Gravity as a classical channel and its dissipative generalization

Giovanni Di Bartolomeo
dibartolomeo.giov@gmail.com

University of Trieste

COLMO workshop 2023

ECT* Trento

Two points of view on gravity

1. **Gravity must be quantized**

   Gravitational interaction should be treated quantum mechanically.

2. **Quantum mechanics must be "gravitized"**

   Gravitational interaction should be fundamentally classical and quantum dynamics should accommodate for it.

Two points of view on gravity

1. **Gravity must be quantized**

   Gravitational interaction should be treated quantum mechanically.

2. **Quantum mechanics must be "gravitized"**

   Gravitational interaction should be fundamentally classical and quantum dynamics should accommodate for it.

Is it possible to construct a semi-classical theory of gravity?

Starting Hypothesis:

- **Classical gravity at a fundamental level**

  - It does not produce quantum states superposition
  - It does not produce entanglement

- **Classical gravitational interaction between quantum systems**

  - Wave function collapse due to gravitational interaction

Is it possible to construct a semi-classical theory of gravity?

Starting Hypothesis:

- **Classical gravity at a fundamental level**

  - It does not produce quantum states superposition
  - It does not produce entanglement

- Classical gravitational interaction between quantum systems

  - Wave function collapse due to gravitational interaction

Is it possible to construct a semi-classical theory of gravity?

Starting Hypothesis:

- **Classical gravity at a fundamental level**

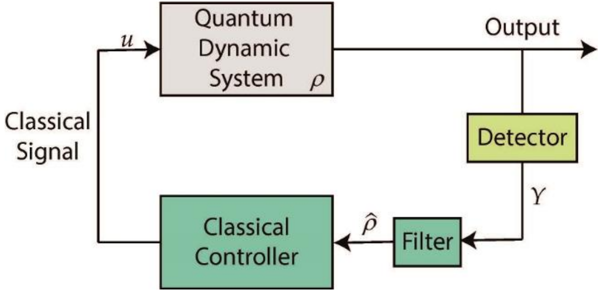
  - It does not produce quantum states superposition
  - It does not produce entanglement

- **Classical gravitational interaction between quantum systems**

  - Wave function collapse due to gravitational interaction

Continuous quantum measurement and feedback

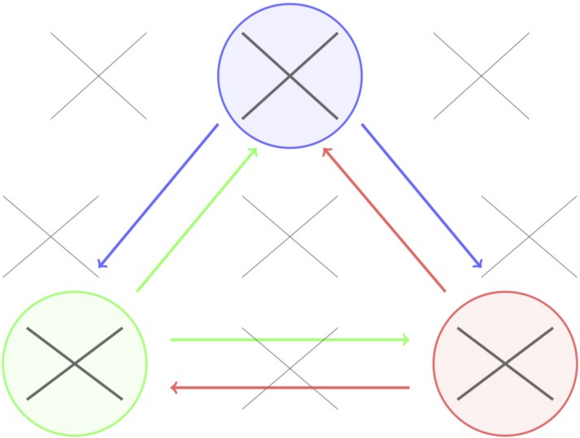
It is a two-step mechanism. Let's consider a quantum system consisting of two or more subsystems.



1. **Continuous measurement**: One or more observables of the system are continuously measured.
2. **Feedback dynamics**: The measurement result is broadcasted to the subsystems through a classical channel.

Tilloy-Diósi (TD) model (A Tilloy and L Diósi, 2018)

Let's consider a system of mass density $\hat{\mu}(\mathbf{x})$:



- Continuous measurement evolution

$$d\,|\varphi_t\rangle_m = \int \frac{d^3x}{2\hbar}\Big(\hat{\mu}(\mathbf{x}) - \langle\hat{\mu}(\mathbf{x})\rangle_t\Big)\,|\varphi_t\rangle\, dW_t(\mathbf{x})$$
$$- \int \frac{d^3x d^3y}{8\hbar^2}\gamma(\mathbf{x}-\mathbf{y})\Big(\hat{\mu}(\mathbf{x}) - \langle\hat{\mu}(\mathbf{x})\rangle_t\Big) \tag{1}$$
$$\times\Big(\hat{\mu}(\mathbf{y}) - \langle\hat{\mu}(\mathbf{y})\rangle_t\Big)\,|\varphi_t\rangle\, dt$$

- Measurement record

$$r(\mathbf{x}) = \langle\hat{\mu}(\mathbf{x})\rangle_t + \hbar\int d^3y\;\gamma^{-1}(\mathbf{x}-\mathbf{y})\frac{dW_t(\mathbf{y})}{dt} \tag{2}$$

- Sourcing the Classical Newtonian gravitational field

$$\nabla_{\mathbf{x}}\Phi(\mathbf{x}) = 4\pi G\, r(\mathbf{x}) \tag{3}$$

- Continuous measurement evolution

$$d|\varphi_t\rangle_m = \int \frac{d^3x}{2\hbar}\Big(\hat{\mu}(\mathbf{x}) - \langle\hat{\mu}(\mathbf{x})\rangle_t\Big)|\varphi_t\rangle\, dW_t(\mathbf{x})$$
$$- \int \frac{d^3x d^3y}{8\hbar^2}\gamma(\mathbf{x}-\mathbf{y})\Big(\hat{\mu}(\mathbf{x}) - \langle\hat{\mu}(\mathbf{x})\rangle_t\Big) \qquad (1)$$
$$\times \Big(\hat{\mu}(\mathbf{y}) - \langle\hat{\mu}(\mathbf{y})\rangle_t\Big)|\varphi_t\rangle\, dt$$

- Measurement record

$$r(\mathbf{x}) = \langle\hat{\mu}(\mathbf{x})\rangle_t + \hbar\int d^3y\ \gamma^{-1}(\mathbf{x}-\mathbf{y})\frac{dW_t(\mathbf{y})}{dt} \qquad (2)$$

- Sourcing the Classical Newtonian gravitational field

$$\nabla_{\mathbf{x}}\Phi(\mathbf{x}) = 4\pi G\, r(\mathbf{x}) \qquad (3)$$

- Continuous measurement evolution

$$d|\varphi_t\rangle_m = \int \frac{d^3x}{2\hbar}\left(\hat{\mu}(\mathbf{x}) - \langle\hat{\mu}(\mathbf{x})\rangle_t\right)|\varphi_t\rangle\, dW_t(\mathbf{x})$$
$$- \int \frac{d^3x d^3y}{8\hbar^2}\gamma(\mathbf{x}-\mathbf{y})\left(\hat{\mu}(\mathbf{x}) - \langle\hat{\mu}(\mathbf{x})\rangle_t\right)$$
$$\times \left(\hat{\mu}(\mathbf{y}) - \langle\hat{\mu}(\mathbf{y})\rangle_t\right)|\varphi_t\rangle\, dt \tag{1}$$

- Measurement record

$$r(\mathbf{x}) = \langle\hat{\mu}(\mathbf{x})\rangle_t + \hbar\int d^3y\ \gamma^{-1}(\mathbf{x}-\mathbf{y})\frac{dW_t(\mathbf{y})}{dt} \tag{2}$$

- Sourcing the Classical Newtonian gravitational field

$$\nabla_{\mathbf{x}}\Phi(\mathbf{x}) = 4\pi G\, r(\mathbf{x}) \tag{3}$$

- Feedback Hamiltonian $\left(V(\mathbf{x}-\mathbf{y}) = -\frac{G}{|\mathbf{x}-\mathbf{y}|}\right)$

$$\Phi(\mathbf{x}) = \int d^3y\, V(\mathbf{x}-\mathbf{y}) r(\mathbf{y}) \tag{4}$$

$$\hat{H}_{fb} = \int d^3x\, \Phi(\mathbf{x}) \hat{\mu}(\mathbf{x}) = \int d^3x\; d^3y\;\; V(\mathbf{x}-\mathbf{y}) \hat{\mu}(\mathbf{x}) r(\mathbf{y}) \tag{5}$$

- Feedback evolution

$$e^{-\frac{i}{\hbar}\hat{H}_{fb}dt} \cdot \left(d\left|\varphi_t\right\rangle_m + \left|\varphi_t\right\rangle\right) \tag{6}$$

- Feedback Hamiltonian $\left(V(\mathbf{x}-\mathbf{y}) = -\frac{G}{|\mathbf{x}-\mathbf{y}|}\right)$

$$\Phi(\mathbf{x}) = \int d^3y\, V(\mathbf{x}-\mathbf{y})r(\mathbf{y}) \tag{4}$$

$$\begin{aligned}\hat{H}_{fb} = \int d^3x\, \Phi(\mathbf{x})\hat{\mu}(\mathbf{x}) = \\ \int d^3x\ d^3y\ V(\mathbf{x}-\mathbf{y})\hat{\mu}(\mathbf{x})r(\mathbf{y})\end{aligned} \tag{5}$$

- Feedback evolution

$$e^{-\frac{i}{\hbar}\hat{H}_{fb}dt} \cdot \left(d\left|\varphi_t\right\rangle_m + \left|\varphi_t\right\rangle\right) \tag{6}$$

- TD model master equation:

$$\frac{d}{dt}\hat{\rho}_t = -\frac{i}{\hbar}[\hat{H}_0 + \hat{H}_{\text{grav}}, \hat{\rho}_t] + \frac{1}{2\hbar}\int d^3x\, d^3y\; V(\mathbf{x}-\mathbf{y})[\hat{\mu}(\mathbf{x}), [\hat{\mu}(\mathbf{y}), \hat{\rho}_t]] \tag{7}$$

where $\hat{H}_{\text{grav}} = \int d^3x\, d^3y\; V(\mathbf{x}-\mathbf{y})\hat{\mu}(\mathbf{x})\hat{\mu}(\mathbf{y})$

1. Quantum Newtonian gravitational interaction is recovered at the level of the master equation. (red)
2. The price to pay is the presence of decoherence effects. (green)

Problem: Violation of the energy conservation

The form of the Newtonian gravitational potential leads to divergences.

- Regularization by using a suitable smearing function

$$V(\mathbf{x}-\mathbf{y}) \to D(\mathbf{x}-\mathbf{y}) = (g \circ V \circ g)(\mathbf{x}-\mathbf{y}) \tag{8}$$

$$g(\mathbf{x}-\mathbf{y}) = \frac{e^{-\frac{|\mathbf{x}-\mathbf{y}|^2}{2R_0^2}}}{(2\pi R_0^2)^{3/2}} \tag{9}$$

- Mean energy of two-particle systems of mass m

$$\langle \hat{H} \rangle_t = \frac{\hbar G m}{2\sqrt{\pi} R_0^3} t \tag{10}$$

where $\hat{H} = \hat{H}_0 + \hat{H}_{\text{grav}}$

Problem: Violation of the energy conservation

The form of the Newtonian gravitational potential leads to divergences.

- Regularization by using a suitable smearing function

$$V(\mathbf{x}-\mathbf{y}) \to D(\mathbf{x}-\mathbf{y}) = (g \circ V \circ g)(\mathbf{x}-\mathbf{y}) \quad (8)$$

$$g(\mathbf{x}-\mathbf{y}) = \frac{e^{-\frac{|\mathbf{x}-\mathbf{y}|^2}{2R_0^2}}}{(2\pi R_0^2)^{3/2}} \quad (9)$$

- Mean energy of two-particle systems of mass m

$$\langle \hat{H} \rangle_t = \frac{\hbar G m}{2\sqrt{\pi} R_0^3} t \quad (10)$$

where $\hat{H} = \hat{H}_0 + \hat{H}_{\text{grav}}$

Dissipative generalization of the TD model

(G Di Bartolomeo, M Carlesso and A Bassi, 2021)

Option one: choosing a different smearing of the mass density to include a momentum operator. A well known approach in literature (A Smirne and A Bassi, 2015). The observable to measure is

$$\hat{L}(\mathbf{x})=\sum_{k=1}^{N}\frac{m}{(2\pi\hbar)^3}\int d^3q\, e^{-\frac{i}{\hbar}\mathbf{q}\cdot(\mathbf{x}-\hat{\mathbf{x}}_k)-\frac{R_0^2}{2\hbar^2}[(1+\alpha_k)\mathbf{q}+2\alpha_k\hat{\mathbf{p}}_k]^2} \tag{11}$$

This choice does not allow to recover the proper Newtonian gravitational interaction:

$$\hat{H}_{\text{int}}=\frac{1}{2}\int d^3x d^3y\, V(\mathbf{x}-\mathbf{y})\left(\hat{\mu}(\mathbf{x})\hat{L}(\mathbf{y})+\hat{L}^\dagger(\mathbf{y})\hat{\mu}(\mathbf{x})\right) \tag{12}$$

$$\hat{H}_{\text{int}}\neq\hat{H}_{\text{grav}} \tag{13}$$

Dissipative generalization of the TD model

(G Di Bartolomeo, M Carlesso and A Bassi, 2021)

Option one: choosing a different smearing of the mass density to include a momentum operator. A well known approach in literature (A Smirne and A Bassi, 2015). The observable to measure is

$$\hat{L}(\mathbf{x}) = \sum_{k=1}^{N} \frac{m}{(2\pi\hbar)^3} \int d^3q \, e^{-\frac{i}{\hbar}\mathbf{q}\cdot(\mathbf{x}-\hat{\mathbf{x}}_k) - \frac{R_0^2}{2\hbar^2}[(1+\alpha_k)\mathbf{q}+2\alpha_k\hat{\mathbf{p}}_k]^2} \tag{11}$$

This choice does not allow to recover the proper Newtonian gravitational interaction:

$$\hat{H}_{\text{int}} = \frac{1}{2}\int d^3x d^3y \, V(\mathbf{x}-\mathbf{y}) \left(\hat{\mu}(\mathbf{x})\hat{L}(\mathbf{y}) + \hat{L}^\dagger(\mathbf{y})\hat{\mu}(\mathbf{x})\right) \tag{12}$$

$$\hat{H}_{\text{int}} \neq \hat{H}_{\text{grav}} \tag{13}$$

Dissipative generalization of the TD model

(G Di Bartolomeo, M Carlesso and A Bassi, 2021)

Option one: choosing a different smearing of the mass density to include a momentum operator. A well known approach in literature (A Smirne and A Bassi, 2015). The observable to measure is

$$\hat{L}(\mathbf{x}) = \sum_{k=1}^{N} \frac{m}{(2\pi\hbar)^3} \int d^3q \, e^{-\frac{i}{\hbar}\mathbf{q}\cdot(\mathbf{x}-\hat{\mathbf{x}}_k) - \frac{R_0^2}{2\hbar^2}[(1+\alpha_k)\mathbf{q}+2\alpha_k\hat{\mathbf{p}}_k]^2} \tag{11}$$

This choice does not allow to recover the proper Newtonian gravitational interaction:

$$\hat{H}_{\text{int}} = \frac{1}{2}\int d^3x d^3y \, V(\mathbf{x}-\mathbf{y}) \left(\hat{\mu}(\mathbf{x})\hat{L}(\mathbf{y}) + \hat{L}^\dagger(\mathbf{y})\hat{\mu}(\mathbf{x})\right) \tag{12}$$

$$\hat{H}_{\text{int}} \neq \hat{H}_{\text{grav}} \tag{13}$$

Option two: We consider the following observable to measure:

$$\hat{L}(\mathbf{x}) = \hat{\mu}(\mathbf{x}) + i\hat{L}_I(\mathbf{x}) \tag{14}$$

- Dissipative TD model master equation:

$$\begin{aligned}\frac{d}{dt}\hat{\rho}_t &= -\frac{i}{\hbar}[\hat{H}_0' + \hat{H}_{grav}, \hat{\rho}_t] \\ &+ \frac{1}{2\hbar}\int d^3x\, d^3y\; D(\mathbf{x}-\mathbf{y})[\hat{\mu}(\mathbf{x}), [\hat{\mu}(\mathbf{y}), \hat{\rho}_t]] \\ &\boxed{+\frac{i}{2\hbar}\int d^3x\, d^3y\; D(\mathbf{x}-\mathbf{y})[\hat{\mu}(\mathbf{x}), \{\hat{L}_I(\mathbf{y}), \hat{\rho}_t\}]} \\ &\boxed{+\frac{1}{4\hbar}\int d^3x\, d^3y\; D(\mathbf{x}-\mathbf{y})[\hat{L}_I(\mathbf{x}), [\hat{L}_I(\mathbf{y}), \hat{\rho}_t]]} \\ &\boxed{+\frac{1}{2\hbar}\int d^3x\, d^3y\; D(\mathbf{x}-\mathbf{y})[\hat{\mu}(\mathbf{x}), [\hat{L}_I(\mathbf{y}), \hat{\rho}_t]]}\end{aligned} \tag{15}$$

Option two: We consider the following observable to measure:

$$\hat{L}(\mathbf{x}) = \hat{\mu}(\mathbf{x}) + i\hat{L}_I(\mathbf{x}) \tag{14}$$

- Dissipative TD model master equation:

$$\begin{aligned}
\frac{d}{dt}\hat{\rho}_t = &-\frac{i}{\hbar}[\hat{H}_0' + \hat{H}_{grav}, \hat{\rho}_t] \\
&+ \frac{1}{2\hbar}\int d^3x\, d^3y\ D(\mathbf{x}-\mathbf{y})[\hat{\mu}(\mathbf{x}), [\hat{\mu}(\mathbf{y}), \hat{\rho}_t]] \\
&\boxed{+\frac{i}{2\hbar}\int d^3x\, d^3y\ D(\mathbf{x}-\mathbf{y})[\hat{\mu}(\mathbf{x}), \{\hat{L}_I(\mathbf{y}), \hat{\rho}_t\}]} \\
&\boxed{+\frac{1}{4\hbar}\int d^3x\, d^3y\ D(\mathbf{x}-\mathbf{y})[\hat{L}_I(\mathbf{x}), [\hat{L}_I(\mathbf{y}), \hat{\rho}_t]]} \\
&\boxed{+\frac{1}{2\hbar}\int d^3x\, d^3y\ D(\mathbf{x}-\mathbf{y})[\hat{\mu}(\mathbf{x}), [\hat{L}_I(\mathbf{y}), \hat{\rho}_t]]}
\end{aligned} \tag{15}$$

Linear limit of the TD model

We start by considering a N point-particle system

$$\hat{\mu}(\mathbf{x}) = m \sum_{k=1}^{N} \delta(\mathbf{x} - \hat{\mathbf{x}}_k) \tag{16}$$

and rewrite the position and momentum operators as

$$\hat{\mathbf{x}}_k = \mathbf{x}_k^{(0)} + \Delta\hat{\mathbf{x}}_k \quad \text{and} \quad \hat{\mathbf{p}}_k = \mathbf{p}_k^{(0)} + \Delta\hat{\mathbf{p}}_k \tag{17}$$

$\Delta\hat{\mathbf{x}}_k$ and $\Delta\hat{\mathbf{p}}_k$ are the quantum fluctuations with respect to the classical position $\mathbf{x}_k^{(0)}$ and momentum $\mathbf{p}_k^{(0)}$. Then, we choose the following form for $\hat{L}_\mathsf{I}(\mathbf{x})$:

$$\hat{L}_\mathsf{I}(\mathbf{x}) = m \sum_{k=1}^{N} \delta\left(\mathbf{x} - \mathbf{x}_k^{(0)} - \frac{\alpha_k}{\hbar}\Delta\hat{\mathbf{p}}_k\right), \tag{18}$$

α_k are real free parameters, $[\alpha_k] = [l^2]$.

Linear limit of the TD model

We start by considering a N point-particle system

$$\hat{\mu}(\mathbf{x}) = m \sum_{k=1}^{N} \delta(\mathbf{x} - \hat{\mathbf{x}}_k) \qquad (16)$$

and rewrite the position and momentum operators as

$$\hat{\mathbf{x}}_k = \mathbf{x}_k^{(0)} + \Delta\hat{\mathbf{x}}_k \quad \text{and} \quad \hat{\mathbf{p}}_k = \mathbf{p}_k^{(0)} + \Delta\hat{\mathbf{p}}_k \qquad (17)$$

$\Delta\hat{\mathbf{x}}_k$ and $\Delta\hat{\mathbf{p}}_k$ are the quantum fluctuations with respect to the classical position $\mathbf{x}_k^{(0)}$ and momentum $\mathbf{p}_k^{(0)}$. Then, we choose the following form for $\hat{L}_\mathrm{I}(\mathbf{x})$:

$$\hat{L}_\mathrm{I}(\mathbf{x}) = m \sum_{k=1}^{N} \delta\left(\mathbf{x} - \mathbf{x}_k^{(0)} - \frac{\alpha_k}{\hbar}\Delta\hat{\mathbf{p}}_k\right), \qquad (18)$$

α_k are real free parameters, $[\alpha_k] = [l^2]$.

Linear limit of the TD model

We start by considering a N point-particle system

$$\hat{\mu}(\mathbf{x}) = m \sum_{k=1}^{N} \delta(\mathbf{x} - \hat{\mathbf{x}}_k) \tag{16}$$

and rewrite the position and momentum operators as

$$\hat{\mathbf{x}}_k = \mathbf{x}_k^{(0)} + \Delta\hat{\mathbf{x}}_k \quad \text{and} \quad \hat{\mathbf{p}}_k = \mathbf{p}_k^{(0)} + \Delta\hat{\mathbf{p}}_k \tag{17}$$

$\Delta\hat{\mathbf{x}}_k$ and $\Delta\hat{\mathbf{p}}_k$ are the quantum fluctuations with respect to the classical position $\mathbf{x}_k^{(0)}$ and momentum $\mathbf{p}_k^{(0)}$. Then, we choose the following form for $\hat{L}_\mathrm{I}(\mathbf{x})$:

$$\hat{L}_\mathrm{I}(\mathbf{x}) = m \sum_{k=1}^{N} \delta\left(\mathbf{x} - \mathbf{x}_k^{(0)} - \frac{\alpha_k}{\hbar}\Delta\hat{\mathbf{p}}_k\right), \tag{18}$$

α_k are real free parameters, $[\alpha_k] = [l^2]$.

We reduce the problem to that of two harmonic oscillators in one dimension at distance d, frequency ω, with mass m and $\alpha_k = \alpha$. In the assumption of small quantum fluctuations we have

- Linear limit of the dissipative TD model

$$\frac{d}{dt}\hat{\rho}_t = -\frac{i}{\hbar}[\hat{H},\hat{\rho}_t] - \sum_{k,j=1}^{2}\frac{iGm^2\alpha\eta_{kj}}{2\hbar^2}[\hat{x}_k,\{\hat{p}_j,\hat{\rho}_t\}]$$

$$-\sum_{k,j=1}^{2}\frac{Gm^2\eta_{kj}}{2\hbar}[\hat{x}_k,[\hat{x}_j,\hat{\rho}_t]] - \sum_{k,j=1}^{2}\frac{Gm^2\alpha\eta_{kj}}{2\hbar^2}[\hat{x}_k,[\hat{p}_j,\hat{\rho}_t]] \quad (19)$$

$$-\sum_{k,j=1}^{2}\frac{Gm^2\alpha^2\eta_{kj}}{4\hbar^3}[\hat{p}_k,[\hat{p}_j,\hat{\rho}_t]]$$

where $\quad \eta_{kj} = \int \frac{d^3q}{2\pi^2\hbar^3}\frac{\tilde{g}^2(\mathbf{q})}{q^2}q_1^2\ e^{\frac{i}{\hbar}q_1(x_k^{(0)}-x_j^{(0)})}$

We reduce the problem to that of two harmonic oscillators in one dimension at distance d, frequency ω, with mass m and $\alpha_k = \alpha$. In the assumption of small quantum fluctuations we have

- Linear limit of the dissipative TD model

$$\frac{d}{dt}\hat{\rho}_t = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}_t] - \sum_{k,j=1}^{2} \frac{iGm^2\alpha\eta_{kj}}{2\hbar^2}[\hat{x}_k, \{\hat{p}_j, \hat{\rho}_t\}]$$

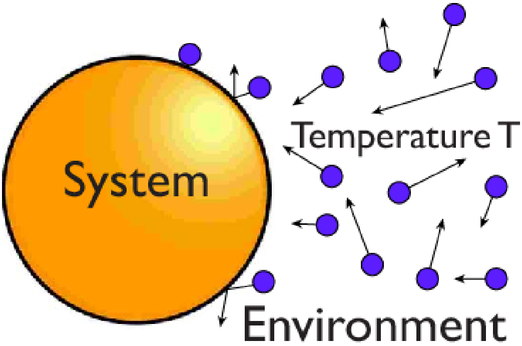
$$- \sum_{k,j=1}^{2} \frac{Gm^2\eta_{kj}}{2\hbar}[\hat{x}_k, [\hat{x}_j, \hat{\rho}_t]] - \sum_{k,j=1}^{2} \frac{Gm^2\alpha\eta_{kj}}{2\hbar^2}[\hat{x}_k, [\hat{p}_j, \hat{\rho}_t]] \quad (19)$$

$$- \sum_{k,j=1}^{2} \frac{Gm^2\alpha^2\eta_{kj}}{4\hbar^3}[\hat{p}_k, [\hat{p}_j, \hat{\rho}_t]]$$

where $$\eta_{kj} = \int \frac{d^3q}{2\pi^2\hbar^3}\frac{\tilde{g}^2(\mathbf{q})}{q^2} q_1^2 \; e^{\frac{i}{\hbar}q_1(x_k^{(0)} - x_j^{(0)})}$$

Analogy with quantum Brownian motion (QBM)

QBM describe the dynamics of open quantum systems in contact with a thermal bath of temperature T.



- QBM master equation

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}[\hat{H}_S, \hat{\rho}] - \sum_{k=1}^{2} \frac{i\lambda_k}{\hbar}[\hat{x}_k, \{\hat{p}_k, \hat{\rho}\}]$$
$$- \sum_{k=1}^{2} \frac{2\lambda_k m_k k_B T}{\hbar^2}[\hat{x}_k, [\hat{x}_k, \hat{\rho}]] - \sum_{k=1}^{2} \frac{\lambda_k}{8 m_k k_B T}[\hat{p}_k, [\hat{p}_k, \hat{\rho}]]$$
$$+ \sum_{k=1}^{2} \frac{\lambda_k k_B T}{\hbar^2 \Lambda}[\hat{x}_k, [\hat{p}_k, \hat{\rho}]] \tag{20}$$

- Asymptotic average energy of a system of two independent harmonic oscillators in the limit of high temperature

$$\langle \hat{H} \rangle_\infty = 2 k_B T \tag{21}$$

- QBM master equation

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}[\hat{H}_S, \hat{\rho}] - \sum_{k=1}^{2} \frac{i\lambda_k}{\hbar}[\hat{x}_k, \{\hat{p}_k, \hat{\rho}\}] - \sum_{k=1}^{2} \frac{2\lambda_k m_k k_B T}{\hbar^2}[\hat{x}_k, [\hat{x}_k, \hat{\rho}]] - \sum_{k=1}^{2} \frac{\lambda_k}{8 m_k k_B T}[\hat{p}_k, [\hat{p}_k, \hat{\rho}]] + \sum_{k=1}^{2} \frac{\lambda_k k_B T}{\hbar^2 \Lambda}[\hat{x}_k, [\hat{p}_k, \hat{\rho}]] \tag{20}$$

- Asymptotic average energy of a system of two independent harmonic oscillators in the limit of high temperature

$$\langle \hat{H} \rangle_\infty = 2 k_B T \tag{21}$$

$$\langle \hat{H} \rangle_\infty = \frac{\hbar^2}{m\alpha} + \frac{\alpha m \omega^2}{2} - \frac{\alpha m^2 G}{2}\eta_- + \frac{G^2\alpha^3 m^5}{4\hbar^2}(\eta^2 + \eta_{12}^2) \tag{22}$$

we can assume that R_0 is small with respect to the distance d between the particles:

$$\eta_{12} \to -2/d^3; \quad \eta_\pm \to \mp 2/d^3; \quad \eta^2 + \eta_{12}^2 \to 4/d^6$$

Finally, we assume

$$\alpha = \frac{m_0}{m}\alpha_0 \tag{23}$$

and take the limit

$$\alpha_0 \to 0 \tag{24}$$

$$\langle \hat{H} \rangle_\infty = \frac{\hbar^2}{m\alpha} + \frac{\alpha m \omega^2}{2} - \frac{\alpha m^2 G}{2}\eta_- + \frac{G^2\alpha^3 m^5}{4\hbar^2}(\eta^2 + \eta_{12}^2) \quad (22)$$

we can assume that R_0 is small with respect to the distance d between the particles:

$$\eta_{12} \to -2/d^3; \quad \eta_\pm \to \mp 2/d^3; \quad \eta^2 + \eta_{12}^2 \to 4/d^6$$

Finally, we assume

$$\alpha = \frac{m_0}{m}\alpha_0 \quad (23)$$

and take the limit

$$\alpha_0 \to 0 \quad (24)$$

$$\langle \hat{H} \rangle_\infty = \frac{\hbar^2}{m\alpha} + \frac{\alpha m \omega^2}{2} - \frac{\alpha m^2 G}{2}\eta_- + \frac{G^2 \alpha^3 m^5}{4\hbar^2}(\eta^2 + \eta_{12}^2) \tag{22}$$

we can assume that R_0 is small with respect to the distance d between the particles:

$$\eta_{12} \to -2/d^3; \quad \eta_\pm \to \mp 2/d^3; \quad \eta^2 + \eta_{12}^2 \to 4/d^6$$

Finally, we assume

$$\alpha = \frac{m_0}{m}\alpha_0 \tag{23}$$

and take the limit

$$\alpha_0 \to 0 \tag{24}$$

Asymptotic behavior of the mean energy

- Asymptotic mean energy

$$\langle \hat{H} \rangle_\infty \simeq \frac{\hbar^2}{m_0 \alpha_0} \quad \text{and} \quad T_{\text{eff}} \simeq \frac{\hbar^2}{2 m_0 \alpha_0 k_B} \tag{25}$$

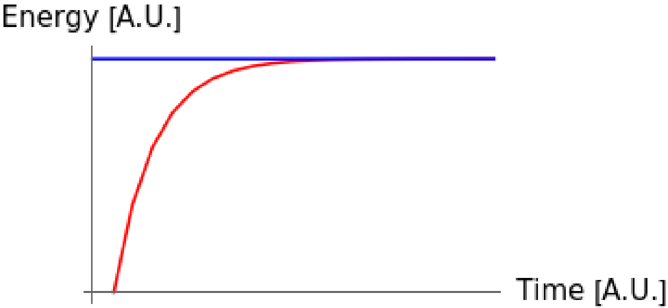


Figura: Time evolution of the mean energy in arbitrary unity with $m_0 = 1,\ G = 1,\ \hbar = 1,\ k_B = 1,\ \omega = 10,\ m = 1,\ d = 1,\ \alpha_0 = 0.01$

Disadvantages of the chosen ansatz for $\hat{L}_\mathrm{I}(\mathbf{x})$

The ansatz we chose for $\hat{L}_\mathrm{I}(\mathbf{x})$ is

$$\hat{L}_\mathrm{I}(\mathbf{x}) = m \sum_{k=1}^{N} \delta\left(\mathbf{x} - \mathbf{x}_k^{(0)} - \frac{\alpha_k}{\hbar}\Delta\hat{\mathbf{p}}_k\right), \tag{26}$$

It works but has some disadvantages:

- It does not have a clear physical meaning.
- It is formulated in first quantization.
- Simple analytic calculations (e.g mean energy) are possible in the linear limit only (already in the one-particle case).

Disadvantages of the chosen ansatz for $\hat{L}_{\mathsf{I}}(\mathbf{x})$

The ansatz we chose for $\hat{L}_{\mathsf{I}}(\mathbf{x})$ is

$$\hat{L}_{\mathsf{I}}(\mathbf{x}) = m \sum_{k=1}^{N} \delta\left(\mathbf{x} - \mathbf{x}_k^{(0)} - \frac{\alpha_k}{\hbar} \Delta \hat{\mathbf{p}}_k\right), \qquad (26)$$

It works but has some disadvantages:

- It does not have a clear physical meaning.
- It is formulated in first quantization.
- Simple analytic calculations (e.g mean energy) are possible in the linear limit only (already in the one-particle case).

Disadvantages of the chosen ansatz for $\hat{L}_{\mathrm{I}}(\mathbf{x})$

The ansatz we chose for $\hat{L}_{\mathrm{I}}(\mathbf{x})$ is

$$\hat{L}_{\mathrm{I}}(\mathbf{x}) = m \sum_{k=1}^{N} \delta\left(\mathbf{x} - \mathbf{x}_k^{(0)} - \frac{\alpha_k}{\hbar} \Delta \hat{\mathbf{p}}_k\right), \tag{26}$$

It works but has some disadvantages:

- It does not have a clear physical meaning.
- It is formulated in first quantization.
- Simple analytic calculations (e.g mean energy) are possible in the linear limit only (already in the one-particle case).

Disadvantages of the chosen ansatz for $\hat{L}_{\mathsf{I}}(\mathbf{x})$

The ansatz we chose for $\hat{L}_{\mathsf{I}}(\mathbf{x})$ is

$$\hat{L}_{\mathsf{I}}(\mathbf{x}) = m\sum_{k=1}^{N} \delta\left(\mathbf{x} - \mathbf{x}_k^{(0)} - \frac{\alpha_k}{\hbar}\Delta\hat{\mathbf{p}}_k\right), \tag{26}$$

It works but has some disadvantages:

- It does not have a clear physical meaning.
- It is formulated in first quantization.
- Simple analytic calculations (e.g mean energy) are possible in the linear limit only (already in the one-particle case).

A general ansatz for many-body systems

(G Di Bartolomeo, M Carlesso , K Piscicchia, C Curceanu, M Derakhshani, L Diósi, 2023)

We found a new general ansatz for many-body systems.

$$\hat{L}(\mathbf{x}) = \hat{\mu}(\mathbf{x}) + i\hat{L}_I(\mathbf{x}) \tag{27}$$

- Second quantization

$$\hat{\mu}(\mathbf{x}) = m\,\hat{\psi}^\dagger(\hat{\mathbf{x}})\hat{\psi}(\hat{\mathbf{x}}); \qquad \hat{L}_I(\mathbf{x}) = -\frac{\hbar\beta}{4}\nabla_{\mathbf{x}}\hat{\mathbf{J}}(\mathbf{x}) \tag{28}$$

$$\hat{\mathbf{J}}(\mathbf{x}) = -i\frac{\hbar}{2}\left(\hat{\psi}^\dagger(\mathbf{x})\nabla_{\mathbf{x}}\hat{\psi}(\mathbf{x}) - \nabla_{\mathbf{x}}\hat{\psi}^\dagger(\mathbf{x})\hat{\psi}(\mathbf{x})\right) \tag{29}$$

- First quantization

$$\hat{\mu}(\mathbf{x}) = m \sum_{k=1}^{N} \delta(\mathbf{x} - \hat{\mathbf{x}}_k); \qquad \hat{L}_I(\mathbf{x}) = -\frac{\hbar\beta}{4} \nabla_{\mathbf{x}} \hat{\mathbf{J}}(\mathbf{x}) \quad (30)$$

$$\hat{\mathbf{J}}(\mathbf{x}) = \frac{1}{2} \sum_{k=1}^{N} \{\mathbf{p}_k, \delta(\mathbf{x} - \hat{\mathbf{x}}_k)\} \quad (31)$$

- Decay of the current

$$\dot{\hat{\mathbf{J}}}(\mathbf{x})|_2 = -\eta \hat{\mathbf{J}}(\mathbf{x}); \qquad \eta = -\frac{\beta m}{2} D''(\mathbf{x})|_{\mathbf{x}=0} \quad (32)$$

Mean energy

- Linear case

In the linear case of two harmonic oscillators in one dimension the calculations are identical to that showed before (once the parameters are changed).

- One-particle case

$$T = T_\beta \frac{1 - \frac{3}{2}(E_{R_0}/k_B T_\beta) + \frac{15}{16}(E_{R_0}/k_B T_\beta)^2}{1 - \frac{9}{8}(E_{R_0}/k_B T_\beta)} \qquad (33)$$

where $T_\beta = 1/\beta$, $E_{R_0} = \frac{\hbar^2}{4mR_0^2}$

Conclusions

- We derived the **dissipative generalization** of the TD model and showed that the system thermalizes to an **effective temperature**.

- Under this perspective, the gravitationally induced stochastic noise acts in an isolated system as a **dissipative medium**, similarly to what a thermal bath does in a typical open quantum system.

- **Subject for future research**: Energy conservation at each time is still lacking. Possible step forward: upgrade the stochastic noises of the protocol to physical dynamical fields. Then, one could in principle be able to conserve the total energy of the system plus the stochastic field.

Conclusions

- We derived the **dissipative generalization** of the TD model and showed that the system thermalizes to an **effective temperature**.
- Under this perspective, the gravitationally induced stochastic noise acts in an isolated system as a **dissipative medium**, similarly to what a thermal bath does in a typical open quantum system.
- **Subject for future research**: Energy conservation at each time is still lacking. Possible step forward: upgrade the stochastic noises of the protocol to physical dynamical fields. Then, one could in principle be able to conserve the total energy of the system plus the stochastic field.

Conclusions

- We derived the **dissipative generalization** of the TD model and showed that the system thermalizes to an **effective temperature**.

- Under this perspective, the gravitationally induced stochastic noise acts in an isolated system as a **dissipative medium**, similarly to what a thermal bath does in a typical open quantum system.

- **Subject for future research**: Energy conservation at each time is still lacking. Possible step forward: upgrade the stochastic noises of the protocol to physical dynamical fields. Then, one could in principle be able to conserve the total energy of the system plus the stochastic field.

The End

Thank you for the attention!

Weak Measurements

$$\hat{M}(r) = \left(\frac{\gamma\Delta t}{2\pi\hbar^2}\right)^{\frac{1}{4}} \int_{-\infty}^{+\infty} da\ e^{-\frac{\gamma\Delta t}{4\hbar^2}(a-r)^2} \left|a\right\rangle \left\langle a\right| \tag{34}$$

is a Gaussian-weighted sum of projectors onto the eigenstates of an Hermitian operator $\hat{A}$, where γ is a real parameter, r is the measurement result, a are the eigenvalues of $\hat{A}$ such that $\hat{A}\left|a\right\rangle = a\left|a\right\rangle$.

Divergency

The decoherence term if expressed in terms of its Fourier transform, is given by

$$\sum_{k,j=1}^{N} m_k m_j \int \frac{d^3k}{(2\pi\hbar)^{3/2}} \tilde{V}(\mathbf{k})[e^{\frac{i}{\hbar}\mathbf{k}\hat{\mathbf{x}}_k}, [e^{-\frac{i}{\hbar}\mathbf{k}\hat{\mathbf{x}}_j}, \hat{\rho}_t]]. \qquad (35)$$

By considering the terms corresponding to $k = j$, which are proportional to

$$\int d^3k \ \tilde{V}(\mathbf{k})\left(2\hat{\rho}_t - e^{\frac{i}{\hbar}\mathbf{k}\hat{\mathbf{x}}_k}\hat{\rho}_t e^{-\frac{i}{\hbar}\mathbf{k}\hat{\mathbf{x}}_k} - e^{-\frac{i}{\hbar}\mathbf{k}\hat{\mathbf{x}}_k}\hat{\rho}_t e^{\frac{i}{\hbar}\mathbf{k}\hat{\mathbf{x}}_k}\right), \qquad (36)$$

we see that the term $\int d^3k \ \tilde{V}(\mathbf{k})$ diverges. Indeed, being $\tilde{V}(\mathbf{k}) = -\frac{4\pi G\hbar^2}{k^2}$, we get

$$\int d^3k \ \tilde{V}(\mathbf{k}) \propto 4\pi \int_0^\infty dk \ = \infty \qquad (37)$$

Extimation of the energy increase

By setting for example $m = 10^{-7}$ kg and $R_0 = 10^{-7}$ m, the rate of change is

$$\frac{\hbar G m}{2\sqrt{\pi} R_0^3} = 1.98 \cdot 10^{-31} \text{J/s}. \tag{38}$$

We have an average energy increase about 10^{-31} J after 1 s that is detectable.

Lost of the transaltional invariance

The drive for the choice of $\hat{L}_I(\mathbf{x})$ is that the resulting dynamics satisfies translational invariance. Indeed, other choices for

$$\sum_k m_k \delta\left(\mathbf{x} - \mathbf{v}_k - \frac{\alpha_k}{\hbar}\Delta\hat{\mathbf{p}}_k\right) \tag{39}$$

would lead to the violation of the translational invariance for any choice of $\mathbf{v}_k$ different from $\mathbf{x}_k^{(0)}$.

Modification of the $\hat{H}_0$ Hamiltonian

$\hat{H}'_0 = \hat{H}_0 + \Delta\hat{H}_0$, where

$$\Delta\hat{H}_0 = -\frac{1}{4\hbar}\int d^3x \int d^3y \, V(\mathbf{x}-\mathbf{y})\{\hat{\mu}(\mathbf{x}), \hat{L}_\mathrm{I}(\mathbf{y})\} \qquad (40)$$

Gravitational interaction Hamiltonian in the Fourier representation

$$\hat{H}_{grav} = \frac{1}{2}\sum_{k,j=1}^{N} m_k m_j \int \frac{d^3k}{(2\pi\hbar)^3}\,\tilde{g}^2(\mathbf{k})\tilde{V}(\mathbf{k})\,e^{\frac{i}{\hbar}\mathbf{k}(\hat{\mathbf{x}}_k-\hat{\mathbf{x}}_j)} \quad (41)$$

Linearized Hamiltonian

$$\hat{H} = \sum_{k=1}^{2} \left(\frac{\hat{p}_k^2}{2m} + \frac{m\Omega^2}{2} \hat{x}_k^2 \right) - Gm^2 \eta_{12} \hat{x}_1 \hat{x}_2, \qquad (42)$$

includes also the linearized quantum gravitational interaction $\hat{H}_{\text{grav}}$ and we defined

$$\Omega^2 = \omega^2 + Gm\eta_{12}. \qquad (43)$$

Anisotropy in the dTD model

- One particle standard TD model

$$\frac{d\rho(\mathbf{p})}{dt} = \frac{m^2}{\hbar^2} \int \frac{d^3k}{(2\pi)^3} D_{\mathbf{k}} \Big(\rho(\mathbf{p} - \hbar\mathbf{k}) - \rho(\mathbf{p}) \Big) \tag{44}$$

$$\frac{m^2}{\hbar^2} \frac{d^3\mathbf{k}}{(2\pi)^3} D_{\mathbf{k}} \tag{45}$$

- One particle dTD model

$$\frac{d\rho(\mathbf{p})}{dt} = \frac{1}{\hbar^2} \int \frac{d^3k}{(2\pi)^3} D_{\mathbf{k}} \Bigg(\left(m - \frac{\beta\hbar^2 k^2}{8} + \frac{\beta\hbar\mathbf{k}}{4}\mathbf{p} \right)^2 \rho(\mathbf{p} - \hbar\mathbf{k}) - \left(m + \frac{\beta\hbar^2 k^2}{8} + \frac{\beta\hbar\mathbf{k}}{4}\mathbf{p} \right)^2 \rho(\mathbf{p}) \Bigg) \tag{46}$$

$$\frac{m^2}{\hbar^2} \frac{d^3\mathbf{k}}{(2\pi)^3} D_{\mathbf{k}} \left(1 + \frac{\beta}{8m} [\mathbf{p}^2 - (\mathbf{p} - \hbar\mathbf{k})^2] \right)^2 \tag{47}$$