Istituto Nazionale di Fisica Nucleare
Sezione di Trieste

UNIVERSITÀ DEGLI STUDI DI TRIESTE

Inflationary cosmology and dynamical collapse models

COLMO: Quantum Collapse Models investigated with Particle, Nuclear, Atomic and Macro systems

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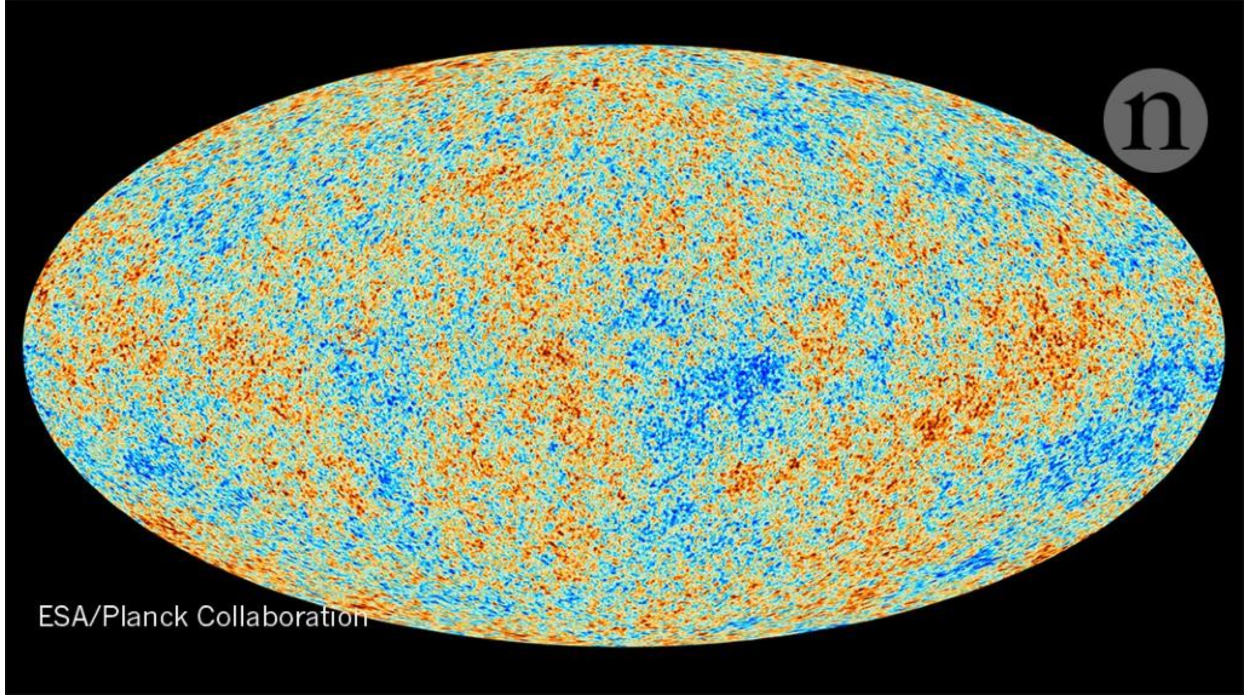
Trento, Italy
5 July 2023

I. Inflation and scalar cosmological perturbations theory

FLRW Cosmology



- The homogeneity and isotropy conditions of the Cosmological Principle lead to the Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dx^2}{1 - Kx^2} + x^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

- Among the problems of the FLRW cosmology, one finds:
  - Horizon problem
  - Flatness problem

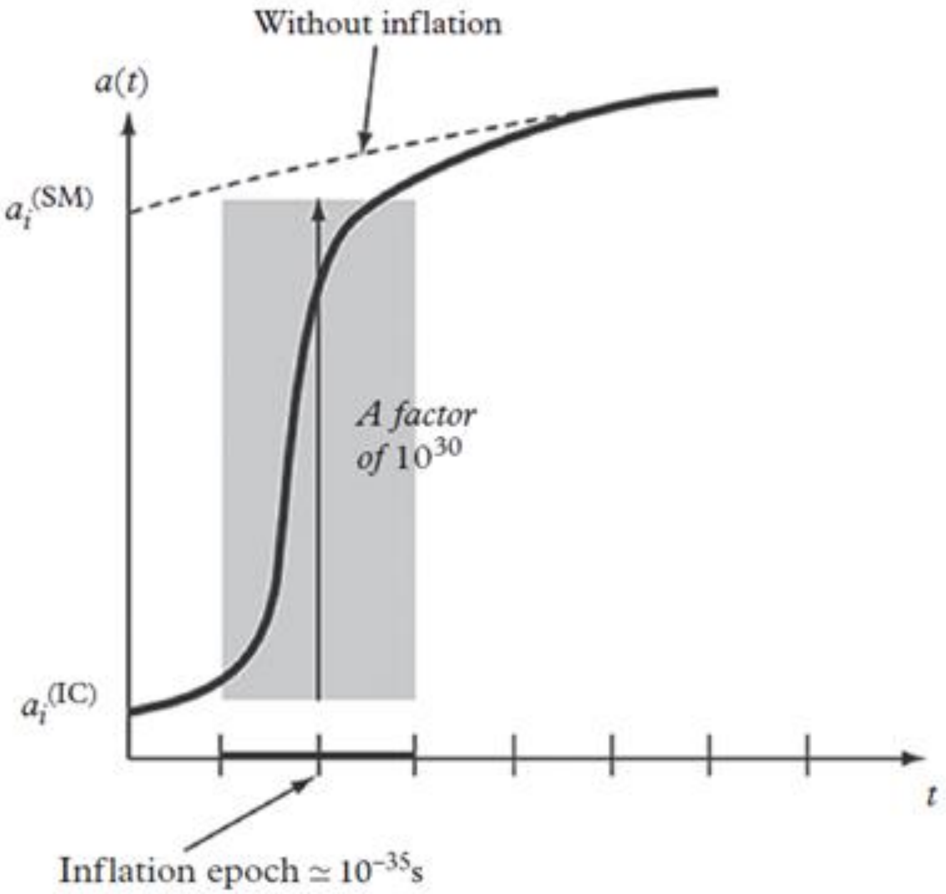
V. Mukhanov, *Physical Foundations of Cosmology*, Cambridge University Press (2005).

Cosmic Inflation: Basic concepts

- Cosmic inflation is an initial era in which the expansion rate $\dot{a}$ is accelerating.
- The evolution of the scalar field driving inflation is given by:

$$\ddot{\varphi} + 3H\,\dot{\varphi} + V'(\varphi) = 0$$

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\left(\frac{1}{2}\,\dot{\varphi}^2 + V(\varphi)\right)$$



D. H Lyth and A. R. Liddle, *The Primordial Density Perturbation. Cosmology, Inflation and the Origin of Structure.* Cambridge University Press (2009).
Credit: T.-P. Cheng, *A College Course on Relativity and Cosmology,* Oxford University Press (2015).

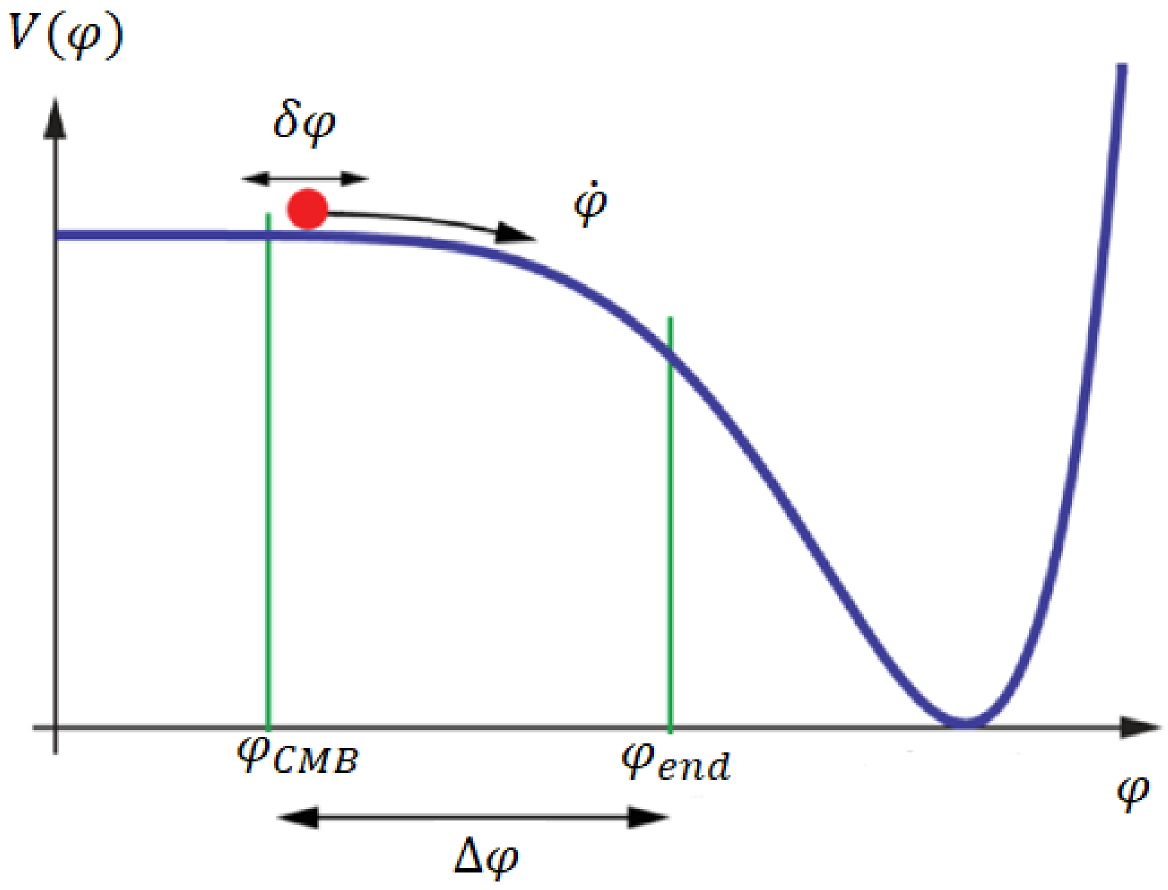
Cosmic Inflation: Basic concepts

- The slow-roll condition implies the following flatness conditions on the potential $V(\varphi)$

$$\epsilon_f(\varphi) = \frac{M_{Pl}^2}{2}\left(\frac{1}{V}\frac{\partial V}{\partial \varphi}\right)^2 \ll 1$$

$$|\eta_f(\varphi)| \simeq M_{Pl}^2 \frac{1}{V}\frac{\partial^2 V}{\partial \varphi^2} \ll 1$$

- If the slow-roll condition is valid, then inflation is guaranteed.



Credit: Adapted from J. Levenga, *Slow-roll inflation and the Hamilton-Jacobi Formalism,* University of Groningen (2019).

Roberston-Walker Metric and scalar perturbations

The most general form of the line element for a background described by the Robertson-Walker metric and scalar perturbations is given by

$$ds^2 = a^2(\eta)\left([1+2\phi]\mathrm{d}\eta^2 - 2B_{|i}\mathrm{d}x^i\mathrm{d}\eta - \left[(1-2\psi)\gamma_{ij} + 2E_{|ij}\right]\mathrm{d}x^i\mathrm{d}x^j\right)$$

One of the gauge-invariant quantities that can be constructed is the so-called Mukhanov-Sasaki variable v, which reads

$$v = a\left[\delta\varphi + \left(\frac{\varphi_0'}{\mathcal{H}}\right)\psi\right]$$

V. Mukhanov, Sov. Phys. JETP **67**, 1297 (1988).
V. Mukhanov, H. Feldman, and R. Brandenberger, Physics Reports **215**, 203 (1992).

Gravity + Scalar field action

In terms of the Mukhanov-Sasaki variable v, the action of gravity + scalar field, up to second order in the perturbations reads:

$$S_2 = \frac{1}{2}\int \mathrm{d}^4x \left(v'^2 - c_s^2 v_{,i}v_{,i} + \frac{z''}{z} v^2 \right)$$

with

$$z = aM_{\mathrm{Pl}}\sqrt{2\epsilon}/c_s$$

Viatcheslav Mukhanov (1956)

Misao Sasaki (1952)

V. Mukhanov, Sov. Phys. JETP **67**, 1297 (1988).
V. Mukhanov, H. Feldman, and R. Brandenberger, Physics Reports **215**, 203 (1992).

Quantization of cosmological perturbations

After quantizing v, one can show that the corresponding field operator $\hat{v}$ satisfies the equation

$$\hat{v}'' - c_s^2 \nabla^2 \hat{v} - \frac{z''}{z} \hat{v} = 0$$

The power spectrum $\mathcal{P}_{\hat{\mathcal{R}}}(k)$ of the comoving curvature perturbation $\hat{\mathcal{R}}$ reads

$$\hat{\mathcal{R}}^2 = \frac{\hat{v}^2}{z^2} \quad \Longrightarrow \quad \langle 0 | \hat{\mathcal{R}}^2(\eta, \boldsymbol{x}) | 0 \rangle = \int \mathrm{d} \ln k \ \mathcal{P}_{\hat{\mathcal{R}}}(k)$$

V. Mukhanov, Sov. Phys. JETP **67**, 1297 (1988).
V. Mukhanov, H. Feldman, and R. Brandenberger, Physics Reports **215**, 203 (1992).

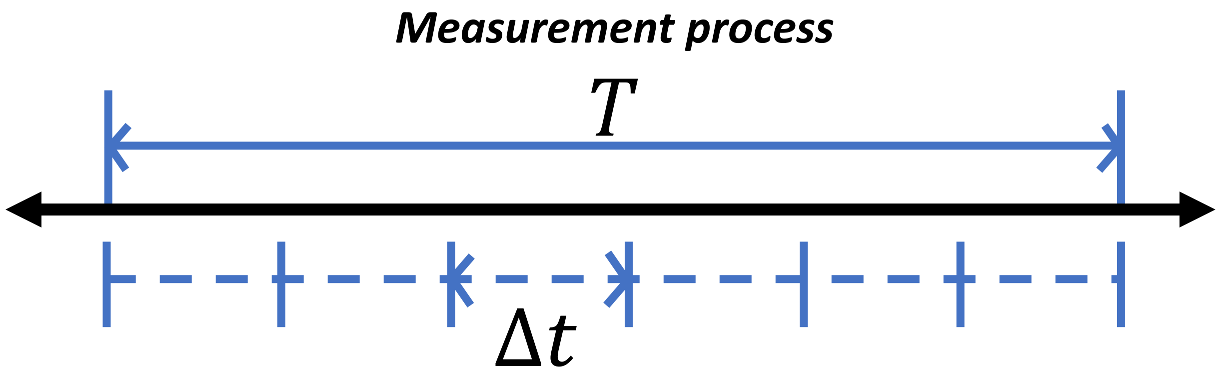
II. Dynamical collapse models

Gaussian continuous measurements

A measurement record

$$r = \langle \hat{a} \rangle + \frac{\hbar}{\sqrt{\gamma}} \frac{\mathrm{d}W_t}{\mathrm{d}t}$$

encodes the definition of a continuous measurement.



A Gaussian quantum continuous measurement of an operator $\hat{a}$ is described through the SDE:

$$\mathrm{d}\,|\psi\rangle = \left\{ -\frac{\gamma}{8\hbar^2} (\hat{a} - \langle \hat{a} \rangle)^2 \mathrm{d}t + \frac{\sqrt{\gamma}}{2\hbar} (\hat{a} - \langle \hat{a} \rangle)\, \mathrm{d}W_t \right\} |\psi\rangle$$

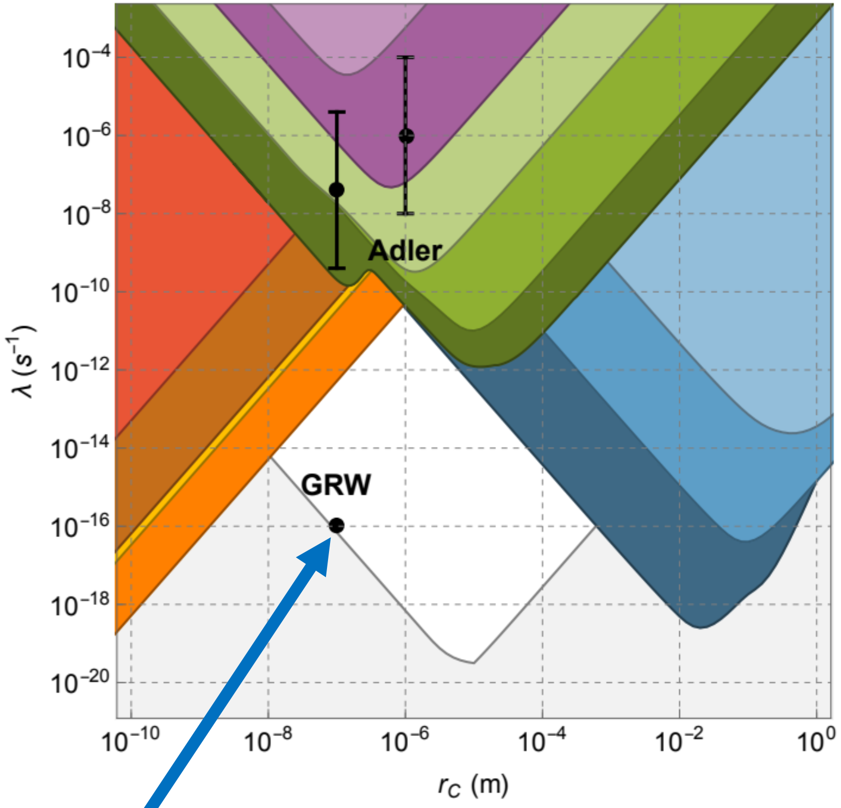
K. Jacobs, *Quantum Measurement Theory and its Applications* (Cambridge University Press, 2014).

Continuous Spontaneous Localization (CSL) model

The most robust dynamical collapse model is the CSL model, which in its mass proportional version is defined at the level of the wavefunction through the following SDE:

$$\mathrm{d}|\psi\rangle = \left[-\frac{i}{\hbar}\hat{H}\mathrm{d}t + \frac{\sqrt{\gamma}}{m_0}\int \mathrm{d}\boldsymbol{x}\,\left[\hat{M}(\boldsymbol{x}) - \langle \hat{M}(\boldsymbol{x})\rangle\right]\mathrm{d}W_t(\boldsymbol{x}) - \frac{\gamma}{2m_0^2}\int \mathrm{d}\boldsymbol{x}\,\left[\hat{M}(\boldsymbol{x}) - \langle \hat{M}(\boldsymbol{x})\rangle\right]^2 \mathrm{d}t \right]|\psi\rangle$$

where $\hat{M}(\boldsymbol{x}) = \frac{m}{(\sqrt{2\pi}r_c)^3}\int \mathrm{d}\boldsymbol{y} e^{-\frac{|\boldsymbol{x}-\boldsymbol{y}|^2}{2r_c^2}}\,\hat{a}^\dagger(\boldsymbol{y})\,\hat{a}(\boldsymbol{y})$



$$\lambda_{\mathrm{GRW}} = 10^{-16}\ \mathrm{s}^{-1}$$

Credit: M. Carlesso, S. Donadi, L. Ferialdi, M. Paternostro, H. Ulbricht and A. Bassi, Nat. Phys. **18**, 243-250 (2022)

Motivations to incorporate collapse models

- How to generalize collapse models within a relativistic context?
- The Cosmological Principle leads to a *natural notion* of time.
- Concepts such as *observers* and *detectors* do not have precise definitions in a primordial Universe.
- Collapse models do not depend on the existence of an observer.
- Collapse models are falsifiable.

Credit: https://hubblesite.org/science/galaxies

Collapse models within a cosmological context

- Bounds for the parameters of the CSL model from heating of the intergalactic medium.
- Candidates to implement an effective cosmological constant.
- Effects of collapse models during inflation and the emergence of cosmic structure.

On the quantum origin of the seeds of cosmic structure
Alejandro Perez[1,2], Hanno Sahlmann[1,3] and Daniel Sudarsky[1,4]
Published 14 March 2006 • 2006 IOP Publishing Ltd
Classical and Quantum Gravity, Volume 23, Number 7

Lower and upper bounds on CSL parameters from latent image formation and IGM heating
Stephen L Adler[1]
Published 7 March 2007 • 2007 IOP Publishing Ltd
Journal of Physics A: Mathematical and Theoretical, Volume 40, Number 12

Cosmological inflation and the quantum measurement problem
Jérôme Martin, Vincent Vennin, and Patrick Peter
Phys. Rev. D **86**, 103524 – Published 26 November 2012

Emergence of inflationary perturbations in the CSL model
Gabriel León & Gabriel R. Bengochea
The European Physical Journal C **76**, Article number: 29 (2016)

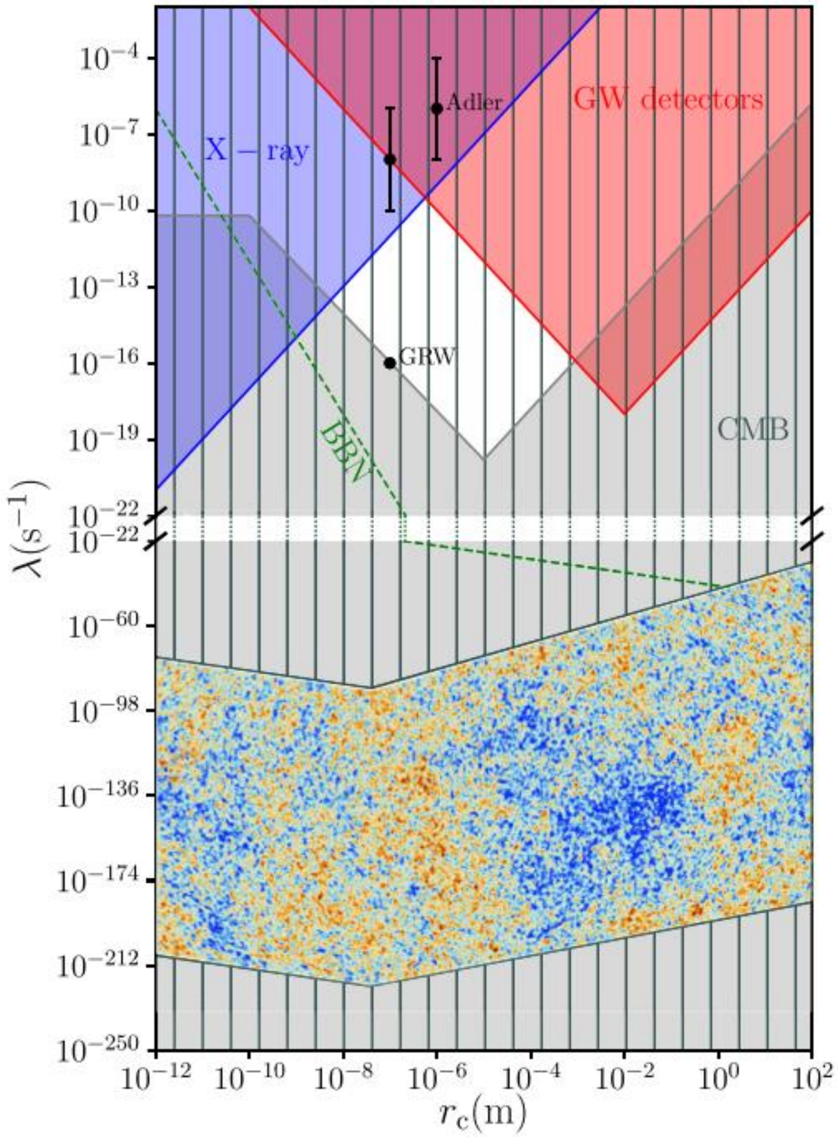
On the choice of the collapse operator in cosmological Continuous Spontaneous Localisation (CSL) theories
Jérôme Martin & Vincent Vennin
The European Physical Journal C **81**, Article number: 516 (2021) | Cite this article

A cosmological arena for collapse models

Cosmic Microwave Background Constraints Cast a Shadow On Continuous Spontaneous Localization Models

Jérôme Martin and Vincent Vennin
Phys. Rev. Lett. **124**, 080402 – Published 26 February 2020

Since the mass density is not uniquely defined in general relativity, this model [CSL] is ambiguous when applied to cosmology. We however show that most natural choices of the density contrast already make current measurements of the cosmic microwave background incompatible with other laboratory experiments.



Credit: J. Martin and V. Vennin, Phys. Rev. Lett. **124**, 080402 (2020).

Effects of Dynamical Collapse Models in Cosmology

We implemented the effects of dynamical collapse models in standard Cosmology, by defining the total Hamiltonian as:

$$\hat{H}_{\text{total}} = \hat{H} + \hat{H}_{\text{DC}}$$

Employing an interaction picture approach, and making use of the *noise trick* of the CSL model, we defined the stochastic Hamiltonian:

$$\hat{H}^{I}_{\text{DC}}(\eta) = \frac{\sqrt{\gamma}}{m_0} \int \mathrm{d}\boldsymbol{x} \xi_\eta(\boldsymbol{x})\, \hat{\mathcal{H}}^{I}_{\text{DC}}(\eta, \boldsymbol{x})$$

where

$$\mathbb{E}[\xi_\eta(\boldsymbol{x})] = 0, \qquad \mathbb{E}[\xi_\eta(\boldsymbol{x})\xi_{\eta'}(\boldsymbol{y})] = \frac{\delta(\eta - \eta')}{a(\eta)} \frac{1}{(4\pi r_c^2)^{3/2}} e^{-a^2(\eta)(\boldsymbol{x}-\boldsymbol{y})^2/(4\, r_c^2)}$$

S. L. Adler and A. Bassi, J. Phys. A **40**, 15083 (2007).

Correction to the Power Spectrum of $\hat{\mathcal{R}}$ during Inflation

We set the collapse operator to be the Hamiltonian density of standard Cosmology, which during the inflationary epoch is given by

$$\hat{\mathcal{H}}^{I}_{\mathrm{DC}}(\eta,\boldsymbol{x}) = \frac{1}{2}\left(\dot{\hat{v}}^2(\eta,\boldsymbol{x}) + \left(\nabla\,\hat{v}(\eta,\boldsymbol{x})\right)^2 - \frac{2}{\eta^2}\hat{v}^2(\eta,\boldsymbol{x})\right)$$

The leading order correction to the power spectrum $\mathcal{P}_{\hat{\mathcal{R}}}$ of $\hat{\mathcal{R}}$ is

$$\mathcal{P}_{\hat{\mathcal{R}}} = \mathcal{P}_{\hat{\mathcal{R}}\,\mathrm{standard}} + \Delta\mathcal{P}_{\hat{\mathcal{R}}}$$

$$\Delta\mathcal{P}_{\hat{\mathcal{R}}} \approx \frac{17}{36}\frac{\lambda H^3_{\mathrm{inf}}}{\pi^2\epsilon_{\mathrm{inf}}\,M^2_{\mathrm{Pl}}m_0^2}\ln\left(\frac{\eta_e}{\eta_0}\right)$$

$$\lambda < 10^7\ \mathrm{s}^{-1}$$

A. Gundhi, JLGR, M. Carlesso, A. Bassi, Phys. Rev. Lett. **127**, 091302 (2021).

Correction to the Power Spectrum of $\hat{\mathcal{R}}$ during the Radiation Dominated Era

Extending the previous analysis to the radiation dominated era, the collapse operator now reads:

$$\hat{\mathcal{H}}^{I}_{\mathrm{DC}}(\eta, \boldsymbol{x}) = \frac{1}{2}\left(\dot{\hat{v}}^2(\eta, \boldsymbol{x}) + \frac{1}{3}\left(\nabla\, \hat{v}(\eta, \boldsymbol{x})\right)^2\right)$$

During this epoch, the leading order correction to the power spectrum $\mathcal{P}_{\hat{\mathcal{R}}}$ of $\hat{\mathcal{R}}$ is now given by

$$\Delta\mathcal{P}_{\hat{\mathcal{R}}} \approx \frac{9}{2}\frac{\lambda H_{\mathrm{inf}}^3 \eta_e^2}{\epsilon_{\mathrm{inf}}^3\, \pi^2\, M_{\mathrm{Pl}}^2 m_0^2 (\eta_r - 2\,\eta_e)^2} \ln\left(\frac{2\eta_e - \eta_r}{\eta_e}\right) \Rightarrow \lambda < 10^{64}\ \mathrm{s}^{-1}$$

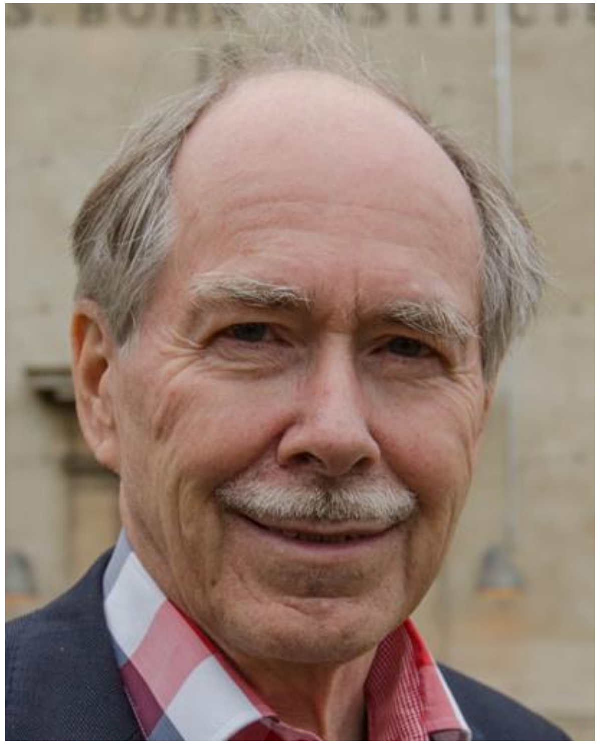
J. Martin and V. Vennin, Phys. Rev. Lett. **124**, 080402 (2020).

Summary and Conclusions

Inflation	Radiation dominated era
$\hat{\mathcal{H}}^{I}_{\mathrm{DC}}(\eta,\boldsymbol{x}) = \frac{1}{2}\left(\dot{\hat{v}}^2(\eta,\boldsymbol{x}) + \left(\nabla\,\hat{v}(\eta,\boldsymbol{x})\right)^2 - \frac{2}{\eta^2}\hat{v}^2(\eta,\boldsymbol{x})\right)$	$\hat{\mathcal{H}}^{I}_{\mathrm{DC}}(\eta,\boldsymbol{x}) = \frac{1}{2}\left(\dot{\hat{v}}^2(\eta,\boldsymbol{x}) + \frac{1}{3}\left(\nabla\,\hat{v}(\eta,\boldsymbol{x})\right)^2\right)$
$\hat{\mathcal{R}}^2 = \frac{1}{2\epsilon\, M_{\mathrm{Pl}}^2}\frac{\hat{v}^2(\eta,\boldsymbol{x})}{a^2(\eta)}$	$\hat{\mathcal{R}}^2 = \frac{1}{12 M_{\mathrm{Pl}}^2}\frac{\hat{v}^2(\eta,\boldsymbol{x})}{a^2(\eta)}$
$\Delta\mathcal{P}_{\hat{\mathcal{R}}} \approx \frac{17}{36}\frac{\lambda H_{\mathrm{inf}}^3}{\pi^2\epsilon_{\mathrm{inf}}\, M_{\mathrm{Pl}}^2 m_0^2}\ln\left(\frac{\eta_e}{\eta_0}\right)$	$\Delta\mathcal{P}_{\hat{\mathcal{R}}} \approx \frac{9}{2}\frac{\lambda H_{\mathrm{inf}}^3\eta_e^2}{\epsilon_{\mathrm{inf}}^3\,\pi^2\, M_{\mathrm{Pl}}^2 m_0^2(\eta_r - 2\,\eta_e)^2}\ln\left(\frac{2\eta_e - \eta_r}{\eta_e}\right)$
$\lambda < 10^7\ \mathrm{s}^{-1}$*	$\lambda < 10^{64}\ \mathrm{s}^{-1}$*

* Compared to $\lambda < 5.6 \times 10^{-90}\ \mathrm{s}^{-1}$ of previous works.

A. Gundhi, JLGR, M. Carlesso, A. Bassi, Phys. Rev. Lett. **127**, 091302 (2021).



Gerardus ‘t Hooft
(05/07/1946)

Nobel Prize in Physics (1999)
“for elucidating the quantum structure of electroweak interactions in physics”

Yoichiro Nambu
(18/01/1921-05/07/2015)

Nobel Prize in Physics (2008)
“for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics”

Thank you for your attention