Spin transport and quantum bounds for unitary fermions

dimensionality, scale invariance and strong interaction

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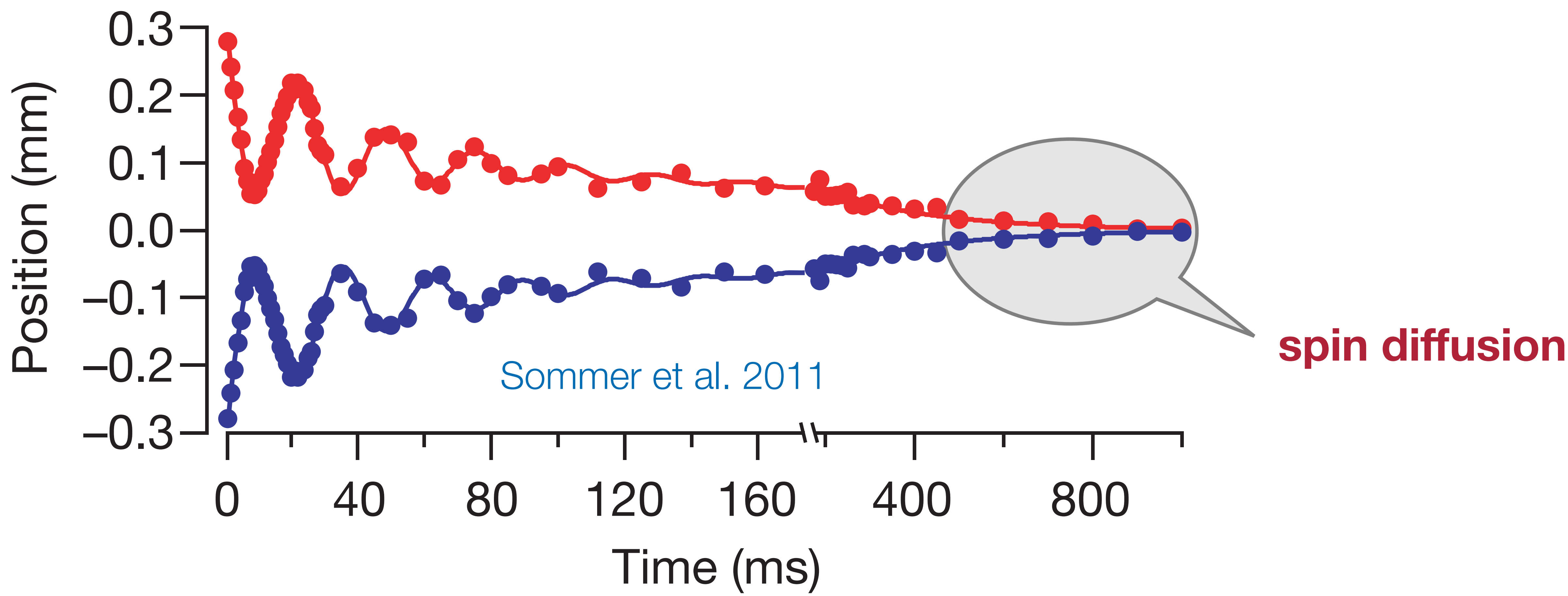
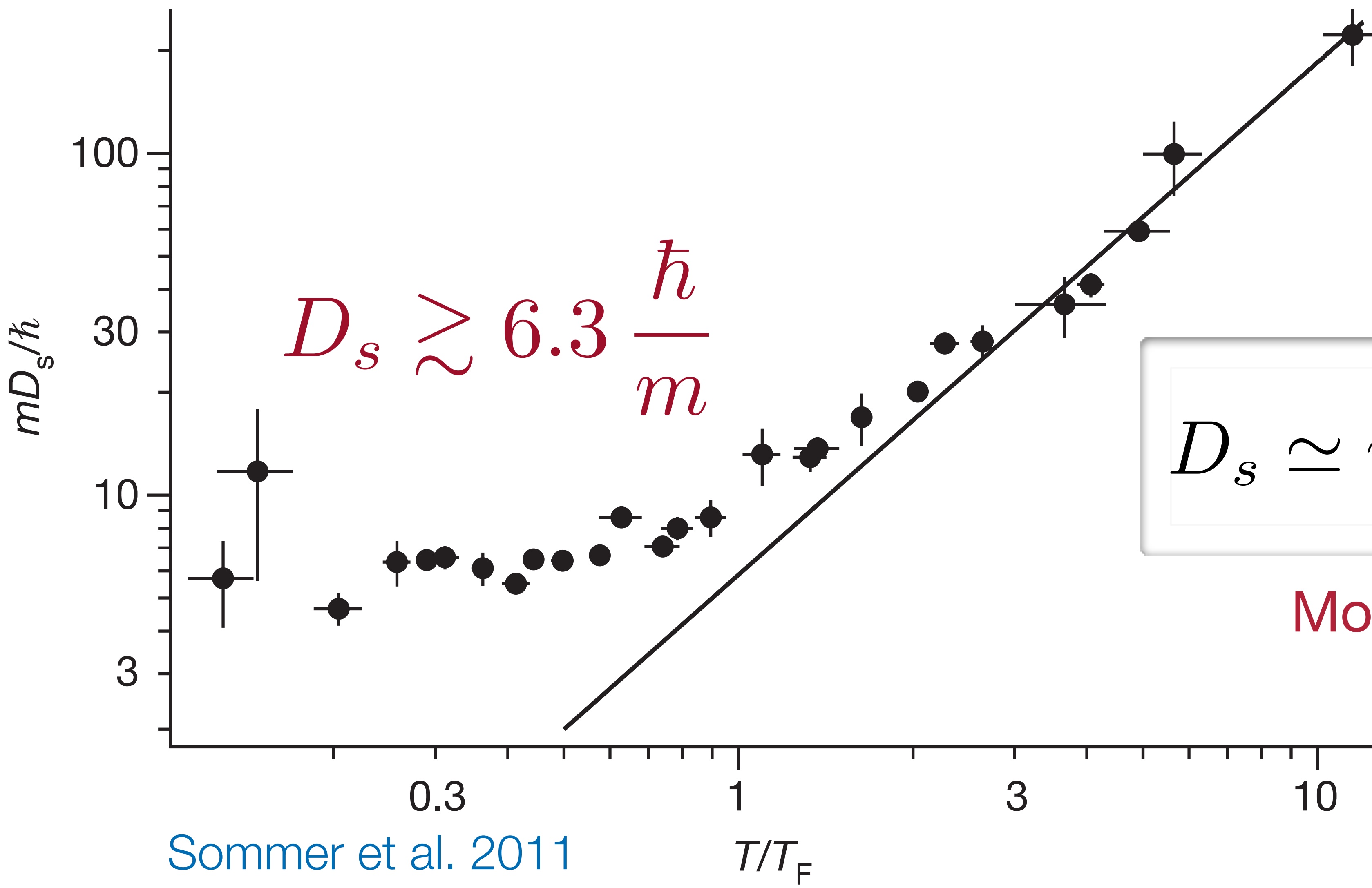
ECT* workshop
Trento, 21 June 2018

Spin diffusion for unitary fermions

- total ⬆+⬇ momentum conserved: particle current conserved
  relative ⬆-⬇ momentum not conserved: **spin current decays**



How slowly can spins diffuse?



$$D_s \gtrsim 6.3\,\frac{\hbar}{m}$$

$$D_s \simeq v_F^2 \tau \simeq \frac{\hbar \ell_{\mathrm{mfp}}}{m\ell} \gtrsim \frac{\hbar}{m}$$

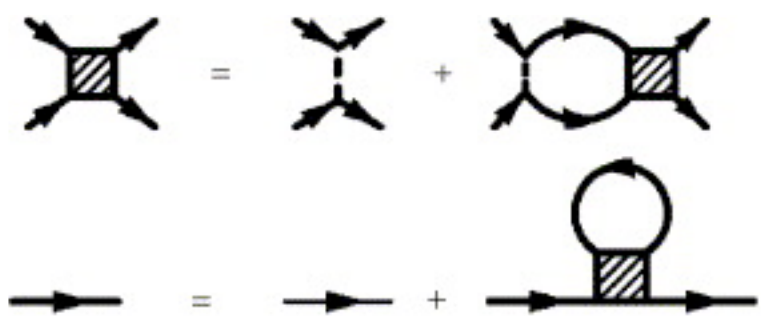
Mott-Ioffe-Regel (MIR) limit

Sommer et al. 2011

experimental parameters: $D_s = \dfrac{\text{area}}{\text{time}} \approx \dfrac{(100\,\mu\text{m})^2}{(1\,\text{second})} \approx \dfrac{\hbar}{m_{\text{Li}}}$

Luttinger-Ward approach

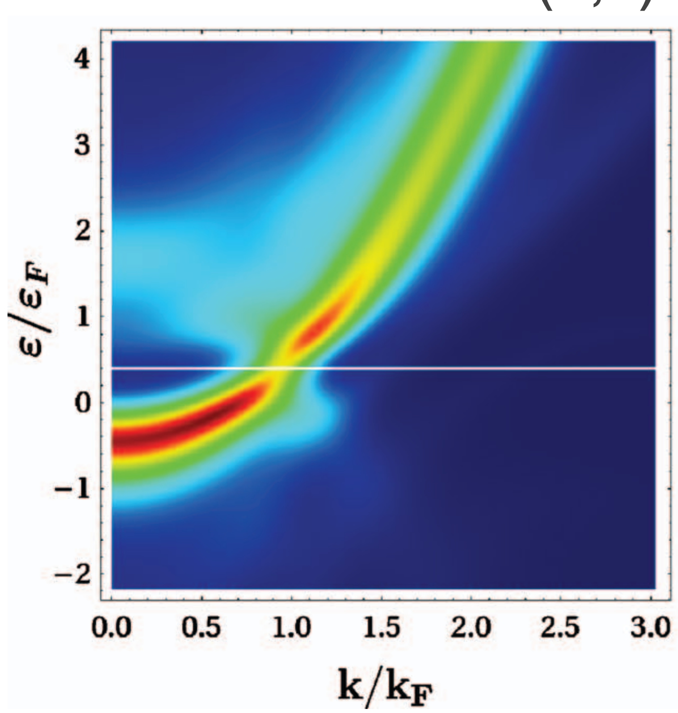
- repeated particle-particle scattering dominant in dilute gas:



self-consistent T-matrix Haussmann 1993, 1994; Haussmann et al. 2007

self-consistent fermion propagator
(300 momenta / 300 Matsubara frequencies)

- spectral function A(k,ε) at Tc



Haussmann et al. 2009

works above and below Tc;
directly in continuum limit

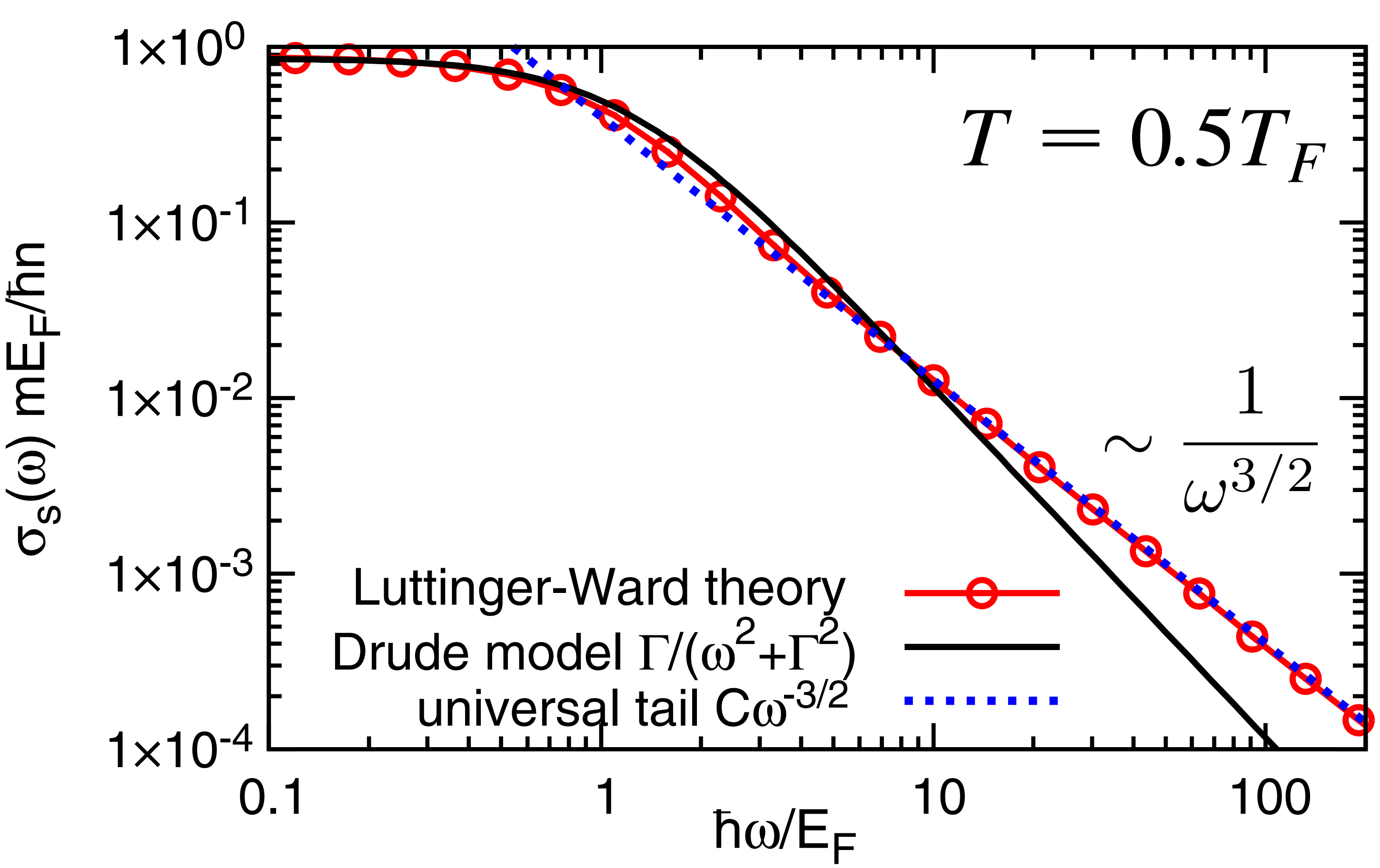
Tc=0.16(1) and ξ=0.36(1)
agree with experiment

conserving: exactly fulfills scale invariance and Tan relations
Enss PRA 2012

Dynamical spin conductivity

$$\sigma_s(\omega) = \frac{1}{\omega}\,\mathrm{Re}\int_0^\infty dt\, e^{i\omega t}\int d^3x\, \langle [j_s^z(\boldsymbol{x},t), j_s^z(0,0)] \rangle$$

with spin current operator $j_s(\boldsymbol{x},t) = j_\uparrow(\boldsymbol{x},t) - j_\downarrow(\boldsymbol{x},t)$



Enss & Haussmann PRL 2012

exact high-frequency tail

Hofmann PRA 2011;
Enss & Haussmann PRL 2012

$$\sigma_s(\omega \to \infty) = \frac{C}{3\pi(m\omega)^{3/2}}$$

Dynamical spin conductivity

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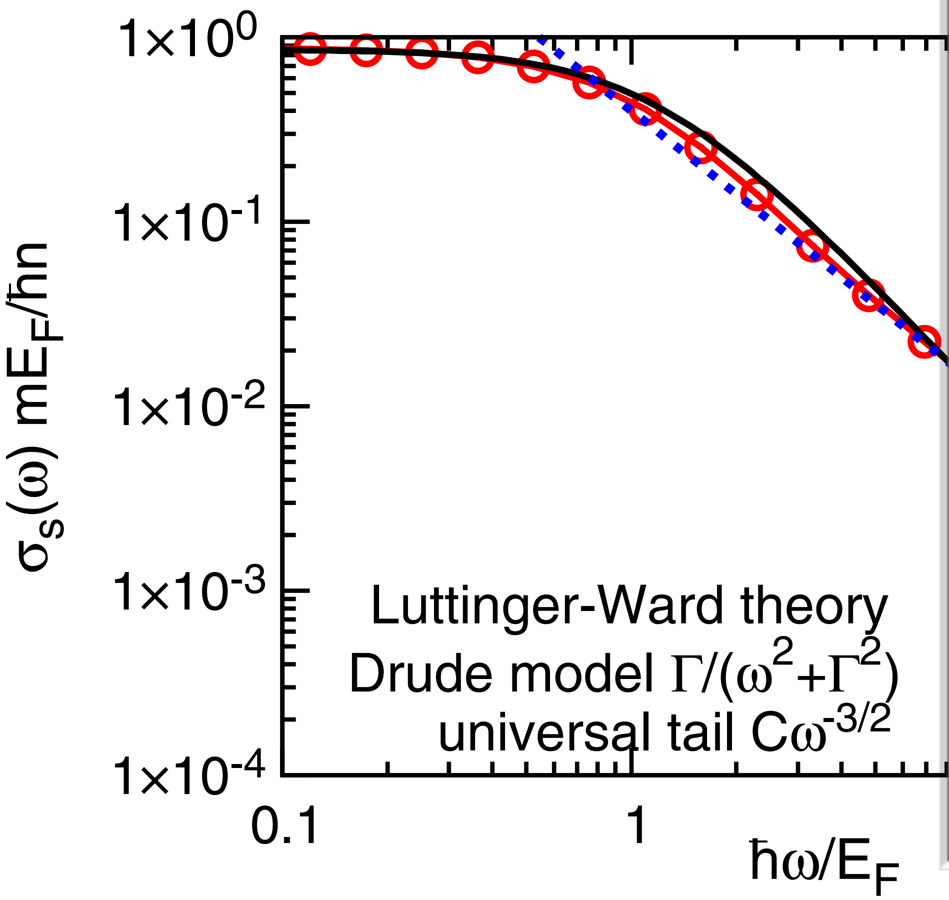
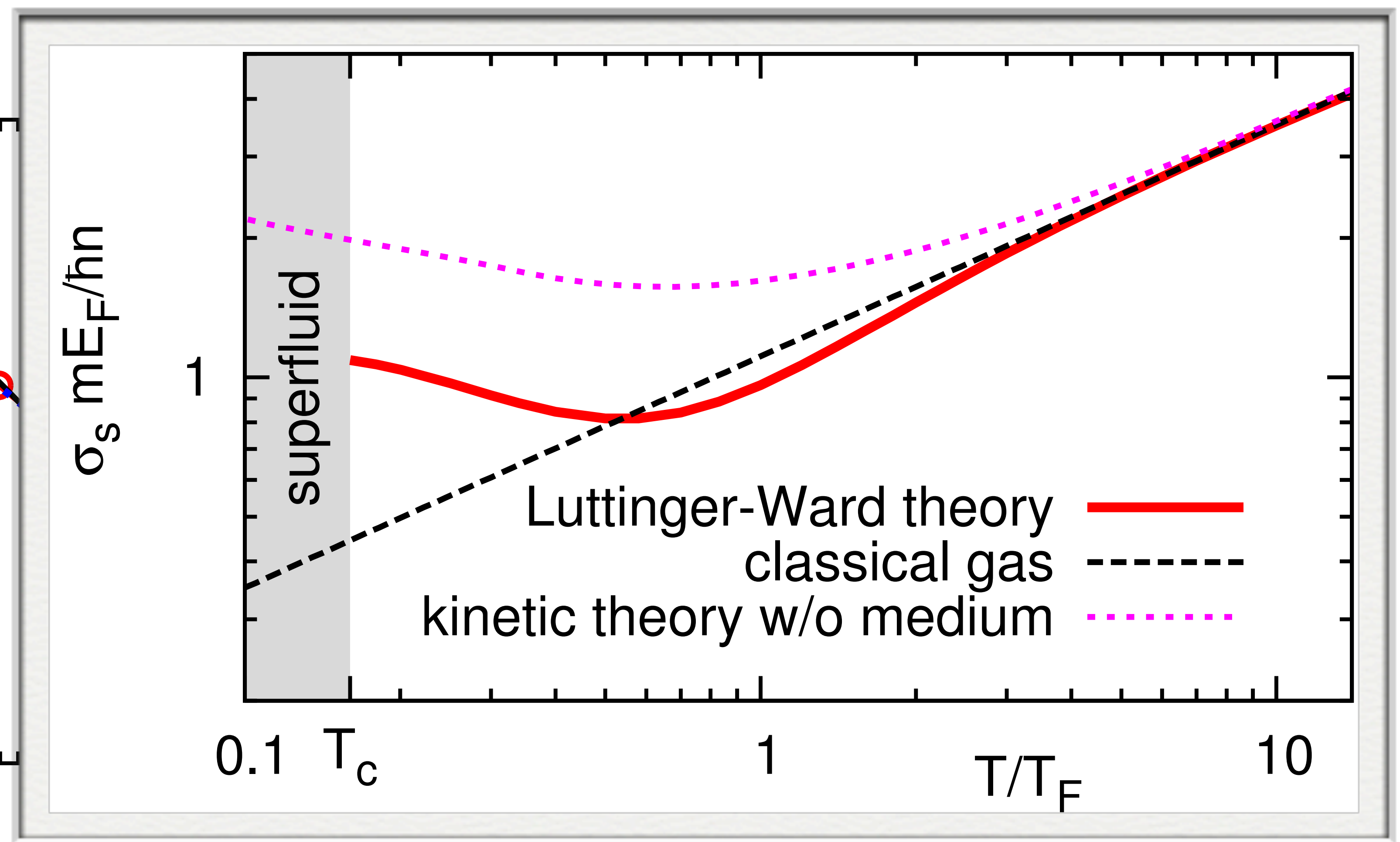
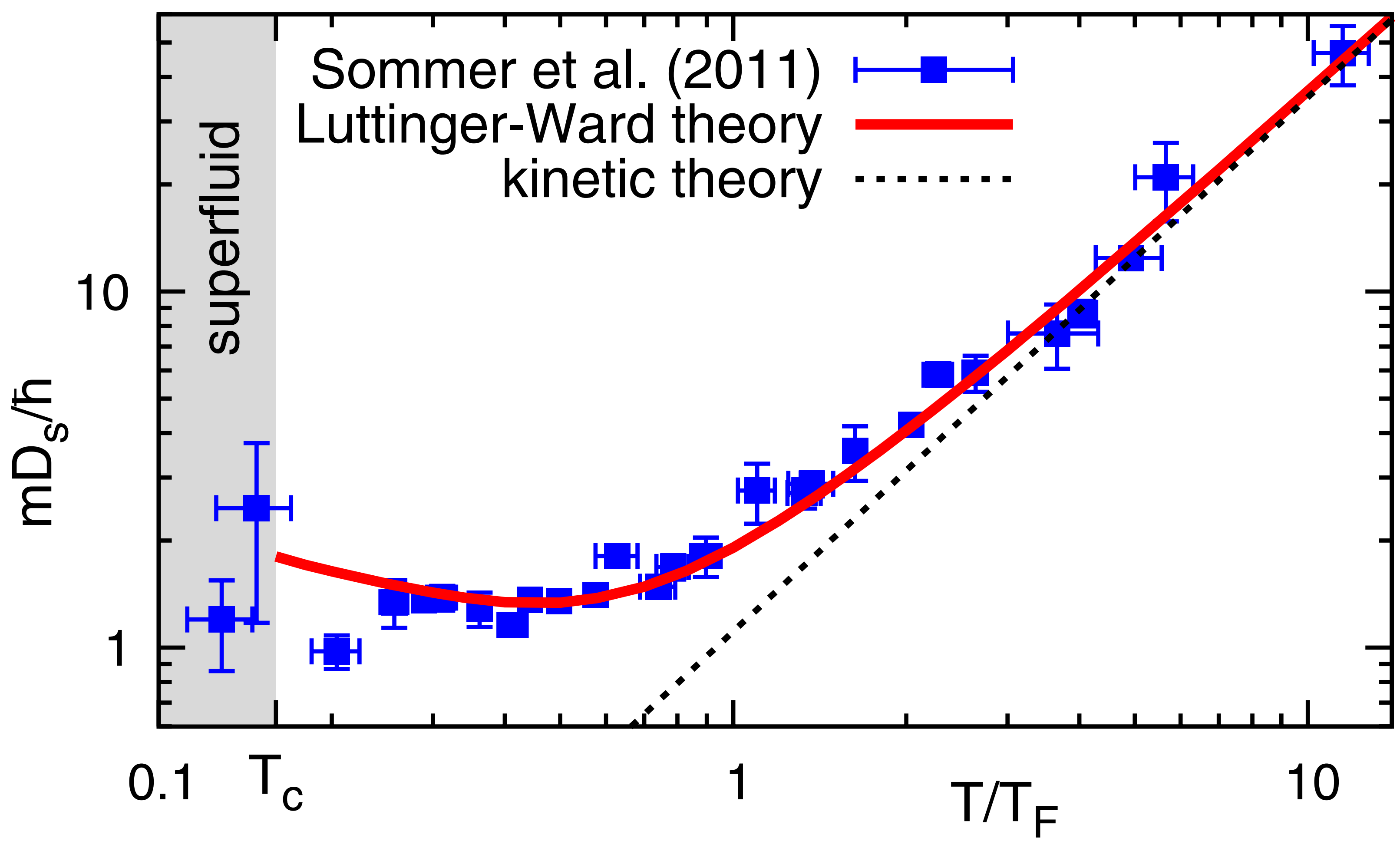




Enss & Haussmann PRL 2012

Spin diffusivity

- obtain diffusivity from Einstein relation, $D_s = \dfrac{\sigma_s}{\chi_s}$



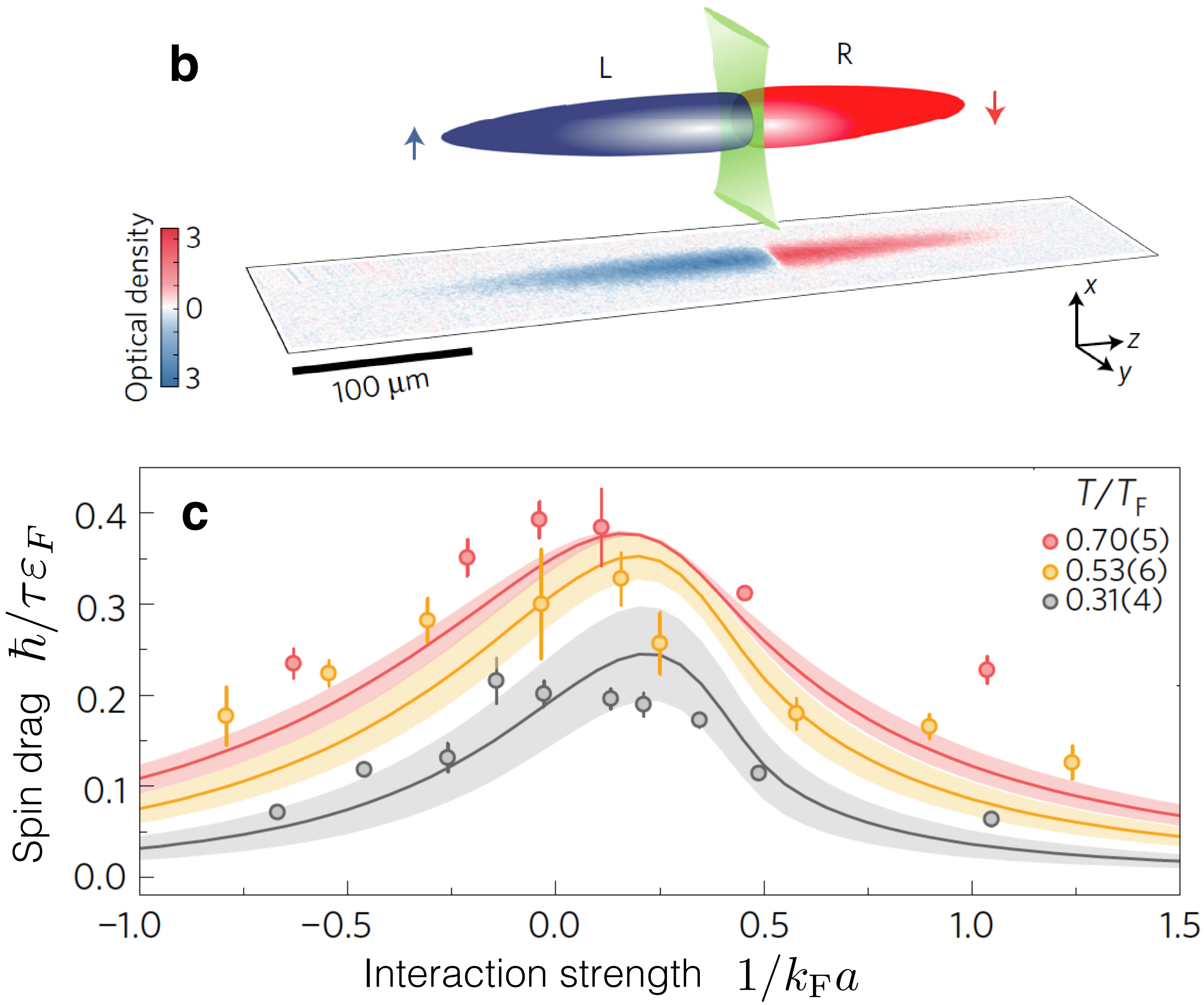
(experiment rescaled from trap to infinite homogeneous box)

minimum $D_s \simeq 1.3 \dfrac{\hbar}{m}$

Enss & Haussmann PRL 2012

see also: Wlazlowski, Magierski, Drut, Bulgac & Roche PRL 2013

Spin drag rate



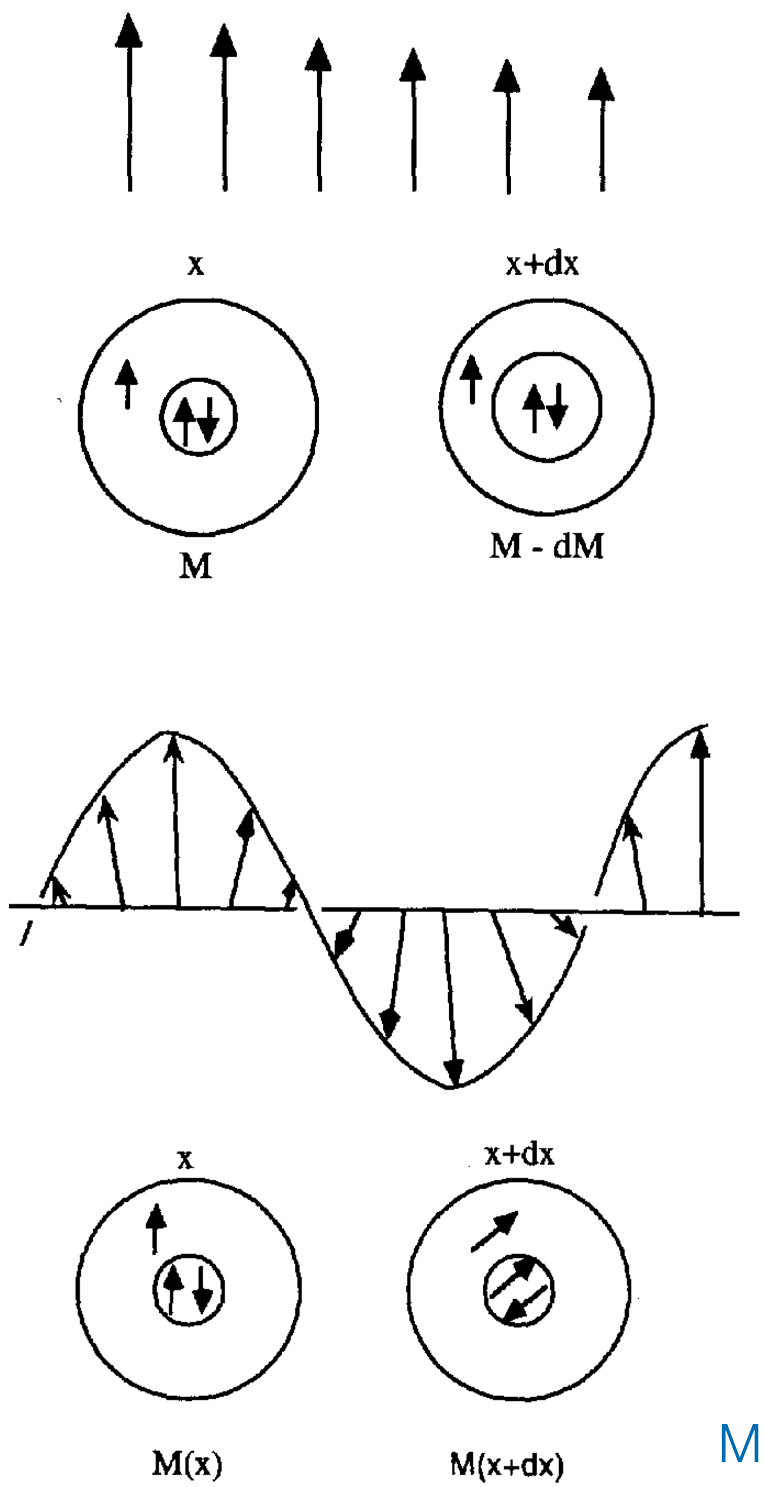
at unitarity:

$$\boxed{\frac{1}{\tau} \lesssim \frac{k_B T}{\hbar}}$$

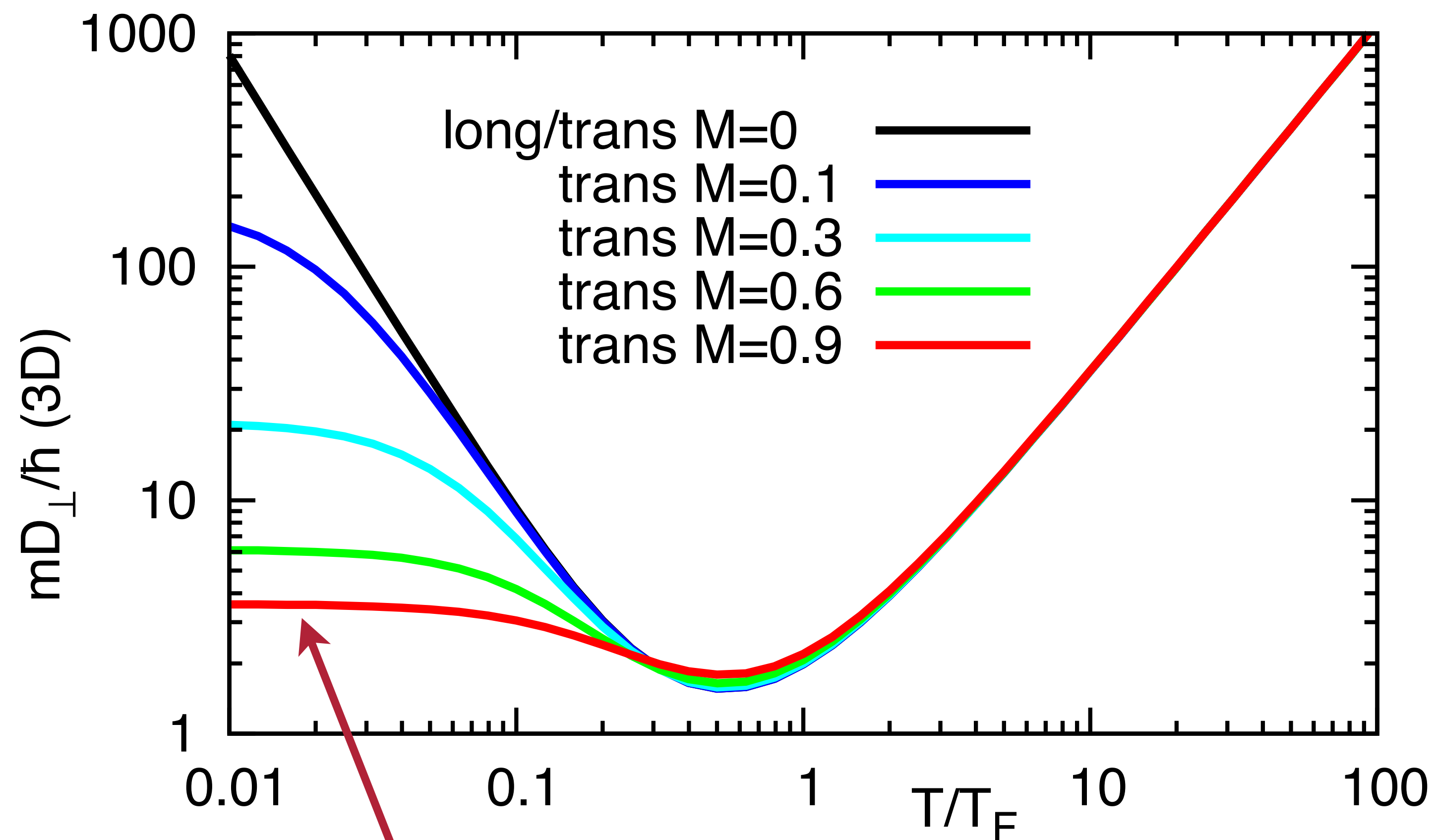
Valtolina, Scazza, Amico, Burchianti, Recati, Enss, Inguscio, Zaccanti & Roati, Nat. Phys. 2017

Transverse spin diffusion

Longitudinal vs transverse spin diffusion



Mullin & Jeon JLTP 1992



Enss PRA 2013

spin polarized & quantum degenerate

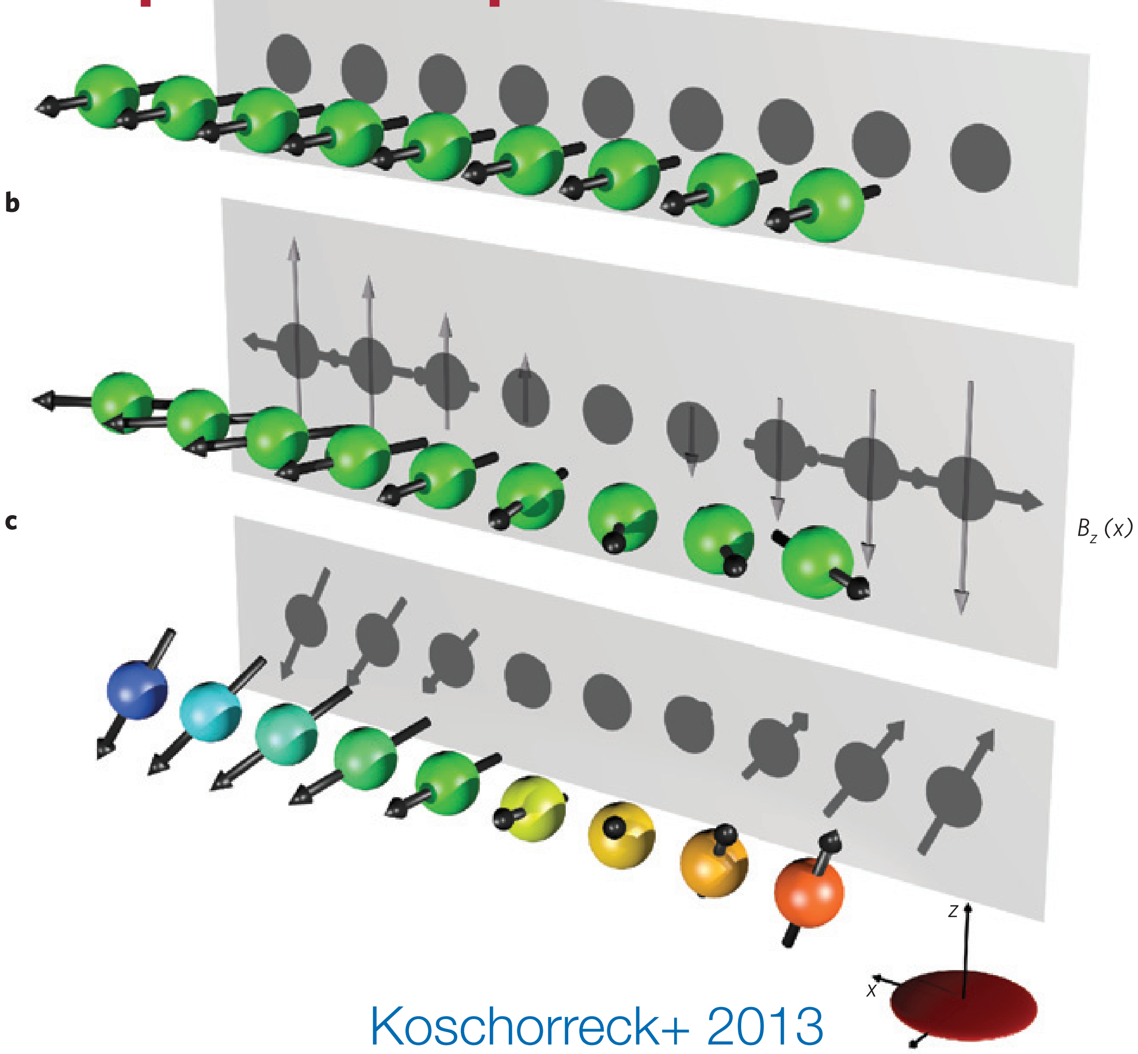
Demagnetization dynamics by spin transport

- **transverse spin current** precesses around local magnetization

$$\boldsymbol{J}_j^{\perp} = -D_{\text{eff}}^{\perp} \nabla_j \boldsymbol{M} - \gamma \boldsymbol{M} \times D_{\text{eff}}^{\perp} \nabla_j \boldsymbol{M}$$

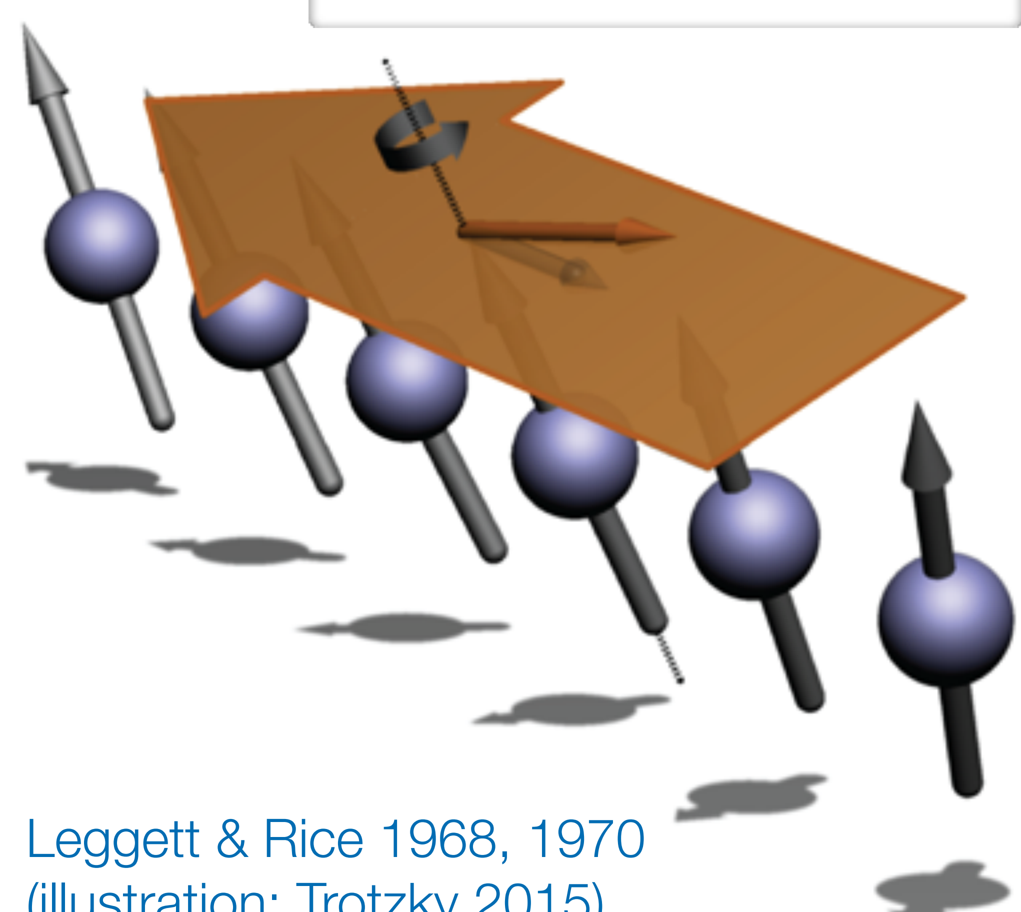
diffusive **reactive (Leggett-Rice)**

- **imprint local perturbation on fluid:**



Koschorreck+ 2013

$$D_{\text{eff}}^{\perp} = \frac{D_0^{\perp}}{1 + \gamma^2 M^2}$$

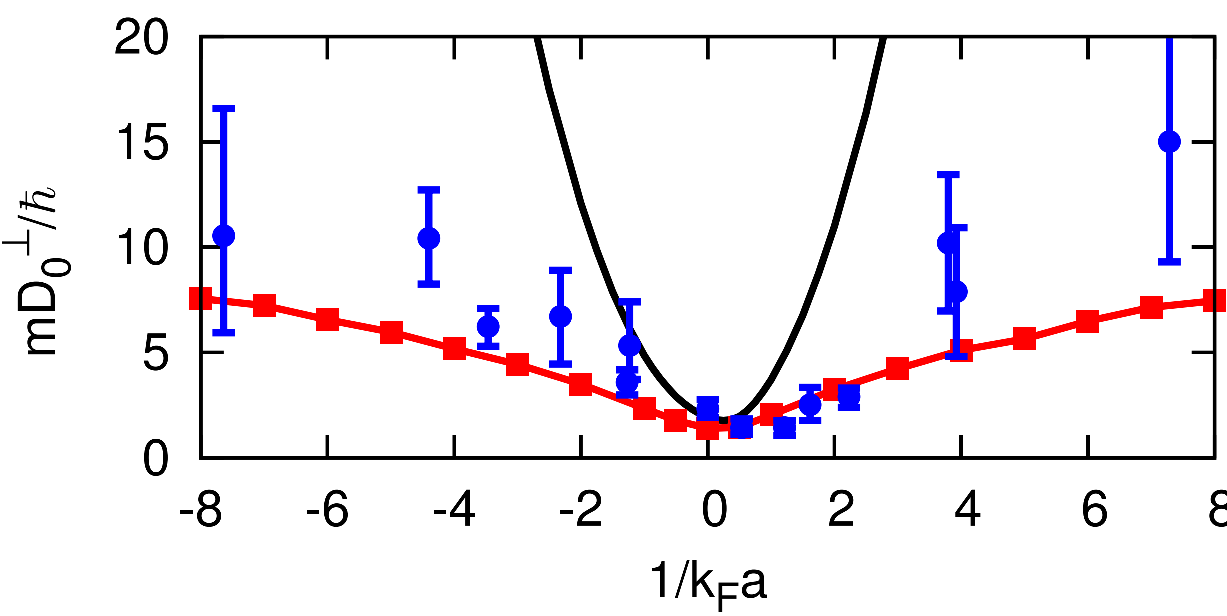


Leggett & Rice 1968, 1970
(illustration: Trotzky 2015)

Transverse diffusion coefficient

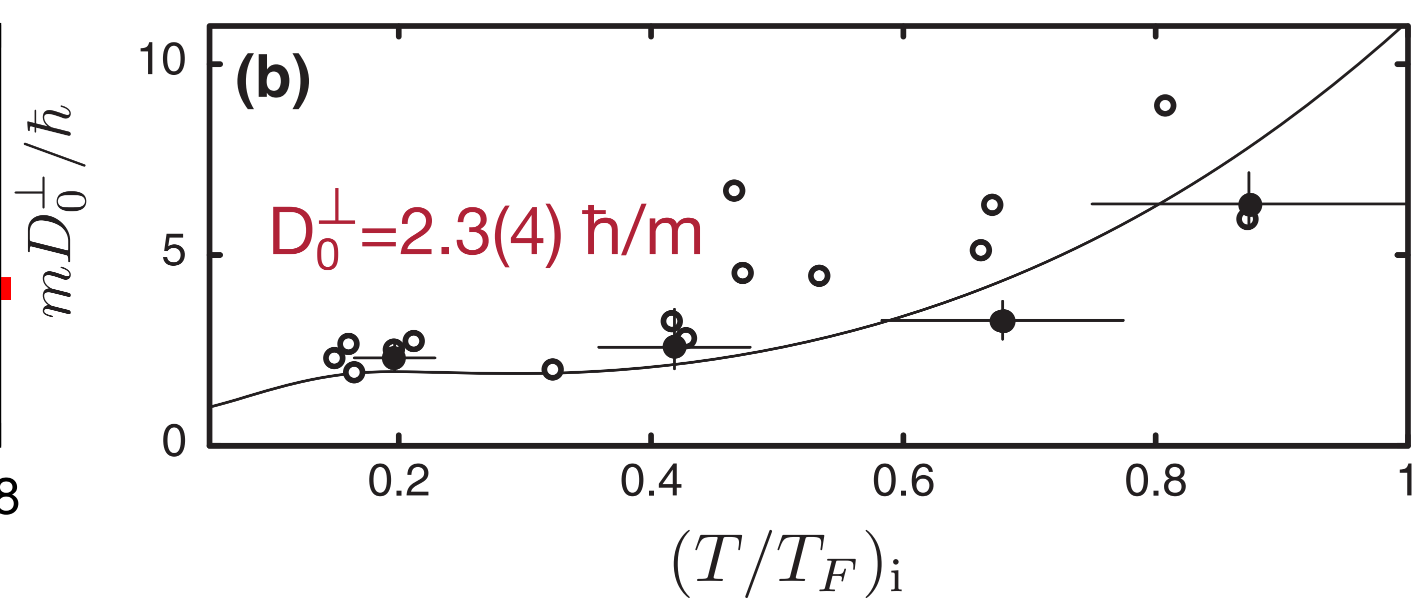
$$D_{\mathrm{eff}}^{\perp} = \frac{D_0^{\perp}}{1 + \gamma^2 M^2}$$

interaction dependence:
minimum near unitarity



Enss, PRA **91**, 023614 (2015)

temperature dependence:
quantum limited diffusion at unitarity



Trotzky *et al.*, PRL **114**, 015301 (2015)

Effective interaction

- precession of spin current around local magnetization m:

Ramsey phase ϕ $\gamma M = -Wm\frac{\tau_\perp}{\hbar}$ $D_0^\perp = \frac{2\varepsilon_F \tau_\perp}{3m^*}$

molecular field

$$\boxed{\lambda = -\frac{\hbar\gamma}{2m^* D_0^\perp} = \frac{3n}{4\varepsilon_F} W}$$

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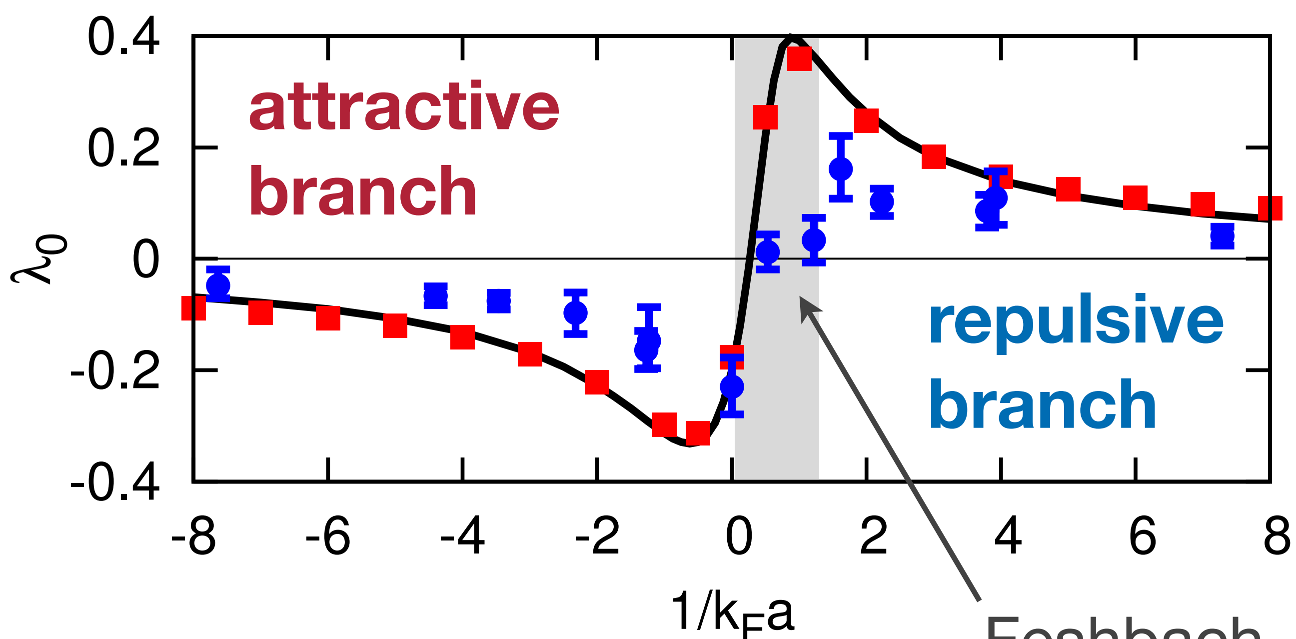


1. **interaction dependence:** sign change near unitarity

2. **Fermi-liquid theory:**

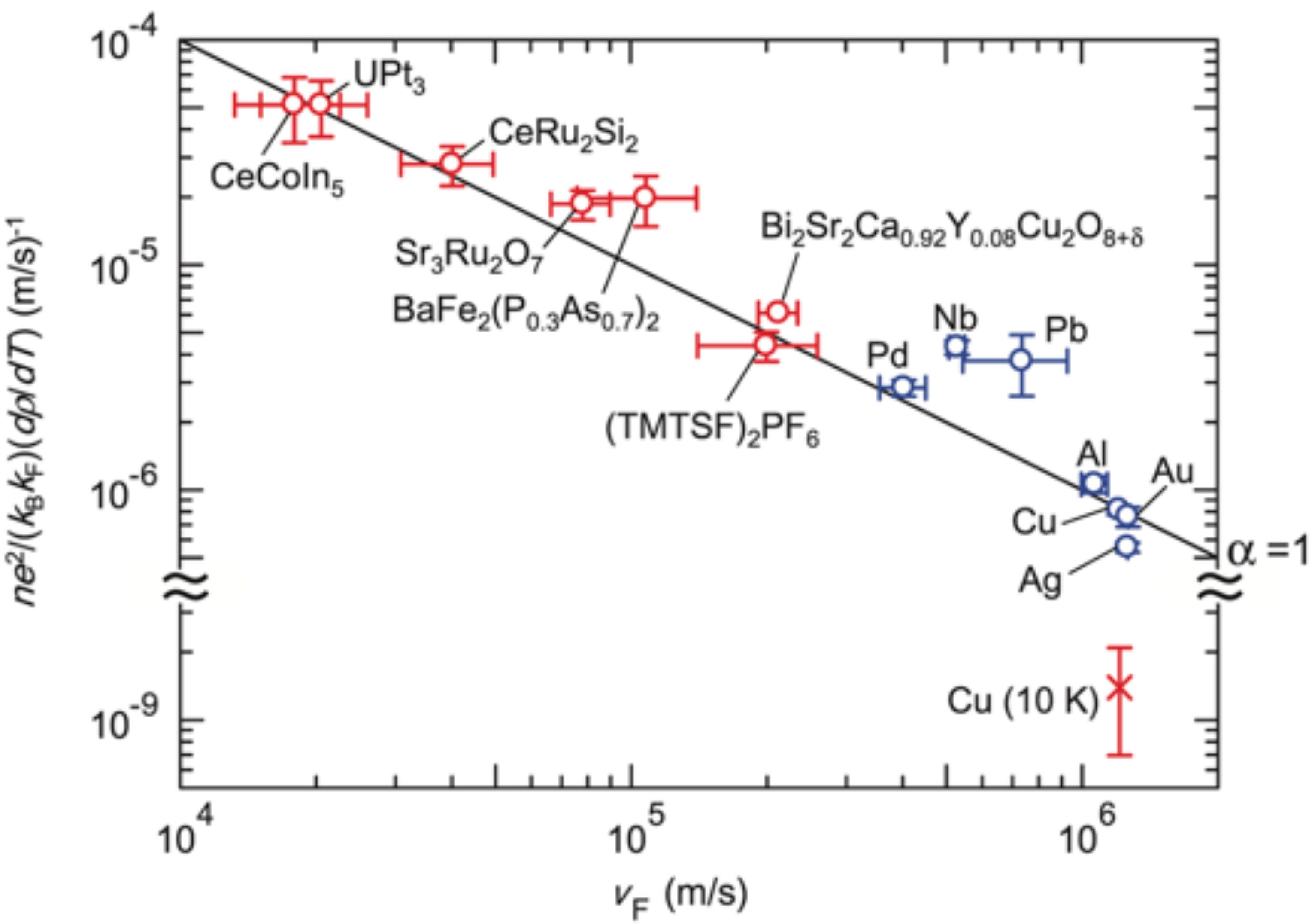
$$\lambda = \frac{1}{1+F_0^a} - \frac{1}{1+F_1^a/3}$$

first measurement of $F_1^a \simeq 0.5$ at unitarity

Enss, PRA **91**, 023614 (2015)
Trotzky *et al.*, PRL **114**, 015301 (2015)

Transport bounds

Transport bounds in solids



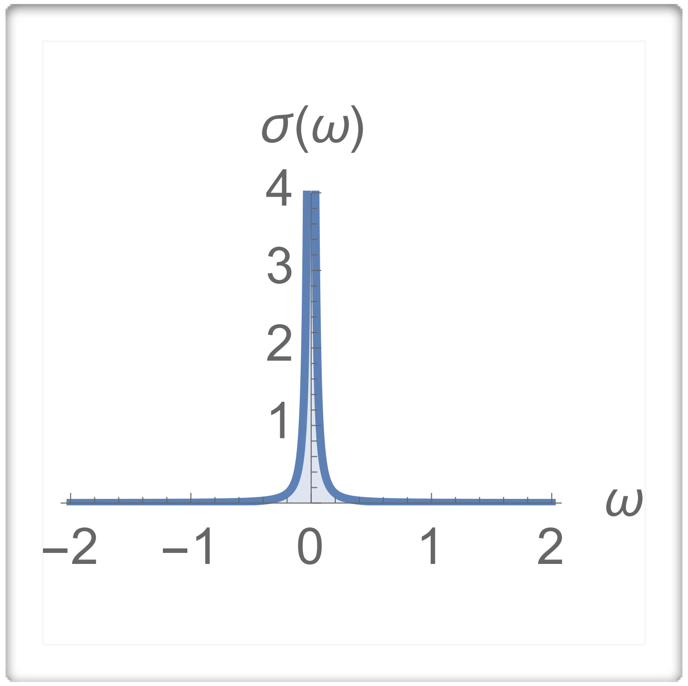
Bruin et al. Science 2013

T-linear resistivity

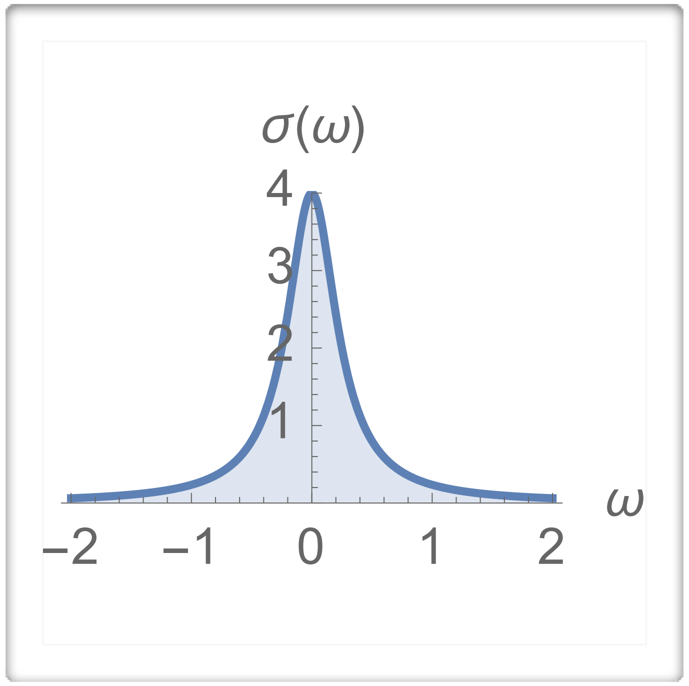
$$\tau_r^{-1} \lesssim \frac{k_B T}{\hbar}$$

Transport regimes

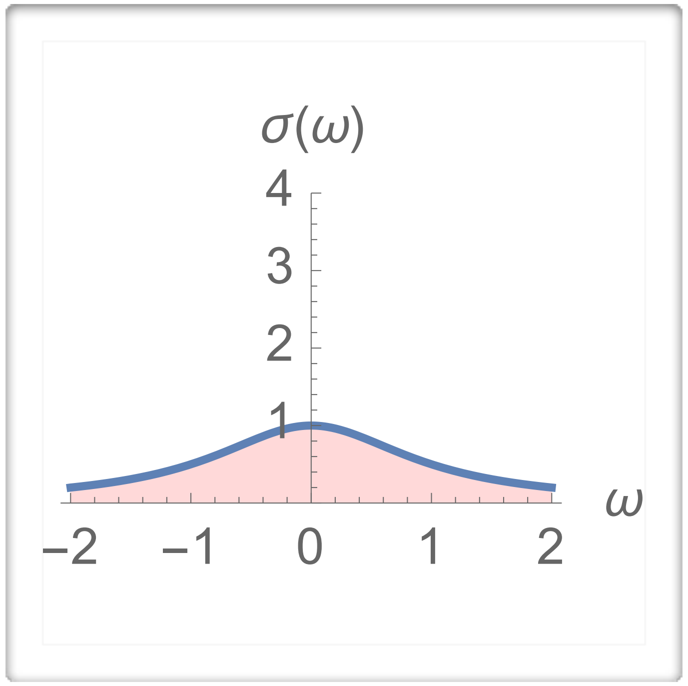
- conserved particle current: $\sigma(\omega)=\delta(\omega)$



- almost conserved current:
  $\sigma(\omega)=\tau/(1+\omega^2\tau^2)$
  by extrinsic processes
  (Umklapp, impurities):
  nonuniversal



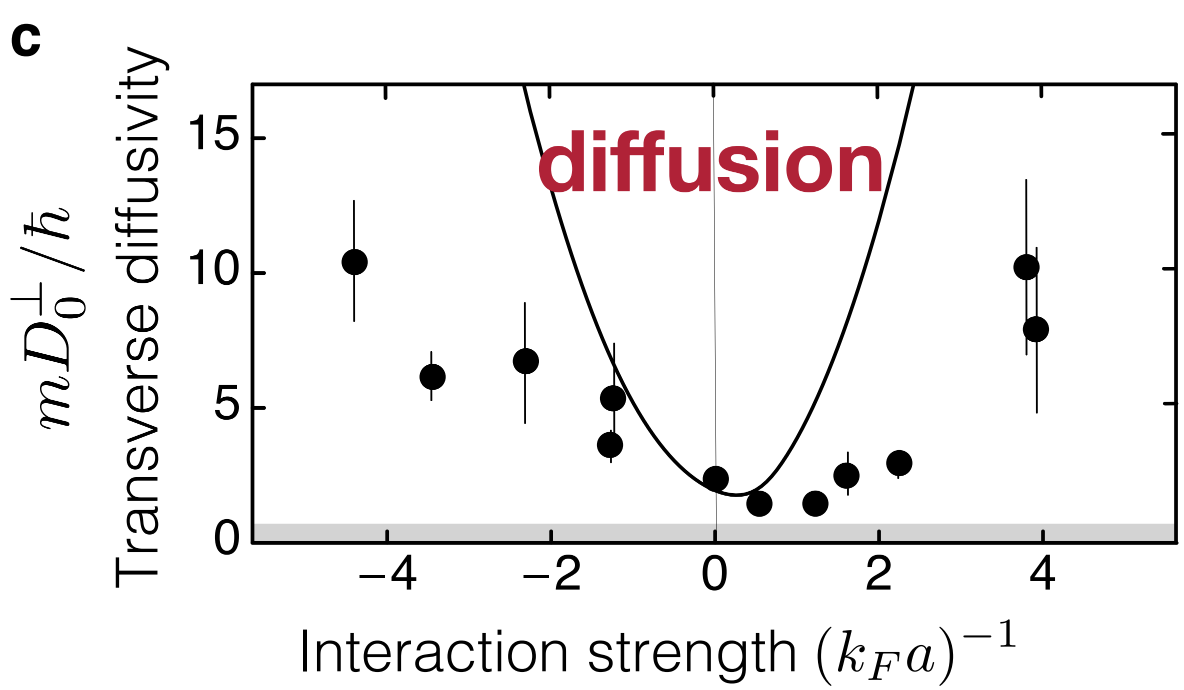
- **nonconserved spin current:**
  quantum limited width $\tau^{-1} \sim T$
  incoherent transport, universal,
  Planckian dissipation



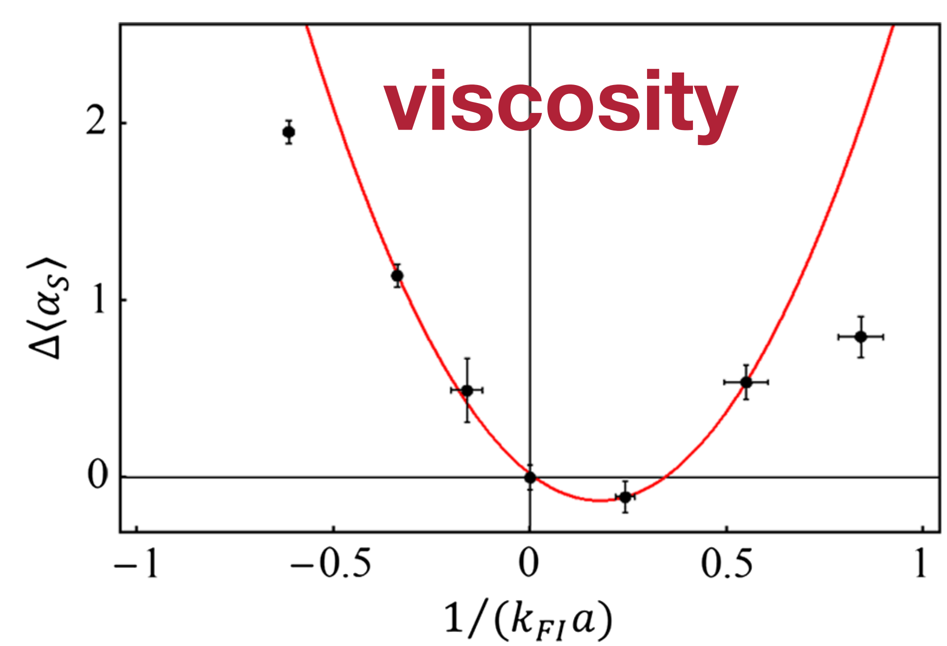
Enss & Thywissen, 1805.05354, Annu. Rev. CMP (2019)

Incoherent transport by Scale invariance?

- Unitary Fermi gas (UFG) quantum limited ✔
- **lower D / viscosity away from unitarity** ✘



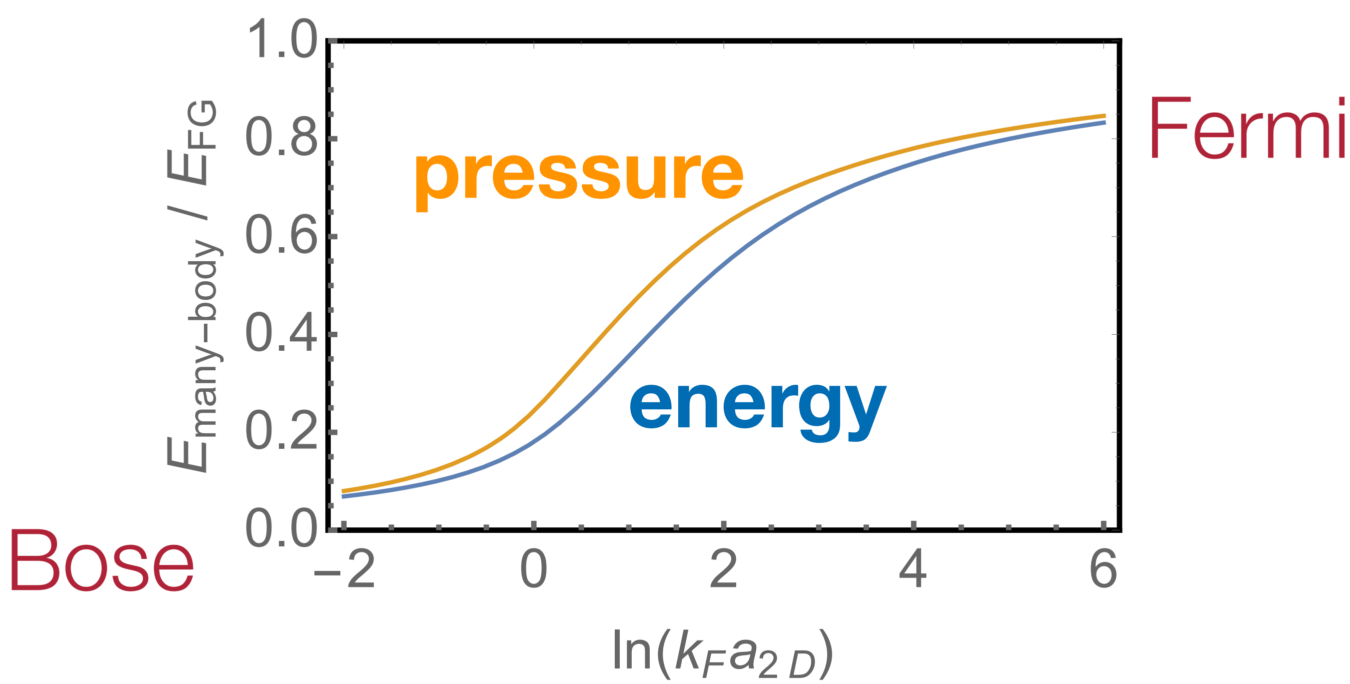
Enss & Thywissen, 1805.05354, Annu. Rev. CMP (2019)

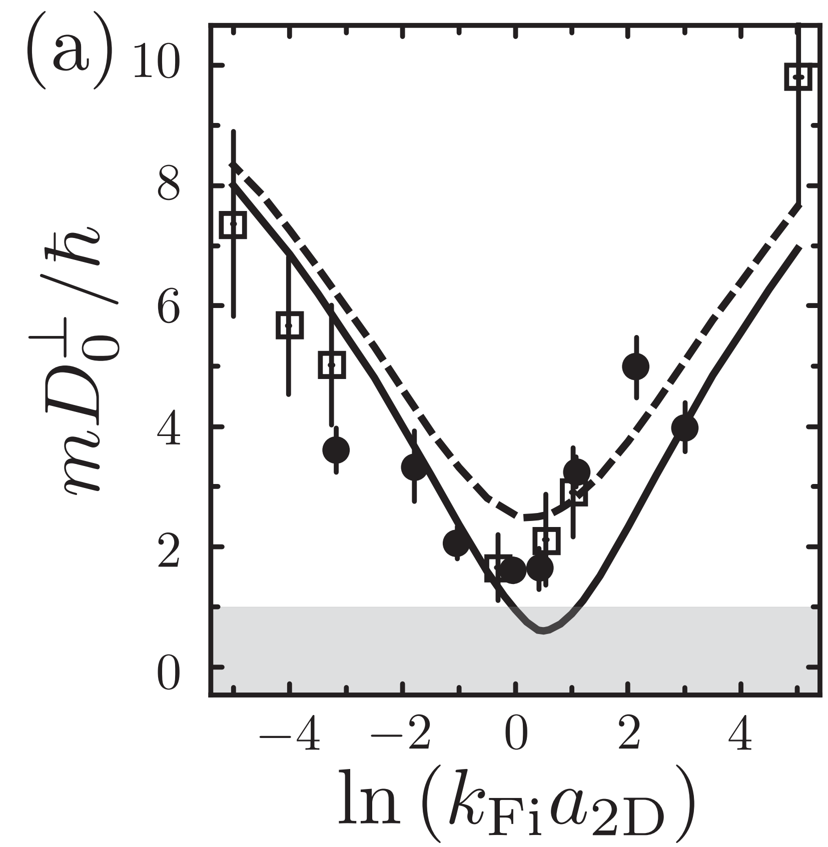


Elliott, Joseph & Thomas PRL 2014

Incoherent transport by Scale invariance?

- 2D: scale invariance most strongly broken in crossover, still lowest D ✘



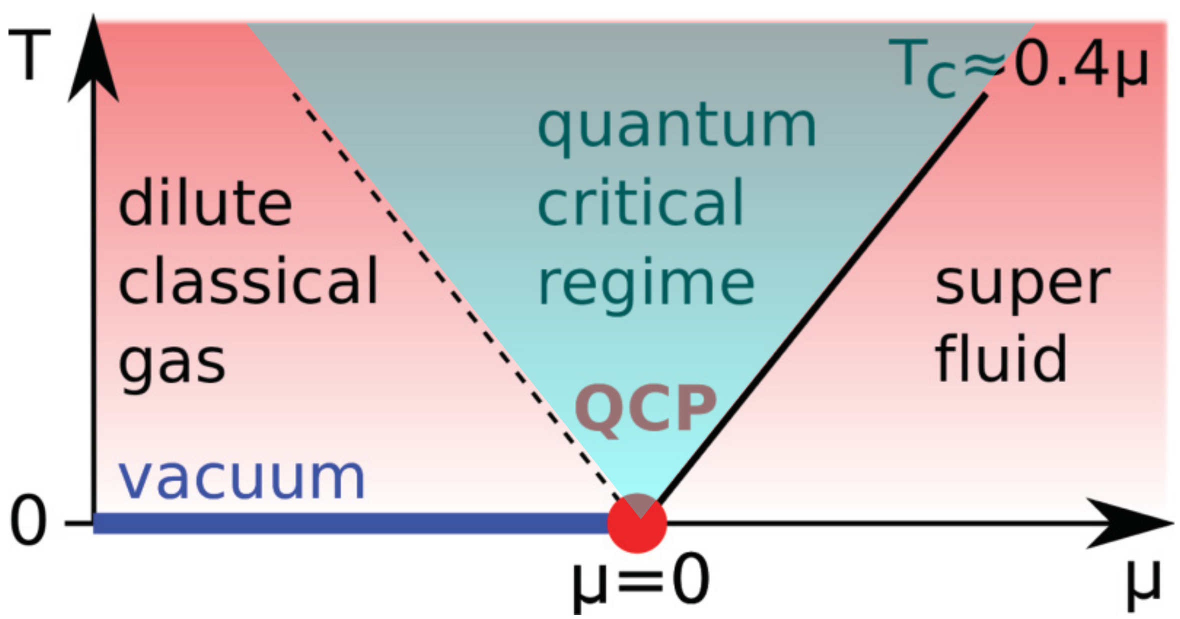


Luciuk, Smale, Böttcher, Sharum, Olsen, Trotzky,
Enss & Thywissen, PRL 118, 130405 (2017)

Bounds by Quantum critical transport?

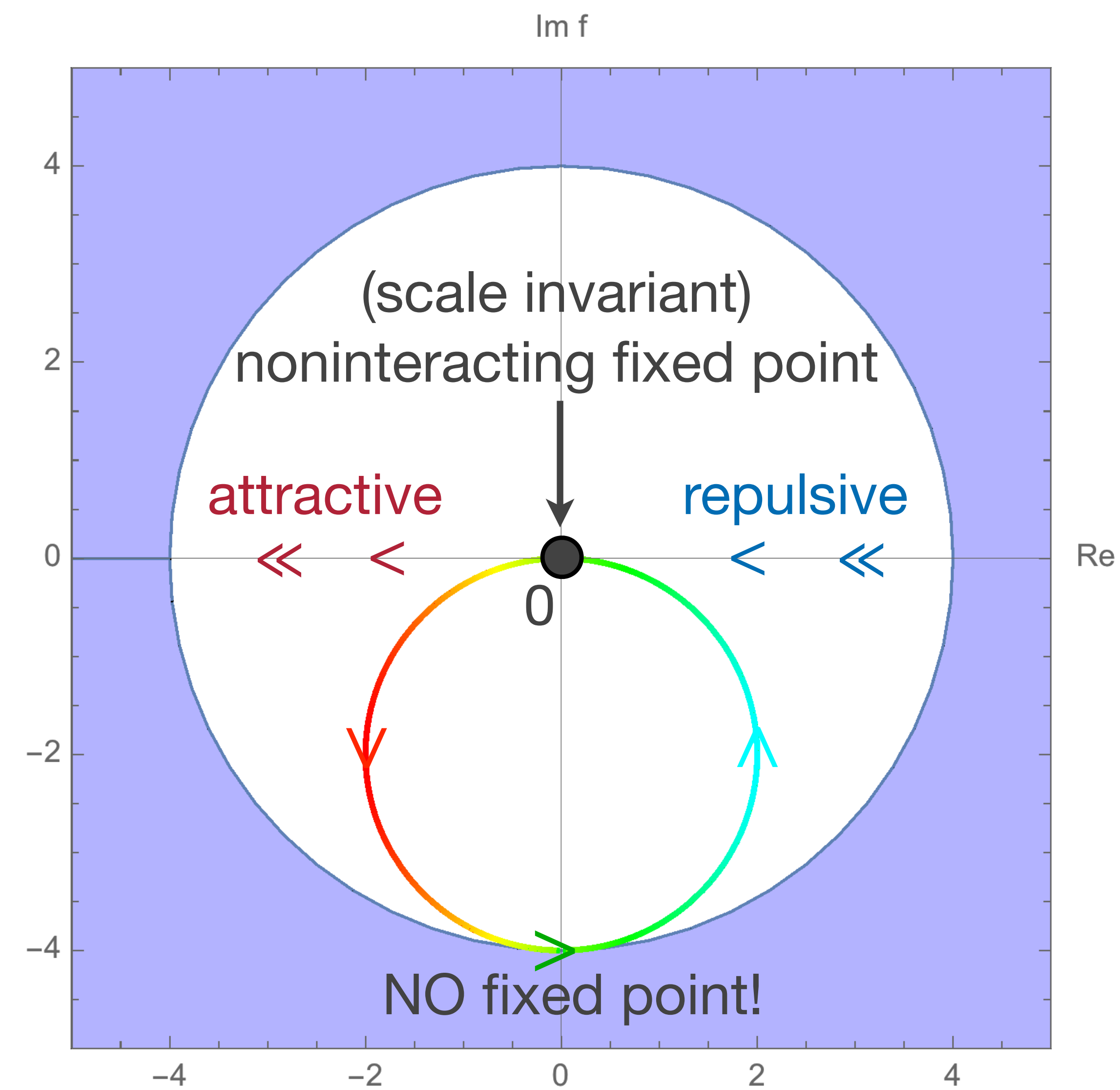
- Unitary Fermi gas in
  quantum critical regime ✔

$$\tau_r^{-1} \lesssim \frac{k_B T}{\hbar}$$



Nikolic, Sachdev 2007; Enss 2012

- **2D: no interacting QCP** ✗



- incoherent metals not always quantum critical (Cu, Au) ✗

Enss & Thywissen, 1805.05354, Annu. Rev. CMP (2019)

Incoherent transport

- **absence of quasiparticles:**

quasiparticles with long lifetime

diffusion far above bound

$$\frac{1}{\tau} \ll \frac{\varepsilon_{\mathrm{qp}}}{\hbar} \qquad \Rightarrow \qquad D \sim \frac{\tau\varepsilon_{\mathrm{qp}}}{m} \gg \frac{\hbar}{m}$$

- **incoherent "metallic" transport:**

$$\boxed{\frac{1}{\tau} \lesssim \frac{k_B T}{\hbar}}$$

- can violate MIR bound parametrically if, e.g., number of carriers decreases

Hartnoll, Nat. Phys. 2015;
Enss & Thywissen, 1805.05354, Annu. Rev. CMP (2019)

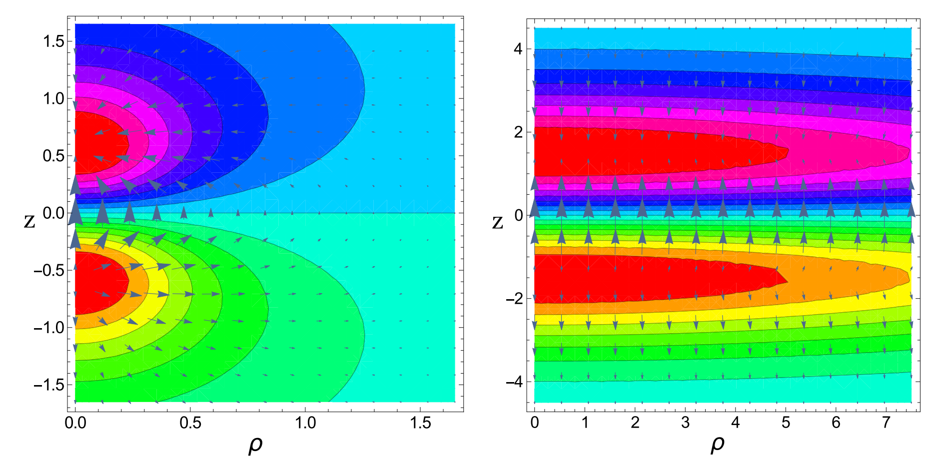
Conclusions

- universal quantum bounds for incoherent spin transport
  for strong scattering, but not necessarily scale invariance/quantum criticality
  Enss & Thywissen, 1805.05354, Annu. Rev. CMP (2019)

$$\boxed{\frac{1}{\tau} \lesssim \frac{k_B T}{\hbar}}$$

$$\boxed{D_{\parallel,\perp} \gtrsim \frac{\hbar}{m}}$$

- map global (trap) dynamics to local transport: hydrodynamics and beyond
  (dense core/dilute corona, quench dynamics,
  small systems; box potential)



Schäfer 2016

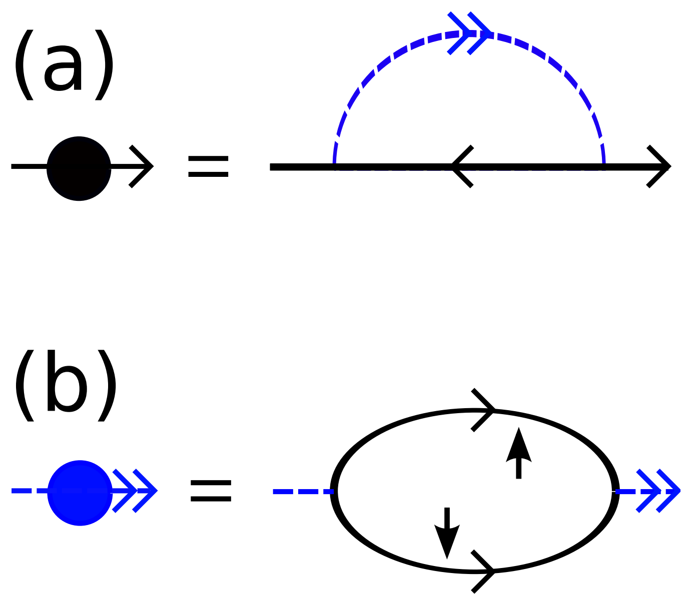
- transport theory:
  - efficient computation for strong coupling
  - superfluid fluctuations near Tc (need vertex corrections)
  - bounds in nonmetallic states (ferromagnet, superfluid)?

Additional material

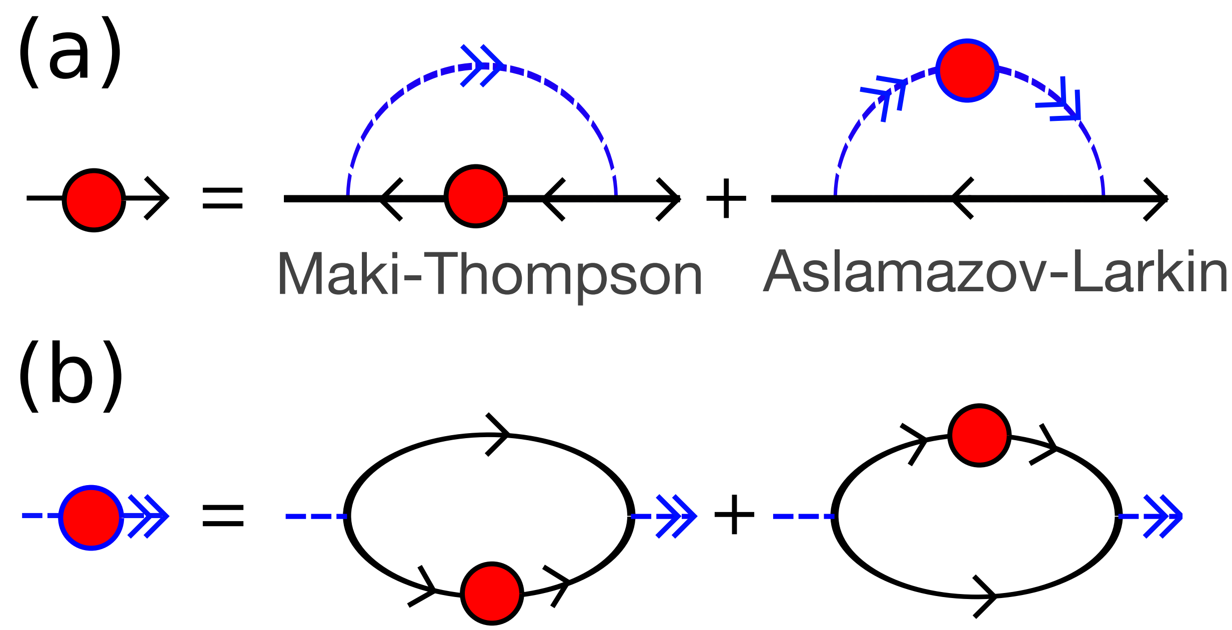
Transport equations

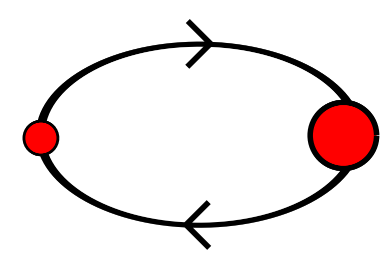
[Enss, Haussmann, Zwerger 2011]

- Single-particle Green functions:



Response to shear perturbations:



- correlation function (Kubo formula): η(ω) = 

- transport via fermionic and bosonic modes: very efficient description, satisfies conservation laws (exact scale invariance and Tan relations)[Enss 2012]

- assumes no quasiparticles: beyond Boltzmann