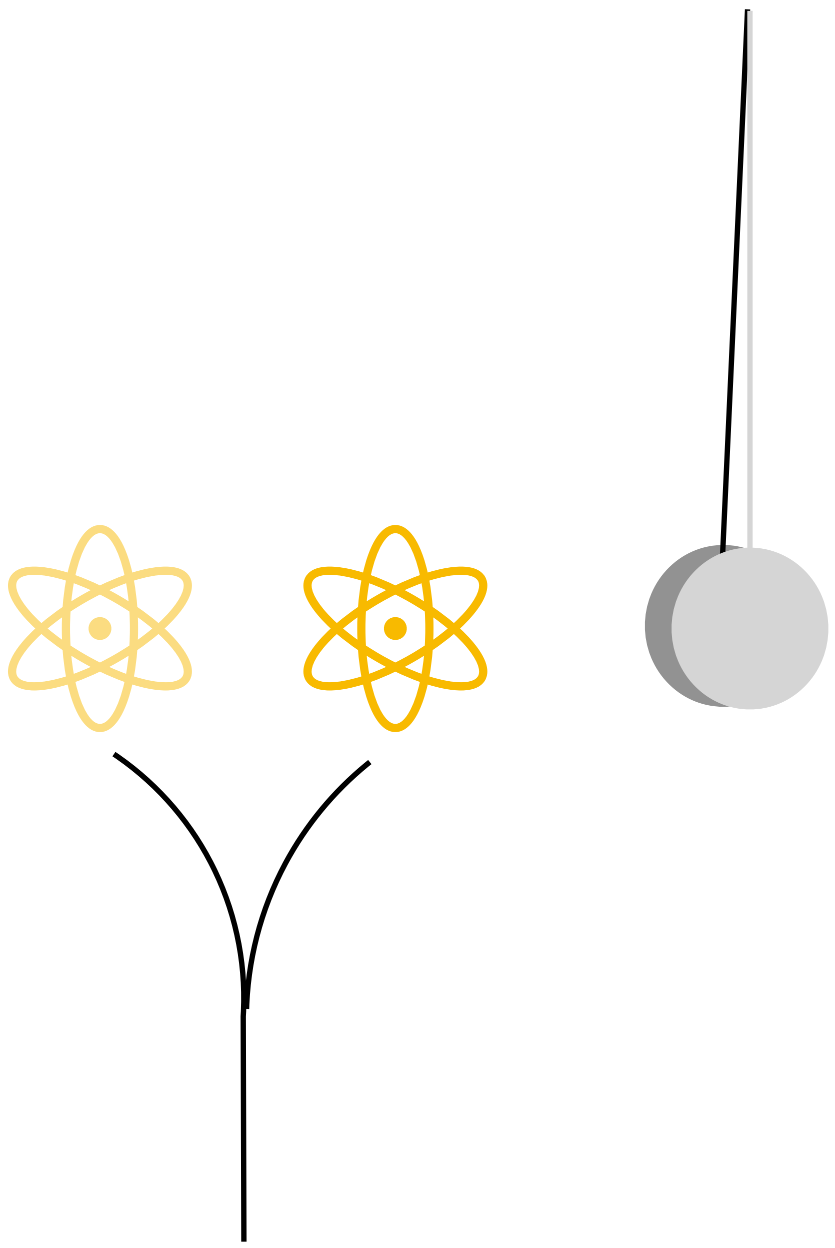
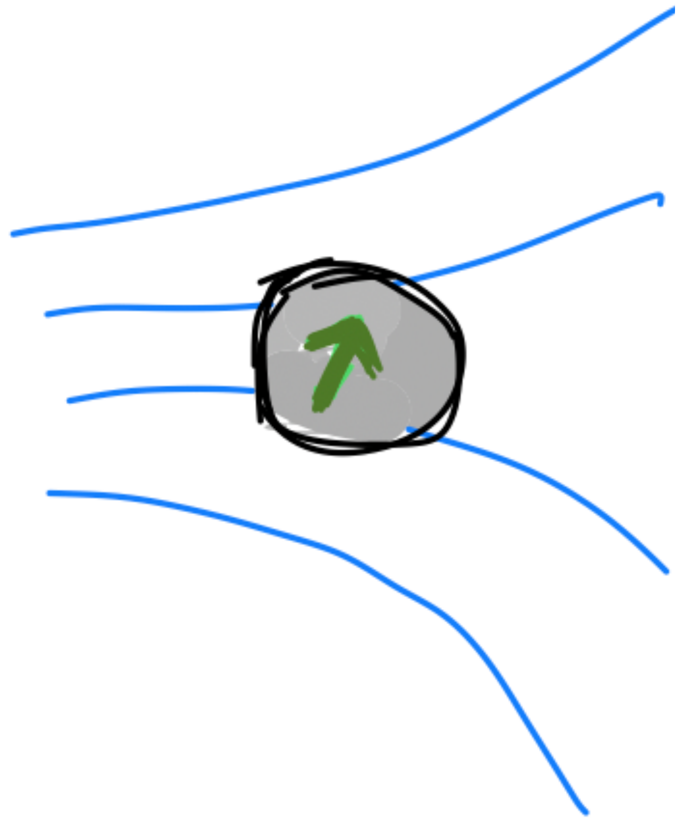
Decoherence of Nanoparticles “multi-Mode Collapse” with Phonons

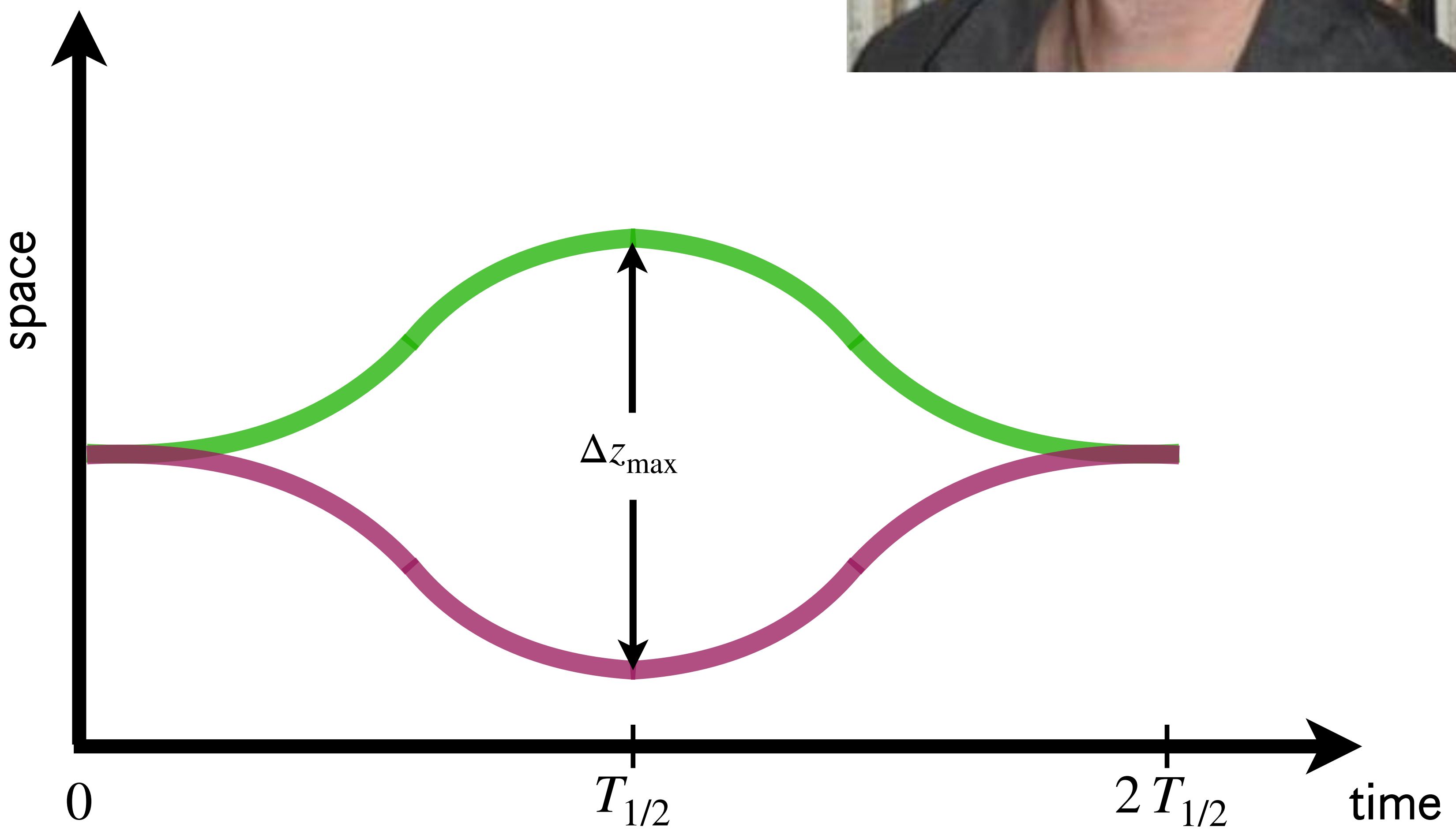
Carsten Henkel & Ron Folman*

University of Potsdam, Institute of Physics and Astronomy, Germany

**Ben Gurion University of the Negev, Department of Physics, Israel*







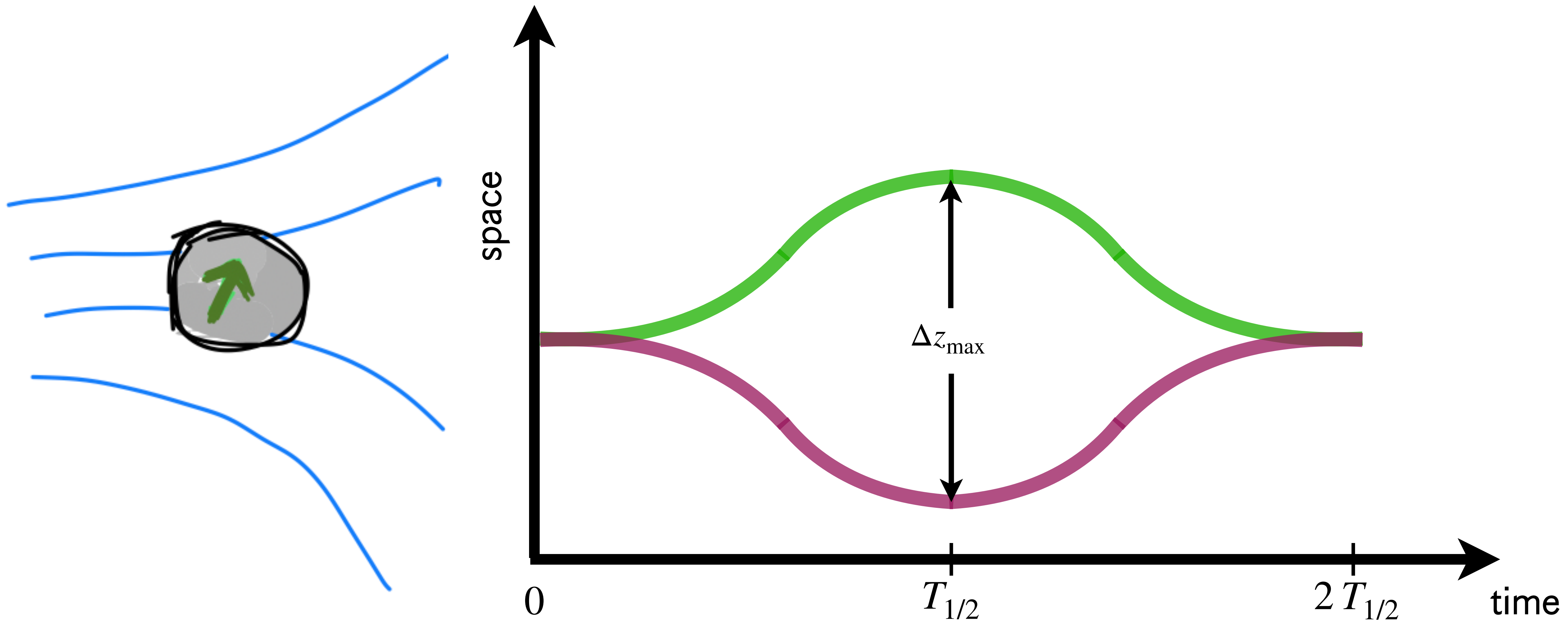
COLMO Trento – 03–07 Jul 2023

Henkel and Ron Folman
AVS Quantum Sci. **4** (2022) 025602

Motivation / Outline

Macroscopic Limits of Quantum Mechanics
— complex system control
— spontaneous collapse

a Probe of Quantum Gravity (?)
— gravity-related collapse (Penrose)
— gravity-based entanglement



Nano-Particle Interferometer
– Stern-Gerlach splitting
– phonon ("internal") decoherence

Henkel & Folman, *AVS Quant. Sci.* **4** (2022) 025602
(Festschrift Nobel Prize Sir R. Penrose)
and *arxiv* 2305.15230

No Access • Submitted: 03 November 2021 • Accepted: 31 January 2022 • Published Online: 15 March 2022

Macroscopic quantum mechanics in gravitational-wave observatories and beyond

AVS Quantum Sci. **4**, 014701 (2022); https://doi.org/10.1116/5.0077548

Roman Schnabel[a)] *and* Mikhail Korobko

Open • Submitted: 23 November 2021 • Accepted: 14 March 2022 • Published Online: 06 April 2022

Many-body probes for quantum features of spacetime

AVS Quantum Sci. **4**, 021402 (2022); https://doi.org/10.1116/5.0079675

Hadrien Chevalier[1,a)], Hyukjoon Kwon[1,2], Kiran E. Khosla[1], Igor Pikovski[3,4], *and* M. S. Kim[1,2,b)]

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No Access • Submitted: 03 November 2021 • Accepted: 31 January 2022 • Publish

Macroscopic quantum mechani tories and beyond

AVS Quantum Sci. 4, 014701 (2022); https://doi.org/10.1116/5.00775

Roman Schnabel[a)] and Mikhail Korobko

Open • Submitted: 23 November 2021 • Accepted: 14 March 2022 • Publish

Many-body probes for quant

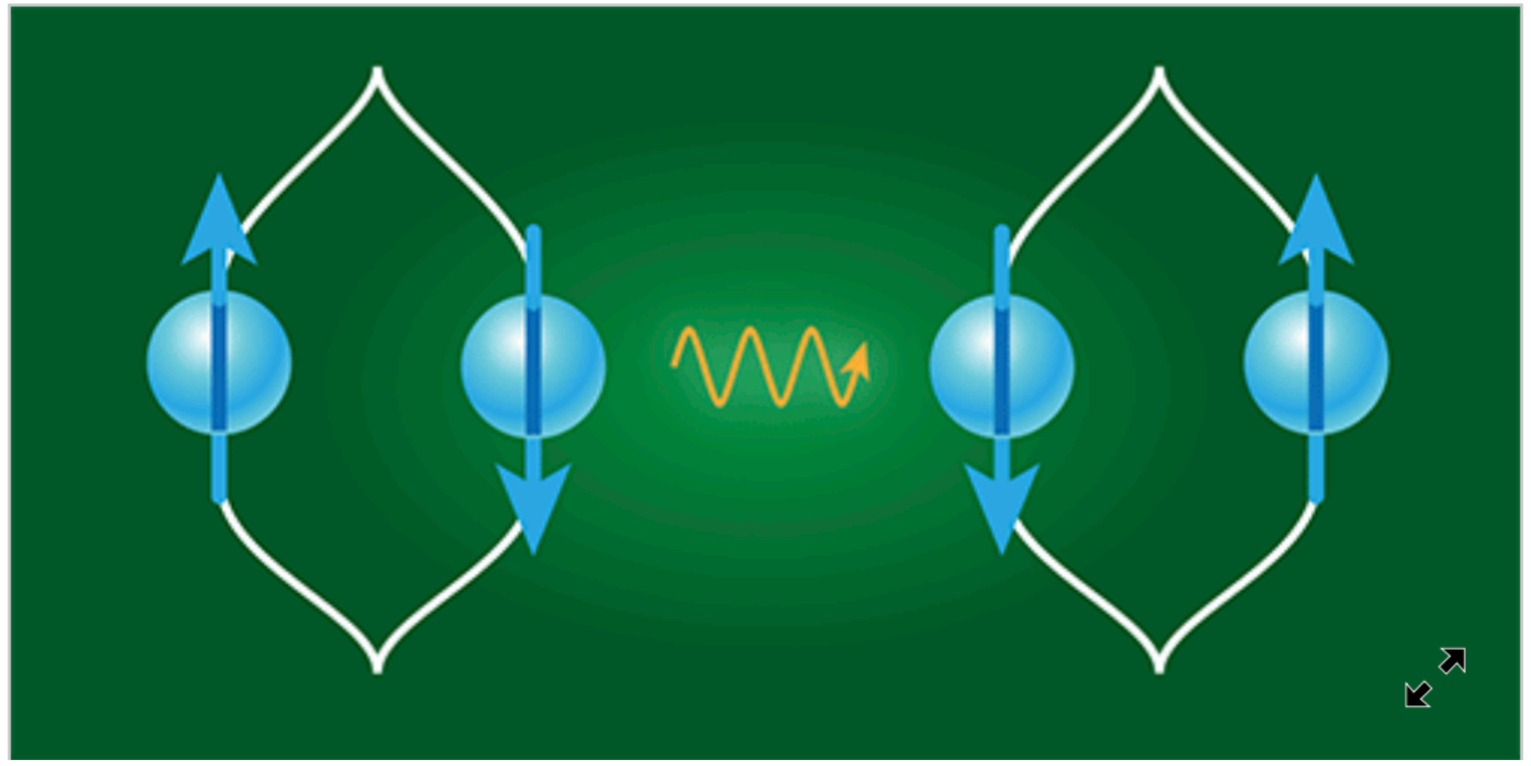
AVS Quantum Sci. 4, 021402 (2022); https://doi.org/10.1116/5.0

Hadrien Chevalier[1,a)], Hyukjoon Kwon[1,2], Kiran E. Khosla[1], Ig

A Test of Gravity's Quantum Side

December 13, 2017 • *Physics* 10, s138

Two proposals describe how to test whether gravity is inherently quantum by measuring the entanglement between two masses.



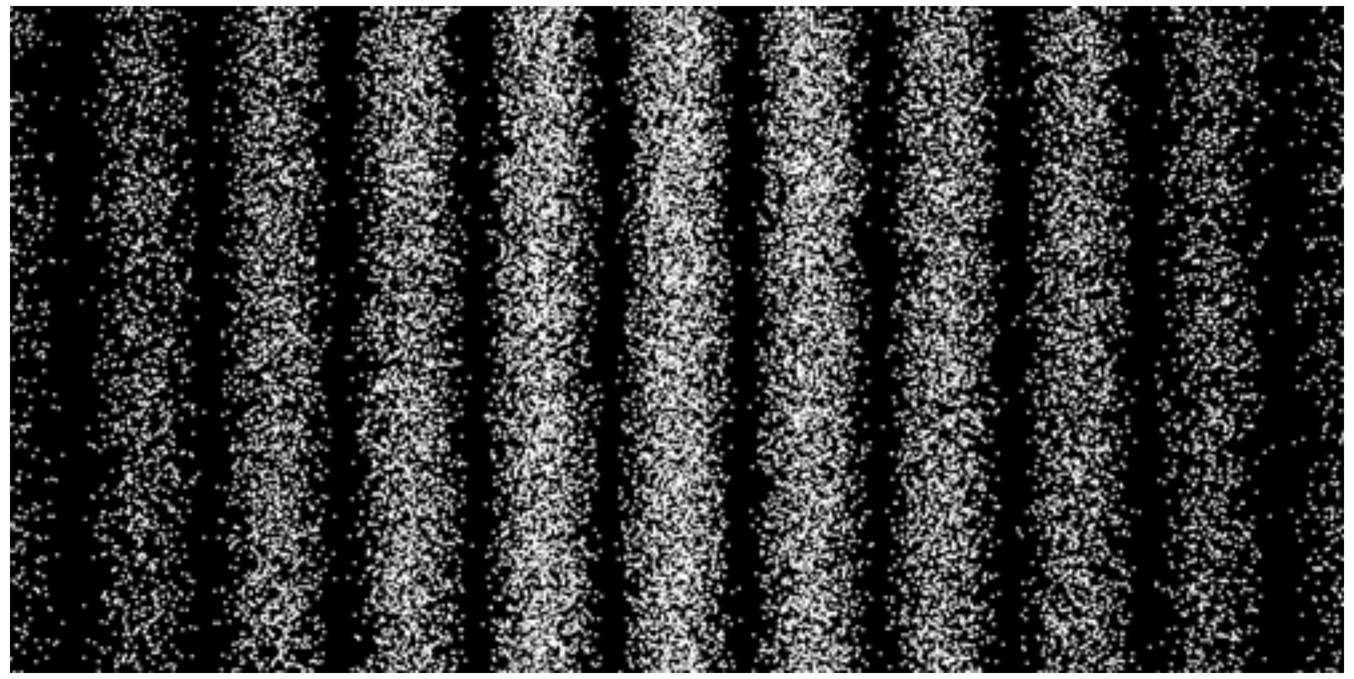
Spin entanglement witness for quantum gravity
S. Bose, A. Mazumdar, G.W. Morley, H. Ulbricht, ...
Phys. Rev. Lett. **119** (2017) 240401

Gravitationally induced entanglement between two massive particles is sufficient evidence of quantum effects in gravity
C. Marletto and V. Vedral
Phys. Rev. Lett. **119** (2017) 240402

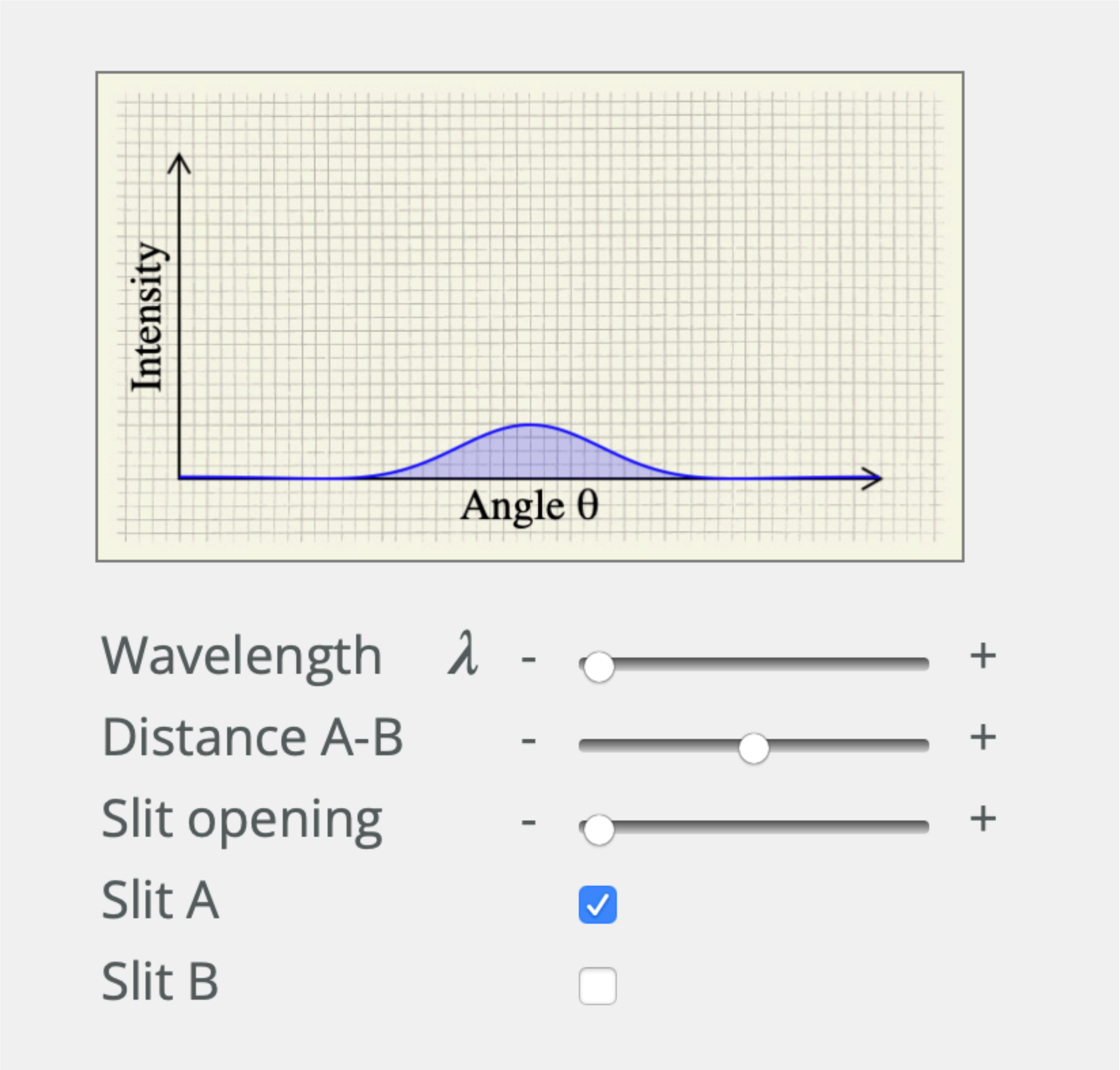
Overview: C. Anastopoulos and Bei-Lok Hu (2018),
AVS Quantum Sci. **4** (2022) 015602

nano-Particle Interferometer

Molecules at a double slit



M. Arndt group (U Vienna)
https://interactive.quantumnano.at



how large is the Limit?

R. Penrose: "QM breaks down on macroscopic scales"

Matter-wave interference of a native polypeptide
A. Shayeghi, P. Rieser, G. Richter, ... and M. Arndt (Vienna)
Nature Commun. **11** (2020) 1447

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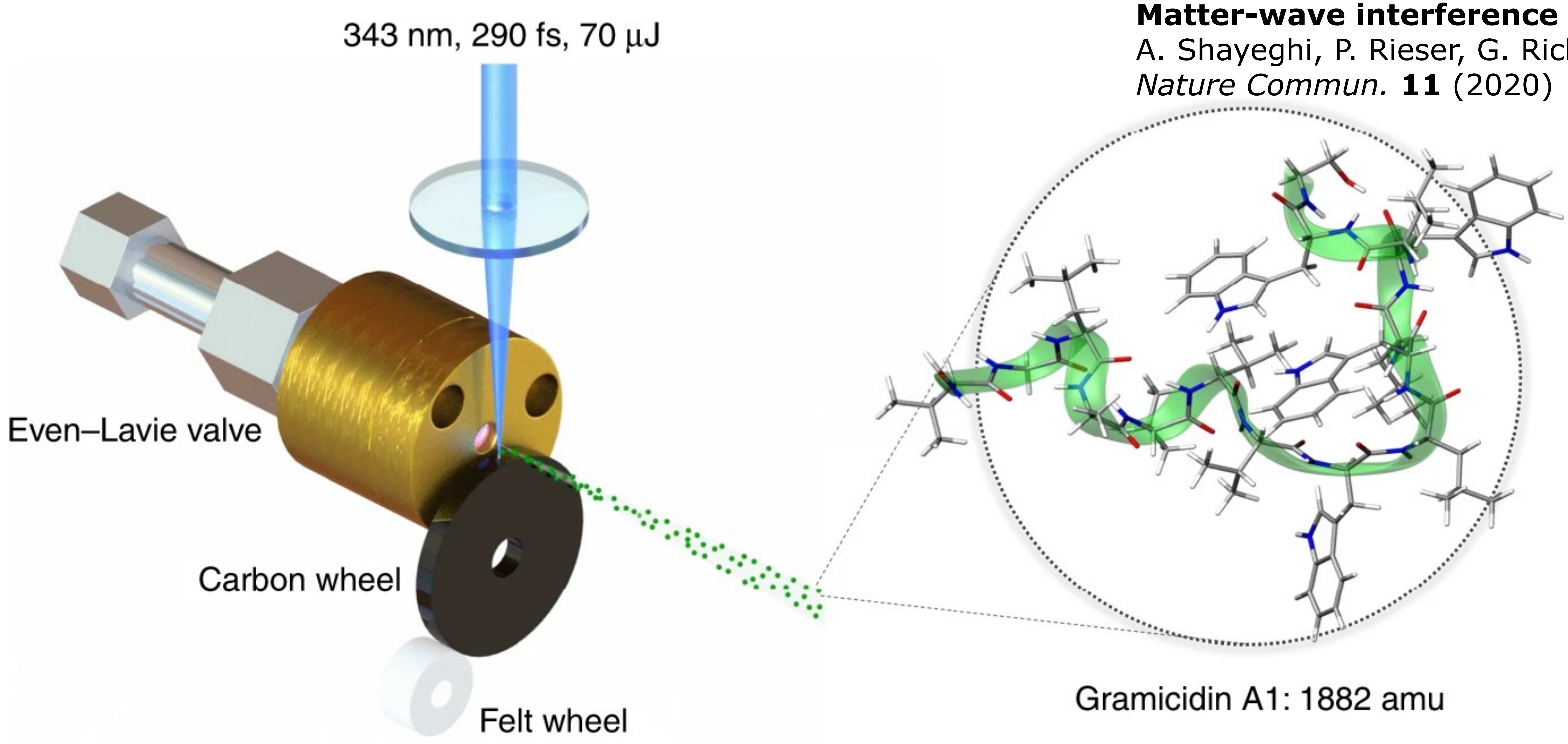

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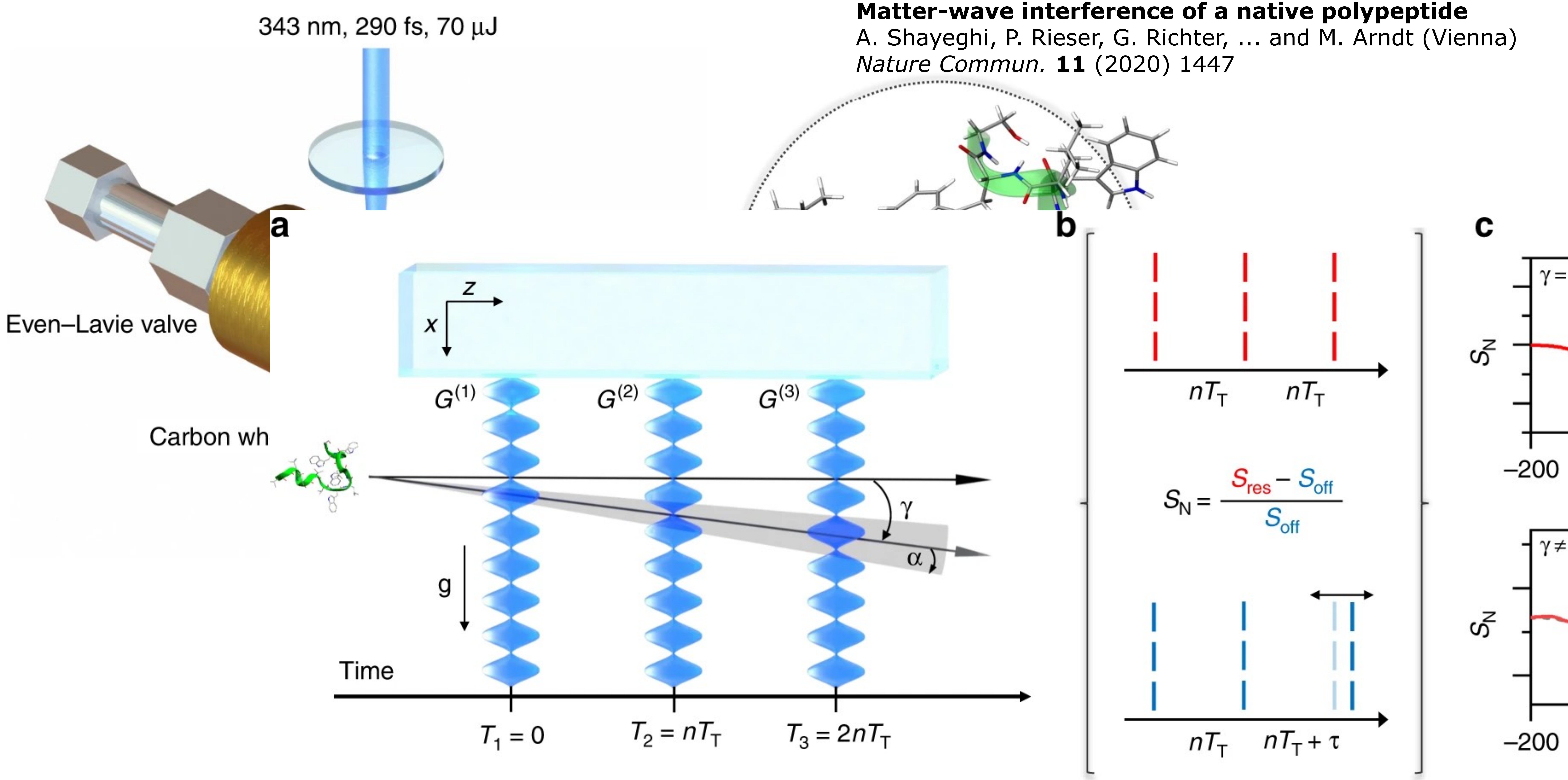

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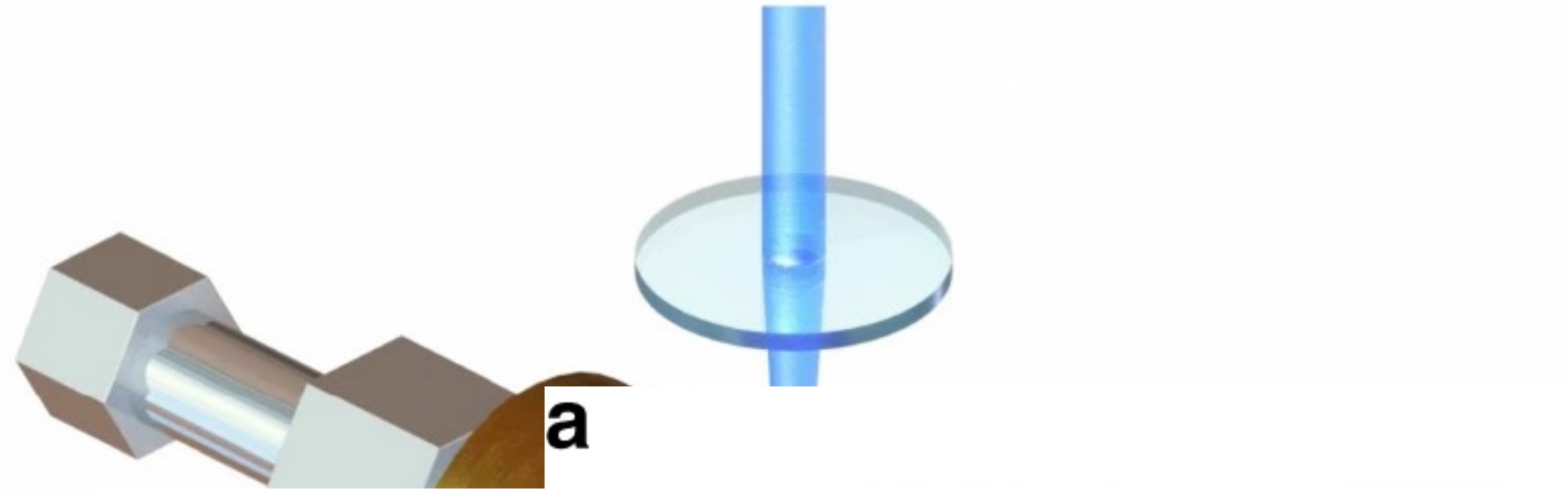
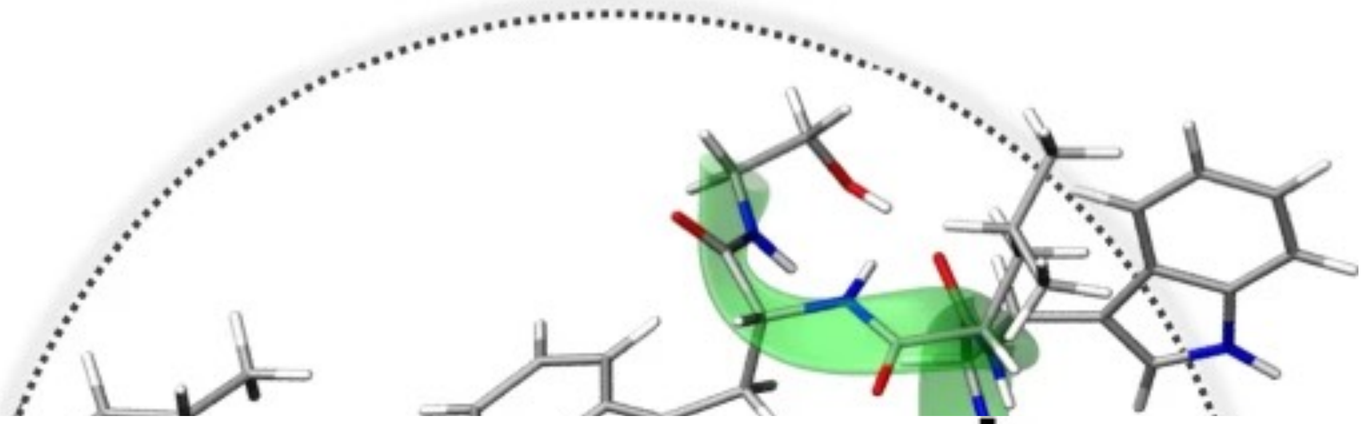
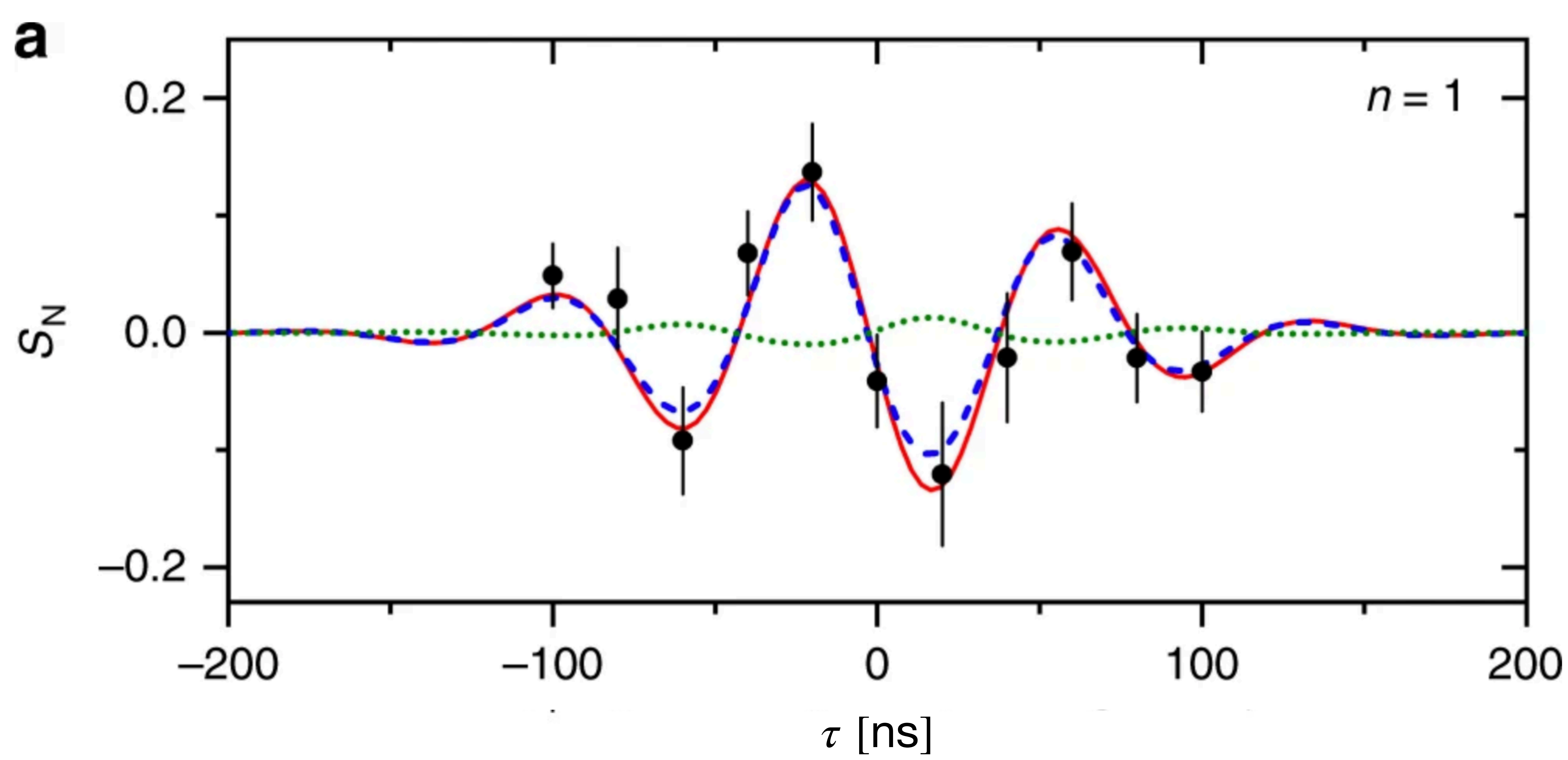


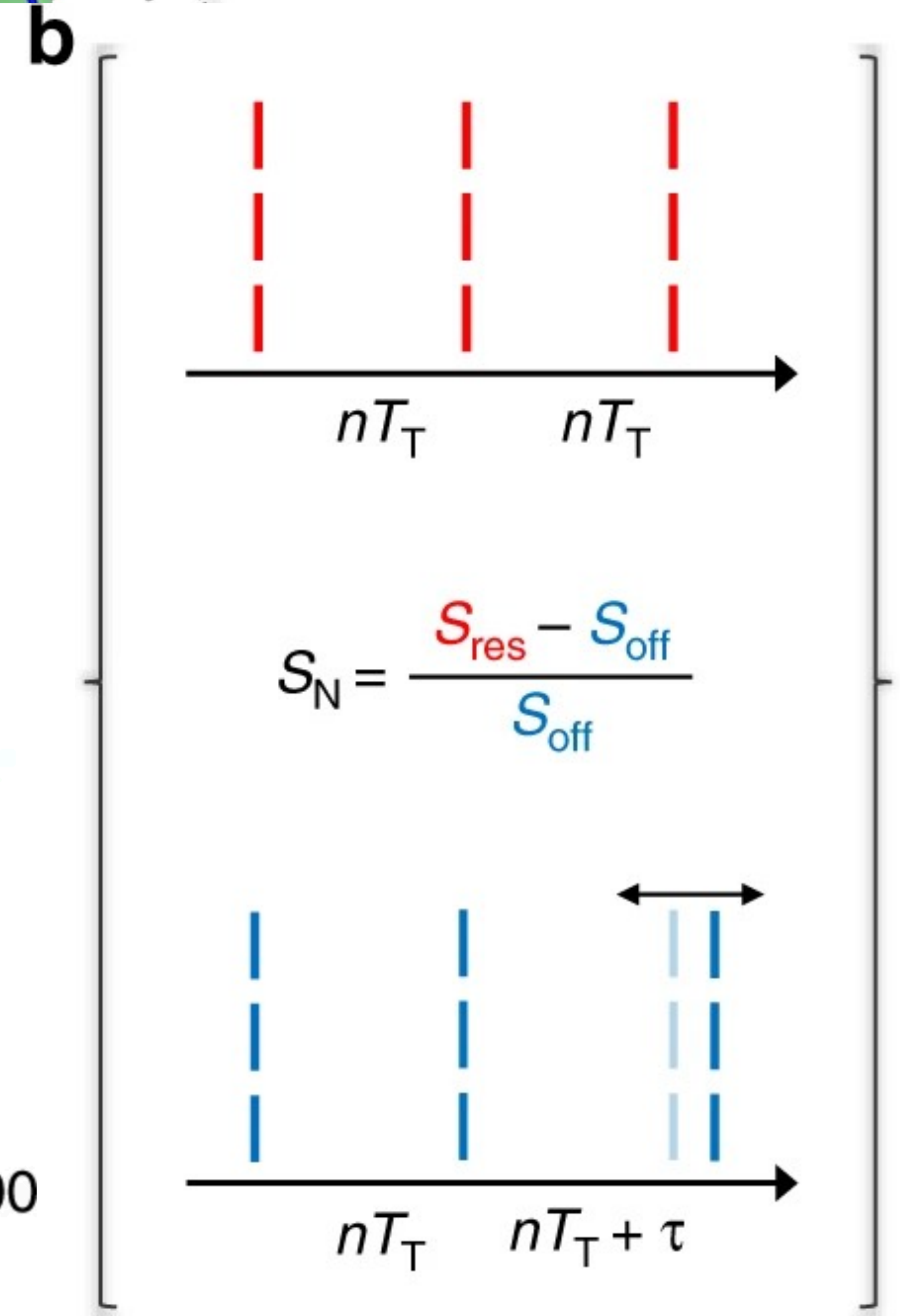


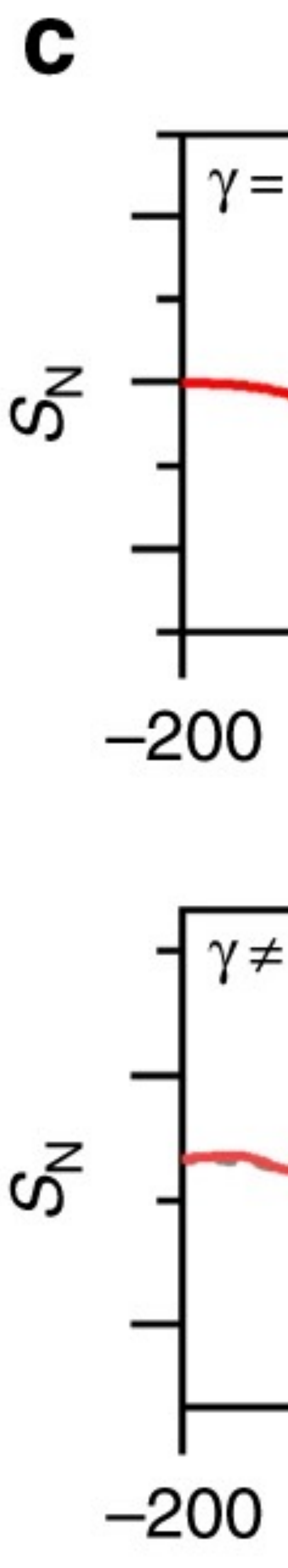
Even–l

Fig. 3: Molecular interference patterns of gramicidin:

From: Matter-wave interference of a native polypeptide

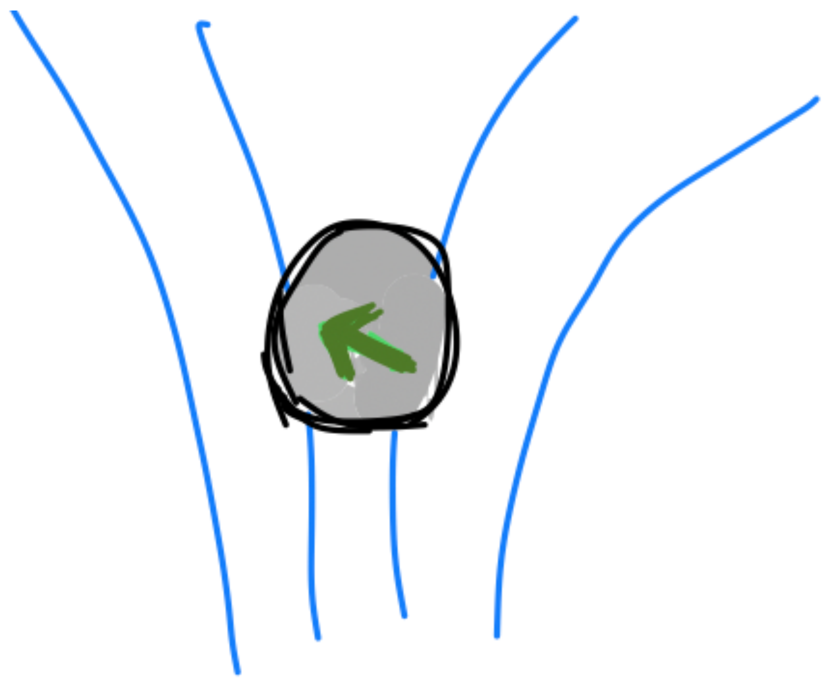






Nano-Particle Interferometer

"May the Force be with the Spin ..."
nano-diamond with one NV centre



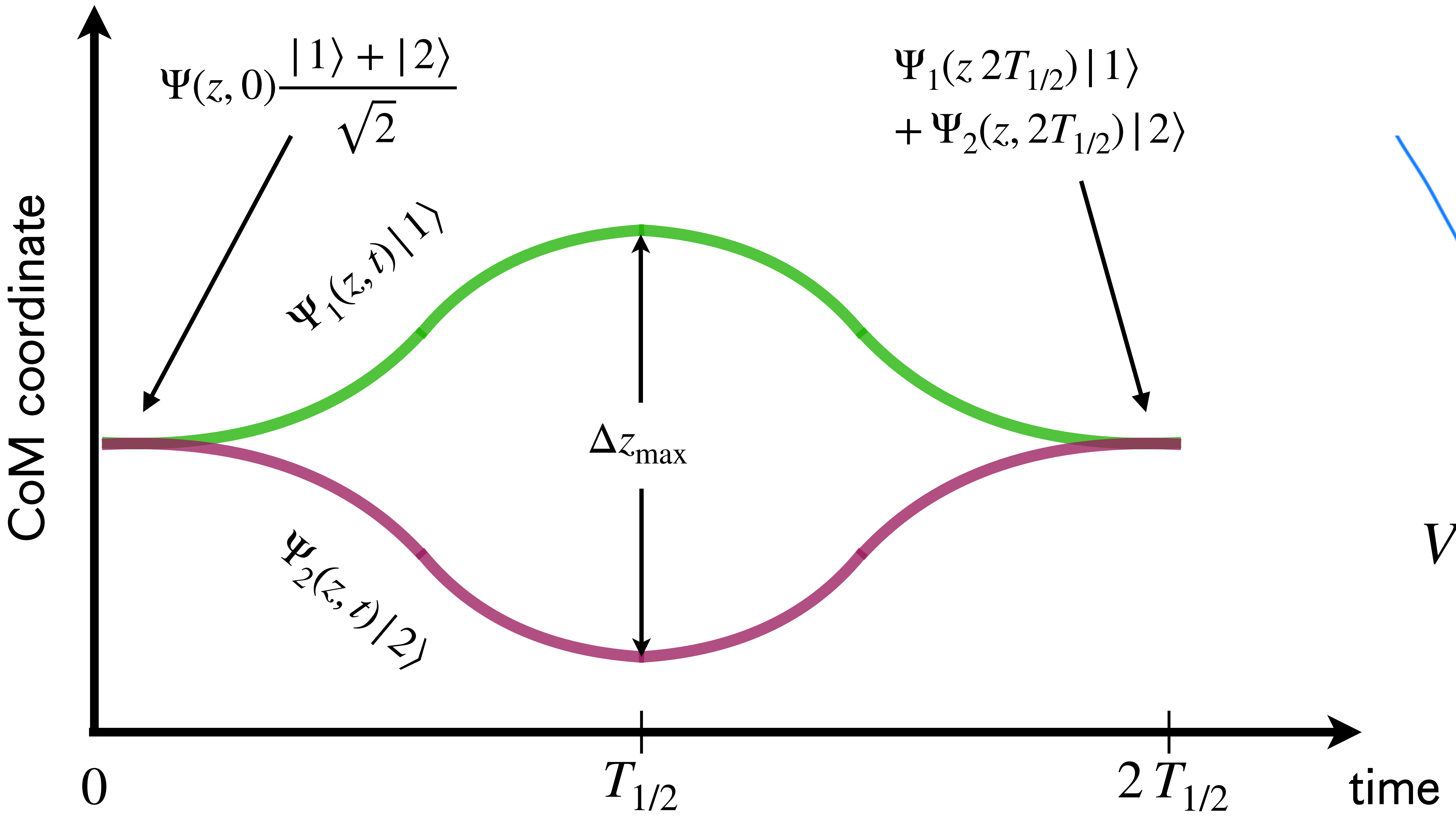
$$V = -\mu \mathbf{S} \cdot \mathbf{B}$$

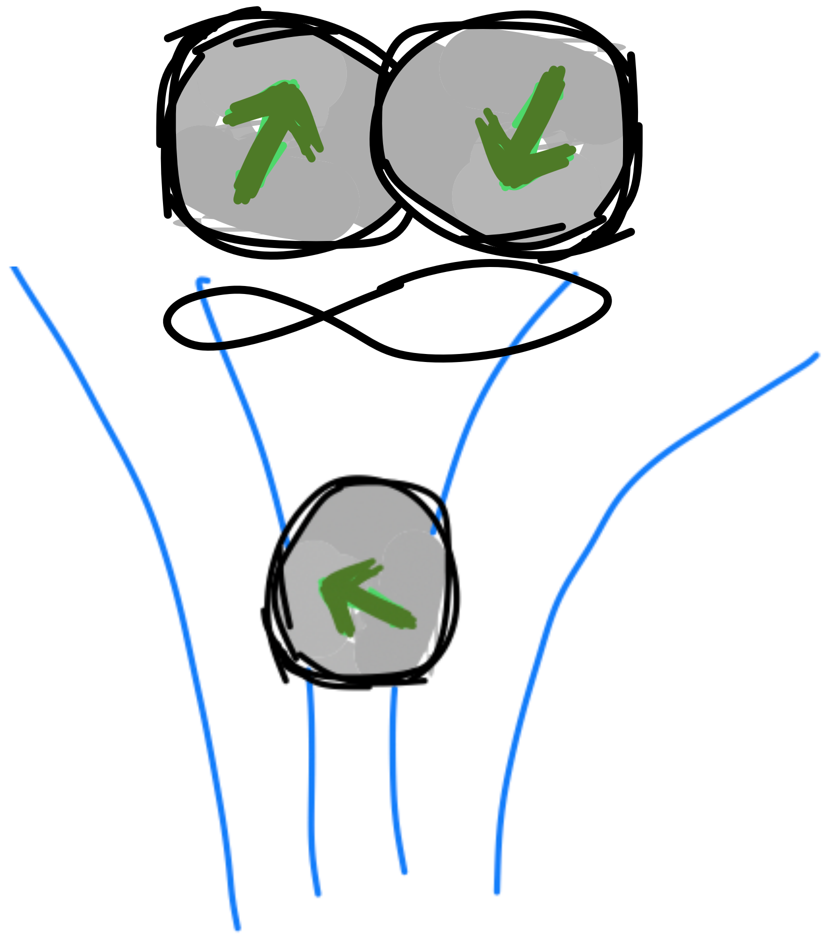
"Constructing nano-object quantum superpositions with a Stern-Gerlach interferometer"
Marshman, Mazumdar, Folman, Bose, *Phys. Rev. Research* **4** (2022) 023087

Nano-Particle Interferometer

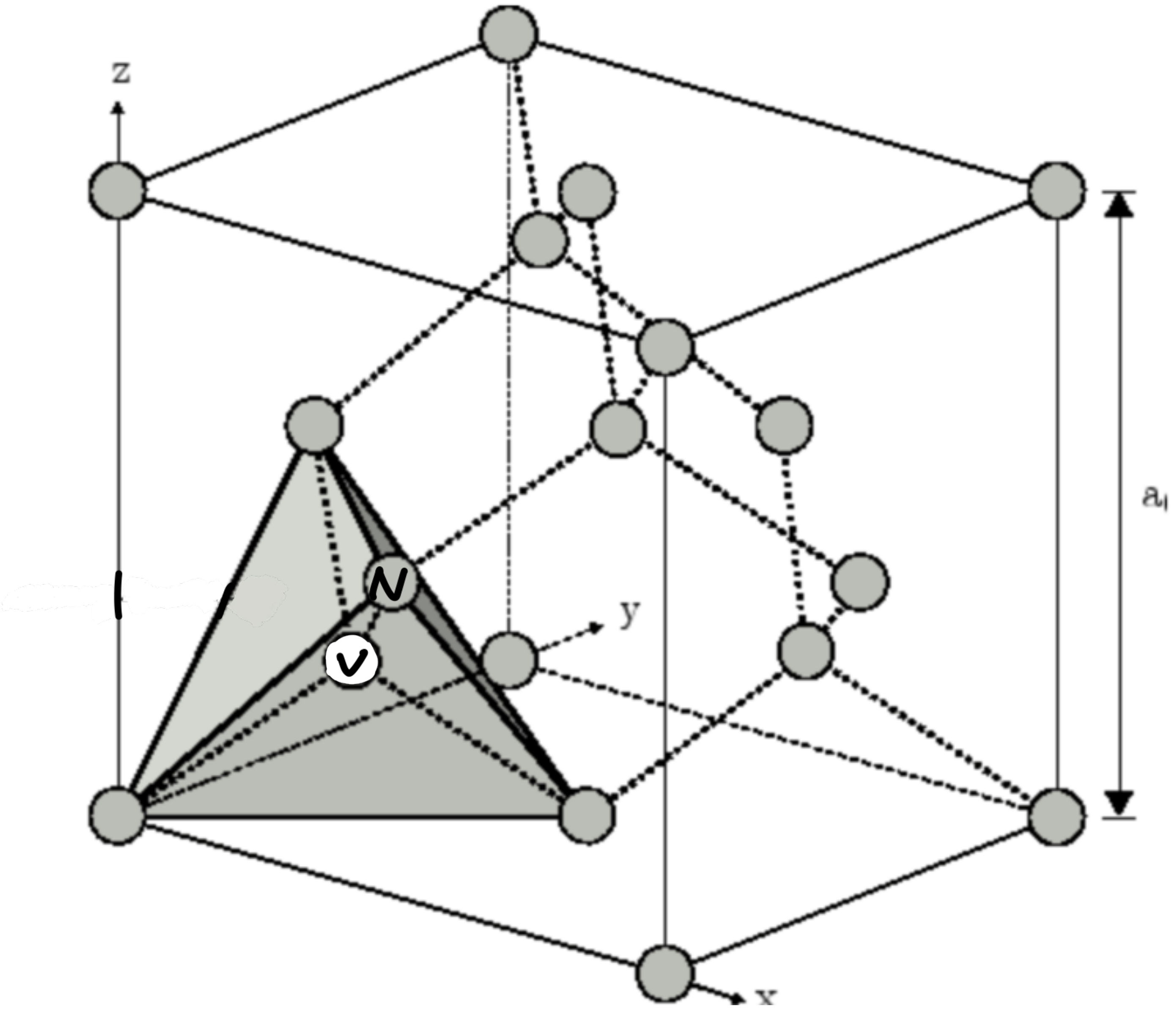
"May the Force be with the Spin ..."
nano-diamond with one NV centre

Stern-Gerlach splitting & "full loop" (in phase space)

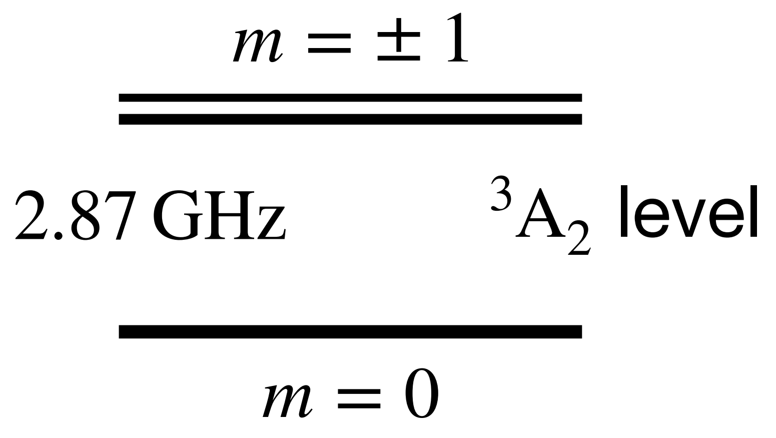




$$V = -\mu \mathbf{S} \cdot \mathbf{B}$$



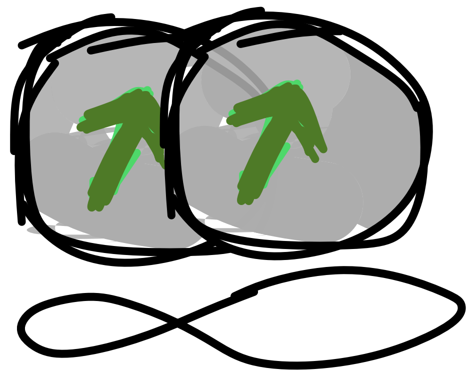
S. E. Ungersböck, PhD TU Wien (2007)



experiment with Rb atoms:
towards testing "quantum gravity"
– Margalit & al., *Science Adv* (2021)

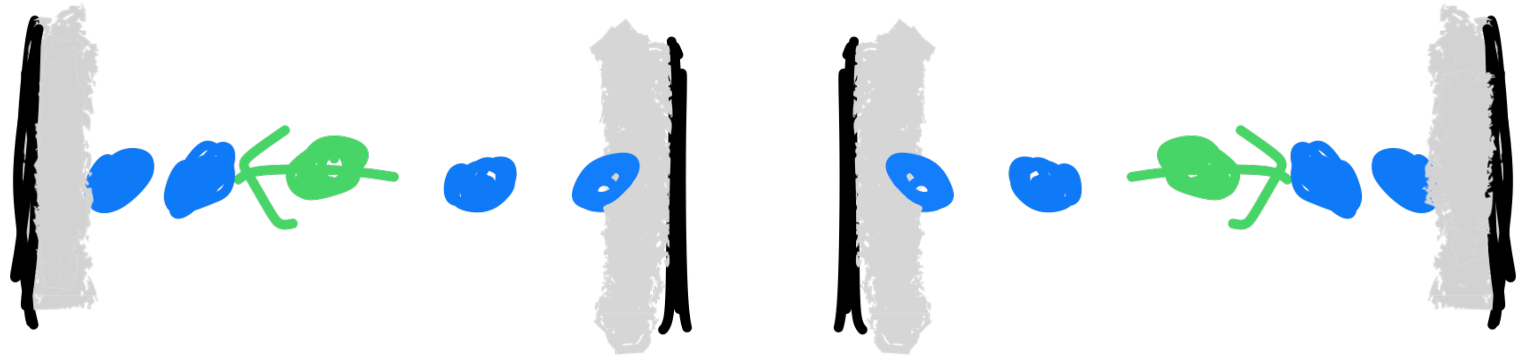
Nano-Particle Interferometer

spatial superposition $|\text{here}\rangle + |\text{there}\rangle$ ... now let’s decohere it:



– take a picture (Heisenberg microscope)
– touch it (gas molecule scattering)
– ...

– remember how you got t/here (“phonons”)



(c 1940) Charles Addams, Pinterest

Interference Contrast

Typical “system + meter” scenario

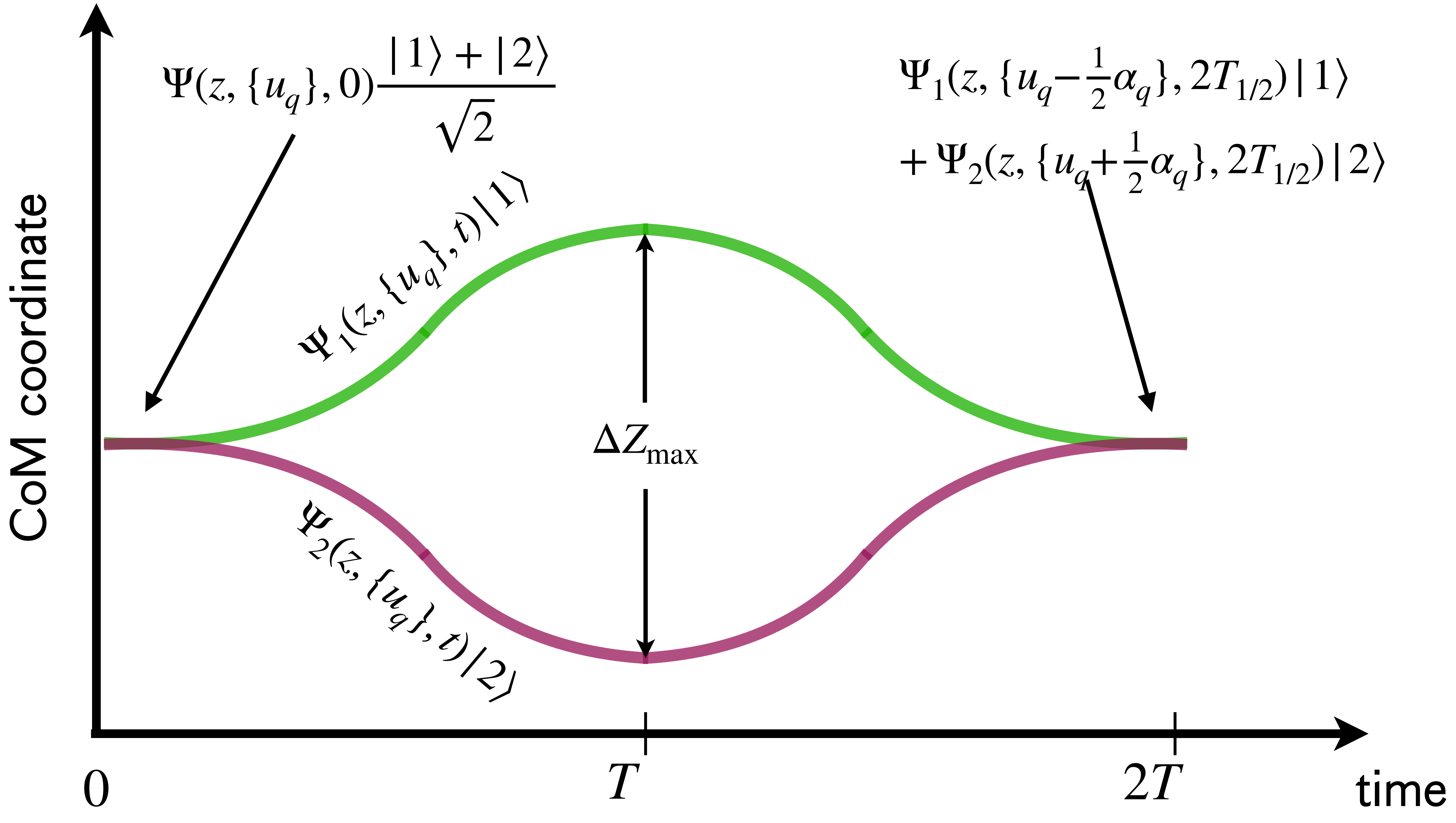
position, spin & phonon state

$|\text{upper}, \uparrow\rangle \otimes |+\frac{1}{2}\alpha_q\rangle + |\text{lower}, \downarrow\rangle \otimes |-\frac{1}{2}\alpha_q\rangle$

phonon states $|\pm\frac{1}{2}\alpha_q\rangle$ entangled

with $|\text{upper}, \uparrow\rangle$ and $|\text{lower}, \downarrow\rangle$

force coupled to phonon mode *q*:
coherent state displacement



Interference Contrast

force coupled to phonon mode q: coherent state displacement

position, spin & phonon state

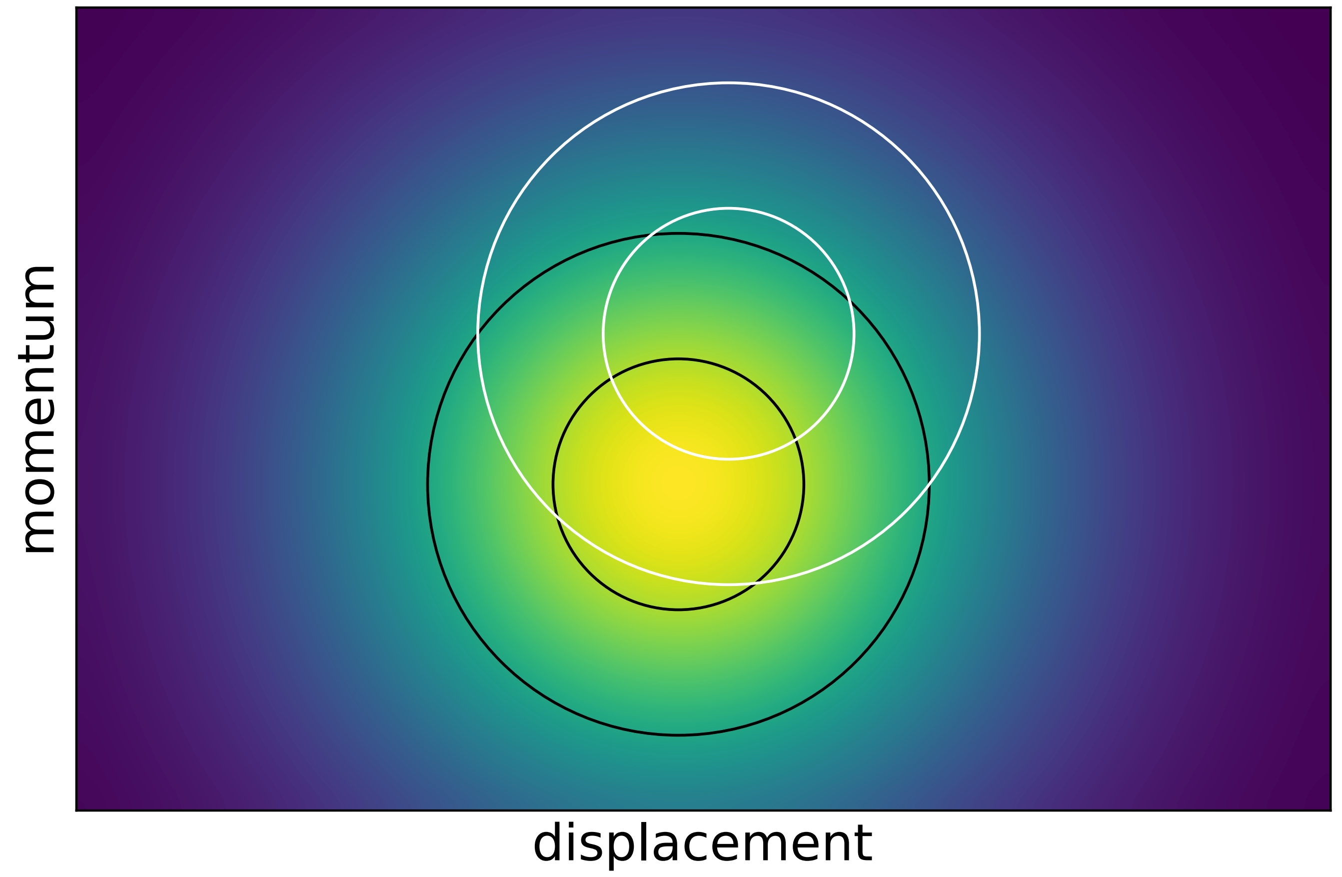
$$|\,\text{upper},\uparrow\,\rangle \otimes |+\tfrac{1}{2}\alpha_q\rangle + |\,\text{lower},\downarrow\,\rangle \otimes |-\tfrac{1}{2}\alpha_q\rangle$$

close the interferometer loop:
phase shift, recombine & rotate spin

$$|\,\text{final},\rightarrow\,\rangle \otimes \frac{1}{\sqrt{2}}\left\{\,|+\tfrac{1}{2}\alpha_q\rangle + e^{i\theta}|-\tfrac{1}{2}\alpha_q\rangle\,\right\} + \text{other}$$

probability of $|\rightarrow\rangle$ vs. θ

$$\frac{1}{2}\|\,\{\cdots\}\,\|^2 = \frac{1}{2} + \frac{1}{2} + \mathrm{Re}\left\{e^{i\theta}\langle+\tfrac{1}{2}\alpha_p|-\tfrac{1}{2}\alpha_p\rangle\right\}$$

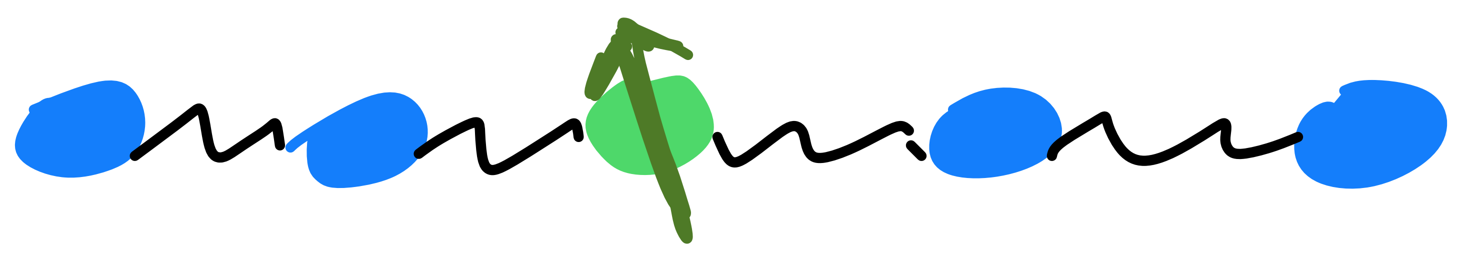


Contrast ↔ overlap of phonon modes

information content: orthogonal / distinguishible?

↔ “collapse” (no superposition)

simple 1D Object: Linear Chain



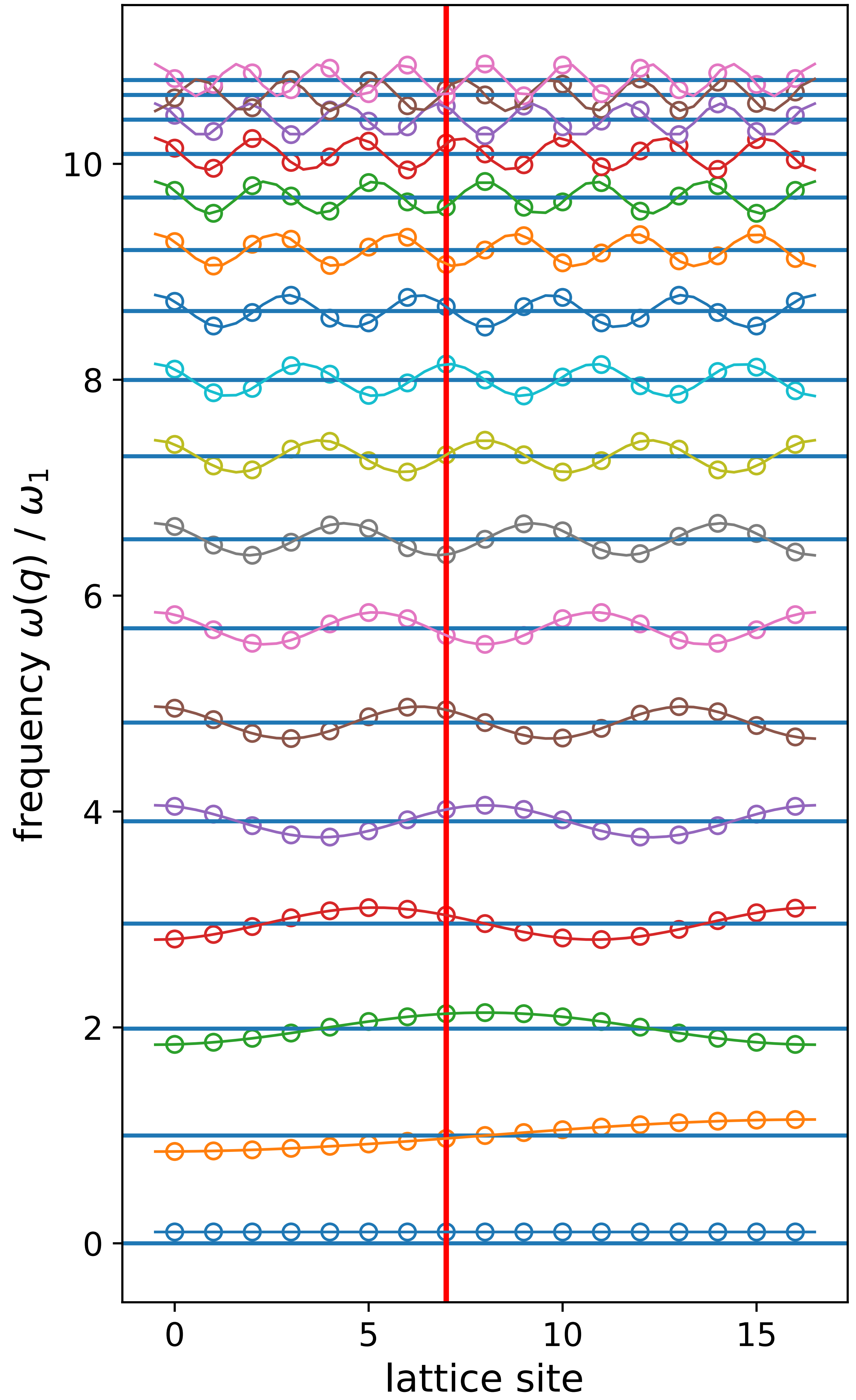
$$H = \sum_i \frac{p_i^2}{2m} + \sum_{\langle i,j \rangle} \frac{k}{2}(x_i - x_j)^2 - \mu \mathbf{S} \cdot \mathbf{B}(x_s)$$

phonon spectrum

$$\omega_q = \left(\frac{4k}{m}\right)^{1/2} \left|\sin \frac{qa}{2}\right| \approx c\,|\,q\,|$$

$$q = \frac{\pi}{L}(0,1,2\ldots)$$ (open boundary conditions)

switch to phonon amplitudes $\{x_i\} \mapsto \{\text{CoM}, u_q\}$



Henkel & Folman (*AVS Quantum Sci.* 2022)

Phonon Displacements ...

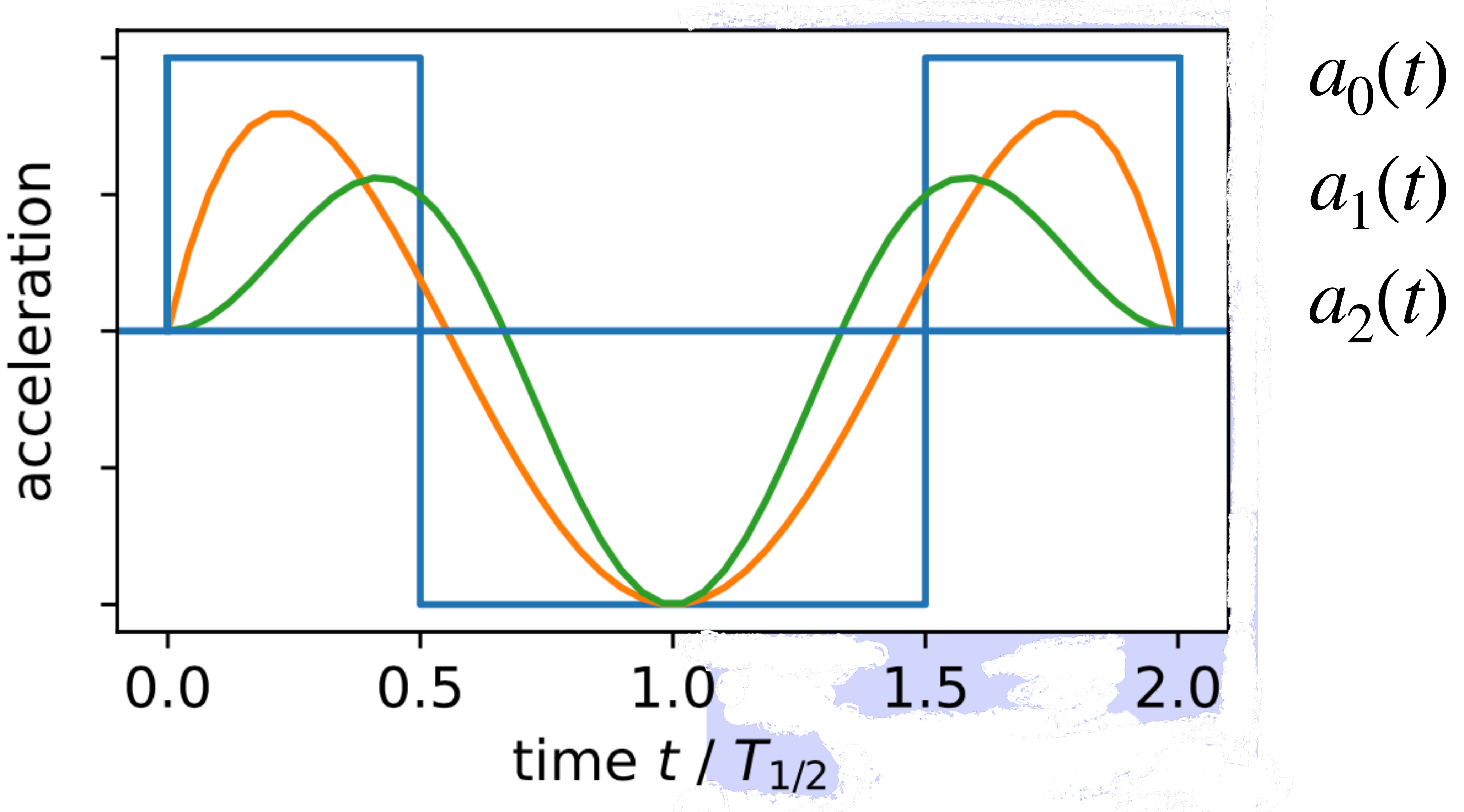
Solve equations of motion

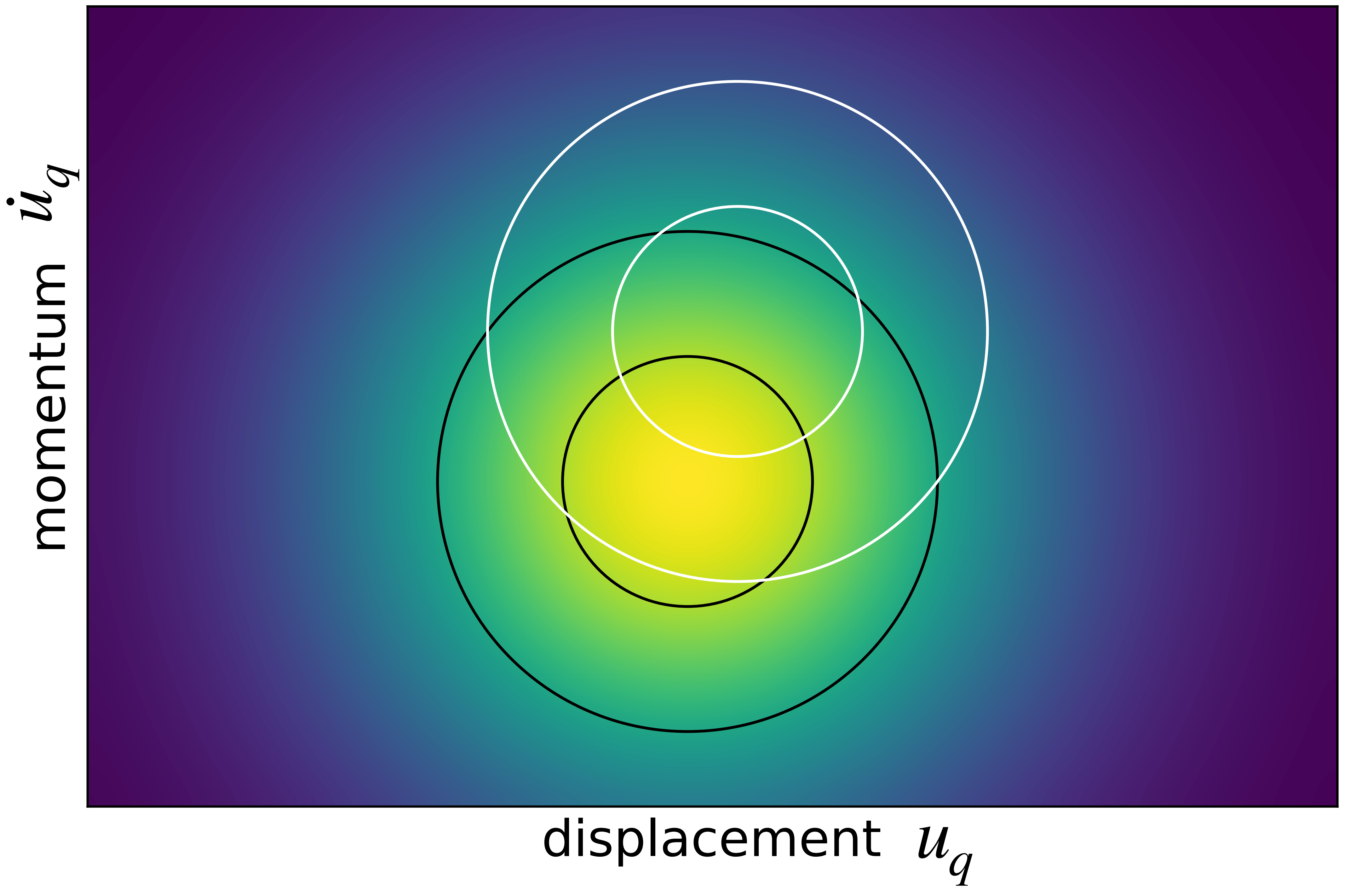
acceleration (CoM)

$$u_q(2T_{1/2}) + \frac{\mathrm{i}}{\omega_q}\dot{u}_q(2T_{1/2}) = \pm\cos[(s+\tfrac{1}{2})qa] \int_0^{2T_{1/2}} \mathrm{d}t\, \frac{\mathrm{i}\, a(t)}{\omega_q}\, \mathrm{e}^{\mathrm{i}\omega_q(2T_{1/2}-t)} + \text{free oscillation}$$

closed loop: both $p(2T) = M\int \mathrm{d}t\, a(t) = 0$ and $z(2T) = 0$

pulsed force (magnetic gradient)





... and Interference Contrast

Phonon states $|\pm\frac{1}{2}\alpha_q\rangle$ entangled with $|\uparrow\rangle$ and $|\downarrow\rangle$

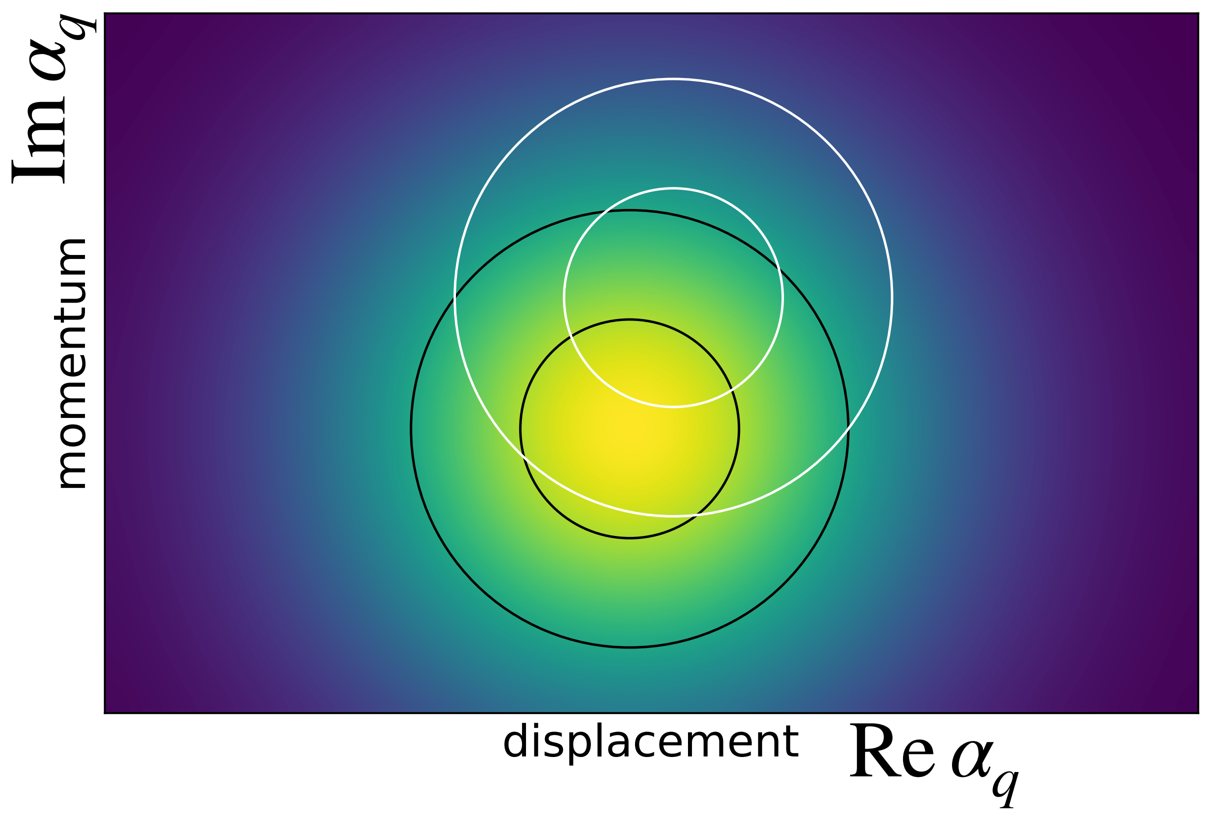
... may become orthogonal

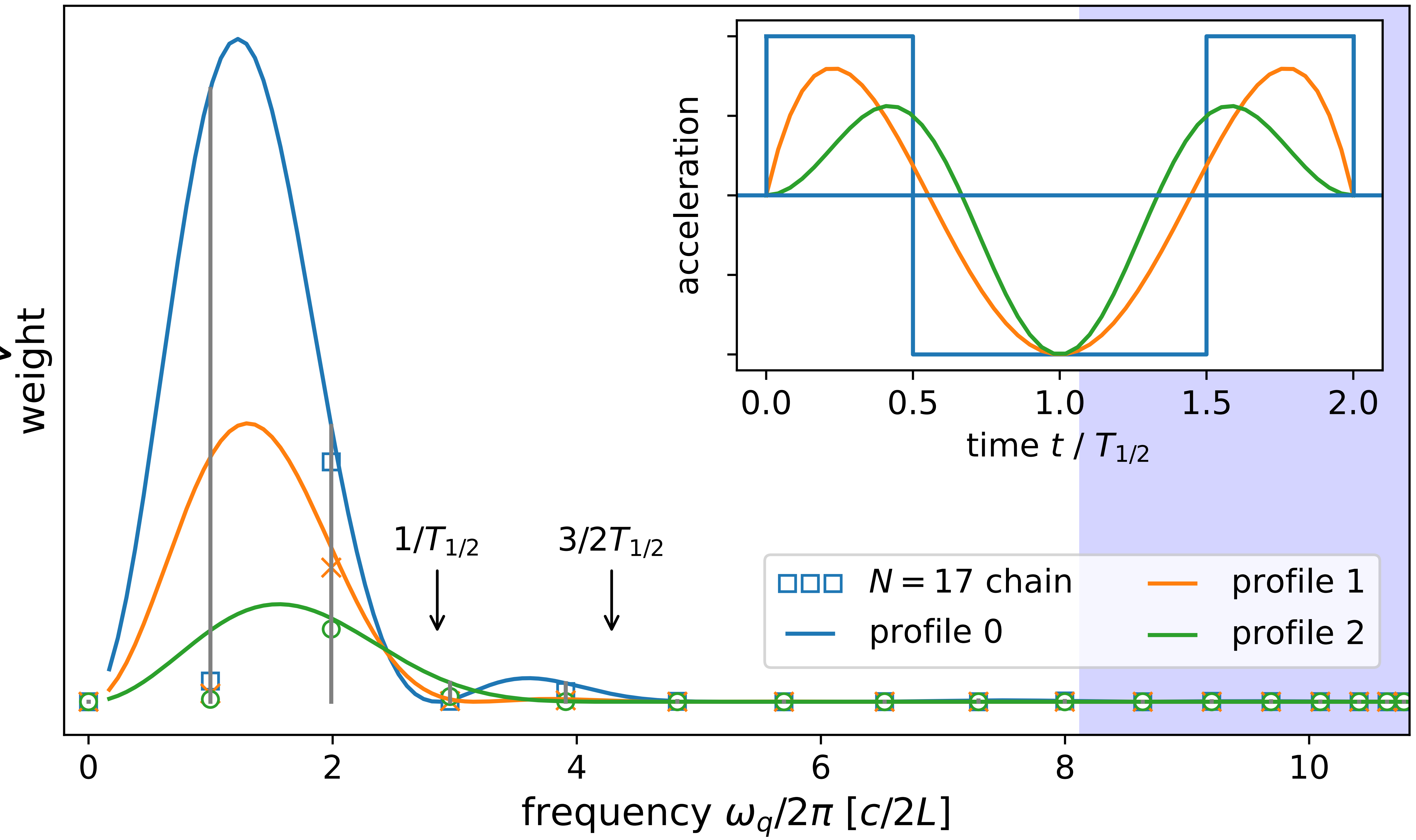
position of spin

$$\alpha_q \sim \left(\frac{m_{\text{eff}}}{\hbar\omega_q}\right)^{1/2} \cos[(s+\tfrac{1}{2})qa] \int_0^{2T_{1/2}} \mathrm{d}t\, a(t)\, \mathrm{e}^{-\mathrm{i}\omega_q t}$$

$$\langle -\tfrac{1}{2}\alpha_q \,|\, \tfrac{1}{2}\alpha_q\rangle \to \mathrm{tr}\big[\rho_q\, D(\alpha_q)\big]$$

$$= \exp\big[-|\alpha_q|^2(2\bar{n}+1)\big]$$





Sum over Phonons

include all phonon modes q, ω_q

... key trick of this calculation:

- thermal equilibrium state $\rho = \bigotimes_q \rho_q(\omega_q/T)$
- total overlap $\prod_q \langle -\tfrac{1}{2}\alpha_q | +\tfrac{1}{2}\alpha_q \rangle$

$$\to \prod_q \mathrm{tr}\left[D(-\tfrac{1}{2}\alpha_q)\,\rho_q(\omega_q/T)\,D(\tfrac{1}{2}\alpha_q)\right]$$

$$= \exp\left[-\sum_q |\alpha_q|^2 (2\bar{n}(\omega_q, T) + 1)\right]$$

potential “orthogonality catastrophe”

Sum over Phonons

total contrast reduction $\exp\left[-\sum_q |\alpha_q|^2 (2\bar{n}(\omega_q, T) + 1)\right]$

display classical limit
just one mode

$$|\alpha|^2 = \frac{m_{\text{eff}}\omega}{2\hbar}u^2 + \frac{m_{\text{eff}}}{2\hbar\omega}\dot{u}^2 = \frac{E_{\text{osc}}}{\hbar\omega}$$

displacements $u, \dot{u}$
converted into energy

$$2\bar{n}(\omega, T) + 1 \approx \frac{kT}{\hbar\omega}$$

$$|\alpha|^2 (2\bar{n}(\omega, T) + 1) \approx \frac{m_{\text{eff}}kT}{2\hbar^2}u^2 = \frac{u^2}{\lambda_{\text{dB}}^2}$$

assuming $\dot{u} = 0$
thermal de Broglie wavelength

applies also for
CoM state

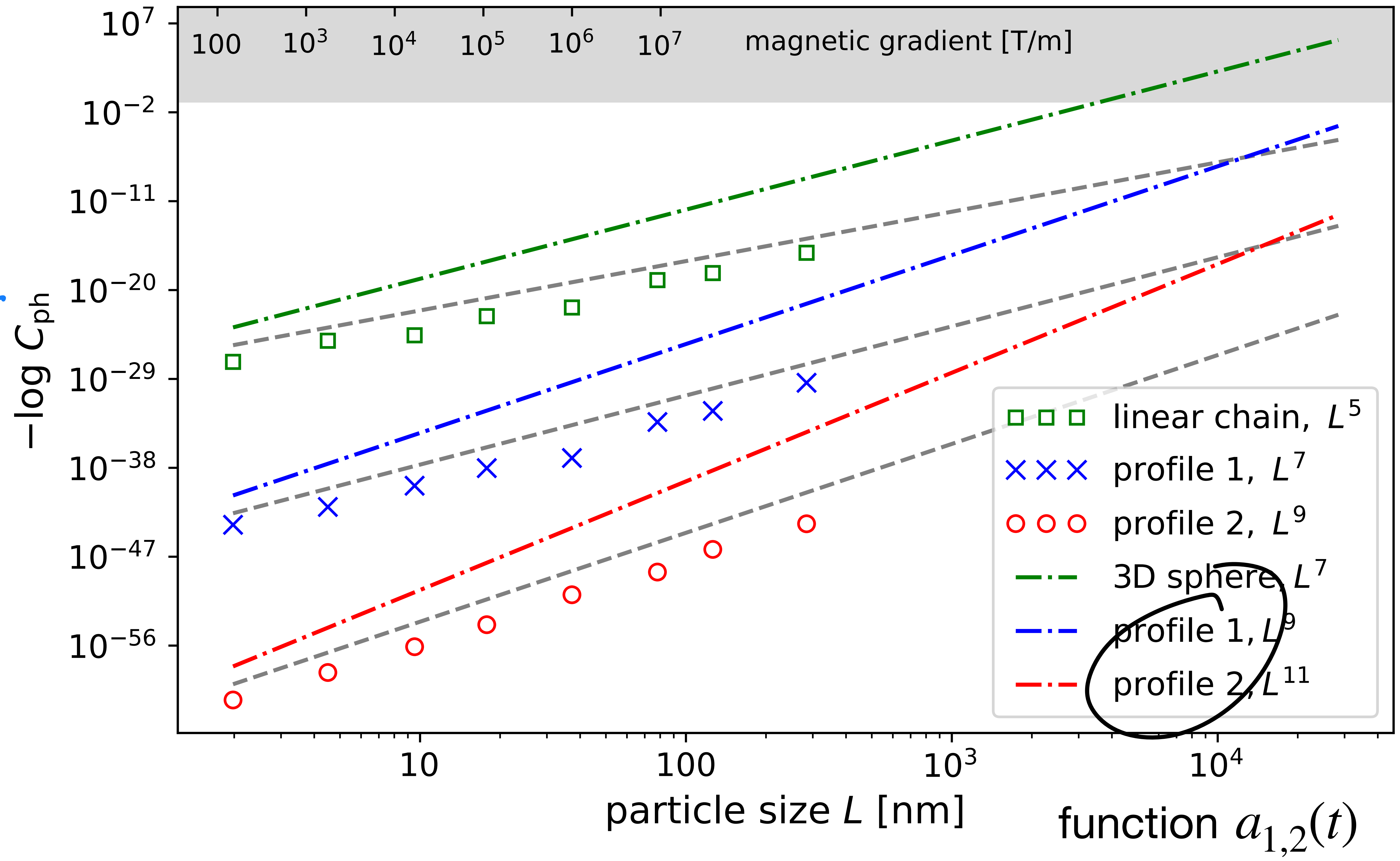
	100 amu	1 nm	1 µm
300 K	4 pm	0.8 pm	0.03 fm
4 K	35 pm	7 pm	0.2 fm

Contrast Scaling with Size

3D sphere

1D chain

60 µs
100 m/s²
293 K



... should work!

Challenges:

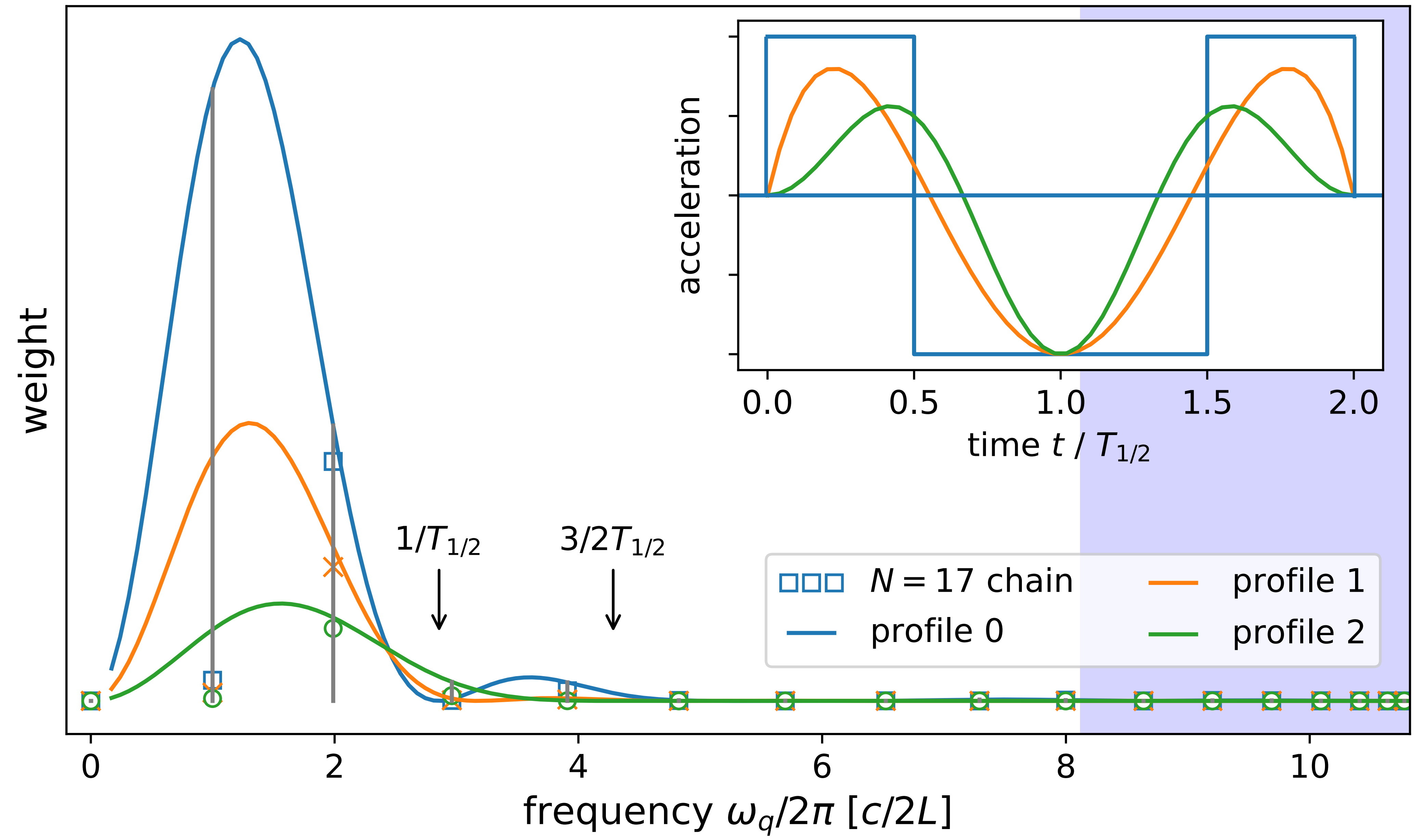
– larger objects
– differential torque (lower rotation frequencies)

→ Japha & Folman (*Phys. Rev. Lett.* 2023)
→ Rusconi & al., (*Phys. Rev. Lett.* 2022)

... towards larger Objects

Parameters so far: “small object”
size $L \ll c\Delta t$ path of sound

- fundamental tone $\omega_1 = \mathcal{O}(\pi c/L)$ dominates



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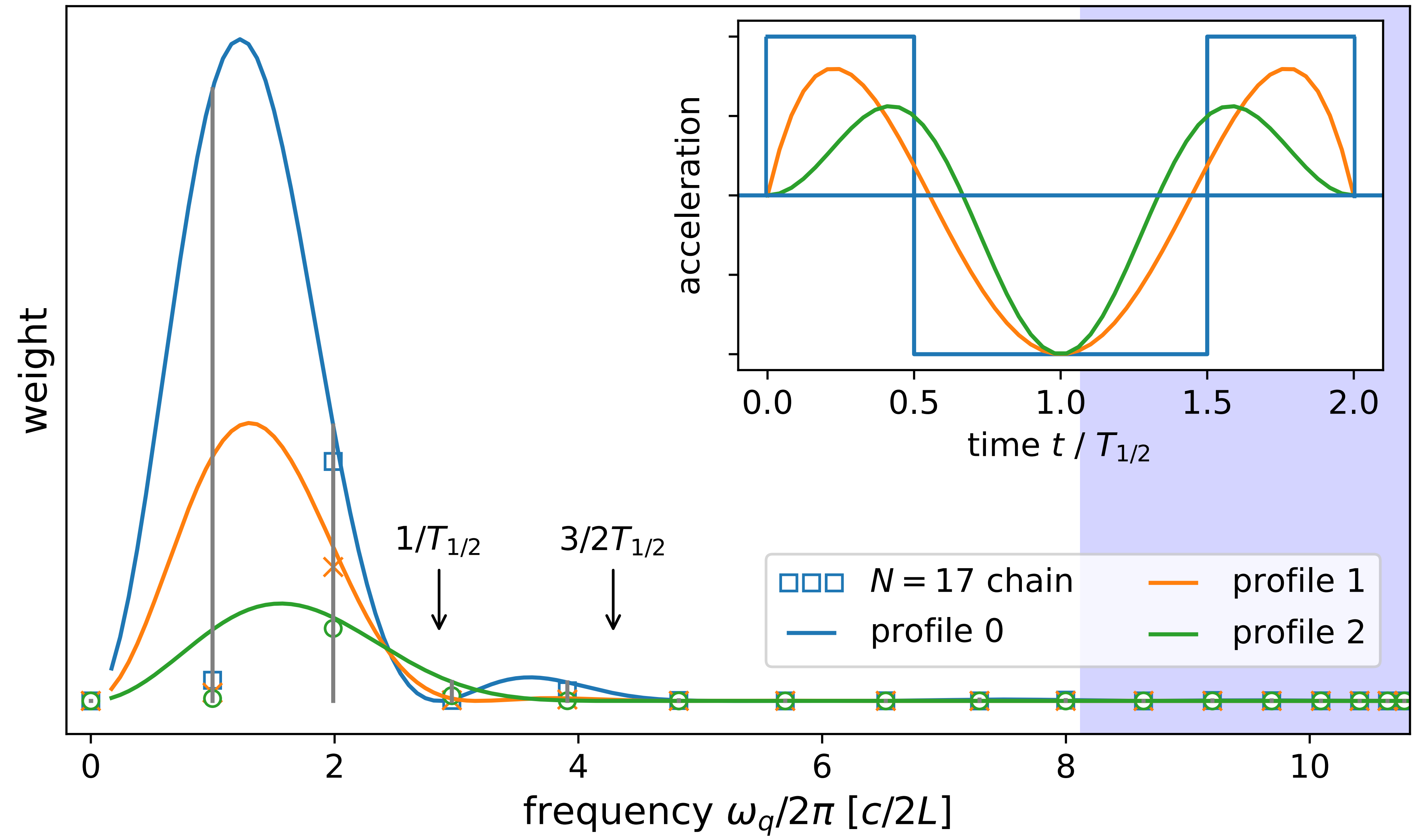
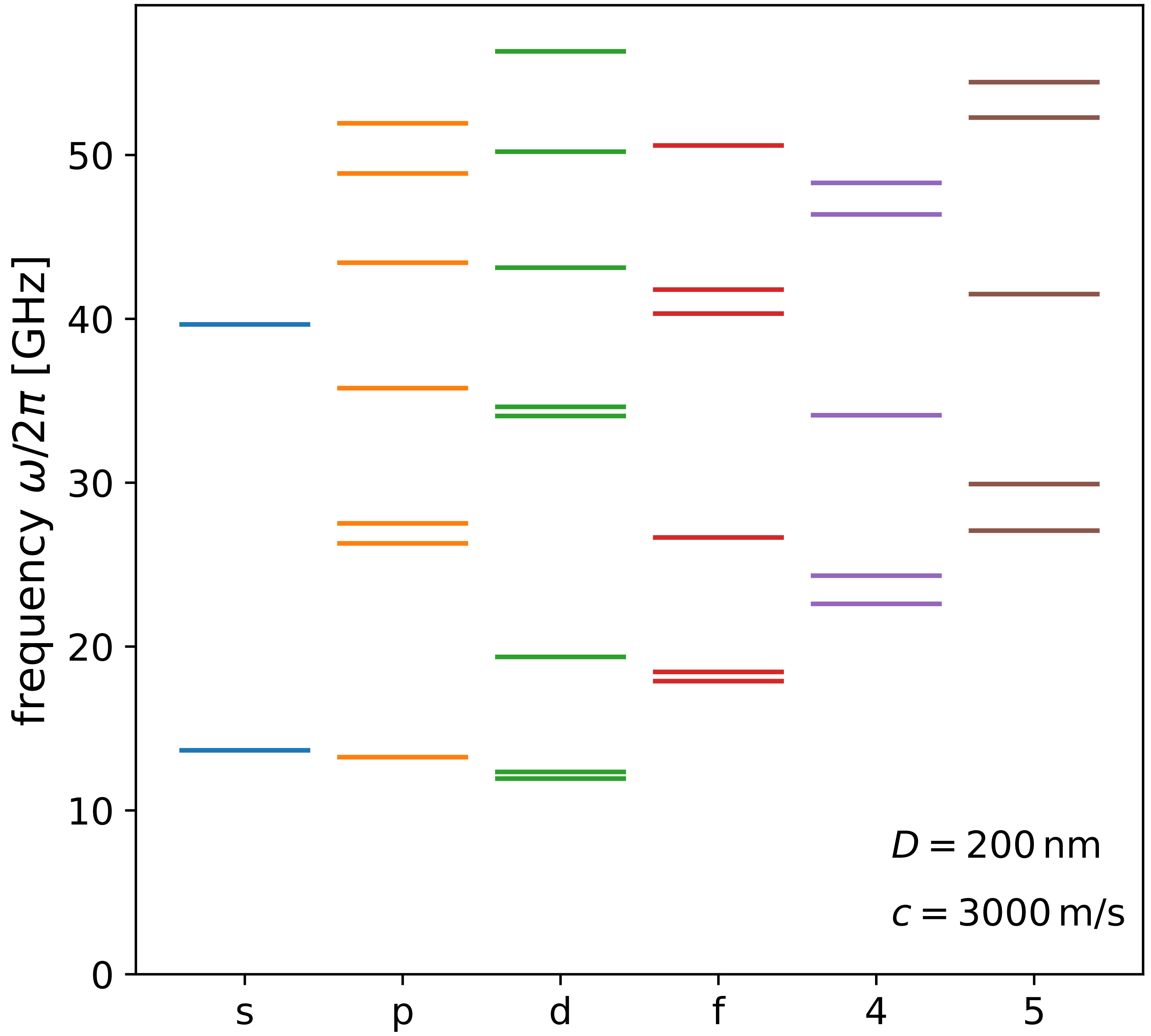
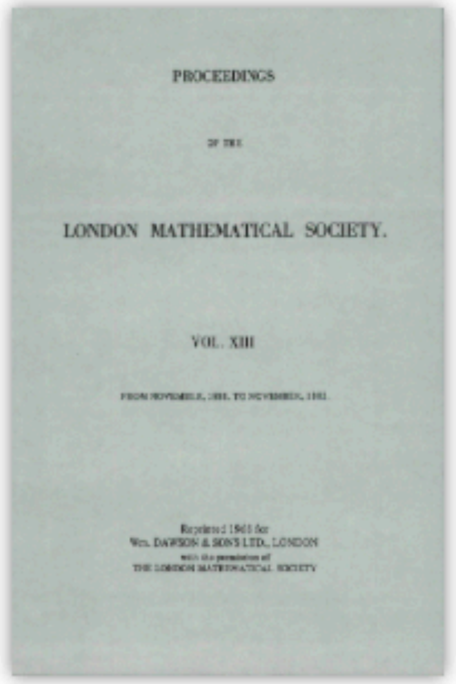


3D object: sphere
“Lamb modes”




PROCEEDINGS
LONDON MATHEMATICAL SOCIETY.
VOL. XIII

Volume s1-13, Issue 1
November 1881
Pages 189-212

Proceedings of the London Mathematical Society
LONDON MATHEMATICAL SOCIETY EST. 1865

Articles | Full Access

On the Vibrations of an Elastic Sphere

Horace Lamb M.A.

First published: November 1881 | https://doi.org/10.1112/plms/s1-13.1.189 |

On the Vibrations of an Elastic Sphere. By HORACE LAMB, M.A.

[*Read May 11th*, 1882.]

The following paper contains an examination into the nature of the fundamental modes of vibration of an elastic sphere by the method employed in a previous communication, "On the Oscillations of a Viscous Spheroid."* The problem here considered is one of considerable

2. The equations of motion of a homogeneous isotropic elastic solid free from external force may be written

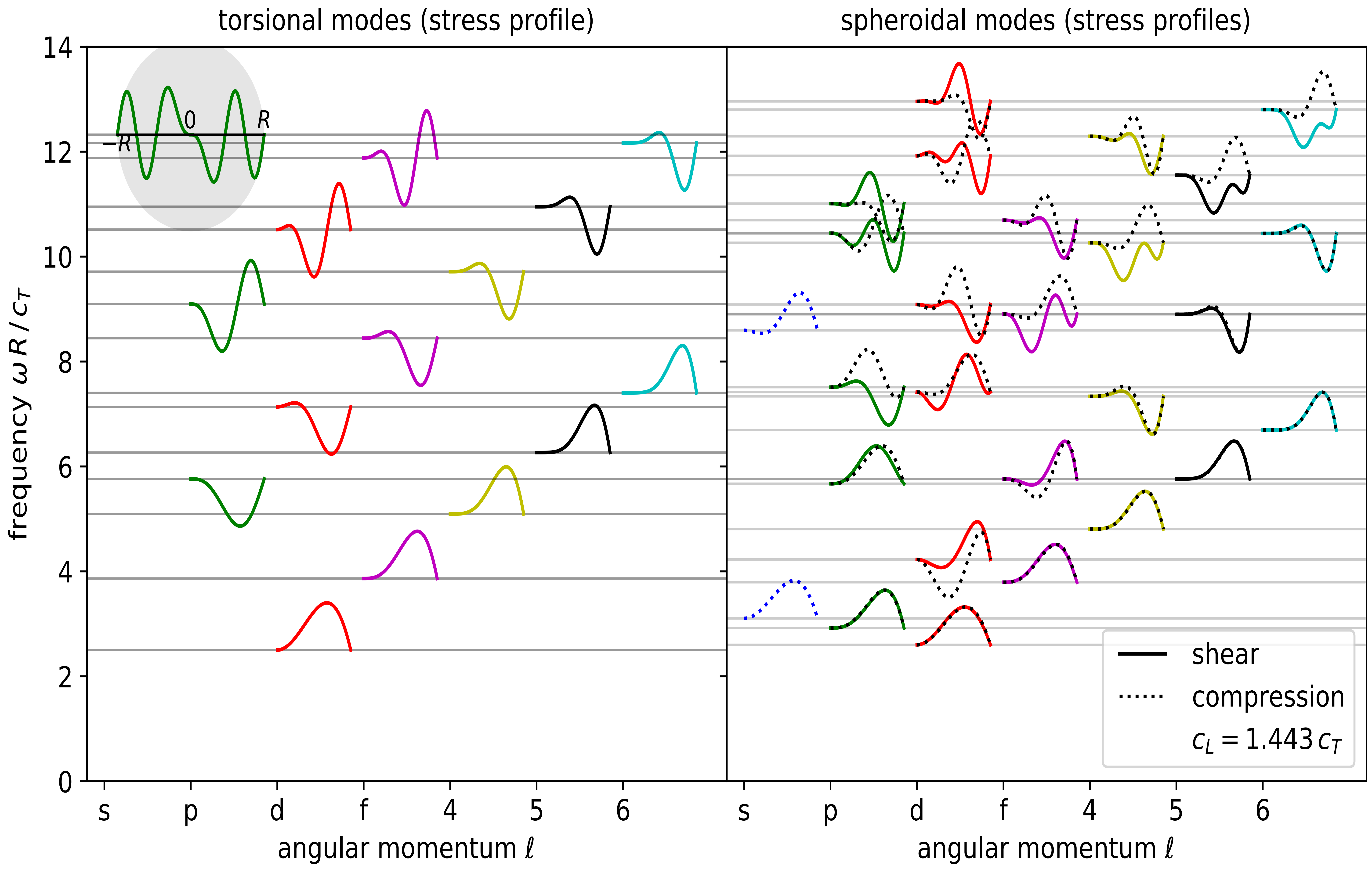
$$\rho\frac{d^2\alpha}{dt^2}=\mathfrak{m}\frac{d\delta}{dx}+\mathfrak{n}\nabla^2\alpha,\quad \rho\frac{d^2\beta}{dt^2}=\mathfrak{m}\frac{d\delta}{dy}+\mathfrak{n}\nabla^2\beta,\quad \rho\frac{d^2\gamma}{dt^2}=\mathfrak{m}\frac{d\delta}{dz}+\mathfrak{n}\nabla^2\gamma \ldots(15).$$

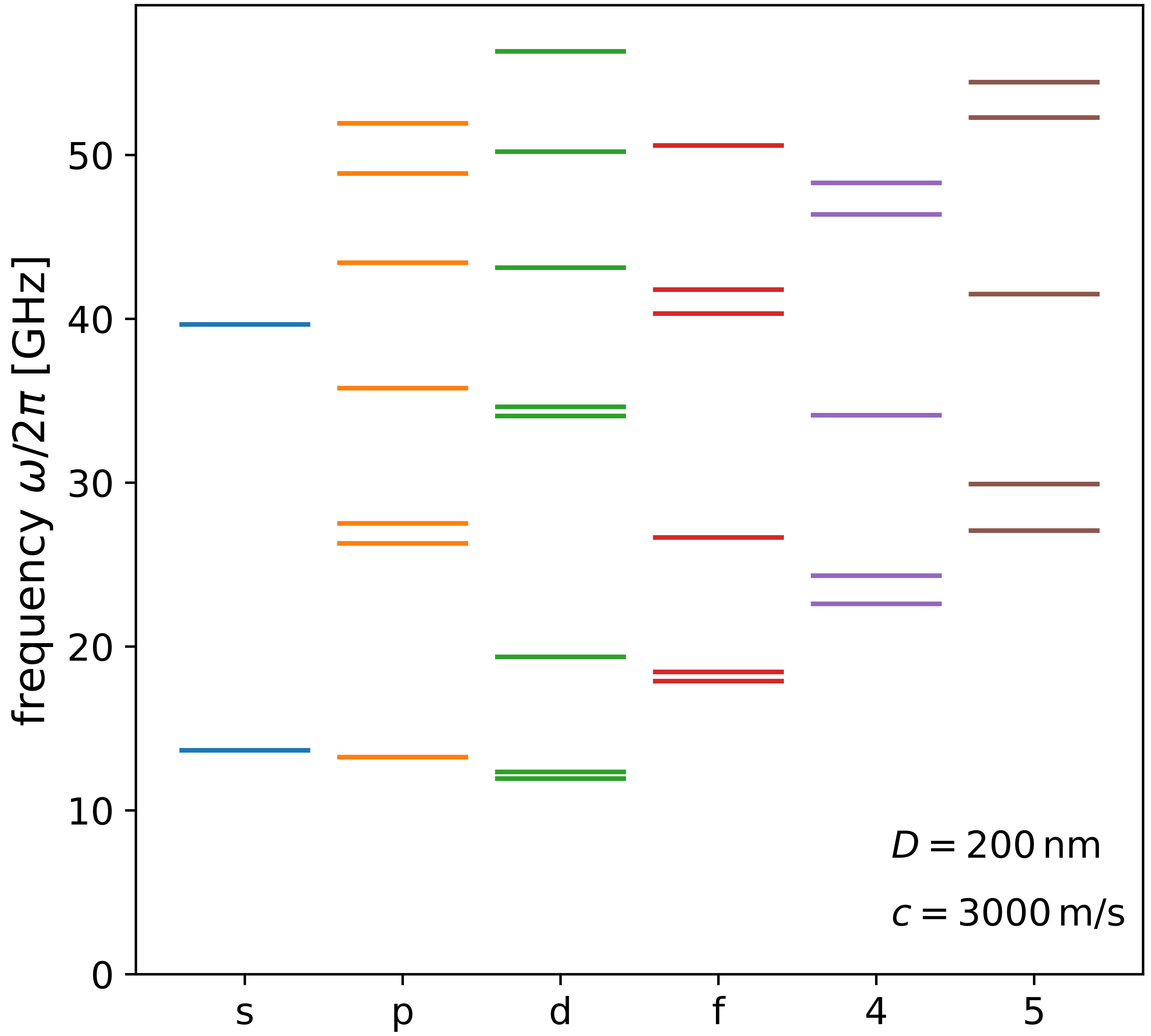
The notation is that of Thomson and Tait, except that $\mathfrak{m}$, $\mathfrak{n}$ are written for the m, n of these writers; viz., α, β, γ are the component displacements, and δ, $= d\alpha/dx+d\beta/dy+d\gamma/dz$, is the dilatation, at the point (x, y, z) of the solid, ρ is the density, $\mathfrak{n}$ the rigidity, and $\mathfrak{m}$ a constant, such that $\mathfrak{m}-\frac{1}{3}\mathfrak{n}$ is the resilience of volume. If the state of stress at

Hydrogen Atom of Sound

displacement field ← scalar mode generators (Lamb 1882)

3D object: sphere
"Lamb modes"



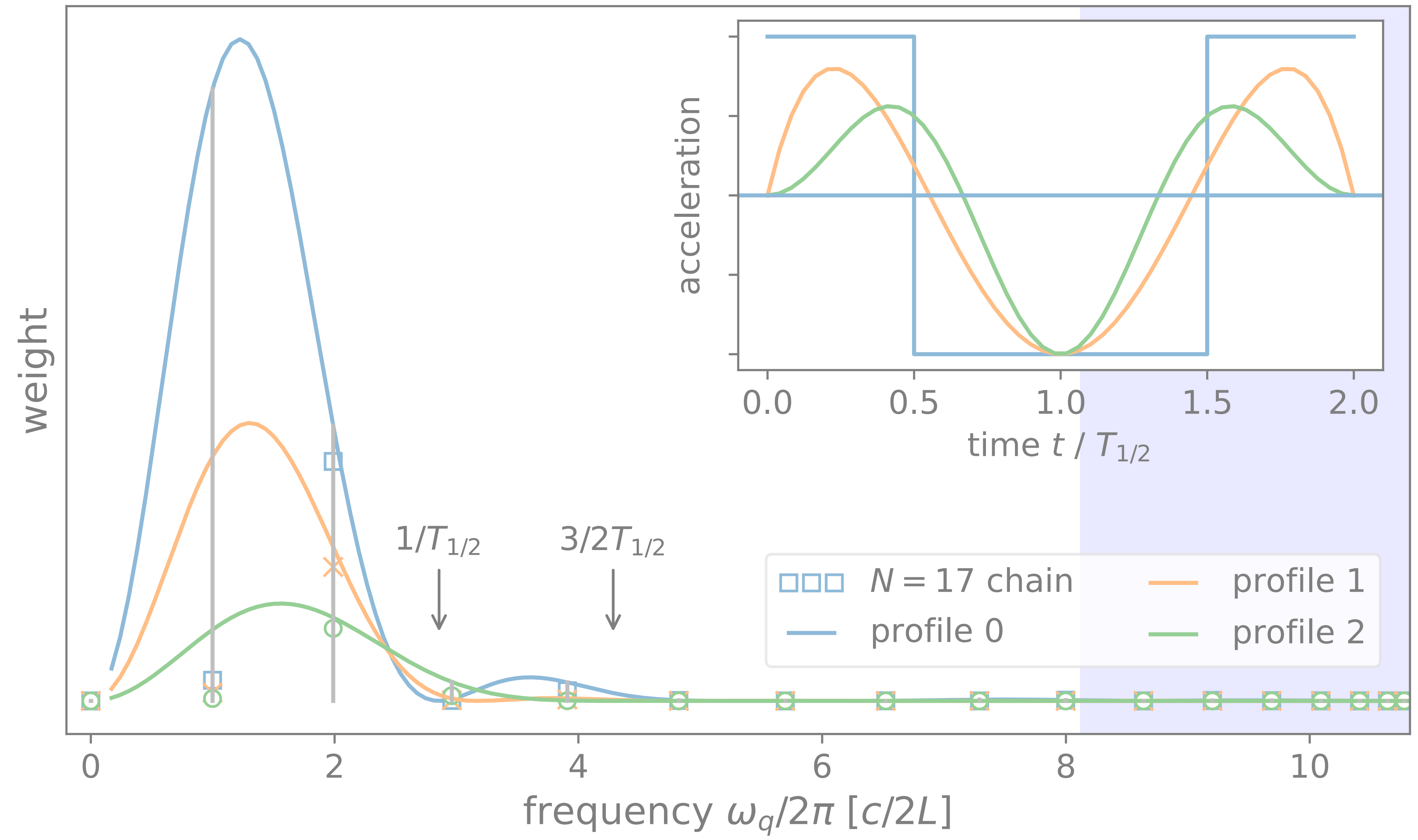


arxiv 2305.15230

... towards larger Objects

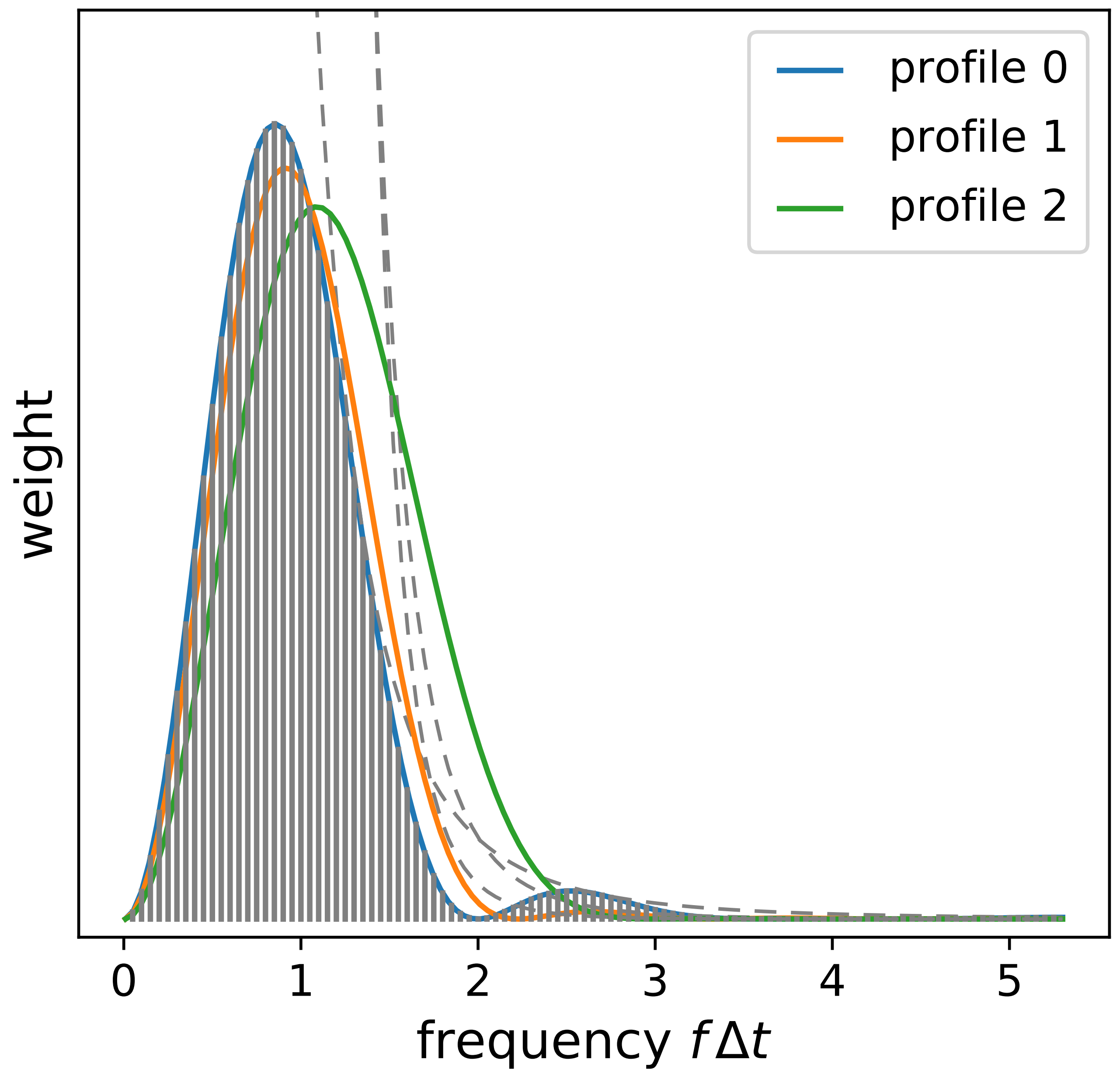
Parameters so far: "small object"
size $L \ll c\Delta t$ path of sound

• fundamental tone $\omega_1 = \mathcal{O}(\pi c/L)$ dominates

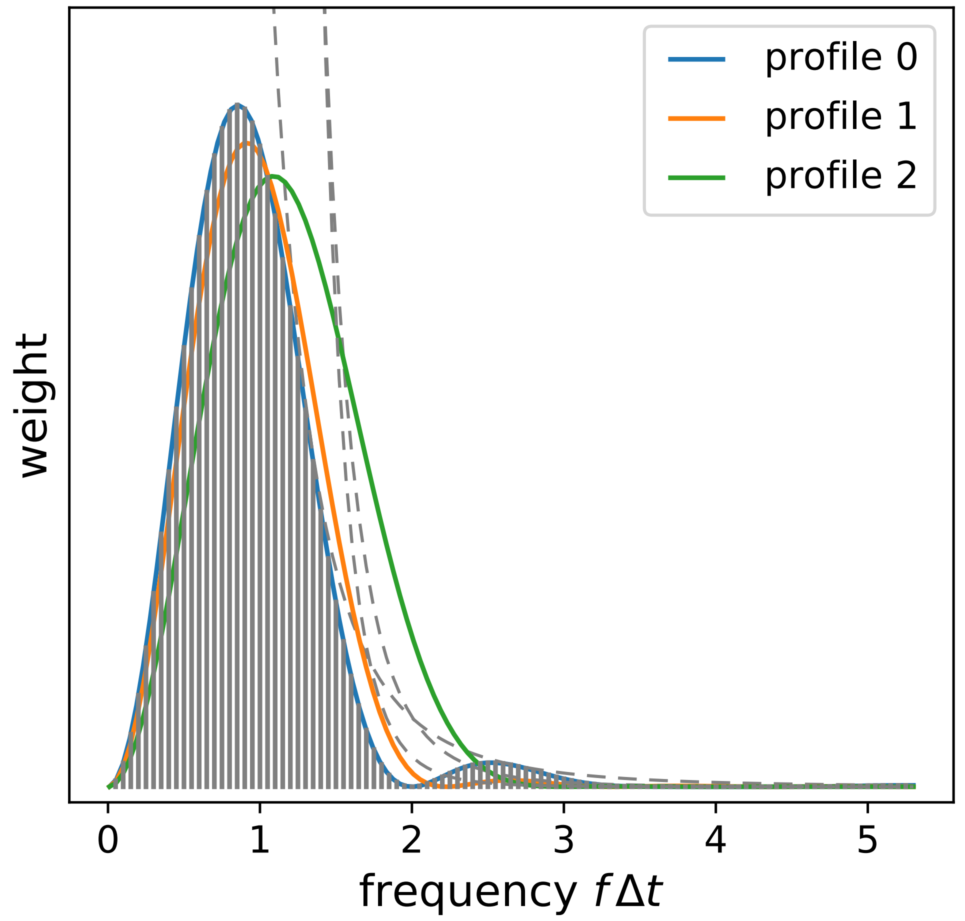


"Large object"
$L \gg c\Delta t \sim c^2/100\, a_{\max} \sim 1\,\mathrm{m}$
with fixed splitting $L/100 \sim \Delta z \sim a_{\max}\Delta t^2$

• all modes relevant



... towards larger Objects



all phonon modes relevant

phonon (int'l) temperature

- thermal equilibrium state $\rho = \bigotimes_q \rho_q(\omega_q/T)$
- total contrast $C = \exp\left[-\sum_q |\alpha_q|^2 (2\bar{n}(\omega_q) + 1) \right]$

$$\sum_q |\alpha_q|^2 \ldots \approx \frac{3ML^3}{2} \int \frac{d^3q}{(2\pi)^3 \hbar\omega_q} \coth(\tfrac{1}{2}\beta\omega_q) |\tilde{a}(\omega_q; T_{1/2})|^2$$

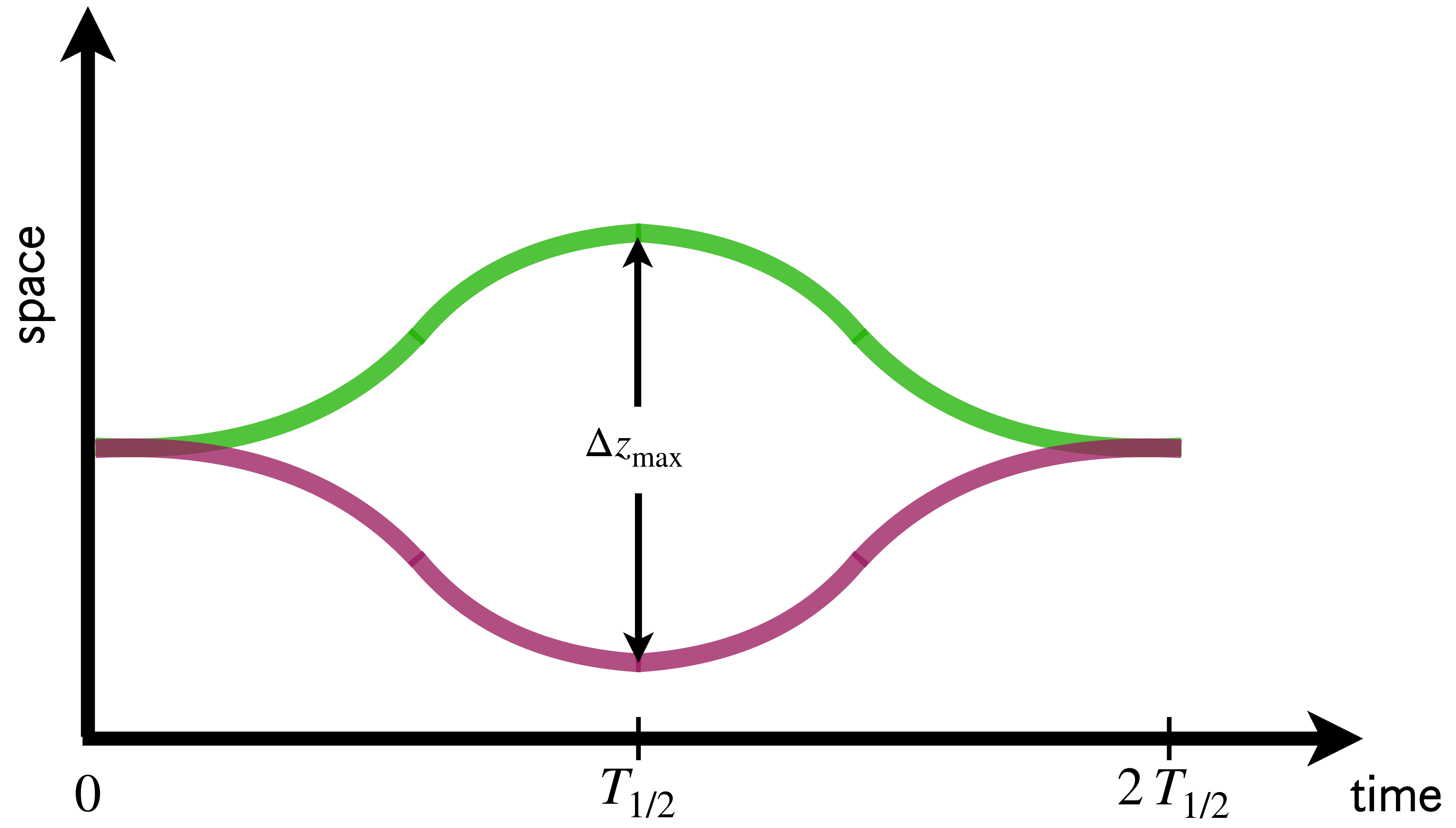
$$\approx \frac{3k_B T ML^3}{\hbar^2 c^3} \int \frac{d\omega_q}{2\pi} |\tilde{a}(\omega_q; T_{1/2})|^2$$

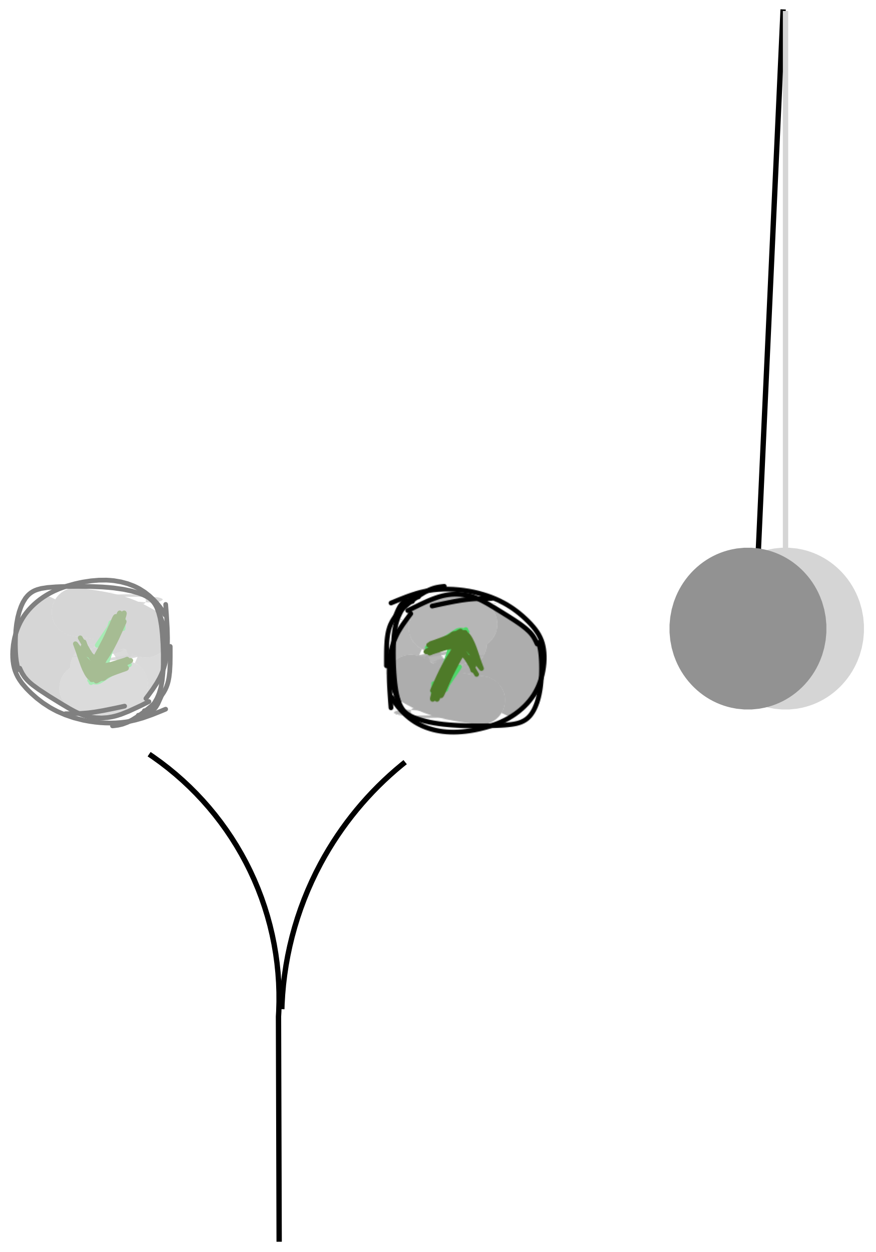
$$\approx \frac{3k_B T ML^3}{\hbar^2 c^3} \int dt |a(t)|^2 \sim \frac{\Delta z_{\max}^2}{\lambda_{dB}^2} \frac{L^3}{(c\Delta t)^3} \gg 1$$

Large object $L \gg c\Delta t$

fixed splitting $L/100 \sim \Delta z_{\max}$

... $C \sim 1$ impossible





Conclusion

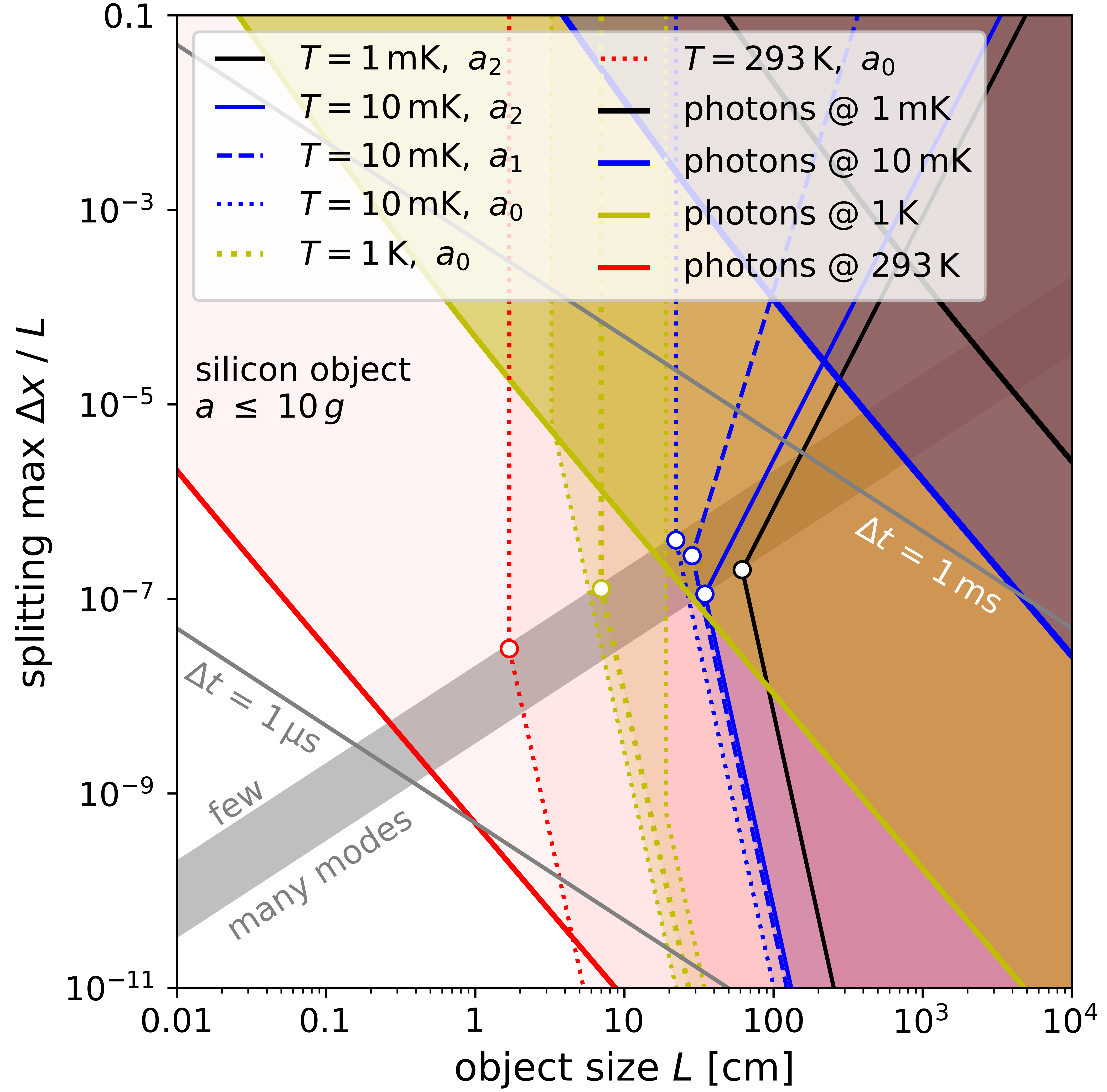
split & recombine composite object
– “quantum anchor”: single impurity spin
– “bath” = internal oscillations / phonons
– little information stored unless large number of modes

“collapse”
= zero contrast in split-path interferometer
= sufficient information stored in phonon state(s)

recent project: generalise to inhomogeneous force
$\mathbf{F}(\mathbf{x}_i) = \mathbf{F}_0 + \delta\mathbf{F}_i$ acting on atom at $\mathbf{x}_i$
→ similar scaling laws
→ stringent requirements on $\delta F / F$ for large objects

Henkel & Folman (*AVS Quantum Sci.* 2022)
(*arxiv* 2305.15230)

Conclusion



arxiv 2305.15230

Blackbody photons

(in harmonic trap)

spectrum. Summing over all modes, the characteristic jump rate due to absorption of blackbody radiation is then given by (cf. Eq. **2** in main text)

$$\gamma_{\mathrm{bb}} = \frac{2\pi^4}{63}\frac{(k_B T)^6}{c^5 \hbar^5 \rho \omega_m} \mathrm{Im}\frac{\epsilon_{\mathrm{bb}}-1}{\epsilon_{\mathrm{bb}}+2}. \qquad \textbf{[S17]}$$

Chang & Zoller group, *PNAS* 2010

this limit the relevant quantity is the localization parameter. This parameter has three contributions given by scattering, emission, and absorption of thermal photons, namely $\Lambda_{\mathrm{bb}} = \Lambda_{\mathrm{bb,sc}} + \Lambda_{\mathrm{bb,e}} + \Lambda_{\mathrm{bb,a}}$, which are given by

$$\Lambda_{\mathrm{bb,e(a)}} = \frac{16\pi^5 c R^3}{189}\left[\frac{k_B T_{i(e)}}{\hbar c}\right]^6 \mathrm{Im}\left[\frac{\epsilon_{\mathrm{bb}}-1}{\epsilon_{\mathrm{bb}}+2}\right]. \qquad (29)$$

Schlosshauer, *Springer* 2007
Romero-Isart, *Phys Rev A* 2011