Causal fermion systems and collapse

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- gives rise to a relativistic effective collapse model,
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- has similarities with CSL model.

Specific features:

- finite propagation speed, i.e. no superluminal travelling
- stochastic and nonlinear terms break Lorentz invariance

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Describes the family of all one-particle wave functions of the system.

Example: Dirac spinors in Minkowski vacuum

- Spacetime M is Minkowski space
- Physical wave functions satisfy the Dirac equation

$$(i\gamma^j \partial_j - m)\, \psi = 0$$

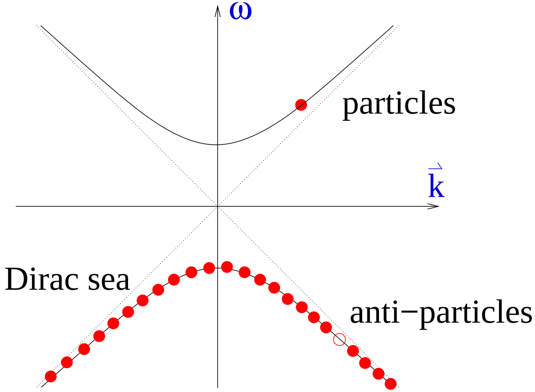
- Convenient to combine the Dirac equations for all physical wave functions as

$$(i\gamma^j \partial_j - m)\, \Psi = 0$$

(recall that $\Psi = \big(\psi^{e_1}, \psi^{e_2}, \cdots\big)$,
thus one equation for the whole family)

Example: Dirac spinors in Minkowski vacuum

- Physical picture: Dirac's hole theory



Example: Dirac spinors in Minkowski space

- This Dirac sea configuration corresponds to a minimizer of the causal action principle.
- Basic question: How do interacting systems in Minkowski space look like? (no gravity, ...)
- Proceed step by step. First example: External potential

$$(i\gamma^j \partial_j + e\gamma^j A_j - m)\, \Psi = 0 \, , \qquad \Box A = 0 \, .$$

All wave functions "feel" the same potential.

Linearized fields in Minkowski space

It turns out that the causal action principle gives rise to a

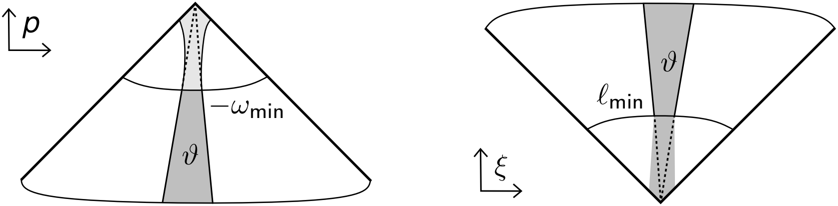
- plethora of fields
- The number N of fields scales like

$$N \simeq \frac{\ell_{\text{min}}}{\varepsilon} \qquad \text{with} \qquad \ell_{\text{Planck}} \ll \ell_{\text{min}} \ll \ell_{\text{macro}}$$

(and ℓ_{Planck} denotes the Planck scale)

- F.F., “Solving the linearized field equations of the causal action principle in Minkowski space,” arXiv:2304.00965 [math-ph]

Linearized fields in Minkowski space



$$\vartheta = \frac{1}{\sqrt{\ell_{\min}\,\omega_{\min}}} \qquad \text{with} \qquad \vartheta_{\min} = \sqrt{\frac{\ell_{\text{Planck}}}{\ell_{\min}}} \,.$$

- Different wave functions “feel” different potentials.
- The low-energy wave functions (i.e. $|\omega| \lesssim \ell_{\text{Planck}}^{-1}$) “feel all the potentials at the same time”.

Linearized fields in Minkowski space

- Coupling is made precise by Dirac equation

$$\Big(i\gamma^j \partial_j + \sum_{a=1}^{N} \gamma_j \, A^j_a \, E_a - m\Big) \Psi = 0$$

  with integral operators E_a, being nonlocal on the scale ℓ_{min}.
- Recall that classical potentials give rise to gauge phases,

$$\Psi(x) \to e^{i\Lambda(x)} \, \Psi(x) \,.$$

  Similarly, the plethora of fields gives rise to many phase factors

$$\Psi(x) \to \sum_{a=1}^{N} \sum_{\alpha=1}^{L} e^{i\Lambda_{a\alpha}(x)} \, \Psi_{a\alpha}(x)$$

- leads to dephasing and decoherence effects

Effective description in Fock spaces

- F.F. and Kamran, N., "Complex structures on jet spaces and bosonic Fock space dynamics for causal variational principles, arXiv:1808.03177 [math-ph], *Pure Appl. Math. Q.* **17** (2021) 55–140
- F.F. and Kamran, N., "Fermionic Fock spaces and quantum states for causal fermion systems," arXiv:2101.10793 [math-ph], *Ann. Henri Poincaré* **23** (2022) 1359–1398
- F.F., Kamran, N. and Reintjes, M., "Entangled quantum states of causal fermion systems and unitary group integrals," arXiv:2207.13157 [math-ph]
- Is ongoing work with C. Dappiaggi, N. Kamran, M. Reintjes.

Effective description in Fock spaces

- System can be described at any time t by a

$$\text{Quantum state} \qquad \omega^t : \mathscr{A} \to \mathbb{C}\,,$$

  where $\mathscr{A}$ is the algebra of observables.
- can be represented on Fock space $\mathcal{F}$ (fermionic and bosonic)

$$\omega^t(A) = \text{Tr}_{\mathcal{F}}\left(\sigma^t A\right) \overset{\text{if pure state}}{=} <\Psi|A|\Psi>$$

- In what follows consider pure state $|\Psi> \in \mathcal{F}$.

Effective description in Fock spaces

- Fock state has a distinguished decomposition

$$|\Psi\rangle = \sum_{\alpha=1}^{L} |\Psi_\alpha\rangle$$

(related to above dephasing and decoherence effects)

- Dynamics described by

$$i\partial_t |\Psi\rangle = H\, |\Psi\rangle$$

with a Hamiltonian of the form

$$H = H_{\text{Dirac}} + H_{\text{stochastic}} + H_{\text{nonlinear}}$$

The stochastic term

$$H_{\text{stochastic}} = \mathbf{B}$$

with $\quad \langle\!\langle B_i \rangle\!\rangle = 0\,, \qquad \langle\!\langle B_i(x)\, dB_j(y) \rangle\!\rangle = \delta_{ij}\, C(x,y)$

and a covariance $C(x,y)$.

- Not Markovian, but covariance decays on a small time scale (presumably $\sim \ell_{\text{min}}$?),
similar to "colored noise"
- Strengh of stochastic term determined by plethora of background fields A_a.

The nonlinear term

$$|\Psi\rangle = \sum_{\alpha=1}^{L} |\Psi_\alpha\rangle \qquad \text{distinguished Fock decomposition}$$

$$H_{\text{nonlinear}} \Psi = \int_{\mathbb{R}^3} \sum_{\alpha=1}^{L} \langle \Psi_\alpha | A(x)\, \Psi_\alpha \rangle_{\mathcal{F}}\, \Psi(x)\, d^3x$$

- position basis is distinguished
- only "diagonal" $\langle \Psi_\alpha | \cdots | \Psi_\alpha \rangle$-terms come up;
- this leads to a "damping" of the nonlinear term by a scaling factor

$$\times \frac{1}{L}$$

- this is why nonlinear term becomes very small

Summary

- Consider causal fermion systems in Minkowski space
- Described by family of fermionic wave functions, encoded in wave evaluation operator Ψ
- Causal action principle gives rise to plethora of fields
- Coupling of these fields yields dephasing and decoherence effects.
- Relativisic model: Dirac dynamics + corrections
  But corrections are not relativistically covariant (depend on microstructure of spacetime).
- Similar to CSL model, we obtain a stochastic and a nonlinear term; detailed scalings still need be worked out (ongoing work also with J. Kleiner and C. Paganini)

www.causal-fermion-system.com

Thank you for your attention!

The role of causality

$$\Psi(x) \;:\; \mathcal{H} \to S_x \qquad \text{wave evaluation operator}$$

$$P(x,y) := \Psi(x)\,\Psi(y)^* \;:\; S_y \to S_x \qquad \text{kernel of fermionic projector}$$

$$A_{xy} := P(x,y)\,P(y,x) \;:\; S_x \to S_x \qquad \text{closed chain}$$

The eigenvalues $\lambda_j^{xy} \in \mathbb{C}$ of A_{xy} determine the causal structure:

Definition (causal structure)

The points $x, y \in \mathcal{F}$ are called

- **spacelike** separated if $|\lambda_j^{xy}| = |\lambda_k^{xy}|$ for all $j, k = 1, \ldots, 2n$
- **timelike** separated if $\lambda_1^{xy}, \ldots, \lambda_{2n}^{xy}$ are all real and $|\lambda_j^{xy}| \neq |\lambda_k^{xy}|$ for some j, k
- **lightlike** separated otherwise

The role of causality

- For causal fermion systems in Minowski space, this causal structure coincides with the usual causal structure, up to corrections involving ℓ_{Planck}.
- This causal structure is relevant for transmitting information.

The role of causality

Plethora of fields A_a, $a = 1, \ldots, L$

- create non-local correlations
- generated decoherence effects and entanglement on macroscopic scale
- collapse phenomena also occur on macroscopic scale

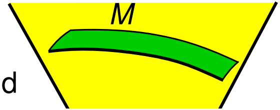
But, as usual, all this cannot be used for signalling.

Quantum spacetimes

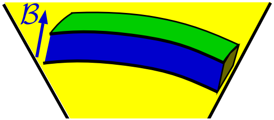
General question: How does an interacting measure look like?

- In more mathematical terms: What is the structure of minimizing measures?

a classical spacetime:
M diffeomorphic to manifold



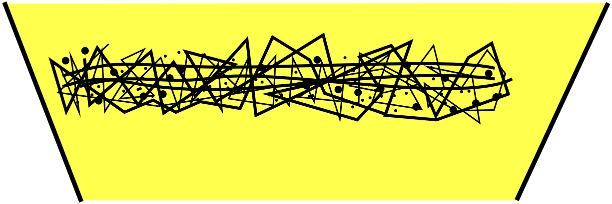
a quantum spacetime:
$M \simeq \mathscr{M} \times \mathcal{B}$



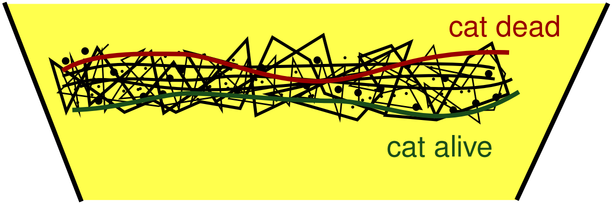
- F.F., "Perturbative Quantum Field Theory in the Framework of the Fermionic Projector"
arXiv:1310.4121 [math-ph], J. Math. Phys. **55** (2014) 042301

Quantum spacetimes

Complicated non-smooth structure expected:



Should account for macroscopic superpositions and entanglement:



General ideas for the construction of a quantum state

Basic questions:

- How can one make this picture precise?
- How can one understand the structure of the interacting measure?
- Goal: Express in the familiar language of quantum field theory.

Working this out leads to quantum states

General ideas for the construction of a quantum state

General setting:

- Two minimizing causal fermion systems
  - $(\mathcal{H}, \mathcal{F}, \rho)$ describing vacuum
  - $(\tilde{\mathcal{H}}, \tilde{\mathcal{F}}, \tilde{\rho})$ describing the interacting spacetime
  - corresponding spacetimes:

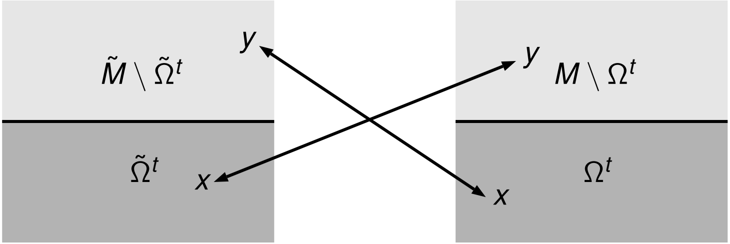
$$M := \text{supp}\ \rho\,, \quad \tilde{M} := \text{supp}\ \tilde{\rho}$$

- Goal: Compare $\tilde{\rho}$ and ρ at time t.
  - Interacting spacetime can be arbitrarily complicated (interacting quantum fields, entanglement, collapse)
  - describe by objects in the vacuum spacetime: free fields, wave functions, ...

General ideas for the construction of a quantum state

- Basic object: Nonlinear surface layer integral
  - identify Hilbert spaces by choosing $V : \mathcal{H} \to \tilde{\mathcal{H}}$ unitary

$$\gamma^{\tilde{\Omega},\Omega}(\tilde{\rho},\rho) := \int_{\tilde{\Omega}} d\tilde{\rho}(x) \int_{M\setminus\Omega} d\rho(y)\, \mathcal{L}(x,y) - \int_{\tilde{M}\setminus\tilde{\Omega}} d\tilde{\rho}(x) \int_{\Omega} d\rho(y)\, \mathcal{L}(x,y)$$



Freedom in identifying the Hilbert spaces

- identification of Hilbert spaces:
  - Choose $V : \mathcal{H} \to \tilde{\mathcal{H}}$ unitary
  - Work exclusively in $\mathcal{H}$
  - But: identification is not canonical, gives freedom

$$\rho \to \mathcal{U}\rho\,, \quad (\mathcal{U}\rho)(\Omega) := \rho(\mathcal{U}^{-1}\Omega\,\mathcal{U})$$

- This freedom can be treated by integrating over $\mathcal{U}$
  - Let $\mathcal{G} \subset \mathrm{U}(\mathcal{H})$ be compact subgroup
  - $\mu_{\mathcal{G}}$ normalized Haar measure on $\mathcal{G}$

The localized refined partition function

Better: Consider a double integral over the unitary group, because it contains more information on relative phases.

- localized refined partition function

$$Z^t(\gamma, V, \tilde{\rho}) := \fint_{\mathcal{G}} d\mu_{\mathcal{G}}(\mathcal{U}_<) \fint_{\mathcal{G}} d\mu_{\mathcal{G}}(\mathcal{U}_>) \exp\Big(\gamma \mathcal{T}^t_V(\tilde{\rho}, T_{\mathcal{U}_<,\mathcal{U}_>}\rho)\Big)$$

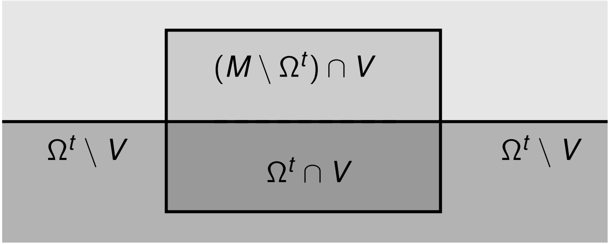
$$\begin{aligned} &\mathcal{T}^t_V(\tilde{\rho}, T_{\mathcal{U}_<,\mathcal{U}_>}\rho) \\ &:= \bigg(\int_{\tilde{\Omega}^t \cap \tilde{V}} d\tilde{\rho}(x) \int_{\Omega^t \cap V} d\rho(y) + \int_{(\tilde{M} \setminus \tilde{\Omega}^t) \cap \tilde{V}} d\tilde{\rho}(x) \int_{(M \setminus \Omega^t) \cap V} d\rho(y) \bigg) \\ &\qquad\qquad \times \big| x\, \mathcal{U}_>\, y\, \mathcal{U}_<^{-1} \big|^2 \,. \end{aligned}$$

- V is a bounded spacetime region and
- γ is a free parameter (more later)

The localized refined partition function

$$\mathcal{T}^t_V(\tilde{\rho}, T_{\mathcal{U}_<,\mathcal{U}_>}\rho) := \bigg(\int_{\tilde{\Omega}^t \cap \tilde{V}} d\tilde{\rho}(x) \int_{\Omega^t \cap V} d\rho(y) + \int_{(\tilde{M} \setminus \tilde{\Omega}^t) \cap \tilde{V}} d\tilde{\rho}(x) \int_{(M \setminus \Omega^t) \cap V} d\rho(y) \bigg) \times \big| x\, \mathcal{U}_>\, y\, \mathcal{U}_<^{-1} \big|^2 \,.$$

- Localized: Consider finite spatial volume and finite time ("laboratory").



- Take infinite volume limit of surrounding region.

The localized refined state

Quantum state $\omega^t_V : \mathscr{A} \to \mathbb{C}$,

$$\omega^t_V(A) = \mathrm{Tr}\left(\sigma^t A\right) = <\Psi|A|\Psi>$$

- Define with insertions in the partition function, symbolically

$$\omega^t_V(\cdots) := \frac{1}{Z^t_V(\gamma, V, \tilde{\rho})} \fint_{\mathcal{G}} d\mu_{\mathcal{G}}(\mathcal{U}_<) \fint_{\mathcal{G}} d\mu_{\mathcal{G}}(\mathcal{U}_>) \\ \times \exp\Big(\gamma \mathcal{T}^t_V\big(\tilde{\rho}, T_{\mathcal{U}_<,\mathcal{U}_>}\rho\big)\Big) \, (\cdots)$$

- Compute group integrals asymptotically with saddle point methods.

Application to the quantum state

- One gets families of saddle points (phase freedom)
- Each saddle point can be analyzed explicitly in the asymptotics $\ell_{\text{Planck}} \searrow 0$ (ultraviolet regularization is taken out; formalism of the continuum limit)

$$\sigma^t = \sum_{a \in \mathfrak{S}} c_a \,\Big| \sum_{\alpha \in \mathfrak{T}_a} \Psi^{\mathcal{F}}_{a\alpha} \Big\rangle \Big\langle \sum_{\beta \in \mathfrak{T}_a} \Psi^{\mathcal{F}}_{a\beta} \Big|$$

- In particular: state is positive, i.e. $\omega^t_V(A^* A) \geq 0$.
- Possible to describe macroscopic superpositions and general entangled states,

$$\omega^t(A) = \langle (\Psi_{\text{dead}} + \Psi_{\text{alive}}) \,|\, A \,|\, (\Psi_{\text{dead}} + \Psi_{\text{alive}}) \rangle_{\mathcal{F}}$$

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Outlook: Dynamics of the quantum state

- Construction so far gives ω^t for all t
- Next steps:
  - Construct time evolution for the density operator

$$\mathfrak{L}^t_{t_0} \;:\; \sigma^{t_0} \rightarrow \sigma^t$$

  - Is there a unitary time evolution on the Fock space?

$$\omega^t = U^t_{t_0}\, \omega^{t_0}\, \big(U^t_{t_0}\big)^{-1}$$

  - Answer: Yes. In this limiting case one gets QFT including loop diagrams, but with intrinsic regularization on scale ℓ_{Planck}.
  - There are nonlinear corrections. Connection to collapse phenomena?
- Is ongoing work with C. Dappiaggi, N. Kamran, M. Reintjes and J. Kleiner.

Underlying physical principles

- local gauge principle:
  freedom to perform local unitary transformations of the spinors
- Pauli exclusion principle:
  Choose orthonormal basis $\psi_1, \ldots, \psi_f$ of $\mathcal{H}$. Set

  $$\Psi = \psi_1 \wedge \cdots \wedge \psi_f \, ,$$

  gives equivalent description by Hartree-Fock state.
- the "equivalence principle":
  symmetry under "diffeomorphisms" of M
  (note: M merely is a topological measure space)

Spacetime and causal structure are emergent

Interpretation in terms of spacetime events

- operators in $\mathcal{F}$ can be interpreted as "possible local correlation operators" or simply as possible events
- operators in M are the events realized in spacetime
- spacetime is made up of all the realized events
- the physical equations relate the events to each other

For details on this connection:

- F.F, J. Fröhlich, C. Paganini, C. and M. Oppio, "Causal fermion systems and the ETH approach to quantum theory," arXiv:2004.11785 [math-ph] (2020)

www.causal-fermion-system.com

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