Time-Dependent Dynamics of Fermionic Superfluids: from cold atomic gases, to nuclei and neutron stars

Aurel Bulgac
University of Washington

Collaborators: Michael M. Forbes (WSU/Seattle)
S. Jin (Seattle)
Yuan-Lung (Alan) Luo (Seattle, now at a start-up)
Piotr Magierski (Warsaw/Seattle)
Kenneth J. Roche (PNNL/Seattle)
Rishi Sharma (Triumf, now Tata)
Ionel Stetcu (UW, now LANL)
Yongle Yu (Seattle, now Wuhan, PRC)
Sukjin Yoon (Seattle, now at APCTP)
Gabriel Wlazłowski (Seattle/Warsaw)

I will tell you why quantum hydrodynamics or GPE are not good enough for many phenomena in fermionic superfluids

Anderson-Higgs mode
TDDFT for fermionic superfluids
Selfbound superfluid liquid drops, two phase trasitions
Polarized unitary Fermi gas
Unitary Fermi Supersolid
Generating of quantized vortices, their crossing and recombination
Quantum Shock waves
Vortex rings, domain walls, solitonic vortex, etc.
Quantum turbulence
Pinning and anti-pinning of vortices in neutron star crust and glitches
Collisions of superfluid nuclei
Dynamics of fragmented condensates
Nuclear fission
Coulomb excitation of nuclei with relativistic heavy ions
Including dissipation and fluctuations into TDDFT

One option is the two-fluid hydrodynamics (here at T=0, only one fluid)

N.B. There is no quantum statistics in two-fluid hydrodynamics

$$\frac{\partial n(\vec{r},t)}{\partial t} + \vec{\nabla}\bullet\left[\vec{v}(\vec{r},t)n(\vec{r},t)\right] = 0$$

$$m\frac{\partial \vec{v}(\vec{r},t)}{\partial t} + \vec{\nabla}\left\{\frac{m\vec{v}^2(\vec{r},t)}{2} + \mu\left[n(\vec{r},t)\right] + V_{ext}(\vec{r},t)\right\} = 0$$

Troubles:

- **These are classical equations, <u>no Planck's constant</u>, thus no quantized vortices (unless one imposes by hand quantization)**
- **No physically clear physical mechanism to describe superfluid to normal transition (no role for the critical velocity)**

Two-fluid hydrodynamics + vortex quantization is equivalent to a ``Bohr model" of a superfluid

Another option is the phenomenological Ginzburg-Landau model or the Gross-Pitaevskii equation:

$$i\hbar e^{i\gamma}\frac{\partial\Psi(\vec{r},t)}{\partial t} = -\frac{\hbar^2\Delta\Psi(\vec{r},t)}{2M} + U\left(\left|\Psi(\vec{r},t)\right|^2\right)\Psi(\vec{r},t) + V_{ext}(\vec{r},t)\Psi(\vec{r},t) + fluct.$$

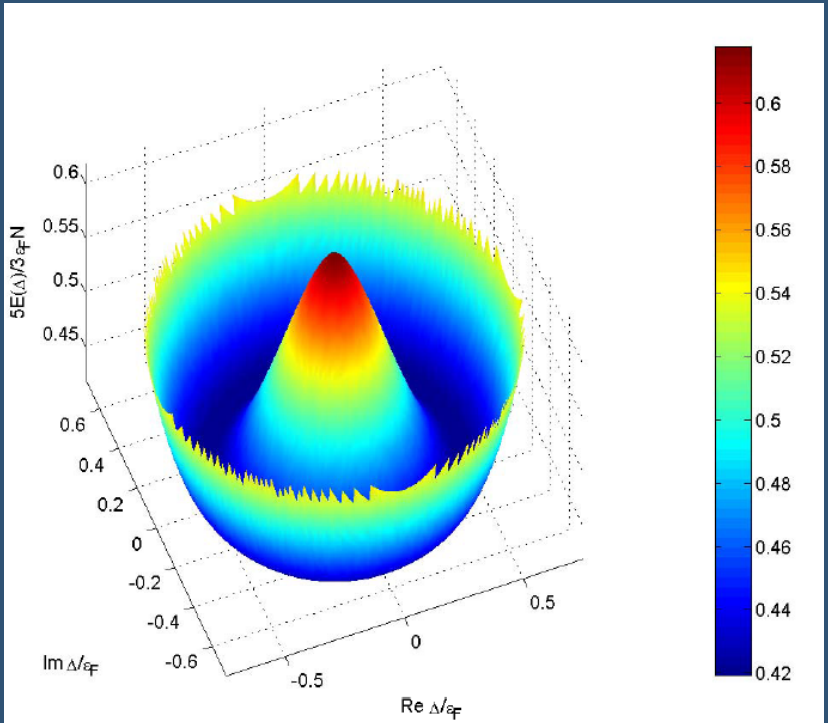
Troubles:

- **GLE valid only for temperatures near and below the critical temperature**
- **Even though is a quantum approach, it describes only the superfluid phase. There is no Cooper pair breaking mechanism**
- **GPE was the only microscopic equation available until recently, valid for a superfluid of weakly interacting bosons at T=0**

Other issues:

There are a number of modes, such as the Anderson-Higgs mode, which cannot be describes in either of these phenomenological approaches.

Energy of a Fermi system as a function of the pairing gap: Anderson-Higgs mode



Both fail

$$\dot{n} + \vec{\nabla}\cdot\left[\vec{\mathrm{v}} n\right] = 0$$

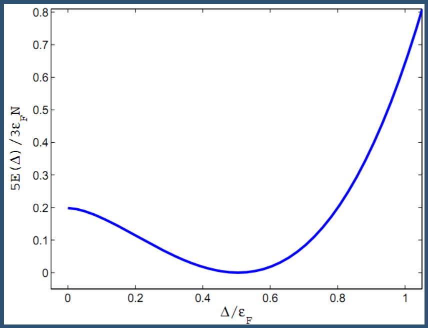
$$m\dot{\vec{\mathrm{v}}} + \vec{\nabla}\left\{\frac{m\vec{\mathrm{v}}^2}{2} + \mu\left[n\right]\right\} = 0$$

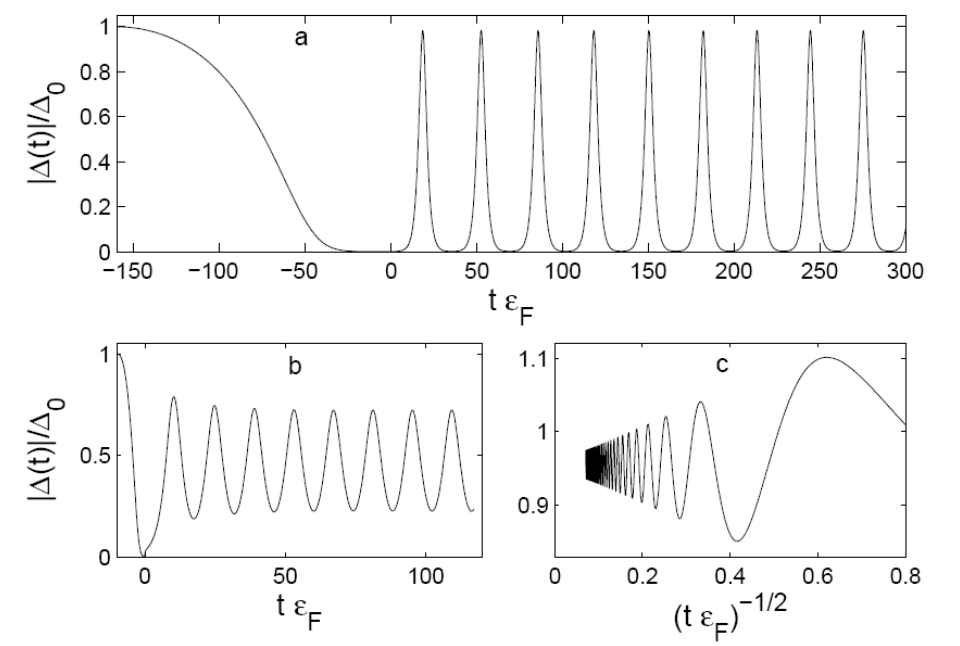
Landau's two-fluid hydrodynamics

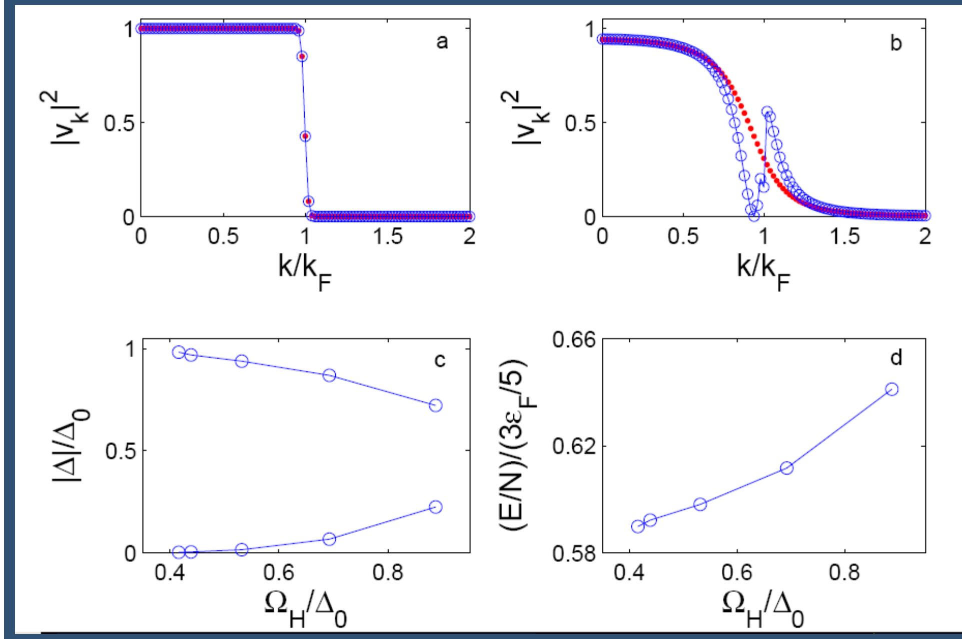
$$i\hbar e^{i\gamma}\dot{\Psi}(\vec{r},t) = -\frac{\hbar^2}{4m}\Delta\Psi(\vec{r},t) + U\left(\left|\Psi(\vec{r},t)\right|^2\right)\Psi(\vec{r},t)$$

Ginzburg-Landau-like equation

Response of a unitary Fermi system to changing the scattering length with time





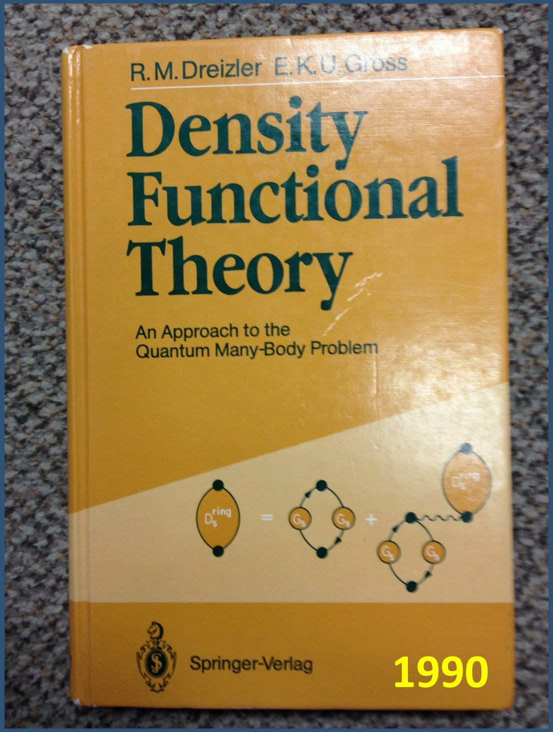


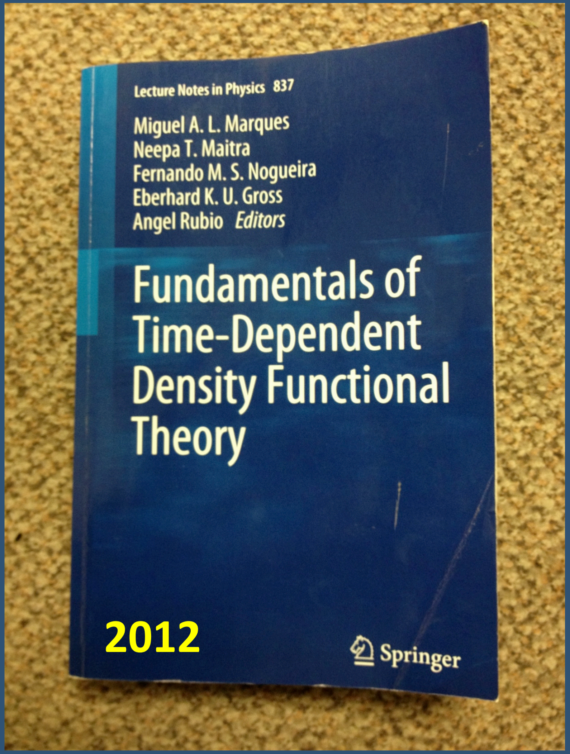
- All these modes have a very low frequency below the pairing gap, a very large amplitude and very large excitation energy

- None of these modes can be described either within two-fluid hydrodynamics or Ginzburg-Landau like approaches

Bulgac and Yoon, Phys. Rev. Lett. 102, 085302 (2009)

Main Theoretical Tool



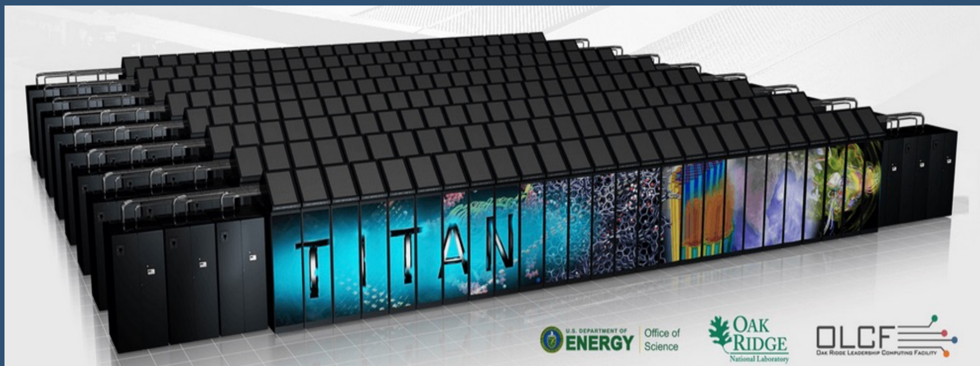


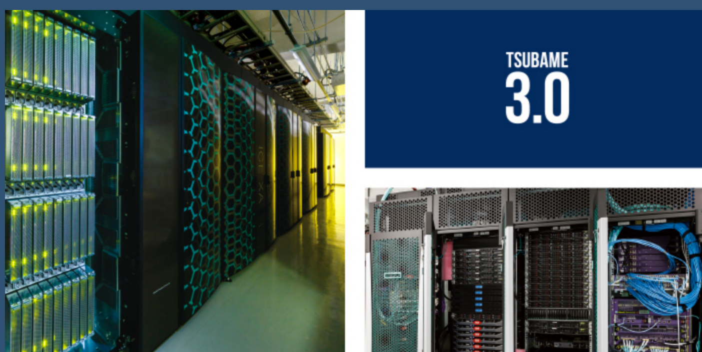
DFT has been developed and used mainly to describe normal (non-superfluid) electron systems – <u>50 years old theory</u>, Kohn and Hohenberg, 1964

A new local extension of DFT to superfluid systems and time-dependent phenomena was developed

Review: A. Bulgac, *Time-Dependent Density Functional Theory and Real-Time Dynamics of Fermi Superfluids*, Ann. Rev. Nucl. Part. Sci. 63, 97 (2013)

The Main Computational Tool





Cray XK7, ranked at peak ≈ 27 Petaflops (Peta – 10^{15})

On Titan there are 18,688 GPUs which provide 24.48 Petaflops !!!
and 299,008 CPUs which provide only 2.94 Petaflops.

A single GPU on Titan performs the same amount of FLOPs as approximately 134 CPUs.

Jaguar, Titan, Piz Daint, Tsubame 3.0, and Summit in the future

Kohn-Sham theorem (1965)

$$H = \sum_{i}^{N} T(i) + \sum_{i<j}^{N} U(ij) + \sum_{i<j<k}^{N} U(ijk) + \ldots + \sum_{i}^{N} V_{ext}(i)$$

$$H\Psi_0(1,2,\ldots N) = E_0\Psi_0(1,2,\ldots N)$$

$$n(\vec{r}) = \left\langle \Psi_0 \left| \sum_{i}^{N} \delta(\vec{r}-\vec{r}_i) \right| \Psi_0 \right\rangle$$

Injective map (one-to-one)

$$\Psi_0(1,2,\ldots N) \Leftrightarrow V_{ext}(\vec{r}) \Leftrightarrow n(\vec{r})$$

$$E_0 = \min_{n(\vec{r})} \int d^3r \left\{ \frac{\hbar^2}{2m^*(\vec{r})}\tau(\vec{r}) + \varepsilon\left[n(\vec{r})\right] + V_{ext}(\vec{r})n(\vec{r}) \right\}$$

$$n(\vec{r}) = \sum_{i}^{N} \left|\varphi_i(\vec{r})\right|^2, \qquad \tau(\vec{r}) = \sum_{i}^{N} \left|\vec{\nabla}\varphi_i(\vec{r})\right|^2$$

THEOREM: There exist an universal functional of particle density alone independent of the external potential

Normal Fermi systems only!

However, not everyone is normal!

The SLDA (DFT) energy density functional for unitary Fermi gas

Dimensional arguments, renormalizability, Galilean invariance, and symmetries ***determine the functional (energy density)***

$$\varepsilon(\vec{r}) = \frac{\hbar^2}{m}\left\{\left[\alpha\frac{\tau_c(\vec{r})}{2} + \gamma\frac{\left|\nu_c(\vec{r})\right|^2}{n^{1/3}(\vec{r})}\right] + \beta\frac{3(3\pi^2)^{2/3}n^{5/3}(\vec{r})}{5}\right\} - \frac{\hbar^2}{m}(\alpha-1)\frac{\vec{j}^{\,2}(\vec{r})}{2n(\vec{r})}$$

$$\Delta(\vec{r}) = \frac{\hbar^2}{m}\tilde{\Delta}(\vec{r})$$

$$n(\vec{r}) = 2\sum_{0<E_k<E_c}\left|\mathrm{v}_\mathrm{k}(\vec{r})\right|^2, \quad \tau_c(\vec{r}) = 2\sum_{0<E_k<E_c}\left|\vec{\nabla}\mathrm{v}_\mathrm{k}(\vec{r})\right|^2,$$

$$\nu_c(\vec{r}) = \sum_{0<E<E_c}\mathrm{u}_\mathrm{k}(\vec{r})\mathrm{v}^*_\mathrm{k}(\vec{r}) \quad \Leftarrow \ \text{divergent without a cutoff, need RG}$$

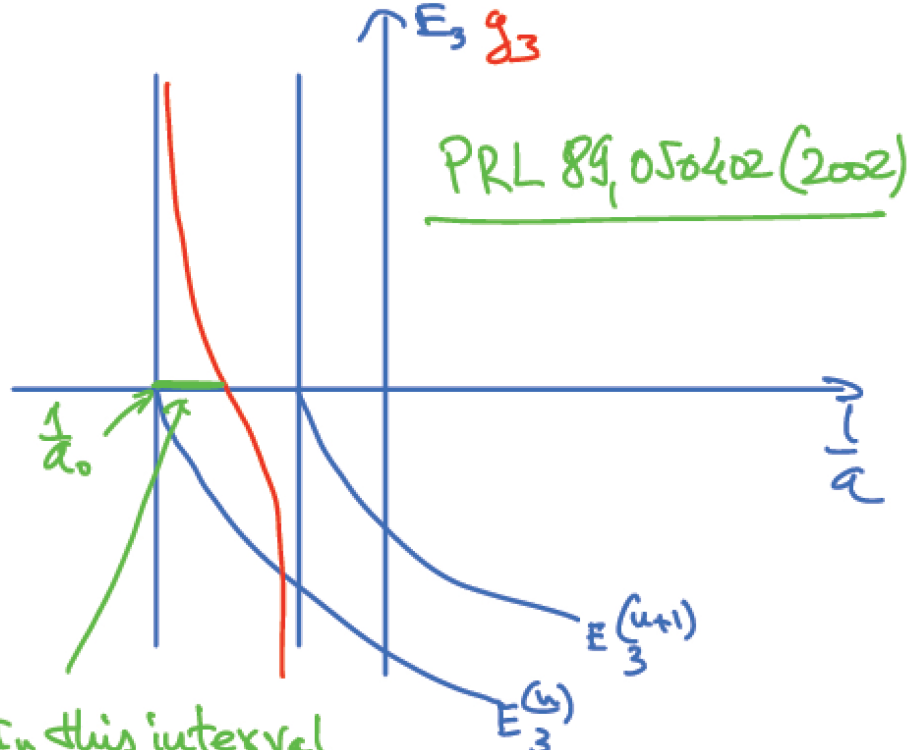
Three dimensionless constants α, β, and γ determining the functional are extracted from QMC for homogeneous systems by fixing the total energy, the pairing gap and the effective mass

The unitary Fermi gas and the dilute Bose gas are the only superfluids for which a microscopic framework exist to describe both statics and dynamics

Normal State			
(N_a, N_b)	E_{FNDMC}	E_{ASLDA}	(error)
(3,1)	6.6 ± 0.01	6.687	1.3%
(4,1)	8.93 ± 0.01	8.962	0.36%
(5,1)	12.1 ± 0.1	12.22	0.97%
(5,2)	13.3 ± 0.1	13.54	1.8%
(6,1)	15.8 ± 0.1	15.65	0.93%
(7,2)	19.9 ± 0.1	20.11	1.1%
(7,3)	20.8 ± 0.1	21.23	2.1%
(7,4)	21.9 ± 0.1	22.42	2.4%
(8,1)	22.5 ± 0.1	22.53	0.14%
(9,1)	25.9 ± 0.1	25.97	0.27%
(9,2)	26.6 ± 0.1	26.73	0.5%
(9,3)	27.2 ± 0.1	27.55	1.3%
(9,5)	30 ± 0.1	30.77	2.6%
(10,1)	29.4 ± 0.1	29.41	0.034%
(10,2)	29.9 ± 0.1	30.05	0.52%
(10,6)	35 ± 0.1	35.93	2.7%
(20,1)	73.78 ± 0.01	73.83	0.061%
(20,4)	73.79 ± 0.01	74.01	0.3%
(20,10)	81.7 ± 0.1	82.57	1.1%
(20,20)	109.7 ± 0.1	113.8	3.7%
(35,4)	154 ± 0.1	154.1	0.078%
(35,10)	158.2 ± 0.1	158.6	0.27%
(35,20)	178.6 ± 0.1	180.4	1%

Superfluid State			
(N_a, N_b)	E_{FNDMC}	E_{ASLDA}	(error)
(1,1)	2.002 ± 0	2.302	15%
(2,2)	5.051 ± 0.009	5.405	7%
(3,3)	8.639 ± 0.03	8.939	3.5%
(4,4)	12.573 ± 0.03	12.63	0.48%
(5,5)	16.806 ± 0.04	16.19	3.7%
(6,6)	21.278 ± 0.05	21.13	0.69%
(7,7)	25.923 ± 0.05	25.31	2.4%
(8,8)	30.876 ± 0.06	30.49	1.2%
(9,9)	35.971 ± 0.07	34.87	3.1%
(10,10)	41.302 ± 0.08	40.54	1.8%
(11,11)	46.889 ± 0.09	45	4%
(12,12)	52.624 ± 0.2	51.23	2.7%
(13,13)	58.545 ± 0.18	56.25	3.9%
(14,14)	64.388 ± 0.31	62.52	2.9%
(15,15)	70.927 ± 0.3	68.72	3.1%
(1,0)	1.5 ± 0.0	1.5	0%
(2,1)	4.281 ± 0.004	4.417	3.2%
(3,2)	7.61 ± 0.01	7.602	0.1%
(4,3)	11.362 ± 0.02	11.31	0.49%
(7,6)	24.787 ± 0.09	24.04	3%
(11,10)	45.474 ± 0.15	43.98	3.3%
(15,14)	69.126 ± 0.31	62.55	9.5%

Bulgac, Forbes, and Magierski, Lecture Notes in Physics (2012)



In this interval

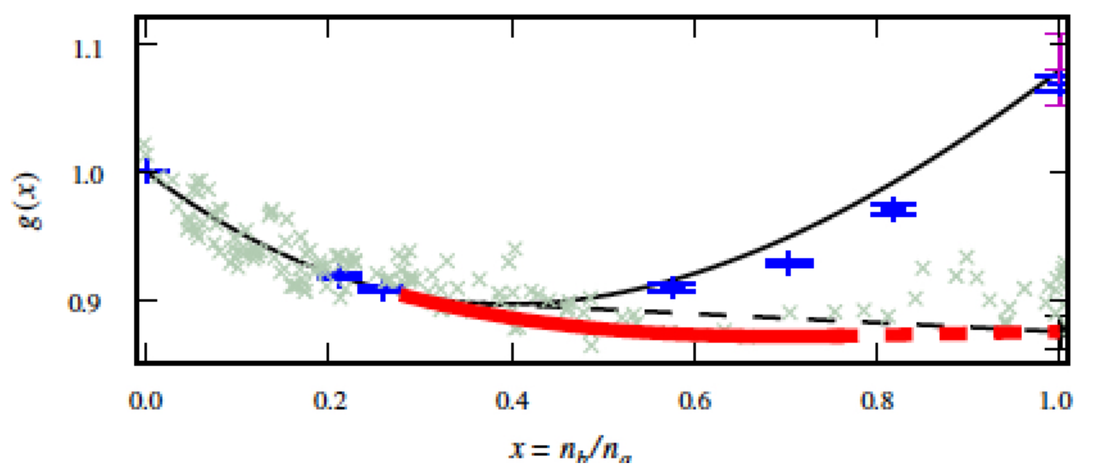
$$g_2 = \frac{4\pi \hbar^2 a}{m} < 0$$

$$g_3 = \frac{12\pi \hbar^2 a^4}{m}\left[d_1 + d_2 \tan\left(s_0 \ln\left|\frac{a}{a_0}\right| + \frac{\pi}{2}\right)\right]$$

$$\rho_0 = -\frac{3g_2}{2g_3}, \quad E_0 = \frac{1}{2} g_2 \rho^2 + \frac{1}{6} g_3 \rho^3$$

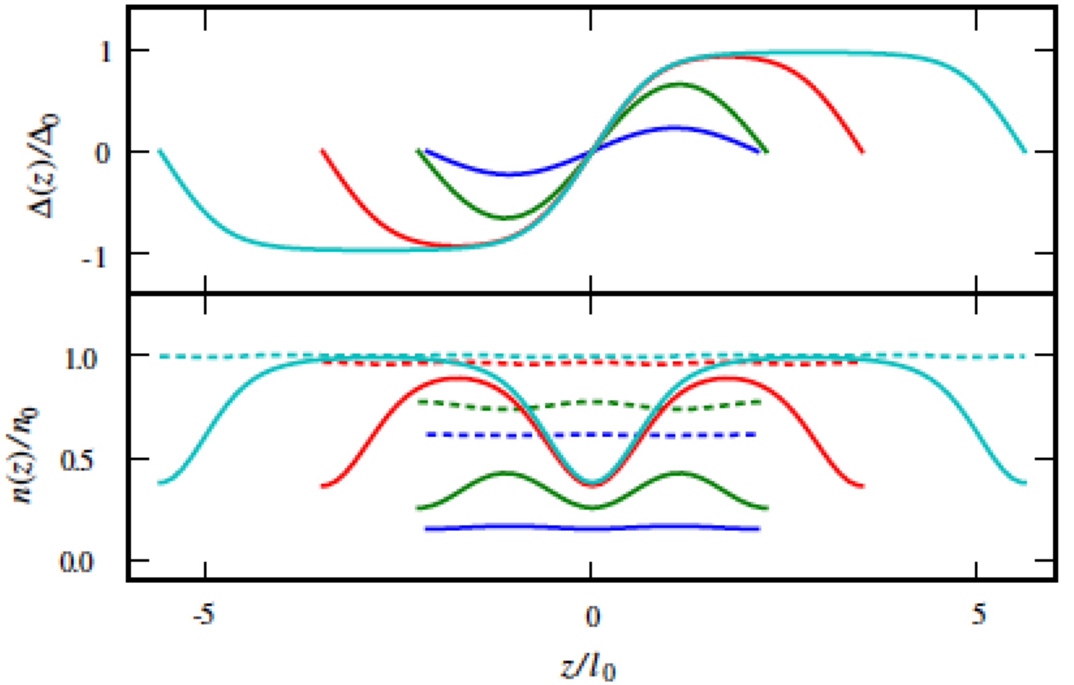
$$\rho_0 = \frac{1}{2|a|^3} \times \frac{1}{d_1 + d_2 \tan\left(s_0 \ln\left|\frac{a}{a_0}\right| + \frac{\pi}{2}\right)}, \quad d_2 < 0$$

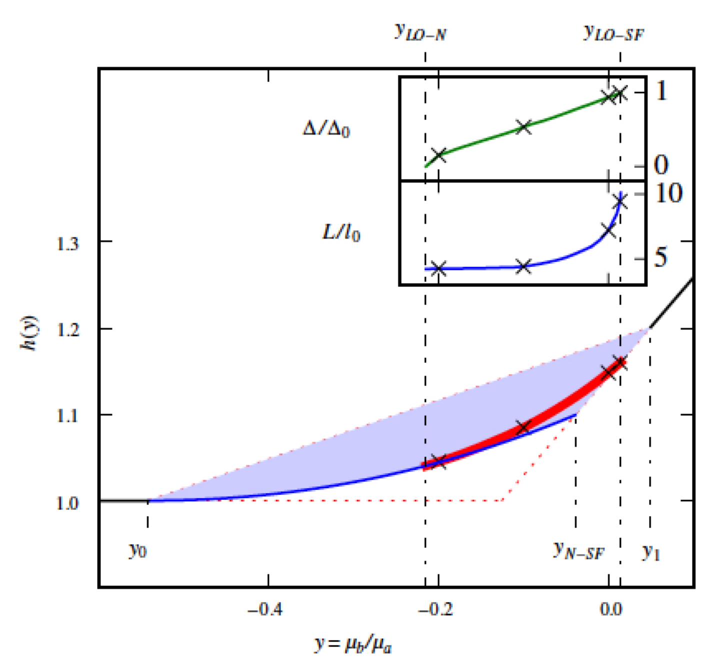
Unitary Fermi Supersolid: The Larkin-Ovchinnikov Phase
Bulgac and Forbes, Phys. Rev.Lett. 101, 215301 (20108)



$$E(n_a, n_b) = \frac{3}{5}\frac{\hbar^2}{2m}\left(6\pi^2\right)^{2/3}\left[n_a g\left(\frac{n_b}{n_a}\right)\right]^{5/3}$$

$$P(\mu_a, \mu_b) = \frac{2}{5}\left(\frac{2m}{\hbar^2}\right)^{3/2}\left[\mu_a h(y)\right]^{5/2}\frac{1}{6\pi^2}$$





Formalism for Time-Dependent Phenomena

"The time-dependent density functional theory is viewed in general as a reformulation of the exact quantum mechanical time evolution of a many-body system when only one-body properties are considered."

A.K. Rajagopal and J. Callaway, Phys. Rev. B 7, 1912 (1973)
V. Peuckert, J. Phys. C 11, 4945 (1978)
E. Runge and E.K.U. Gross, Phys. Rev. Lett. 52, 997 (1984)

http://www.tddft.org

$$E(t) = \int d^3r \left[\varepsilon(n(\vec{r},t), \tau(\vec{r},t), \nu(\vec{r},t), \underline{\vec{j}(\vec{r},t)}) + V_{ext}(\vec{r},t)\, n(\vec{r},t) + \ldots \right]$$

$$\begin{cases} [h(\vec{r},t) + V_{ext}(\vec{r},t) - \mu]\mathrm{u}_i(\vec{r},t) + [\Delta(\vec{r},t) + \Delta_{ext}(\vec{r},t)]\mathrm{v}_i(\vec{r},t) = i\hbar \dfrac{\partial \mathrm{u}_i(\vec{r},t)}{\partial t} \\ [\Delta^*(\vec{r},t) + \Delta^*_{ext}(\vec{r},t)]\mathrm{u}_i(\vec{r},t) - [h(\vec{r},t) + V_{ext}(\vec{r},t) - \mu]\mathrm{v}_i(\vec{r},t) = i\hbar \dfrac{\partial \mathrm{v}_i(\vec{r},t)}{\partial t} \end{cases}$$

For time-dependent phenomena one has to add currents.
Galilean invariance determines the dependence on currents.

TDSLDA equations

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{n\uparrow}(\vec{r},t) \\ u_{n\downarrow}(\vec{r},t) \\ v_{n\uparrow}(\vec{r},t) \\ v_{n\downarrow}(\vec{r},t) \end{pmatrix} = \begin{pmatrix} \hat{h}_{\uparrow\uparrow}(\vec{r},t)-\mu & \hat{h}_{\uparrow\downarrow}(\vec{r},t) & 0 & \Delta(\vec{r},t) \\ \hat{h}_{\downarrow\uparrow}(\vec{r},t) & \hat{h}_{\downarrow\downarrow}(\vec{r},t)-\mu & -\Delta(\vec{r},t) & 0 \\ 0 & -\Delta^*(\vec{r},t) & -\hat{h}^*_{\uparrow\uparrow}(\vec{r},t)+\mu & -\hat{h}^*_{\uparrow\downarrow}(\vec{r},t) \\ \Delta^*(\vec{r},t) & 0 & -\hat{h}^*_{\downarrow\uparrow}(\vec{r},t) & -\hat{h}^*_{\downarrow\downarrow}(\vec{r},t)+\mu \end{pmatrix}\begin{pmatrix} u_{n\uparrow}(\vec{r},t) \\ u_{n\downarrow}(\vec{r},t) \\ v_{n\uparrow}(\vec{r},t) \\ v_{n\downarrow}(\vec{r},t) \end{pmatrix}$$

- The system is placed on a large 3D spatial lattice (adequate representation of continuum)
- Derivatives are computed with FFTW (this insures machine accuracy) and is very fast
- Fully self-consistent treatment with fundamental symmetries respected (isospin, gauge, Galilean, rotation, translation)
- Adams-Bashforth-Milne fifth order predictor-corrector-modifier integrator
  Effectively a sixth order method
- No symmetry restrictions
- *Number of PDEs is of the order of the number of spatial lattice points – from 10,000s to 1-2,000,000*

$$\propto 4\left(\frac{2p_c L}{2\pi\hbar}\right)^3 = 4N_x N_y N_z$$

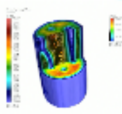
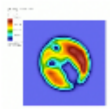
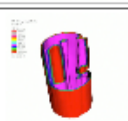
- SLDA/TDSLDA (DFT) is formally by construction like meanfield HFB/BdG
- The code was implemented on Jaguar, Titan, Franklin, Hopper, Edison, Hyak, Athena
- Initially Fortran 90, 95, 2003 …, presently C, CUDA, and obviously MPI, threads, etc.

Several hours of videos

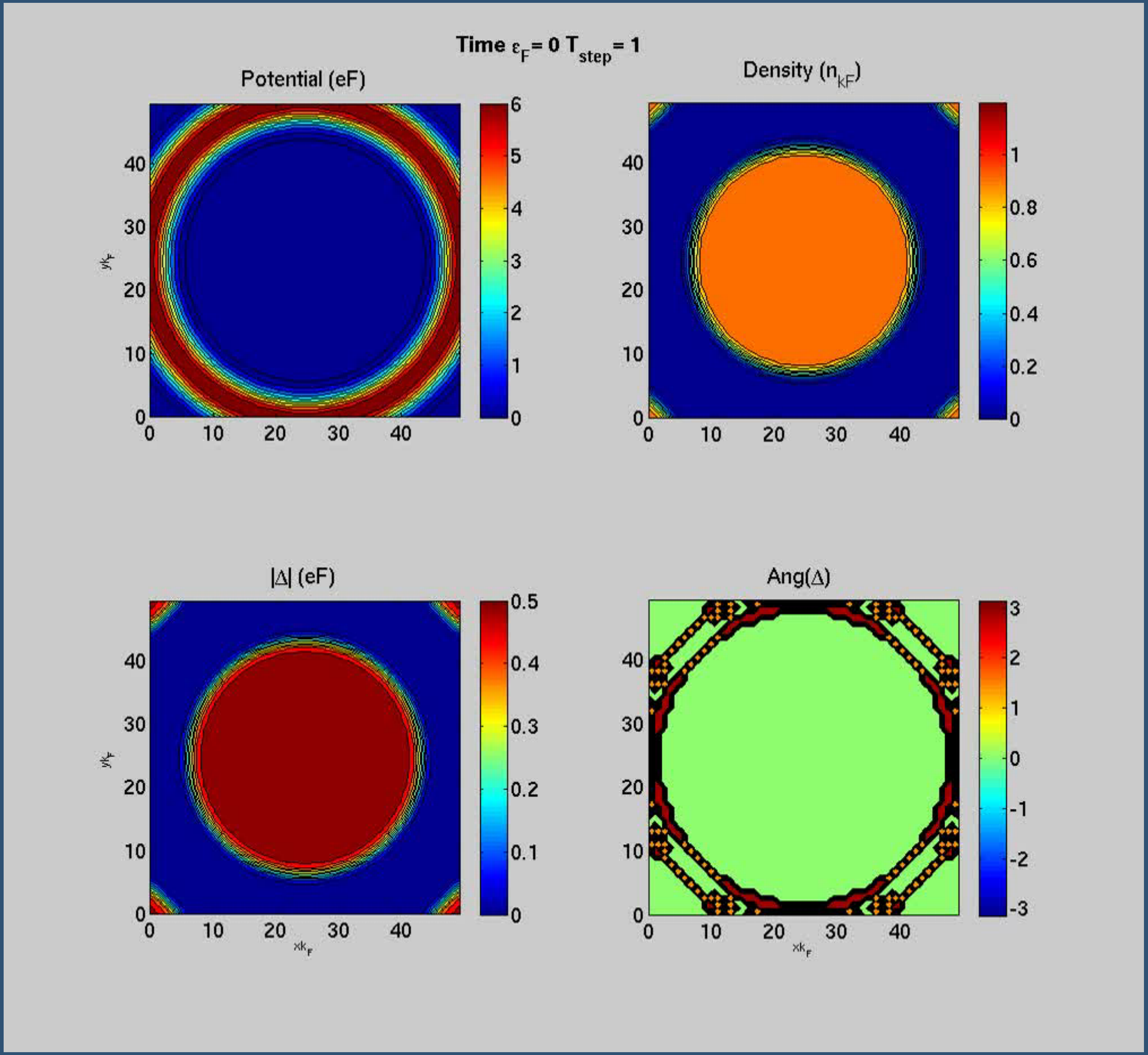
The Superfluid Local Density Approximation Applied to Unitary Fermi Gases -Supplementary Material

All simulations can be found here: http://www.phys.washington.edu/groups/qmbnt/UFG. The simulations can be categorized by the excitations: ball and rod, centered ball, centered small ball, centered big ball, centered supersonic ball, off-centered ball, and twisted stirrer. The following table matches simulations with numerical experiments. In several studies, we present multiple perspectives of the event as well as different plotting schemes to reveal different features of the dynamics.

3D Simulations

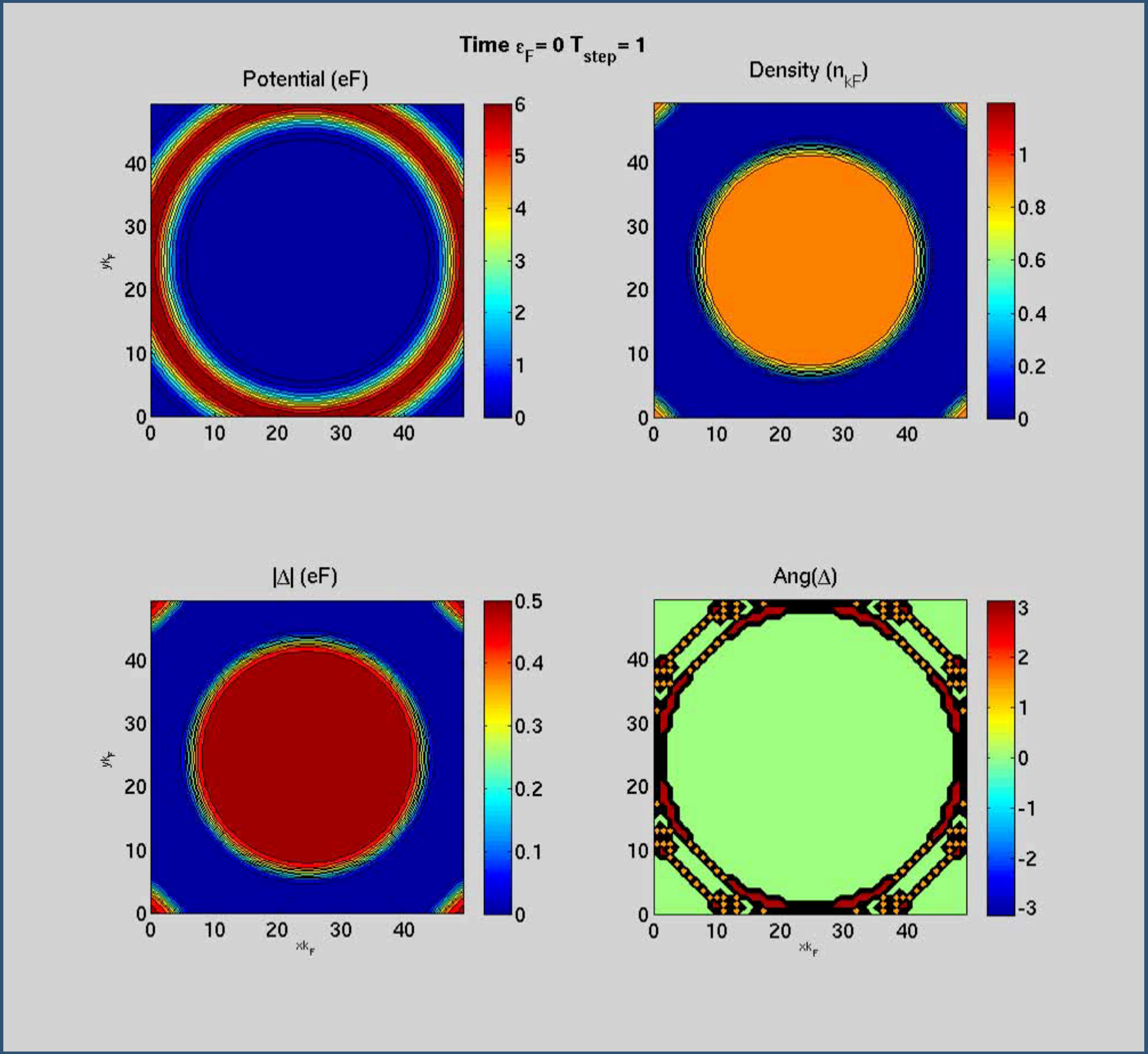
Excitation	Link	Description
Ball and Rod		
	nt-ball-rod-dns.m4v	density volume plot of magnitude of pairing field; front facing with quarter segment slice; 5m28s duration (20.9 MB)
	nt-ball-rod-dns-pln.m4v	density volume plot of magnitude of pairing field; 2D slice; 5m28s duration (9.8MB)
	nt-ball-rod-thin-angl.m4v	density contour plot of magnitude of pairing field focused on vortices ; angled front-facing with quarter segment slice; 5m28s duration (12.8MB)
Centered Ball		
	nt-ball-c.m4v	density contour plot of magnitude of pairing field focused on vortices; full geometry ; 3m29s

A. Bulgac, Y.-L. Luo, P. Magierski, K.J. Roche, Y. Yu
Science, 332, 1288 (2011)

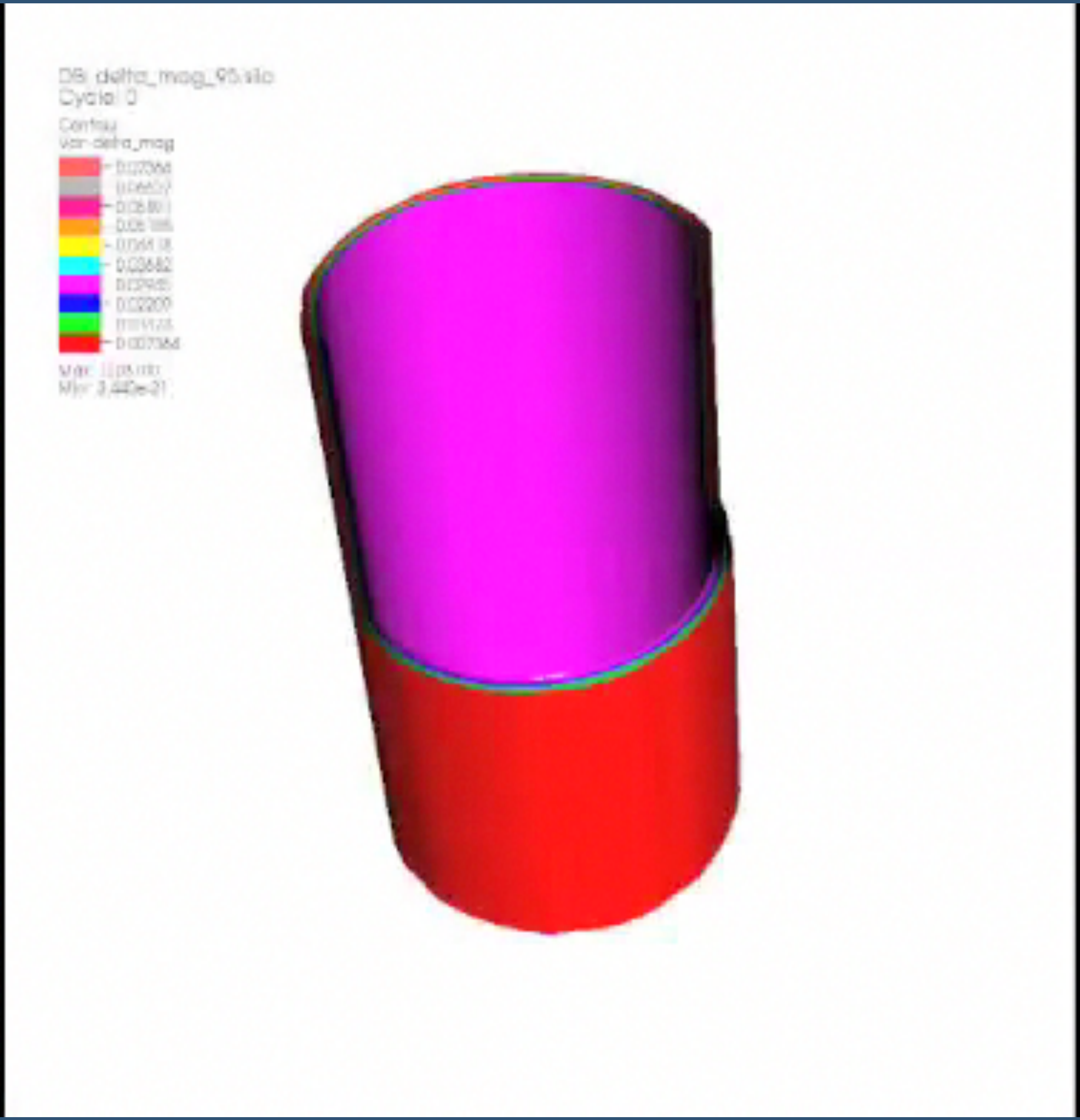


Time ε_F= 0 T_{step}= 1
Potential (eF)
Density (n_{kF})
|Δ| (eF)
Ang(Δ)
yk_F
xk_F

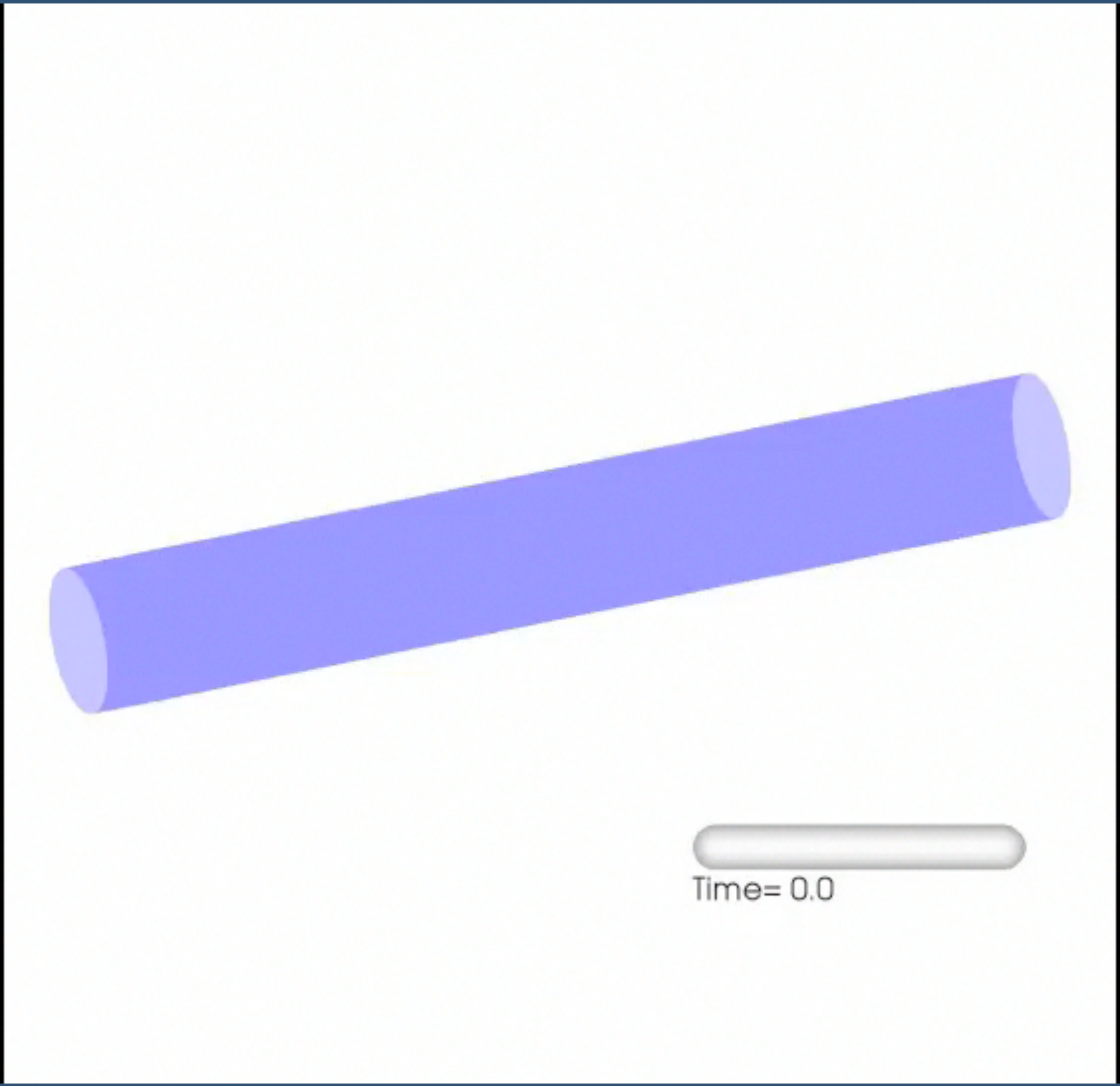
Movie


Time ε_F= 0 T_{step}= 1
Potential (eF)
Density (n_{kF})
|Δ| (eF)
Ang(Δ)
yk_F
xk_F

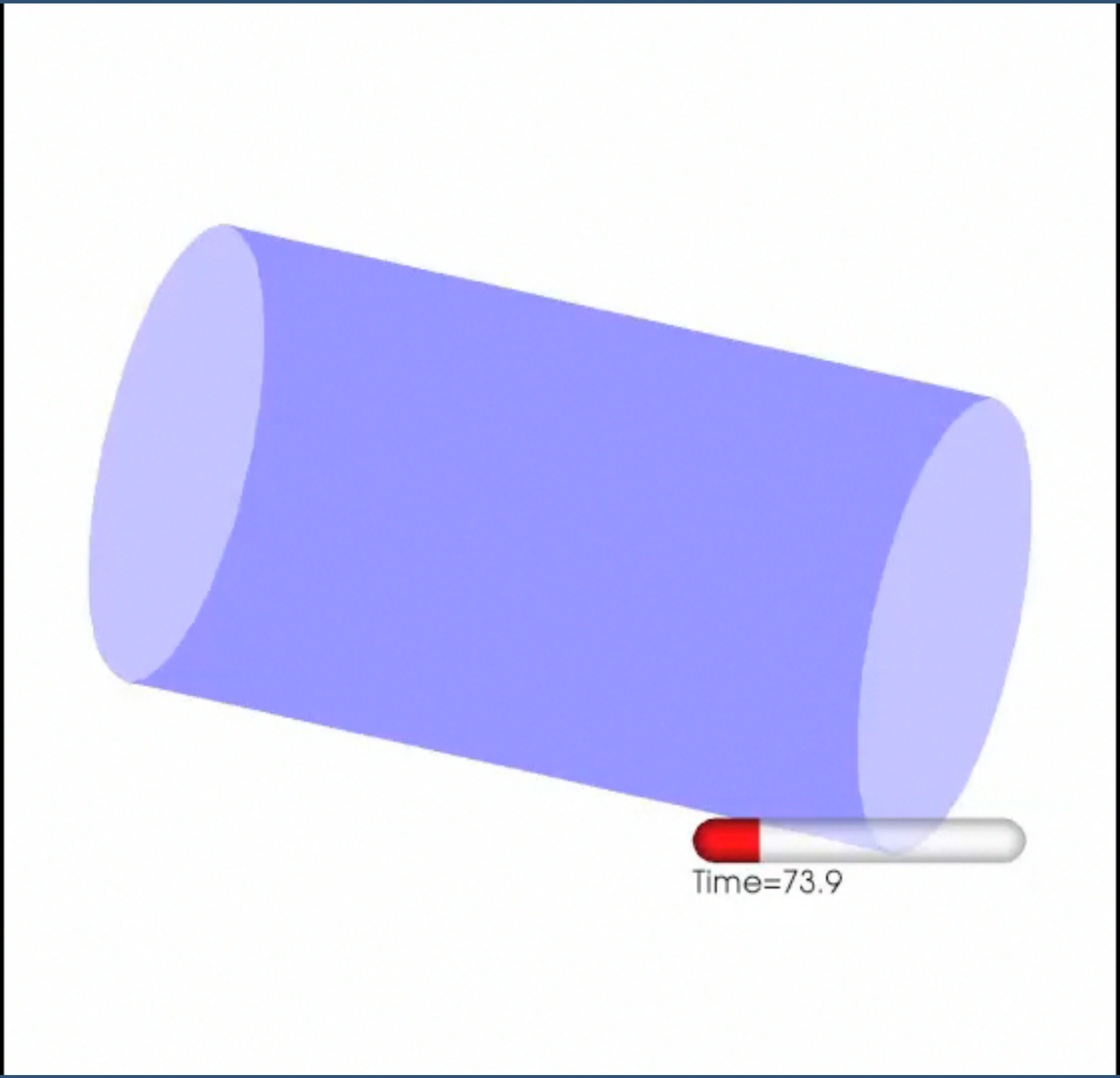
Movie



Movie



Movie



Movie

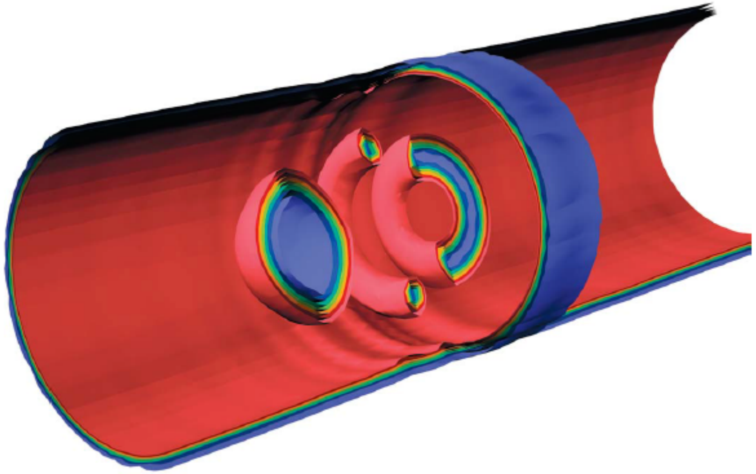


Fig. 2. A spherical projectile flying along the symmetry axis leaves in its wake two vortex rings.

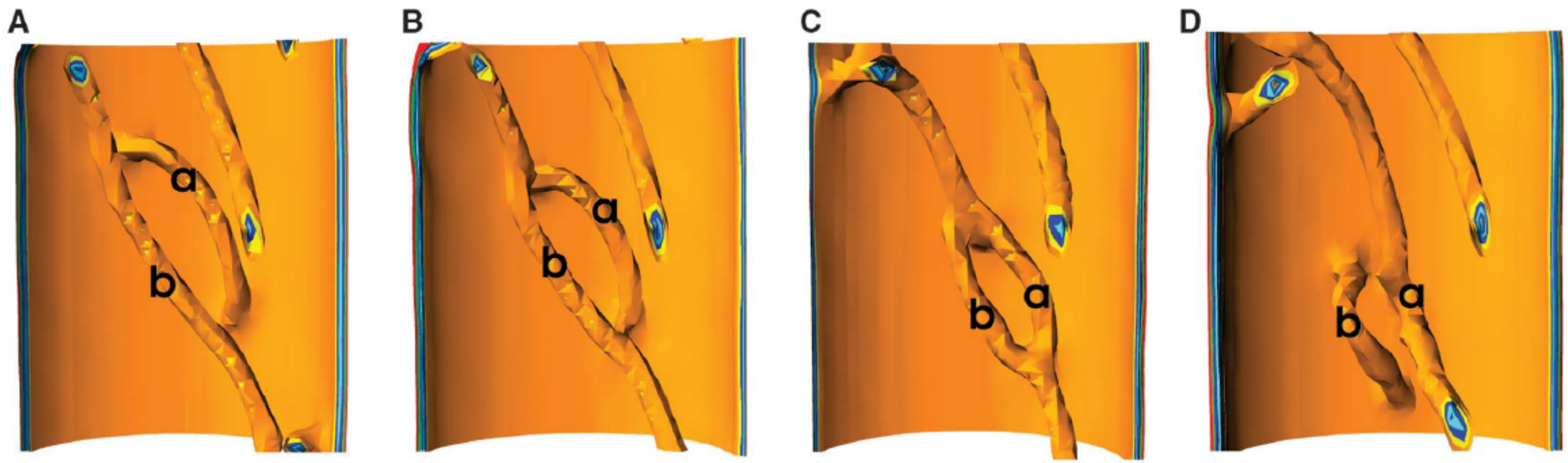
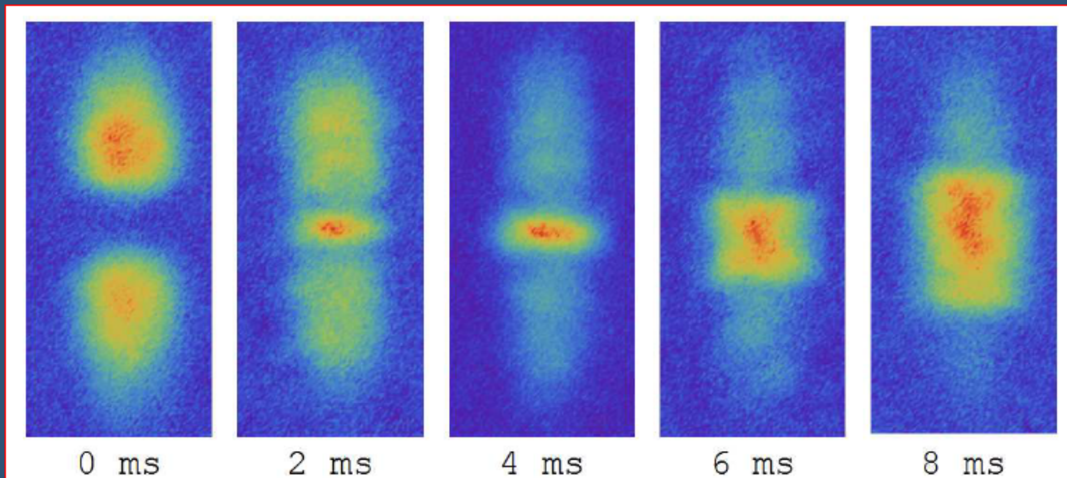
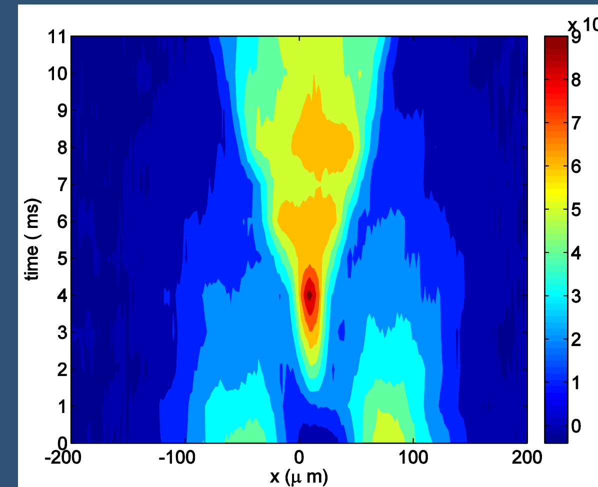


Fig. 3. (**A** to **D**) Two vortex lines approach each other, connect at two points, form a ring and exchange between them a portion of the vortex line, and subsequently separate. Segment (a), which initially belonged to the vortex line attached to the wall, is transferred to the long vortex line (b) after reconnection and vice versa.

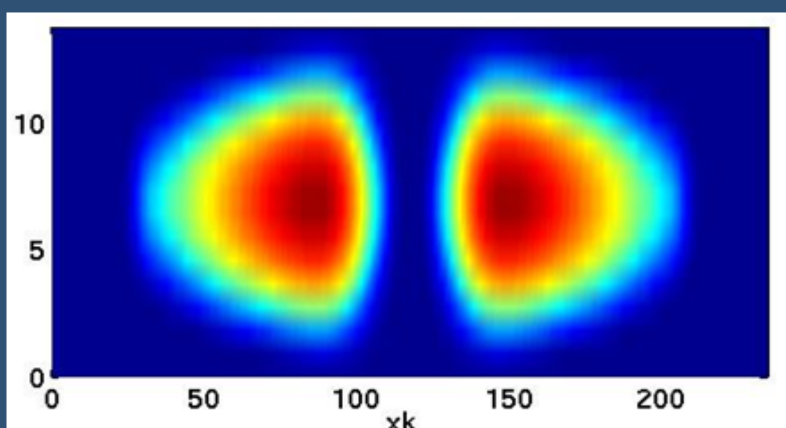
A. Bulgac, Y.-L. Luo, P. Magierski, K.J. Roche, Y. Yu
Science, 332, 1288 (2011)

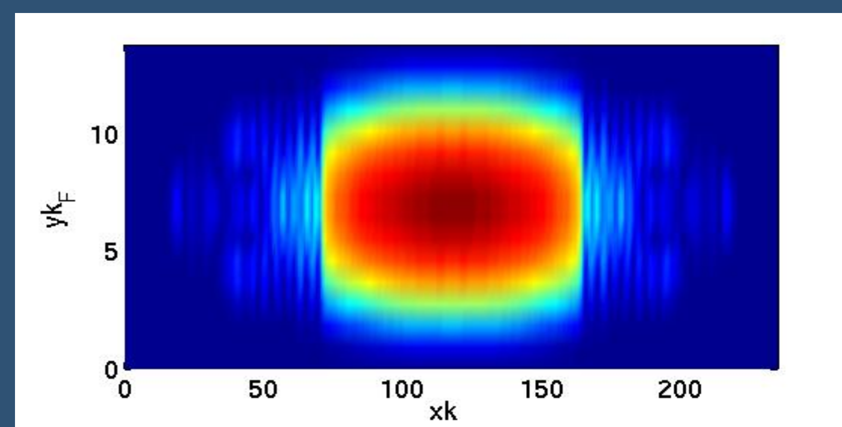




Observation of shock waves in a strongly interacting Fermi gas

J. Joseph, J.E. Thomas, M. Kulkarni, and A.G. Abanov PRL 106, 150401 (2011)

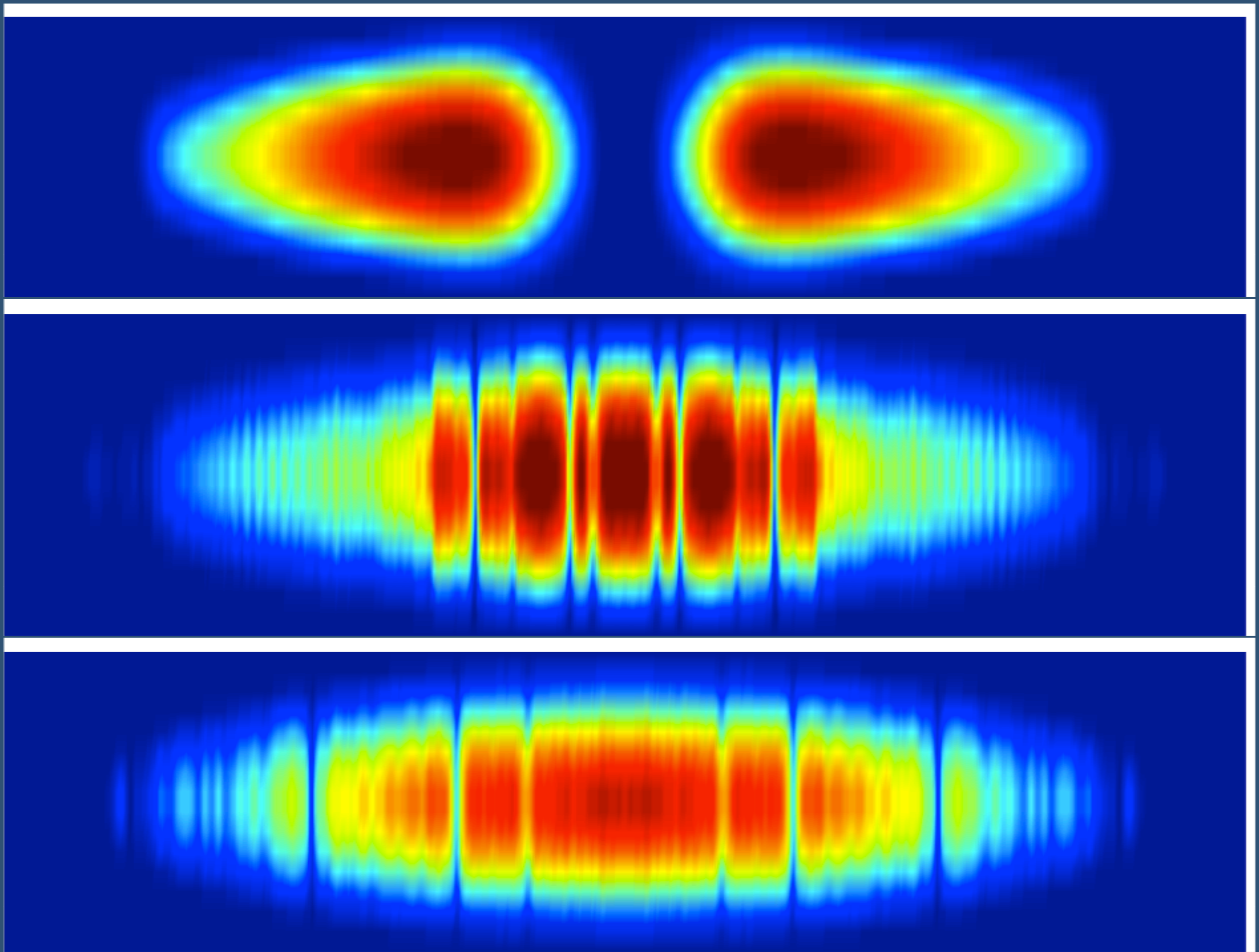




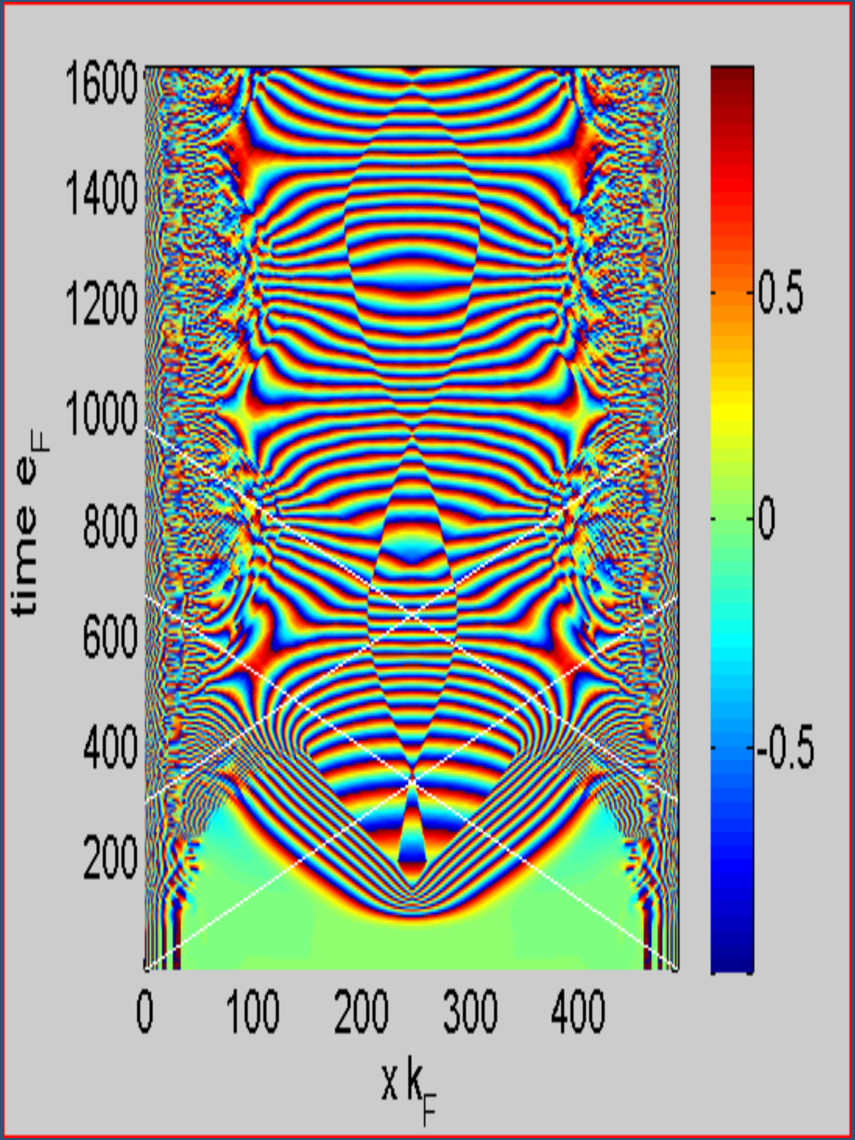
Number density of two colliding cold Fermi gases in TDSLDA

Bulgac, Luo, and Roche, Phys. Rev. Lett. 108, 150401 (2012)

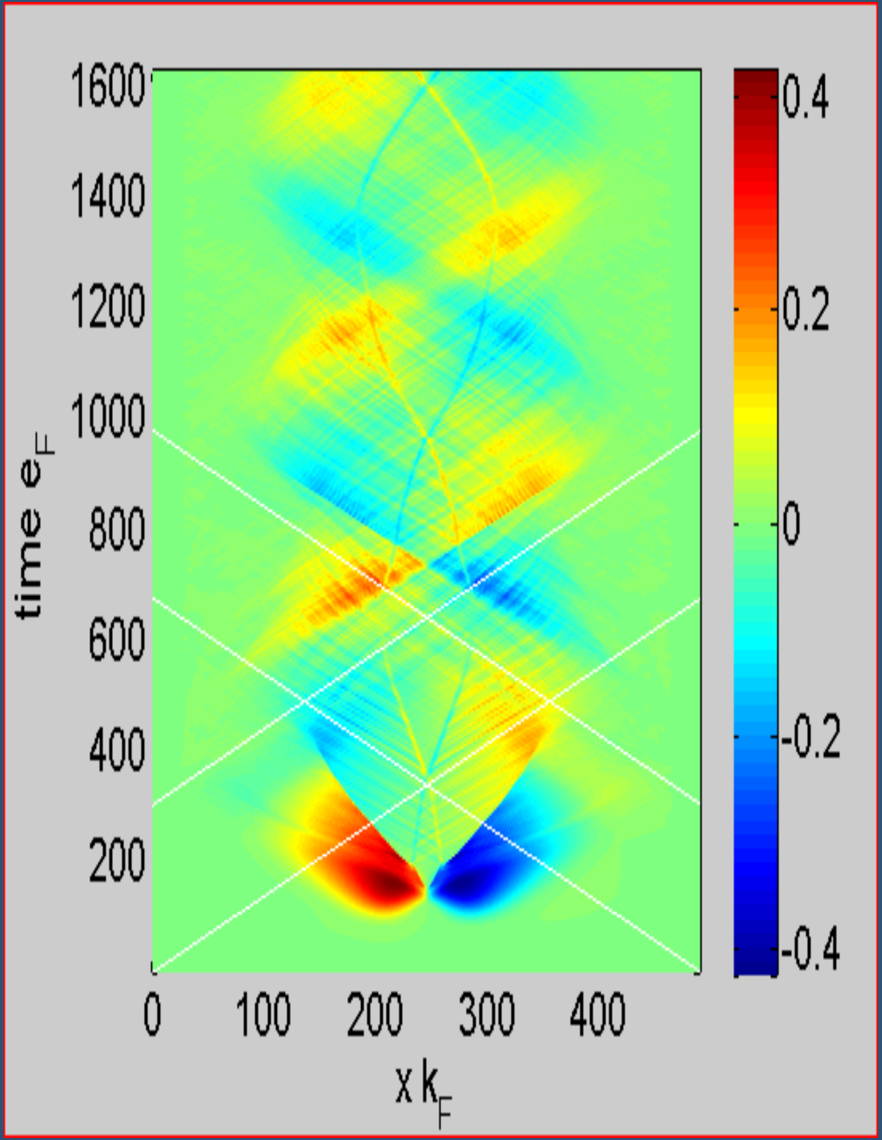
Collision of clouds with larger aspect ratio



Dark solitons/domain walls and shock waves in the collision of two UFG clouds



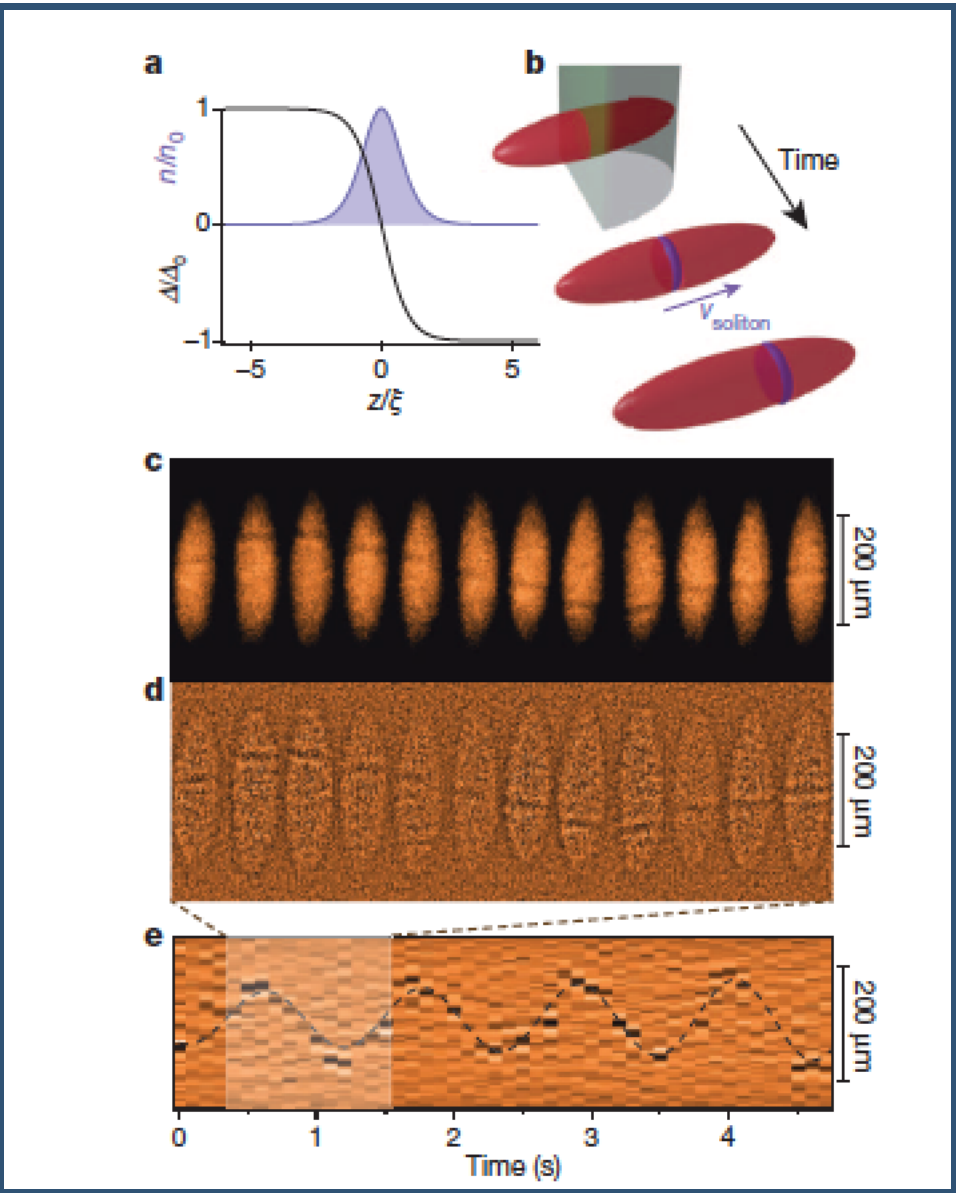
Phase of the pairing gap normalized to ε_F

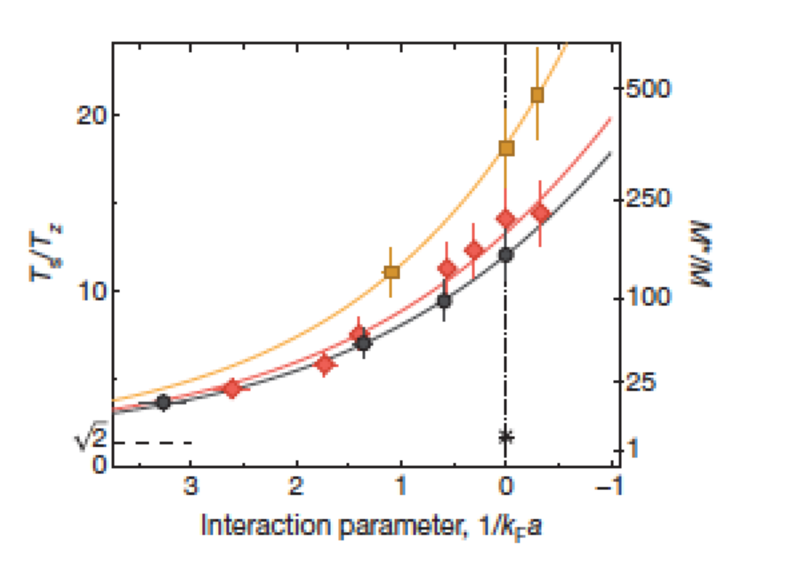


Local velocity normalized to Fermi velocity

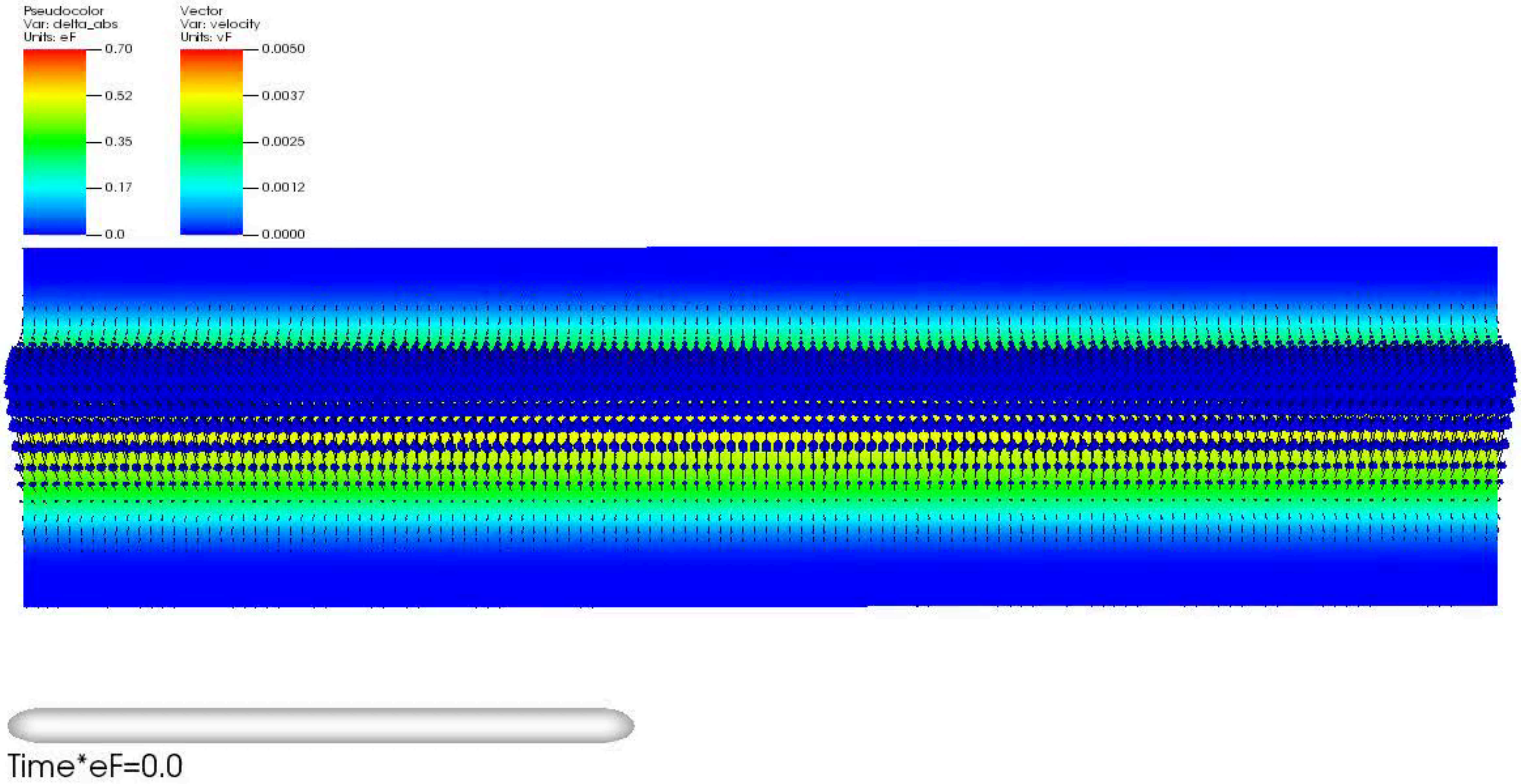
Heavy solitons in a fermionic superfluid

Tarik Yefsah[1], Ariel T. Sommer[1], Mark J. H. Ku[1], Lawrence W. Cheuk[1], Wenjie Ji[1], Waseem S. Bakr[1] & Martin W. Zwierlein[1]





TDSLDA



Construction of ground state (adiabatic switching with quantum friction), generation of a domain wall using an optical knife, followed by the spontaneous formation of a vortex ring.
Aproximately 1270 fermions on a 48x48x128 spatial lattice, ≈ 260,000 complex PDEs,
≈ 309,000 time-steps, 2048 GPUs on Titan, 27.25 hours of wall time (initial code)
Wlazłowski et al, Phys. Rev. Lett. 112, 025301 (2014)

Vortex rings

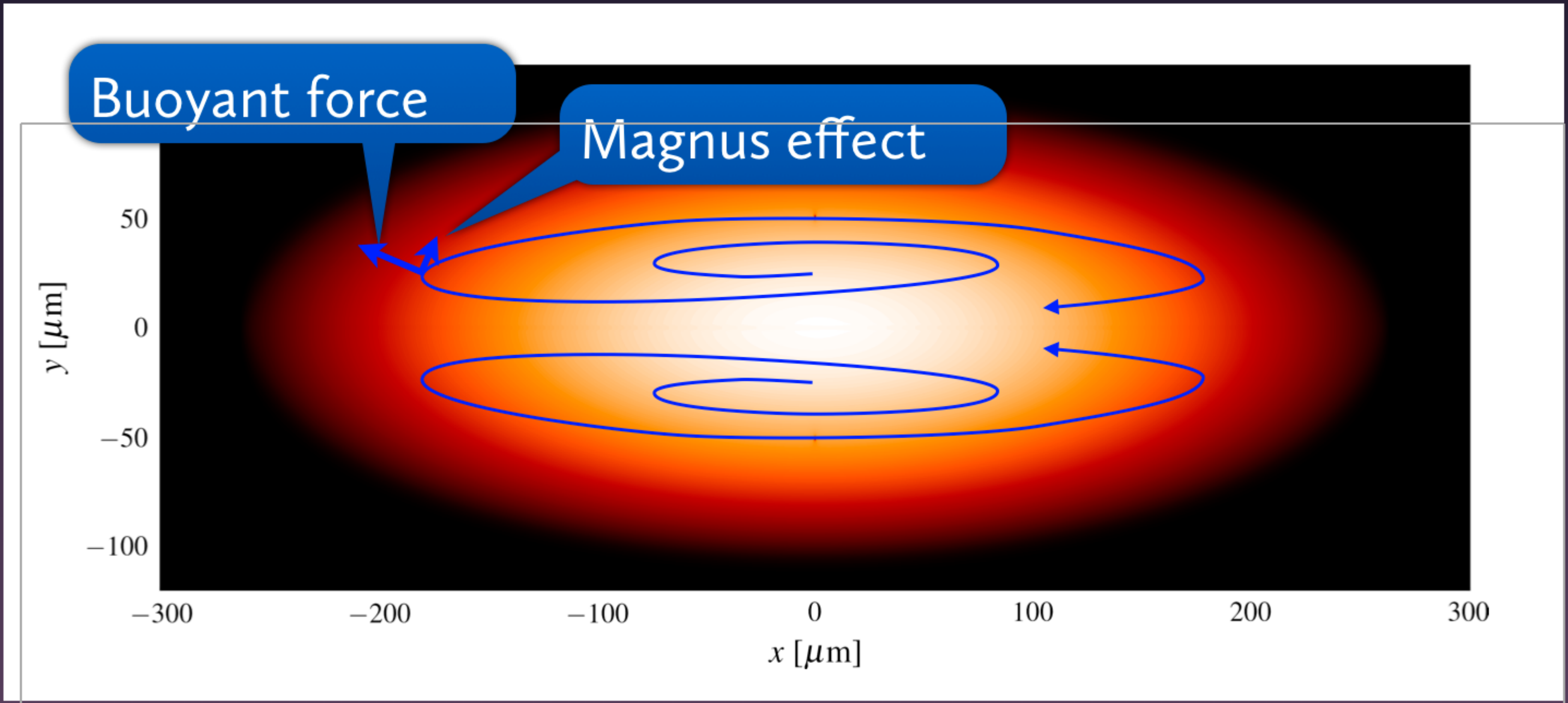
$$E \approx \frac{mn\kappa^2}{2} R \ln \frac{R}{l_{coh}}, \qquad \kappa \text{ - circulation}$$

$$p \approx mn\kappa\pi R^2$$

$$\text{v} = \frac{dE}{dp} \approx \frac{\kappa}{4\pi R} \ln \frac{R}{l_{coh}}$$

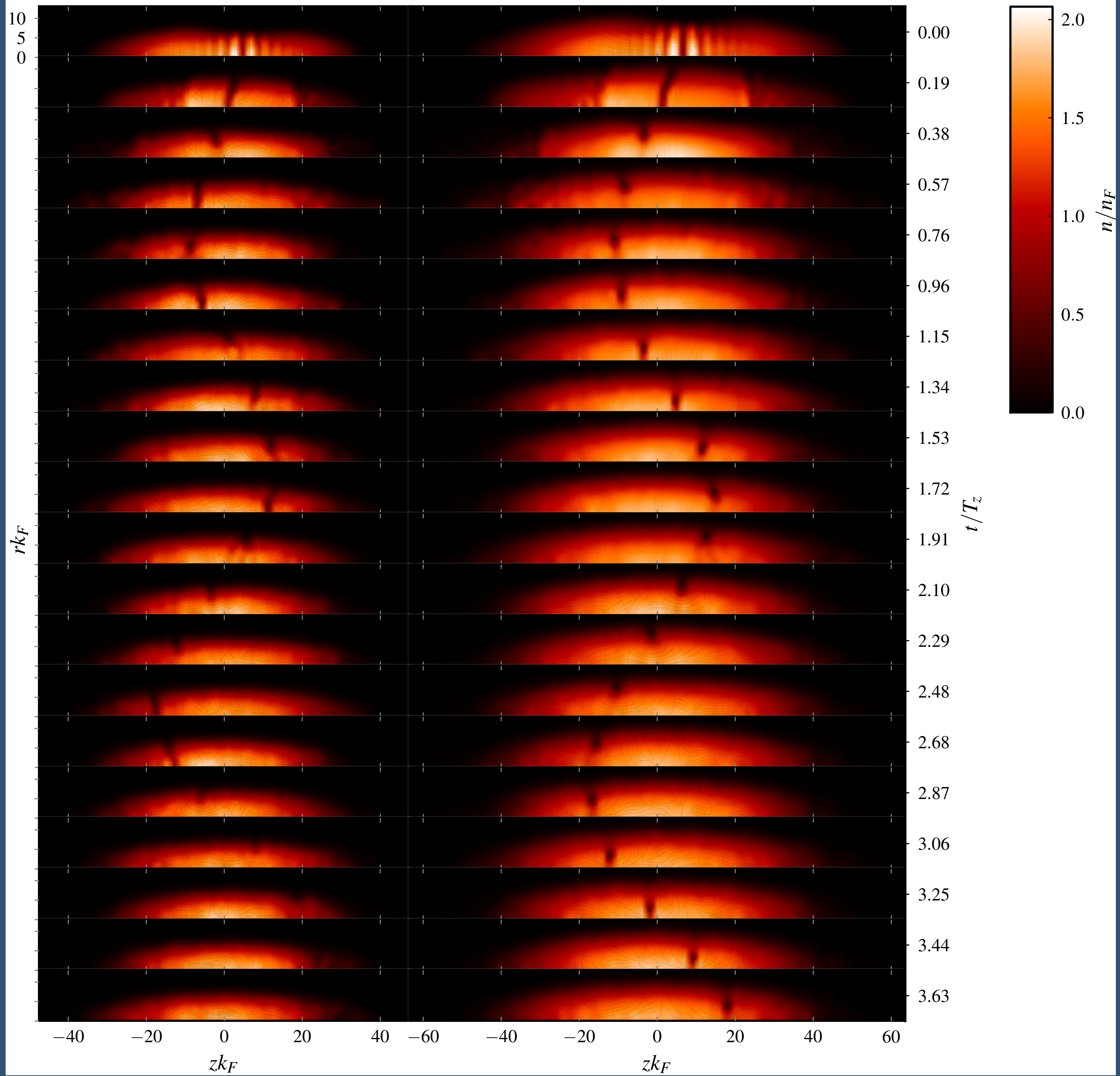
The bigger the vortex ring is the slower it moves

Vortex Ring Motion



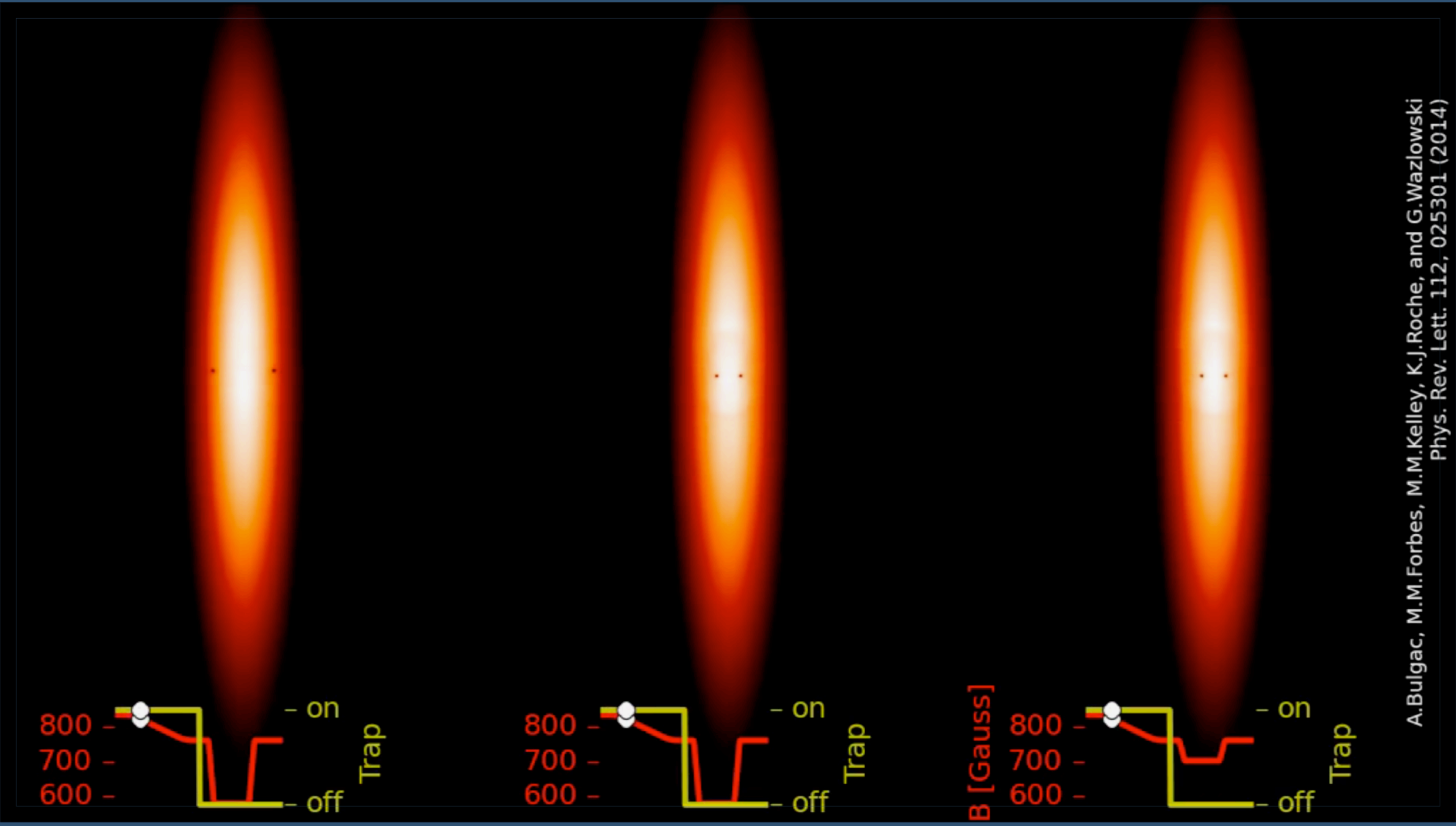
Vortex ring motion (here in the presence of "thermal" noise, hence the inverse decay)

TDSLDA

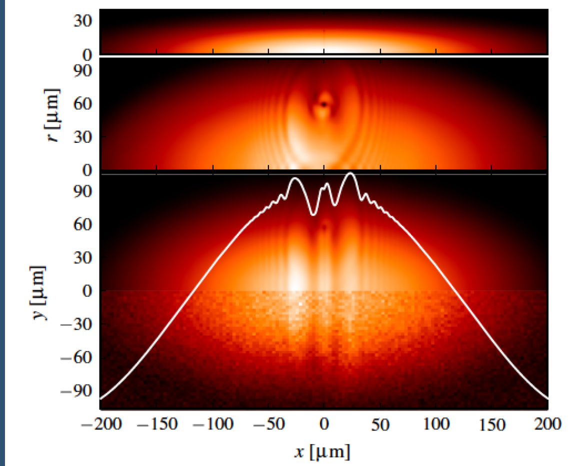


10
5
0
rk_F
0.00
0.19
0.38
0.57
0.76
0.96
1.15
1.34
1.53
1.72
1.91
2.10
2.29
2.48
2.68
2.87
3.06
3.25
3.44
3.63
t/T_z
2.0
1.5
1.0
0.5
0.0
n/n_F
−40 −20 0 20 40
zk_F
−60 −40 −20 0 20 40 60
zk_F

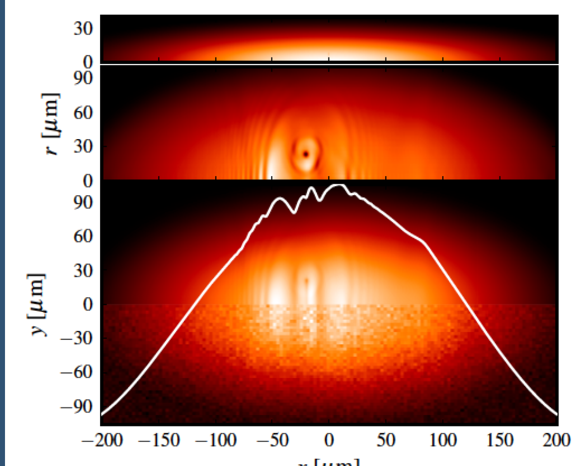
Imaging the vortex ring in experiment (movie)



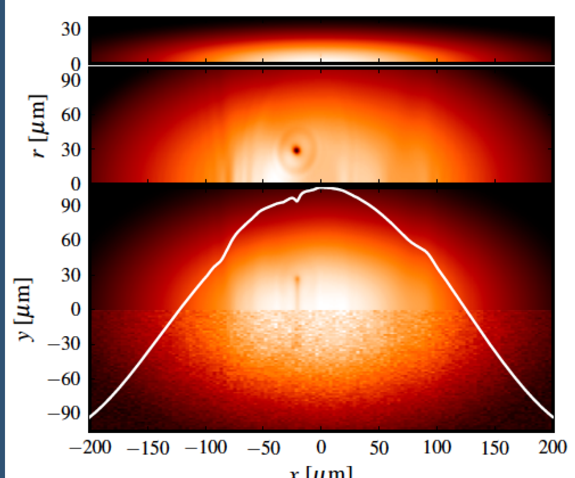
Large ring **Small ring** **Too large B_{min}**



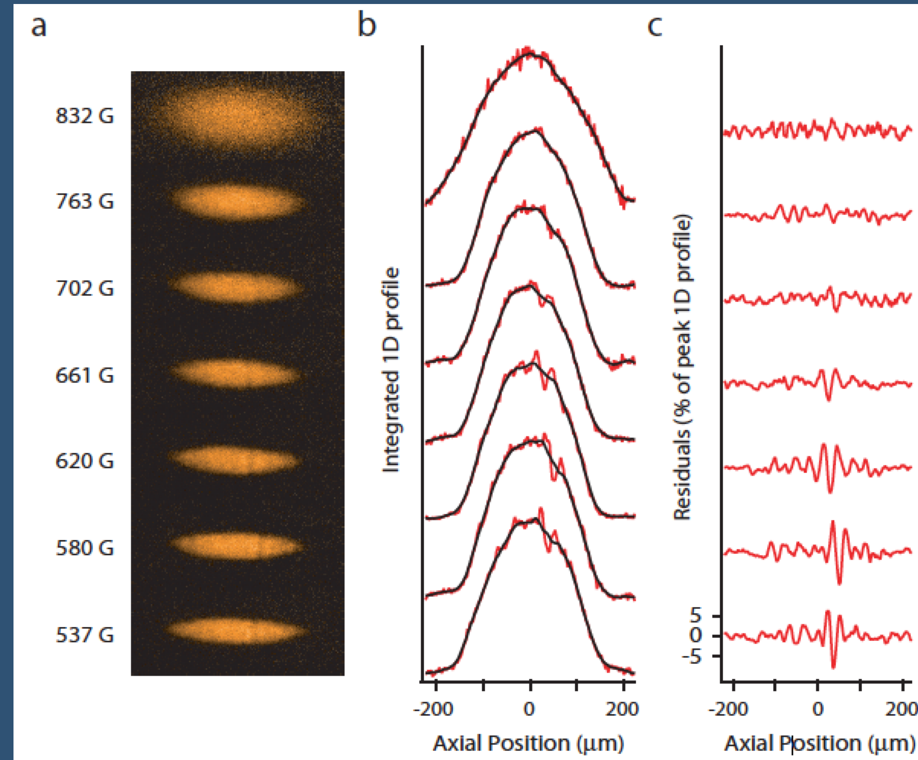
Large ring



Small ring



Insufficient ramping of magnetic field



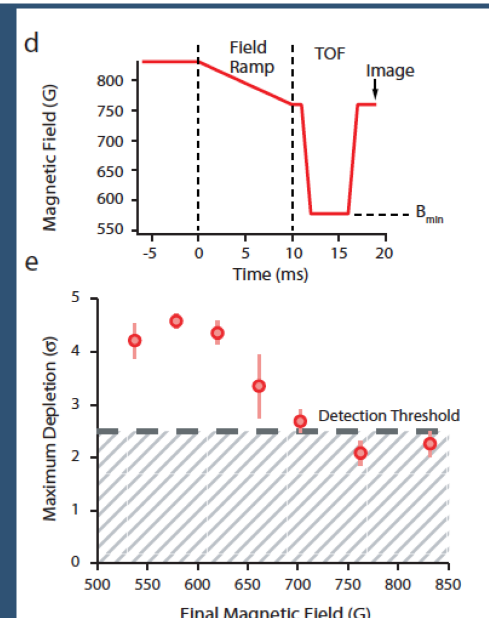
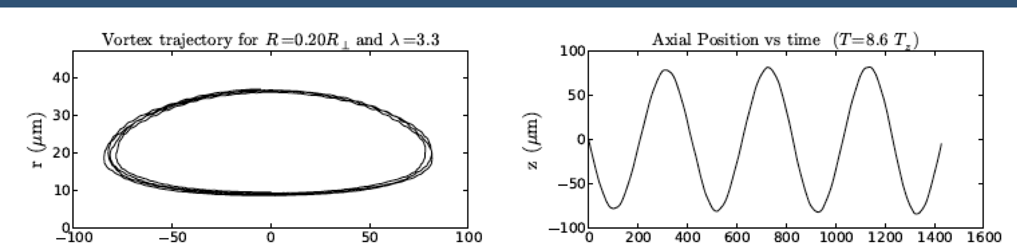


TABLE I. Dependence of the oscillation period on aspect ratio for a vortex ring imprinted with $R_0 = 0.30 R_\perp$ at resonance. Note that the ETF consistently underestimates the period by about a factor of 0.56.

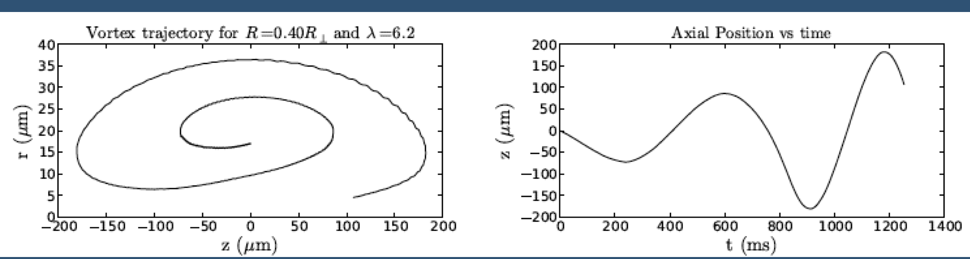
Aspect ratio	ETF period	Observed period [18]
$\lambda = 3.3$	$T = 9.9 T_z$	$T = 18(2) T_z$
$\lambda = 6.2$	$T = 8.4 T_z$	$T = 14(2) T_z$
$\lambda = 15$	$T = 6.7 T_z$	$T = 12(2) T_z$

TABLE II. Benchmark of the ETF periods to the SLDA periods for sizes $24 \times 24 \times 96$, $32 \times 32 \times 128$, and $48 \times 48 \times 128$.

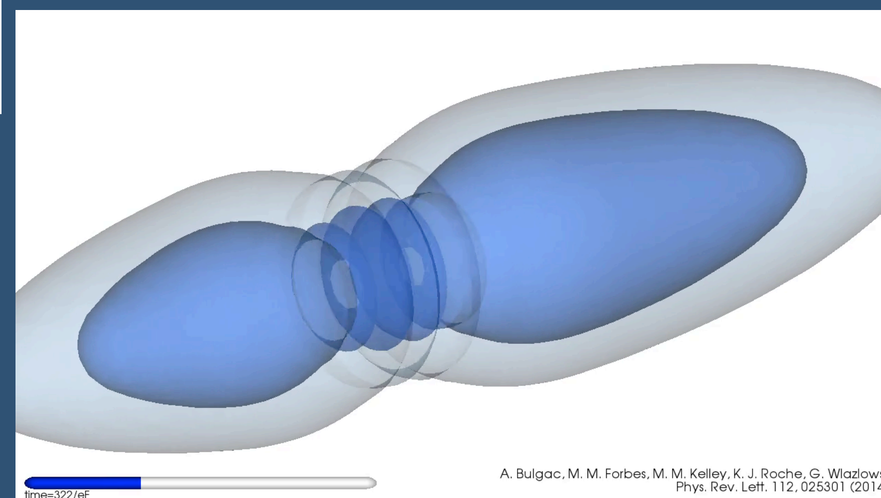
Size	T_{ETF}	T_{SLDA}	$T_{\mathrm{SLDA}}/T_{\mathrm{ETF}}$
$24 \times 24 \times 96$	$1.4 T_z$	$1.7 T_z$	1.2
$32 \times 32 \times 128$	$1.6 T_z$	$1.9 T_z$	1.2
$48 \times 48 \times 128$	$1.9 T_z$	$2.6 T_z$	1.4



Near harmonic motion close to T=0 (very small number of phonons)

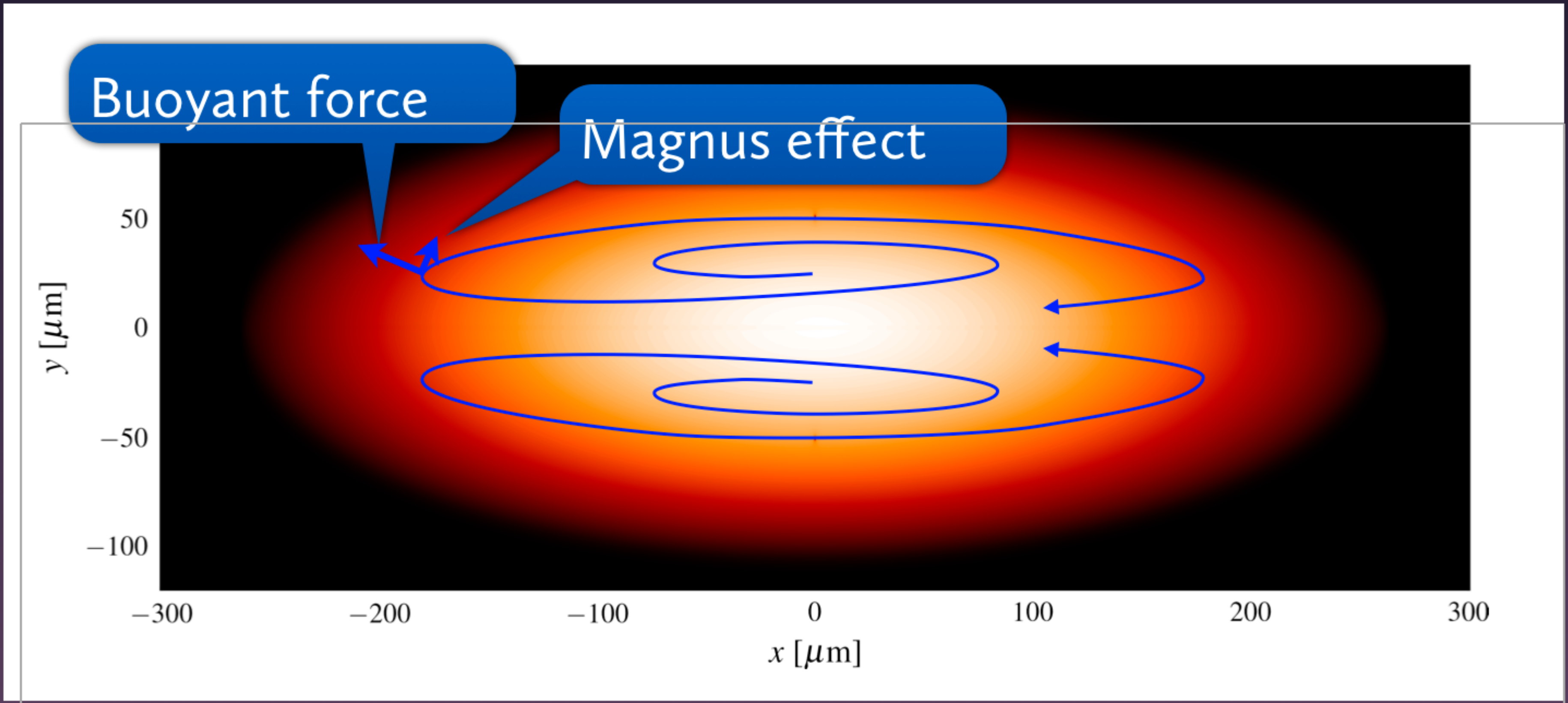


Anti-damping of the motion in the presence of a considerable number of phonons



TDSLDA (movie)

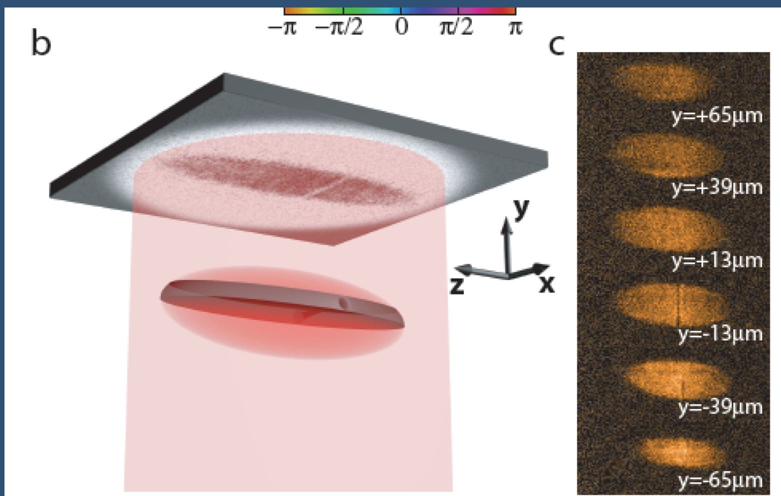
Vortex Ring Motion

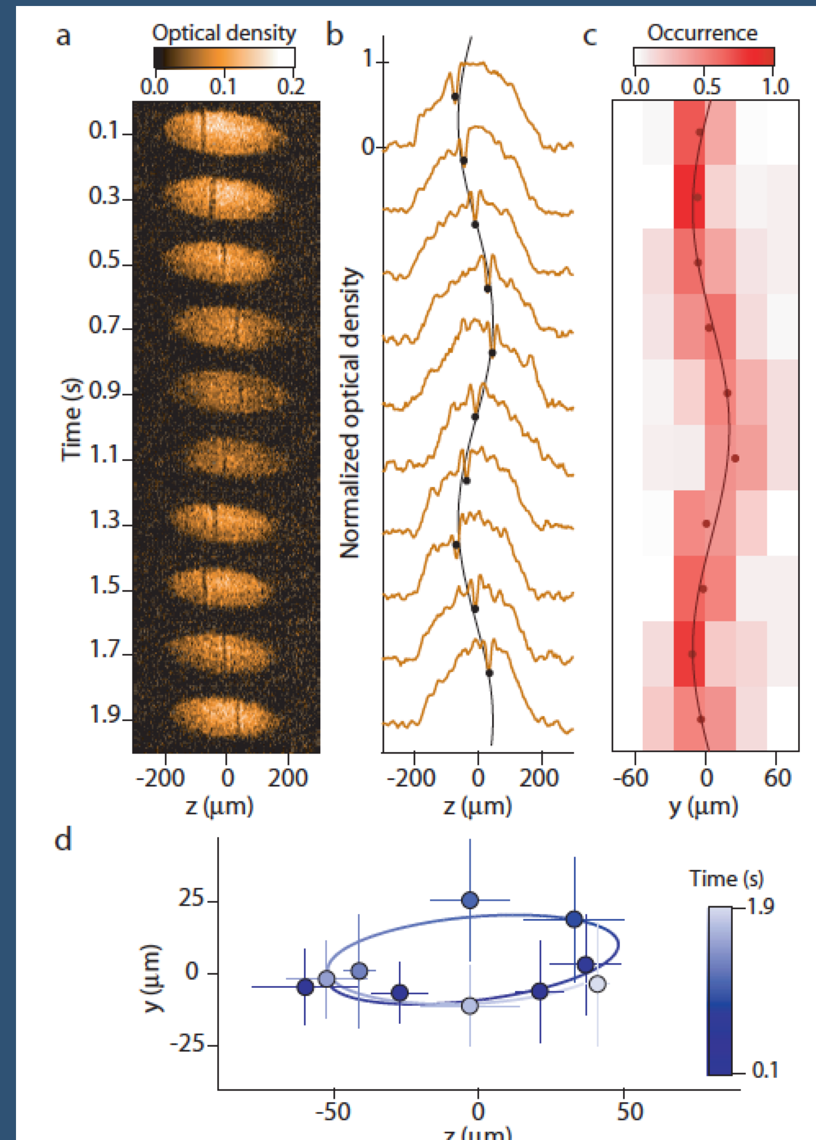


Vortex ring motion (here in the presence of “thermal” noise, hence the inverse decay)

The 2014 MIT experiment:

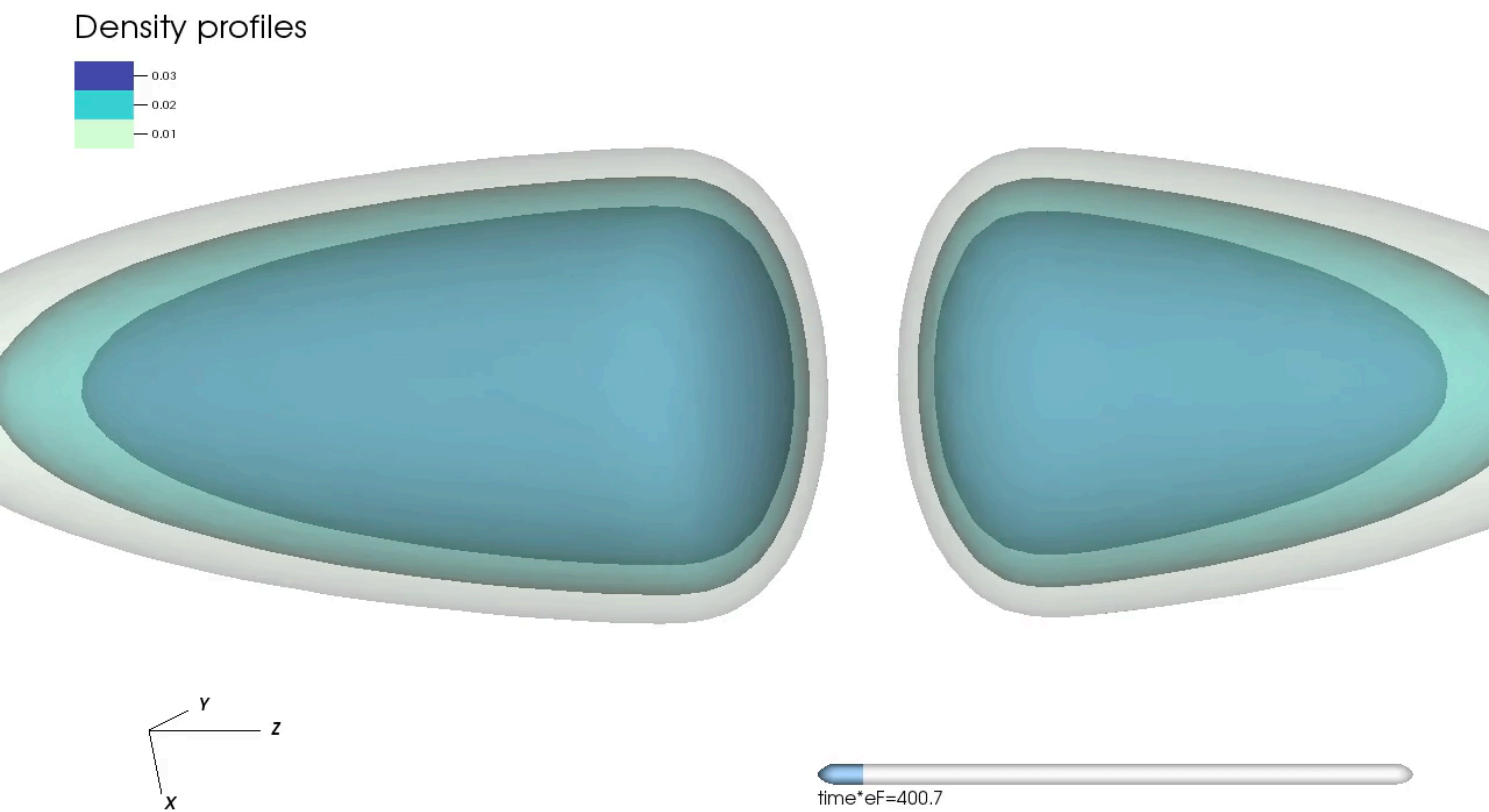
Motion of a Solitonic Vortex in the BEC-BCS Crosover
Ku, Ji, Mukherjee, Guardado-Sanchez, Cheuk, Yefsah, Zwierlein
Phys. Rev. Lett. 113, 065301 (2014)



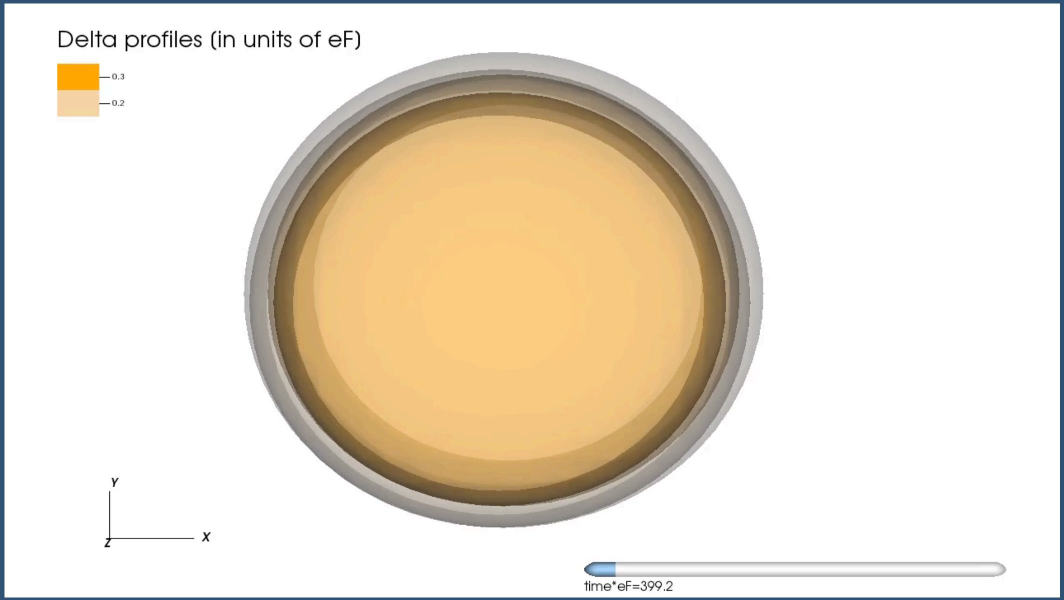


- <u>In this case the trap is triaxial, the long and medium axes horizontal</u>
- <u>The excitation in this case has the width of a vortex line (it is not wide as it was in the previous experiment, different imaging procedure)</u> and it is a horizontal vortex aligned with the medium axis
- The period is again much larger than that of a domain wall
- Motion is again almost harmonic and the trajectory is very similar to that of the vortex ring

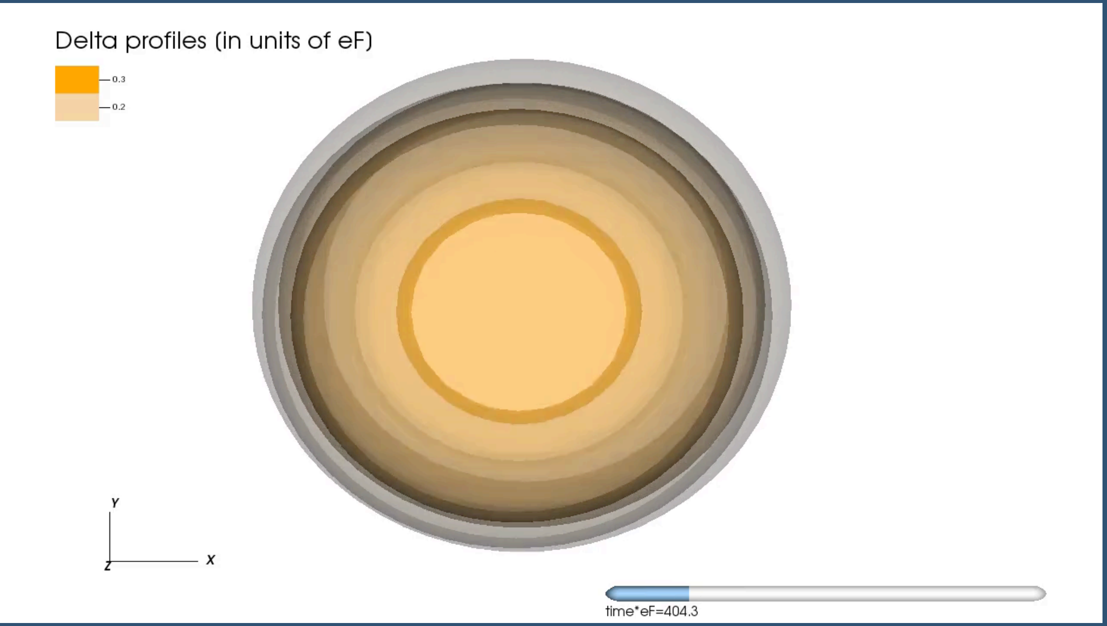
What TDSLDA tells us in the case of an axially non-symmetric trap, similar to the 2014 MIT experiment? (movie)



In agreement with the new experiment, when axial symmetry is broken a domain wall, converts to a vortex ring, which shortly becomes a vortex line.



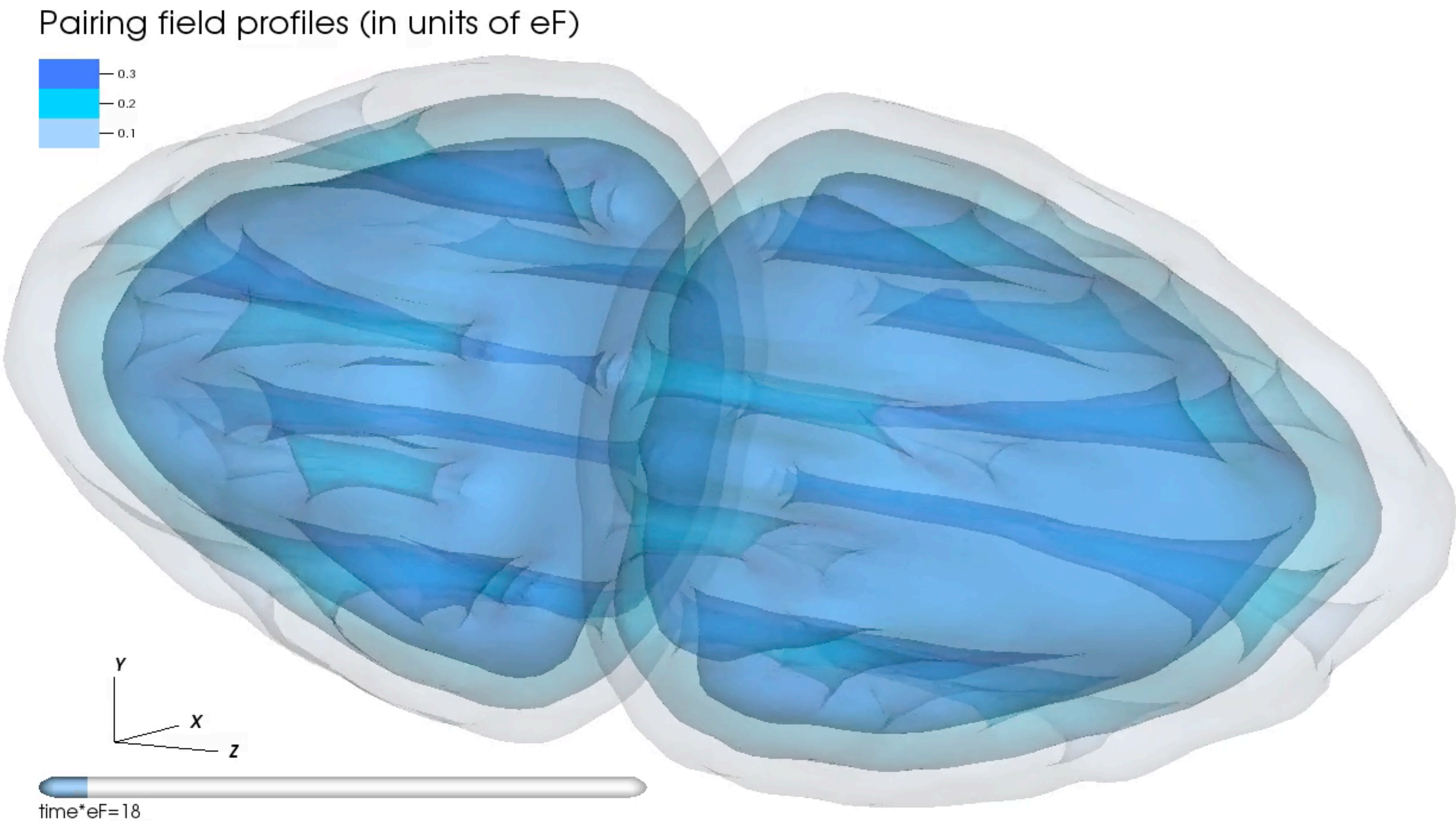
View along the long axis
(y-axis vertical, movie)



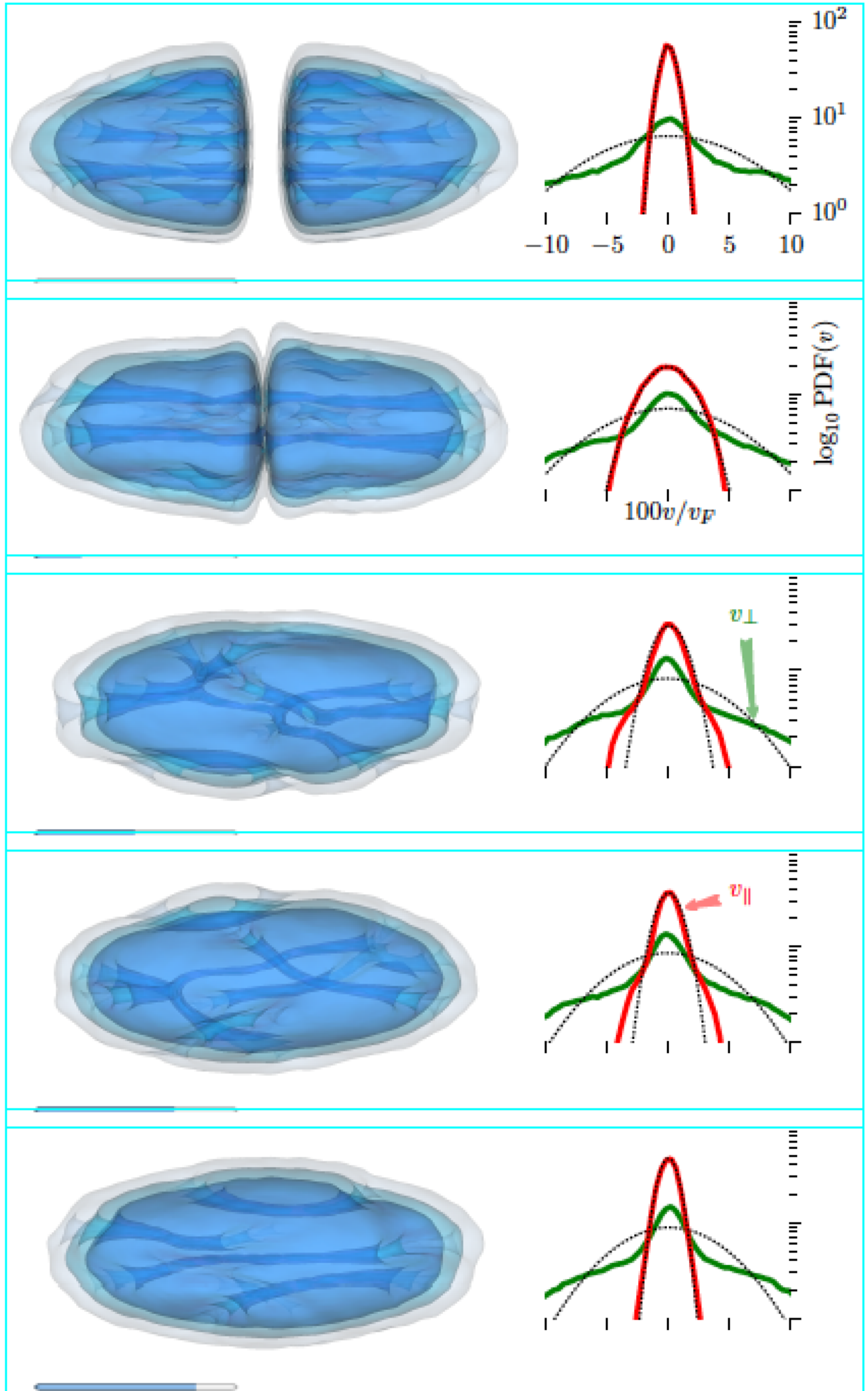
In a slightly different geometry one can put directly in evidence in great detail the crossing and reconnection of vortex lines, the mechanism envisioned by Feynman in 1955 as the route to Quantum Turbulence (movie)

Classical Turbulence

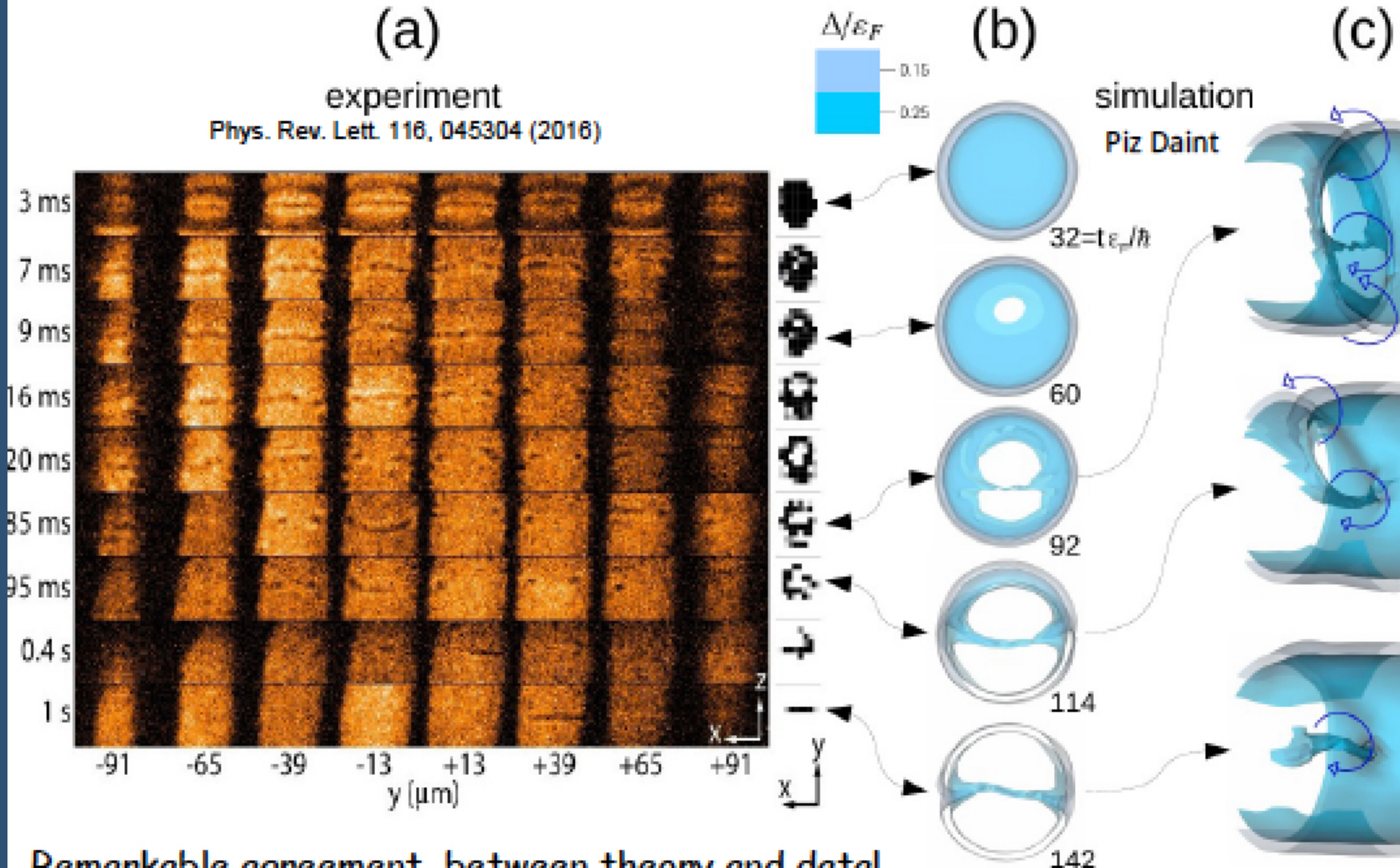
Exciting quantum turbulence in a unitary Fermi gas in a trap



Wlazłowski et al, arXiv:1404.1038



10^2
10^1
10^0
-10 -5 0 5 10
$\log_{10} \mathrm{PDF}(v)$
$100v/v_F$
$v_\perp$
$v_\parallel$

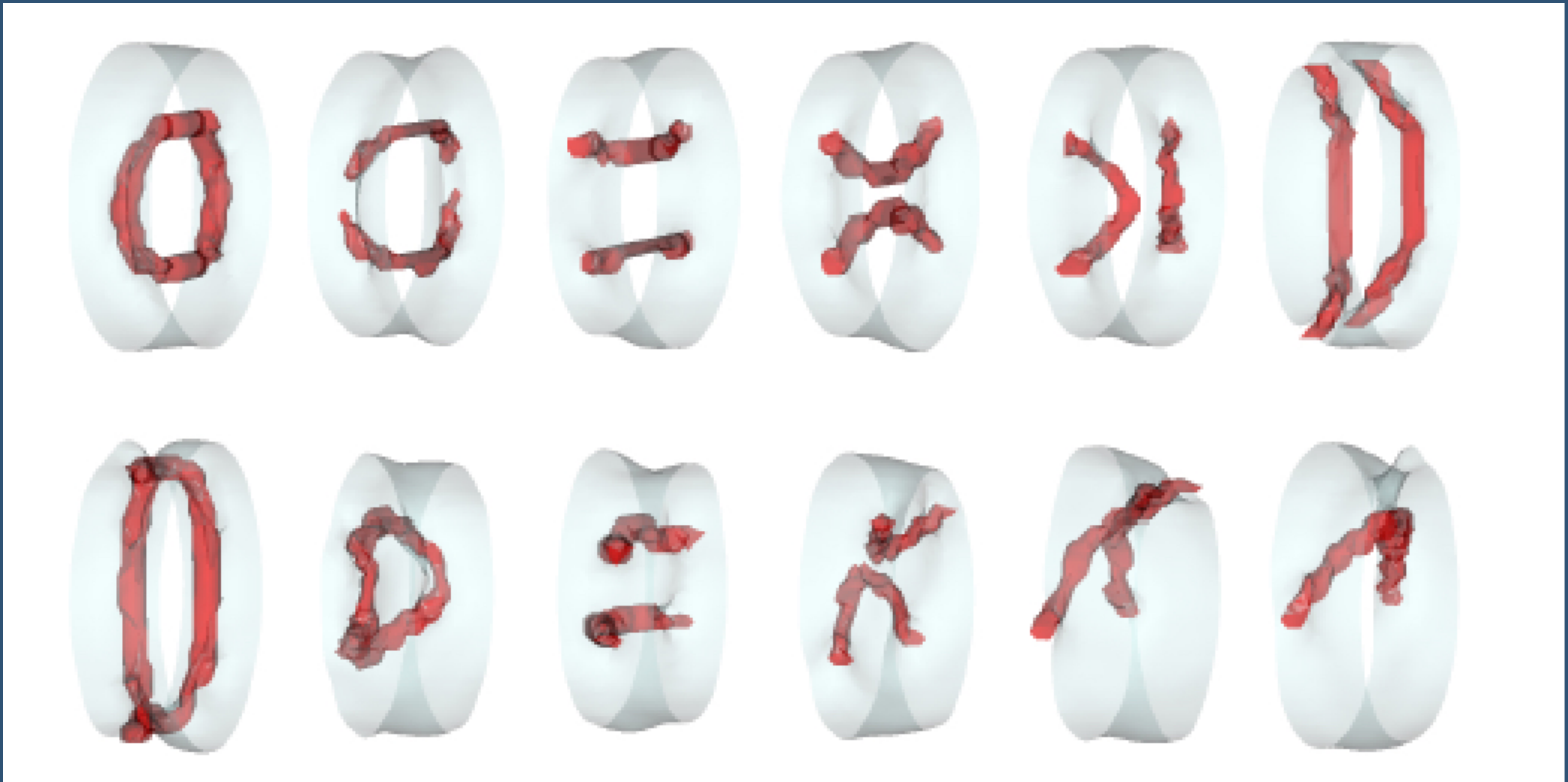


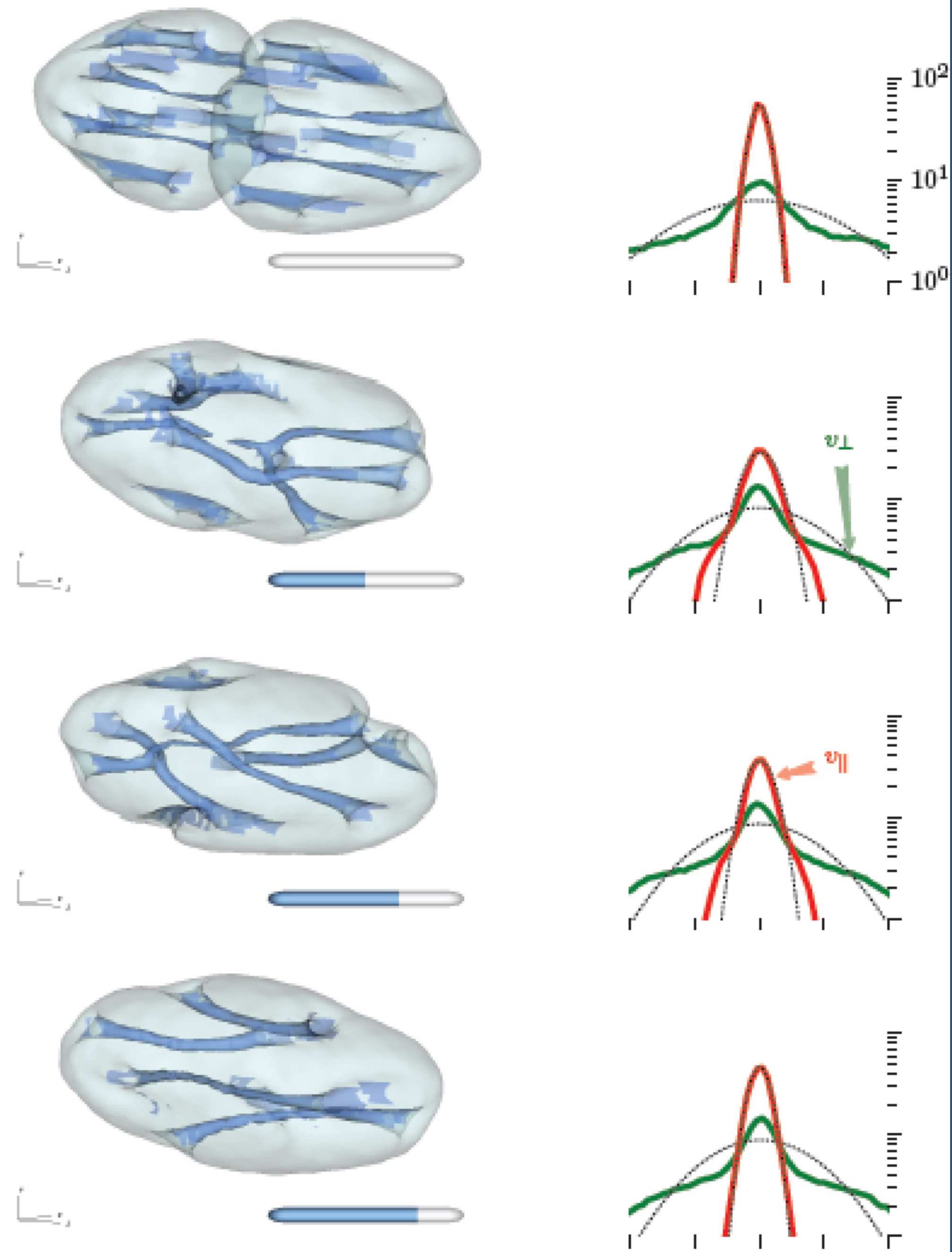
Remarkable agreement between theory and data!

Other approaches fail!

G. Wlazłowski, K. Sekizawa, M. Marchwiany, P. Magierski, arXiv:1711.05803

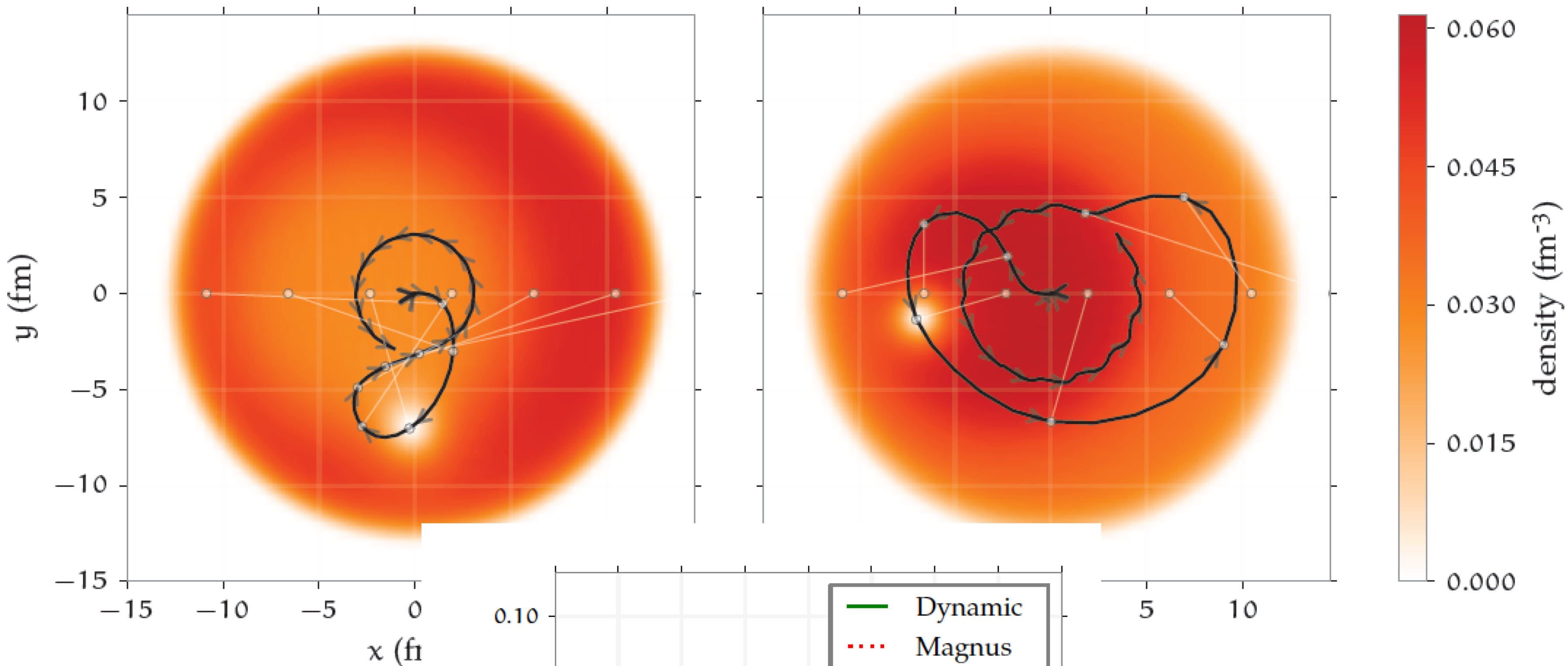
Quantum Turbulence
Crossing and reconenctions of quantized vortices
Feynman (1956)

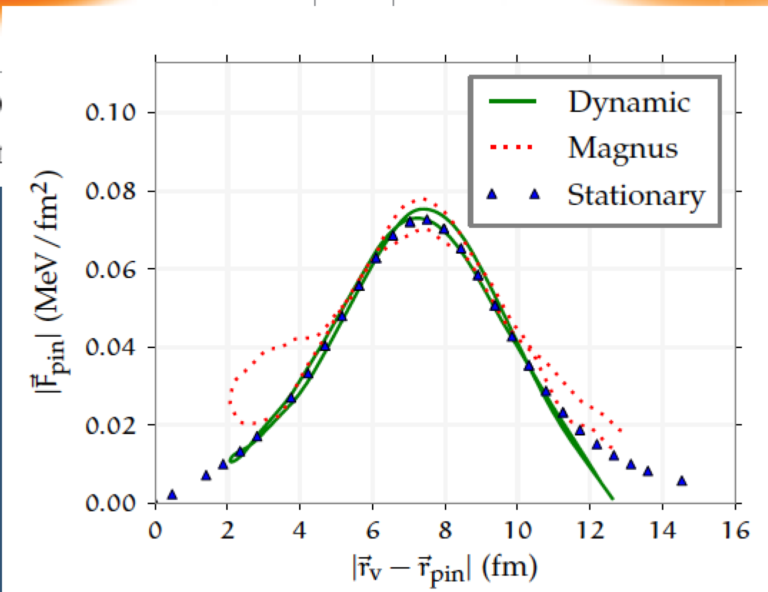




10^2
10^1
10^0
$v_\perp$
$v_\parallel$

How to compute the pinning energy of a vortex on nucleus in the neutron star crust





Attraction

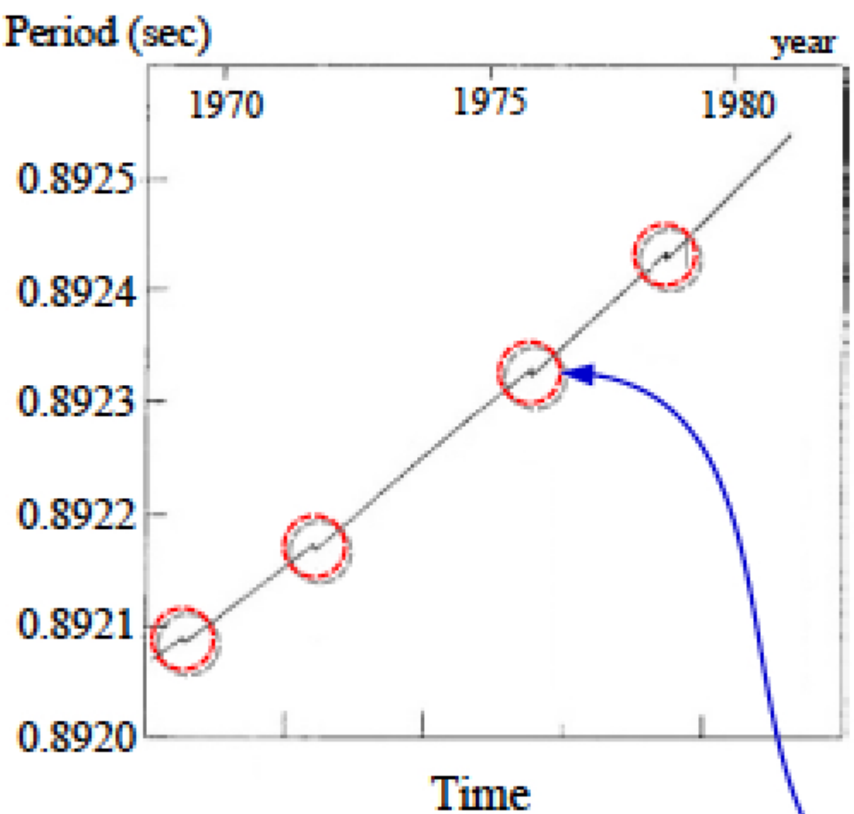
Repulsion

Bulgac, Forbes, and Sharma, Phys. Rev. Lett. 110, 241102 (2013)

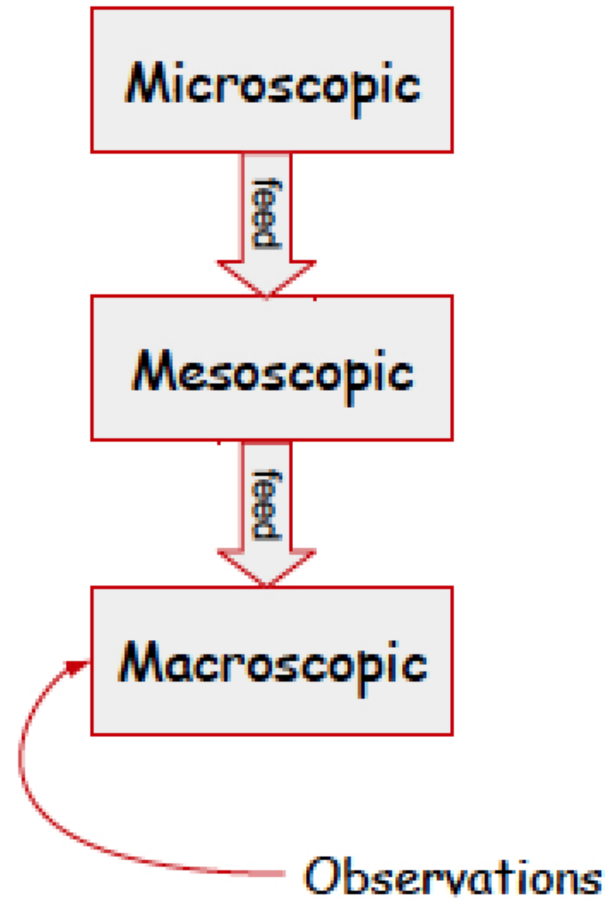
Our motivation: Glitch
(a sudden increase of the rotational frequency)

Hierarchy of theories:

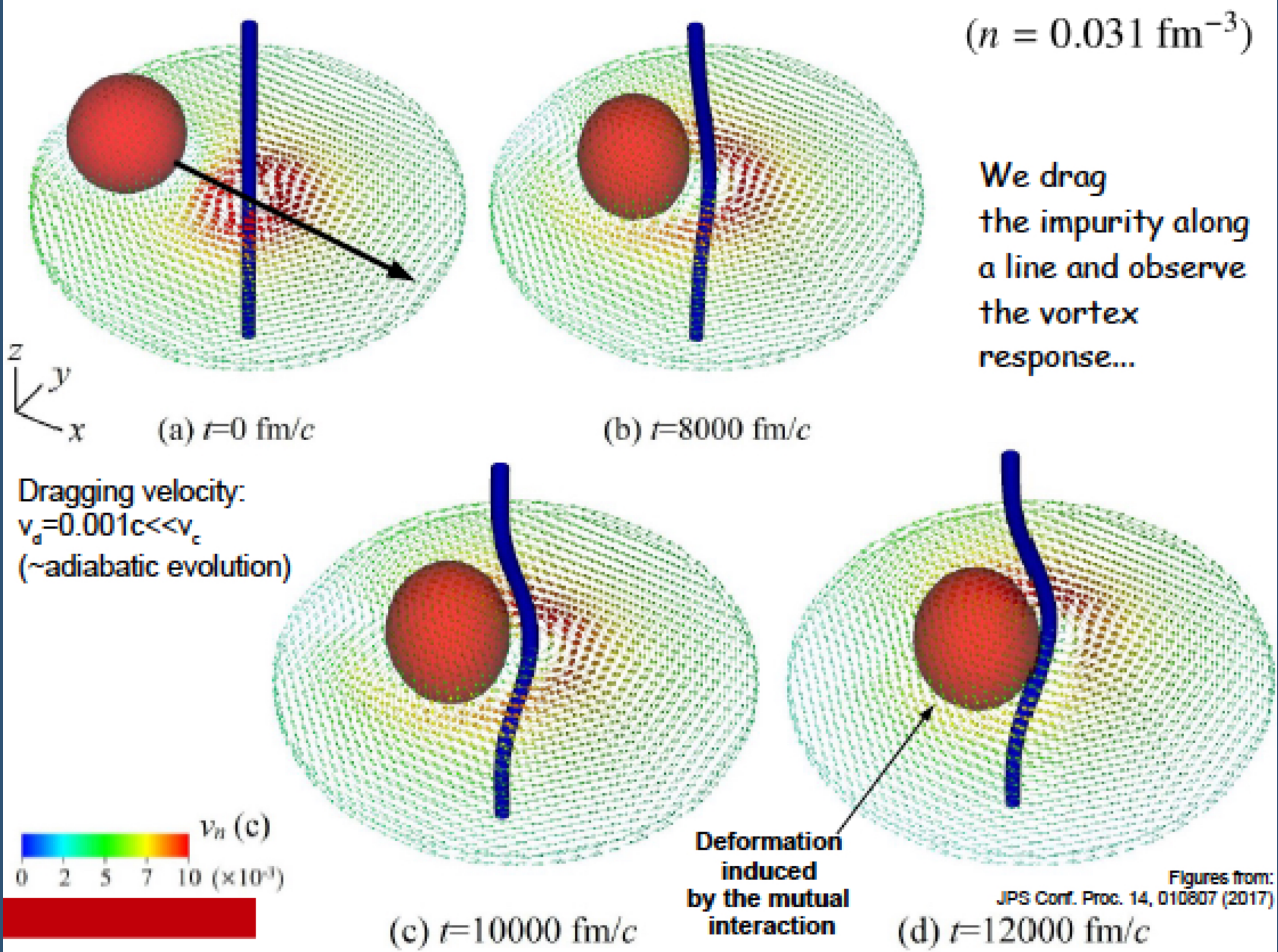
- Glitches in the Vela pulsar



V.B. Bhatia, A Textbook of Astronomy and Astrophysics with Elements of Cosmology, Alpha Science, 2001.



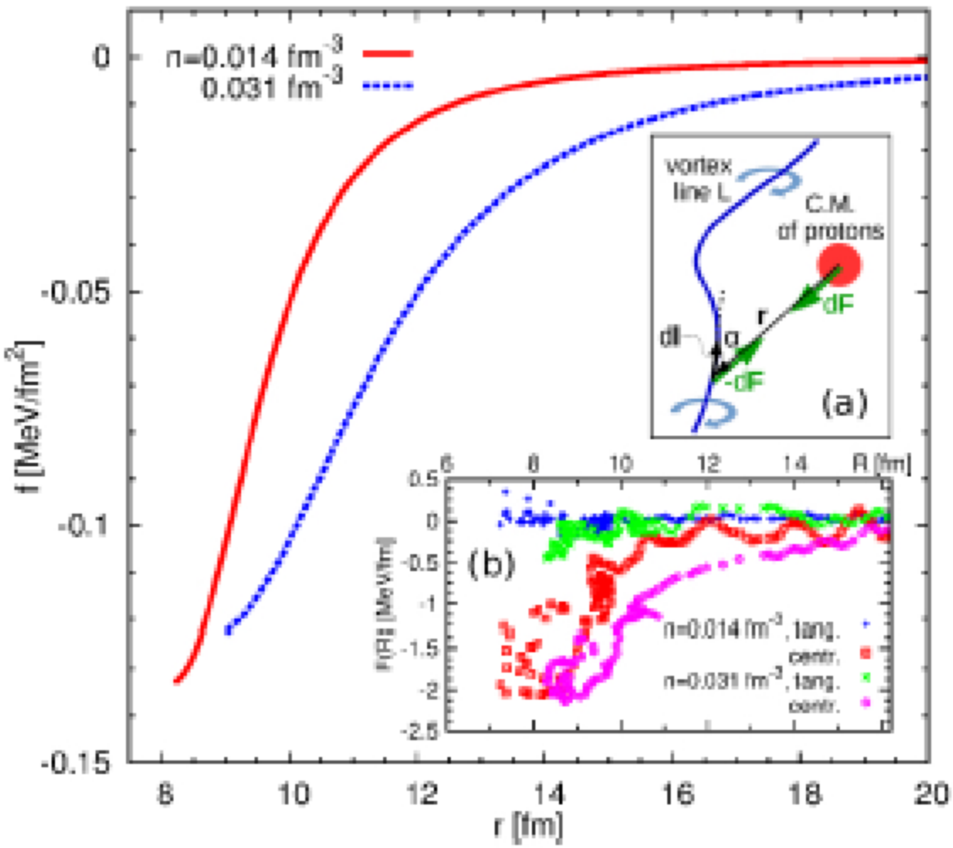
- **Vortex model**
  (P. W. Anderson and N. Itoh, Nature 256 (1975
  - Presently the standard picture for pulsar glitches

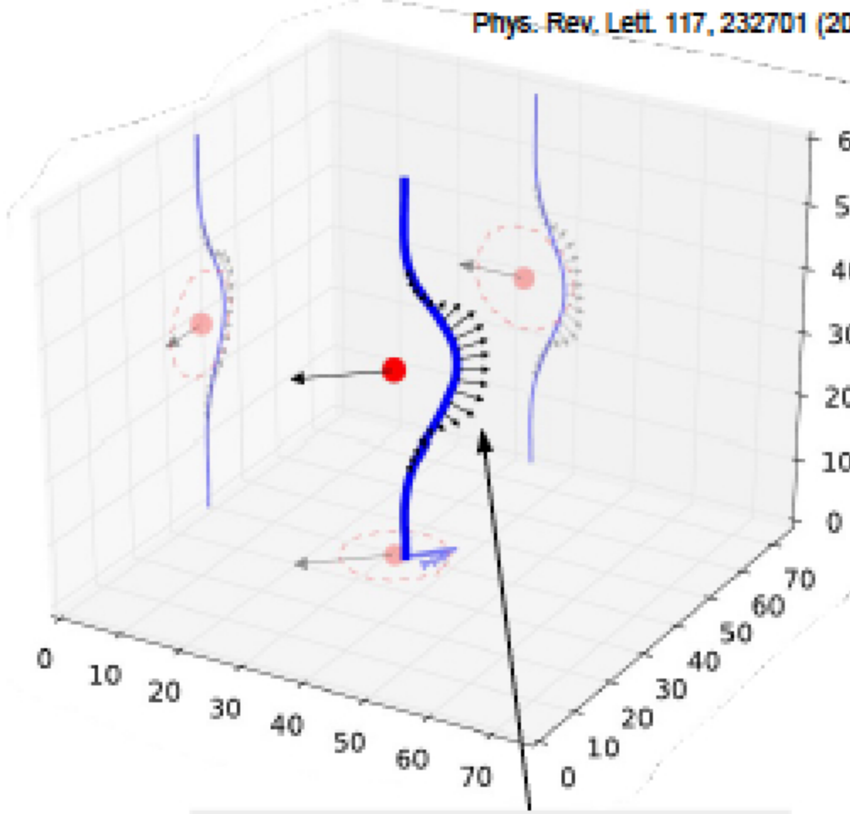


(n = 0.031 fm^{-3})
We drag the impurity along a line and observe the vortex response...
(a) t=0 fm/c
(b) t=8000 fm/c
Dragging velocity:
v_d=0.001c<<v_c
(~adiabatic evolution)
v_n (c)
0 2 5 7 10 (×10^{-3})
(c) t=10000 fm/c
Deformation induced by the mutual interaction
(d) t=12000 fm/c
Figures from:
JPS Conf. Proc. 14, 010807 (2017)

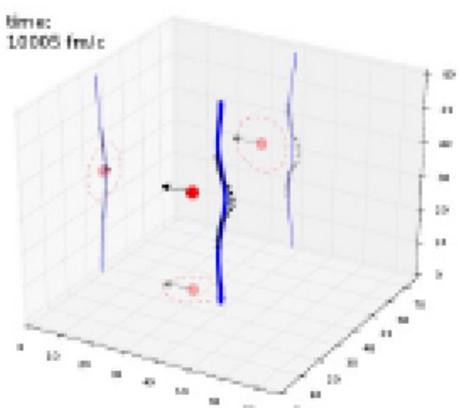
Force per unit length

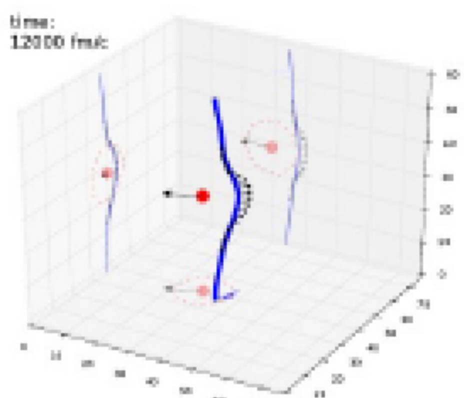
Phys. Rev. Lett. 117, 232701 (201

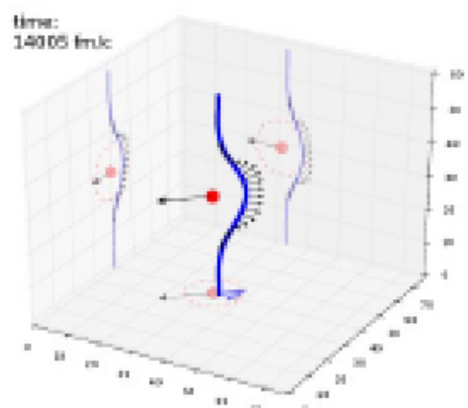


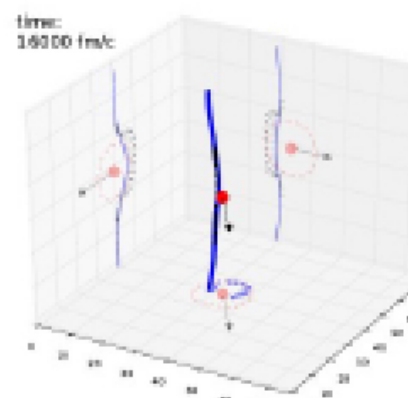


Only part close to nucleus surface contributes...

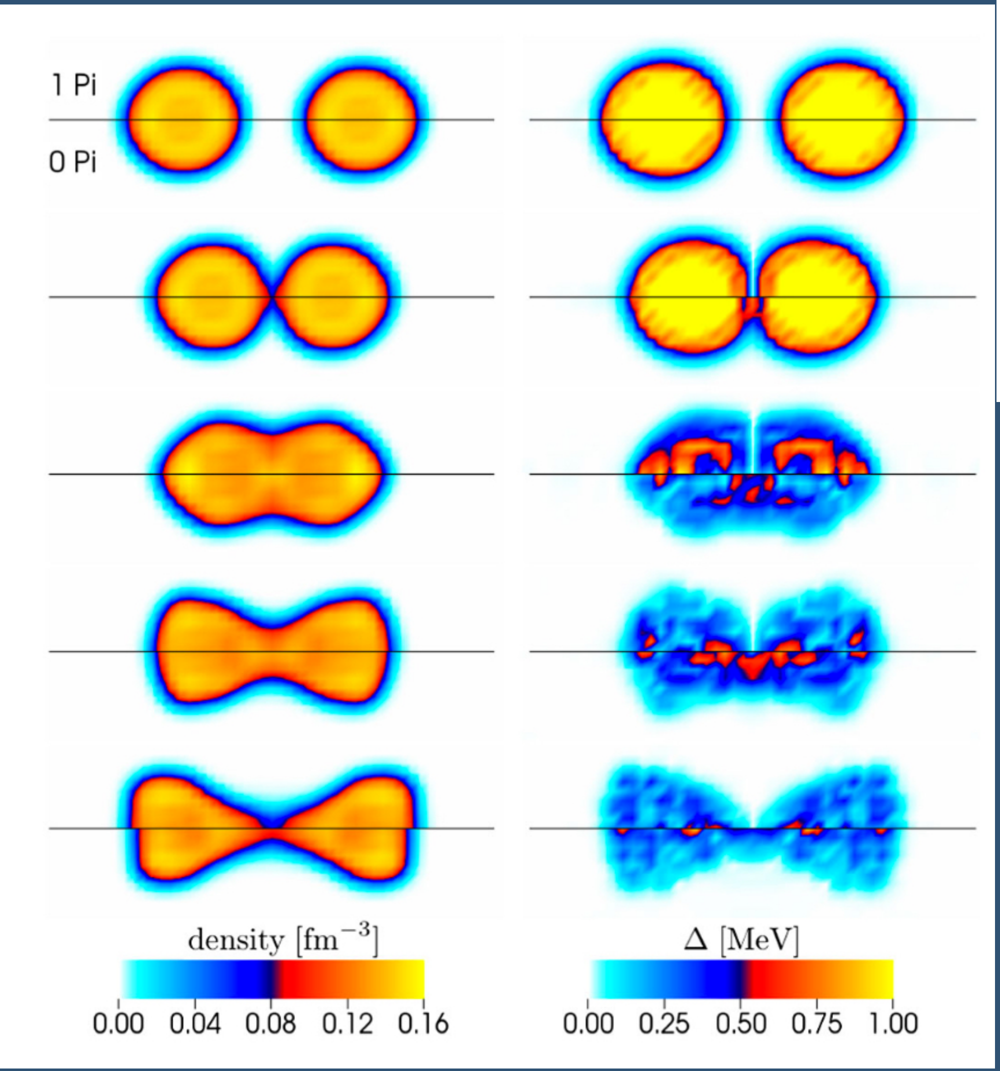


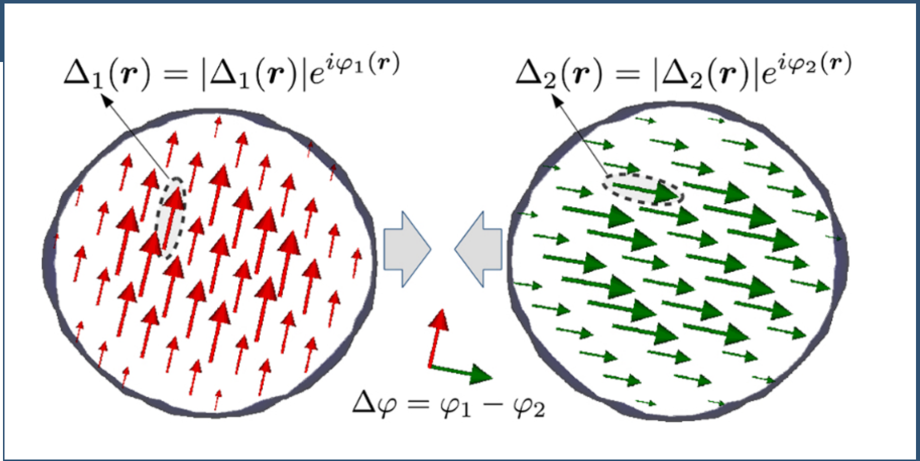


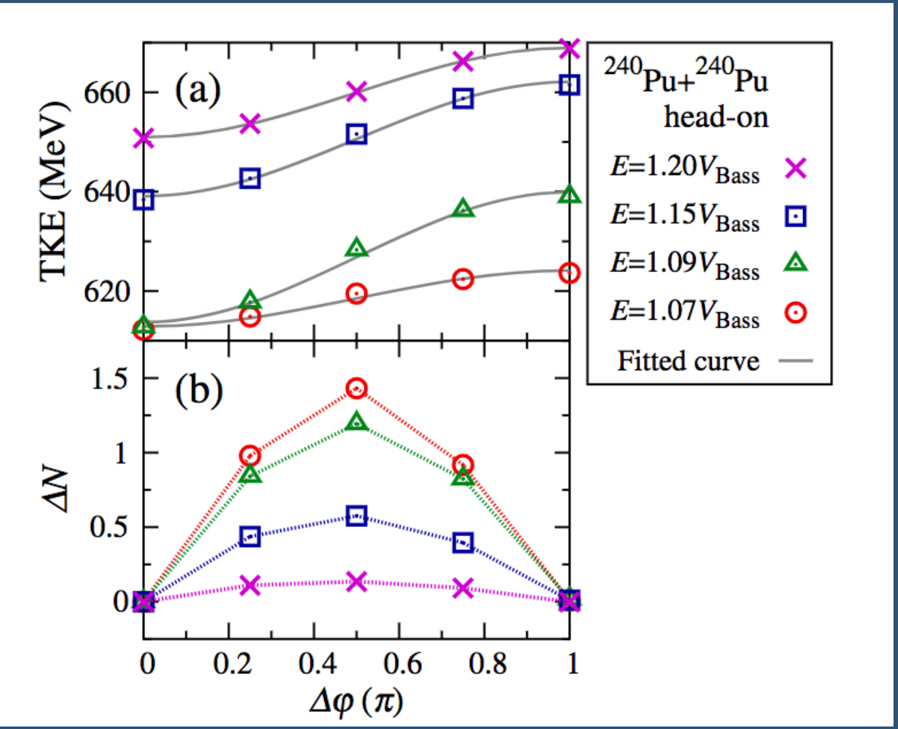




Collisions of superfluid nuclei

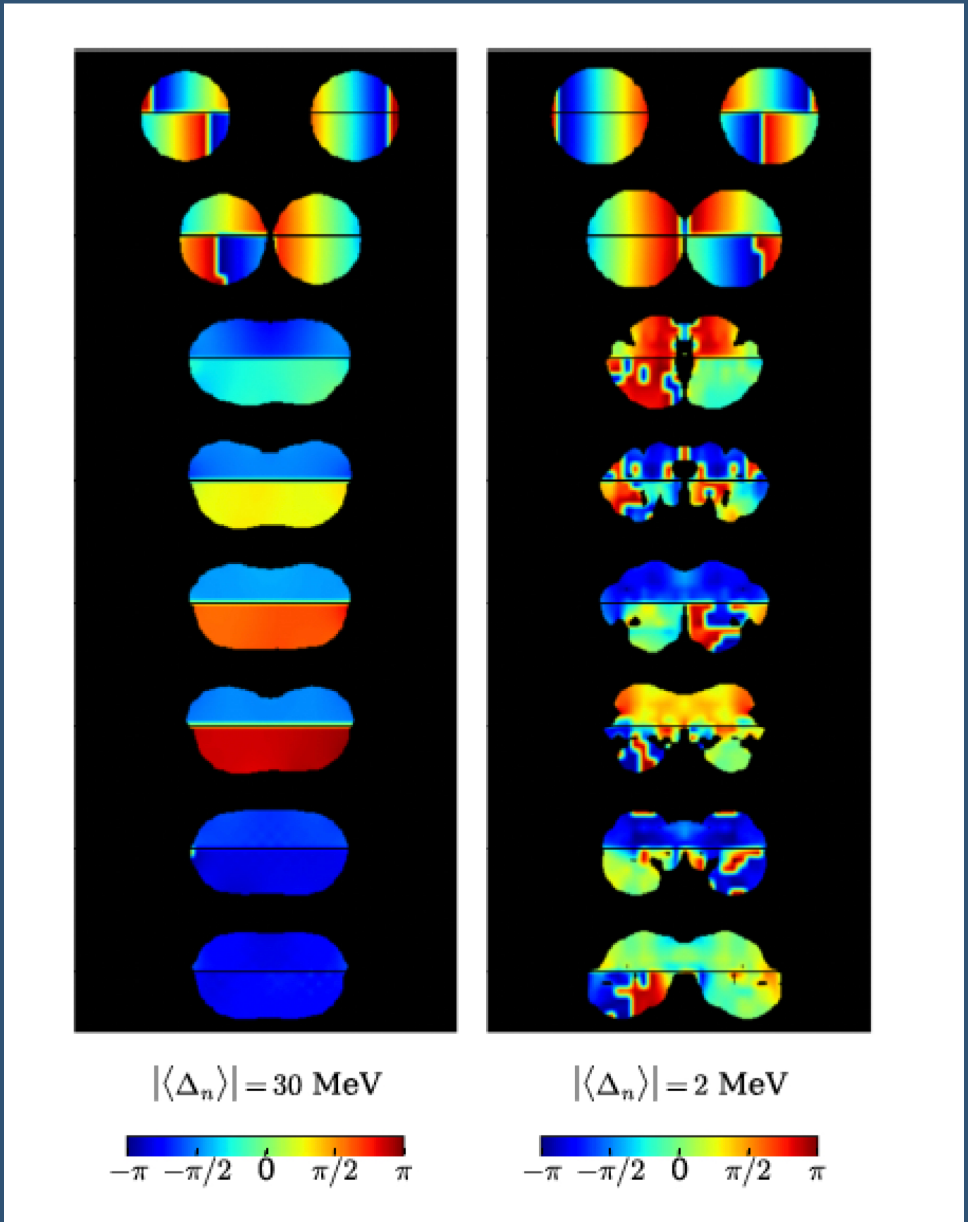






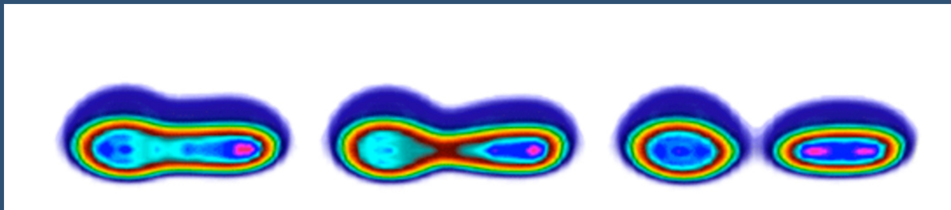
Magierski, et al, Phys. Rev. Lett. 119, 042501 (2017)

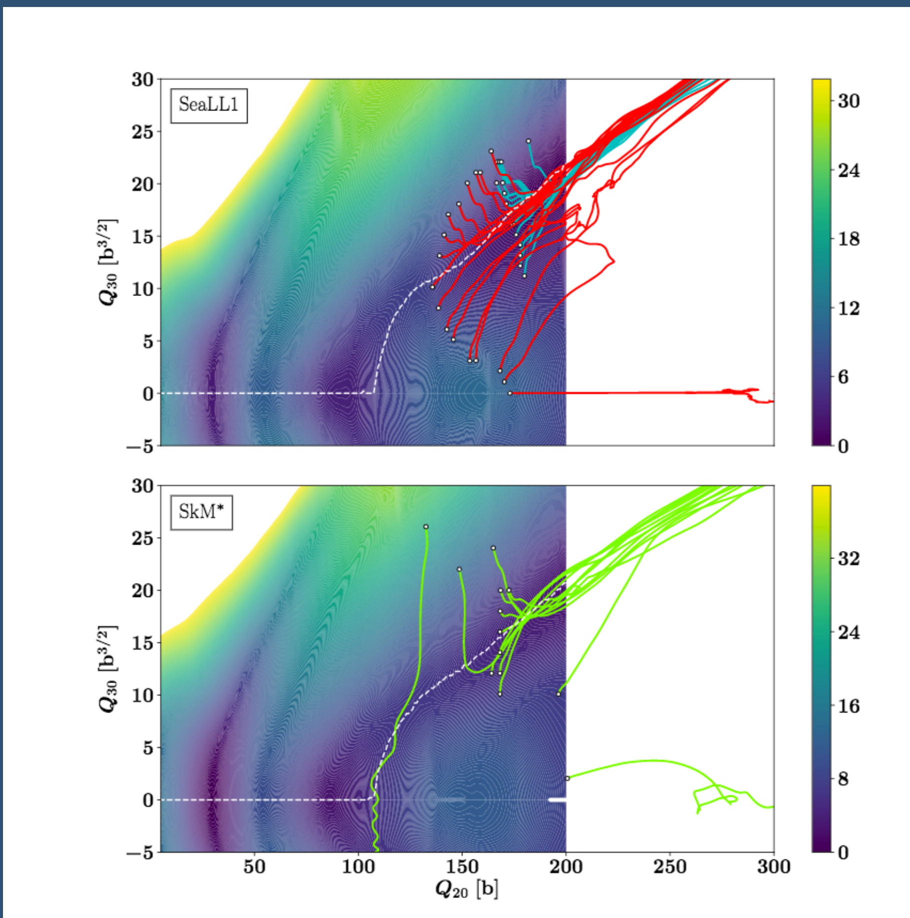
Collisions of superfluid nuclei

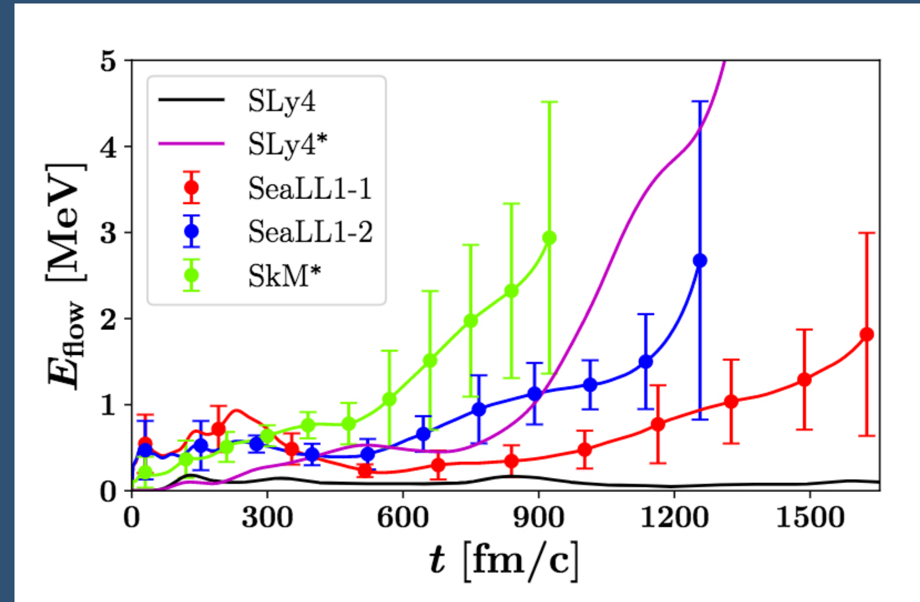


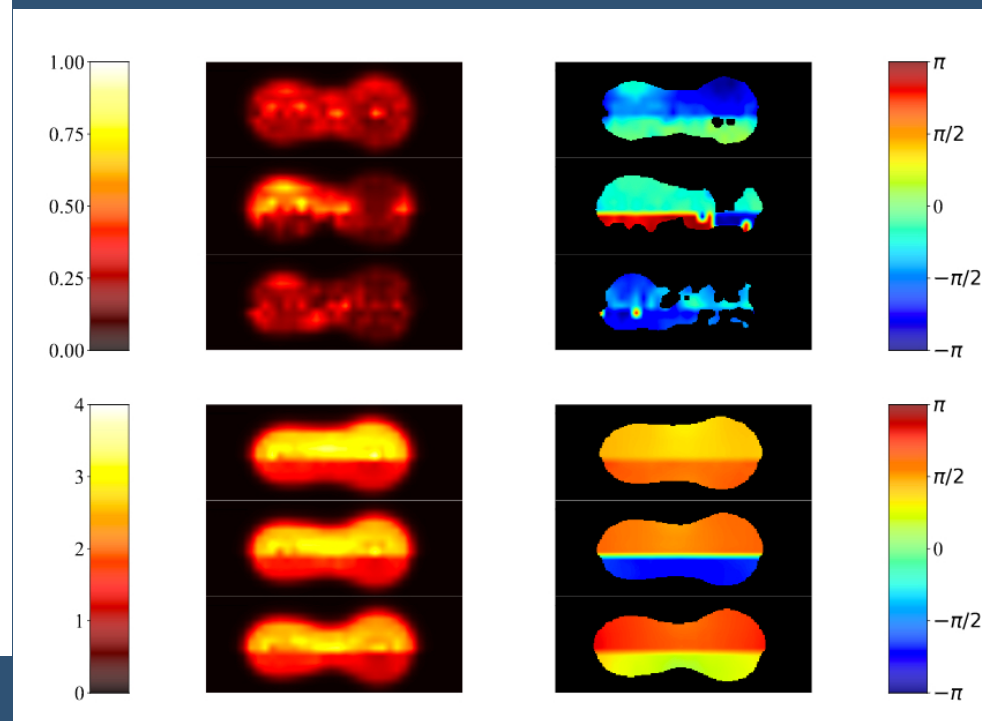
Bulgac and Jin, Phys. Rev. Lett. 119,052501 (2017)

Nuclear Fission



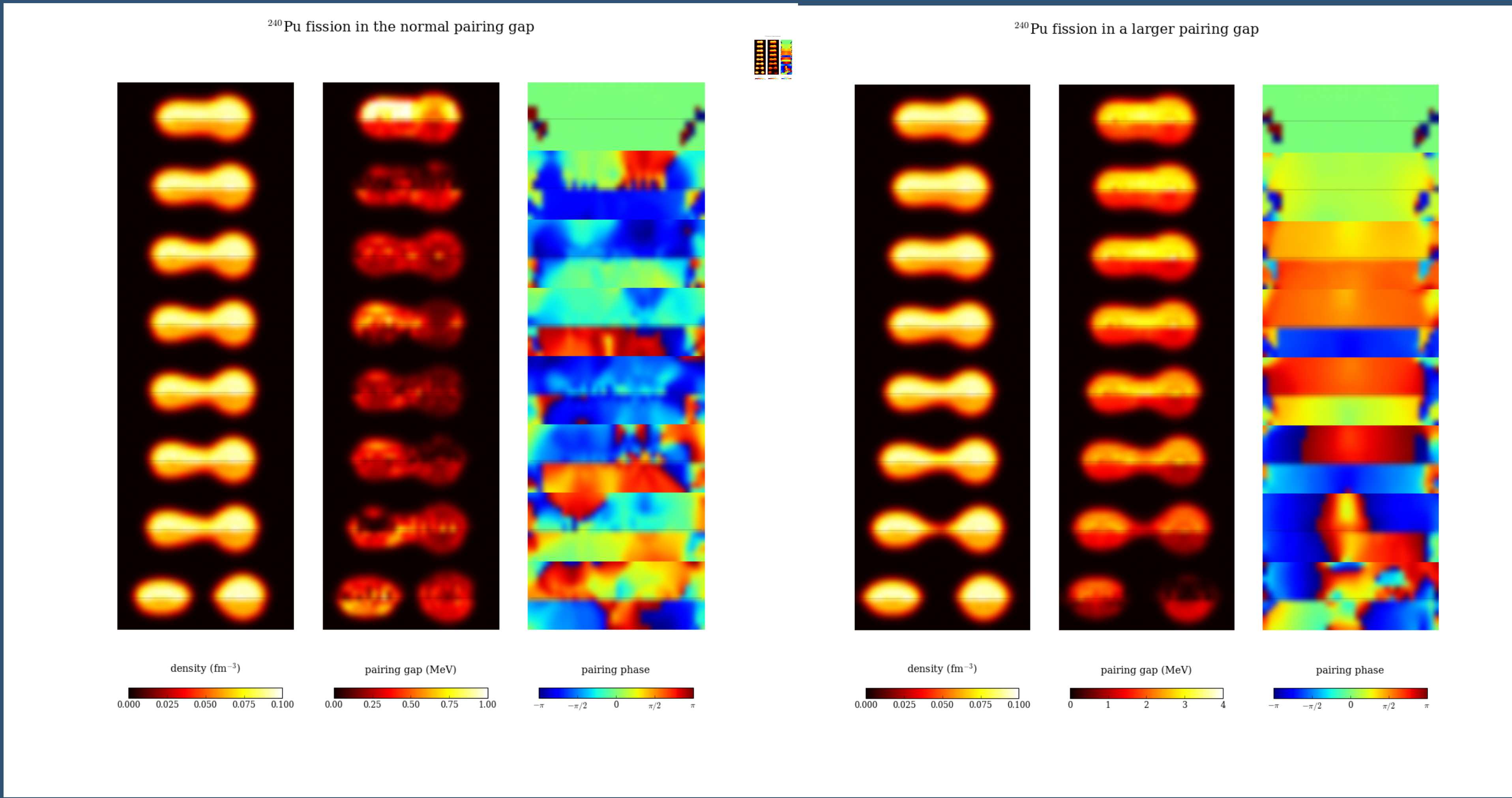






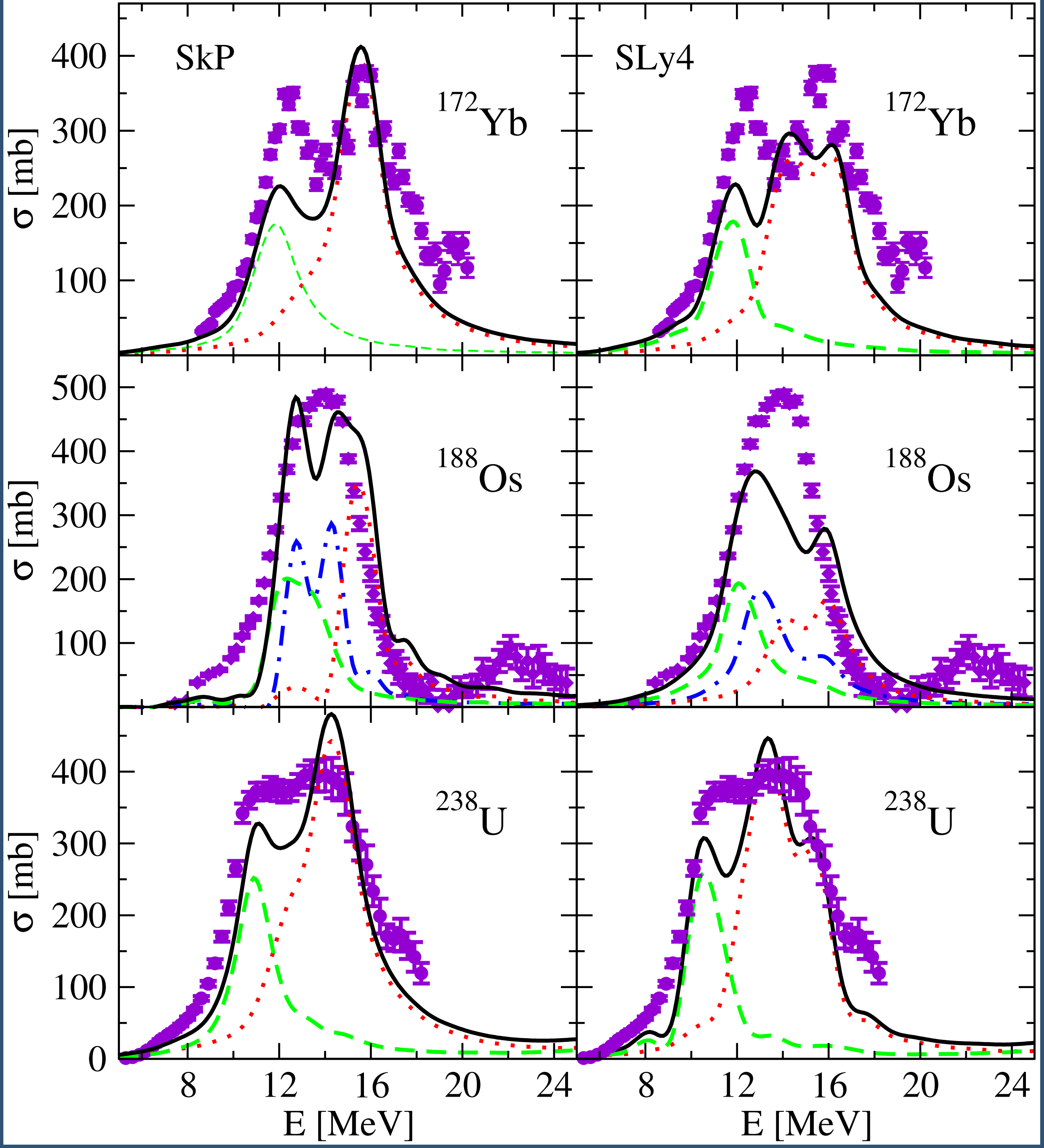
Bulgac et al, arXiv:1806.00694

How important pairing is?



Normal pairing strength
Saddle-to-scission 14,000 fm/c

Enhanced pairing strength
Saddle-to-scission 1,400 fm/c !!!



Giant Dipole Resonance deformed and superfluid nuclei

Osmium is triaxial, and both protons and neutrons are superfluid.

Stetcu, *et al.*, Phys. Rev. C 84, 051309(R) (2011)

Including dissipation and fluctuations

Classically, Langevin equation:

$$m\ddot{x}(t) = F - \gamma m\dot{x}(t) + m\xi(t),$$

$$\langle \xi(t) \rangle = 0, \quad \langle \xi(t)\xi(t') \rangle = \Gamma\delta(t-t'),$$

$$\dot{x}(t) = \mathrm{v}(0)\exp(-\gamma t) + \frac{F}{m\gamma}\left(1-\exp(-\gamma t)\right) + \int_0^t dt'\xi(t)\exp(-\gamma(t-t')),$$

$$\langle \mathrm{v}(t) \rangle \to \frac{F}{m\gamma}, \quad \langle\langle \mathrm{v}^2(t) \rangle\rangle \to \frac{\Gamma}{2\gamma} = \frac{T}{m}$$

Quantum mechanically, Lindblad equation

$$i\hbar\dot{\rho} = \left[H, \rho\right] - i\left(W\rho + \rho W\right) + i\sum_{k,l} h_{kl} A_k \rho A_l^{\dagger},$$

$$W = W^{\dagger} = \frac{1}{2}\sum_{k,l} h_{kl} A_l^{\dagger} A_k, \quad h_{kl} = h_{lk}^{*}, \quad \mathrm{Tr}\dot{\rho} = 0.$$

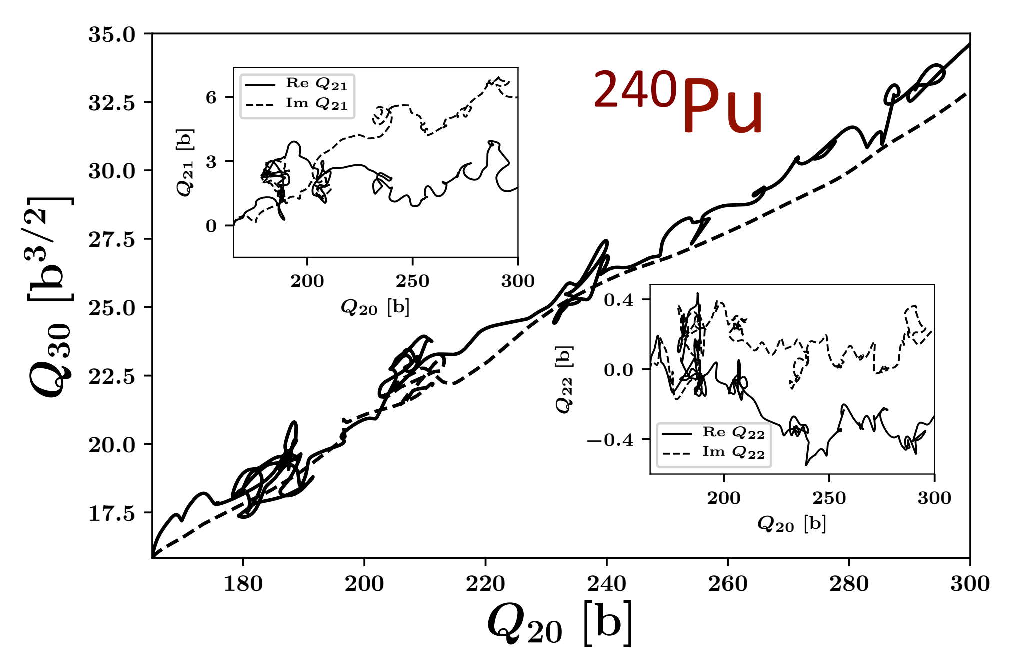
A much better and simpler solution:
A quantum Hermitian “Langevin” equation

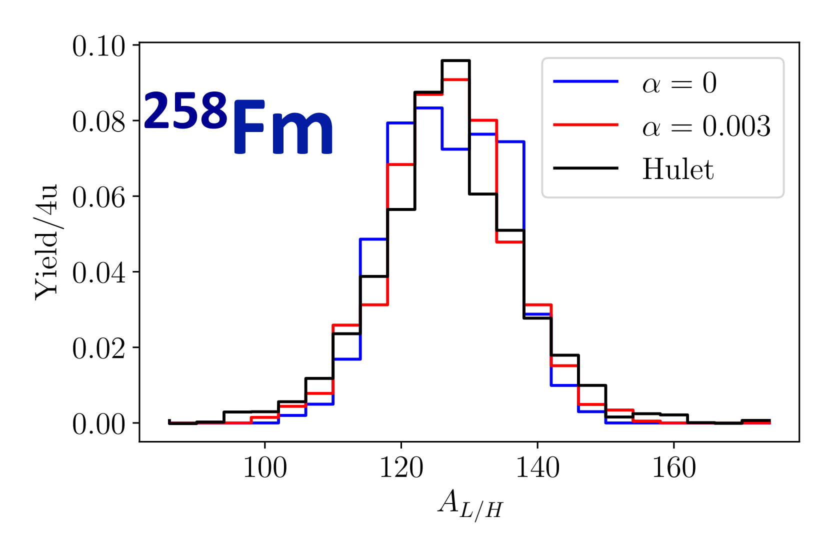
Quantum friction

$$i\hbar\dot{\psi}_k(\vec{r},t) = h\left[n(\vec{r},t)\right]\psi_k(\vec{r},t) + \gamma\left[n(\vec{r},t)\right]\dot{n}(\vec{r},t)\psi_k(\vec{r},t)$$

$$-\frac{1}{2}\left[\vec{u}(\vec{r},t)\cdot\vec{p} + \vec{p}\cdot\vec{u}(\vec{r},t)\right]\psi_k(\vec{r},t) + \varsigma(\vec{r},t)\psi_k(\vec{r},t)$$

“Stochastic fields”





Mass yields