FONDAZIONE BRUNO KESSLER

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ECT*

A fine measurement of the Pauli Exclusion Principle with VIP-2

4 July 2023

Alessio Porcelli

COLMO

Quantum collapse models investigated with particle, nuclear, atomic and macro systems workshop

Why Pauli Exclusion Principle?

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WE DON'T KNOW WHY

Fermi-Dirac and Bose-Einstein are distinct

Beyond Standard Model…

Reasons of Pauli's Exclusion Principle (PEP)

- **Particle nature? Green's general quantum field:** paronic particles
  - Order 1: fermionic/bosonic fields
  - Order>1: parafermionic/parabosonic fileds
  - **Messiah-Greenberg Super-Selection**: no fermion/boson decays into parafermion/paraboson (and vice-versa)
  - **Paronic**: a mixture of fermionic/bosonic and parefermionic/parabosonic states

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- **Interactions result? Non-Commutative Quantum Gravity**
  - **θ-Poincaré**: distortion of Lorentz symmetry (visible in a two identical particles system)

$$\left|\alpha,\alpha\right\rangle = \left\langle a^\dagger,\alpha\right\rangle \left\langle a^\dagger,\alpha\right\rangle \left|0\right\rangle = \int \frac{d^d p_1}{2p_{10}} \frac{d^d p_2}{2p_{20}}\, e^{-\frac{i}{2} p_{1\mu}\theta^{\mu\nu} p_{2\nu}} \alpha(p_1) c^\dagger(p_1) \alpha(p_2) c^\dagger(p_2)$$

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Time (θ_{00})

Space distortion (θ_{ji})

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Time: θ_{00}

Space-Time mix distortion: $(\theta_{j0} = \theta_{0i})$

Space distortion: θ_{ji}

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Time

Space-Time mix distortion ($\theta_{j0} = \theta_{0i}$)

Space distortion

Magnetic Scenario: $\theta_{0i} = 0$
only space-sector distortions

Electric Scenario: $\theta_{0i} \neq 0$
also space-time mixing

Signature

Anti-/symmetric commutativity with a coefficient β

$$a^\dagger \left|0\right\rangle = \left|1\right\rangle \quad a^\dagger \left|1\right\rangle = \beta \left|2\right\rangle \quad a^\dagger \left|2\right\rangle = 0$$

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In a system of two fermions (i.e., two electrons),
PEP is violated with an amplitude probability of $\boldsymbol{\beta^2/2}$

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…but for NCQC and Quon algebra connection we use $\delta^2 = \beta^2/2$ instead:

$$a_i a_j^\dagger - q(E) a_j^\dagger a_i = \delta_{ij}$$

with $q(E) = 2\delta(E)^2 - 1$

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(different for the two θ_{0i} scenarios)

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[Further details in Fabrizio Napolitano's Talk]

How about so far?

- **Ramberg and Snow (1988):** $\beta^2/2 \lesssim 10^{-26}$
- **DAMA (2009):** $\beta^2/2 \lesssim 10^{-47}$
- **Borexino (2011):** $\beta^2/2 \lesssim 10^{-60}$

Models scenarios implications

❑ Democratic scenario

all type of particles have the same degree of violation β

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Models scenarios implications

✗ **Democratic scenario**

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☐ **Despotic scenario**

each type of particle has its degree of violation β_i

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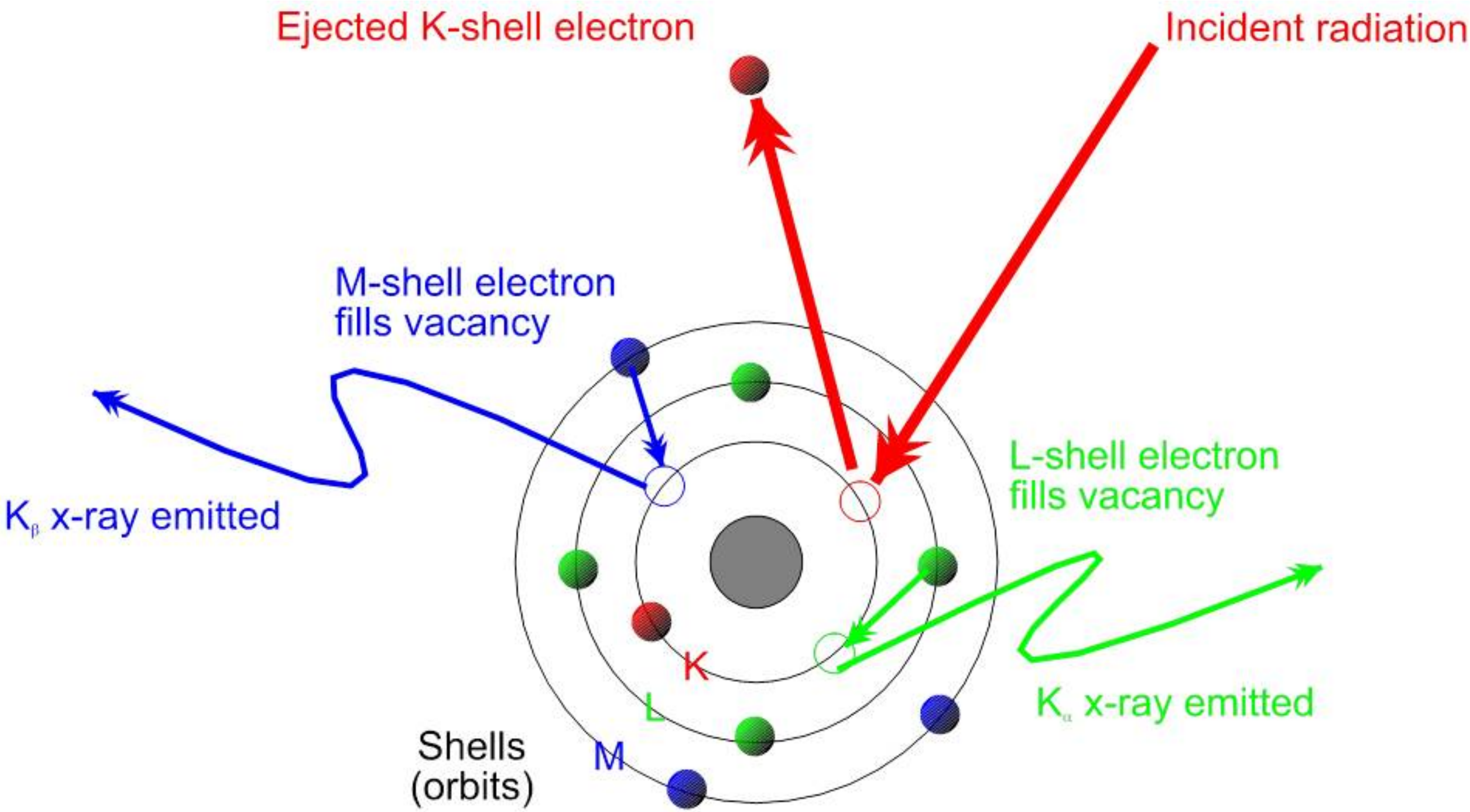
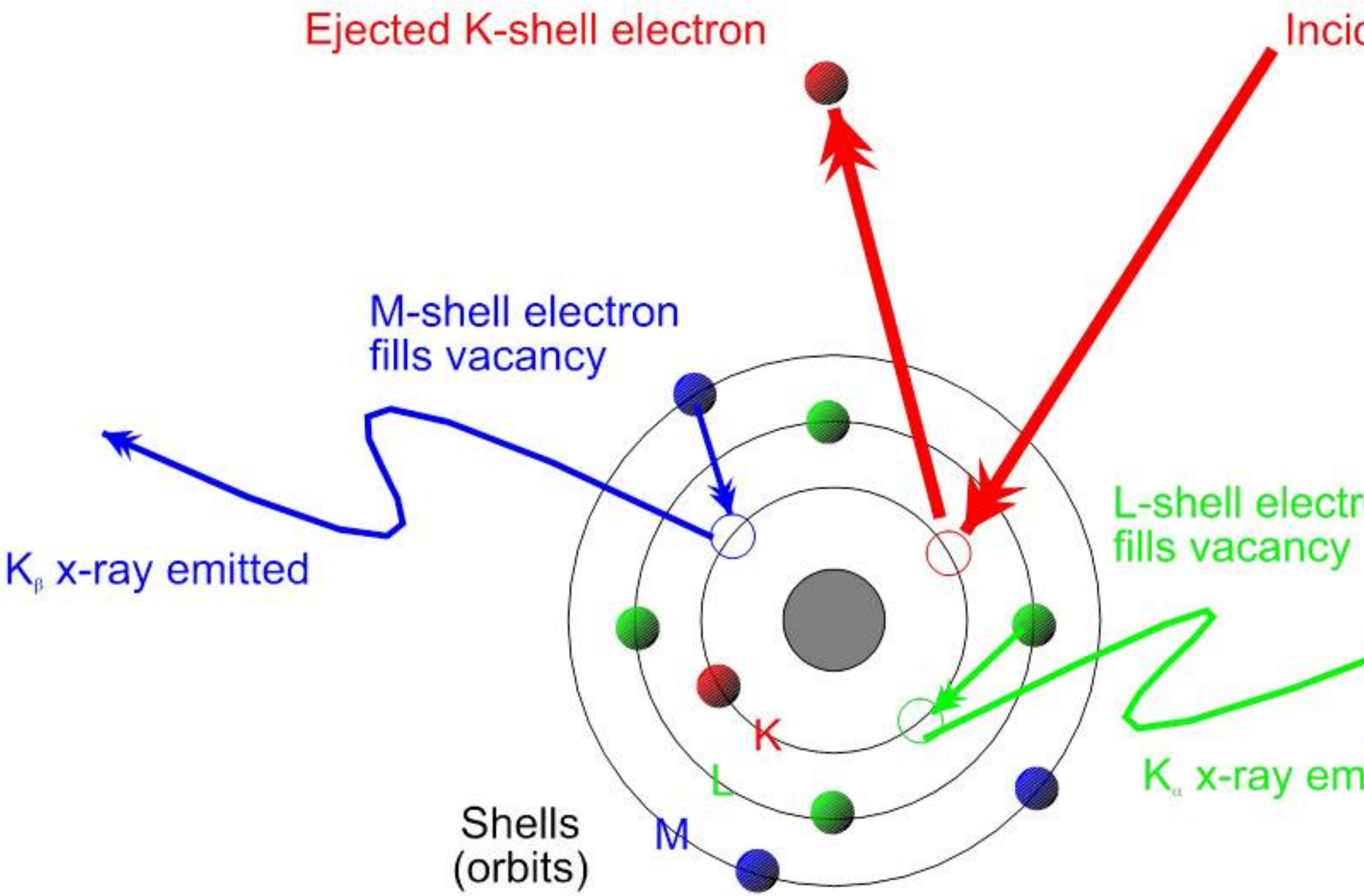
X-Rays

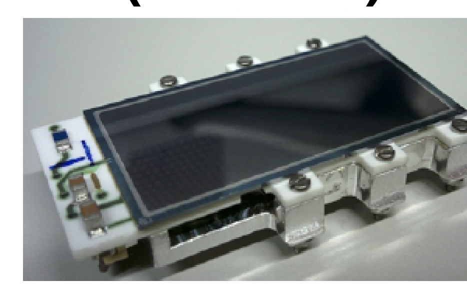


Ejected K-shell electron
Incident radiation
M-shell electron
fills vacancy
K_β x-ray emitted
L-shell electron
fills vacancy
K_α x-ray emitted
K
L
M
Shells
(orbits)

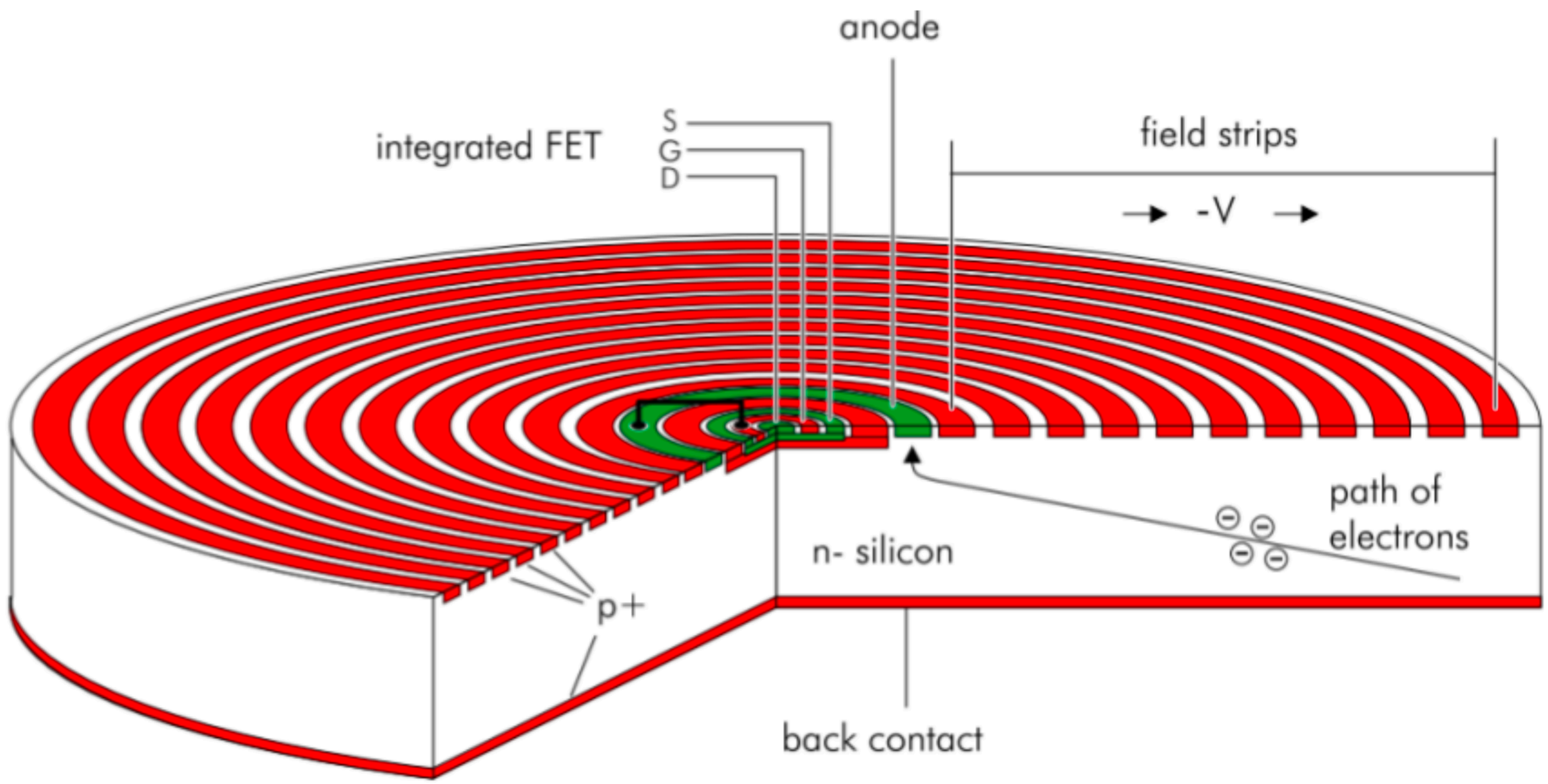
X-Rays



Silicon
Drift
Detector
(**SDD**)



SDDs



- Based on sideward depletion
- Charge particle or photon hits the silicon wafer
  - electron-hole pairs are generated
  - free electrons move to the anode following the lower potential due to the concentric electrodes
- The amount of charge collected by the anode is proportional to the energy of the radiation (X-Rays range)

VIP group

VIolation of **P**auli Exclusion Principle

Open Systems
testing newly injected electrons

- **VIP**
  - **VIP-2**
    - **VIP-3**
- **GATOR**

Close System
testing spontaneous emissions

- **VIP-Lead**
- **BEGe**

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[see Fabrizio Napolitano's]

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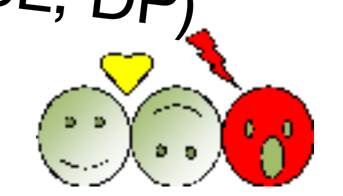
[see Fabrizio Napolitano's]

+Wave Function Collapse (CSL, DP)

[see Kristian Piscicchia's talks]



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[THIS WORK] VIP-2

[Future] **VIP-3** **GATOR**

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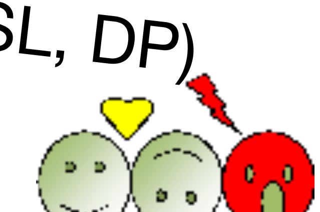
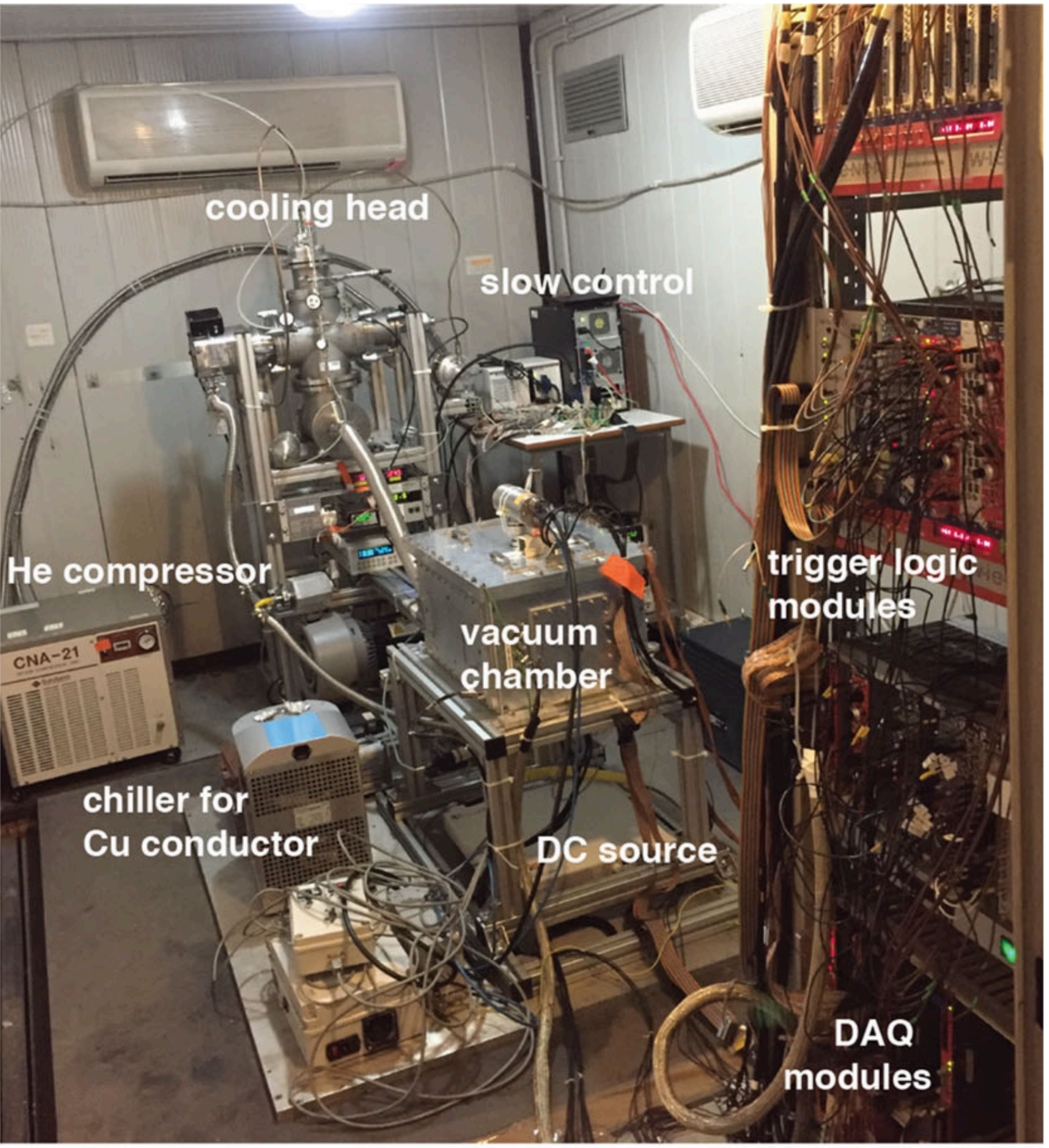
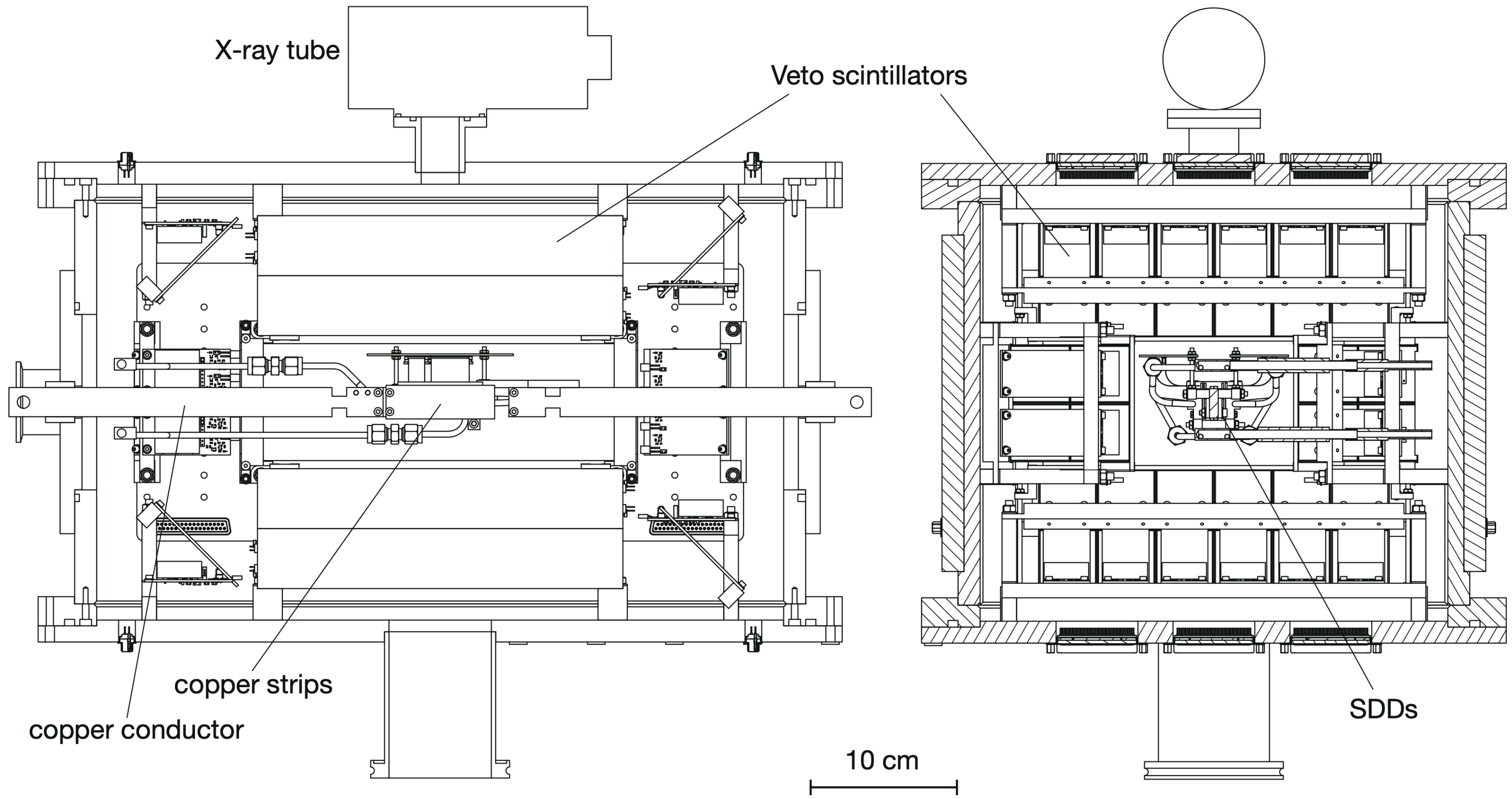


VIP-2

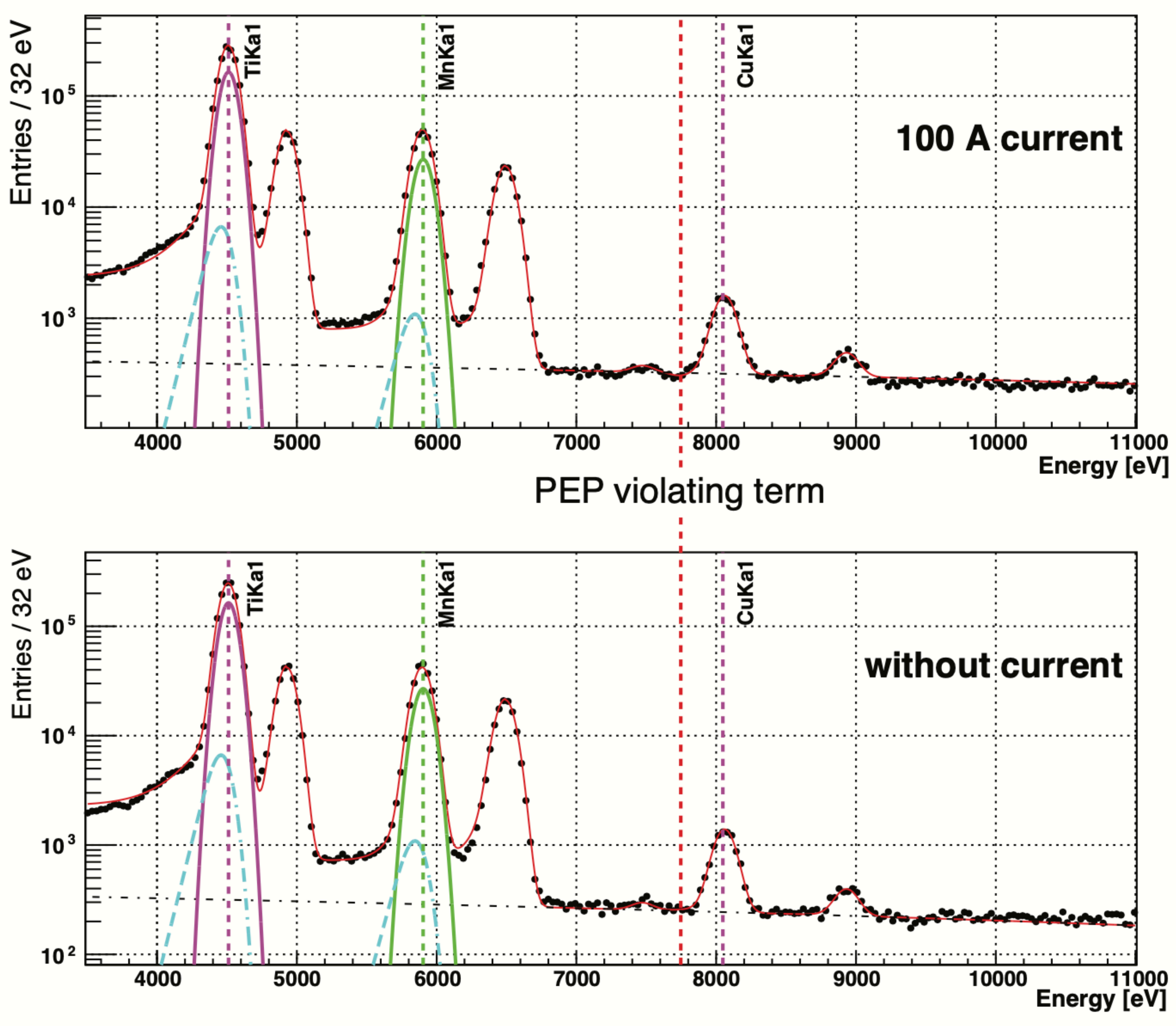




- **Target**: Copper strips
  - WITHOUT CURRENT configuration: regime case (stable states: background)
  - WITH CURRENT configuration (180 A): dynamic case (PEP violation through electron capture)
- **SDD**: 32 detectors by SDDs, stably kept @ -170^{+1}_{-0} °C even with the current in Cu
- **@LNGS Underground** (beneath Gran Sasso Mountain – IT): ~1400 m of rock shielding

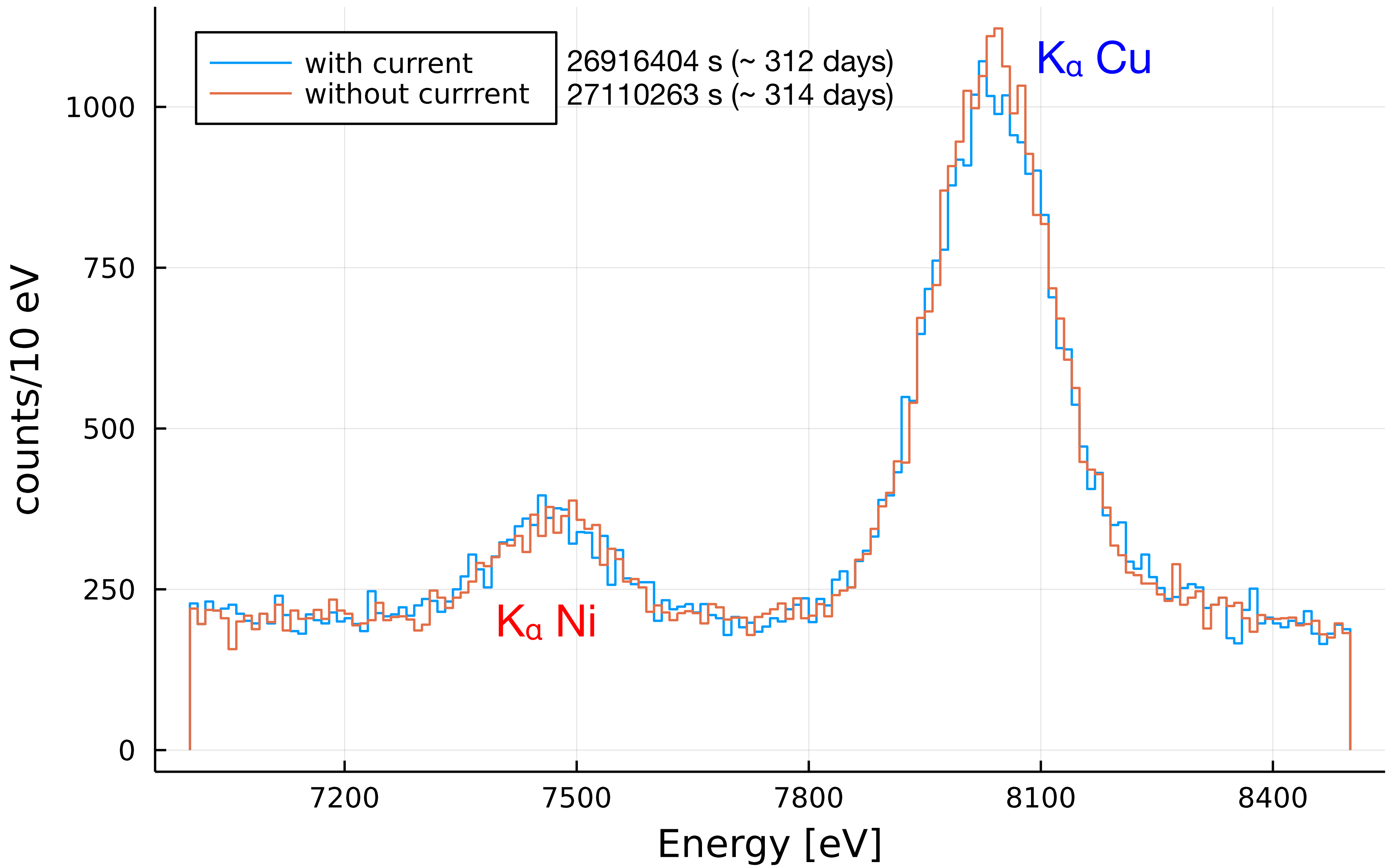
Calibration

Fe-55 source, with a 25 µm thick Titanium foil



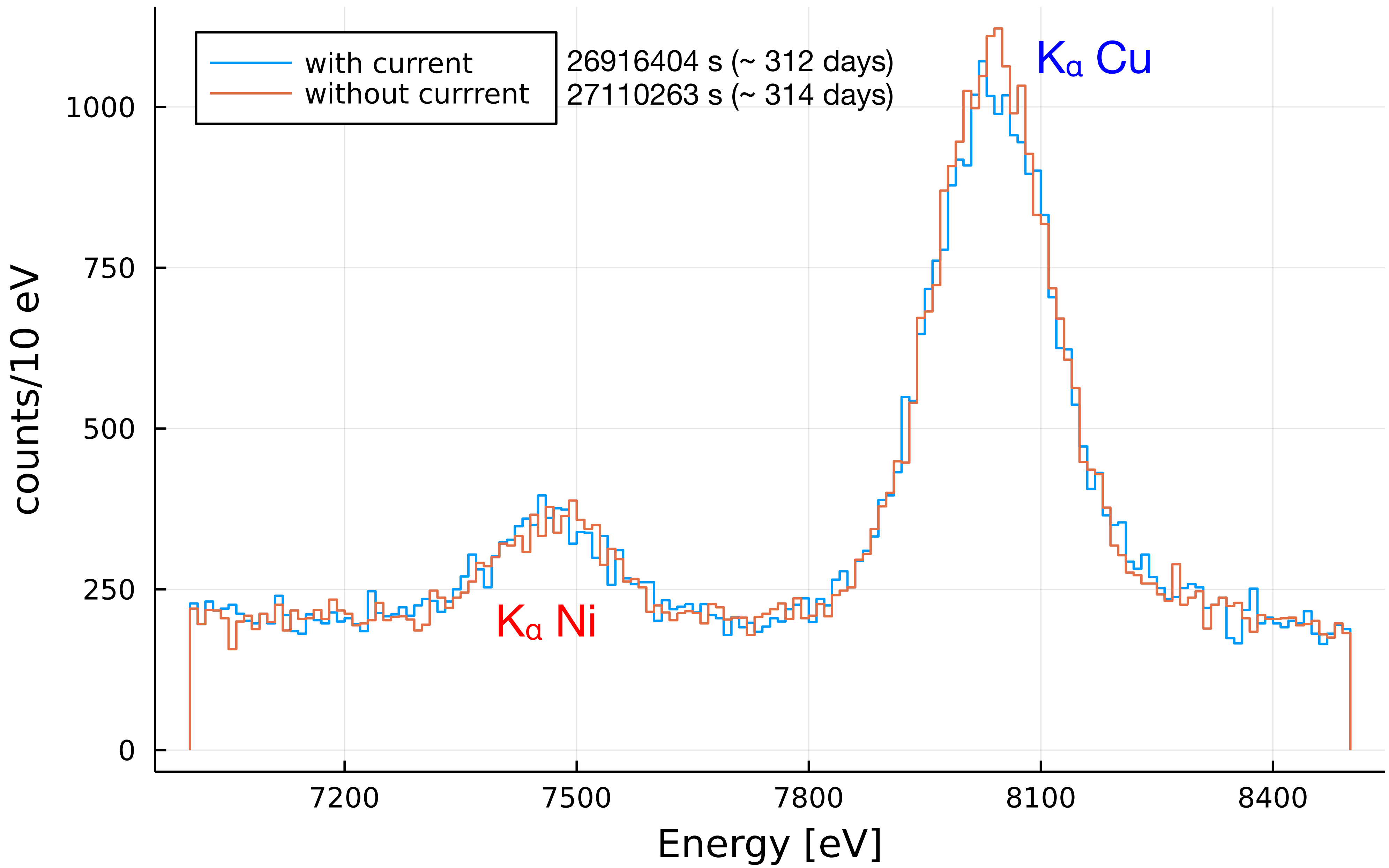
[hyperbolic calibration]

Data model


with current
without currrent
26916404 s (~ 312 days)
27110263 s (~ 314 days)
Kα Cu
Kα Ni
counts/10 eV
0
250
500
750
1000
7200
7500
7800
8100
8400
Energy [eV]

Data model

$$\mathscr{F}^{woc}(\boldsymbol{\theta}, \boldsymbol{y}) = y_1 \times Ni(\theta_1, \theta_2) + y_2 \times Cu(\theta_3, \theta_4) + y_3 \times \mathrm{pol}_1(\theta_5)$$



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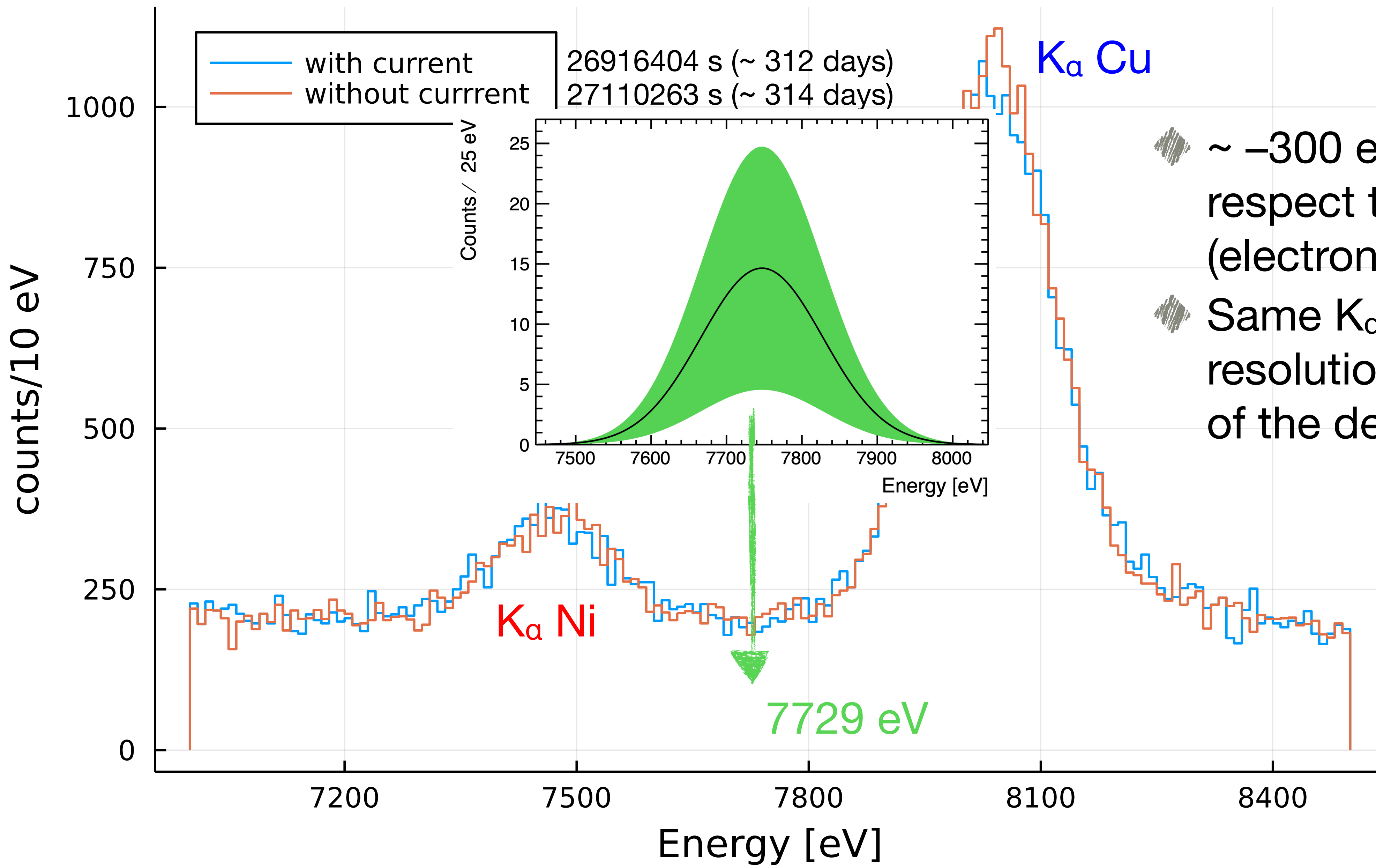

- ~ –300 eV with respect to Kα Cu (electron shielding)
- Same Kα Cu resolution (i.e., σ of the detector)

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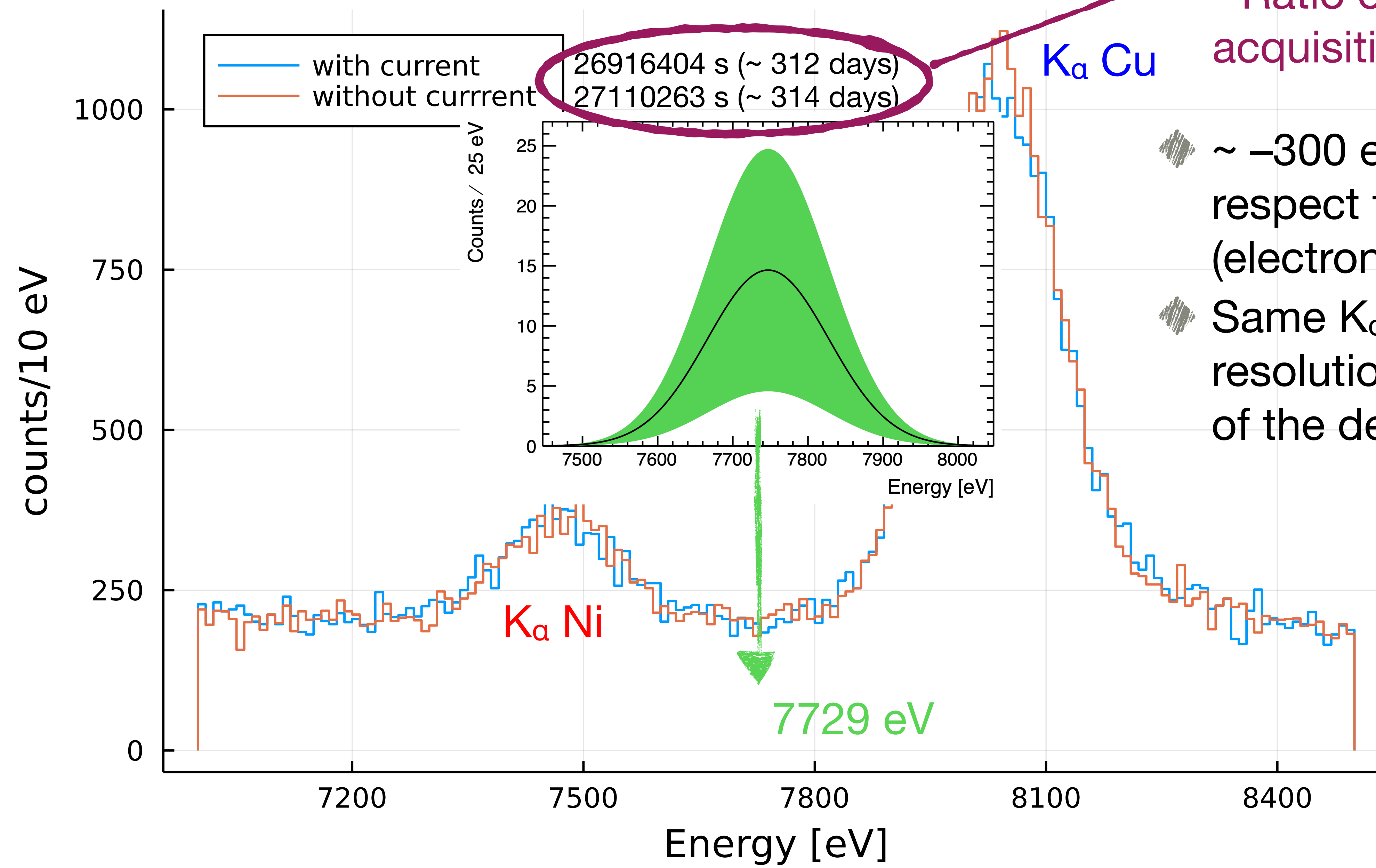
$$\mathcal{L}(\mathcal{D}^{wc}, \mathcal{D}^{woc} \,|\, \boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S}) = \text{Poiss}(\mathcal{D}^{wc} \,|\, \mathcal{F}^{wc}(\boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S})) \times \text{Poiss}(\mathcal{D}^{woc} \,|\, \mathcal{F}^{woc}(\boldsymbol{\theta}, \boldsymbol{y} \times \mathcal{R}))$$

Ratio of data acquisition time



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Bayesian approach

$$p(\boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S} \,|\, \mathcal{D}^{wc}, \mathcal{D}^{woc}) = \frac{\mathcal{L}(\mathcal{D}^{wc}, \mathcal{D}^{woc} \,|\, \boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S})\, p(\boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S})}{\int d\boldsymbol{\theta} d\boldsymbol{y} \mathcal{L}(\mathcal{D}^{wc}, \mathcal{D}^{woc} \,|\, \boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S})\, p(\boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S})}$$

$$p(\mathcal{S} \,|\, \mathcal{D}^{wc}, \mathcal{D}^{woc}) = \int p(\boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S} \,|\, \mathcal{D}^{wc}, \mathcal{D}^{woc})\, d\boldsymbol{\theta} d\boldsymbol{y}$$

- Priors of $\boldsymbol{\theta}$ and $\boldsymbol{y}$ are Gaussians: statistical fluctuations around known values
- Prior of $\mathcal{S}$ is flat, limited from previous experiments
- Systematic uncertainties included

Bayesian approach

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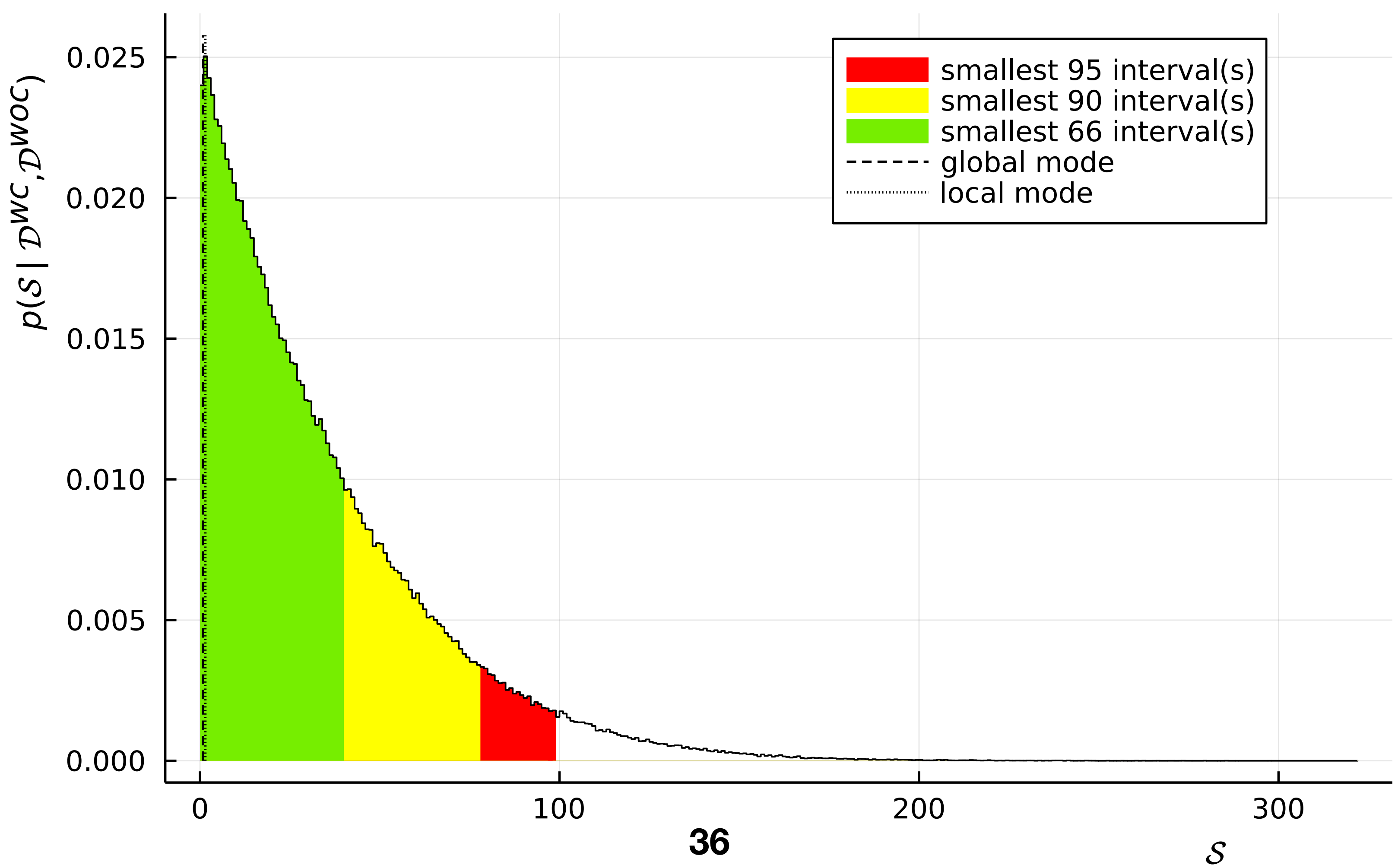
Integrals with
Markov Chain Monte Carlo method



Modified frequentist: CL_s

$$\mathcal{L}(\boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S}) = \mathcal{L}(\mathcal{D}^{wc}, \mathcal{D}^{woc} | \boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S}) p(\boldsymbol{\theta}, \boldsymbol{y}, \mathcal{S})$$

$$t_{\mathcal{S}} = -2 \ln \Lambda(\mathcal{S}) = -2 \ln \frac{\mathcal{L}(\hat{\hat{\boldsymbol{\theta}}}, \hat{\hat{\boldsymbol{y}}}, \mathcal{S})}{\mathcal{L}(\hat{\boldsymbol{\theta}}, \hat{\boldsymbol{y}}, \hat{\mathcal{S}})} \qquad p_{\mathcal{S}} = \int_{t_{obs}}^{\infty} f(t_{\mathcal{S}} | \mathcal{S}) dt_{\mathcal{S}} \qquad \text{CL}_\text{s} = \frac{p_{\mathcal{S}}}{1 - p_0} < 1 - \text{C.L.}$$

one-sided Likelihood Test statistic

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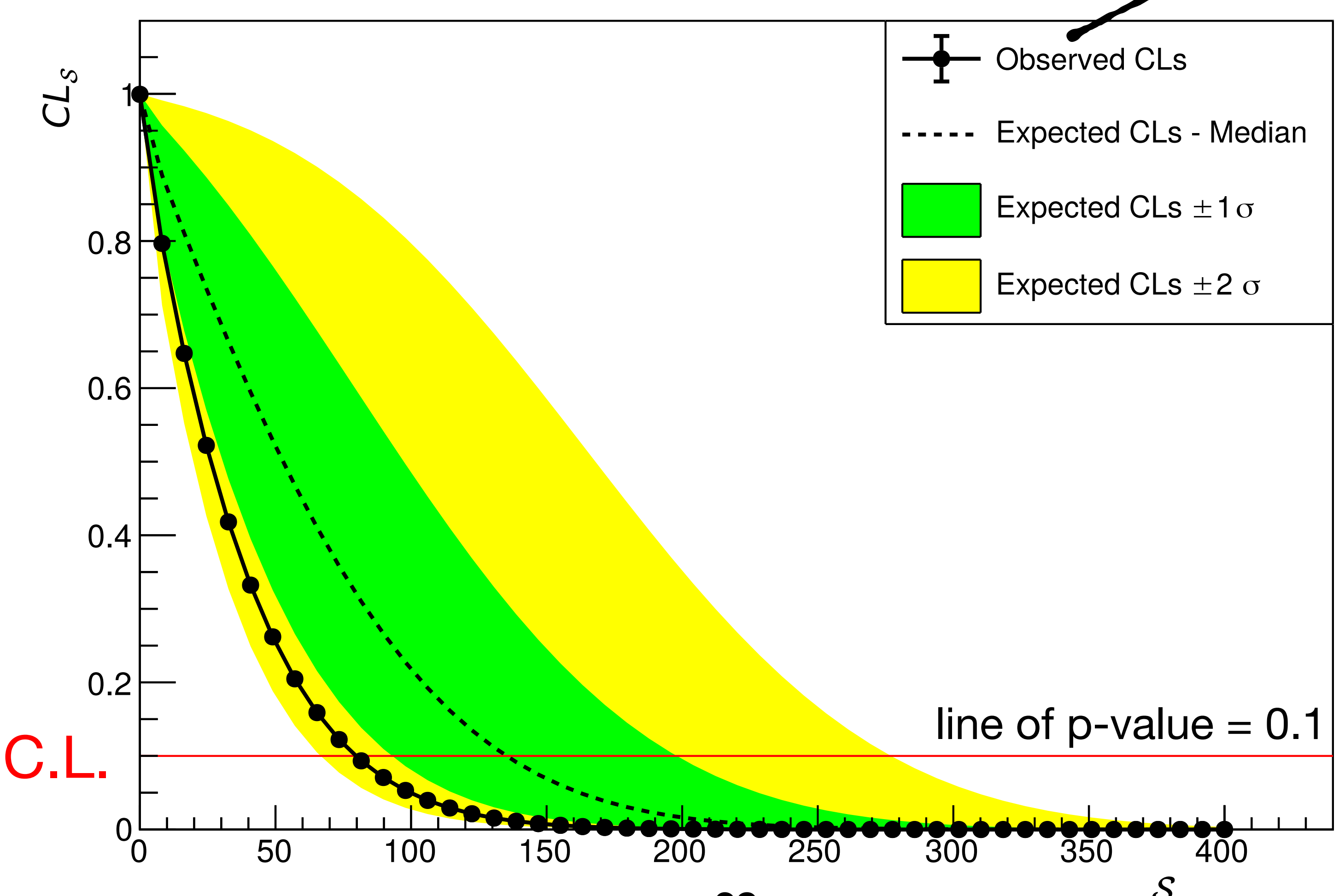
one-sided Likelihood Test statistic

Computation with RooFit

CLs expected with measured $\mathscr{S}$



90% of C.L.

CLs expected in case of $\mathscr{S} = 0$ but measured $\mathscr{S}$

Modified frequentist: $\mathrm{CL_S}$

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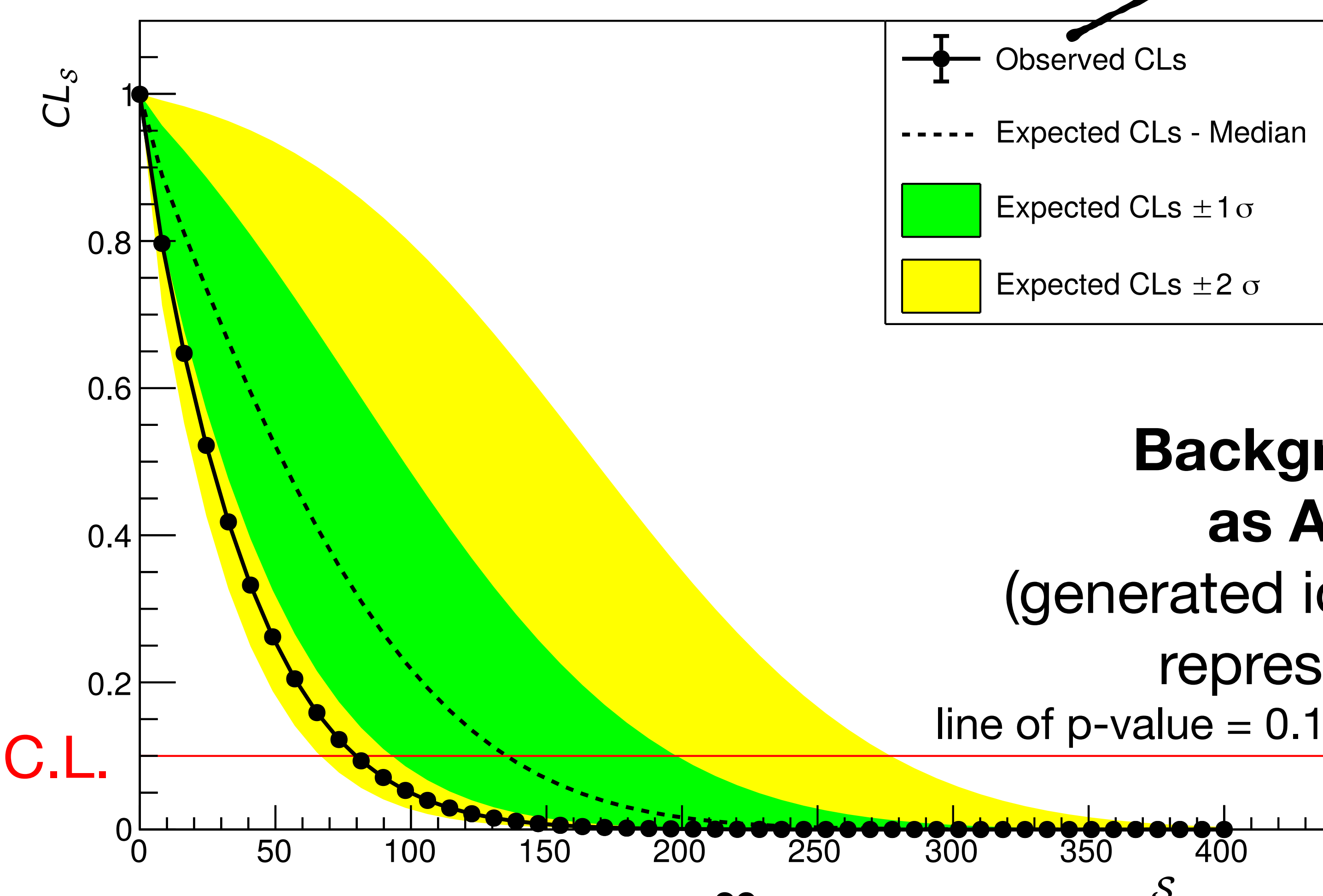
Background Hypothesis as Asimov Dataset

(generated ideal dataset most likely representing the model)



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From $\mathcal{S}$ to $\beta^2/2$

$$\mathcal{S} \simeq \frac{\beta^2}{2} \cdot N_{\text{new}} \cdot \frac{N_{\text{int}}}{10} \cdot 7.25 \times 10^{-2}$$

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efficiency simulated: considered X-ray absorption + geometry acceptance + SDDs efficiency

Number of interactions; every ~10 interactions, 1 cascade

Newly injected electrons!

$$\sum_{i}^{\text{runs}} I_i \Delta t_i / e \; (= I\Delta t/e \text{ for simplicity})$$

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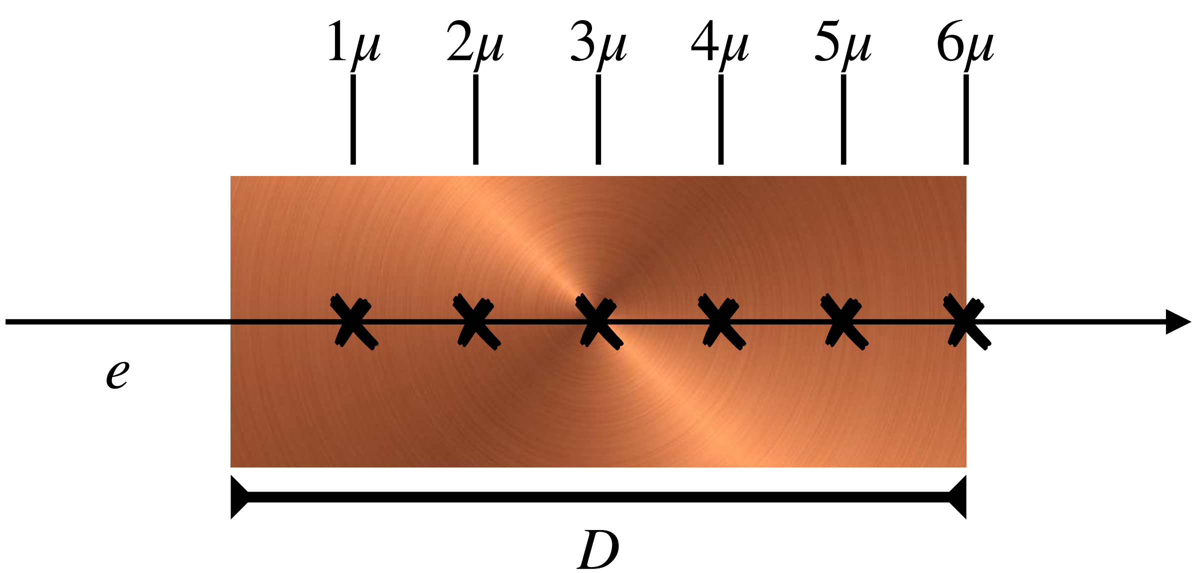
N_{int} is the normalization that decides the order of magnitude of $\beta^2/2$

Let’s discuss e–atoms interaction Models!

N_{int}

Linear Scattering

Through Copper Resistance, we know the average interaction length μ



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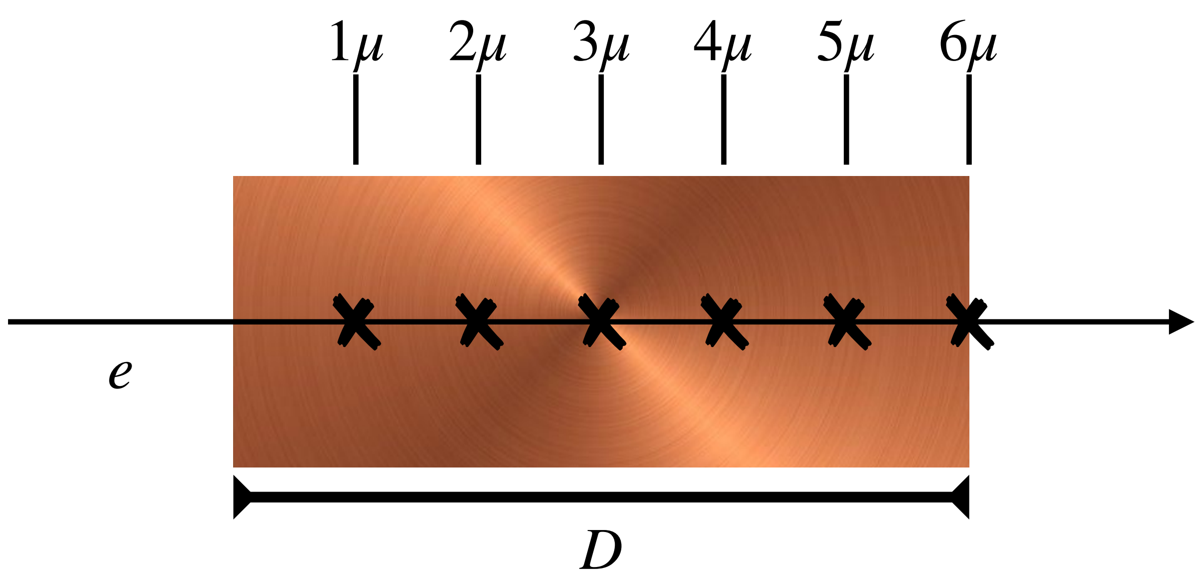

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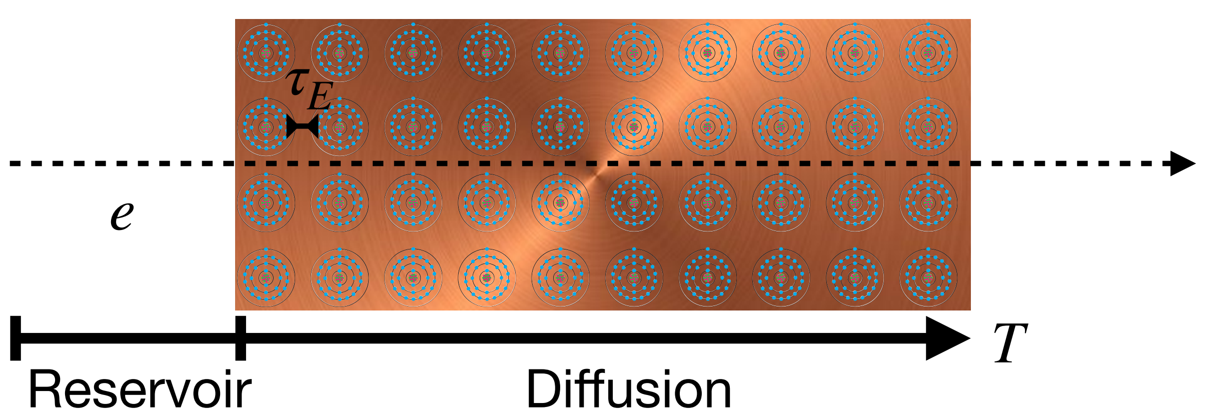
Close Encounters

Through Diffusion-Transport theory and Copper atomic density:

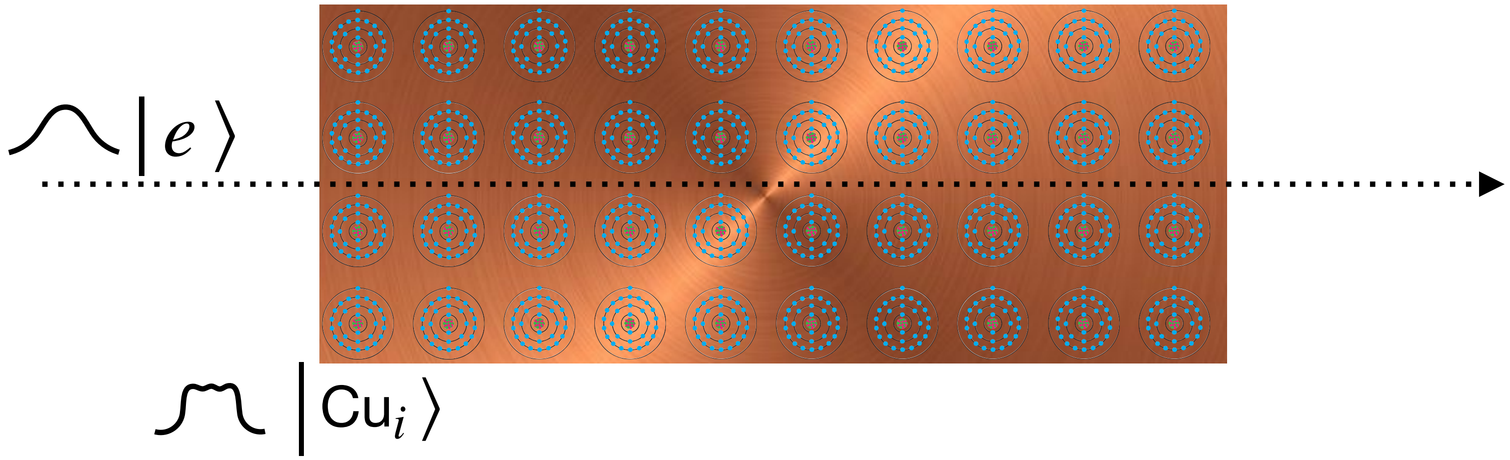
- the average time τ_E on atomic encounter for a diffused electron
- the average time T of target crossing by an electron



$$N_{\text{int}} = T/\tau_E \simeq 4.29 \times 10^{17}$$

$$\Rightarrow \frac{\beta^2}{2} \lessapprox 10^{-43}$$

TO DO: a quantum N_{int}?



How many interactions between Cu atomic and electron fields occur?

Outlook

How to finely measure a Pauli Exclusion Principle Violation?

VIP (past), **VIP-2 (current)**, and VIP-3 and GATOR (future)

- **X-Rays emissions:**
  - **SDDs** well versed for this measurement
  - Accurate **calibration**
- **Rigorous Data Analysis**
  - **Bayesian** Approach: well established and reliable
  - Modified Frequenstist **CL_s**: optimal test (sensible to small parameters fluctuations)
  - …use other observables (i.e., Time) to further refine the measurement
- **Electron-atoms interaction modelling (N_{int})**
  - **Linear Scattering**: due to phonons and lattice irregularities
    - ✓ Safest hypothesis
    - ✗ Largely underestimation of how many interactions an electron does
  - **Close Encounters**: a more realistic model of e-atom encounters, but still approximated
    - ✓ 12 order of magnitudes larger than Linear Scattering!
  - **This is the key element to improve the measurement!**

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THANK YOU