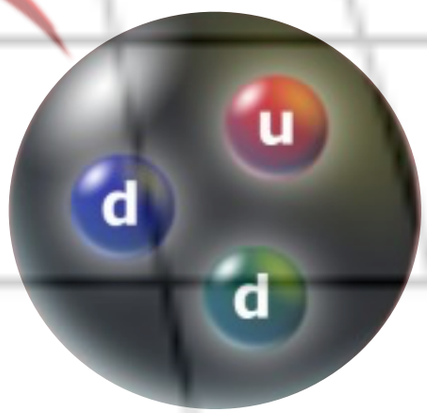
From QCD to cold atoms: what can we learn from the lattice?



Amy Nicholson
UNC, Chapel Hill

Exploring Nuclear Physics with Ultracold atoms
ECT, Jun. 22, 2018*

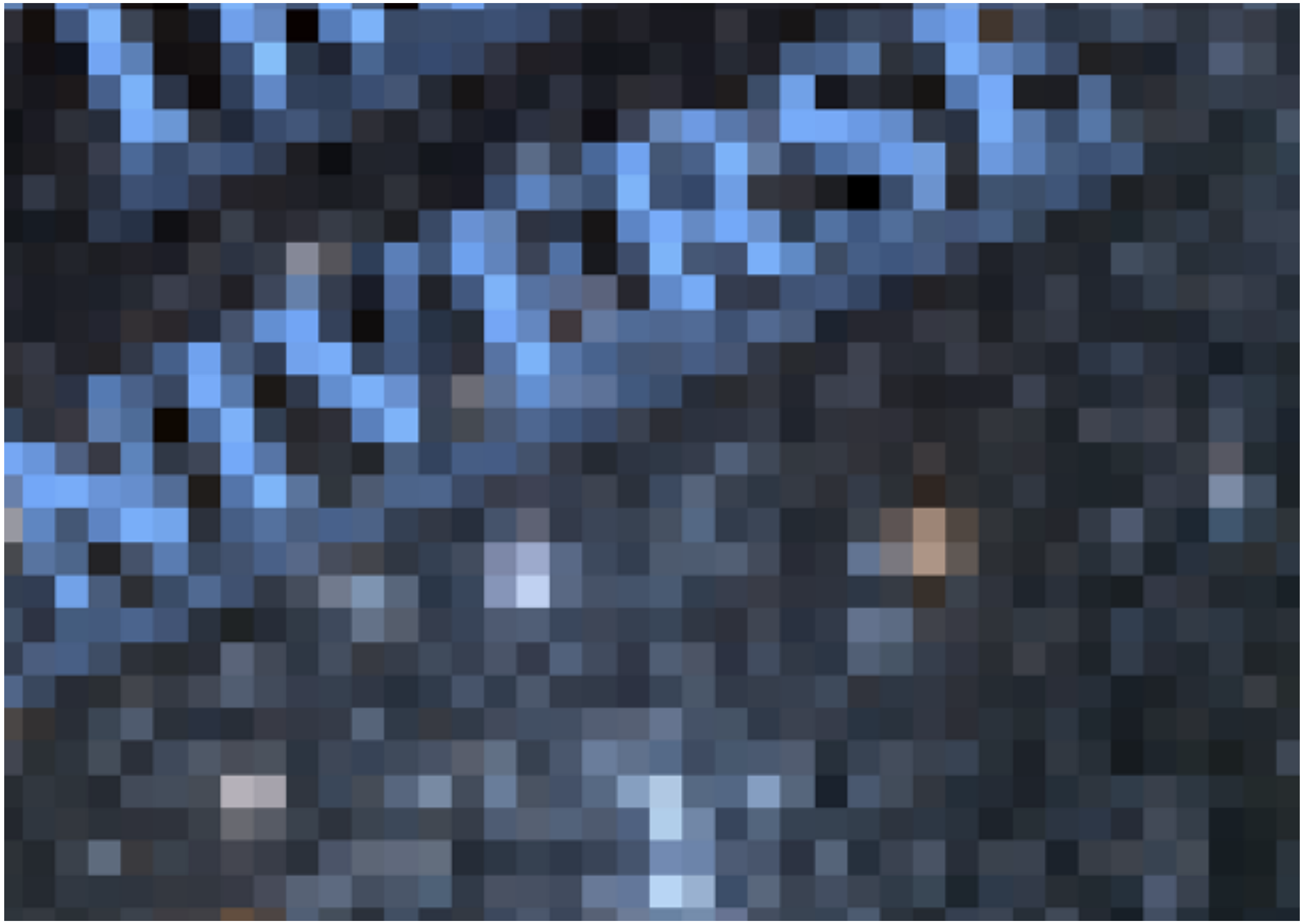
Lattice QCD

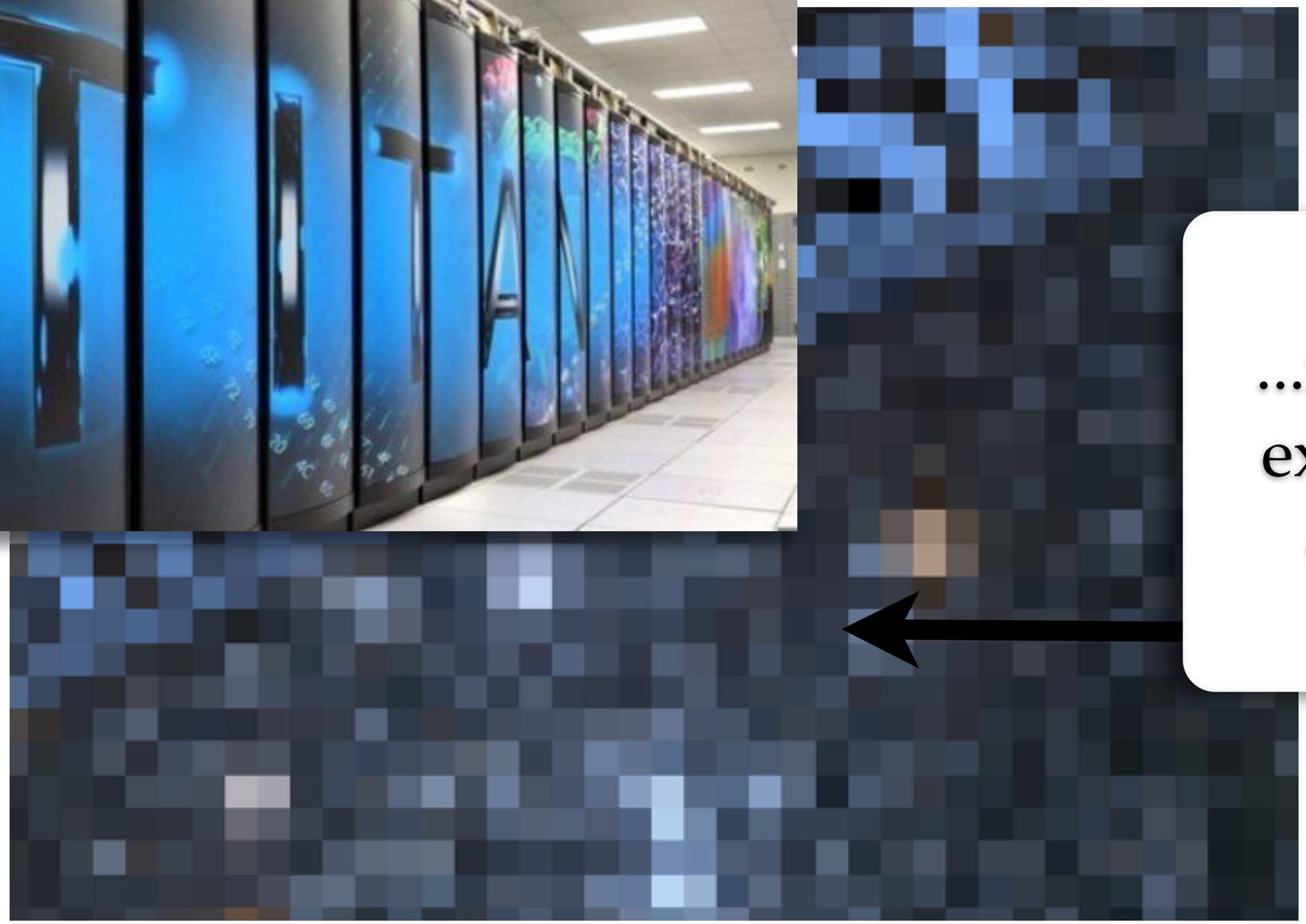
Numerical solution to QCD
Not a model:
All uncertainties are quantifiable and may be systematically removed

QCD does NOT fit on a computer!

THE UNIVERSE

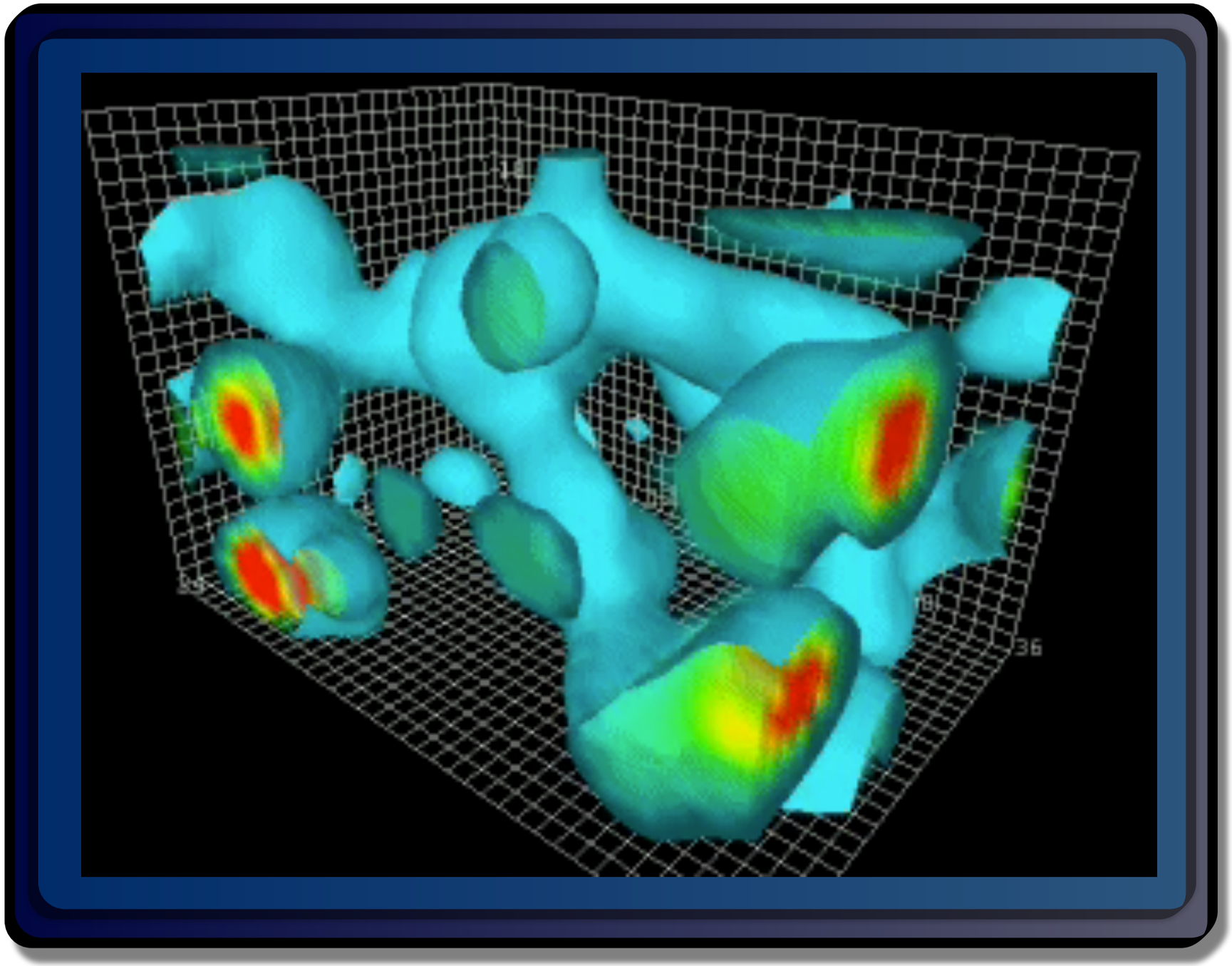
space
time





Now it fits?
...no! - finite but extremely large dimensional integral

Animation by Derek Leinweber



Monte Carlo sampling of "vacuum"
(gluons, quark/anti-quark pairs)

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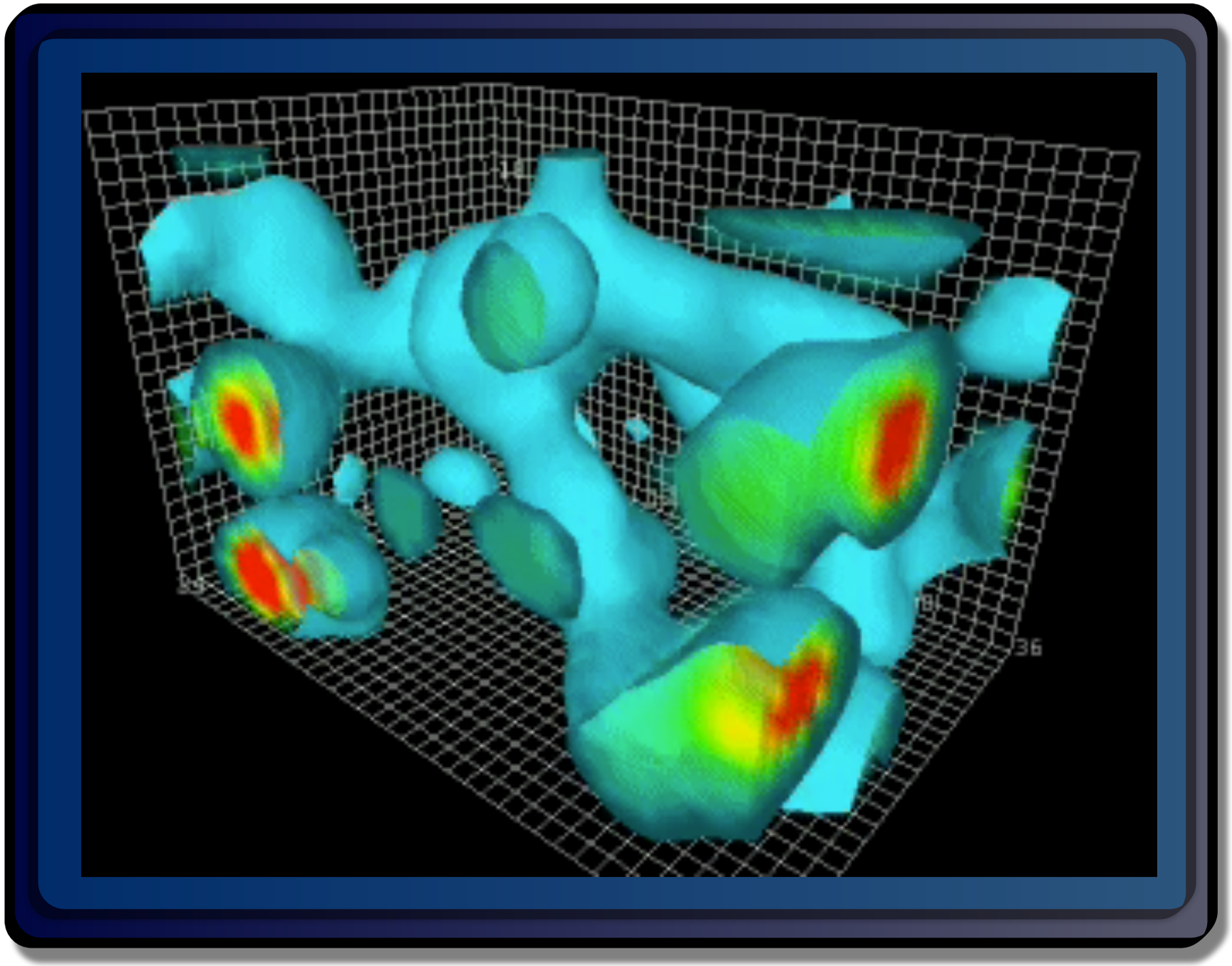

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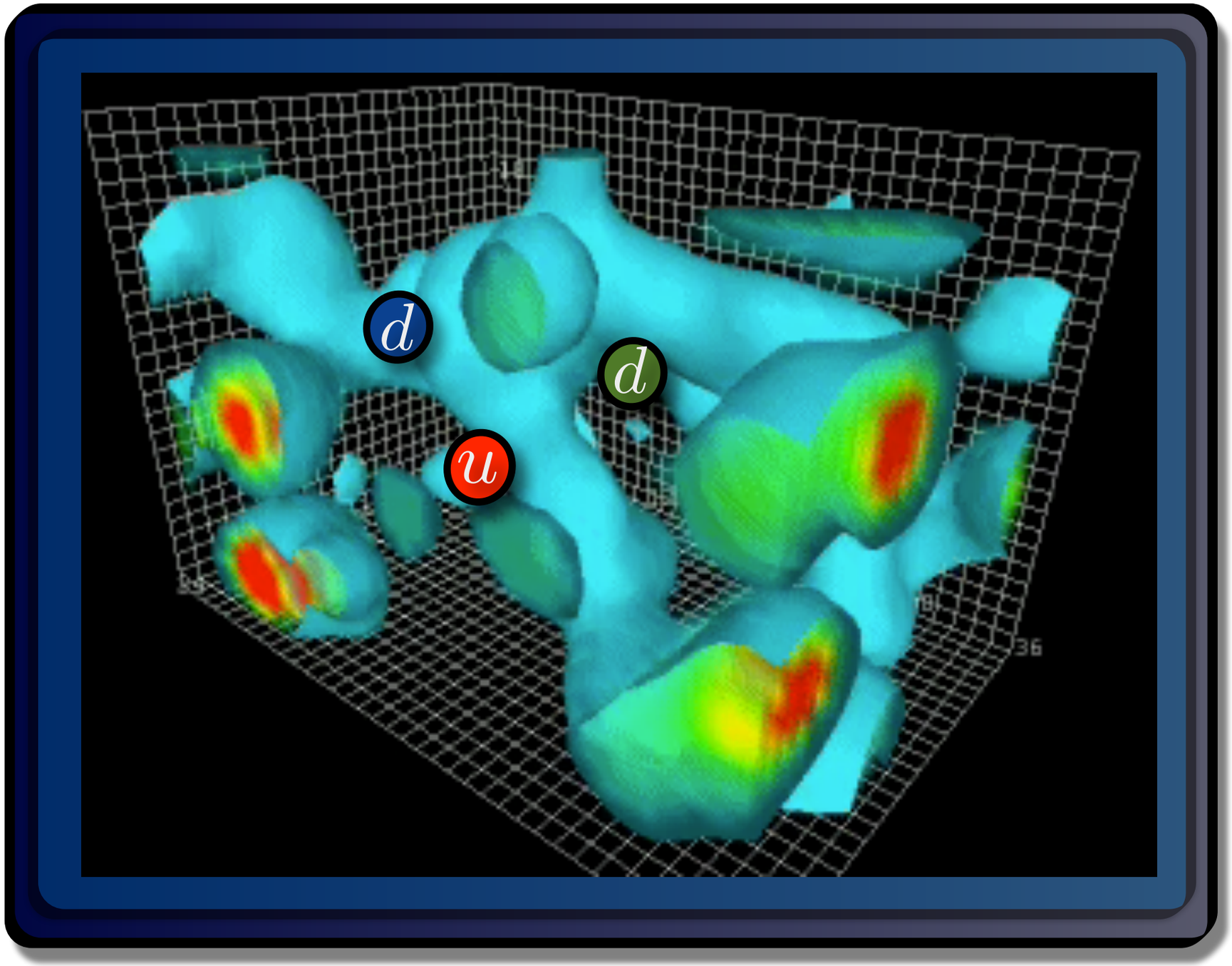
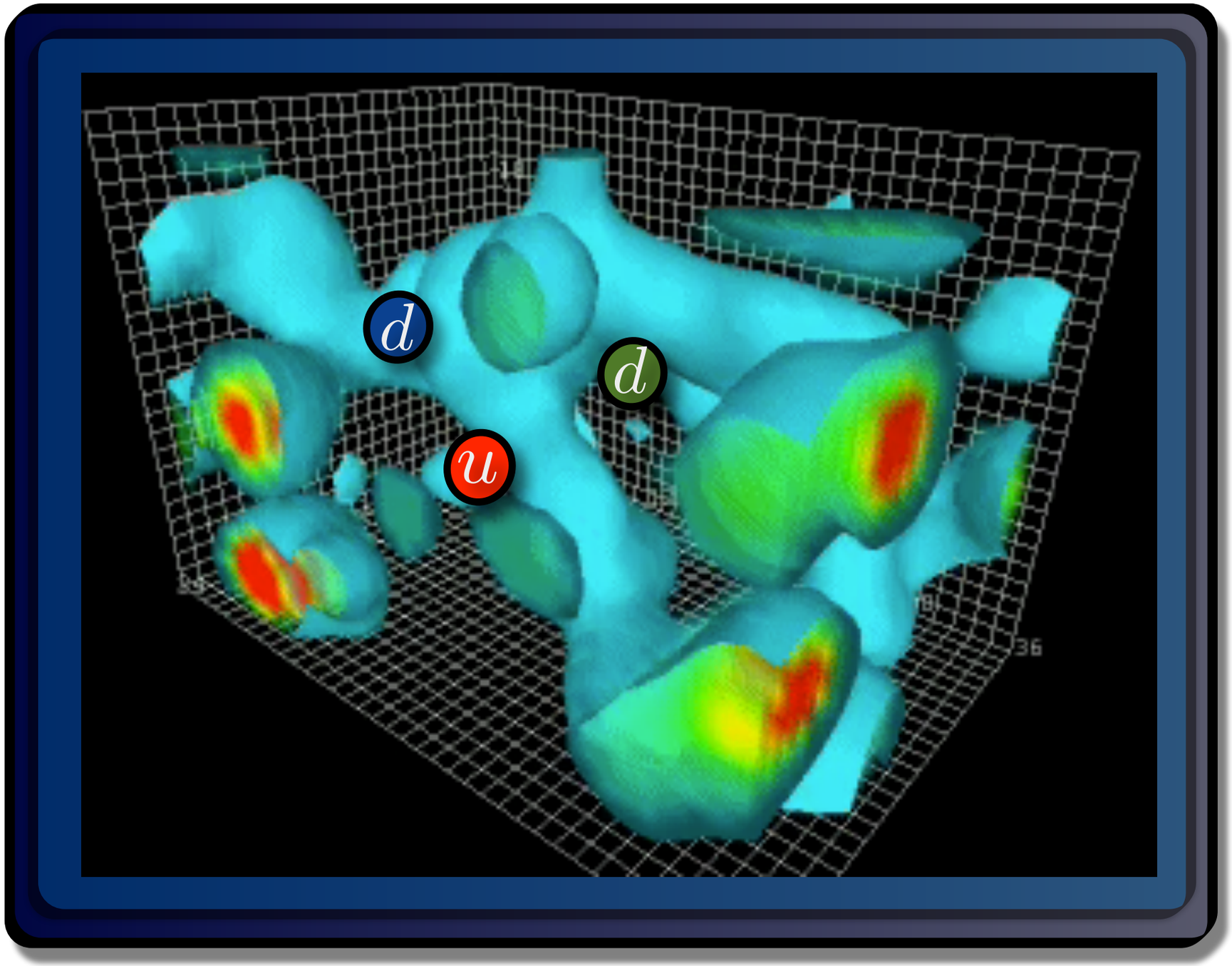


Insert probes & measure correlations

Animation by Derek Leinweber



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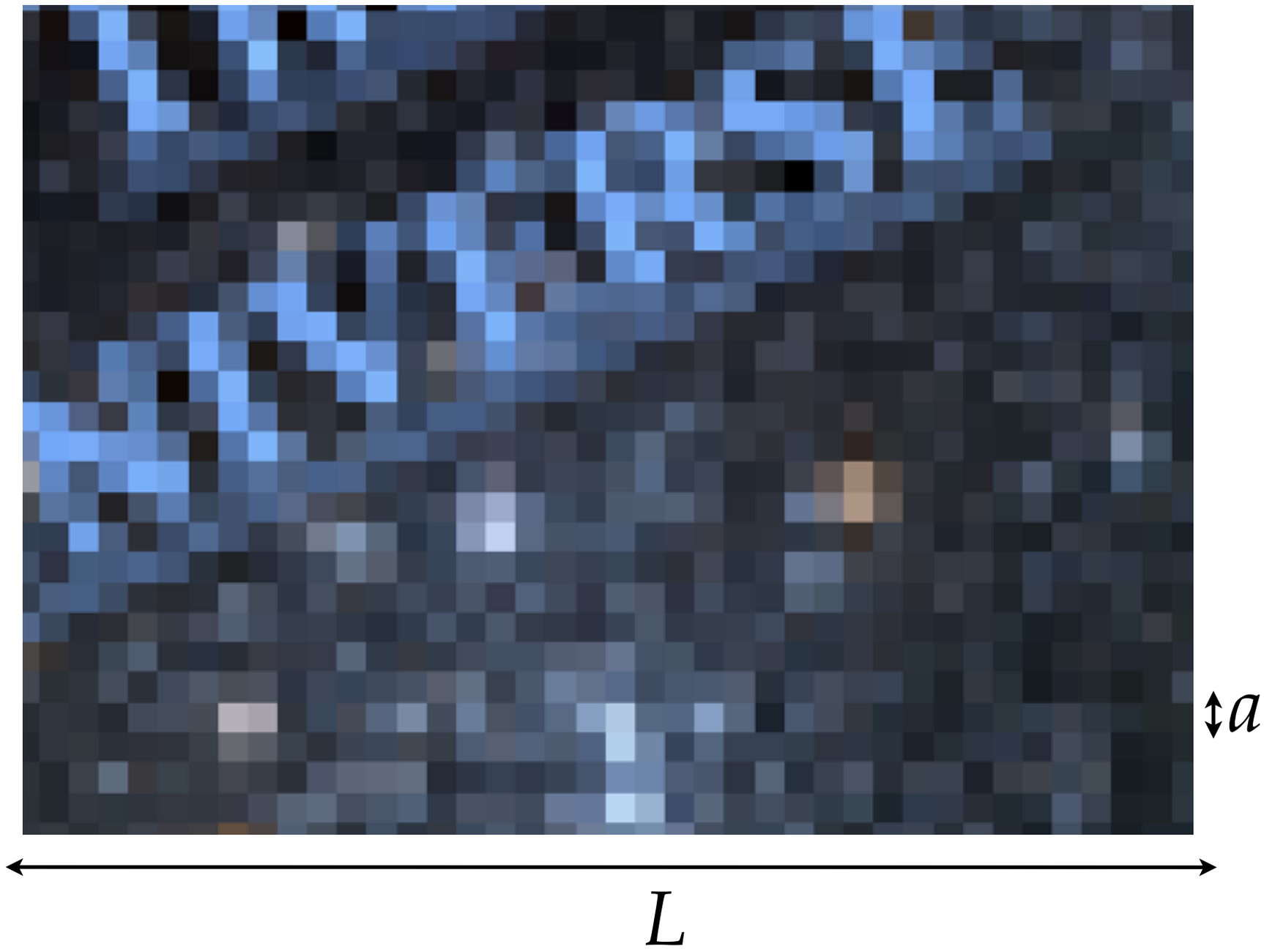
a
L

$L \to \infty$

THE UNIVERSE

$$L \to \infty$$

$$a \to 0$$

THE UNIVERSE

$$L \to \infty$$

$$a \to 0$$

$$m_q \to m_q^{\text{phys}}$$

Precision era for (single hadron) LQCD

Neutron-proton mass difference: accurate to 300 KeV (BMW 2015)

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REPORT

Ab initio calculation of the neutron-proton mass difference

Sz. Borsanyi[1], S. Durr[1,2], Z. Fodor[1,2,3,*], C. Hoelbling[1], S. D. Katz[3,4], S. Krieg[1,2], L. Lellouch[5], T. Lippert[1,2], A. Portelli[5,6], K. K. Szabo[1,2], B. C. Toth[1]

[1]Department of Physics, University of Wuppertal, D-42119 Wuppertal, Germany.

[2]Jülich Supercomputing Centre, Forschungszentrum Jülich, D-52428 Jülich, Germany.

[3]Institute for Theoretical Physics, Eötvös University, H-1117 Budapest, Hungary.

[4]Lendület Lattice Gauge Theory Research Group, Magyar Tudományos Akadémia–Eötvös Loránd University, H-1117 Budapest, Hungary.

[5]CNRS, Aix-Marseille Université, Université de Toulon, CPT UMR 7332, F-13288, Marseille, France.

[6]School of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, UK.

*Corresponding author. E-mail: fodor@bodri.elte.hu

Science 27 Mar 2015:
Vol. 347, Issue 6229, pp. 1452-1455

Axial charge of the nucleon: g_A = 1.271(13) (CalLat 2018)

nature
International journal of science

Altmetric: 114 More detail »

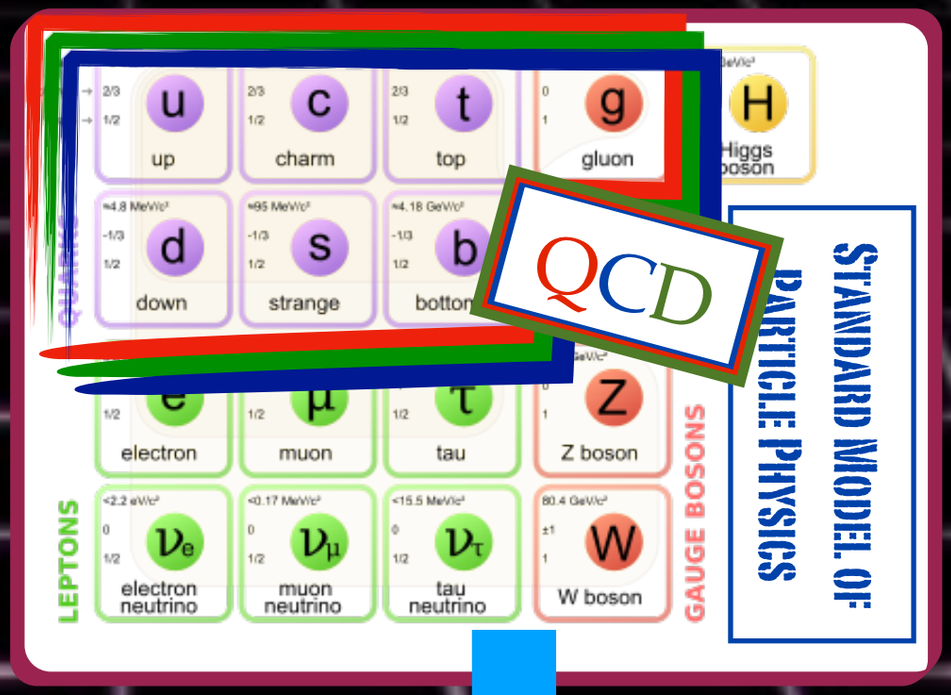
Letter | Published: 30 May 2018

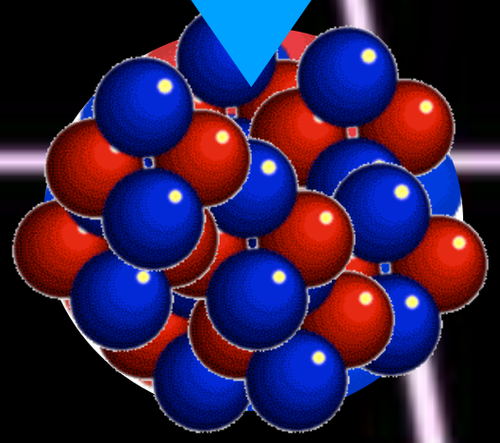
A per-cent-level determination of the nucleon axial coupling from quantum chromodynamics

C. C. Chang, A. N. Nicholson, E. Rinaldi, E. Berkowitz, N. Garron, D. A. Brantley, H. Monge-Camacho, C. J. Monahan, C. Bouchard, M. A. Clark, B. Joó, T. Kurth, K. Orginos, P. Vranas & A. Walker-Loud

Nature **558**, 91–94 (2018) | Download Citation

We know from experiment that NN interactions are finely-tuned. What can LQCD teach us?





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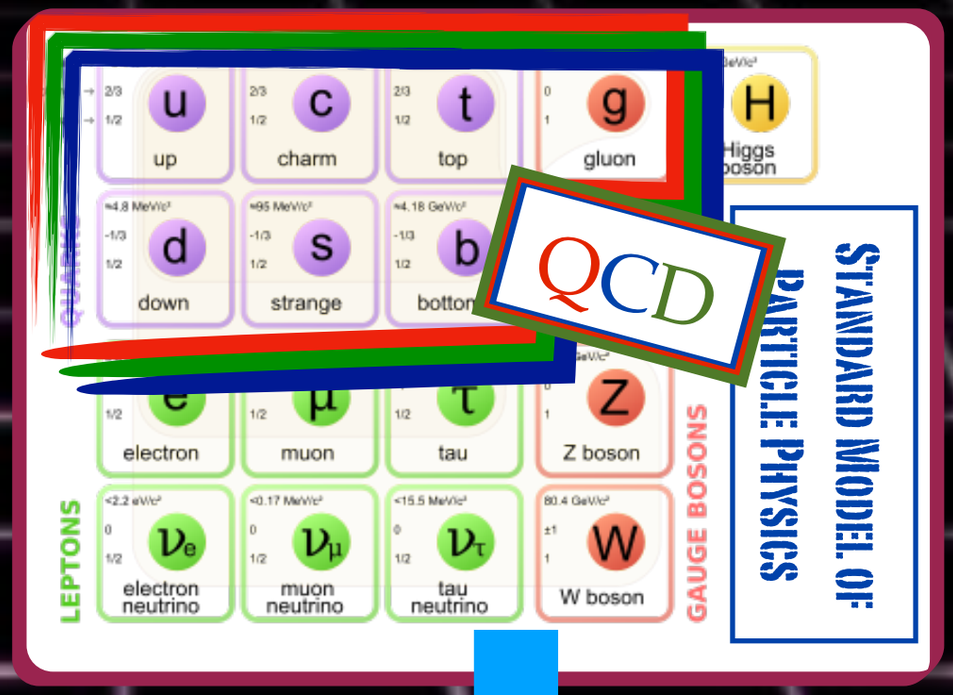

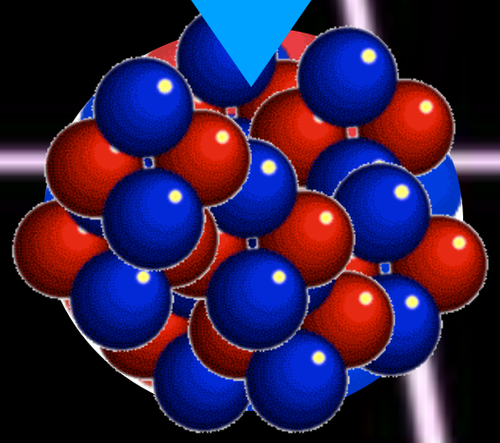

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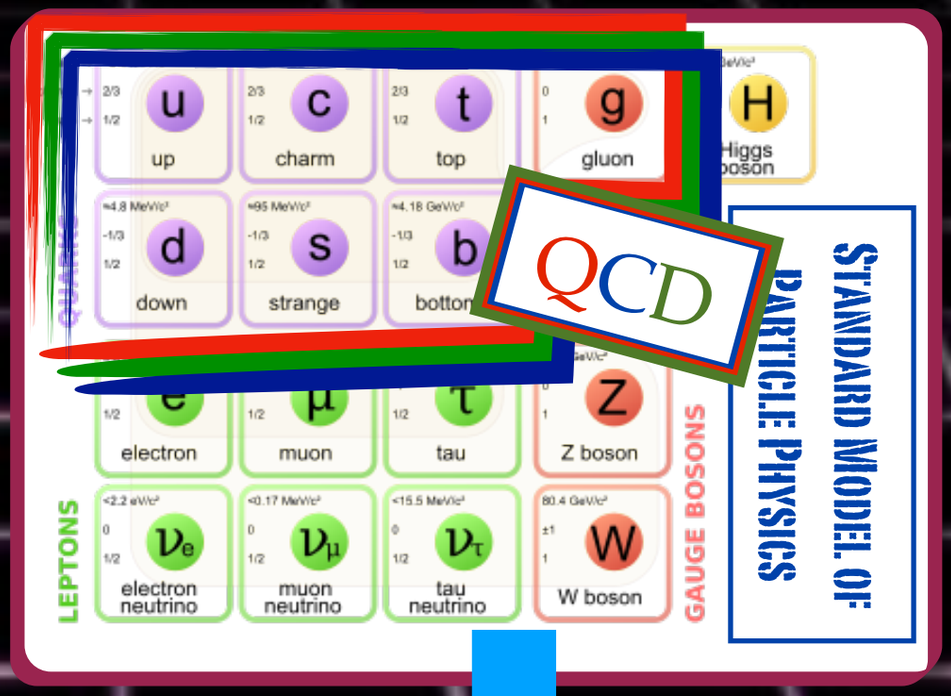

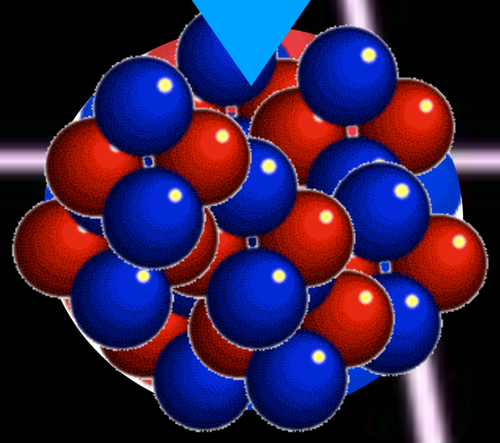

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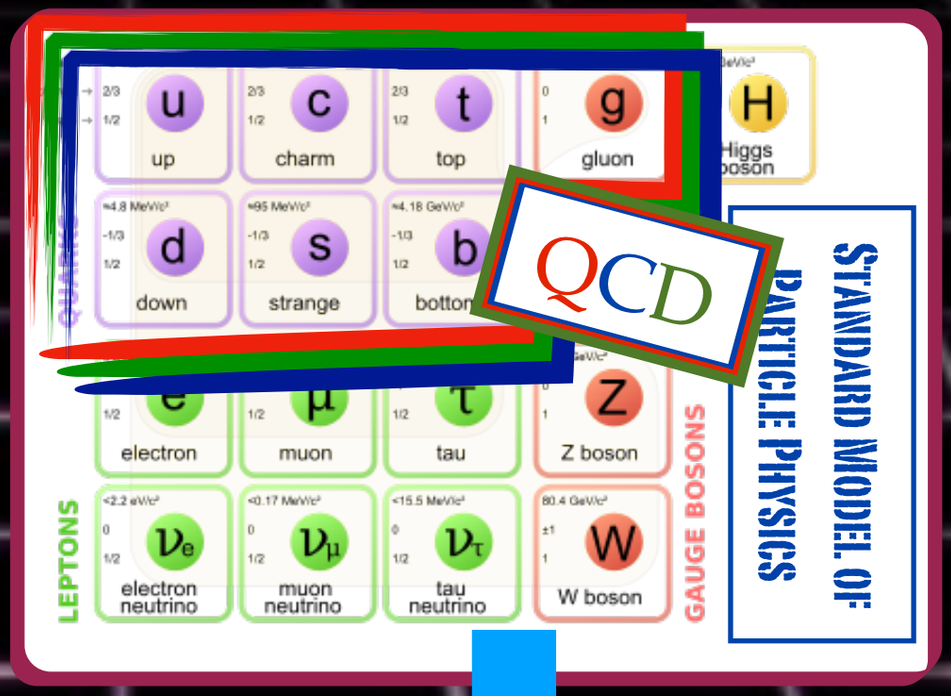

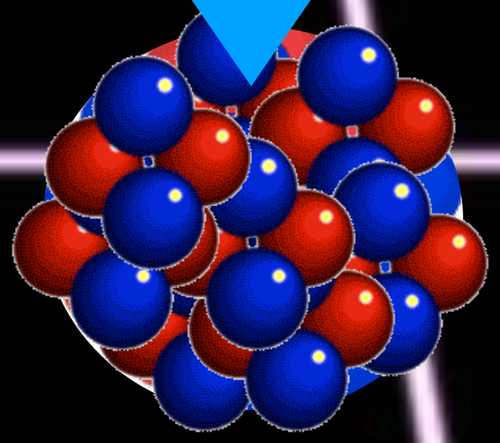

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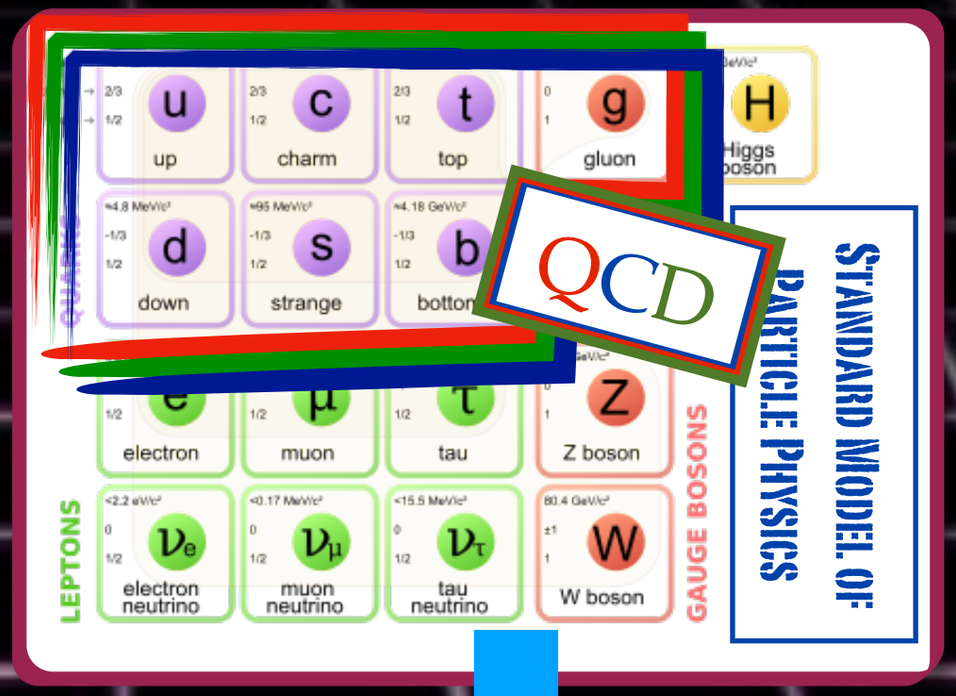
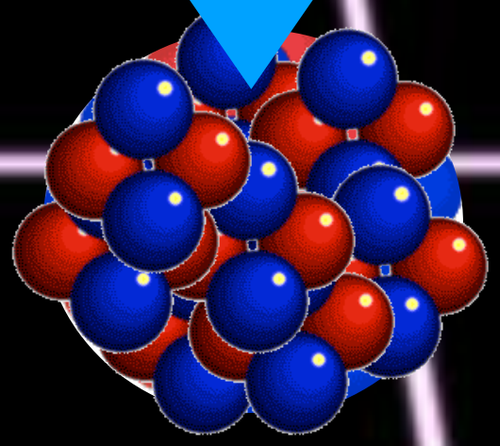
Q: Does fine-tuning in larger systems (halo nuclei, alpha clusters,….) come purely from strong interactions, or from the interplay between QCD & QED?

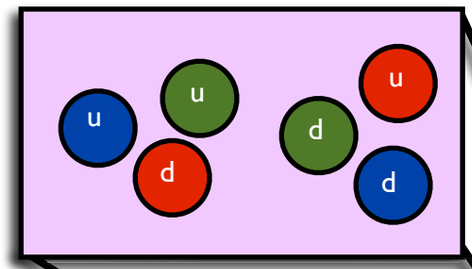


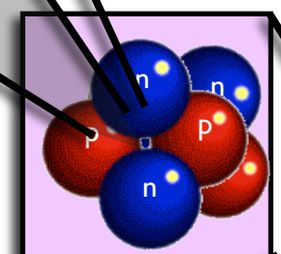


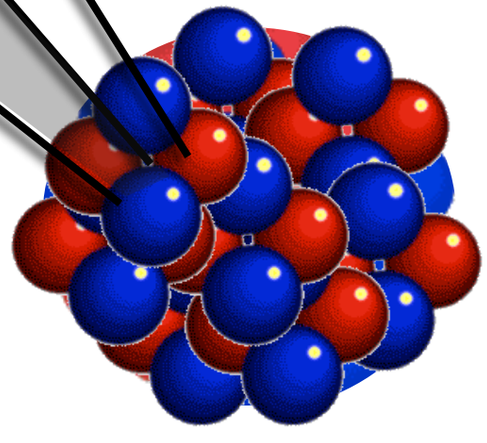
LQCD

ab initio

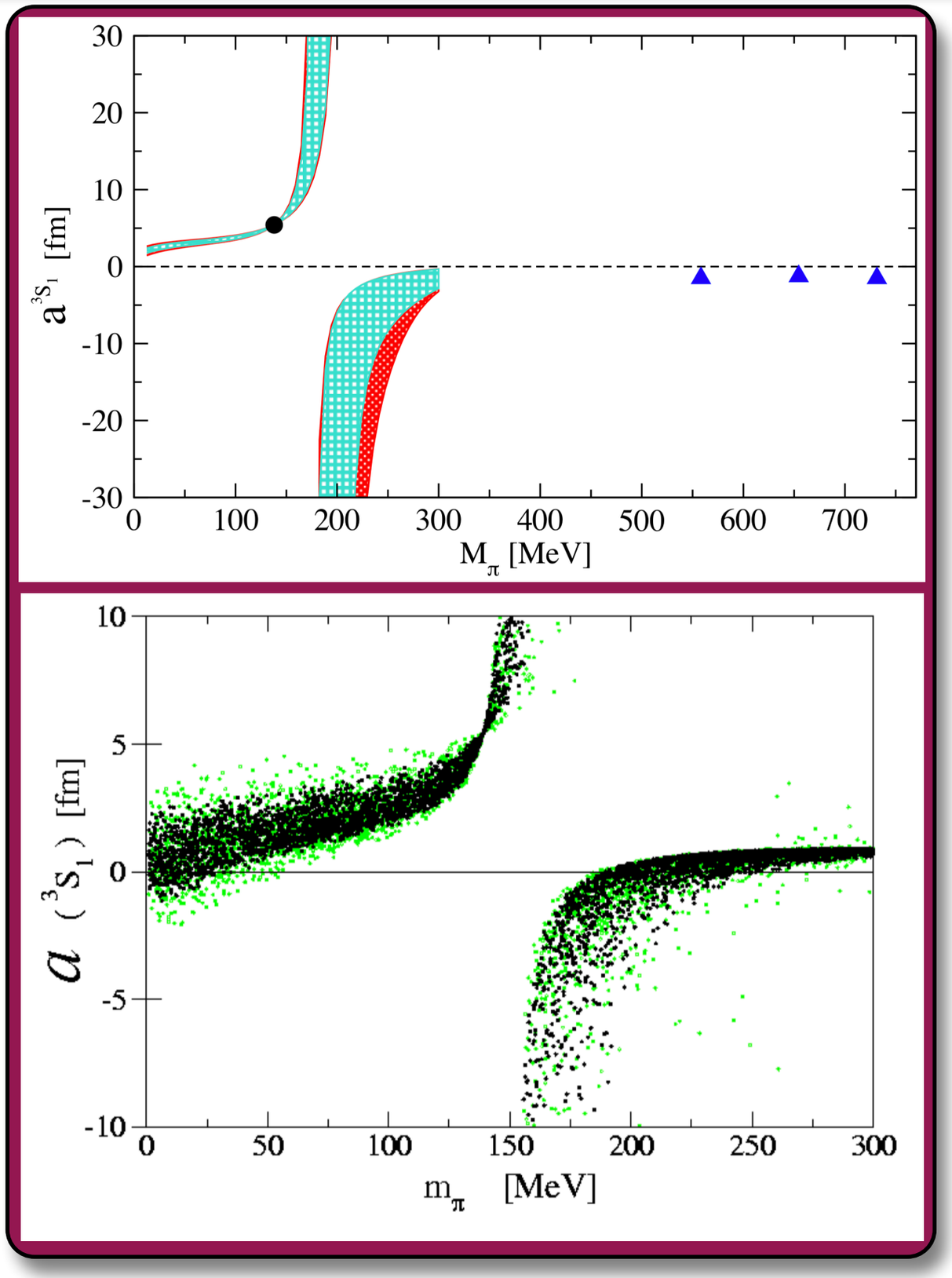
many-body





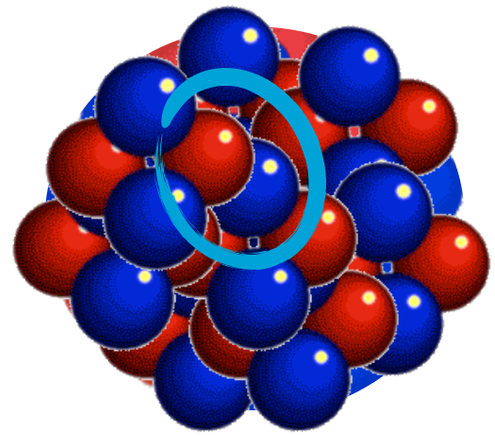


S-wave scattering length vs. m_π: EFT prediction

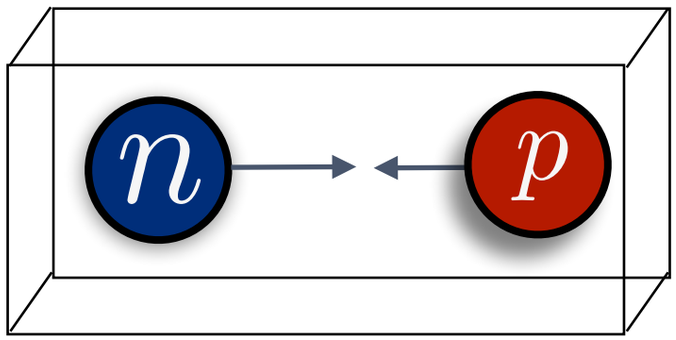


Epelbaum, Meissner, Glockle 2002
Savage, Beane 2002

Two methods for calculating few-body interactions from LQCD:



Spectroscopy + Lüscher Method

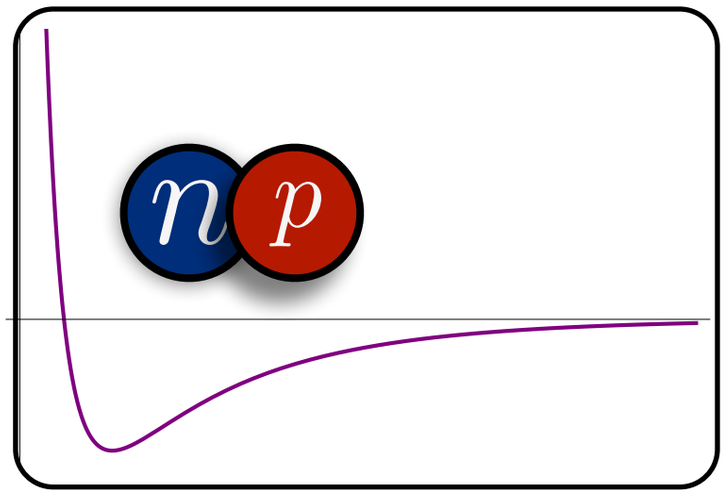


NPLQCD

CalLat

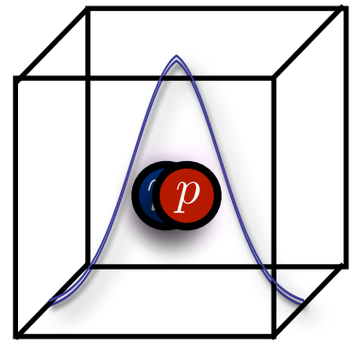
Yamazaki, et. al.

Potential Method



Hadrons to Atomic nuclei
HAL
from Lattice QCD

Spectroscopy

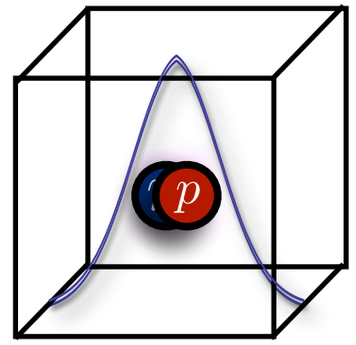


- Energies simple to calculate from correlation functions at large Euclidean time:

$$\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle = \langle \mathcal{O}(0)e^{-Ht}\mathcal{O}(0)\rangle = \sum_n |\langle 0|\mathcal{O}|n\rangle|^2 e^{-E_n t}$$

- Bound states: infinite volume extrapolation gives binding energies

Spectroscopy



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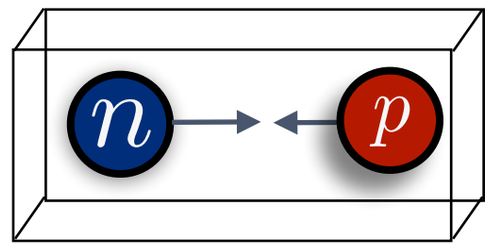
Lüscher

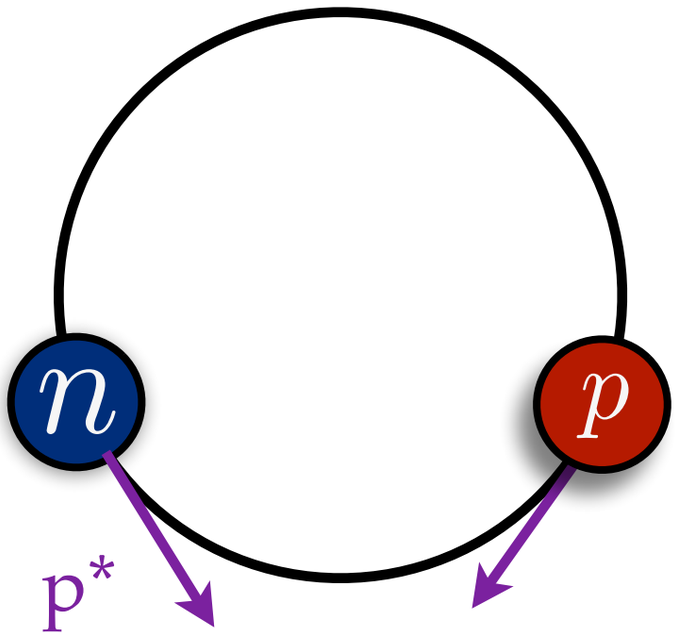


- Direct scattering "experiments" not possible in finite volume/ Euclidean time

- Lüscher: measure discrete spectra of interacting particles in a box, and infer the interaction (scattering phase shift)

Example: 1-d

Slide stolen unabashedly from R. Briceno



Wavefunction outside the interaction region:

$$\psi(x) \sim e^{ip^* x + i2\delta(p^*)}$$

Infinite volume scattering phase shift

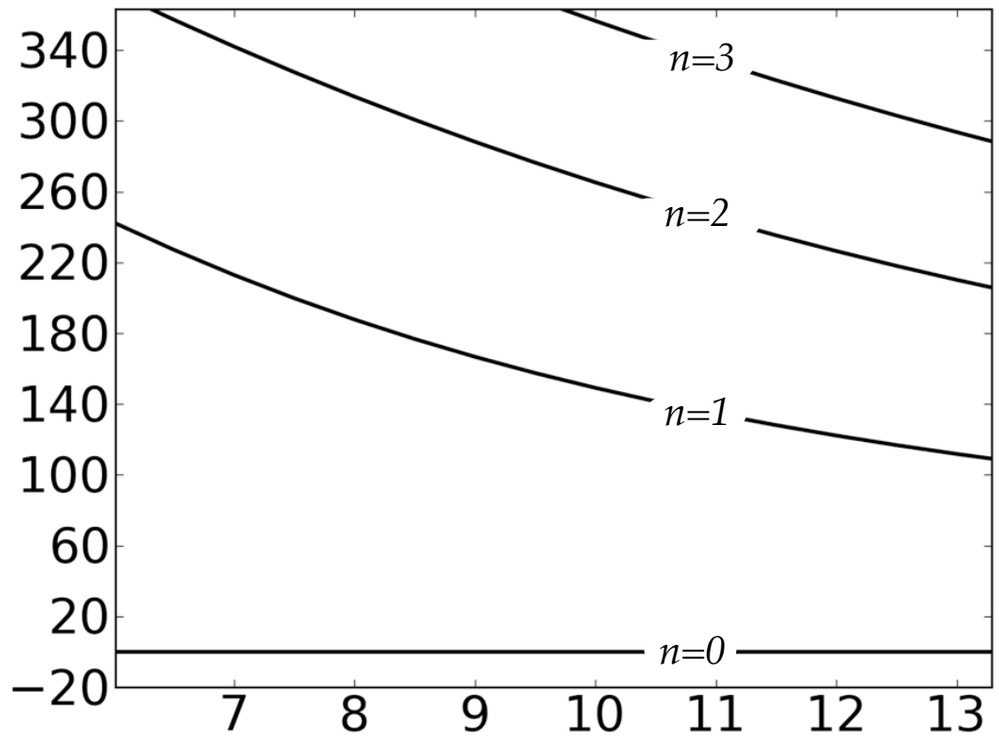
- Periodicity: $\psi(L) = \psi(0)$
- Quantization condition: $\boxed{L\ p_n^* + 2\delta(p_n^*) = 2\pi n}$

Example: 1-d

Slide stolen unabashedly from R. Briceno

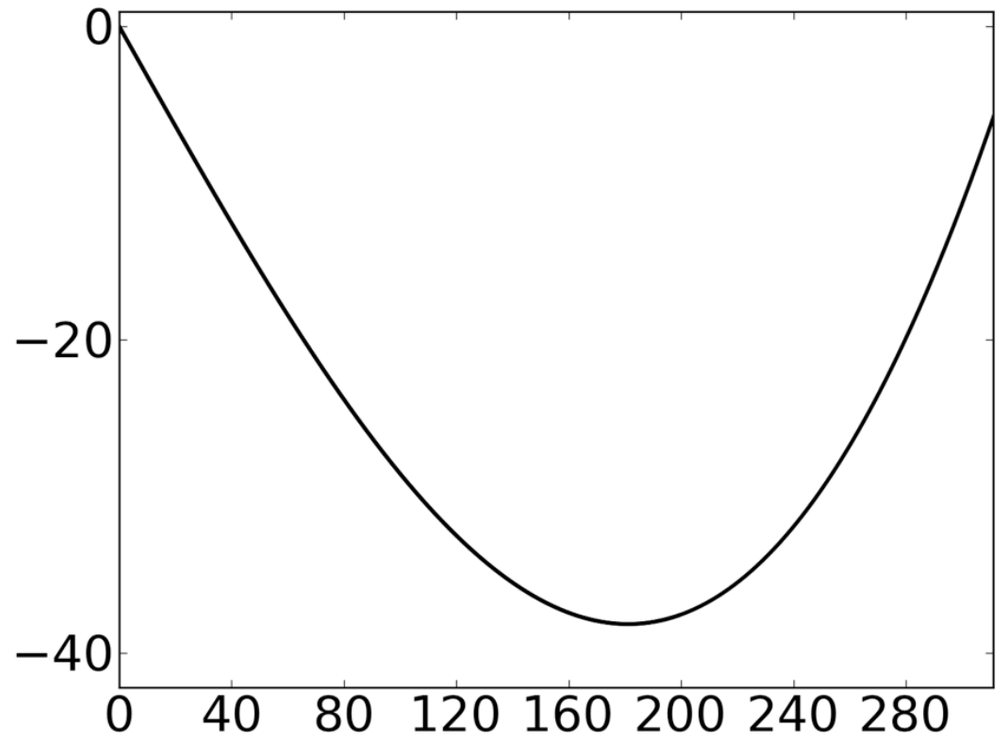
$$L\ p_n^* + 2\delta(p_n^*) = 2\pi n$$

p^* [MeV]



L [fm]

δ [degrees]



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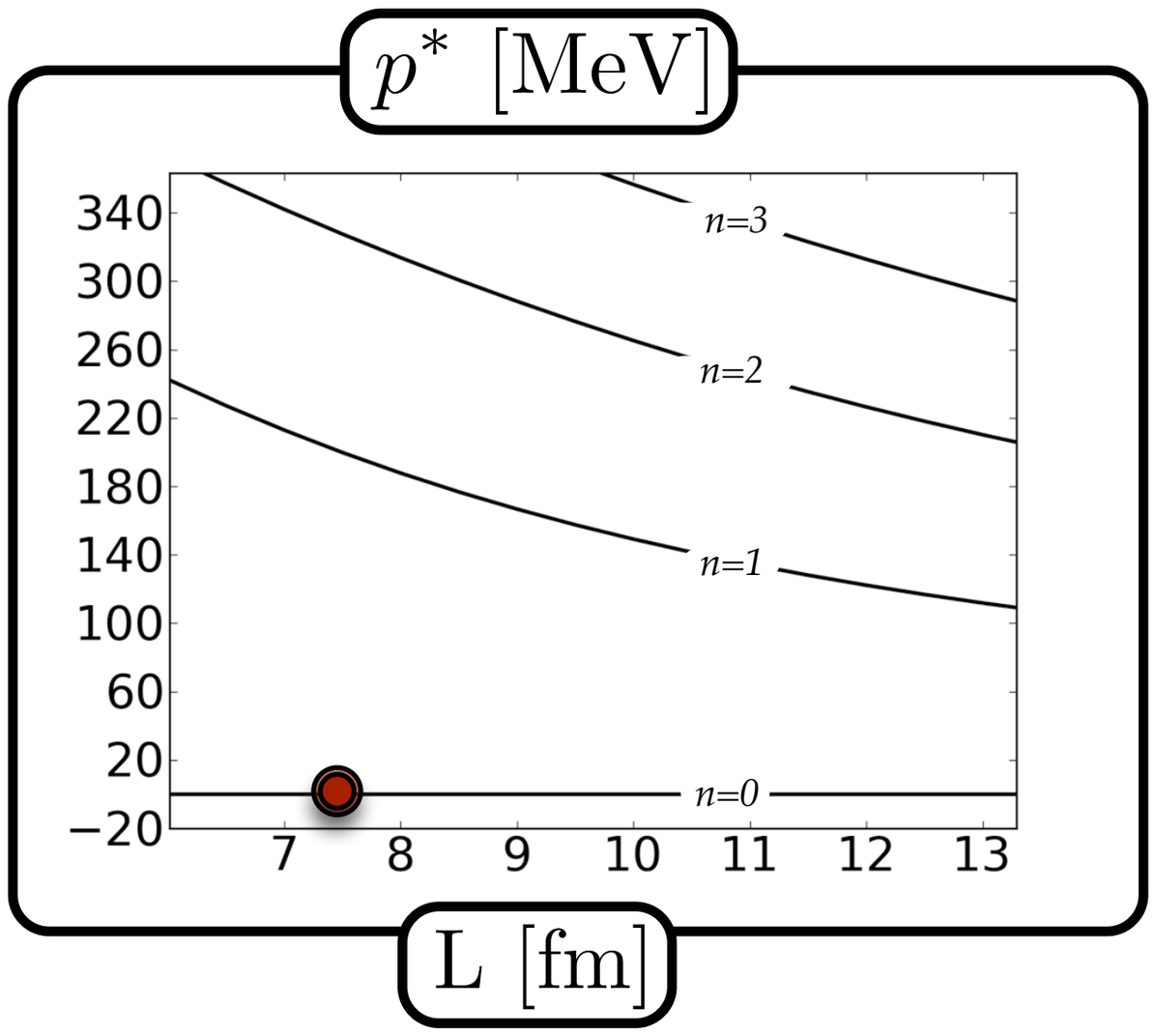

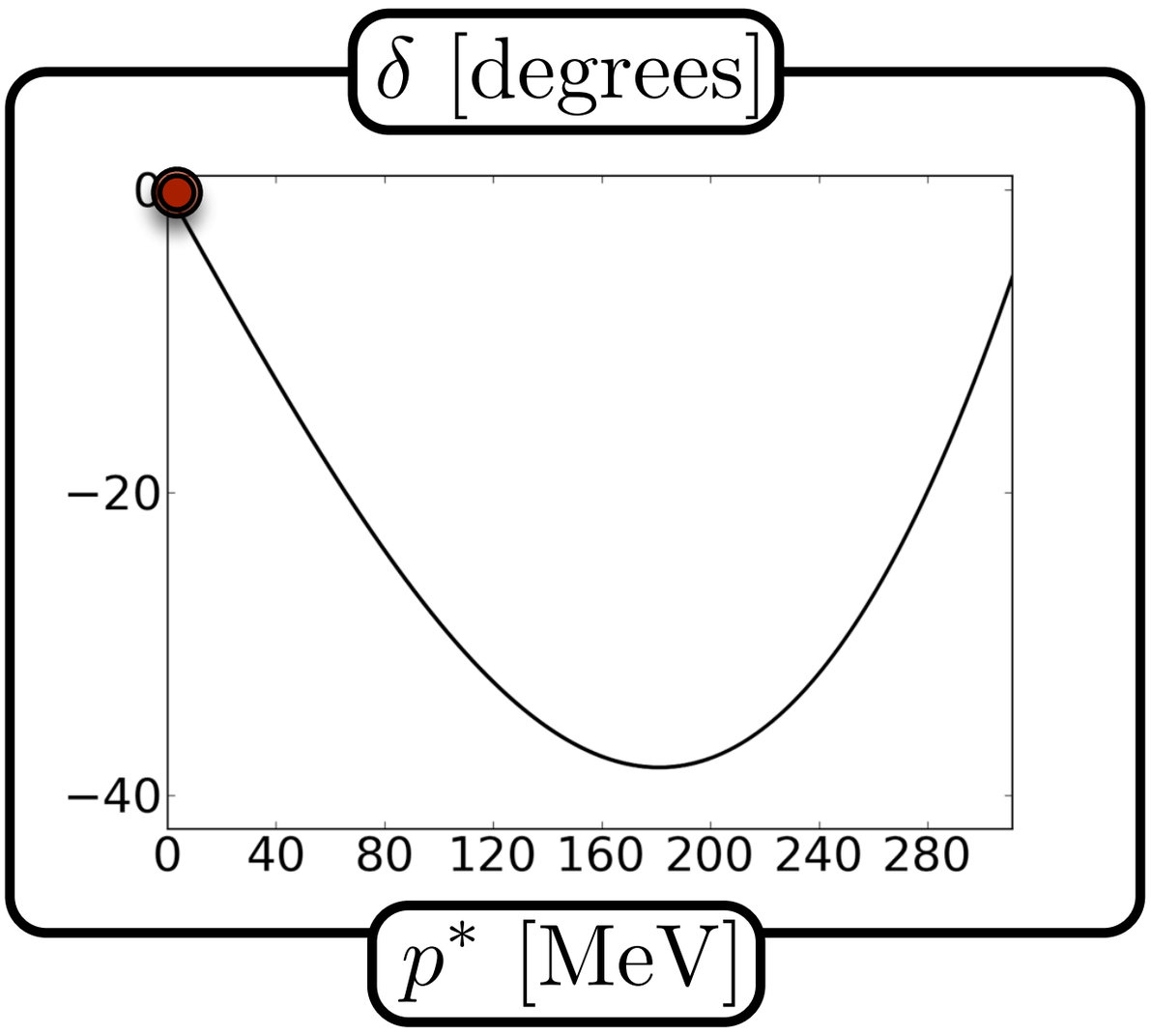

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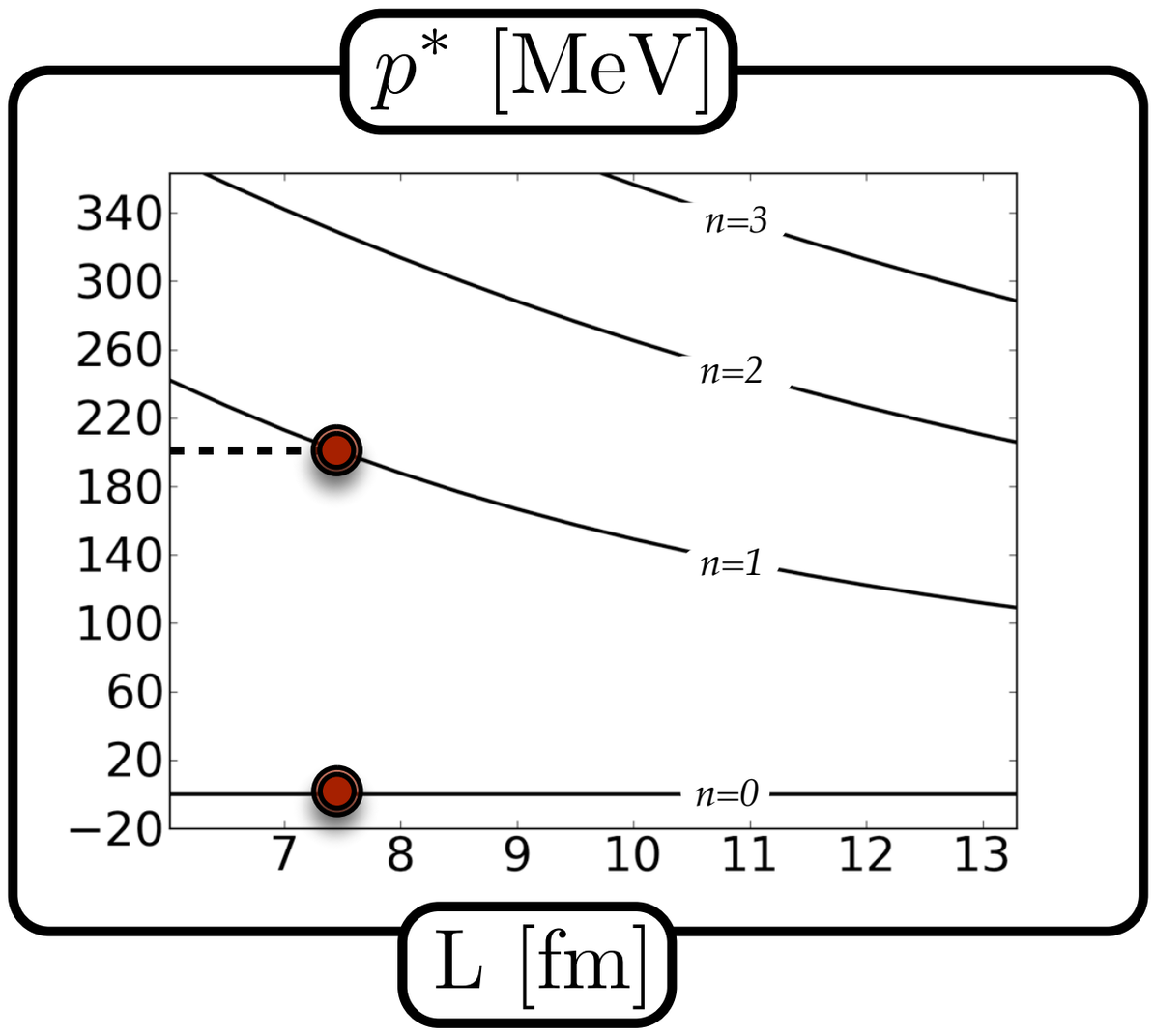

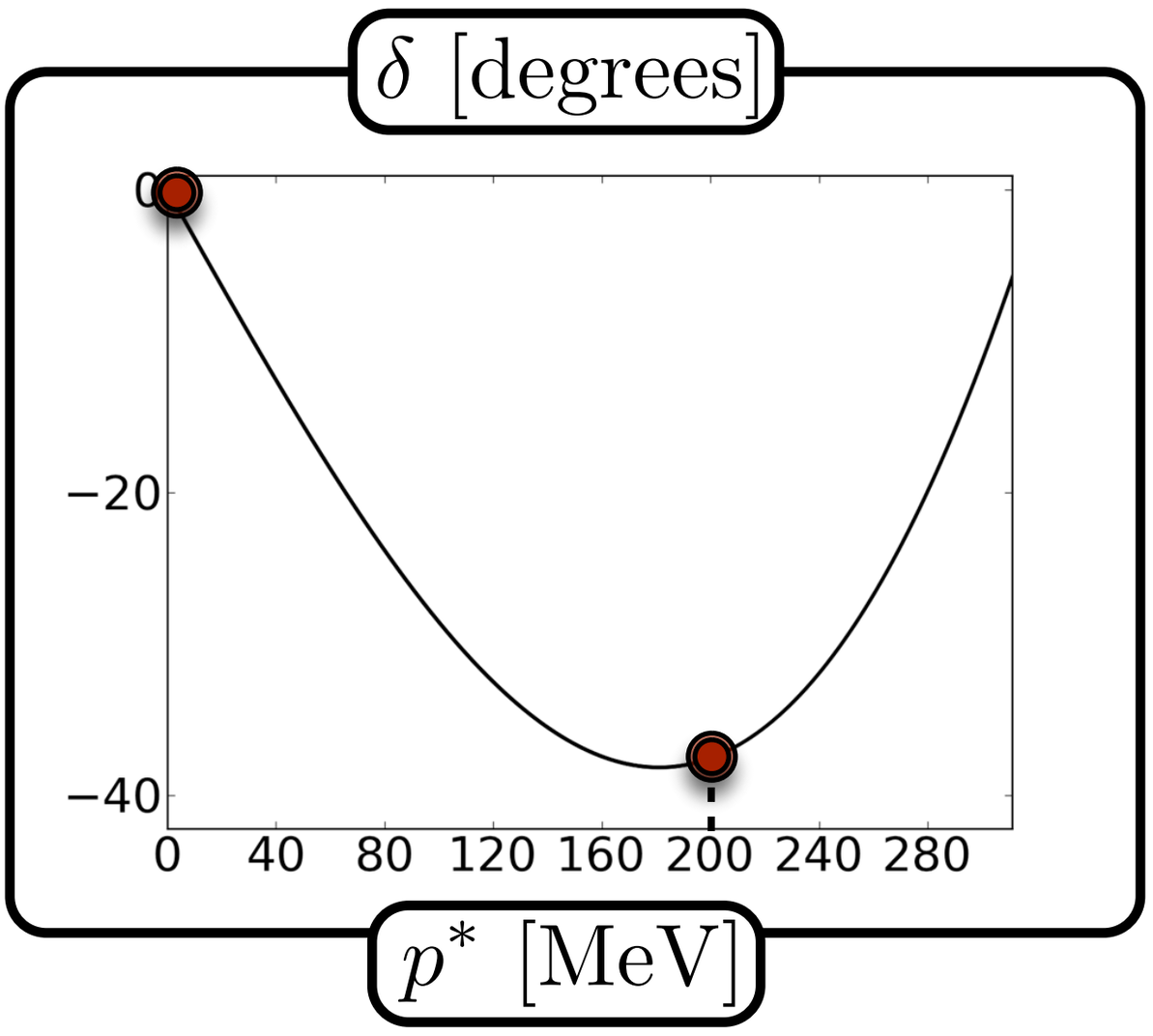

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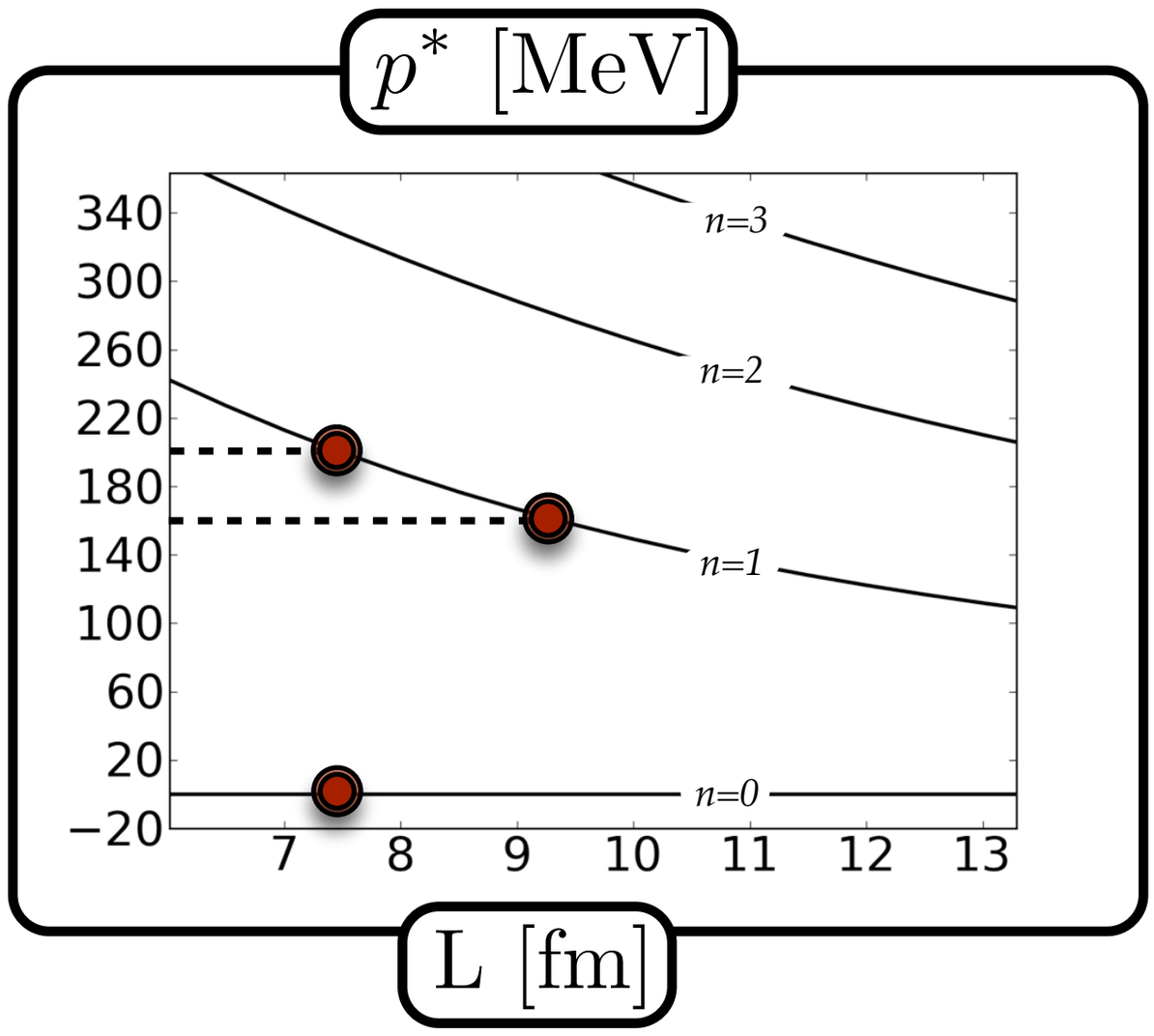

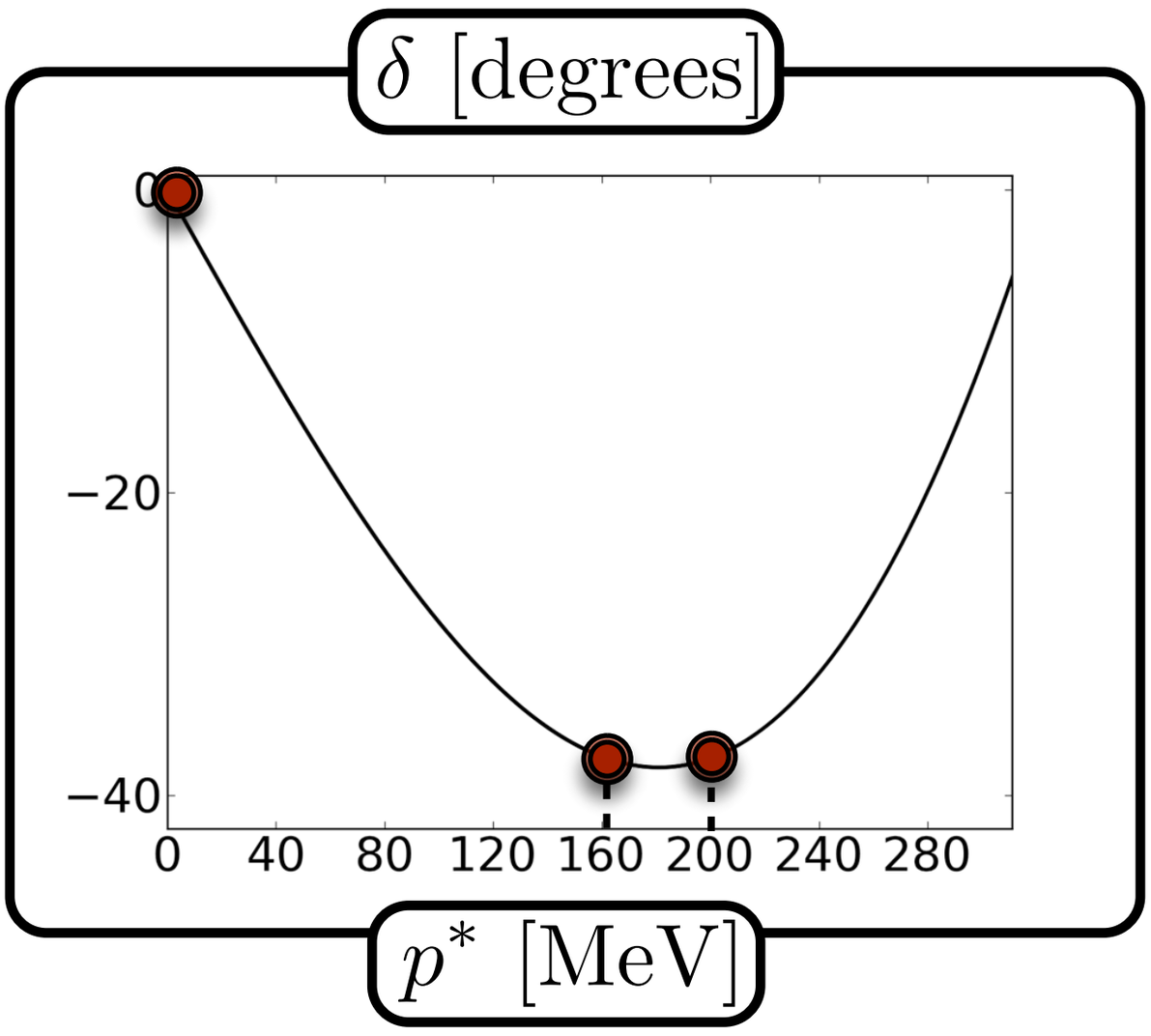

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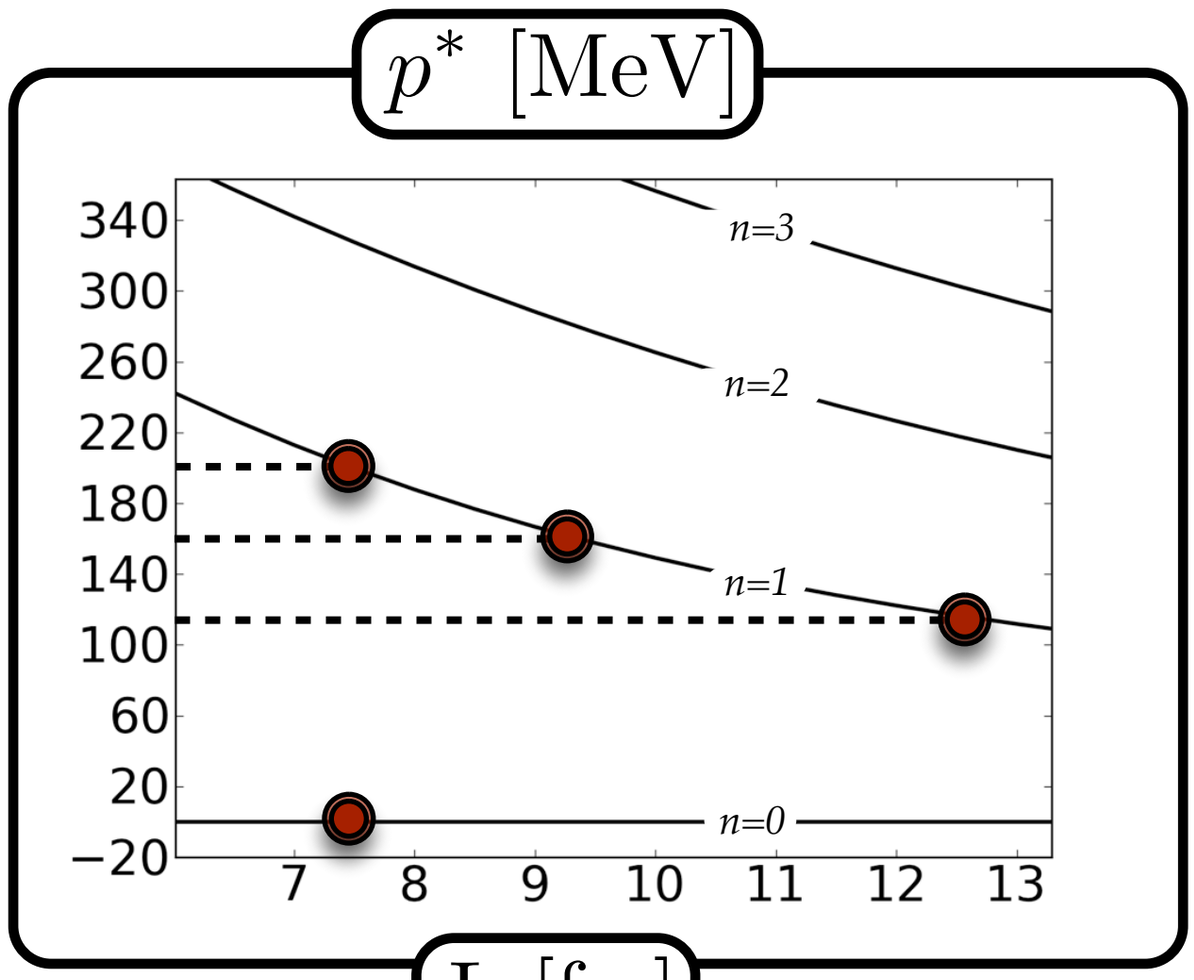
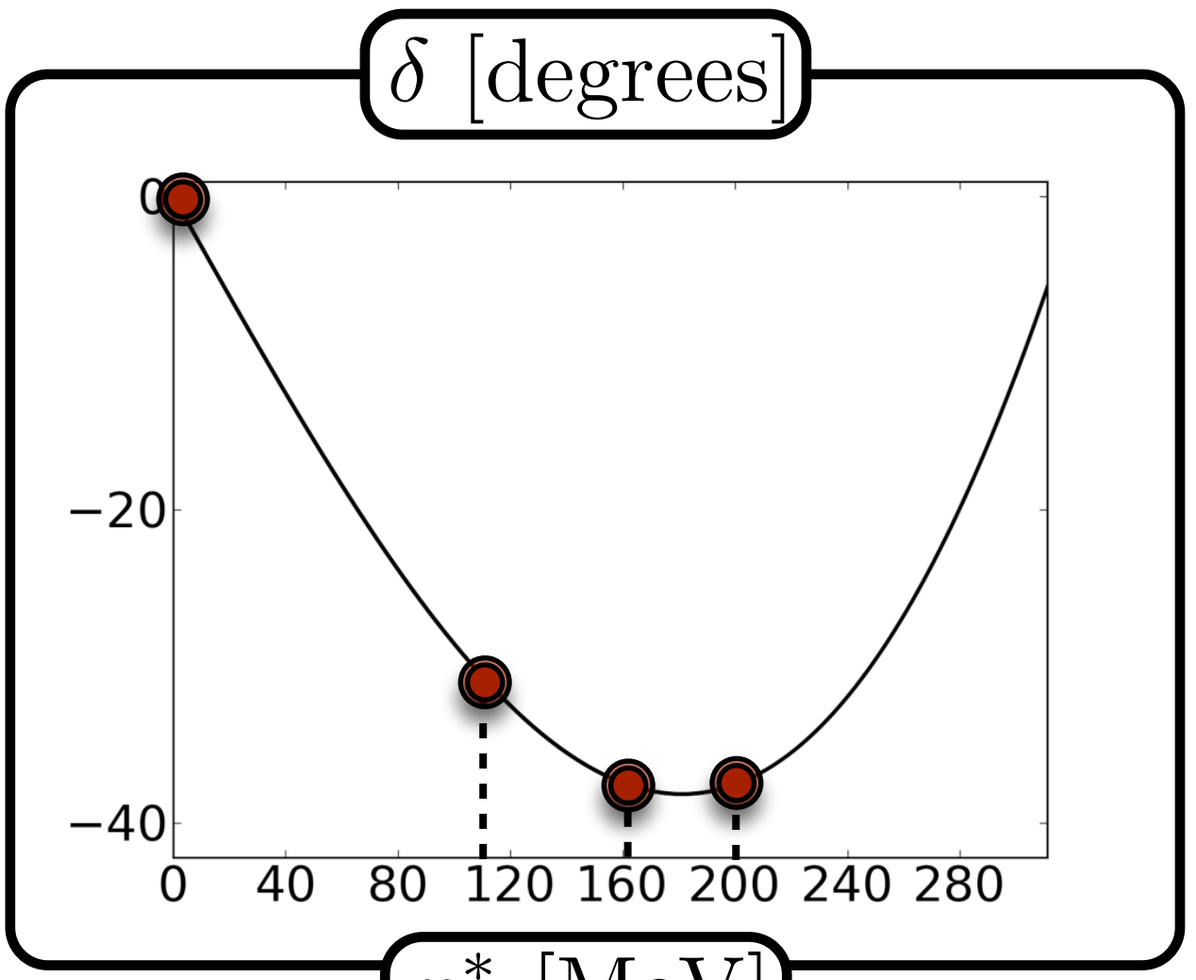


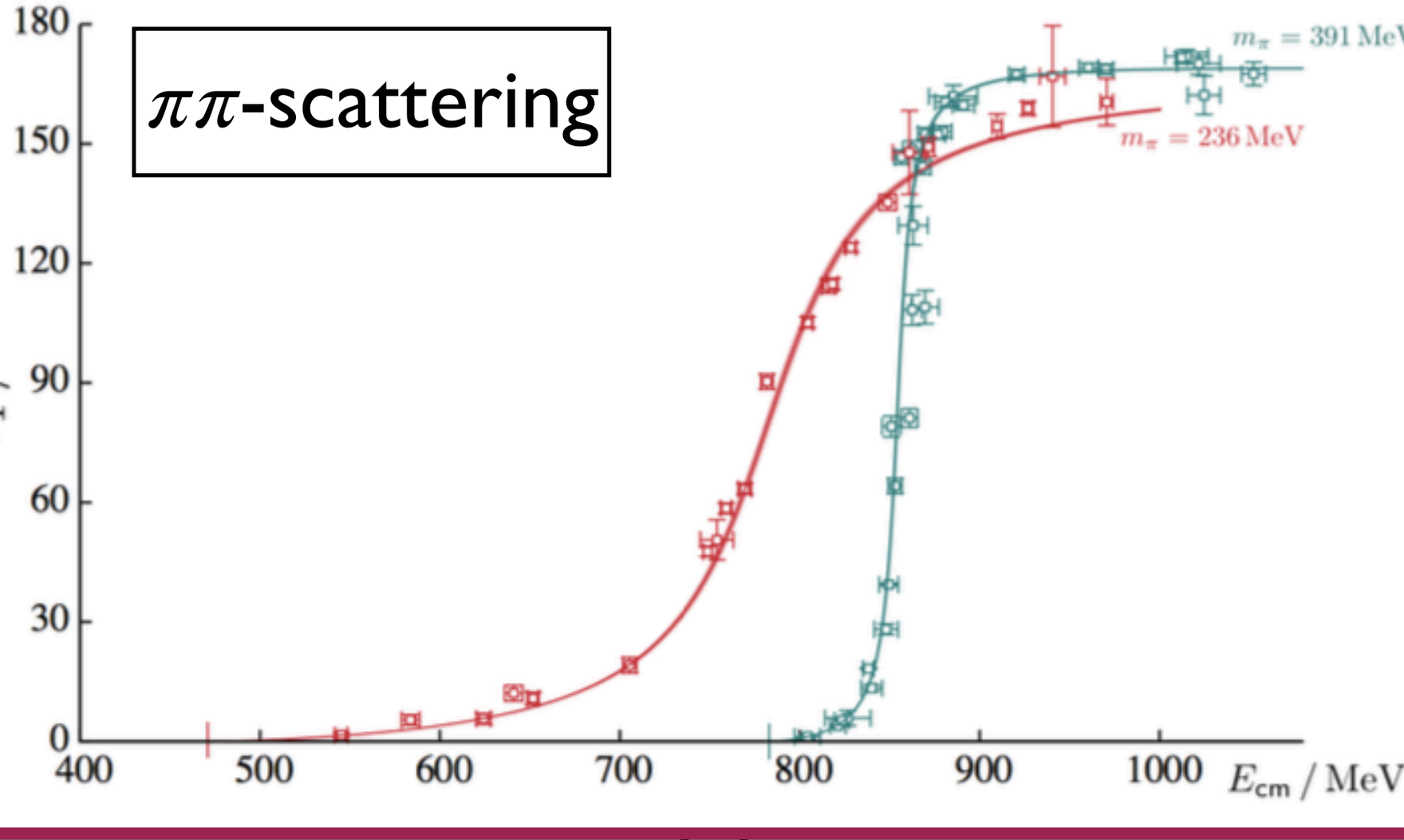


Example: 1-d

D. J. Wilson, R. A. Briceno, J. J. Dudek, R. G. Edwards and C. E. Thomas, Phys. Rev. D 92, 094502 (2015)

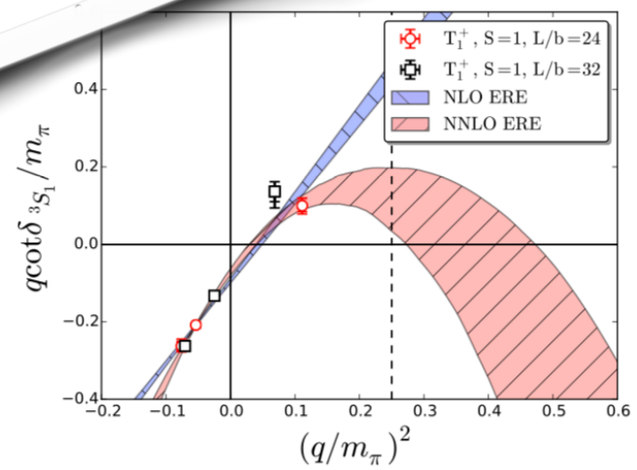
$$L = p^* + 2\delta(p^*) = 2\pi n$$

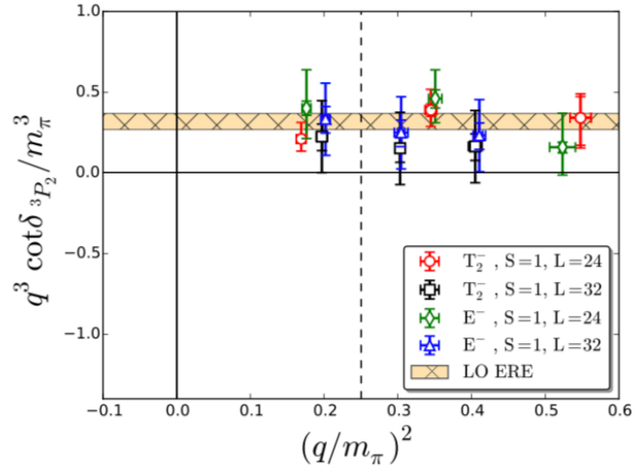
$\pi\pi$-scattering

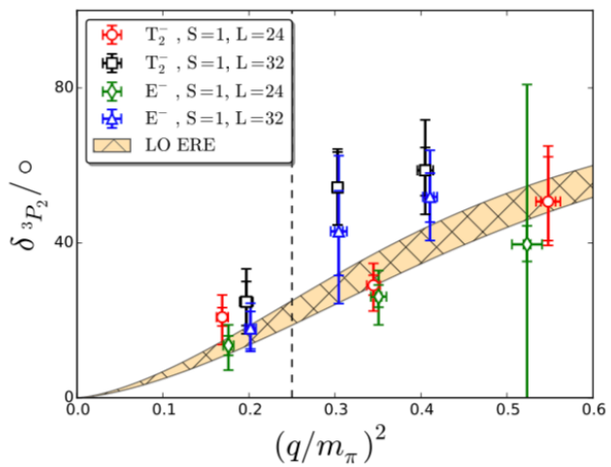


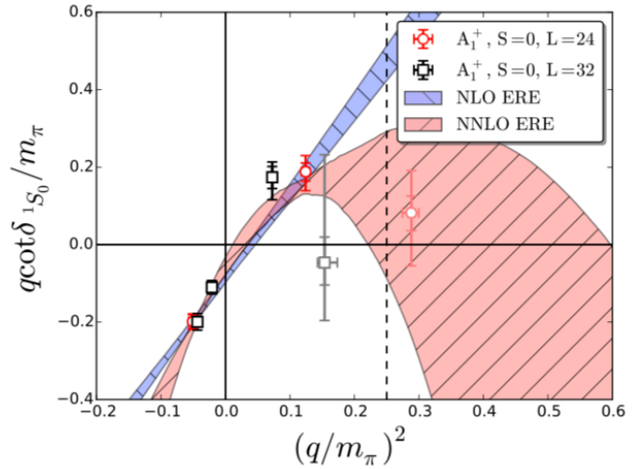
LQCD

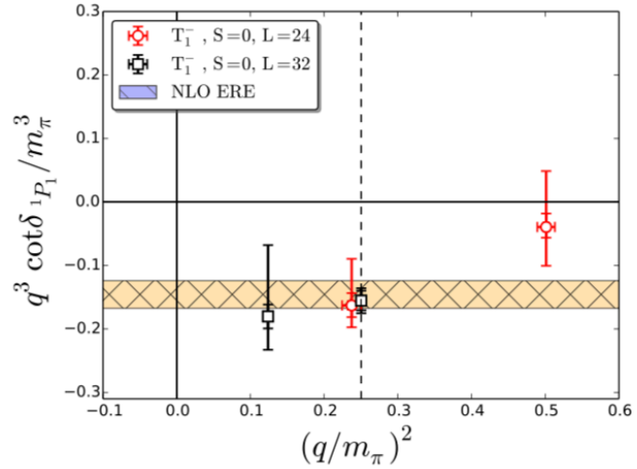
NN scattering at $m_\pi \sim 800$ MeV

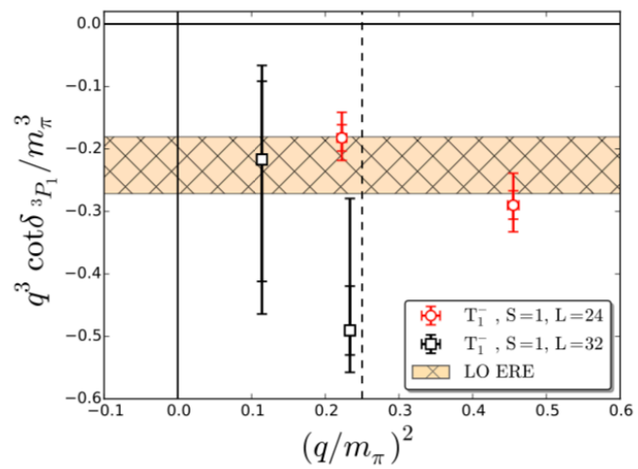


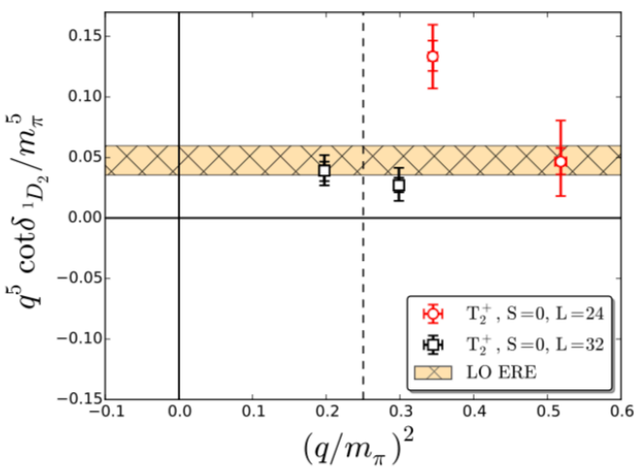


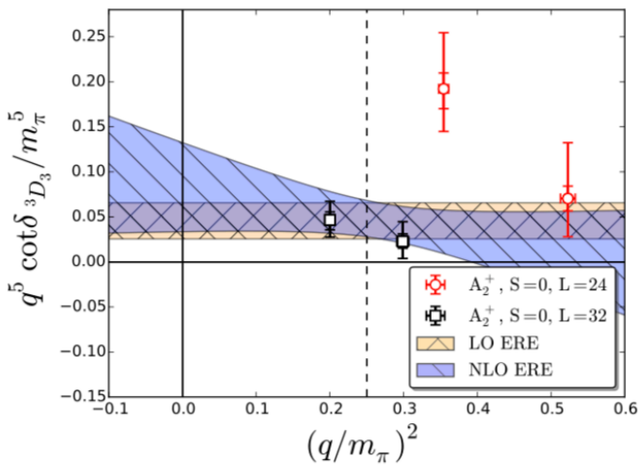


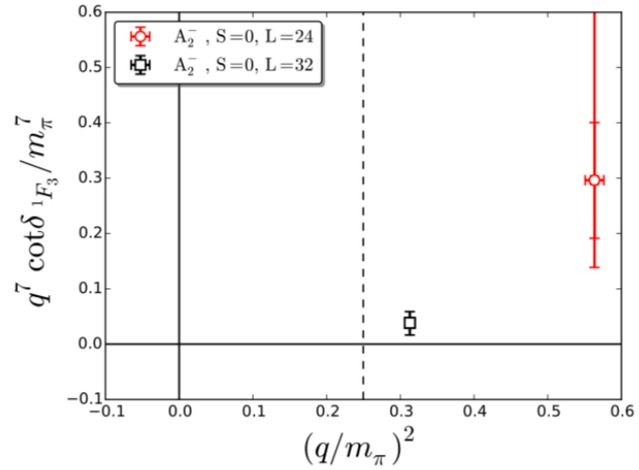






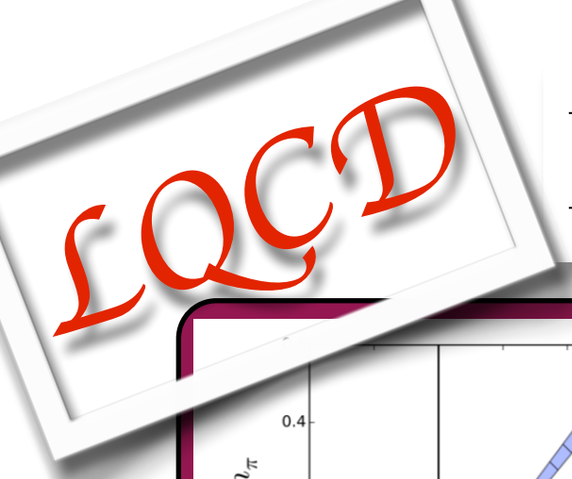


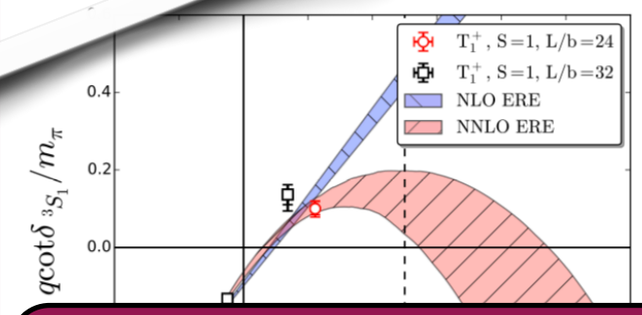


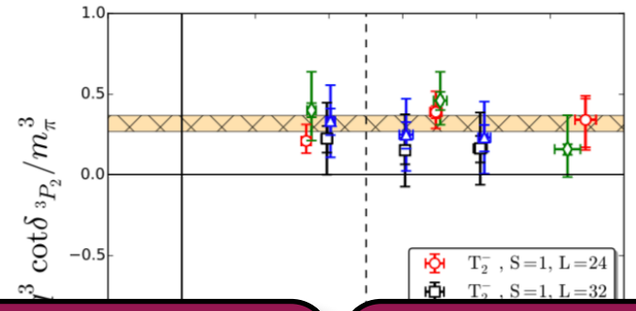


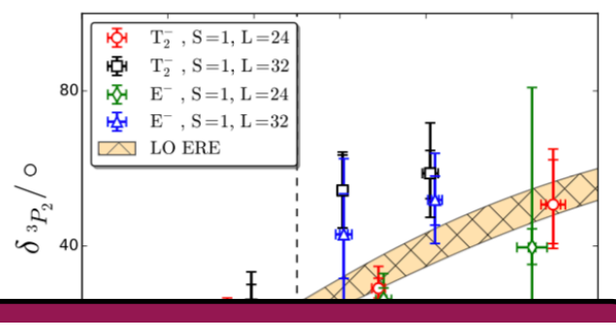
Callat (2015)

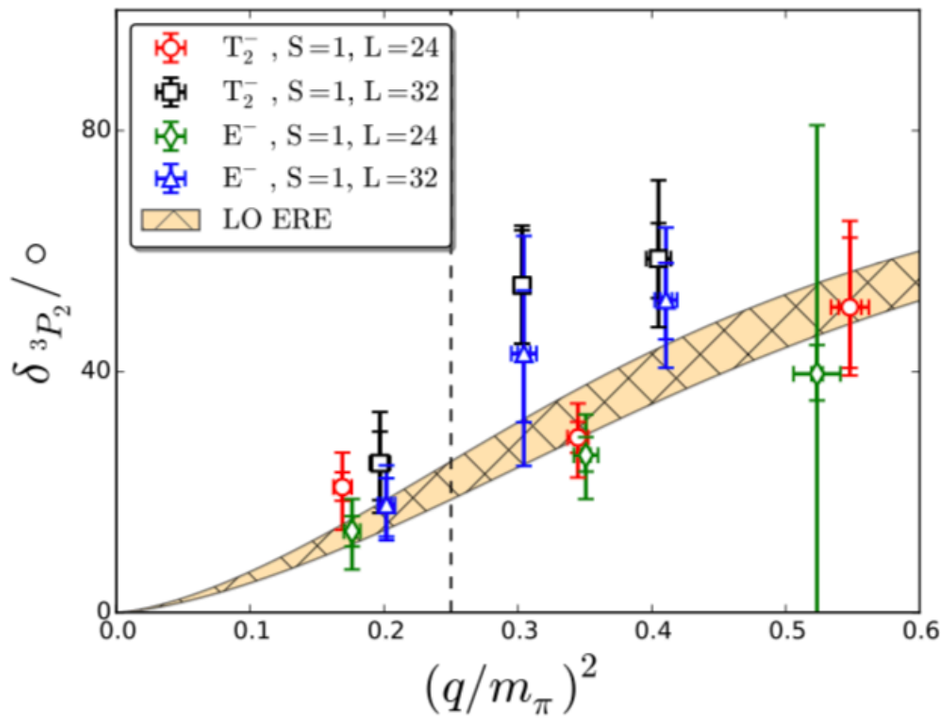
NN scattering at $m_\pi \sim 800$ MeV





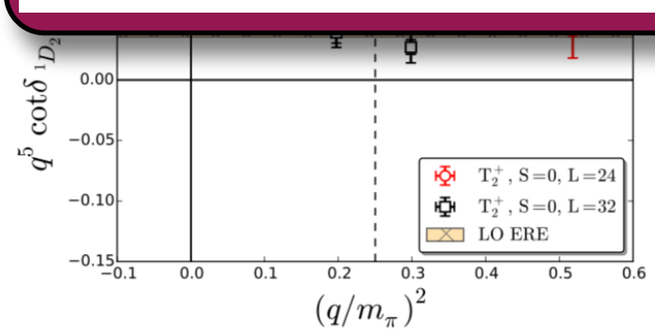


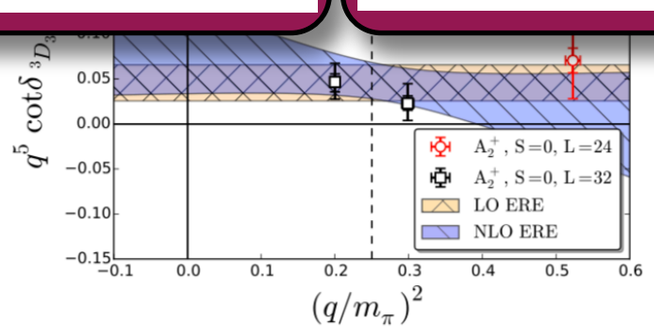


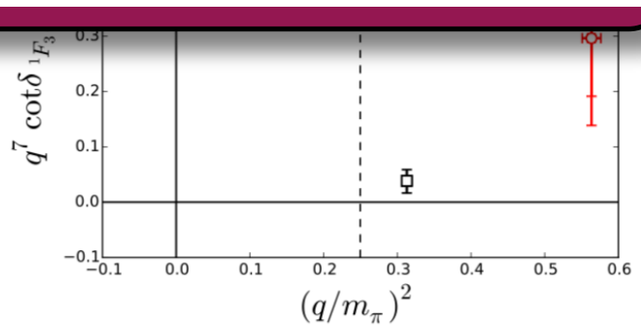


Interplay between $^1S_0$ & $^3P_2$ important in neutron star cores

$^3P_2$ looks qualitatively similar to experiment, but more strongly attractive - what does this mean for pairing?



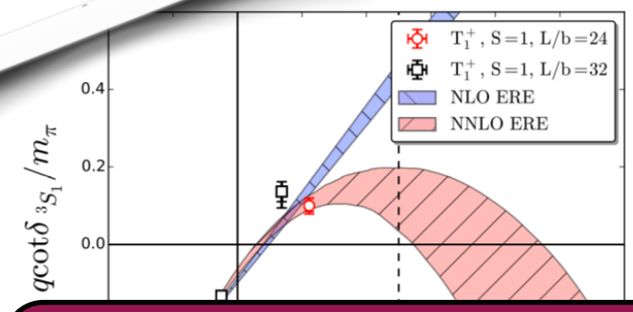


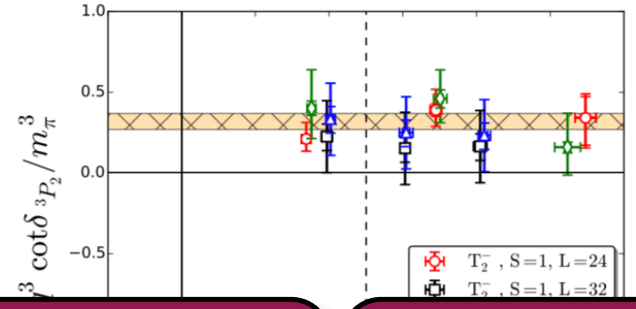


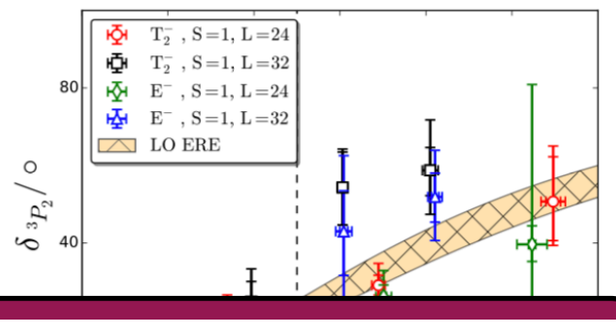
Callat (2015)

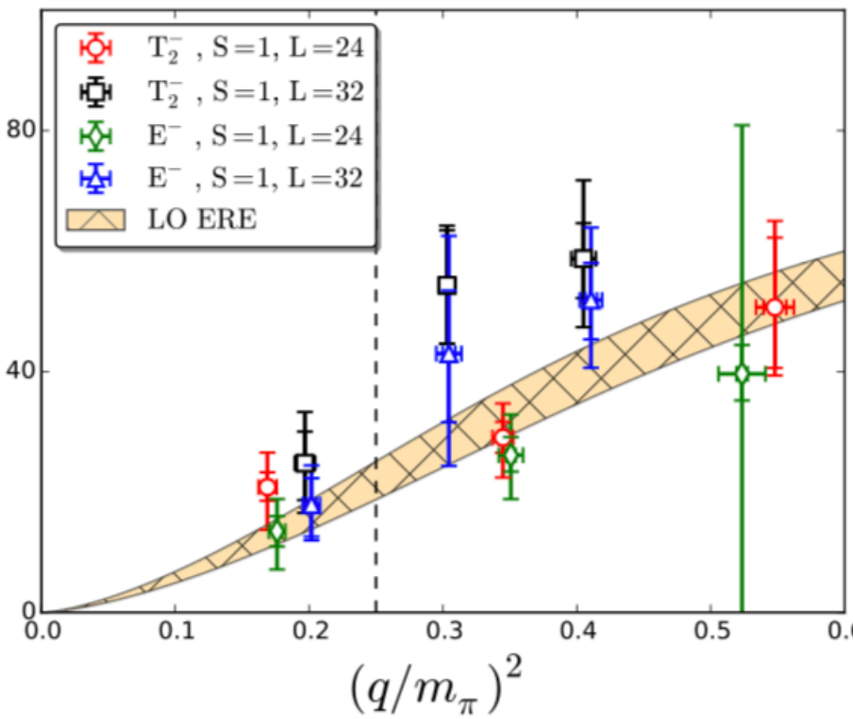
LQCD

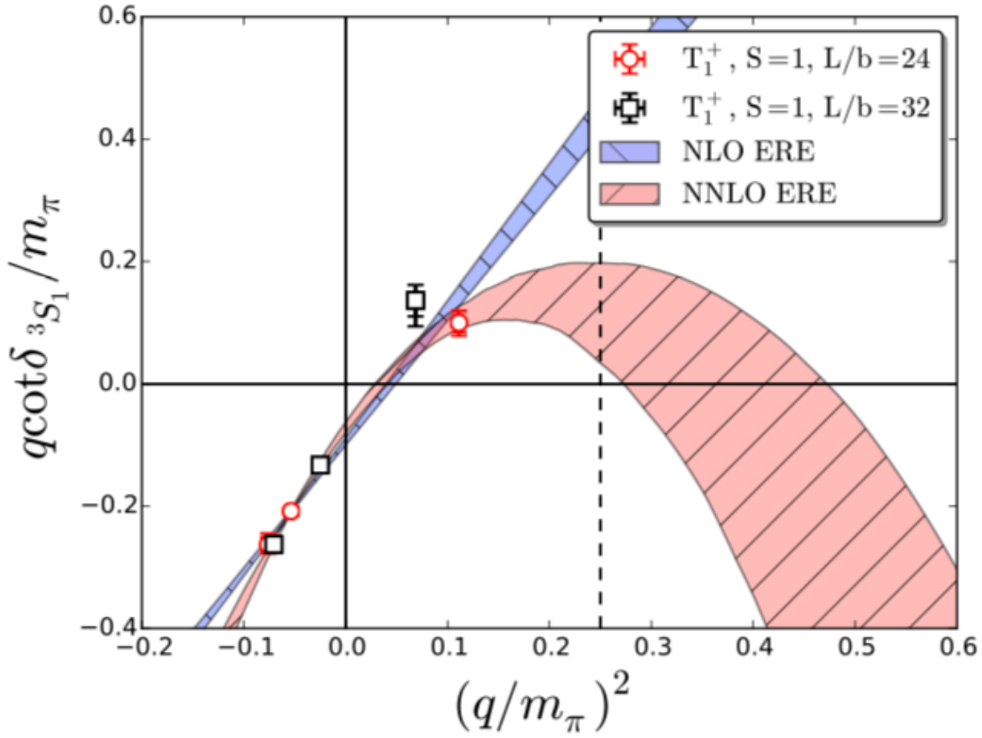
NN scattering at $m_\pi \sim 800$ MeV

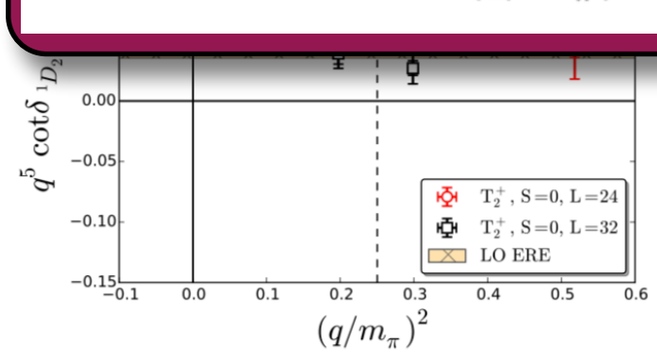


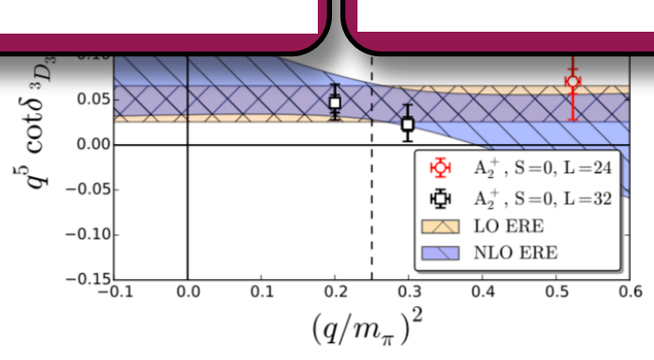


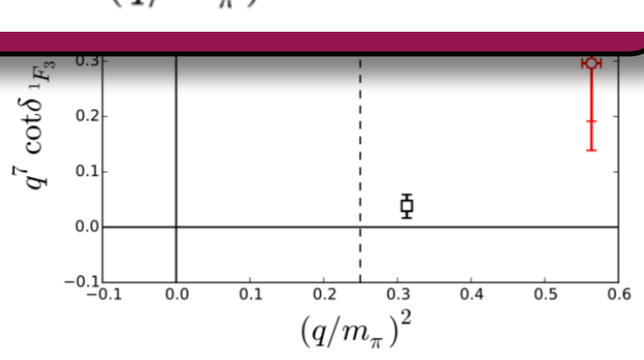






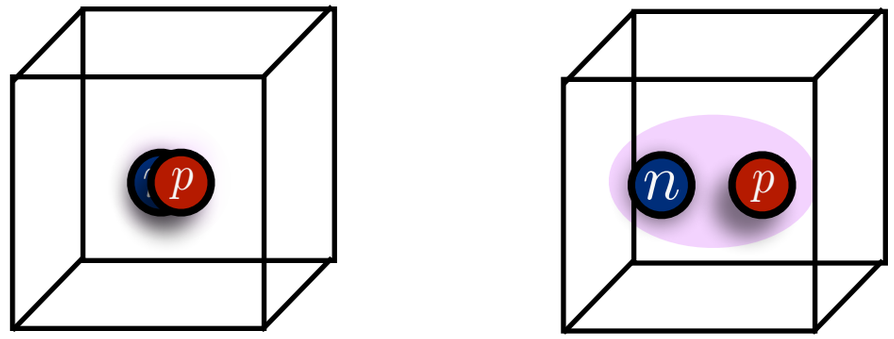






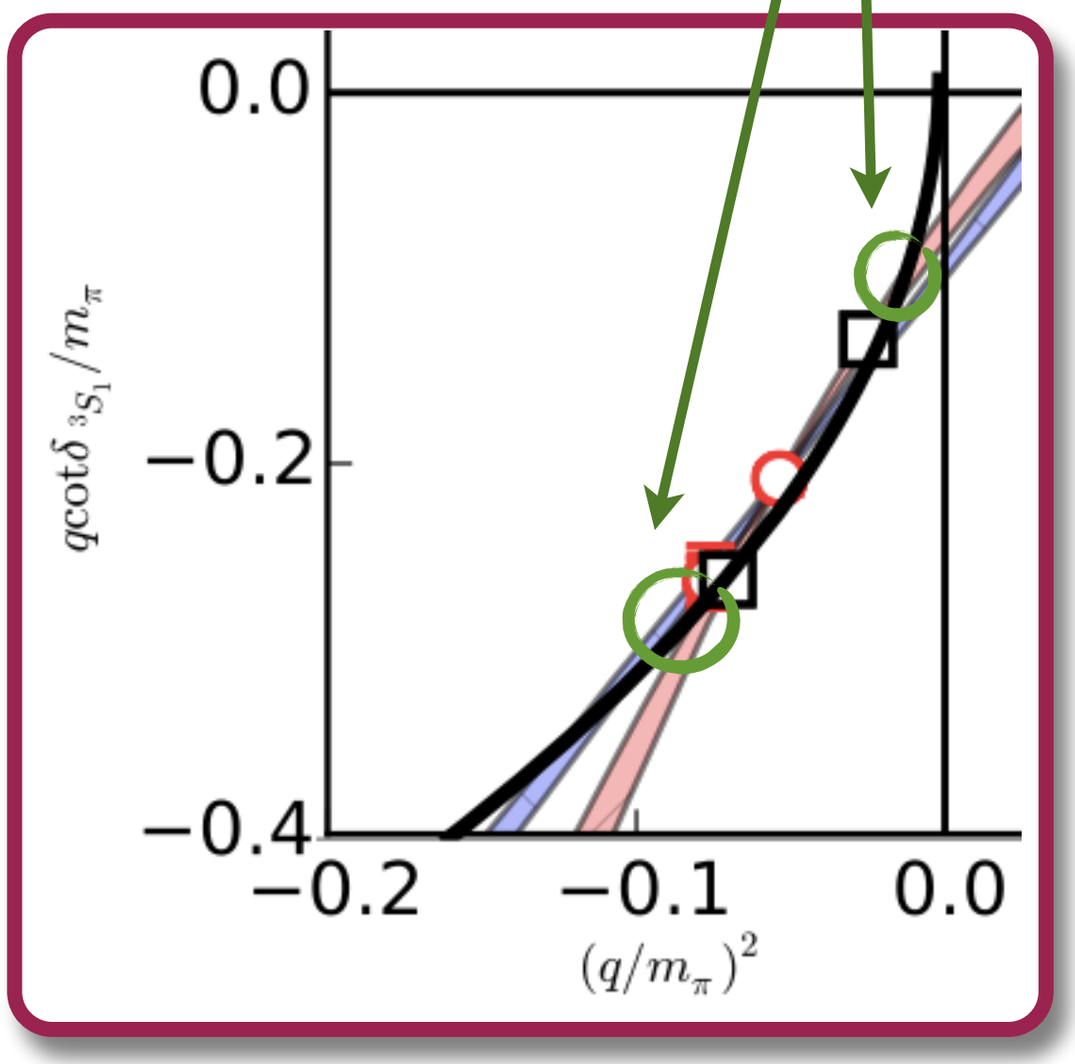
Callat (2015)

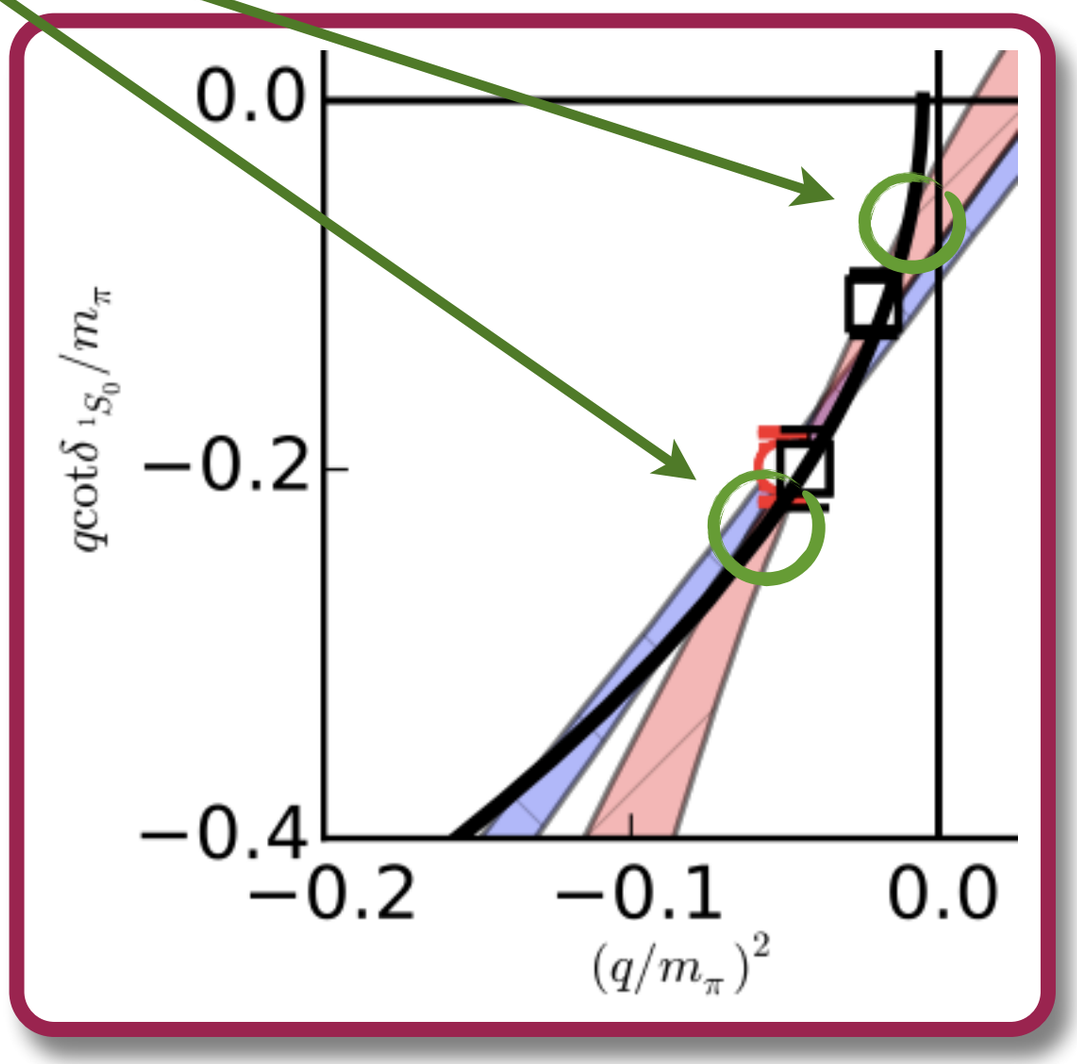
Binding energies: Lüscher method



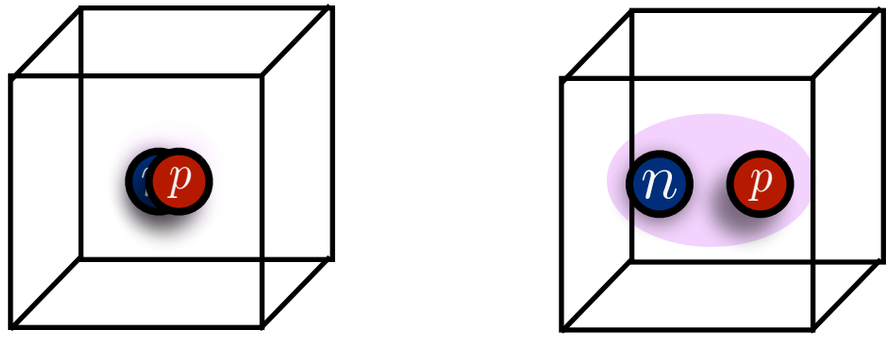
$$p\cot\delta = ip$$

NNLO crossings

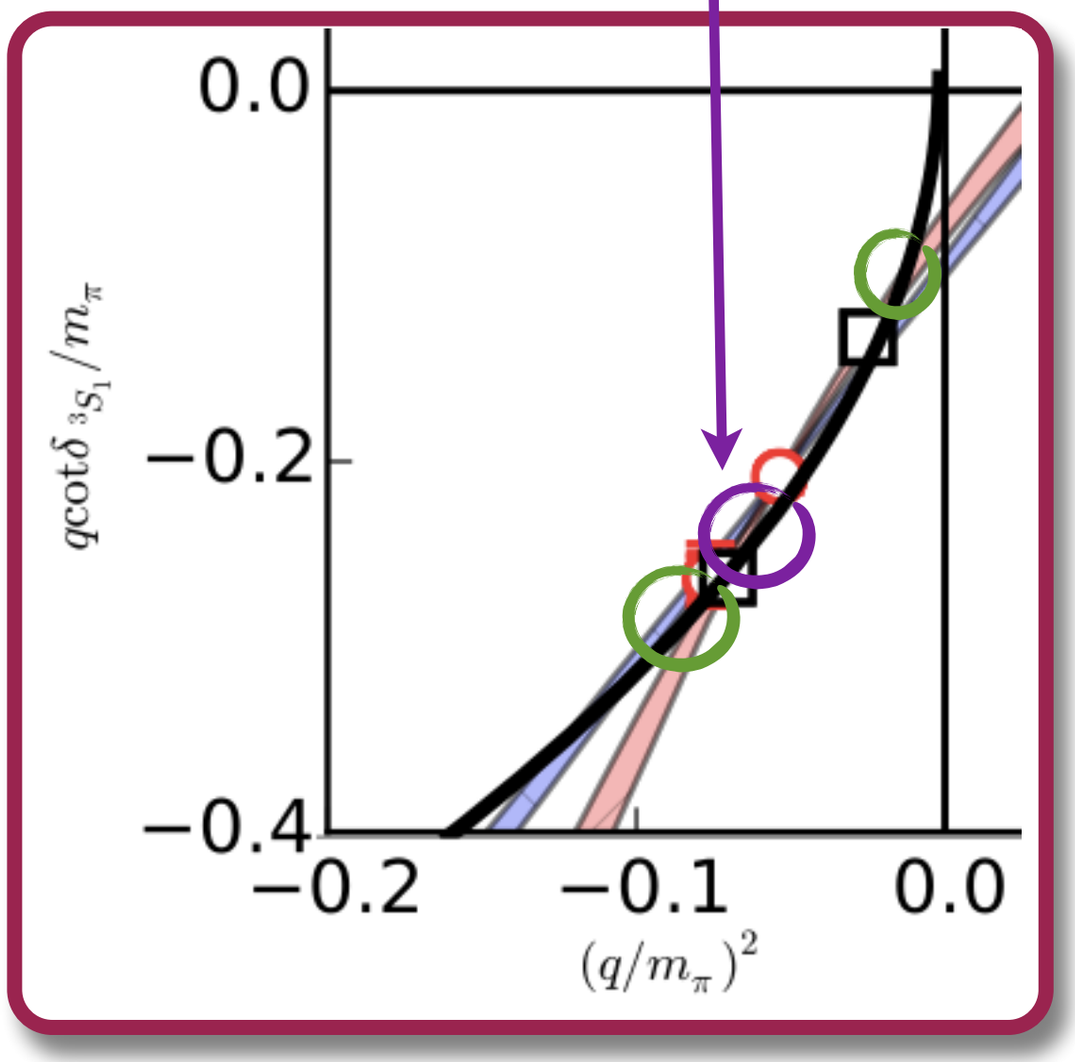


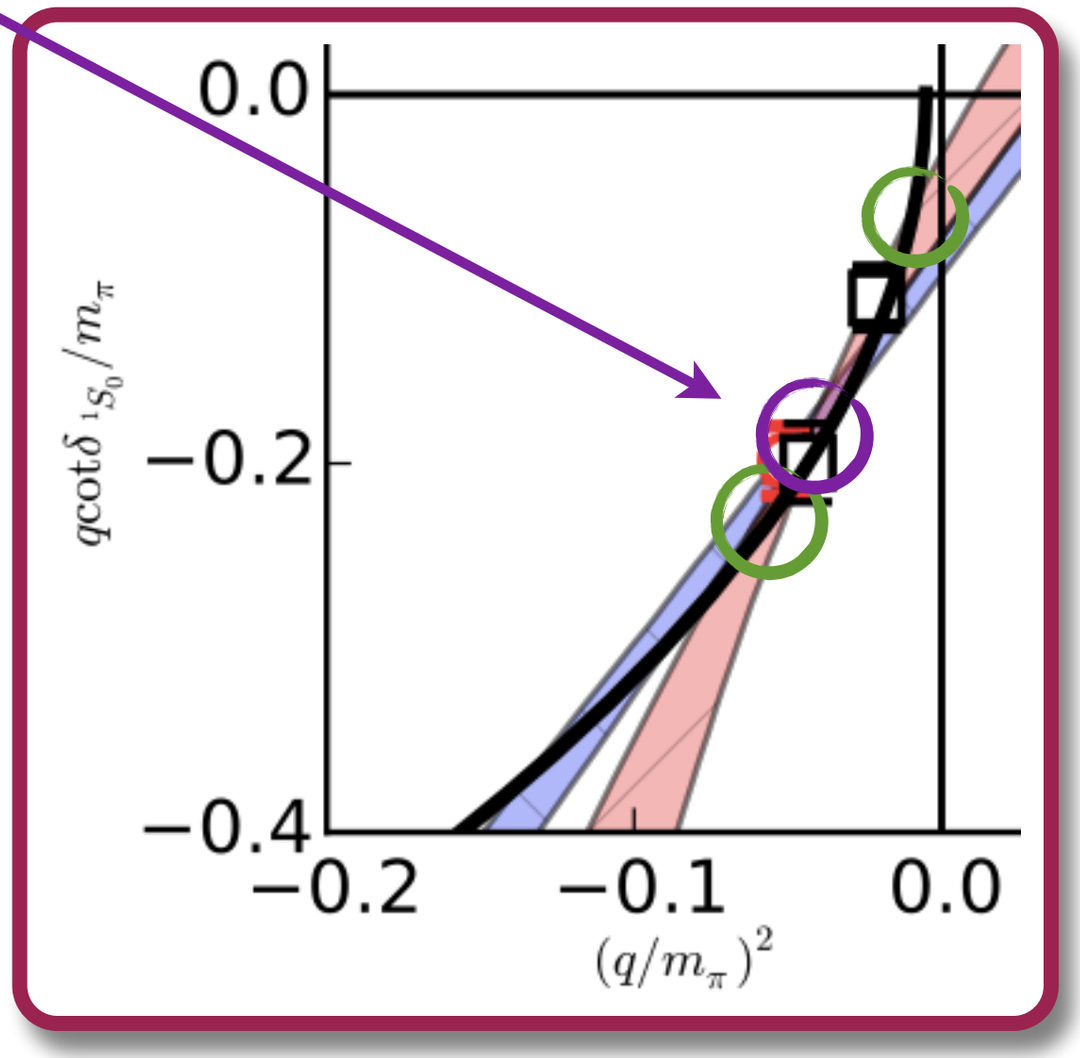


Binding energies: Lüscher method

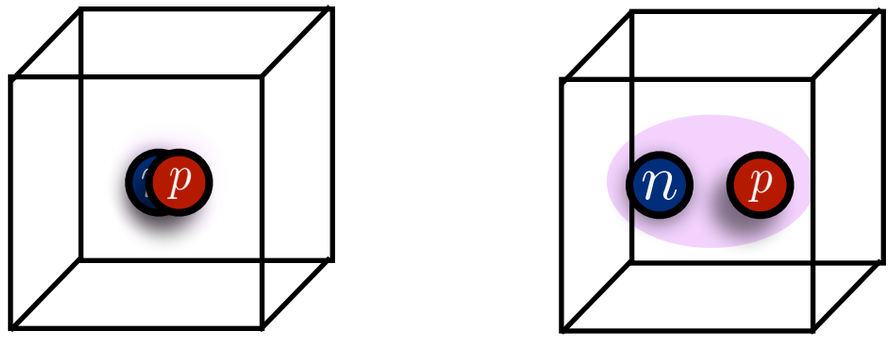


$$p\cot\delta = ip$$

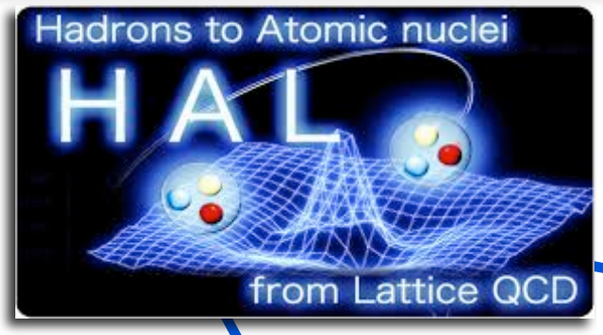


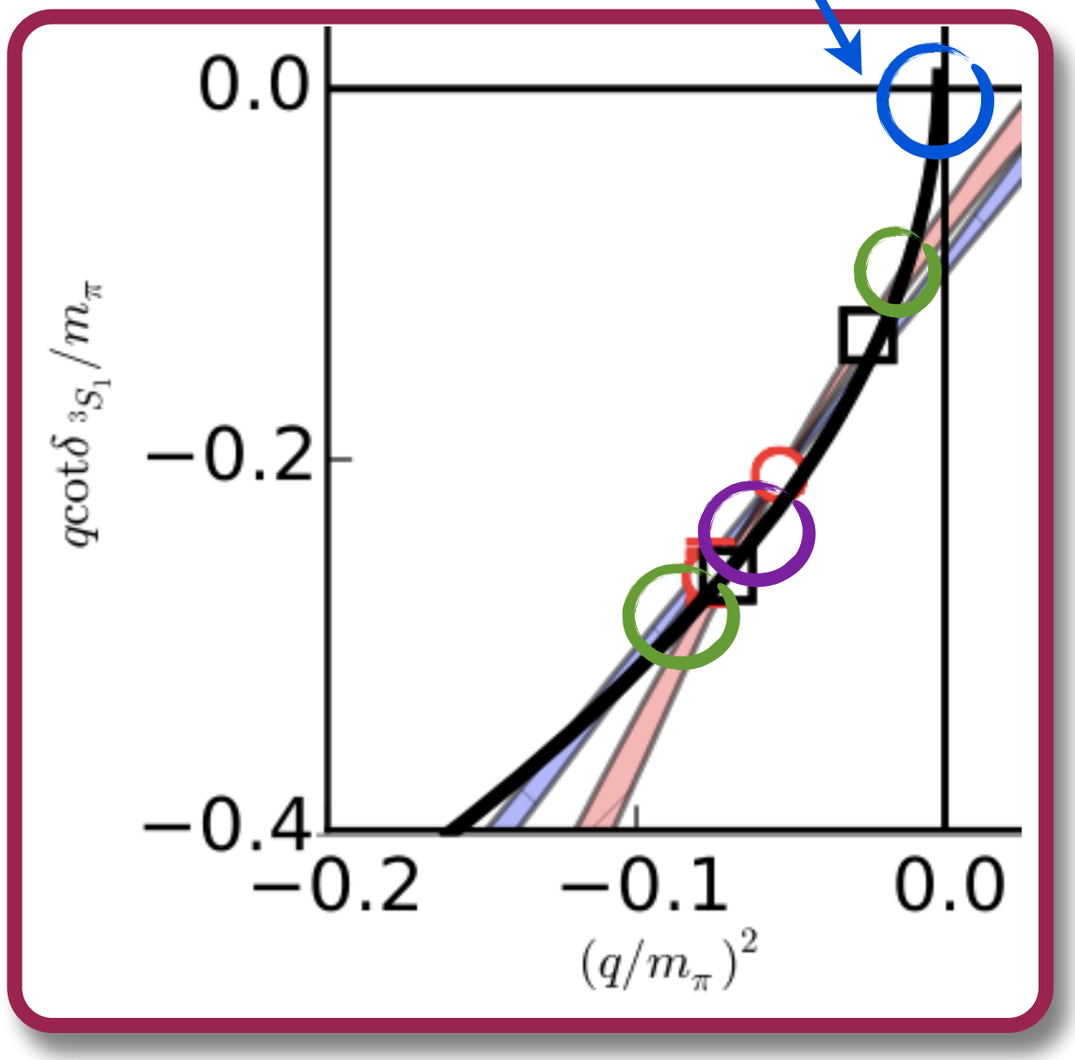


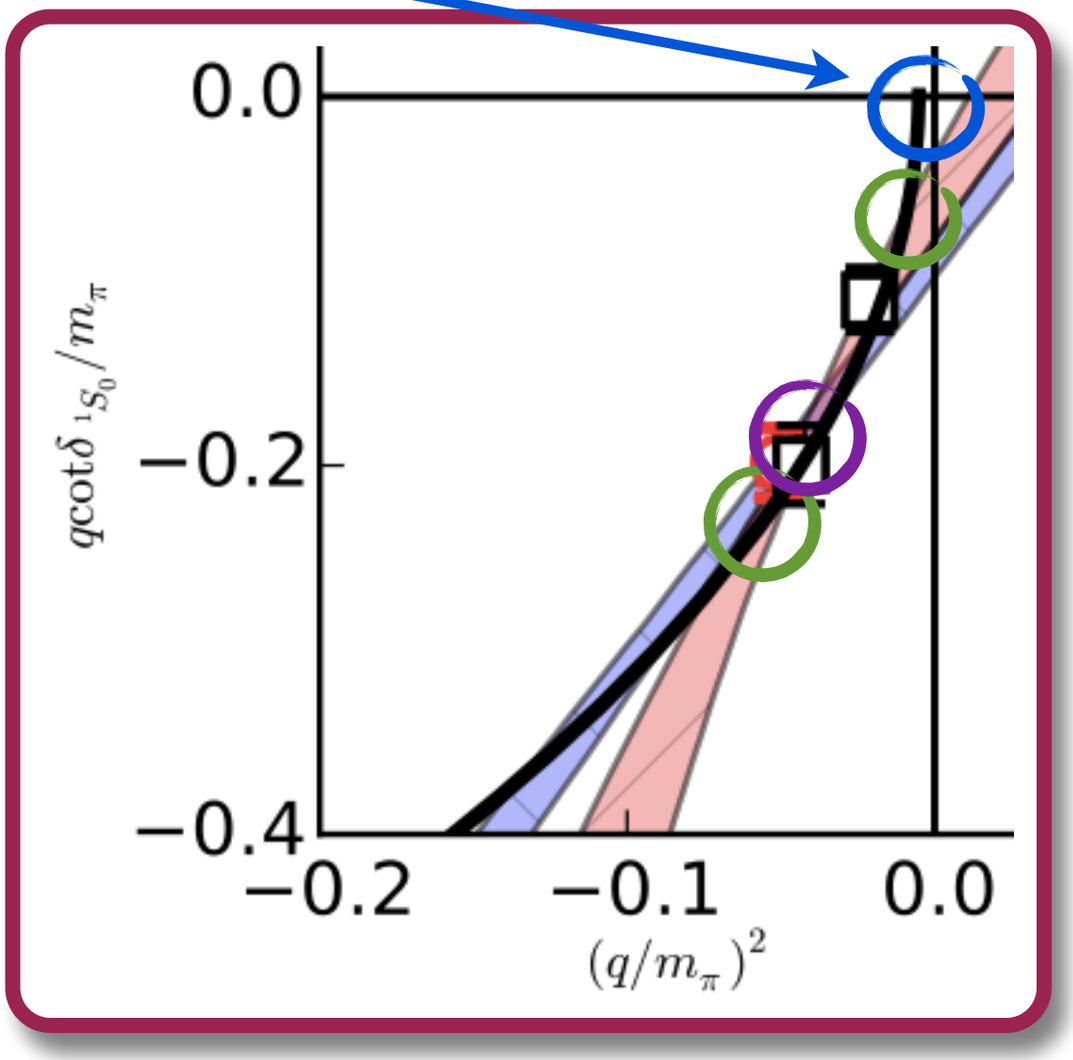
Binding energies: potential method



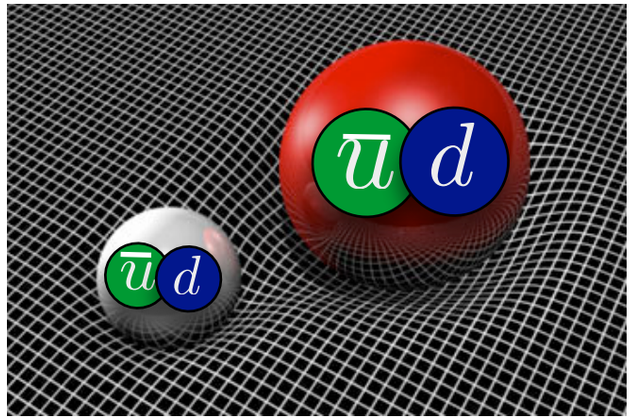
$$p\cot\delta = ip$$

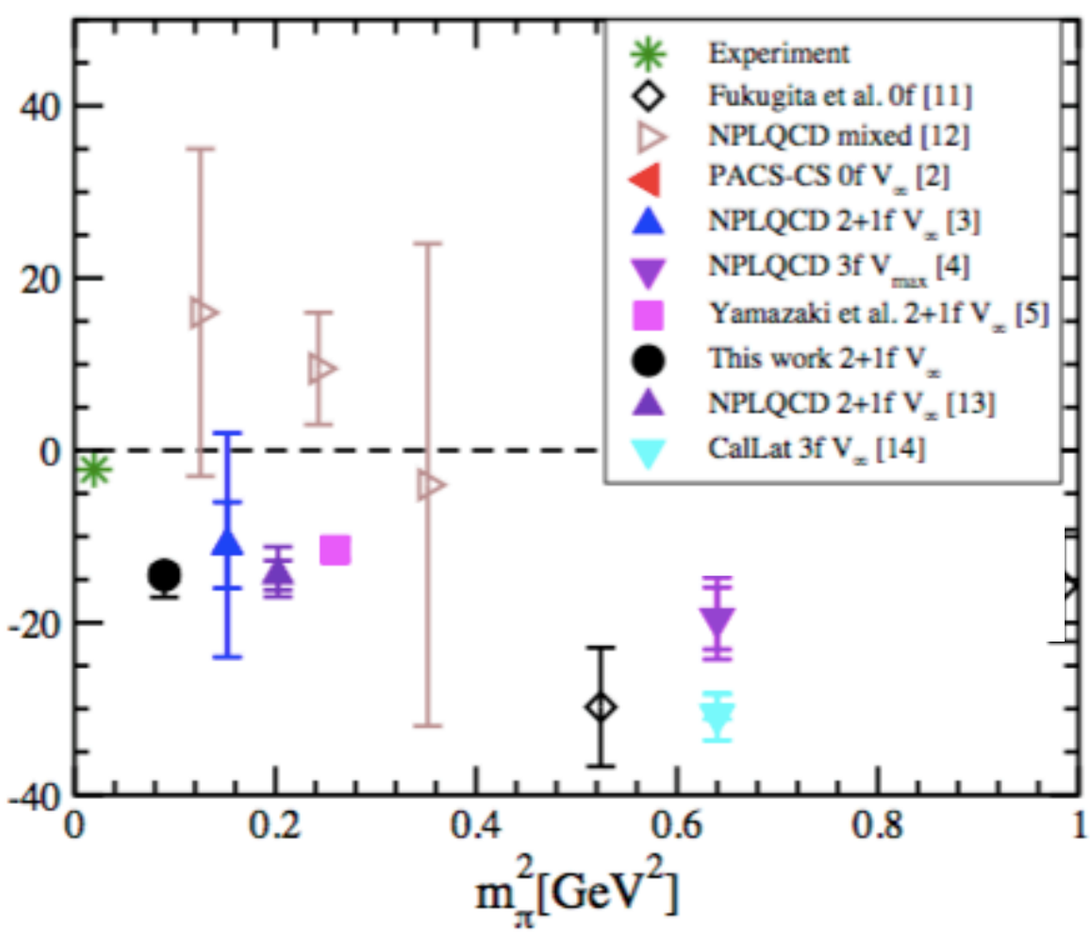


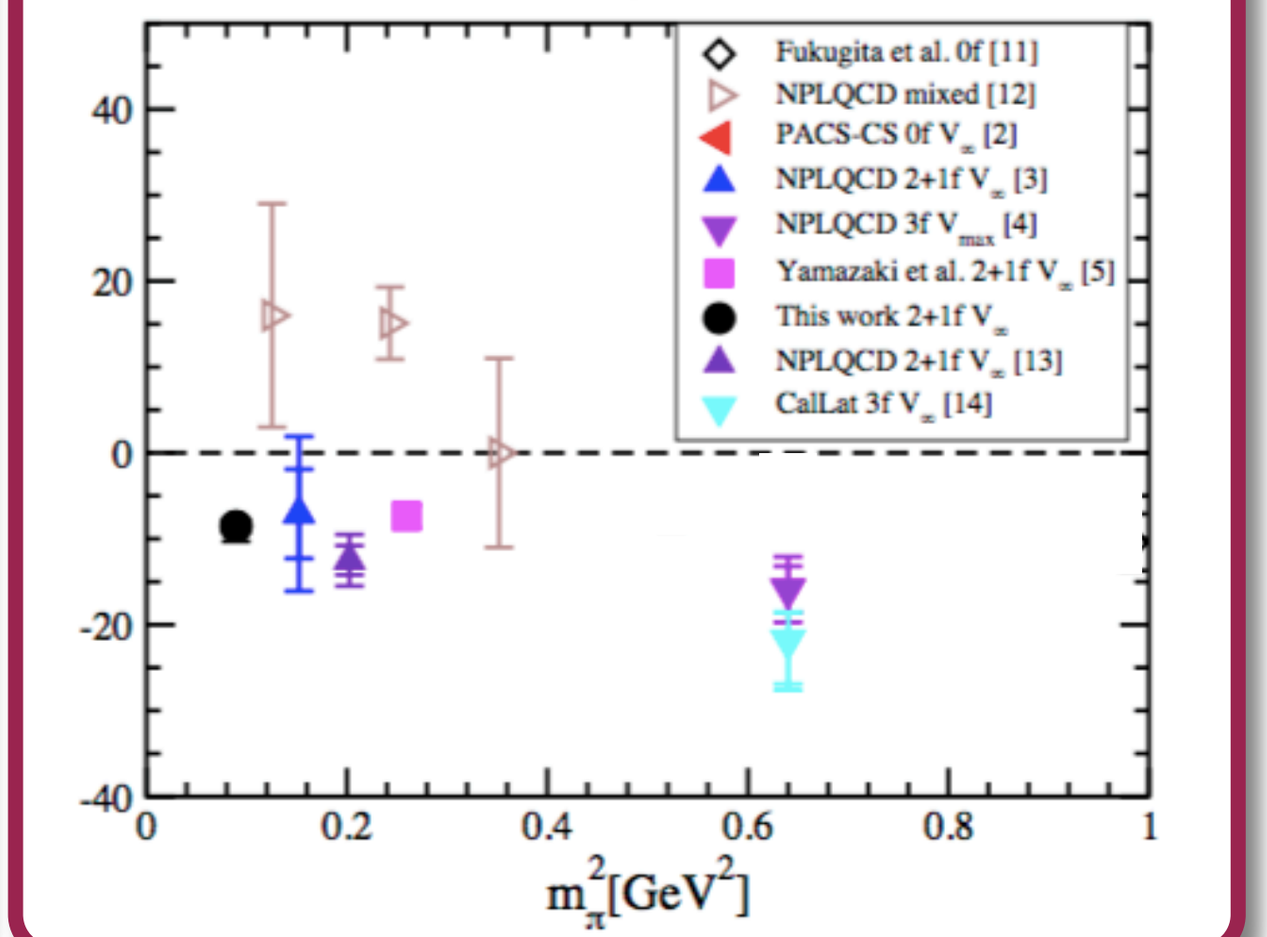




NN Binding energies

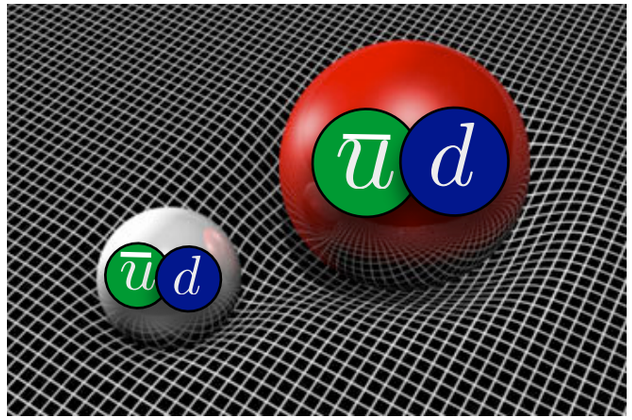


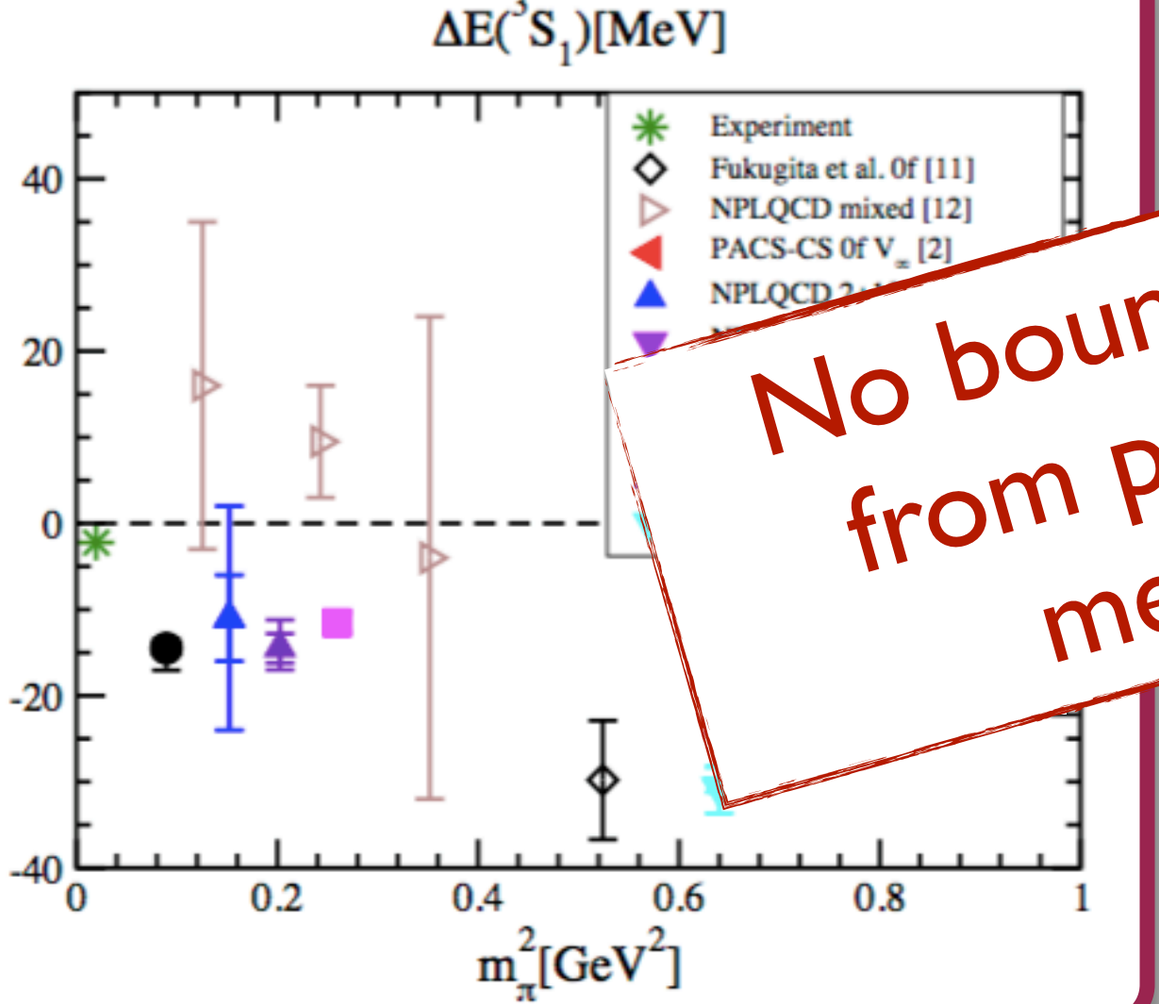


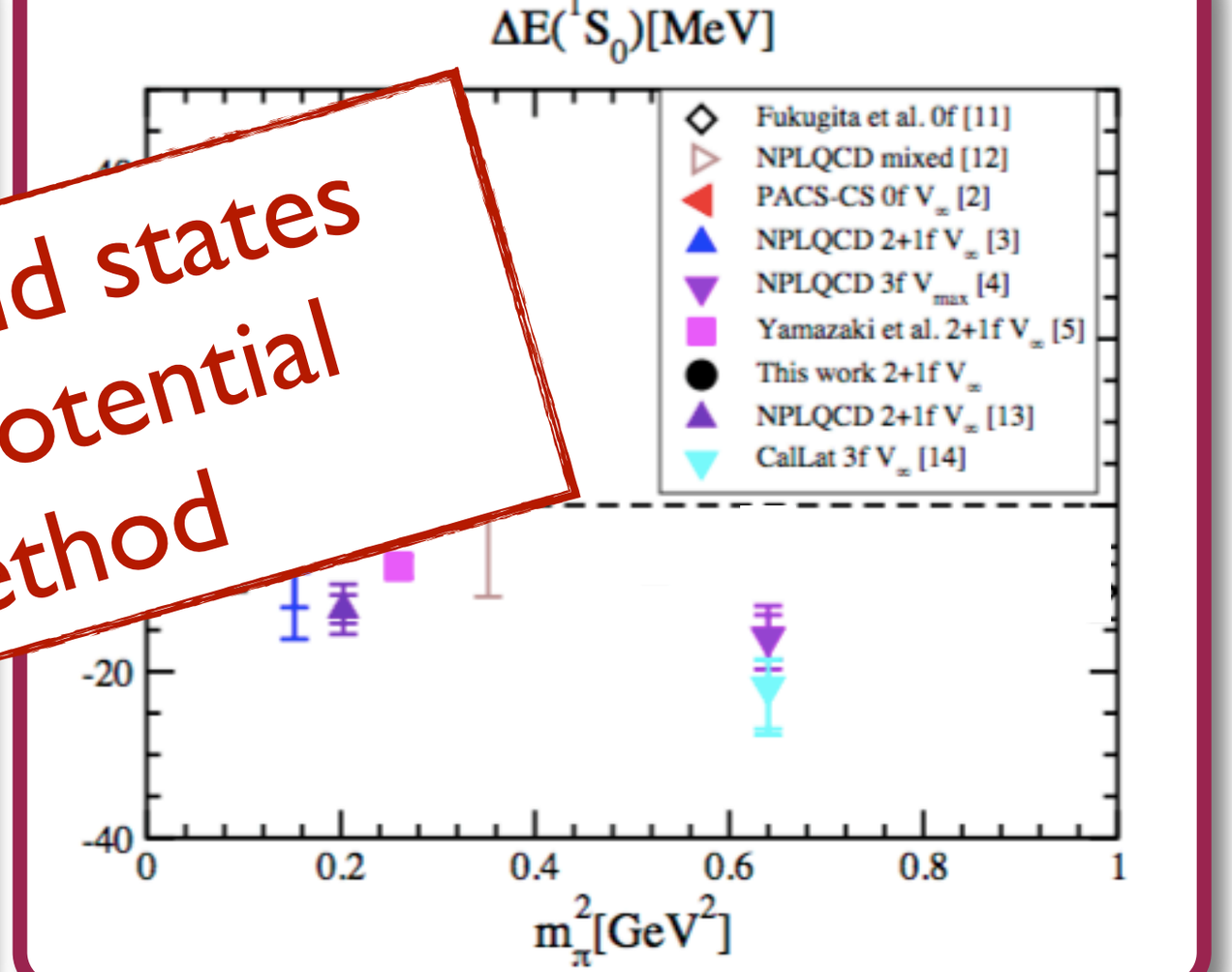


T. Yamazaki, arXiv:1511.09179

NN Binding energies







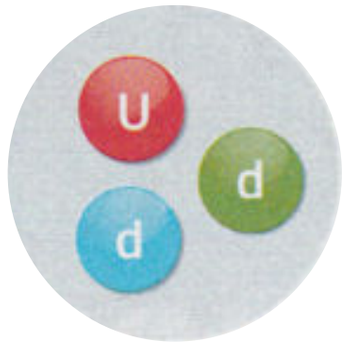
No bound states from potential method

T. Yamazaki, arXiv:1511.09179

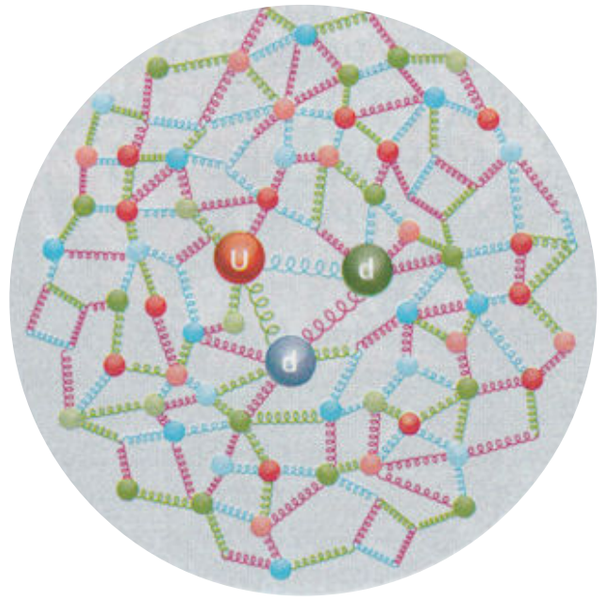
Lüscher: only real issue could be spectroscopy

$$\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle = \langle \mathcal{O}(0)e^{-Ht}\mathcal{O}(0)\rangle = \sum_n |\langle 0|\mathcal{O}|n\rangle|^2 e^{-E_n t}$$

$\mathcal{O}$:



ψ_n :



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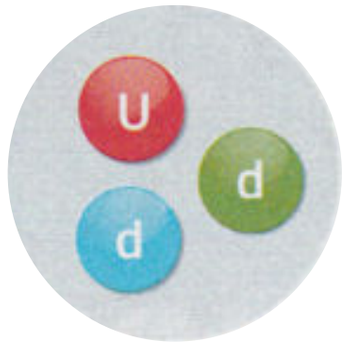
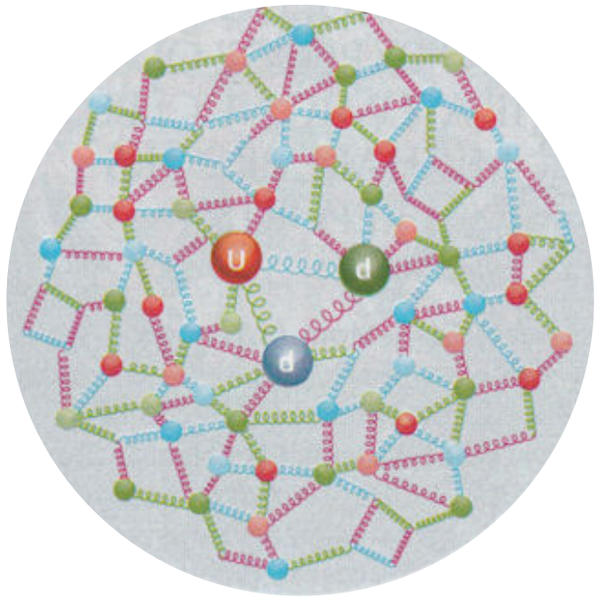
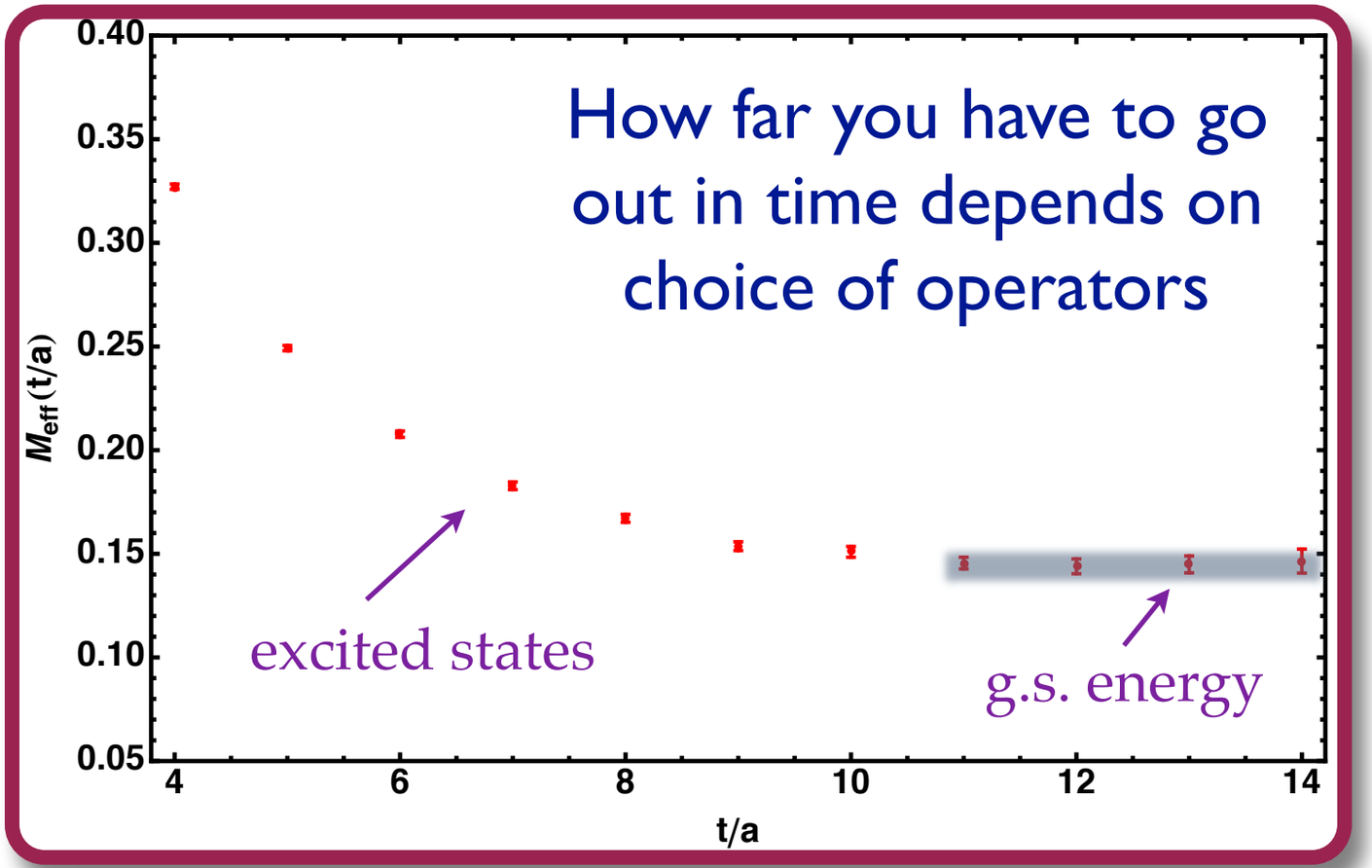


ψ_n :



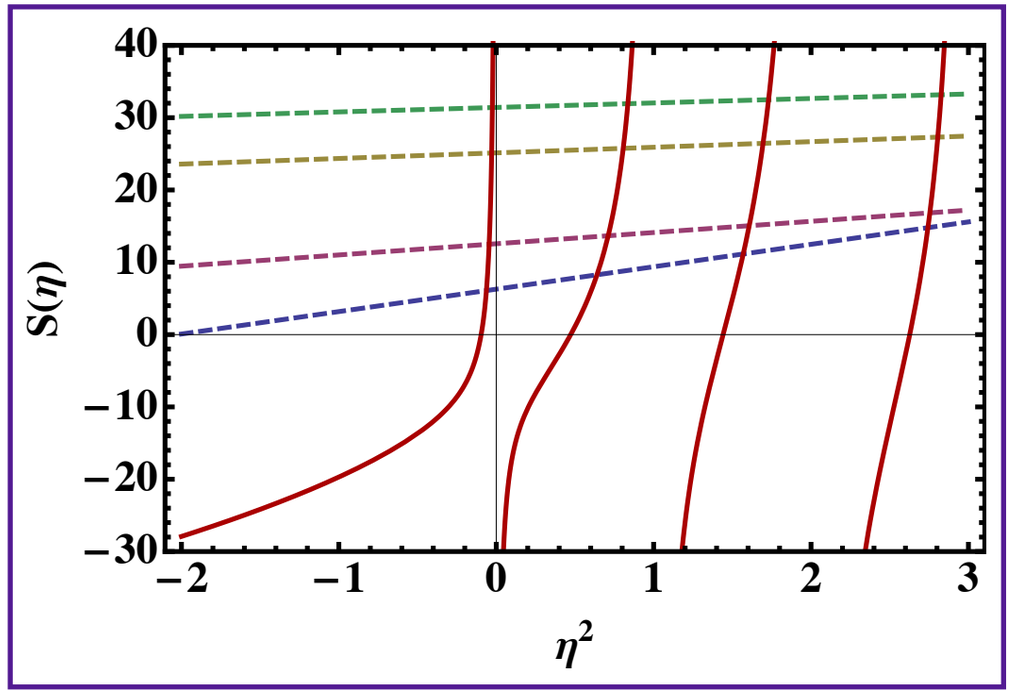
Effective mass plot:

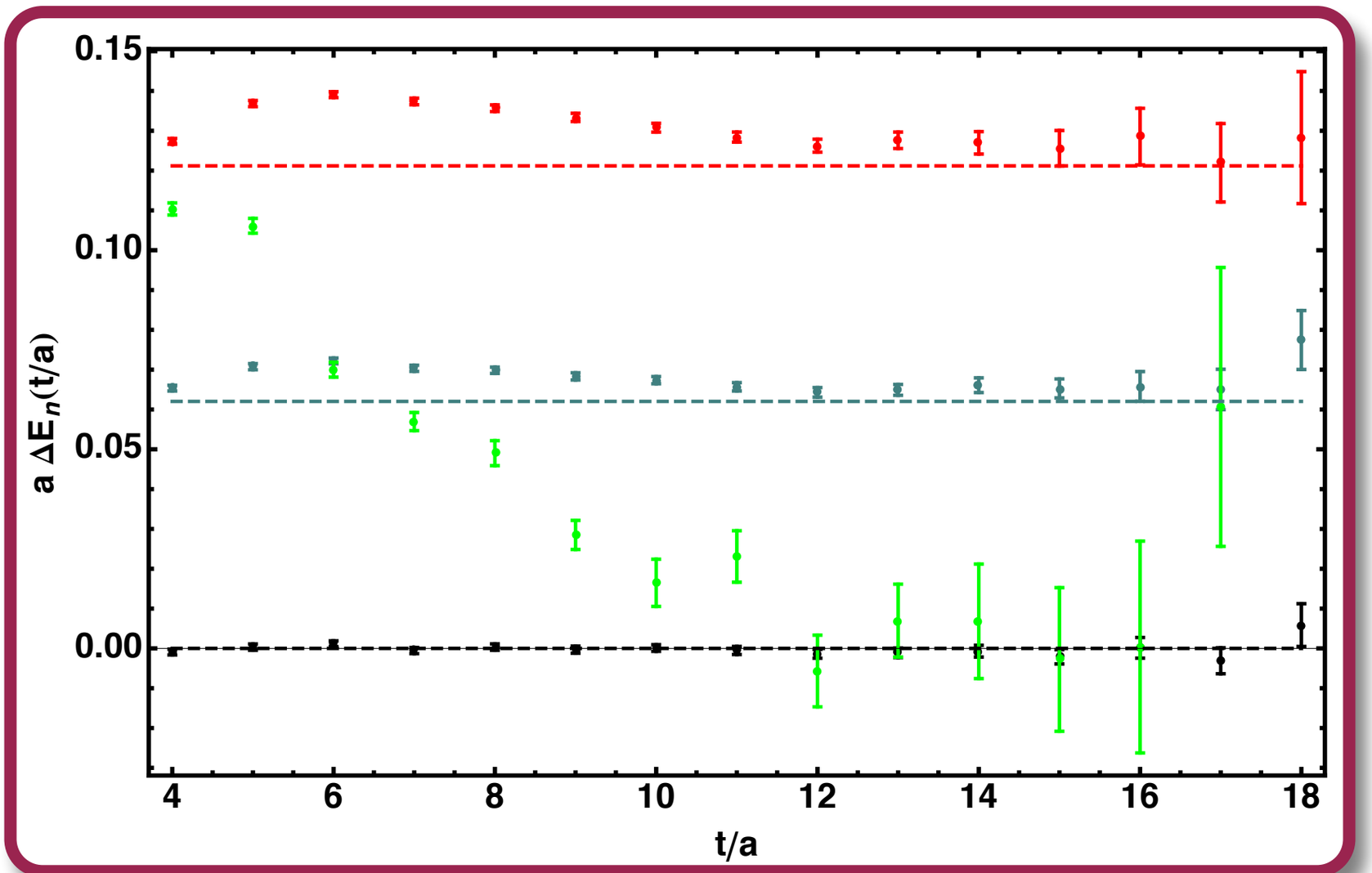
$$M_{\text{eff}} \equiv \ln \frac{C(t)}{C(t+1)} \underset{t\to\infty}{\longrightarrow} E_0$$



Trying to pull off tiny correction compared to large nucleon mass:

$\Delta E = E_{NN} - 2E_N$





$$p \cot \delta(p) = \frac{1}{\pi L} S[(pL/2\pi)^2]$$

Lüscher (1986)

Trying to pull off tiny correction compared to large nucleon mass:

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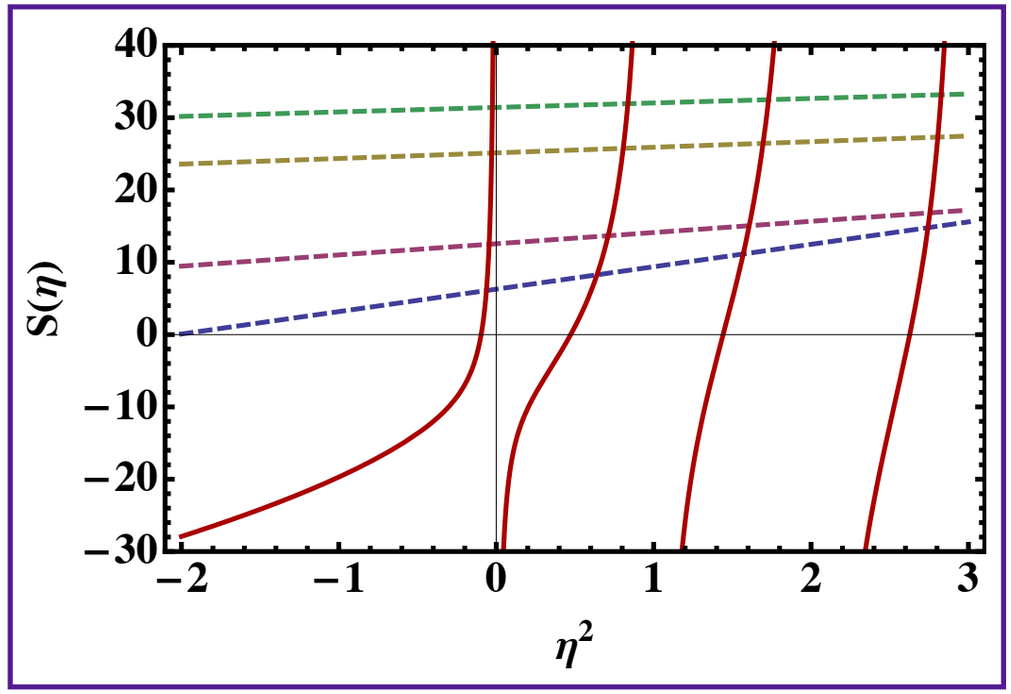

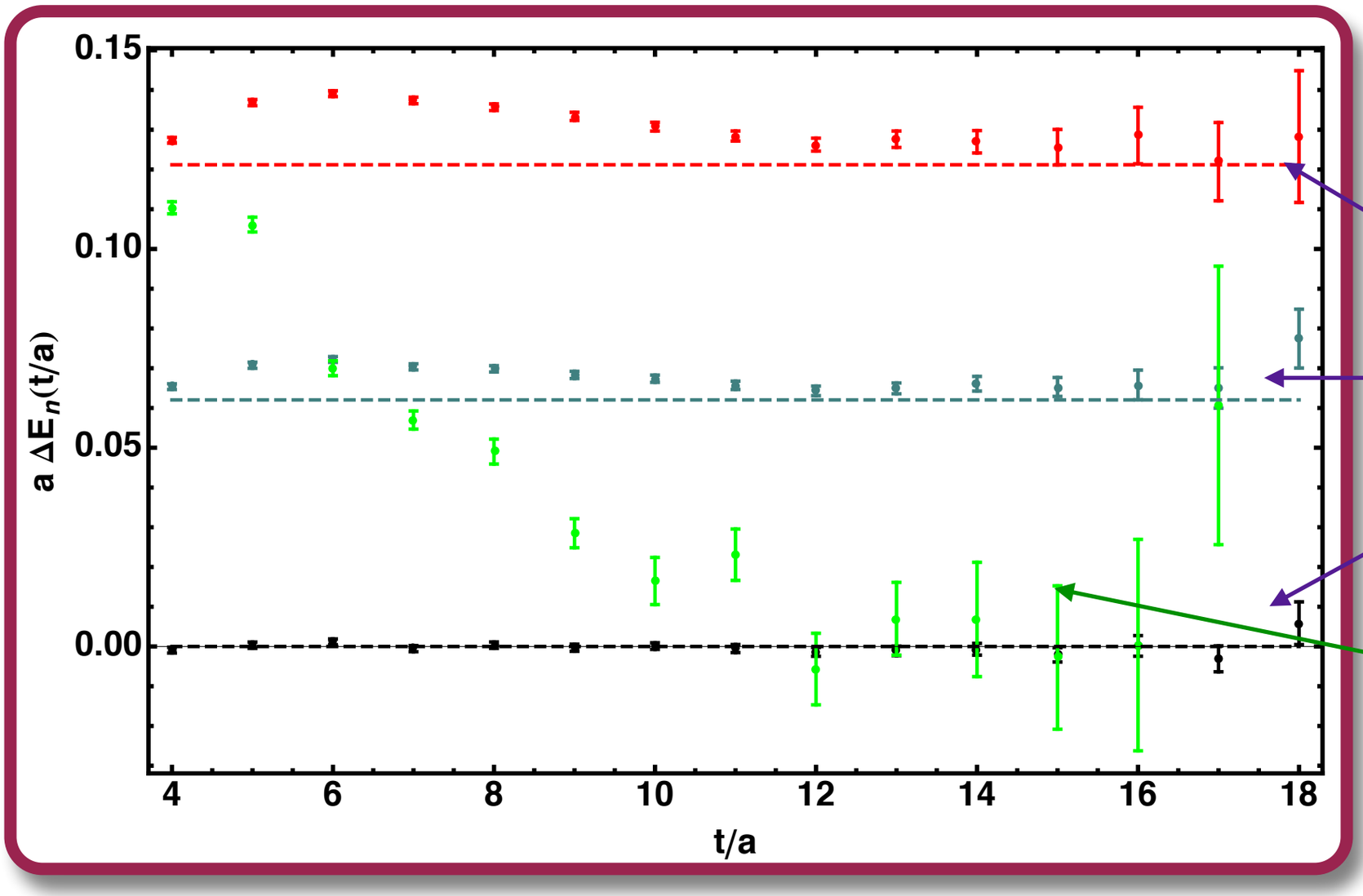

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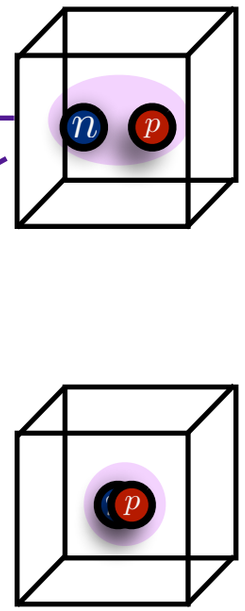
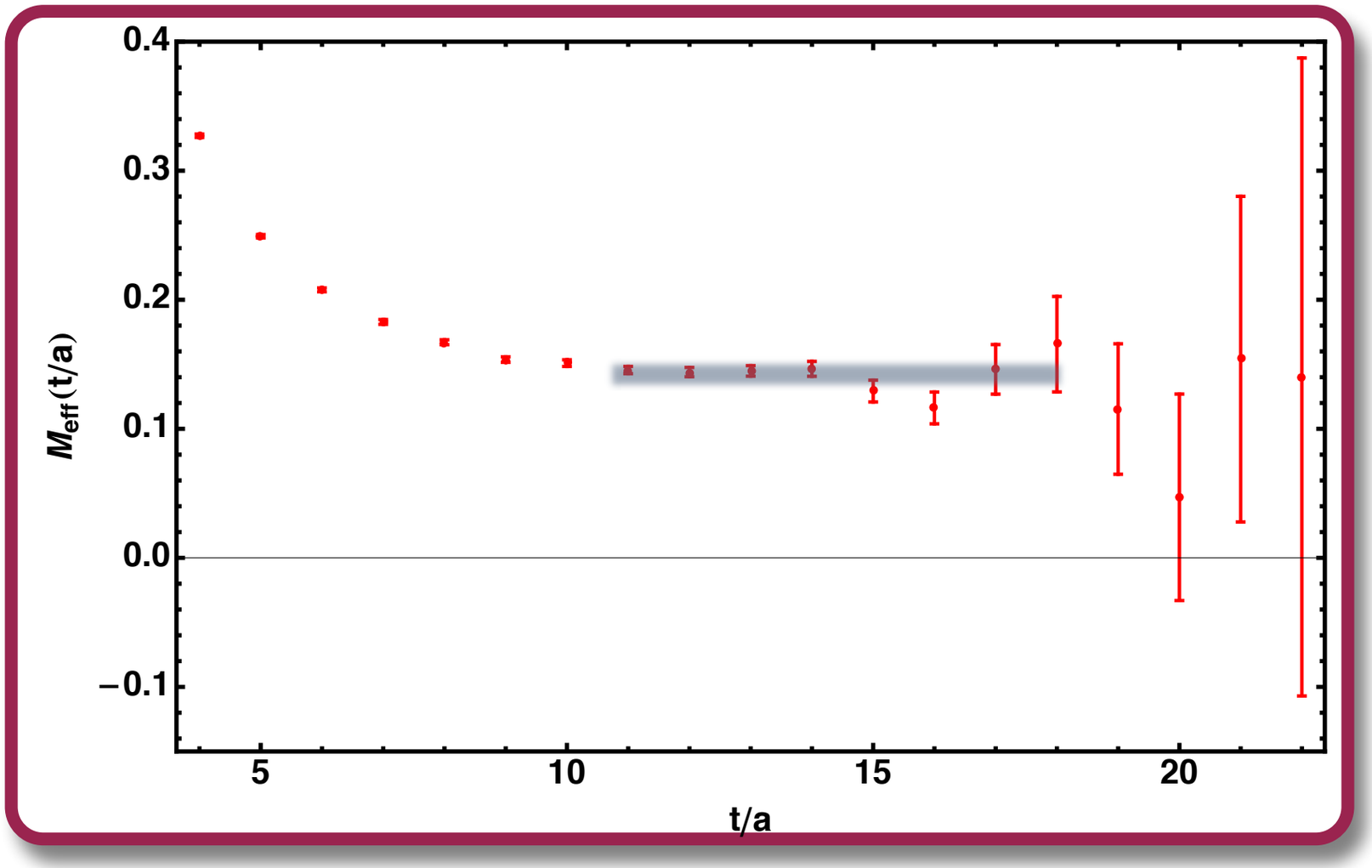
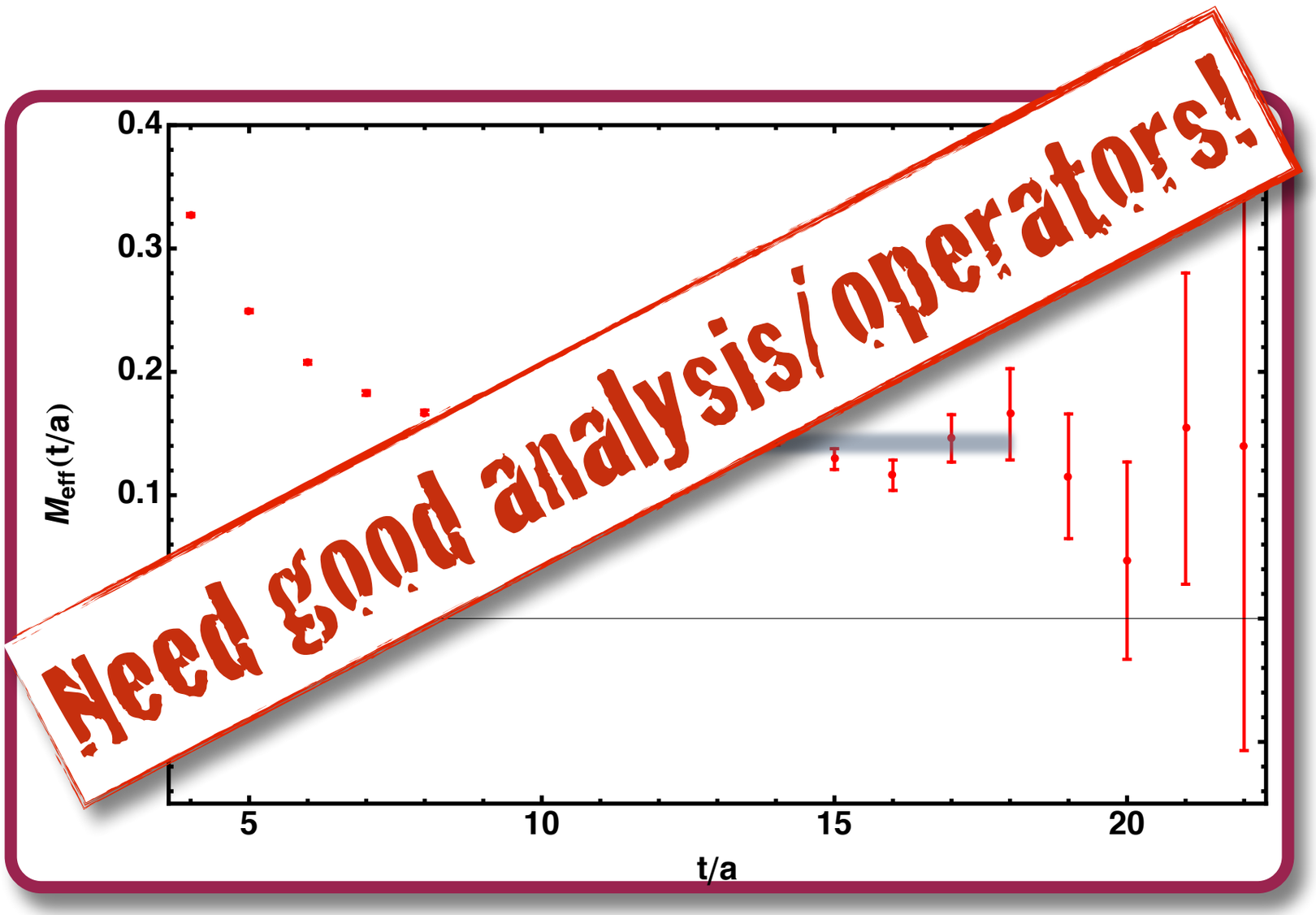


Nucleons: Signal-to-noise



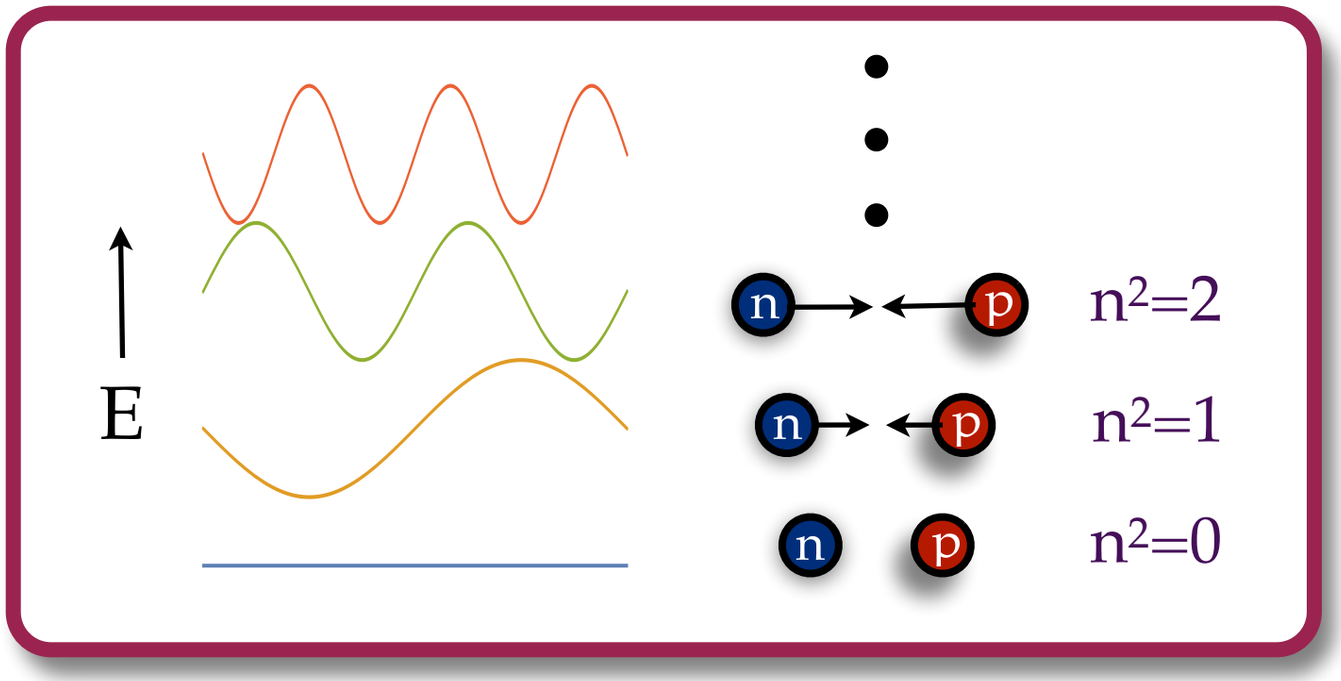
$$\sim e^{A(M_n - 3/2 m_\pi)t}$$

Nucleons: Signal-to-noise



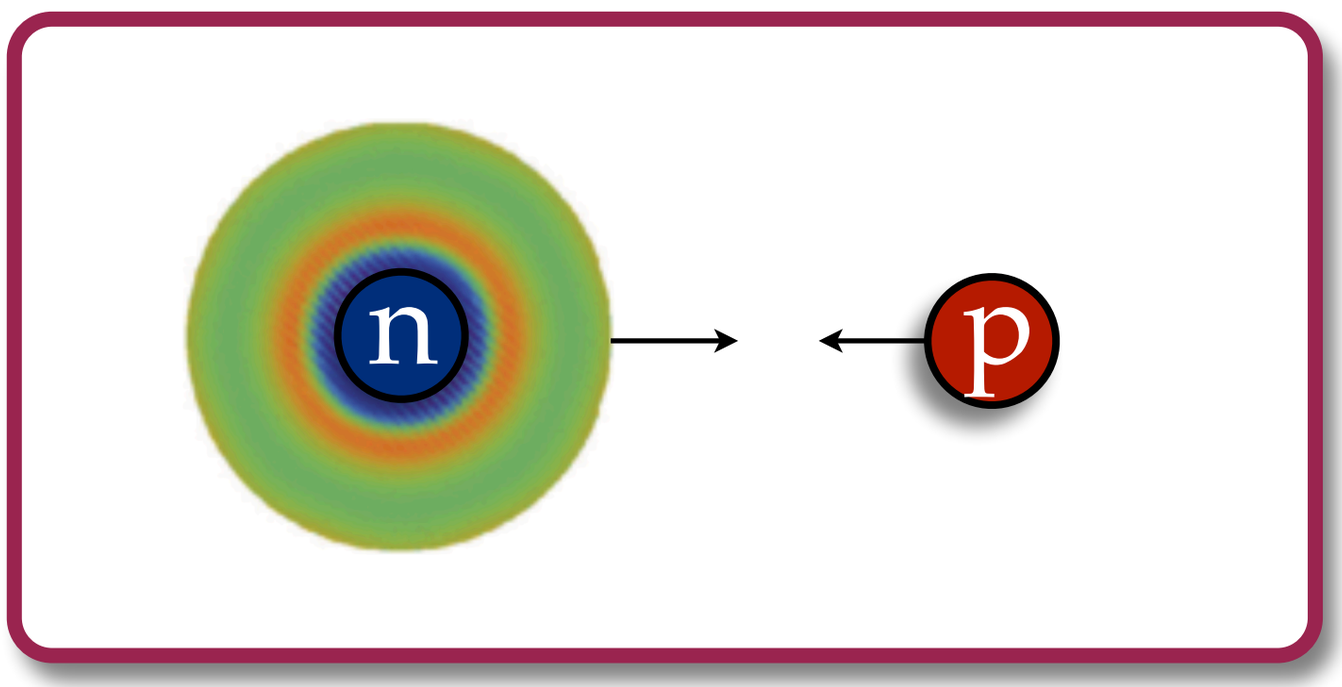
$$\left.\right\} \sim e^{A(M_n - 3/2m_\pi)t}$$

Excited state contamination



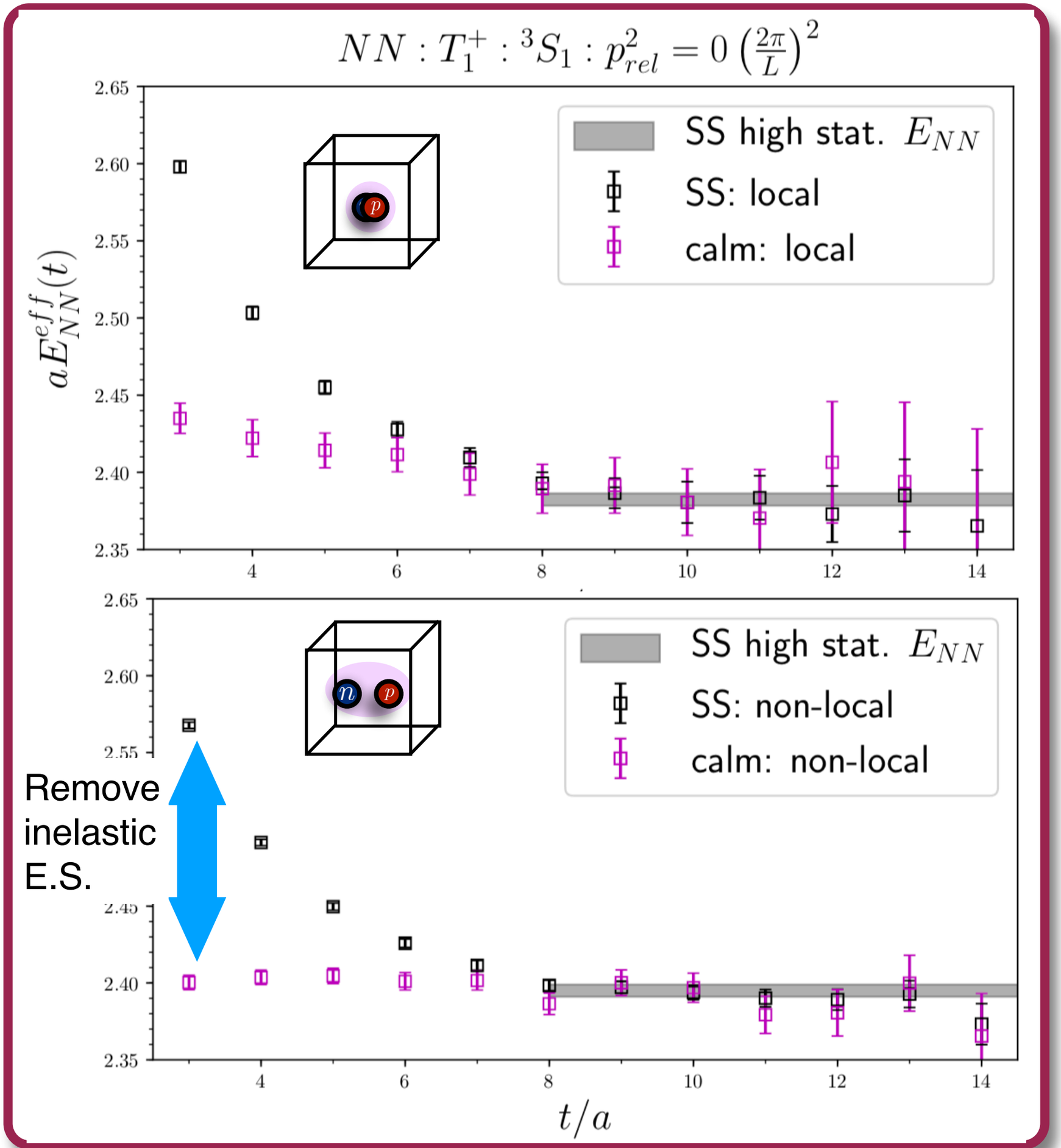
Elastic scattering
(2-body)

$\Delta E \sim 50$ MeV



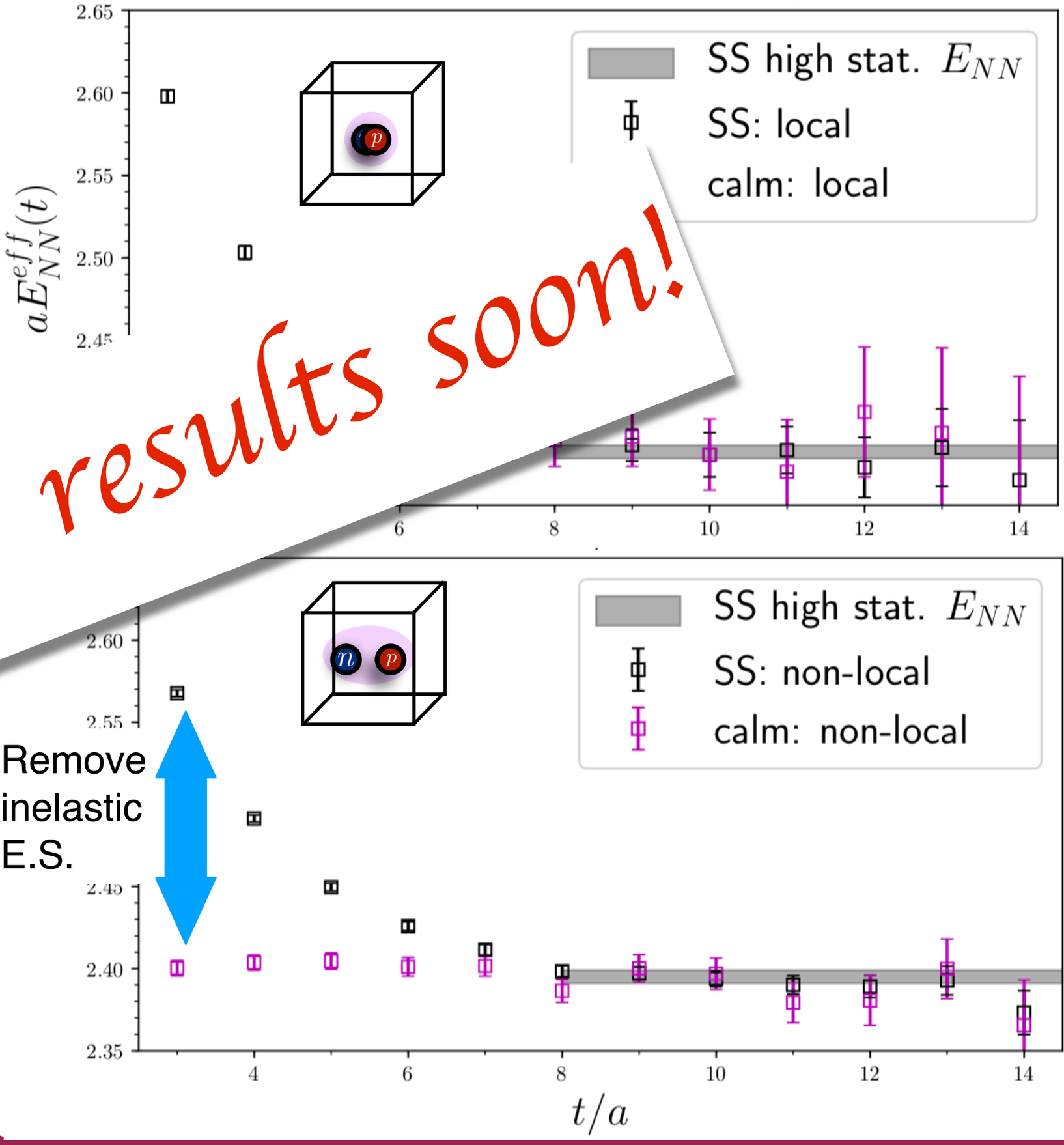
Inelastic single body

$\Delta E \sim m_\pi$


$NN : T_1^+ : {}^3S_1 : p^2_{rel} = 0 \left(\frac{2\pi}{L}\right)^2$
$aE^{eff}_{NN}(t)$
SS high stat. E_{NN}
SS: local
calm: local
p
SS high stat. E_{NN}
SS: non-local
calm: non-local
n
p
Remove inelastic E.S.
Remove elastic E.S.
t/a

CalLat (2017)

$NN : T_1^+ : {}^3S_1 : p^2_{rel} = 0 \left(\frac{2\pi}{L}\right)^2$



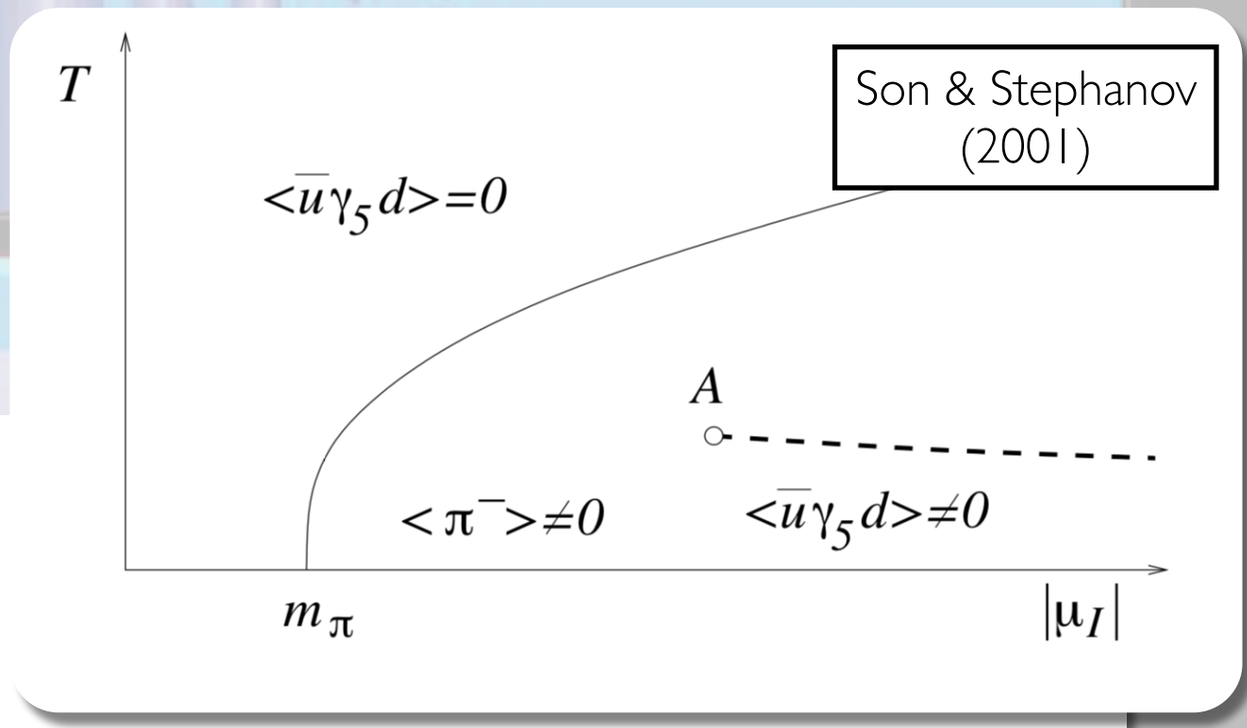
New results soon!

Remove inelastic E.S.

Remove elastic E.S.

CalLat (2017)

π/K condensation

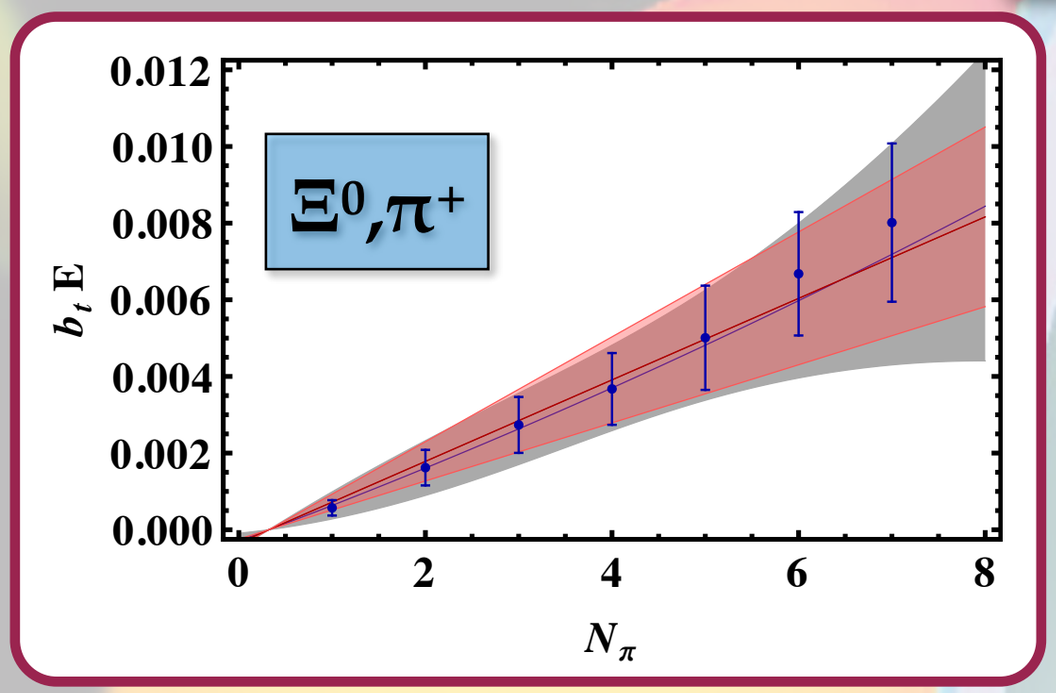


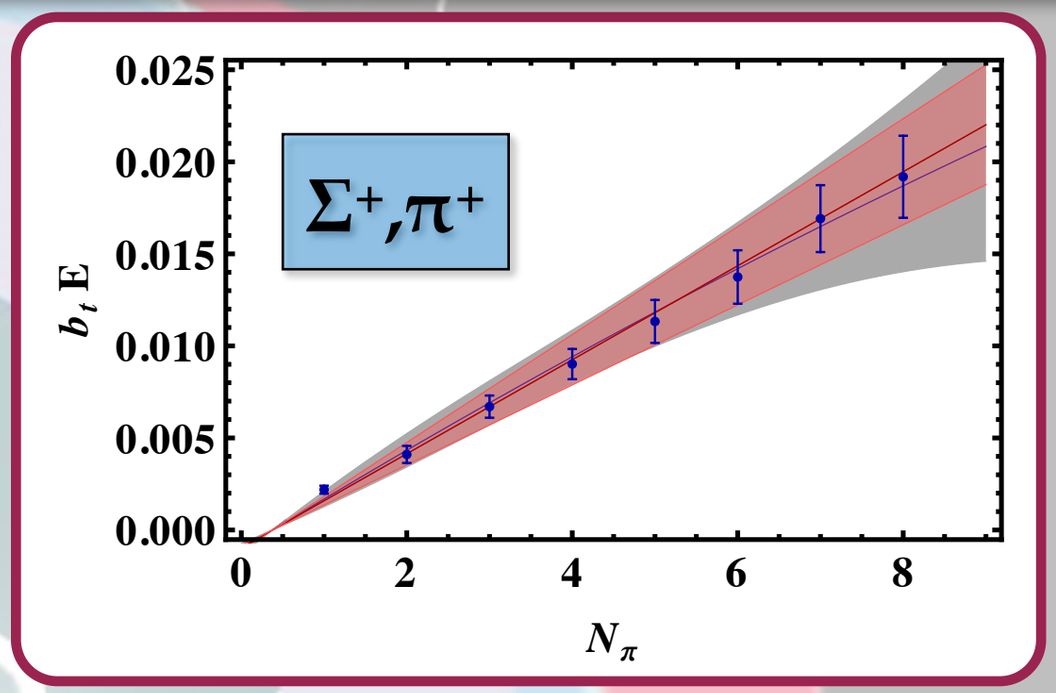
- Pions/kaons may condense at baryon densities relevant for cores of neutron stars
  - May be important for understanding QCD equation of state
- QCD at finite isospin/kaon chemical potential has a rich phase diagram
- Whether or not this occurs in nature depends on how nucleons are modified in the presence of the Bose condensate
- Many mesons are doable from lattice QCD!
  - Sometimes….the most interesting channels are very difficult
- Calculation of the nucleon mass, as well as 2- and 3-body interactions in the presence of a meson condensate
  - $m_\pi \sim 390$ MeV
  - Canonical ensemble: vary density of mesons by adding them one at a time

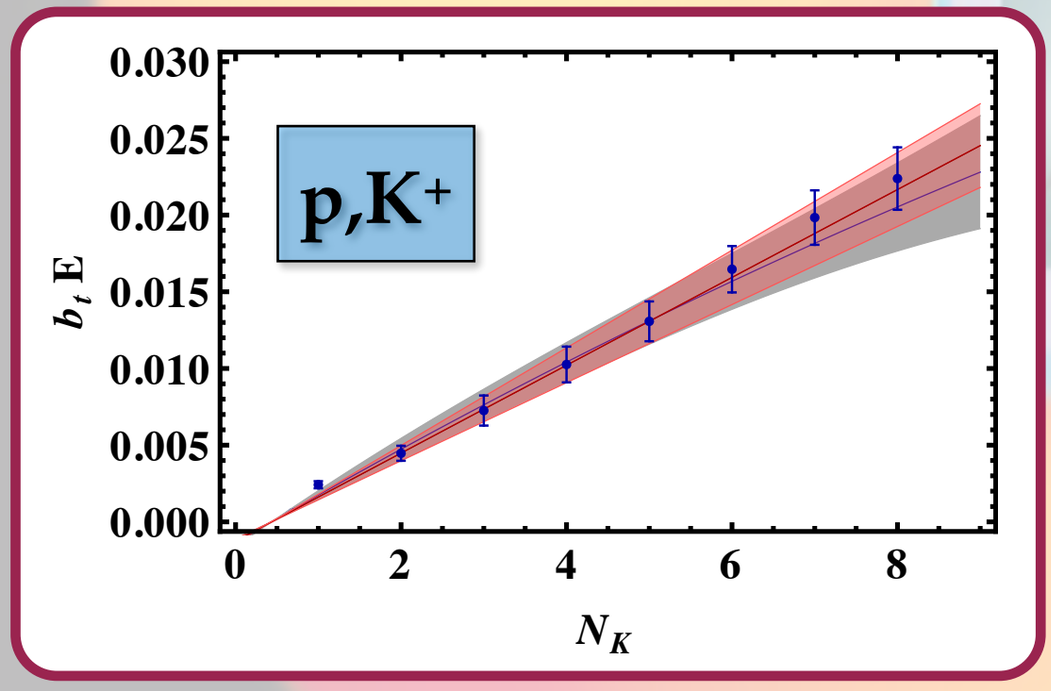
π/K condensation

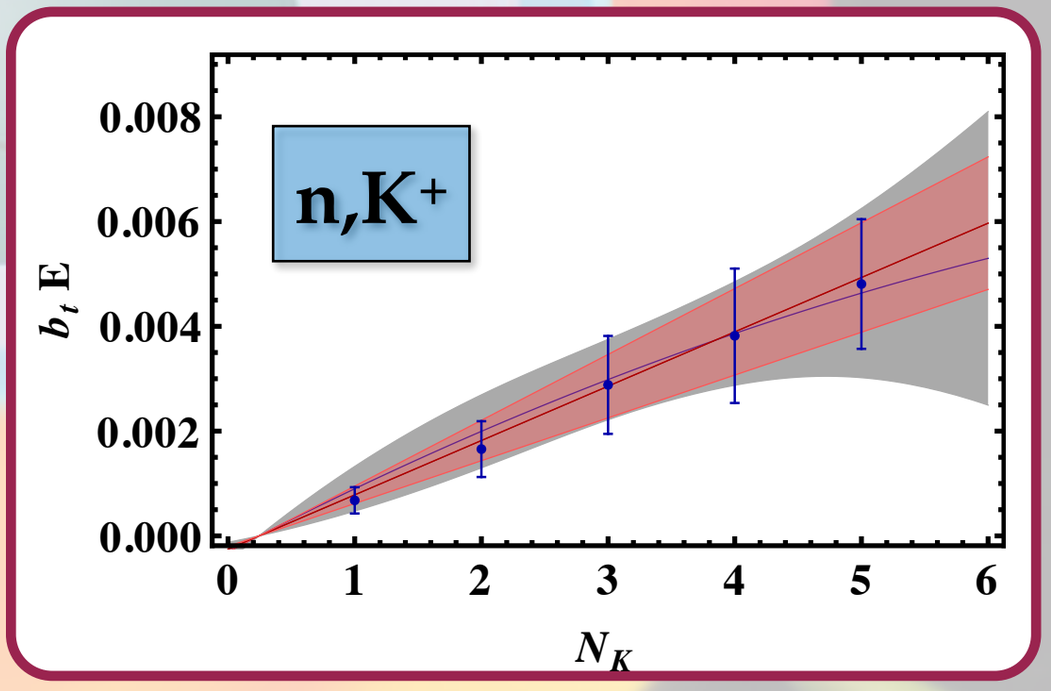
W. Detmold, AN (2013)

$$M_n(\mu_K) = M_{n^{(0)}} - \frac{\mu_K}{2}\cos\alpha + (2b_0 + b_D - b_F)m_K^2\cos\alpha + \frac{1}{4}(b_1 - b_2 + b_3 + b_4 - b_5 + b_6 + 2b_7 + 2b_8)\mu_K \sin^2\alpha$$





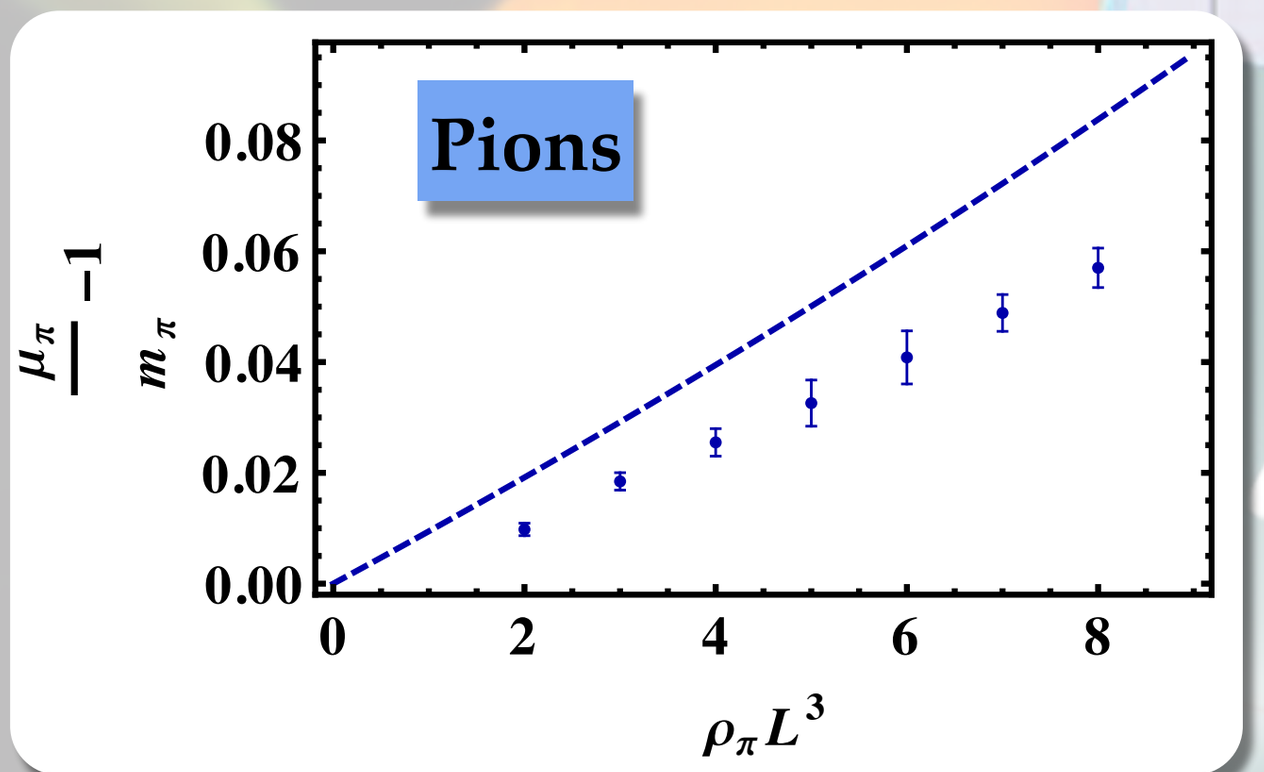


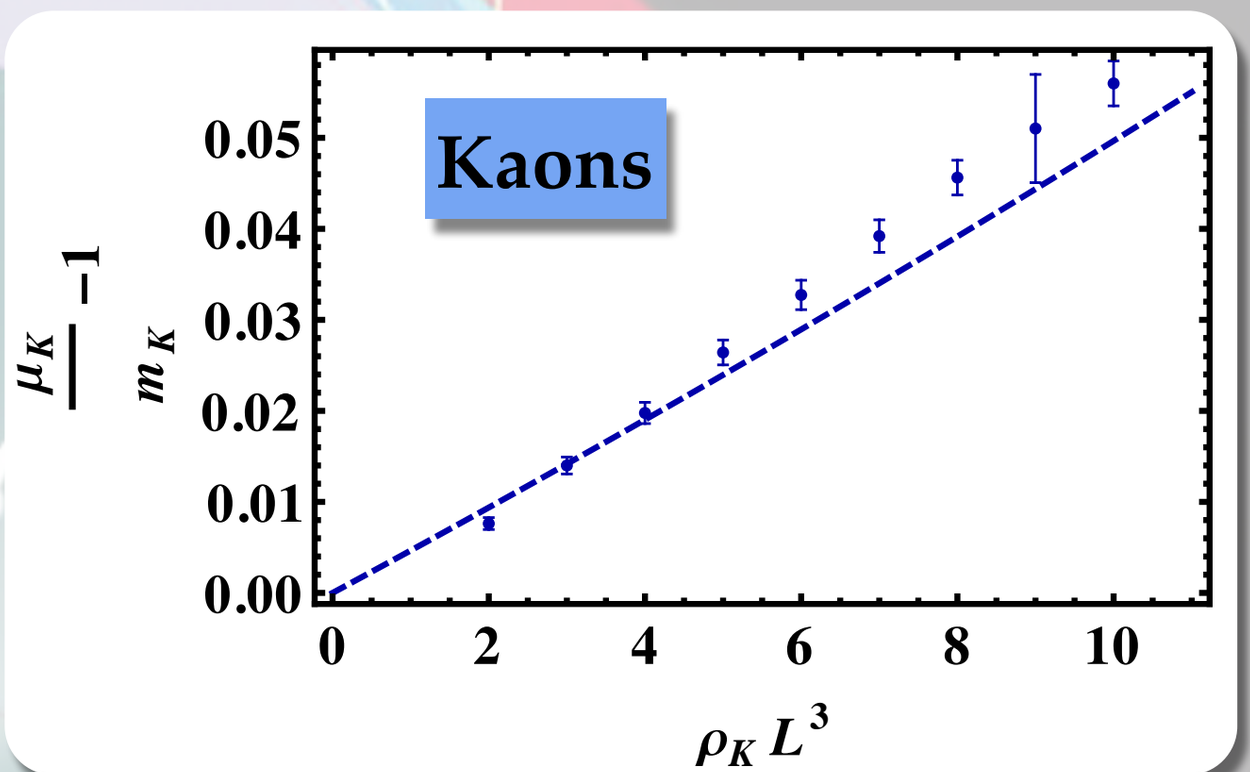


Equation of state

$$\rho_{\pi,K} = f^2_{\pi,K}\mu_{\pi,K}\left(1 - \frac{m^4_{\pi,K}}{\mu^4_{\pi,K}}\right)$$

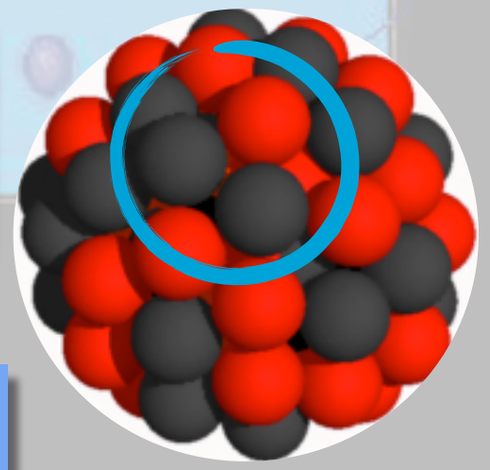
Son & Stephanov (2001)





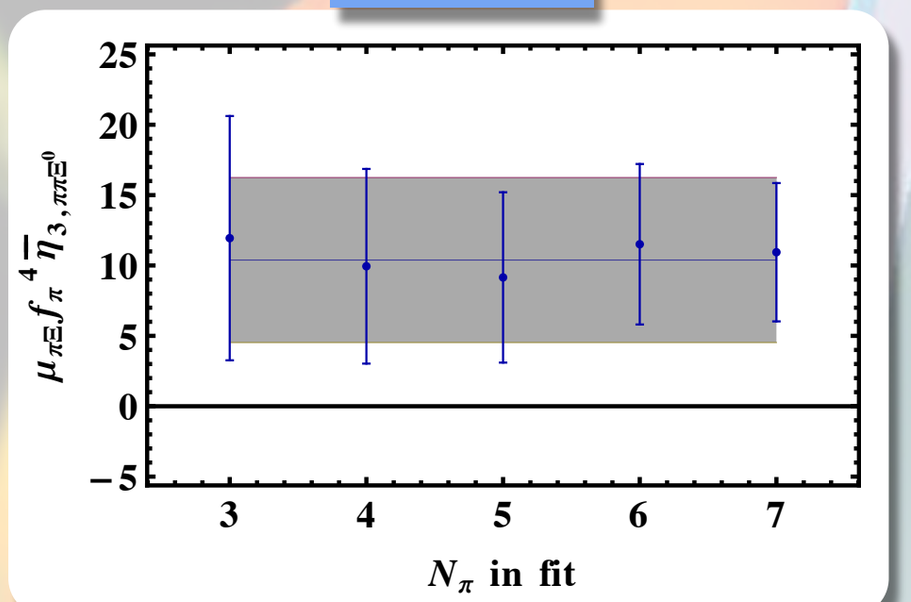
W. Detmold, AN (2013)

Meson-meson-baryon 3-body interactions

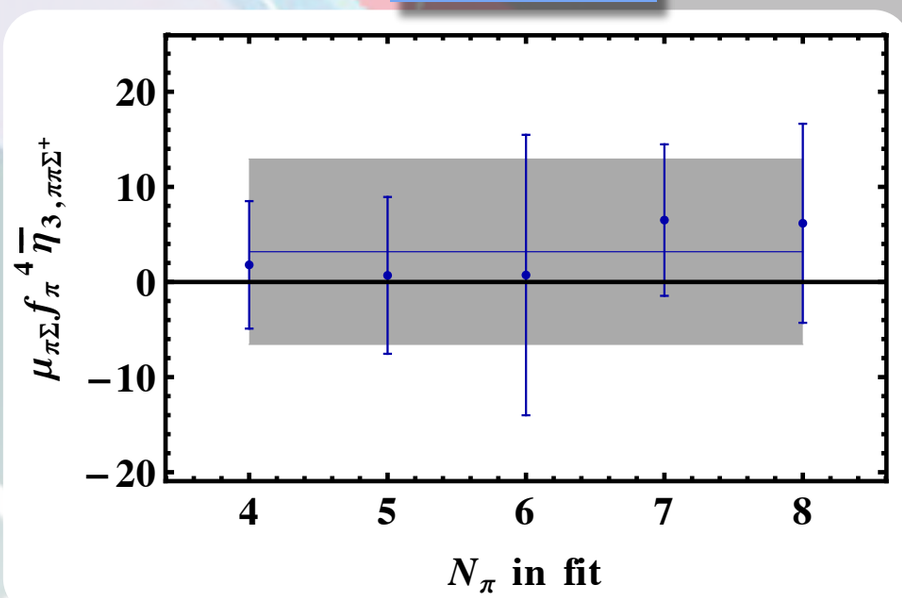


W. Detmold, AN (2013)

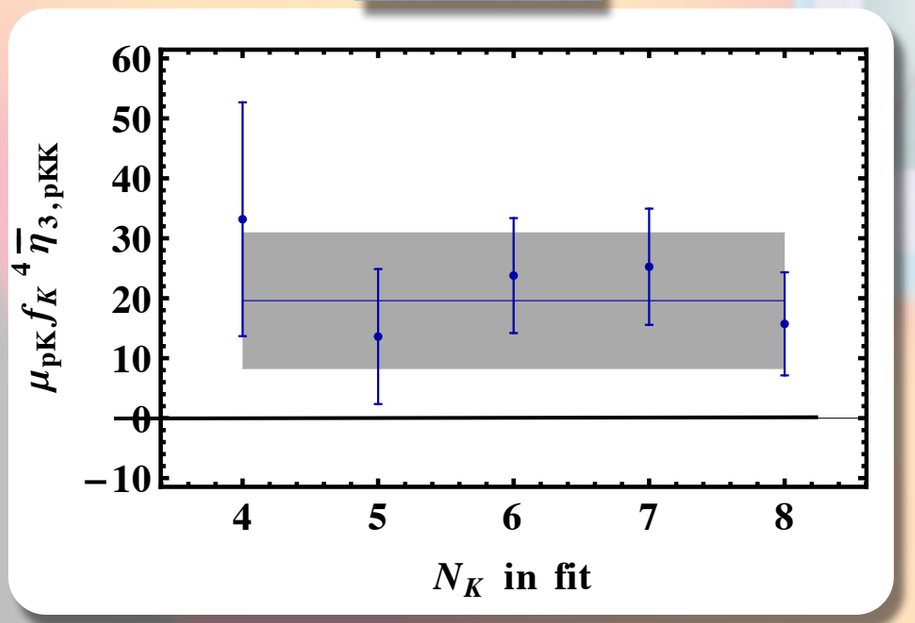
$\Xi^0\pi^+\pi^+$



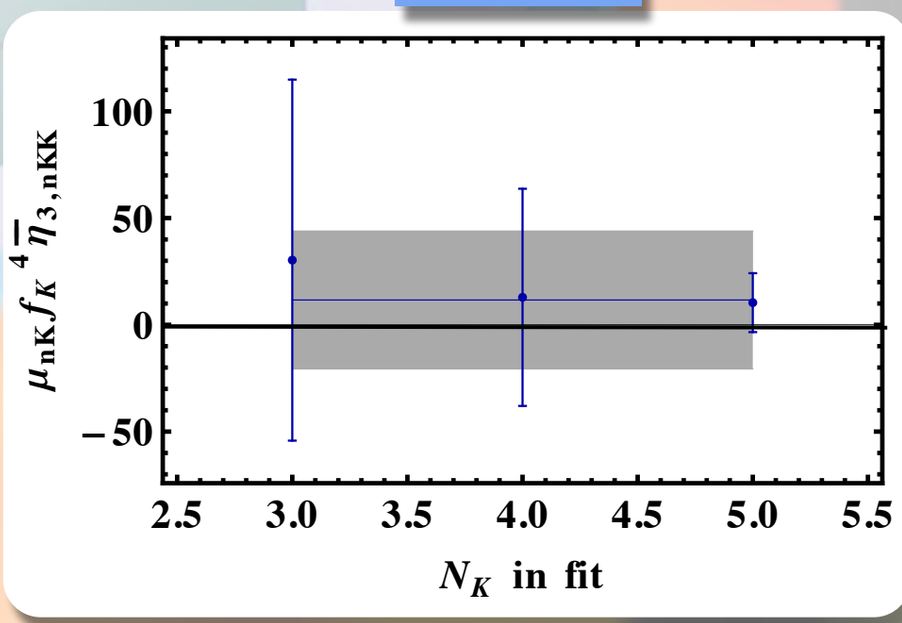
$\Sigma^+\pi^+\pi^+$



pK^+K^+



nK^+K^+

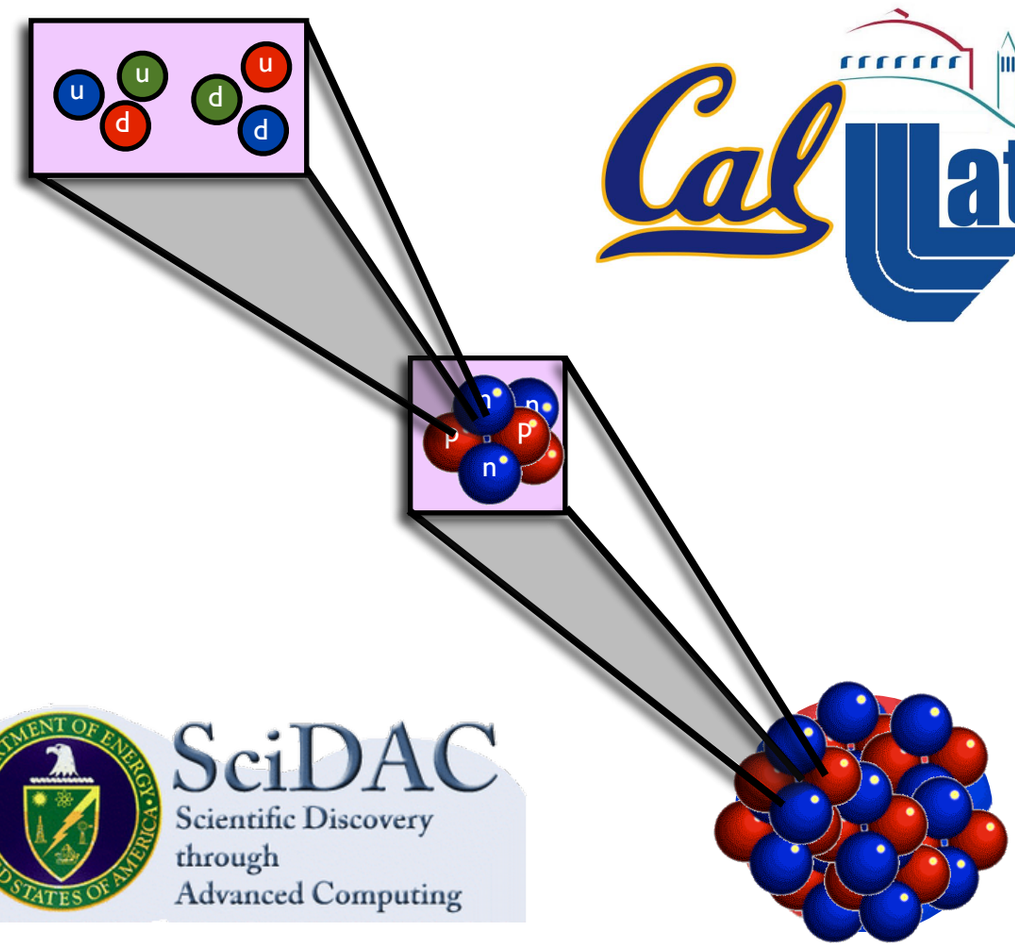


What else I would like to calculate

- n-K^- interactions / condensates
- ρ condensation?
- nucleon-hyperon scattering
- meson-nucleon-nucleon systems (Efimov states?)
- 3 nucleon interactions
    - developing EFT-based method for extracting 3-body interactions without the need for Lüscher
- ….

Jülich	Evan Berkowitz
LBL/UCB	Chia Cheng Chang
	Thorsten Kurth
	Mark Strother
	André Walker-Loud
NVIDIA	Kate Clark
UNC	Amy Nicholson
JLab	Balint Jóo
RIKEN/BNL	Enrico Rinaldi
LLNL	Arjun Gambhir
	Pavlos Vranas

red = postdoc
blue = grad student



CalLat

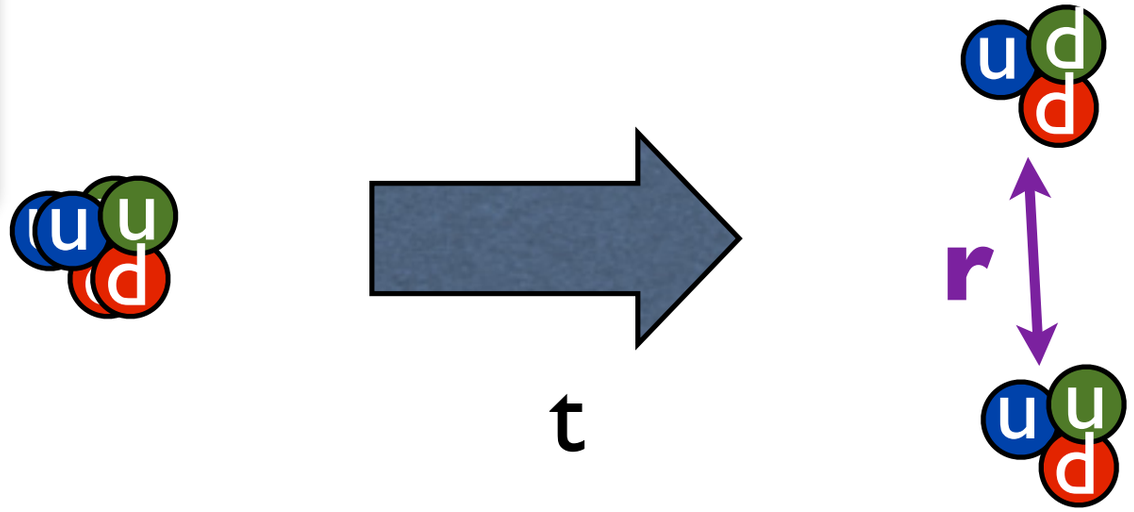
SciDAC
Scientific Discovery through Advanced Computing

NERSC

U.S. DEPARTMENT OF ENERGY
INCITE
LEADERSHIP COMPUTING

OLCF
Oak Ridge Leadership Computing Facility

Potential method



3. Use derivative expansion to determine the leading order potential:

$$U(\mathbf{r},\mathbf{r}') = V_C(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}') + \mathcal{O}(\nabla_{\mathbf{r}}^2/\Lambda^2)$$

$$V_C(\mathbf{r}) \simeq \frac{\mathbf{p}^2}{2\mu} + \lim_{t\to\infty} \frac{1}{2\mu} \frac{\nabla_{\mathbf{r}}^2 C_{NN}(\mathbf{r},t)}{C_{NN}(\mathbf{r},t)}$$

$$\left[\frac{\mathbf{p}^2}{2\mu} - H_0\right] \psi_{\mathbf{p}}(\mathbf{r}) = \int d^3r' U(\mathbf{r},\mathbf{r}')\psi_{\mathbf{p}}(\mathbf{r}') \longleftarrow \psi_0(\mathbf{r})$$

Potential method

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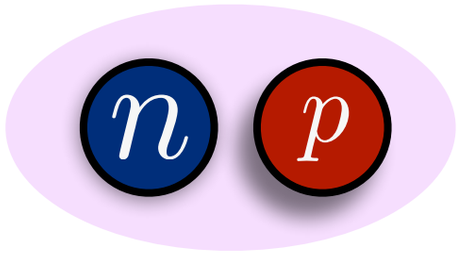
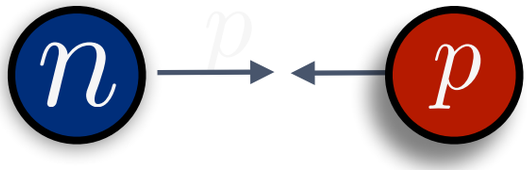
Binding energies



Phase shifts



$$\left[\frac{\mathbf{p}^2}{2\mu} - H_0\right]\psi_{\mathbf{p}}(\mathbf{r}) = \int d^3r' U(\mathbf{r},\mathbf{r}')\psi_{\mathbf{p}}(\mathbf{r}') \longleftarrow \psi_0(\mathbf{r})$$

Some comparisons between methods

Luscher

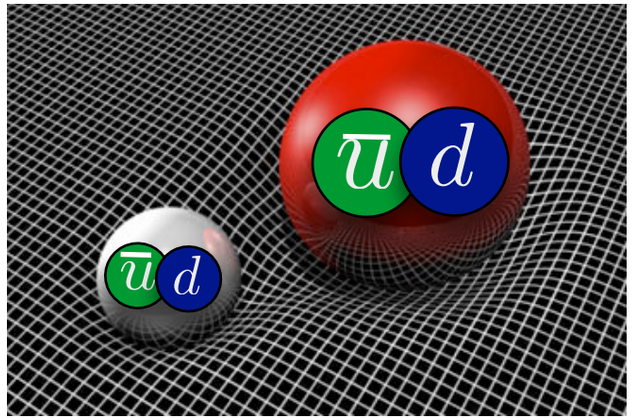
- discrete phase shifts
- need ground state saturation
- no volume extrapolation
- no uncontrolled approximations

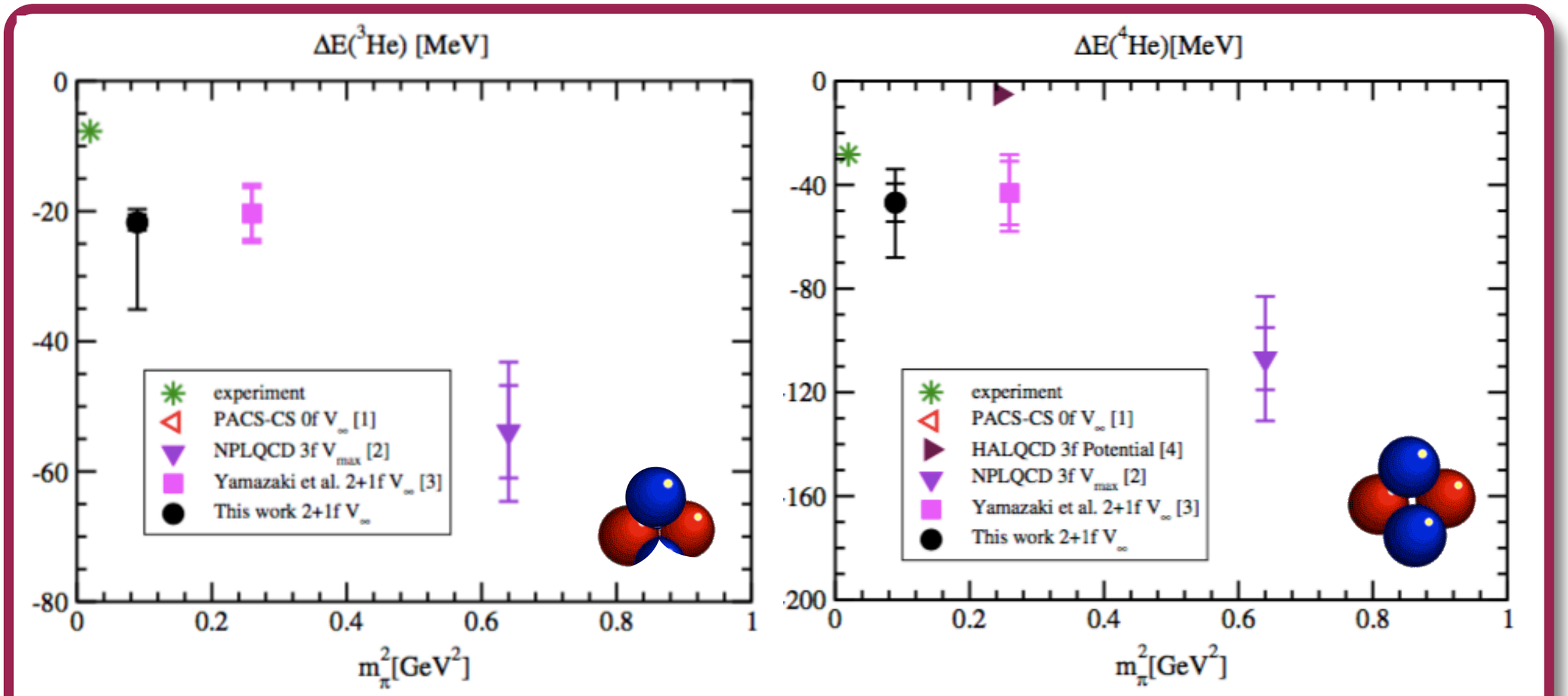
Potential

- nearly continuous phase shifts
- only need elastic state saturation
- need volume extrapolation
- cutoff in gradient expansion

$$U(\mathbf{r},\mathbf{r}') = V_C(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}') + \mathcal{O}(\nabla^2_{\mathbf{r}}/\Lambda^2)$$

Few-body systems





T. Yamazaki, arXiv:1511.09179