Causal fermion systems
and collapse

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causal fermion systems can describe non-smooth spacetime structures; in particular, spacetimes involving fluctuating fields
Summary of basic features

- **causal fermion systems** can describe **non-smooth spacetime structures**; in particular, spacetimes involving **fluctuating fields**
- **causal action principle** describes **nonlinear dynamics**
causal fermion systems can describe non-smooth spacetime structures; in particular, spacetimes involving fluctuating fields
causal action principle describes nonlinear dynamics

Main message of this talk:

- gives rise to a relativistic effective collapse model,
- has similarities with CSL model.
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### Main message of this talk:

- Gives rise to a **relativistic effective collapse model**, and has similarities with **CSL model**.

### Specific features:

- **finite propagation speed**, i.e. no superluminal travelling
- **stochastic** and **nonlinear terms** break Lorentz invariance
A few basic structures
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- Complex Hilbert space \((\mathcal{H}, \langle . | . \rangle_{\mathcal{H}})\),
describes “occupied one-particle states” of the system
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  Spinor bundle: to every \(x \in M\) there is an associated complex vector space \(S_x M\) (spin space)
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- Every Hilbert space vector \(u \in \mathcal{H}\) is represented by a spinorial wave function (physical wave function)

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\psi^u \in \Gamma(M, SM) , \quad \psi^u(x) \in S_{xM} .
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- Basic object for what follows: wave evaluation operator

\[ \psi : \mathcal{H} \to \Gamma(M, SM) , \quad \psi u := \psi^u \]
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  \[\Psi : \mathcal{H} \to \Gamma(M, SM)\, , \quad \Psi u := \psi^u\]
- In simple terms, choosing an orthonormal basis \((e_i)\),
  \[\Psi = (\psi^{e_1}, \psi^{e_2}, \ldots)\]
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Describes the family of all one-particle wave functions of the system.
Example: Dirac spinors in Minkowski vacuum

- Spacetime $M$ is Minkowski space
- Physical wave functions satisfy the Dirac equation
  \[
  (i\gamma^j \partial_j - m) \psi = 0
  \]
- Convenient to combine the Dirac equations for all physical wave functions as
  \[
  (i\gamma^j \partial_j - m) \Psi = 0
  \]
  (recall that $\Psi = (\psi^{e_1}, \psi^{e_2}, \ldots)$, thus one equation for the whole family)
Example: Dirac spinors in Minkowski vacuum

- Physical picture: Dirac’s hole theory

\[ \omega \]

\[ \vec{k} \]

- Dirac sea
- Particles
- Anti-particles

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This Dirac sea configuration corresponds to a minimizer of the causal action principle.

Basic question: How do interacting systems in Minkowski space look like? (no gravity, . . .)

Proceed step by step. First example: External potential

\[(i \gamma^j \partial_j + e \gamma^j A_j - m) \psi = 0, \quad \Box A = 0.\]

All wave functions “feel” the same potential.
It turns out that the causal action principle gives rise to a plethora of fields.

The number $N$ of fields scales like

$$N \sim \frac{\ell_{\text{min}}}{\varepsilon}$$

with

$$\ell_{\text{Planck}} \ll \ell_{\text{min}} \ll \ell_{\text{macro}}$$

(and $\ell_{\text{Planck}}$ denotes the Planck scale)

Different wave functions “feel” different potentials.

The low-energy wave functions (i.e. $|\omega| \lesssim \ell_{\text{Planck}}^{-1}$) “feel all the potentials at the same time”.
Linearized fields in Minkowski space

- Coupling is made precise by Dirac equation

\[ (i \gamma^j \partial_j + \sum_{a=1}^N \gamma_j A^j_a E_a - m) \psi = 0 \]

with integral operators $E_a$, being nonlocal on the scale $\ell_{\text{min}}$.

- Recall that classical potentials give rise to gauge phases,

\[ \psi(x) \rightarrow e^{i\Lambda(x)} \psi(x). \]

Similarly, the plethora of fields gives rise to many phase factors

\[ \psi(x) \rightarrow \sum_{a=1}^N \sum_{\alpha=1}^L e^{i\Lambda_{a\alpha}(x)} \psi_{a\alpha}(x) \]

- leads to dephasing and decoherence effects
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Effective description in Fock spaces

- Is ongoing work with C. Dappiaggi, N. Kamran, M. Reintjes.
System can be described at any time \( t \) by a quantum state \( \omega^t : \mathcal{A} \rightarrow \mathbb{C} \), where \( \mathcal{A} \) is the algebra of observables.

The system can be represented on Fock space \( \mathcal{F} \) (fermionic and bosonic)

\[
\omega^t(A) = \text{Tr}_{\mathcal{F}} (\sigma^t A) \quad \text{if pure state} \quad \langle \psi | A | \psi \rangle
\]

In what follows consider pure state \( |\psi\rangle \in \mathcal{F} \).
Fock state has a **distinguished decomposition**

\[ |\Psi\rangle = \sum_{\alpha=1}^{L} |\Psi_\alpha\rangle \]

(related to above **dephasing and decoherence effects**)

**Dynamics** described by

\[ i\partial_t |\Psi\rangle = H |\Psi\rangle \]

with a Hamiltonian of the form

\[ H = H_{\text{Dirac}} + H_{\text{stochastic}} + H_{\text{nonlinear}} \]
The stochastic term

\[ H_{\text{stochastic}} = B \]

with \( \langle B_i \rangle = 0 \), \( \langle B_i(x) \, dB_j(y) \rangle = \delta_{ij} \, C(x, y) \)

and a covariance \( C(x, y) \).

- Not Markovian, but covariance decays on a small time scale (presumably \( \sim \ell_{\text{min}} \)?)
  similar to “colored noise”
- Strength of stochastic term determined by plethora of background fields \( A_a \).
The nonlinear term

\[ |\Psi> = \sum_{\alpha=1}^{L} |\Psi_{\alpha}> \quad \text{distinguished Fock decomposition} \]

\[ H_{\text{nonlinear}} \psi = \int_{\mathbb{R}^3} \sum_{\alpha=1}^{L} <\Psi_{\alpha}|A(x)\psi_{\alpha}>_{\mathcal{F}} \psi(x) \, d^3x \]

- position basis is distinguished
- only “diagonal” \( <\Psi_{\alpha}| \cdots |\Psi_{\alpha}>\)-terms come up;
- this leads to a “damping” of the nonlinear term by a scaling factor
  \[ \times \frac{1}{L} \]
- this is why nonlinear term becomes very small
Consider causal fermion systems in Minkowski space

Described by family of fermionic wave functions, encoded in wave evaluation operator $\Psi$

Causal action principle gives rise to plethora of fields

Coupling of these fields yields dephasing and decoherence effects.

Relativistic model: Dirac dynamics + corrections
But corrections are not relativistically covariant (depend on microstructure of spacetime).

Similar to CSL model, we obtain a stochastic and a nonlinear term; detailed scalings still need be worked out (ongoing work also with J. Kleiner and C. Paganini)
www.causal-fermion-system.com

Thank you for your attention!
The role of causality

\[ \psi(x) : \mathcal{H} \to S_x \]  
wave evaluation operator

\[ P(x, y) := \psi(x) \psi(y)^* : S_y \to S_x \]  
kernel of fermionic projector

\[ A_{xy} := P(x, y) P(y, x) : S_x \to S_x \]  
closed chain

The eigenvalues \( \lambda_{xy}^{j} \in \mathbb{C} \) of \( A_{xy} \) determine the causal structure:

**Definition (causal structure)**

The points \( x, y \in \mathcal{F} \) are called

- **spacelike** separated if \( |\lambda_{xy}^{j}| = |\lambda_{xy}^{k}| \) for all \( j, k = 1, \ldots, 2n \)
- **timelike** separated if \( \lambda_{xy}^{1}, \ldots, \lambda_{xy}^{2n} \) are all real and \( |\lambda_{xy}^{j}| \neq |\lambda_{xy}^{k}| \) for some \( j, k \)
- **lightlike** separated otherwise
For causal fermion systems in Minowski space, this causal structure coincides with the usual causal structure, up to corrections involving $\ell_{\text{Planck}}$.

This causal structure is relevant for transmitting information.
The role of causality

Plethora of fields $A_a$, $a = 1, \ldots, L$

- create non-local correlations
- generated decoherence effects and entanglement on macroscopic scale
- collapse phenomena also occur on macroscopic scale

But, as usual, all this cannot be used for signalling.
Quantum spacetimes

General question: How does an interacting measure look like?

- In more mathematical terms: What is the structure of minimizing measures?

![Diagram showing classical and quantum spacetime]

- A classical spacetime: \( M \) diffeomorphic to manifold
- A quantum spacetime: \( M \simeq M \times B \)

- F.F., “Perturbative Quantum Field Theory in the Framework of the Fermionic Projector”
Quantum spacetimes

Complicated non-smooth structure expected:

Should account for macroscopic superpositions and entanglement:
Basic questions:

- How can one make this picture precise?
- How can one understand the structure of the interacting measure?
- Goal: Express in the familiar language of quantum field theory.

Working this out leads to quantum states
General ideas for the construction of a quantum state

General setting:

- Two minimizing causal fermion systems
  - $\mathcal{H}, \mathcal{F}, \rho$ describing vacuum
  - $\tilde{\mathcal{H}}, \tilde{\mathcal{F}}, \tilde{\rho}$ describing the interacting spacetime
- Corresponding spacetimes:
  \[
  M := \text{supp } \rho, \quad \tilde{M} := \text{supp } \tilde{\rho}
  \]

Goal: Compare $\tilde{\rho}$ and $\rho$ at time $t$.

- Interacting spacetime can be arbitrarily complicated (interacting quantum fields, entanglement, collapse)
- Describe by objects in the vacuum spacetime: free fields, wave functions, ...
General ideas for the construction of a quantum state

- Basic object: **Nonlinear surface layer integral**
  
  - identify Hilbert spaces by choosing $V : \mathcal{H} \to \tilde{\mathcal{H}}$ unitary

$$
\gamma_{\tilde{\Omega},\Omega}(\tilde{\rho}, \rho) := \int_{\tilde{\Omega}} d\tilde{\rho}(x) \int_{\tilde{M} \setminus \Omega} d\rho(y) \mathcal{L}(x, y) \\
- \int_{\tilde{M} \setminus \tilde{\Omega}} d\tilde{\rho}(x) \int_{\Omega} d\rho(y) \mathcal{L}(x, y)
$$
identification of Hilbert spaces:

- Choose $V : \mathcal{H} \rightarrow \tilde{\mathcal{H}}$ unitary
- Work exclusively in $\mathcal{H}$
- But: identification is not canonical, gives freedom

\[ \rho \rightarrow \mathcal{U}\rho , \quad (\mathcal{U}\rho)(\Omega) := \rho(\mathcal{U}^{-1}\Omega\mathcal{U}) \]

This freedom can be treated by integrating over $\mathcal{U}$

- Let $G \subset U(\mathcal{H})$ be compact subgroup
- $\mu_G$ normalized Haar measure on $G$
The localized refined partition function

Better: Consider a double integral over the unitary group, because it contains more information on relative phases.

- localized refined partition function

\[ Z^t(\gamma, V, \tilde{\rho}) := \int_{G} d\mu_g(U_<) \int_{G} d\mu_g(U_>) \exp(\gamma \mathcal{T}^t_V(\tilde{\rho}, T_{U_<,U_>\rho})) \]

\[ \mathcal{T}^t_V(\tilde{\rho}, T_{U_<,U_>\rho}) \]

\[ := \left( \int_{\tilde{\Omega}^t \cap \tilde{V}} d\tilde{\rho}(x) \int_{\Omega^t \cap V} d\rho(y) + \int_{(\tilde{M} \setminus \tilde{\Omega}^t) \cap \tilde{V}} d\tilde{\rho}(x) \int_{(M \setminus \Omega^t) \cap V} d\rho(y) \right) \]

\[ \times |x_{U_>} y_{U_<}^{-1}|^2. \]

- \( V \) is a bounded spacetime region and
- \( \gamma \) is a free parameter (more later)
The localized refined partition function

\[ \mathcal{I}_V^t(\tilde{\rho}, T_{\mathcal{U}_<,\mathcal{U}_>}) := \left( \int_{\tilde{\Omega}^t \cap \tilde{V}} d\tilde{\rho}(x) \int_{\Omega^t \cap V} d\rho(y) + \int_{(\tilde{\mathcal{M}} \setminus \tilde{\Omega}^t) \cap \tilde{V}} d\tilde{\rho}(x) \int_{(\mathcal{M} \setminus \Omega^t) \cap V} d\rho(y) \right) \times |x \mathcal{U}_> y \mathcal{U}_<^{-1}|^2. \]

- **Localized**: Consider finite spatial volume and finite time (“laboratory”).

- **Take** infinite volume limit of surrounding region.
The localized refined state

Quantum state $\omega^t_V : \mathcal{A} \to \mathbb{C}$,

$$\omega^t_V(A) = \text{Tr} (\sigma^t A) = \langle \Psi | A | \Psi \rangle$$

- Define with insertions in the partition function, symbolically

$$\omega^t_V(\cdots) := \frac{1}{Z^t_V(\gamma, V, \tilde{\rho})} \int_\mathcal{G} d\mu_g(U<) \int_\mathcal{G} d\mu_g(U>) \times \exp\left(\gamma T^t_V(\tilde{\rho}, T_{U<,U>}\rho)\right) (\cdots)$$

- Compute group integrals asymptotically with saddle point methods.
Application to the quantum state

- One gets families of saddle points (phase freedom)
- Each saddle point can be analyzed explicitly in the asymptotics $\ell_{\text{Planck}} \searrow 0$ (ultraviolet regularization is taken out; formalism of the continuum limit)

$$\sigma^t = \sum_{a \in \mathcal{G}} c_a \left| \sum_{\alpha \in \mathcal{I}_a} \Psi_{a\alpha}^{\mathcal{F}} \right\rangle \left\langle \sum_{\beta \in \mathcal{I}_a} \Psi_{a\beta}^{\mathcal{F}} \right|$$

- In particular: state is positive, i.e. $\omega^t_V(A^*A) \geq 0$.
- Possible to describe macroscopic superpositions and general entangled states,

$$\omega^t(A) = \langle (\Psi_{\text{dead}} + \Psi_{\text{alive}}) | A | (\Psi_{\text{dead}} + \Psi_{\text{alive}}) \rangle^{\mathcal{F}}$$

Construction so far gives $\omega^t$ for all $t$

Next steps:

- Construct time evolution for the density operator
  \[ \mathcal{L}^t_{t_0} : \sigma^{t_0} \rightarrow \sigma^t \]

- Is there a unitary time evolution on the Fock space?
  \[ \omega^t = U^t_{t_0} \omega^{t_0} (U^t_{t_0})^{-1} \]

Answer: Yes. In this limiting case one gets QFT including loop diagrams, but with intrinsic regularization on scale $\ell_{\text{Planck}}$.

- There are nonlinear corrections. Connection to collapse phenomena?

Is ongoing work with C. Dappiaggi, N. Kamran, M. Reintjes and J. Kleiner.
Underlying physical principles

- **local gauge principle**: freedom to perform local unitary transformations of the spinors

- **Pauli exclusion principle**: Choose orthonormal basis $\psi_1, \ldots, \psi_f$ of $\mathcal{H}$. Set $\Psi = \psi_1 \wedge \cdots \wedge \psi_f$,

  gives equivalent description by Hartree-Fock state.

- **the “equivalence principle”**: symmetry under “diffeomorphisms” of $M$
  (note: $M$ merely is a topological measure space)

  Spacetime and causal structure are emergent
operators in $\mathcal{F}$ can be interpreted as “possible local correlation operators”
or simply as possible events
operators in $M$ are the events realized in spacetime
spacetime is made up of all the realized events
the physical equations relate the events to each other

For details on this connection:
F.F, J. Fröhlich, C. Paganini, C. and M. Oppio,
“Causal fermion systems and the ETH approach to quantum theory,”
www.causal-fermion-system.com

Thank you for your attention!