Quantum anomaly and scaling dynamics in the 2D Fermi gas

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ECT*
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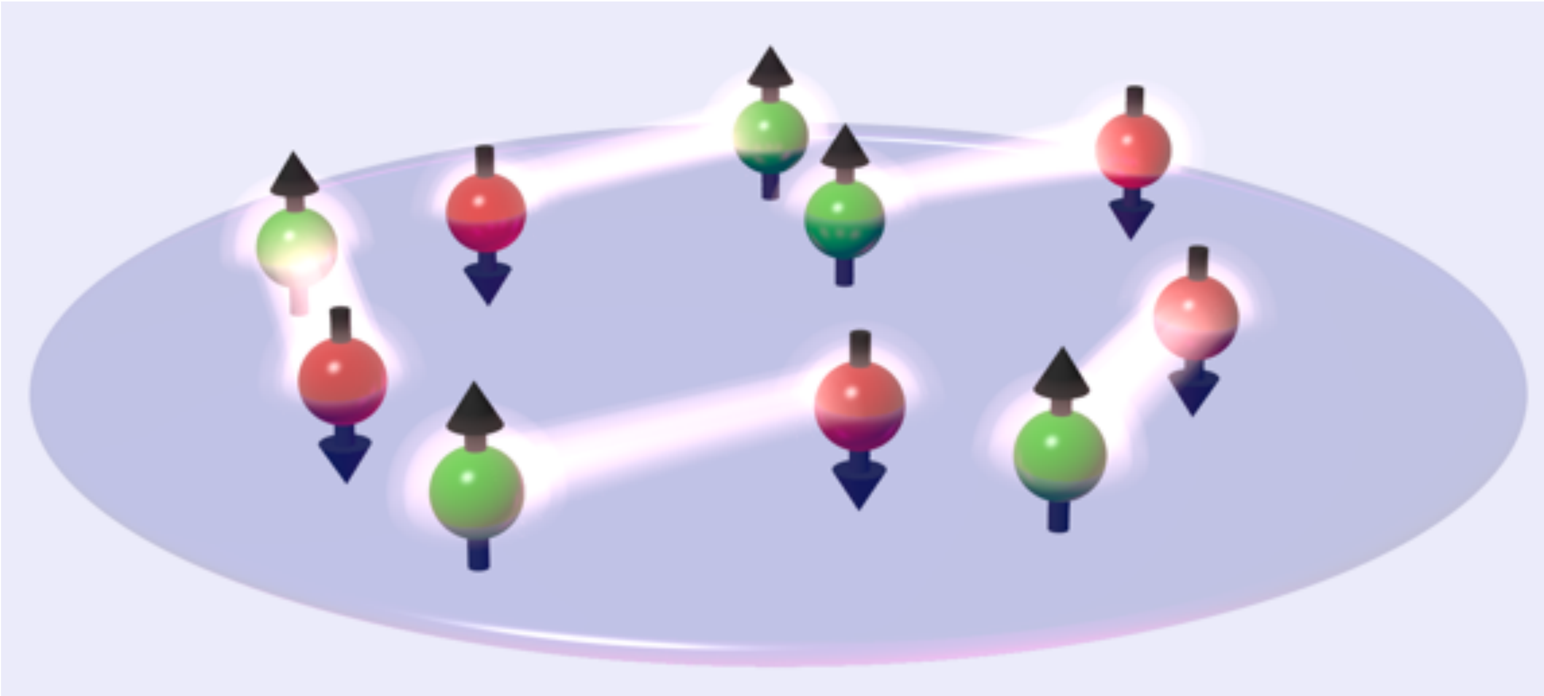
2D Fermi gas



dilute gas of ↑ and ↓ fermions with contact interaction:

$$\mathcal{H} = \int d\mathbf{x} \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \Big(-\frac{\hbar^2\nabla^2}{2m} - \mu_\sigma\Big)\psi_\sigma + g_0 \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow$$

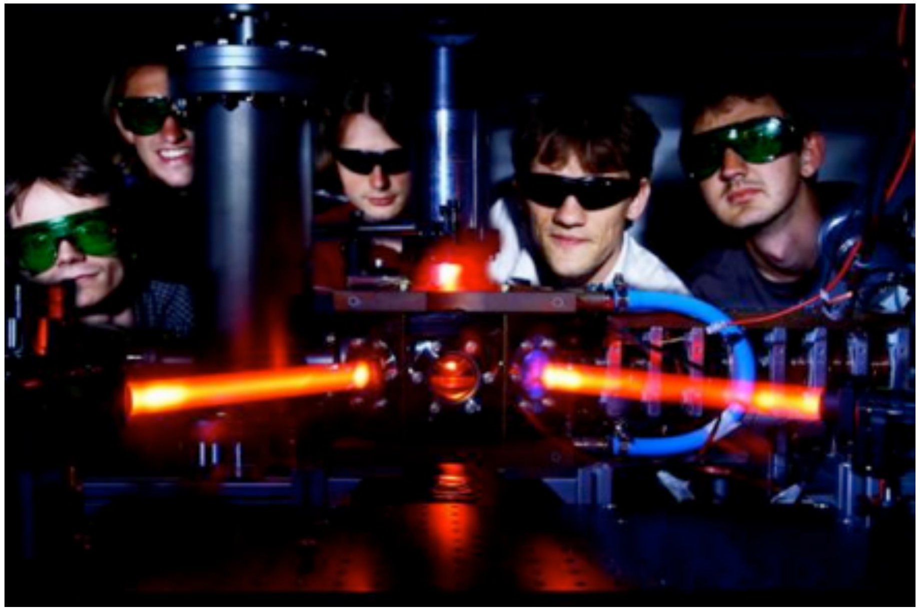
Levinsen & Parish Annu. Rev. CMP 2015

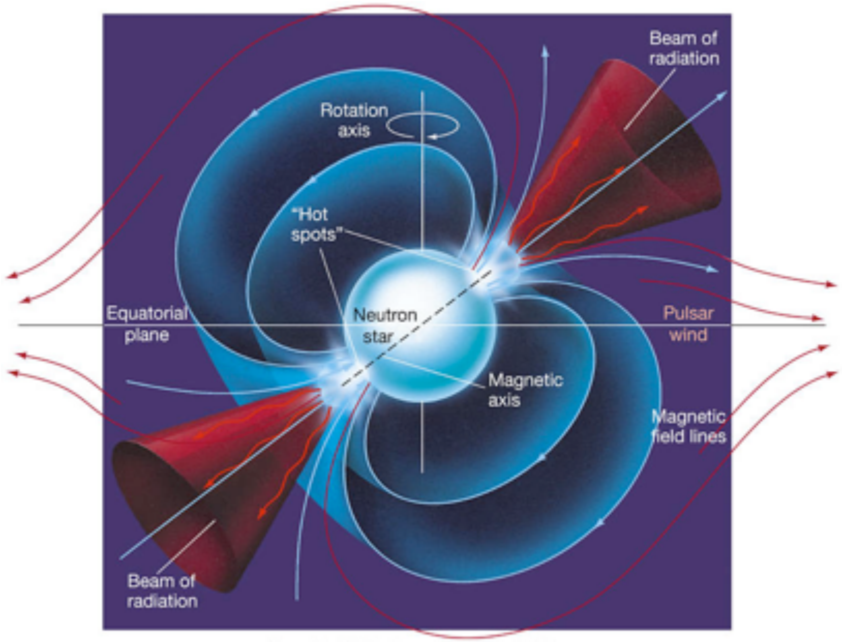
Scale invariance

- self-similar on different length scales: no intrinsic length/energy scale

- statistical mechanics: near phase transition, fluctuations on all length scales
  ➩ no intrinsic scale

- strongly interacting Fermi gas similar in cold atom lab (μm) and neutron star (fm)

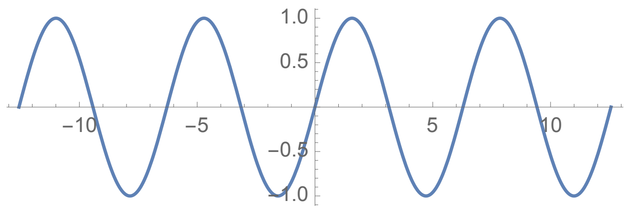




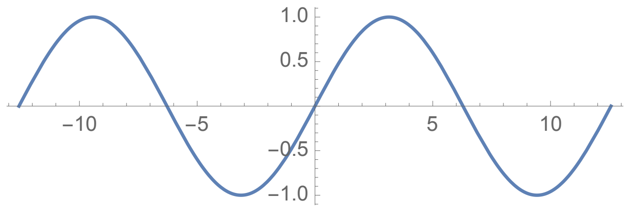
Scale invariance

- ideal gas

$$H = \frac{\boldsymbol{p}^2}{2m}\,, \qquad \psi(\boldsymbol{x}) = e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$



- rescale space by factor λ: $\boldsymbol{x} \mapsto \lambda\boldsymbol{x}$



- solution self-similar with $\boldsymbol{k} \mapsto \frac{1}{\lambda}\boldsymbol{k}$: $\quad e^{i\frac{\boldsymbol{k}}{\lambda}\cdot\lambda\boldsymbol{x}} = e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$

- Hamiltonian scales as $H \mapsto \frac{1}{\lambda^2}H$

Scale invariance

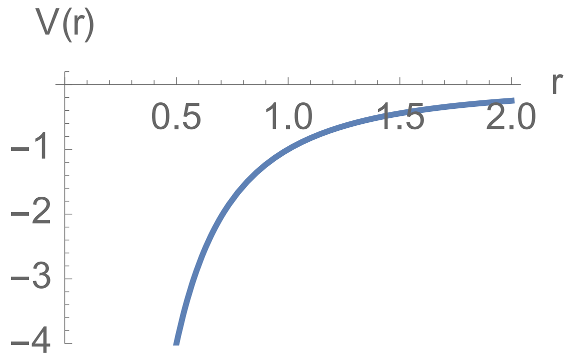
- particles with interaction

$$H = \sum_i \frac{\boldsymbol{p}_i^2}{2m} + \frac{1}{2}\sum_{i\neq j} V(\boldsymbol{x}_i - \boldsymbol{x}_j)$$

- consider power-law interaction

$$V(\boldsymbol{x}) = \frac{g}{|\boldsymbol{x}|^\alpha}, \qquad V(\lambda\boldsymbol{x}) = \frac{1}{\lambda^\alpha} V(\boldsymbol{x})$$

- scaling law



$$H \mapsto \frac{1}{\lambda^2} H_{\text{kin}} + \frac{1}{\lambda^\alpha} H_{\text{int}} \stackrel{!}{=} \frac{1}{\lambda^2} H$$

scale invariant only for inverse square potential α=2 (not often realized)

Scale invariance

- particles with interaction

$$H = \sum_i \frac{\boldsymbol{p}_i^2}{2m} + \frac{1}{2}\sum_{i\neq j} V(\boldsymbol{x}_i - \boldsymbol{x}_j)$$

- **contact interaction**

$$V(\boldsymbol{x}) = g\delta(x)\delta(y)\cdots$$

$$V(\lambda\boldsymbol{x}) = \frac{1}{\lambda^d}V(\boldsymbol{x}) \text{ since } \delta(\lambda x) = \frac{1}{\lambda}\delta(x)$$

- scaling law

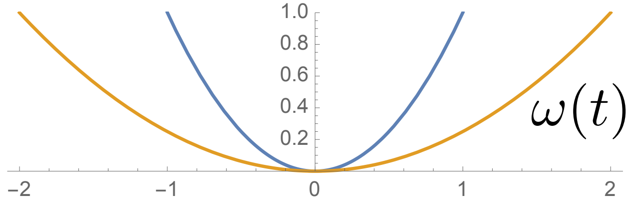
$$H \mapsto \frac{1}{\lambda^2}H_{\text{kin}} + \frac{1}{\lambda^d}H_{\text{int}} \overset{!}{=} \frac{1}{\lambda^2}H$$

scale invariant in d=2 dimensions for any dimensionless coupling g
no intrinsic scale: kinetic and interaction equally important on any scale

Dynamical scaling

- time dependent harmonic oscillator (oscillator length $\ell_{\rm osc} = \sqrt{\hbar/m\omega}$)

$$H = \frac{p^2}{2m} + \frac{m}{2}\omega^2(t)x^2$$



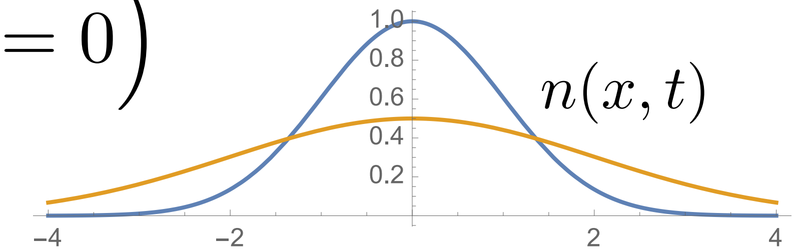
- stationary initial state $\psi(x, t=0)$, impose potential $\omega(t)$:
  dynamical scaling solution

$$\psi(x,t) = \frac{1}{\lambda^{1/2}}\,\psi\left(\frac{x}{\lambda}, t=0\right)\exp\left(i\frac{m\dot{\lambda}}{\hbar\lambda}x^2\right)e^{i\theta}$$

- density profile

$$n(x,t) = |\psi(x,t)|^2 = \frac{1}{\lambda}n\left(\frac{x}{\lambda}, t=0\right)$$

self-similar at all times



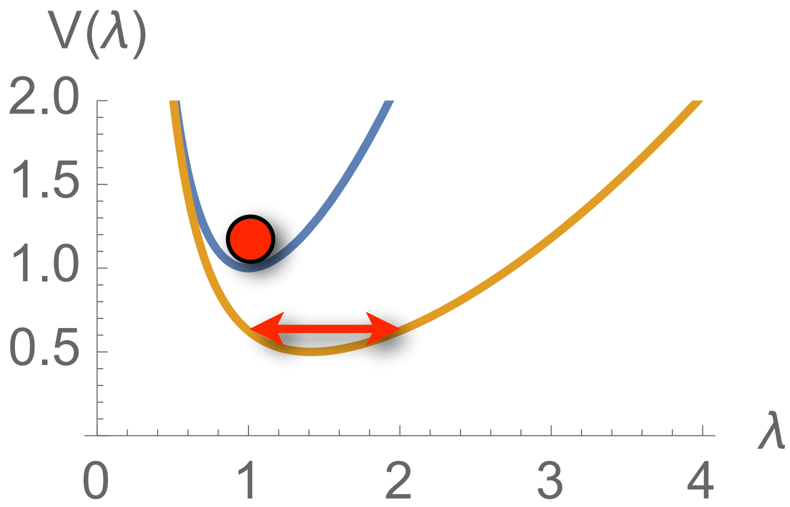
Dynamical scaling

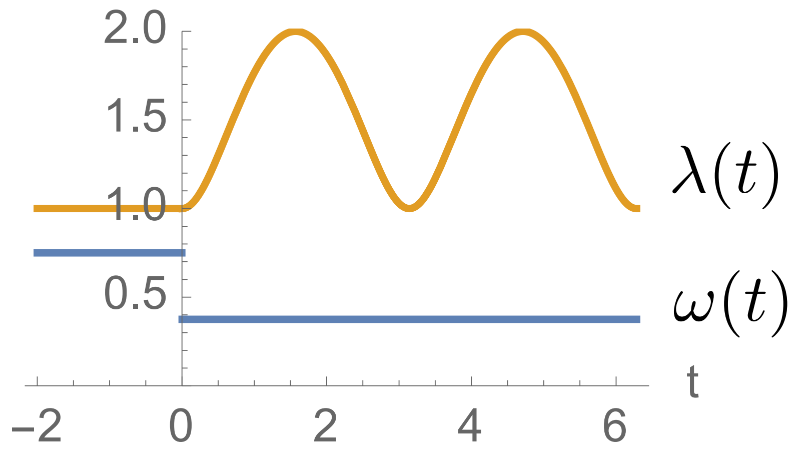
- global scale factor λ(t) governed by *Ermakov equation*

$$\ddot{\lambda}(t) + \omega^2(t)\lambda(t) = \frac{\omega^2(0)}{\lambda^3(t)}, \quad \lambda(0) = 1,\ \dot{\lambda}(0) = 0$$

- quantum time evolution determined by motion of “classical particle” λ:

$$\ddot{\lambda}(t) = -V'(\lambda) \qquad\qquad \lambda(t) = \sqrt{1 + \alpha \sin^2(\omega t)}$$





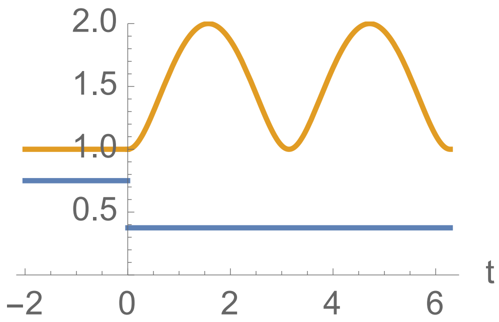
periodic breathing motion at $\omega_{\mathrm{b}} = 2\omega$

Dynamical scaling

- **interacting many-body system** with scale invariance: $\mathbf{X} = (\mathbf{x}_1, \ldots, \mathbf{x}_N)$

$$\psi(\mathbf{X}, t) = \frac{1}{\lambda^{Nd/2}}\, \psi\Big(\frac{\mathbf{X}}{\lambda}, t=0\Big) \exp\Big(i \frac{m\dot{\lambda}}{\hbar\lambda} \mathbf{X}^2\Big) e^{i\theta}$$

- **exact many-body wavefunction** known at all later times



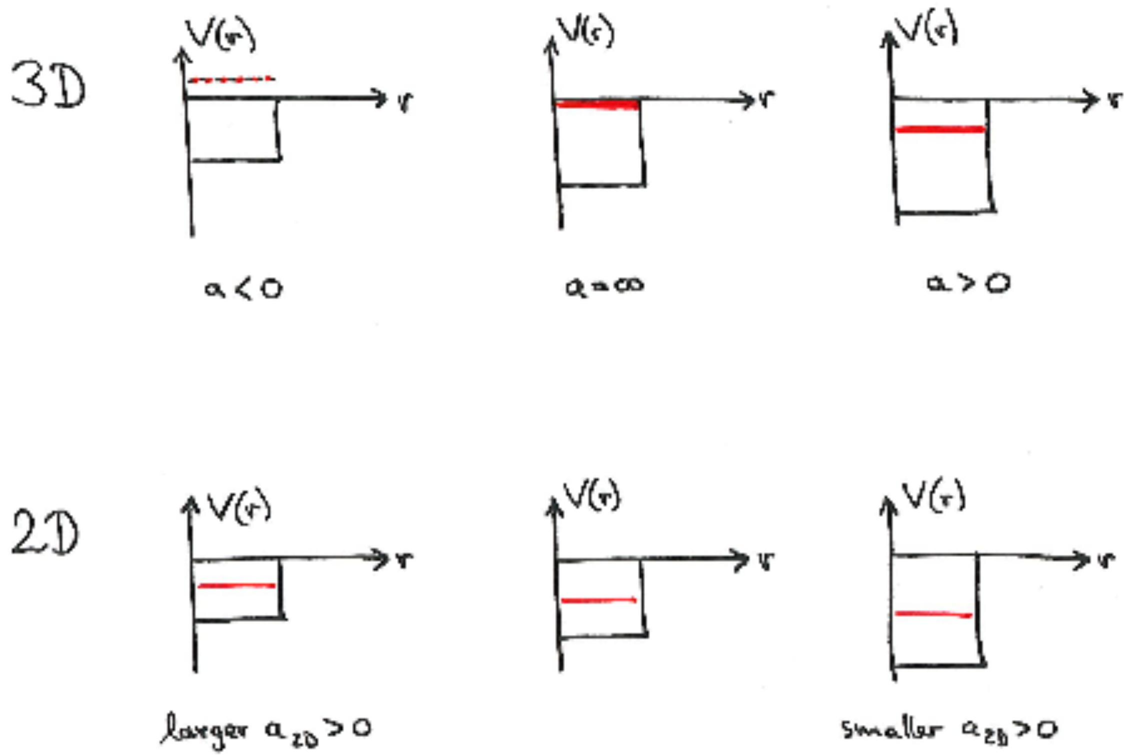
- no equilibration/thermalization:
  dissipationless hydrodynamics ζ=0 (entropy const.)

- hidden symmetry SO(2,1) generates many-body spectrum
  [Pitaevskii & Rosch 1997; Werner & Castin 2006]

- realized exactly in Unitary Fermi gas ($a \to \infty$ no scale) [Werner; expt. challenging]
  approx. in 1D/2D Bose gas [Pitaevskii; Olshanii; expt. Bouchoule 2014]
  2D Fermi gas: control scale invariance, study deviations

Quantum anomaly

- **quantum mechanical scattering** (attractive interaction by potential well)



- always bound state in 2D of size $a_{2\mathrm{D}}$, binding energy scale $\varepsilon_B = \dfrac{\hbar^2}{m a_{2\mathrm{D}}^2}$
  breaks classical scale invariance: quantum anomaly

Scattering amplitude

- two-body scattering amplitude

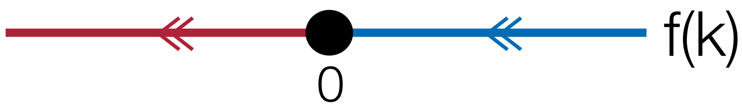
$$f(k) = \frac{2\pi}{i\frac{\pi}{2} - \ln(ka_{2D})}$$

large $ka_{2D} > 1$: see a bound state, attractive
small $ka_{2D} < 1$: can't see it, repulsive

- how does scattering amplitude depend on scale (zoom out)?

$$\frac{d\, f(k)}{d \ln k} = \frac{f(k)^2}{2\pi} \quad (= 0 \text{ scale invariant})$$

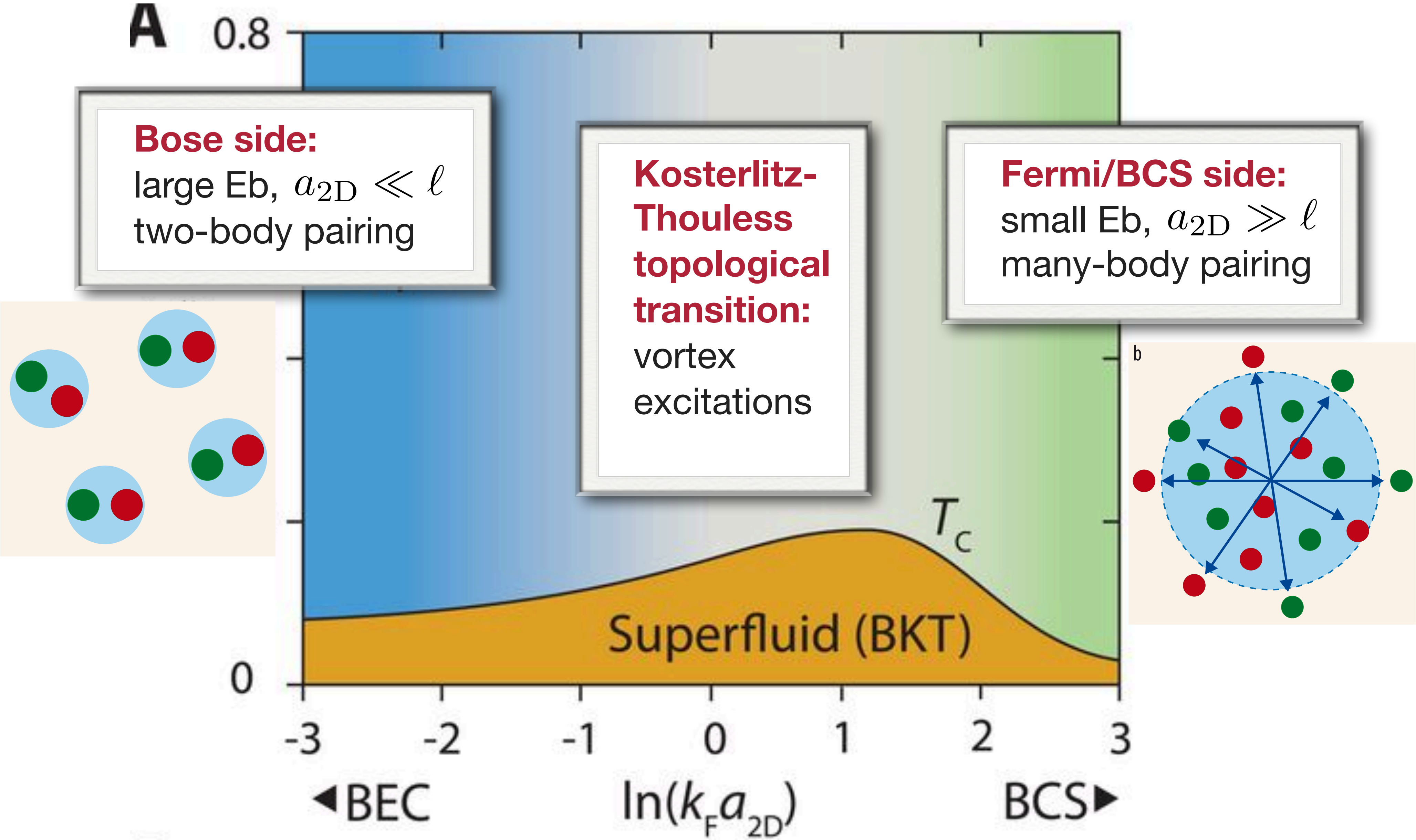
attractive: strong binding   repulsive: asymptotically free



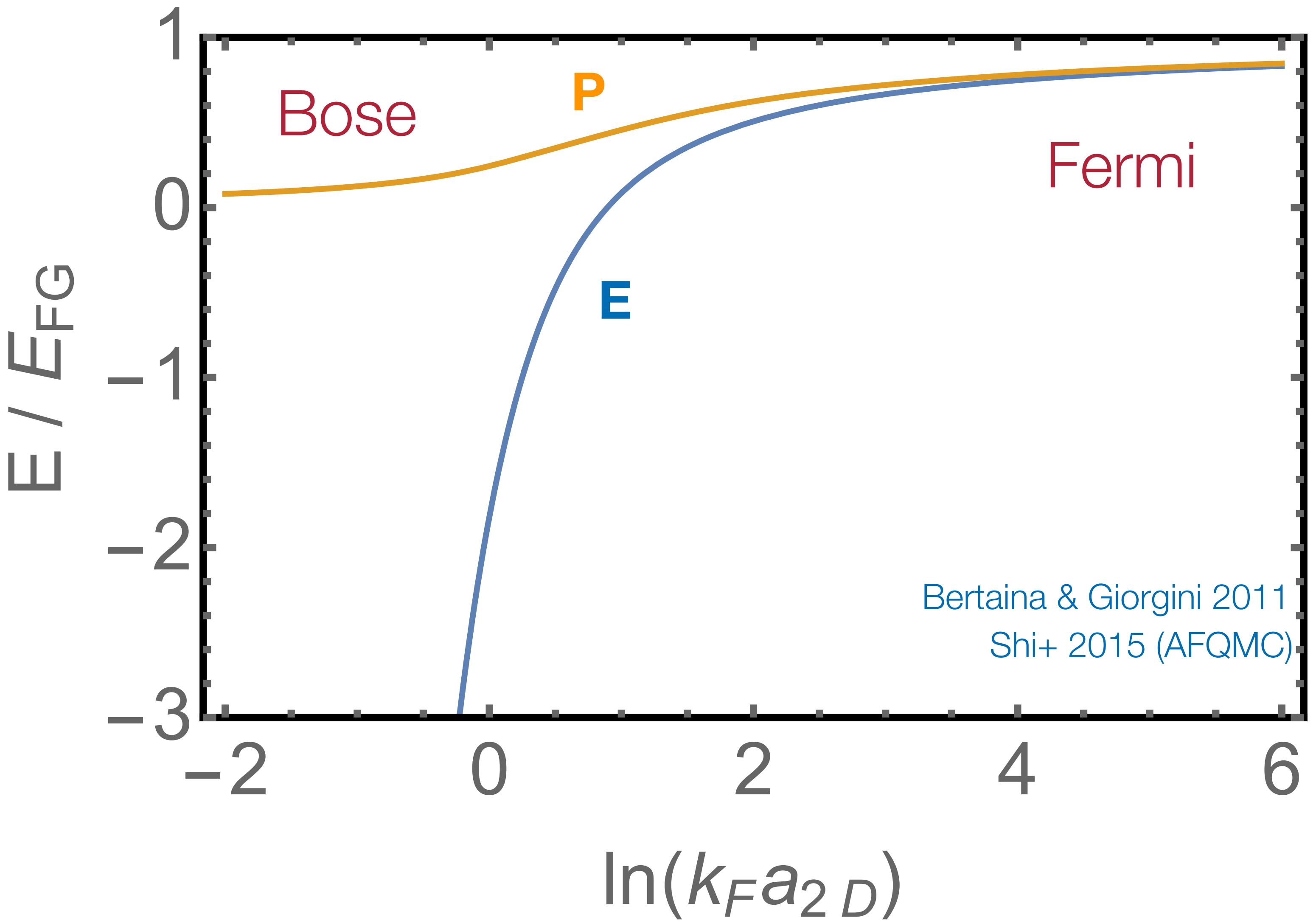
coupling always energy dependent (**log. running coupling**) Holstein 1993

many-body scale $k=k_F$: interaction parameter $\ln(k_F a_{2D})$

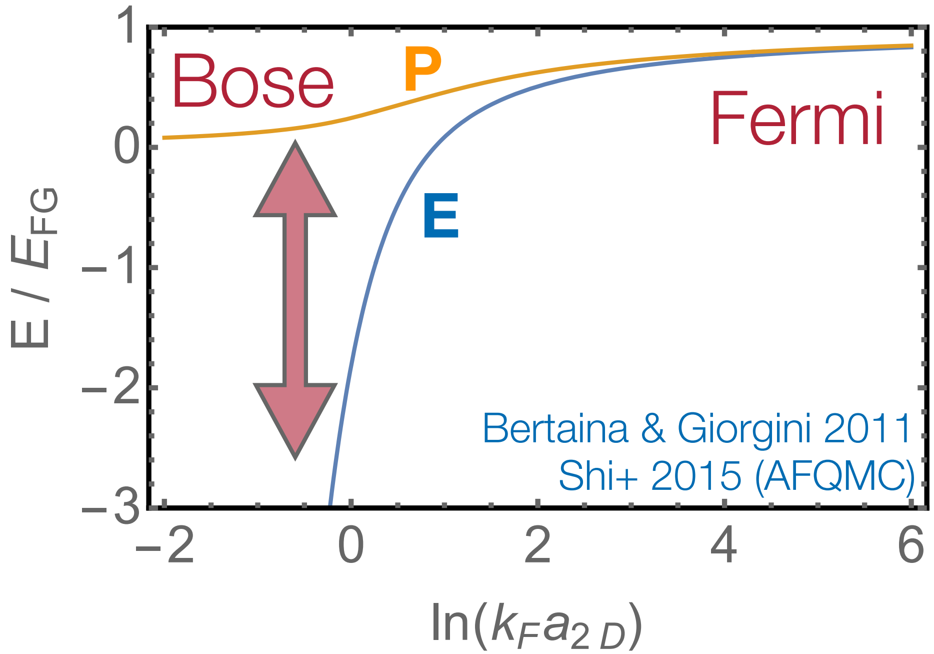
Phase diagram



Thermodynamics & scale invariance



Thermodynamics & scale invariance



scale invariance in 2D:

$$P = E$$

interacting 2D Fermi gas:

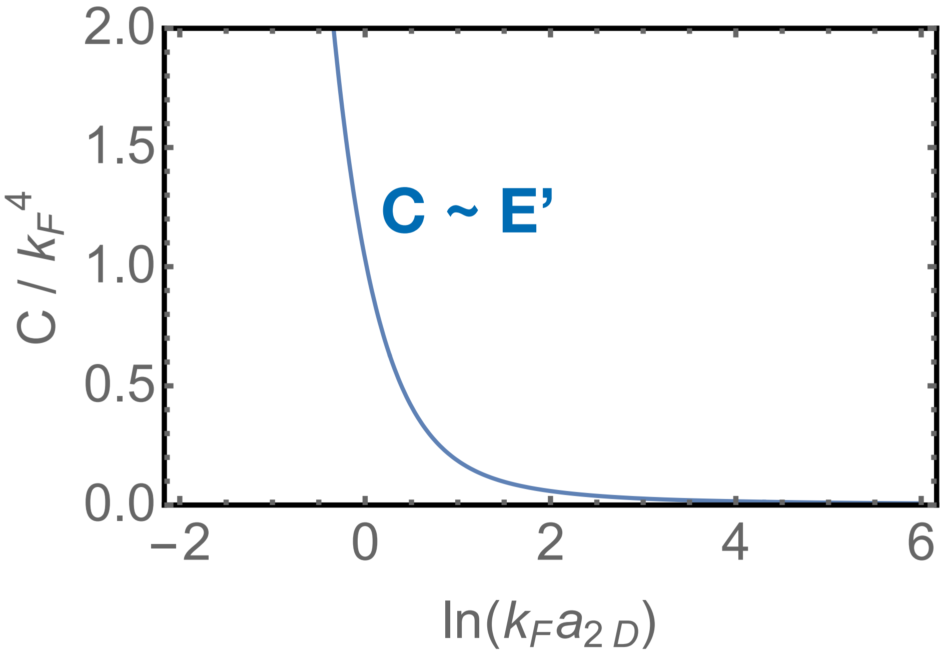
$$P = E + \frac{C}{4\pi m}$$

scale invariance broken

Contact density: probability to find ⬆ and ⬇ in same place

$$C = m^2 g_0^2 \langle \bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow (r) \rangle$$

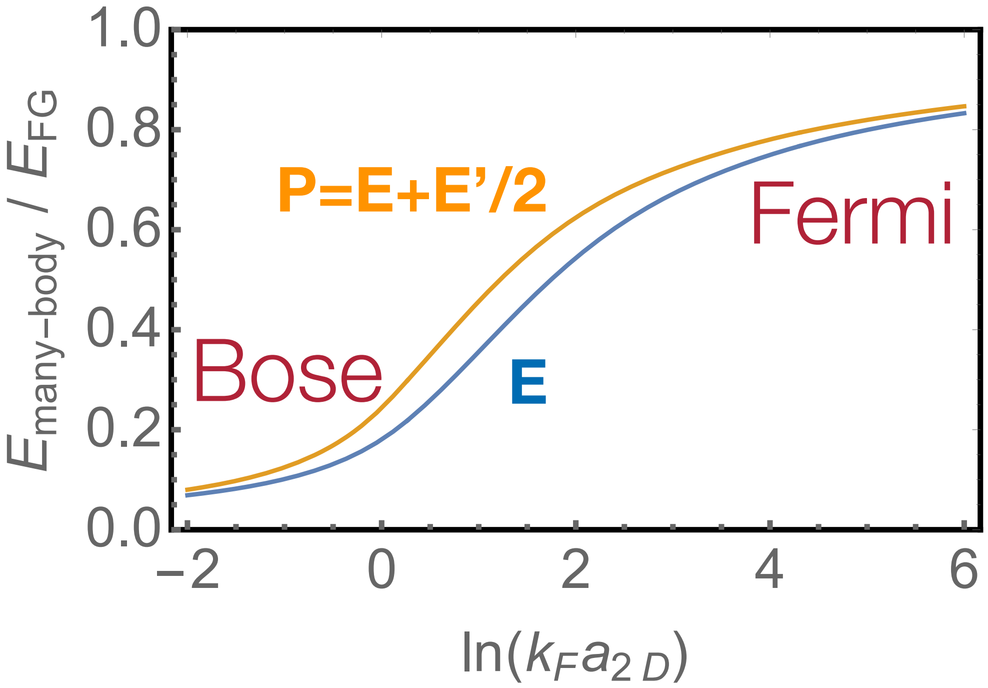
$$\frac{dE}{d \ln a_{2D}} = \frac{C}{2\pi m}$$

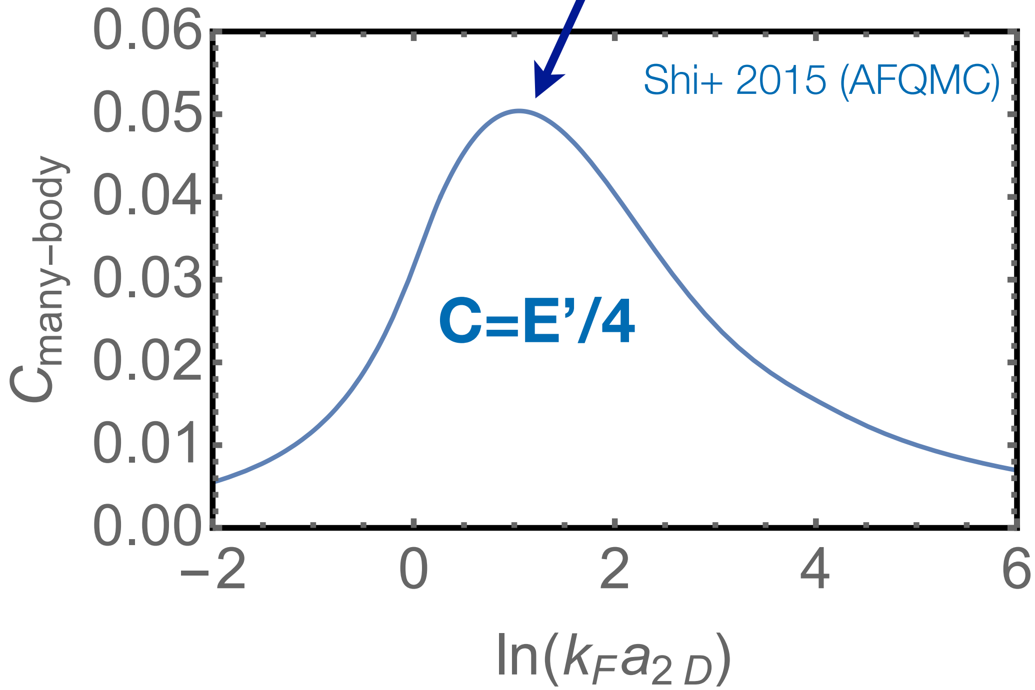


Local **many-body** correlations

subtract two-body binding energy:

$$C_{\text{many-body}} \approx 8 n_\uparrow n_\downarrow$$

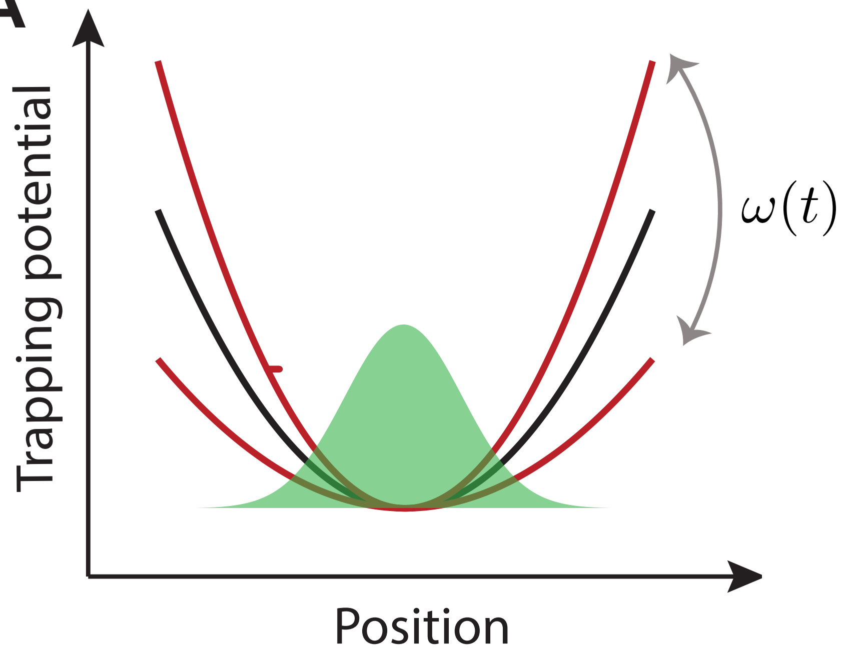




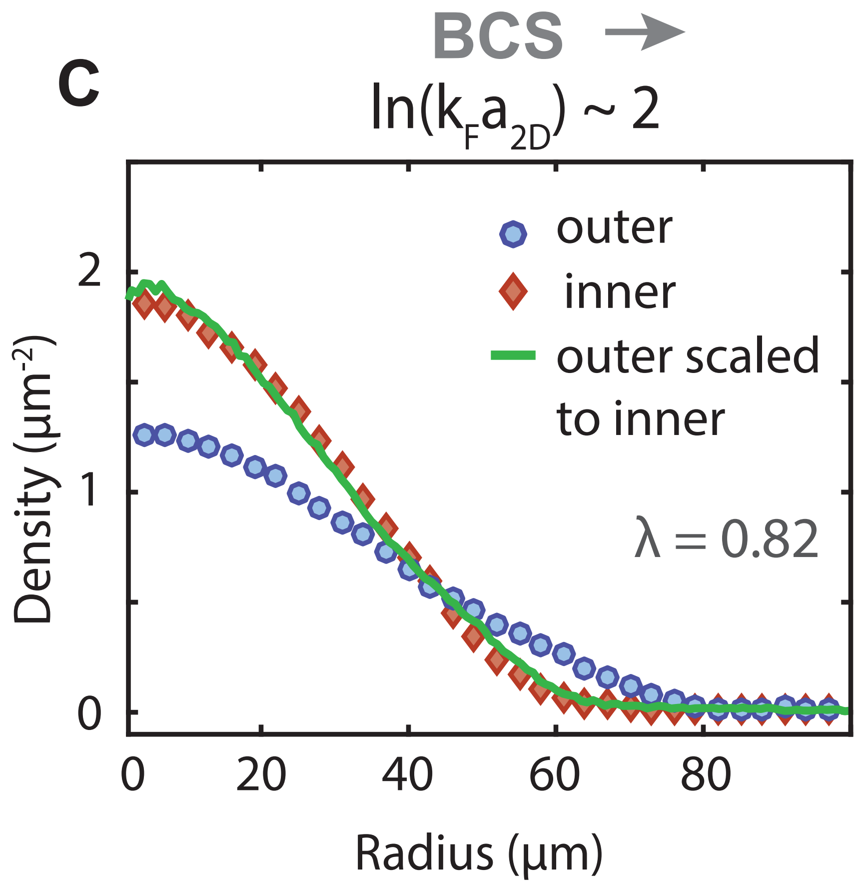
strong local correlation in crossover:
quantify scale invariance breaking

Scaling dynamics

A



C

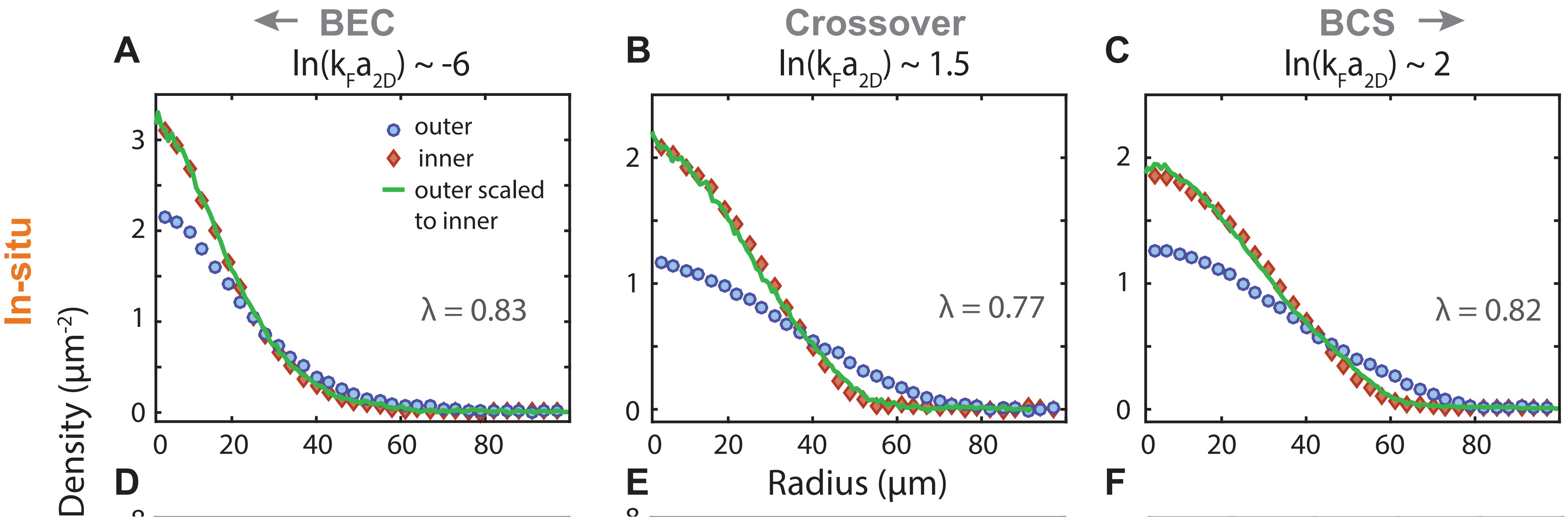


self-similar density profile $n(\boldsymbol{x},t) = \frac{1}{\lambda^2} n\left(\frac{\boldsymbol{x}}{\lambda}, t=0\right)$

P. A. Murthy, N. Defenu, L. Bayha, M. Holten, P. M. Preiss, T. Enss, S. Jochim, *in preparation*

Scaling dynamics

- BEC-BCS crossover

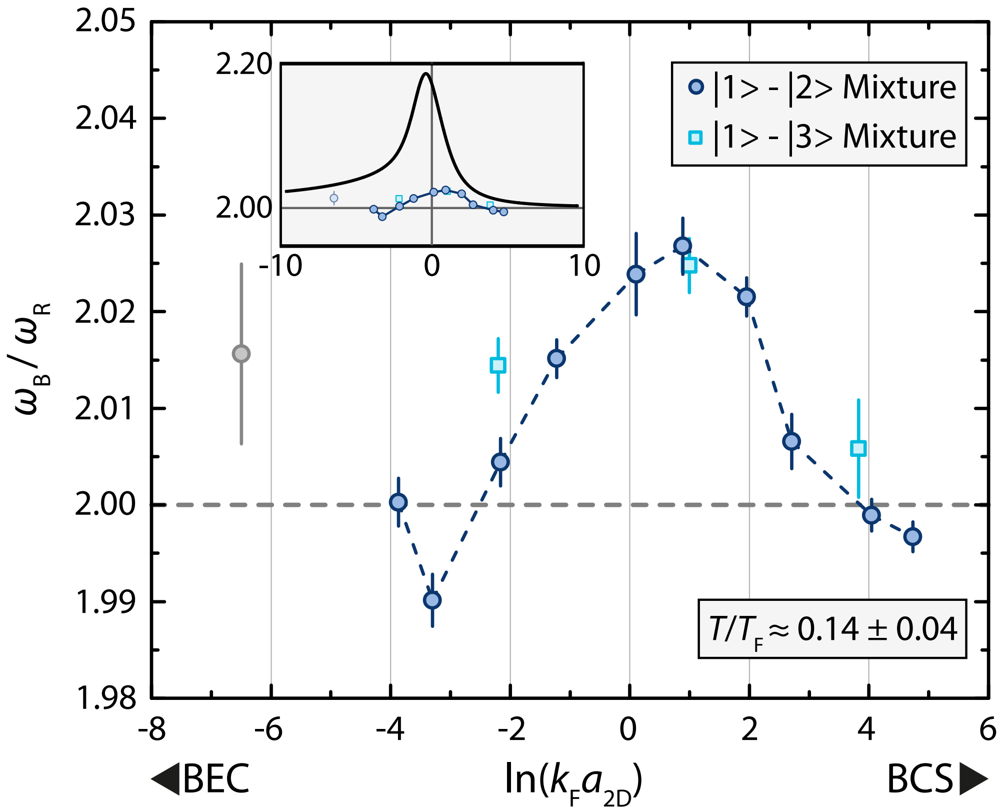


- density profiles satisfy scale invariant prediction!

P. A. Murthy, N. Defenu, L. Bayha, M. Holten, P. M. Preiss, T. Enss, S. Jochim, *in preparation*

Scaling dynamics

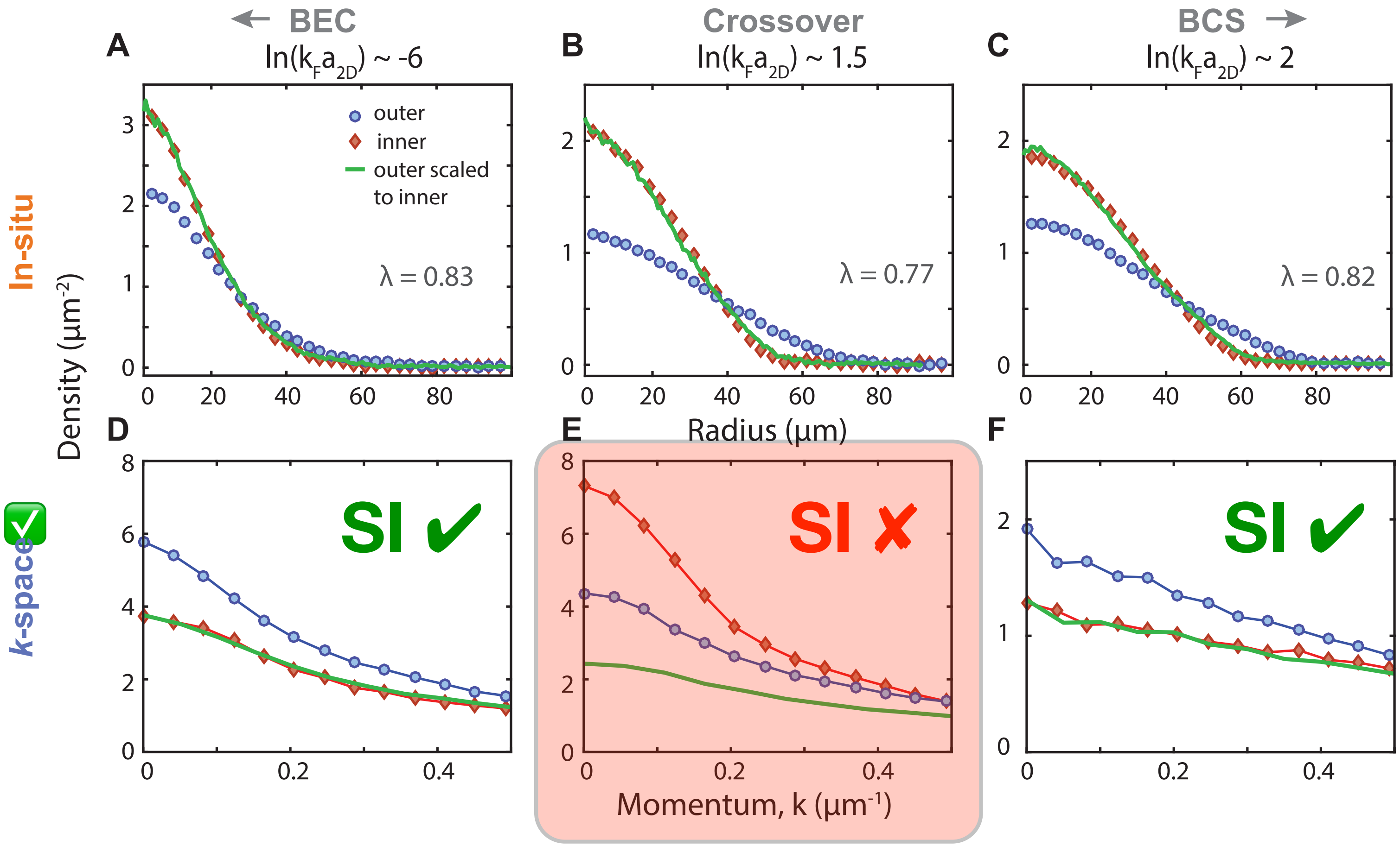
- breathing mode frequency $\omega_\mathrm{b} = 2\omega$?



- significant shift of breathing frequency where scale invariance broken

M. Holten, L. Bayha, A. C. Klein, P. A. Murthy, P. M. Preiss, and S. Jochim, arXiv:1803.08879

Scaling dynamics

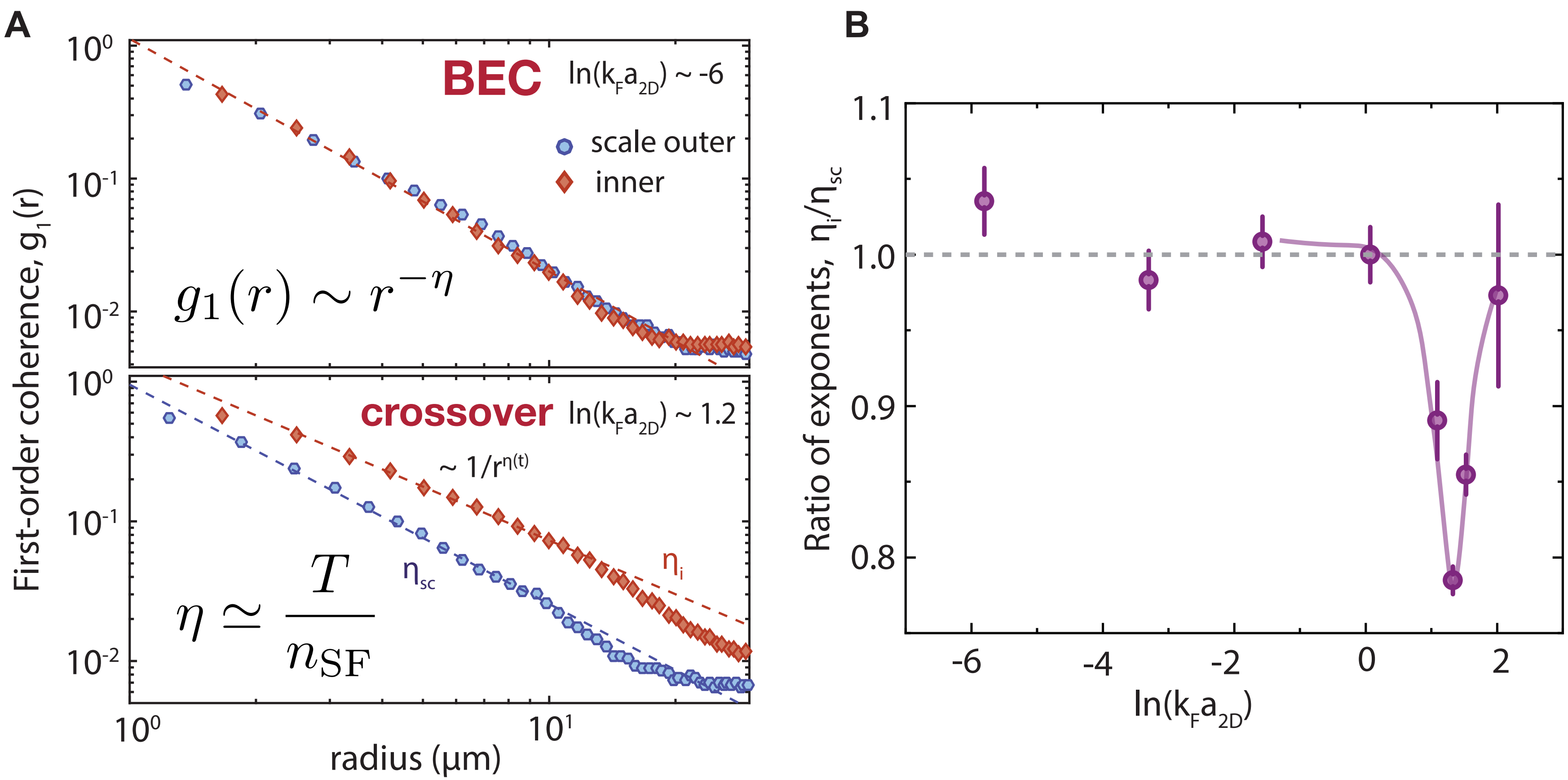


SI: $n(\boldsymbol{k},t) = \lambda^2 n\big(\lambda \boldsymbol{k}, t=0\big)$ at turning points $\dot{\lambda} = 0$

momentum distribution strongly violates scaling prediction in crossover

P. A. Murthy, N. Defenu, L. Bayha, M. Holten, P. M. Preiss, T. Enss, S. Jochim, *in preparation*

Phase correlations



- density scale invariant but **superfluid density n_{SF}** anomalously enhanced: **scale dependence (scaling violation) of critical exponent**

P. A. Murthy, N. Defenu, L. Bayha, M. Holten, P. M. Preiss, T. Enss, S. Jochim, *in preparation*

Conclusion

- 2D classical gas scale invariant, exact scaling dynamics
- **2D Fermi gas: quantum anomaly breaks scale invariance**

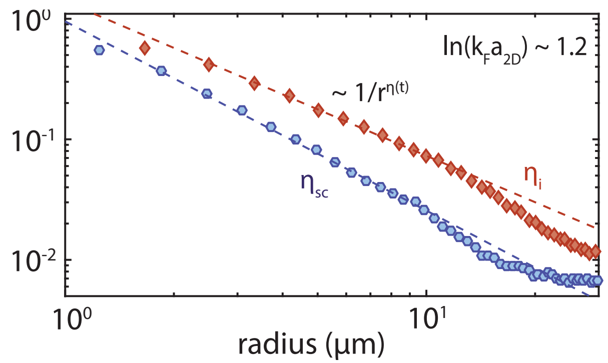
➡ density driven crossover from Bose to Fermi

➡Small breathing frequency corrections.

$$\ell \approx a_{2D}$$

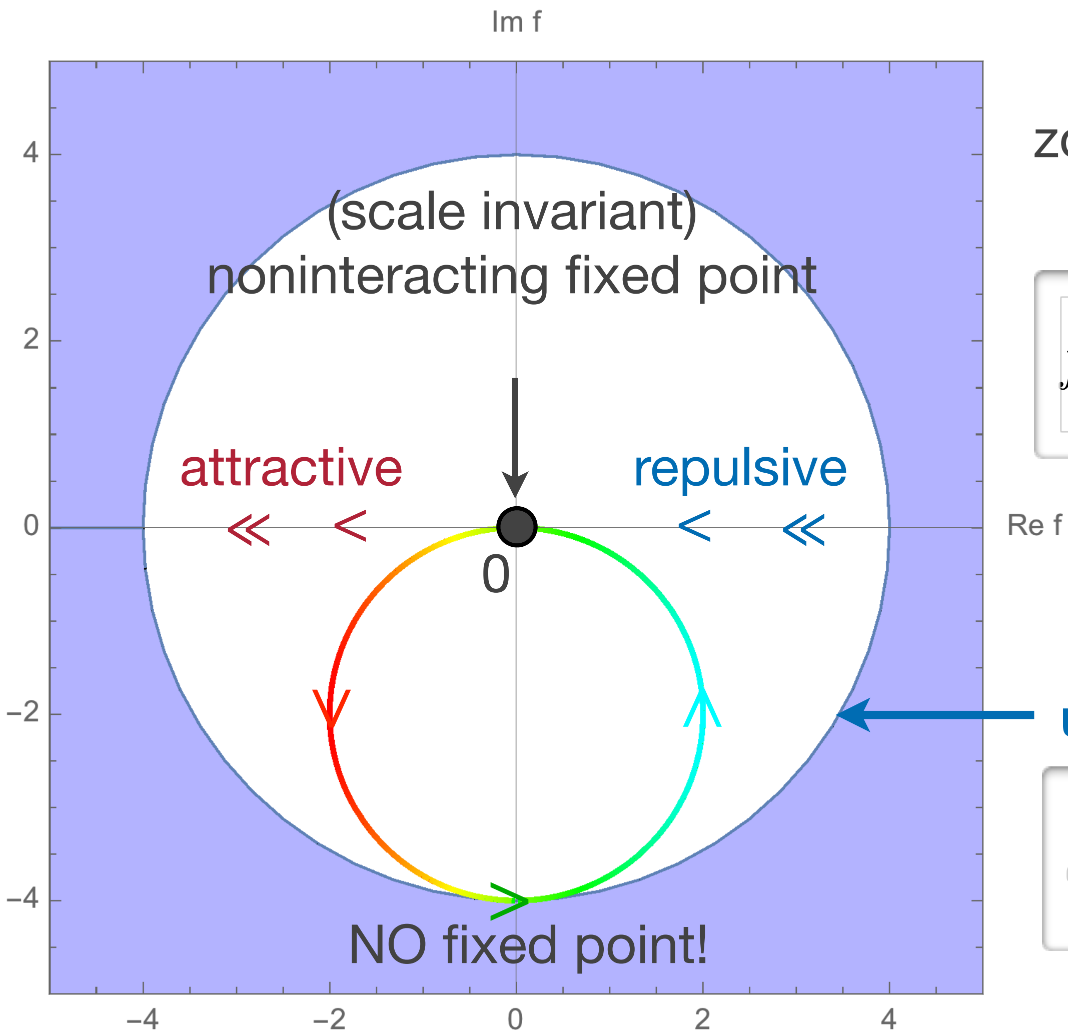
➡Small density profile corrections.

➡'Large' critical properties correction.



Additional material

Scale invariance and scattering



zoom out: $\frac{\partial f(k)}{\partial \ln k} = \frac{f(k)^2}{2\pi}$

$$f(k) = \frac{2\pi}{-\ln(ka) + i\pi/2}$$

unitarity bound:

$$\sigma = \frac{|f(k)|^2}{4k} \leq \frac{16}{4k}$$

at unitarity: 1. Re(f) changes, zero crossing; determines mean-field energy and ɣ

2. σ does not change; transport minimum $D_s \sim \frac{v}{n\sigma} \sim \hbar/m$