Unconventional color superfluidity without quarks: ultracold fermions in the presence of color-orbit and color-flip fields

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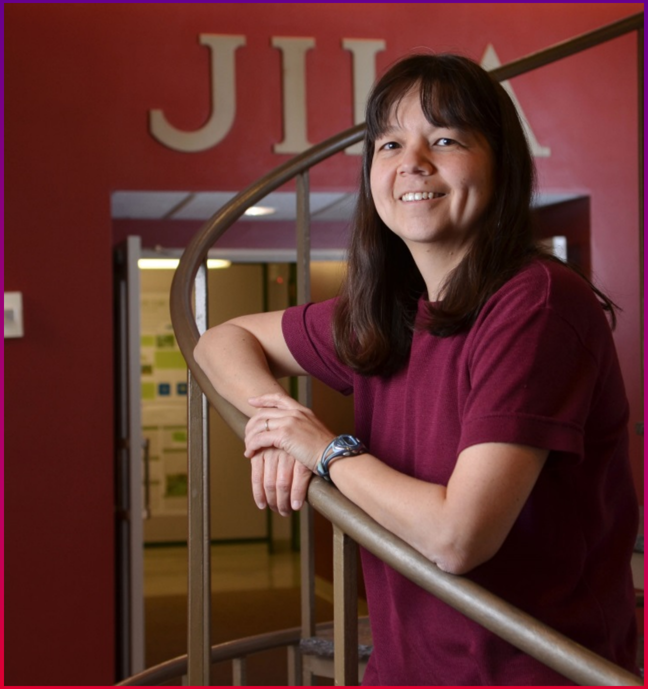
ECT*

Trento, June 20th, 2018

In Memory of Debbie Jin: a colleague and a friend.



Verified evolution from BCS to BEC superfluidity using s-wave Fano-Feshbach resonances in ultra-cold fermions.

Explored possibility of the evolution from BCS to BEC superfluidity using p-wave Fano-Feshbach resonances in ultra-cold cold fermions.

In Memory of Alex Abrikosov: an amazing physicist and mentor

Acknowledgements

Doga Kurkcuoglu

Ian Spielman

Outline of talk

1) Motivation: color superfluidity and ultracold fermions

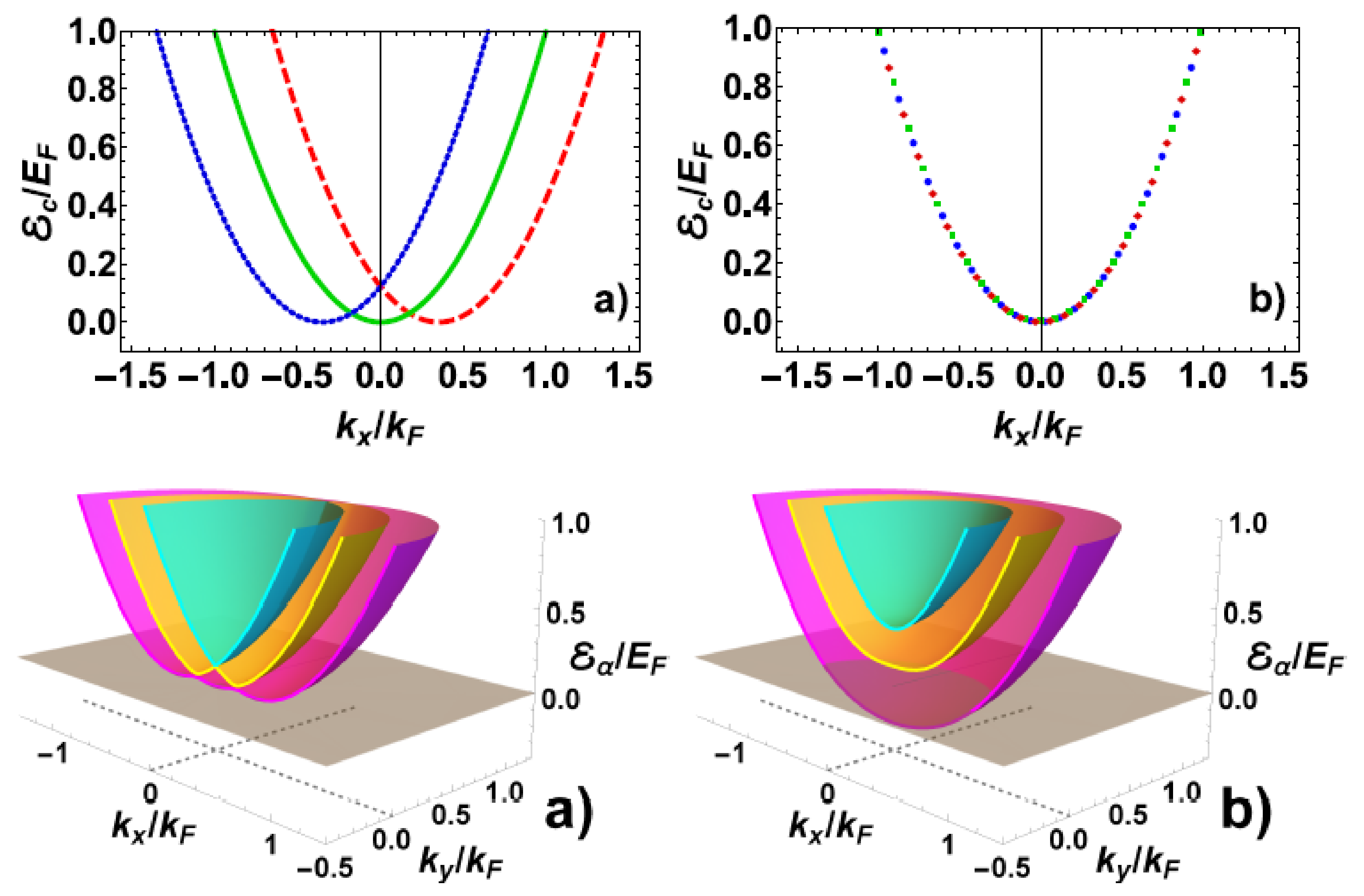
2) Introduction to spin-orbit and color-orbit coupling

3) Interacting fermions with color-orbit and color-flip fields

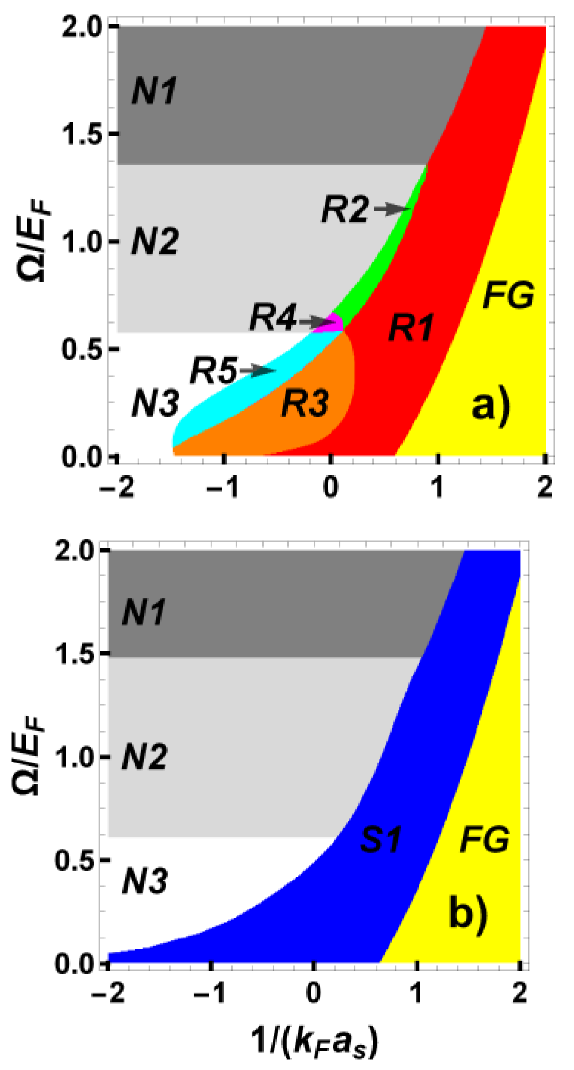
4) Spectroscopic and thermodynamic properties

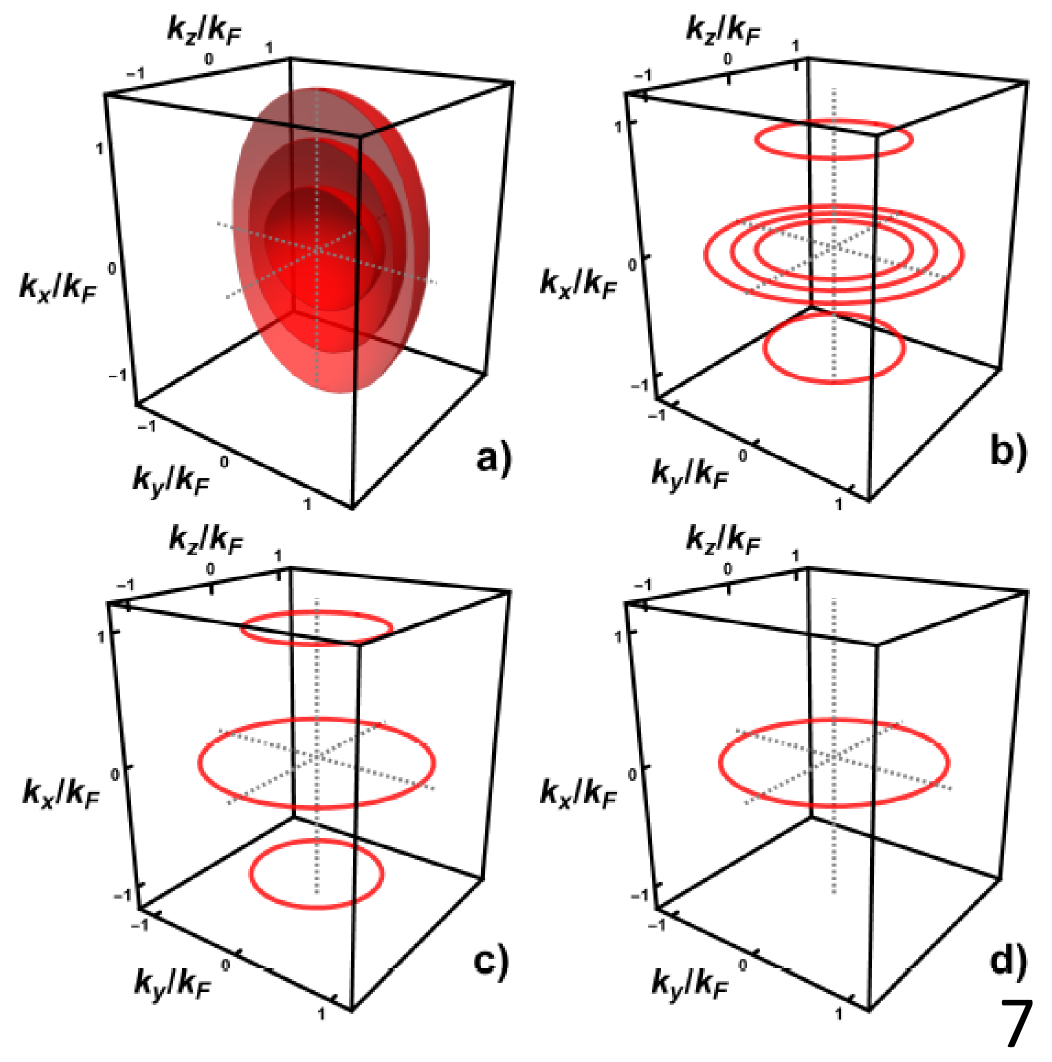
5) Conclusions

Conclusions in pictures: color-orbit and color-flip fields



Conclusions in pictures: color-orbit and color-flip fields





Conclusions in words

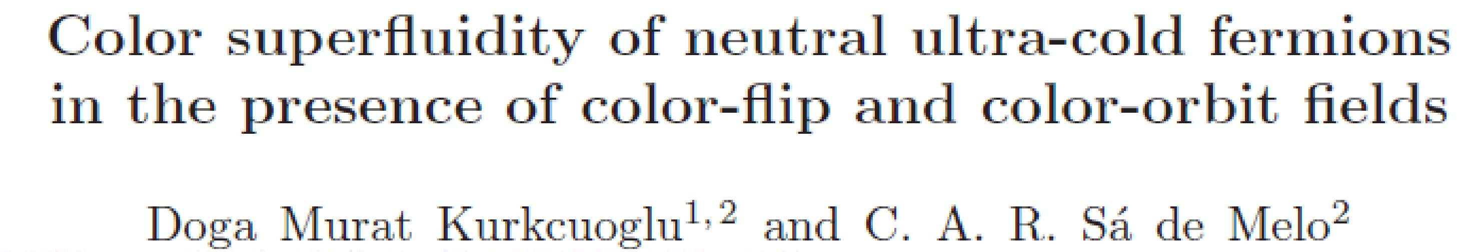
Ultracold fermions with three internal states can exhibit very unusual color superfluidity in the presence of color-orbit and color-flip fields, where SU(3) symmetry is explicitly broken.

The phase diagram of color-flip versus interaction parameter for fixed color-orbit coupling exhibits several topological phases associated with the nodal structure of the quasiparticle excitation spectrum. The phase diagram exhibits a pentacritical point where five nodal superfluid phases merge.

Even for interactions that occur only in the color s-wave channel, the order parameter for superfluidity exhibits singlet, triplet and quintuplet components due to the presence of color-orbit and color-flip fields.

These topological phases can be probed through measurements of spectroscopic properties such as excitation spectra, momentum distributions and density of states.

References for today’s talk

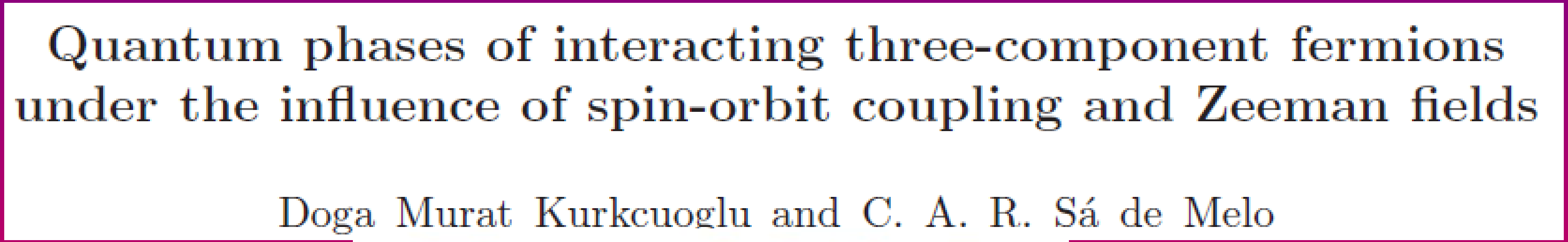


Color superfluidity of neutral ultra-cold fermions in the presence of color-flip and color-orbit fields

Doga Murat Kurkcuoglu[1,2] and C. A. R. Sá de Melo[2]

arXiv:1707.09923v1

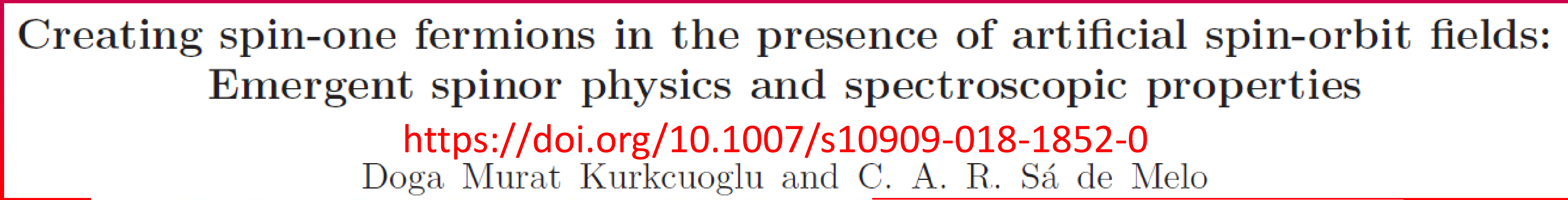
PRA 97, 023632 (2018)



Quantum phases of interacting three-component fermions under the influence of spin-orbit coupling and Zeeman fields

Doga Murat Kurkcuoglu and C. A. R. Sá de Melo

arXiv:1612.02365v1



Creating spin-one fermions in the presence of artificial spin-orbit fields: Emergent spinor physics and spectroscopic properties

https://doi.org/10.1007/s10909-018-1852-0

Doga Murat Kurkcuoglu and C. A. R. Sá de Melo

arXiv:1609.06607v1

JLTP 191,174 (2018)

Outline of Talk

1) Motivation: color superfluidity and ultracold fermions

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3) Interacting fermions with color-orbit and color-flip fields

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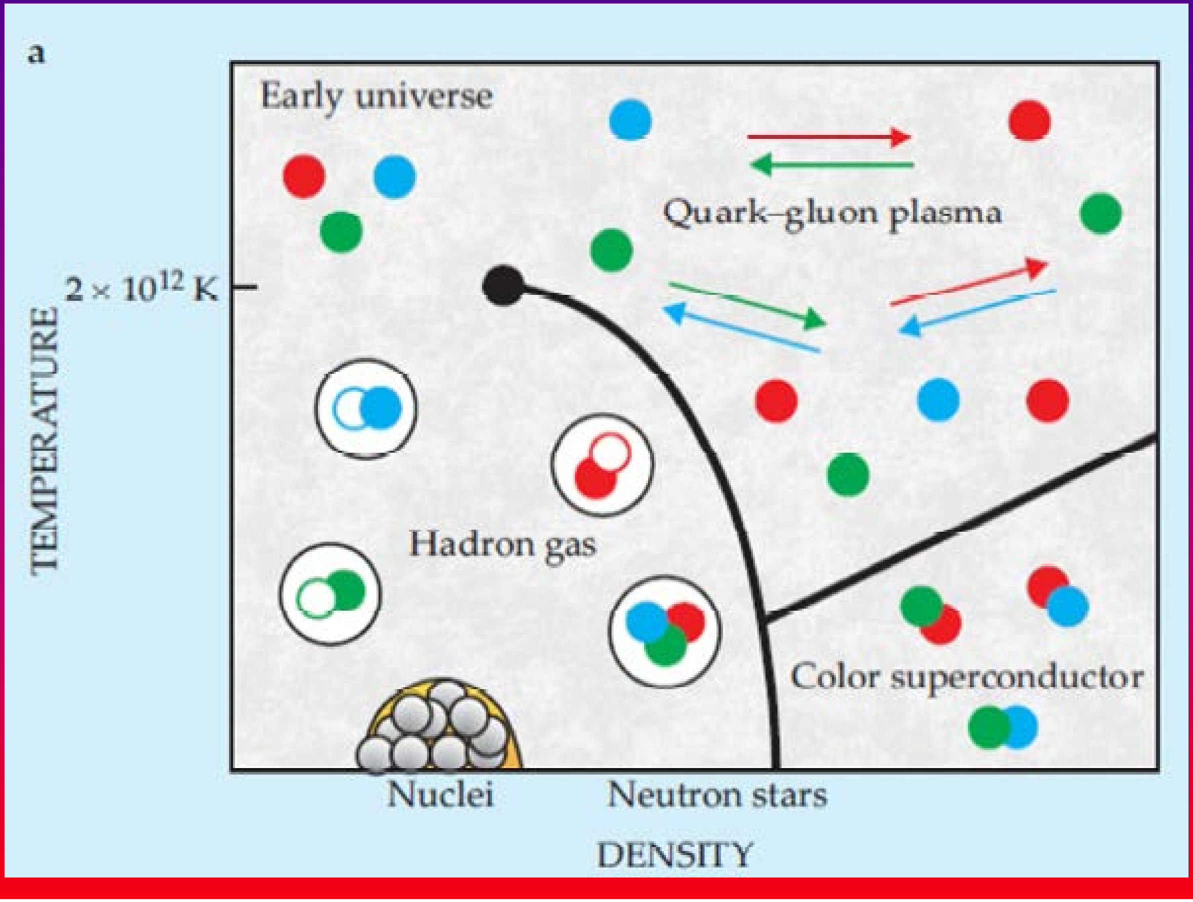
Motivation: color superfluidity and ultracold fermions

- Why studying ultracold fermions is important?

- Because it allows for the exploration of several fundamental properties of matter, such as superfluidity, which is encountered in atomic, condensed matter, nuclear and astrophysics.

Possible phase diagram for Quantum Chromodynamics (QCD)



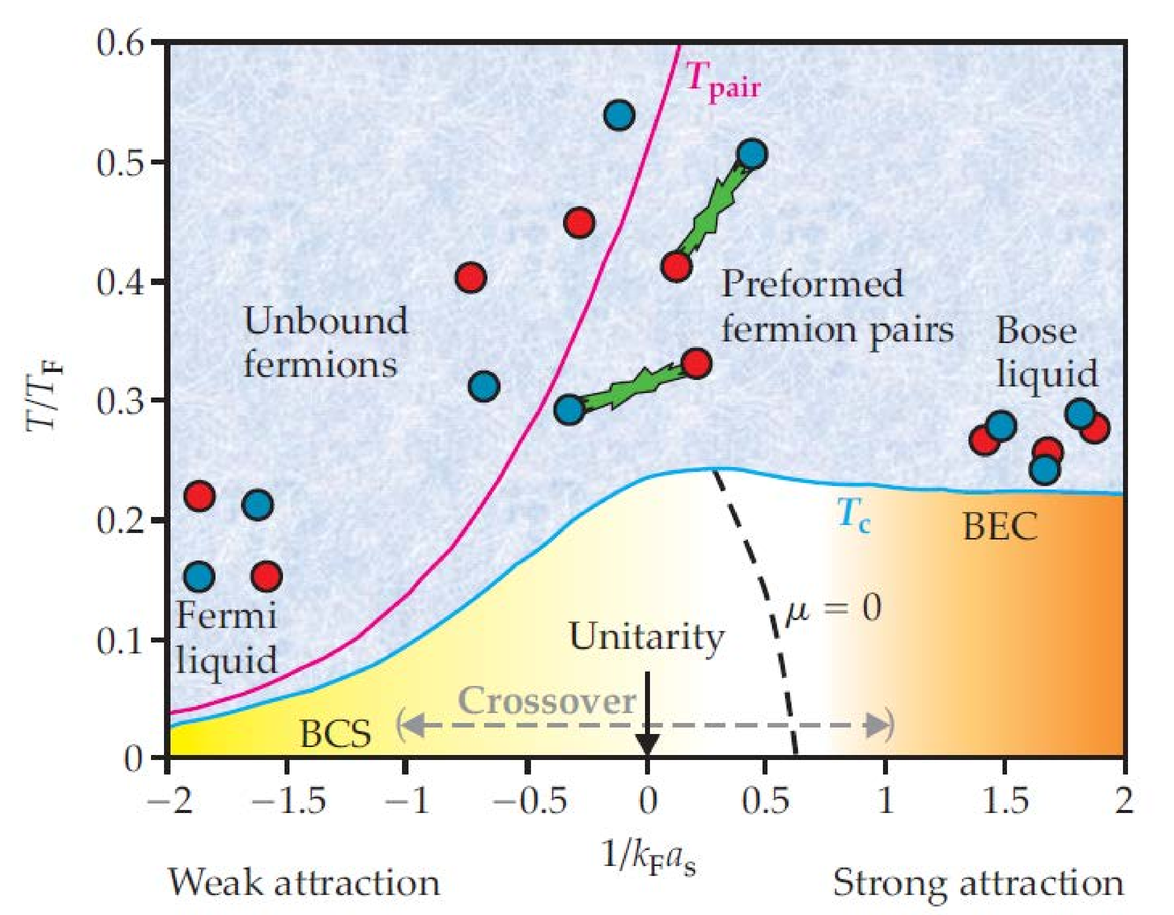
SdM – Physics Today, October (2008)

QCD and ultracold fermions (UCF) with three internal states: SU(3) case

- QCD – gluons mediate interactions
- QCD – s-wave interactions are not controllable
- QCD - quark masses are different
- QCD – quarks are charged
- QCD – quarks have three colors (internal states)

- UCF – contact interactions
- UCF – s-wave interactions are controllable
- UCF – Fermi atoms masses are the same
- UCF – Fermi atoms are neutral
- UCF – Fermi atoms can have three internal states

Ultracold fermions (UCF) with two internal states: SU(2) case



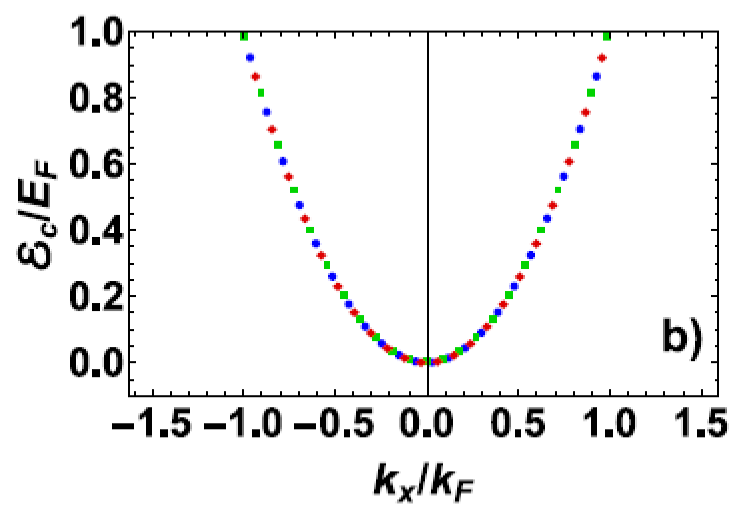
F = 5/2

^{6}Li, ^{40}K

F = 9/2

SdM - Physics Today, October (2008)

Simplest example: colored fermions and single interaction channel

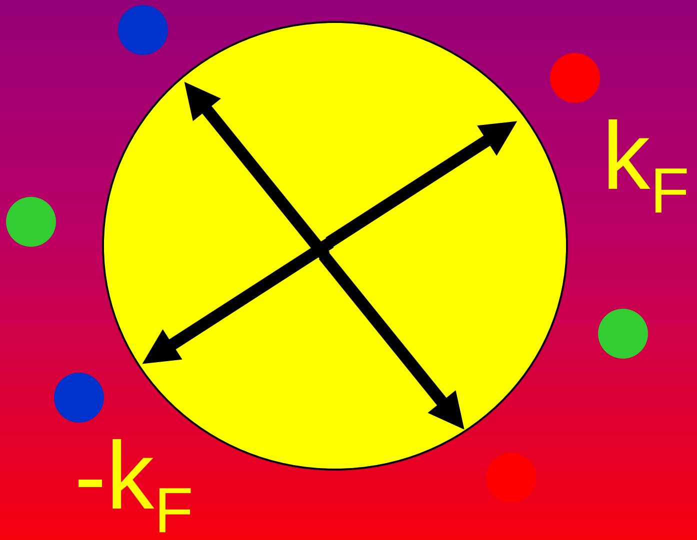


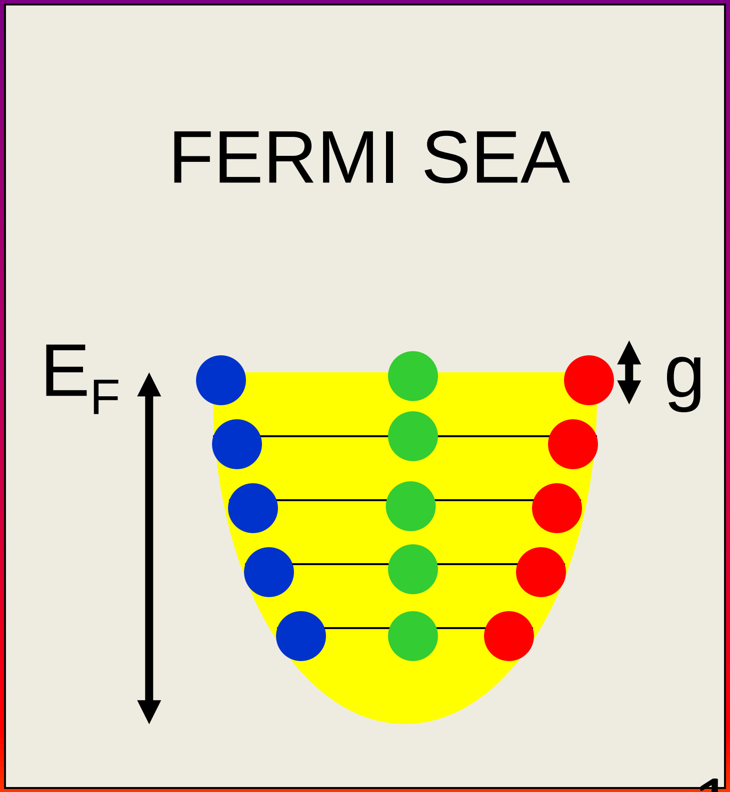
Single channel
only Red and Blue
have contact interactions

Green band is inert: non-interacting

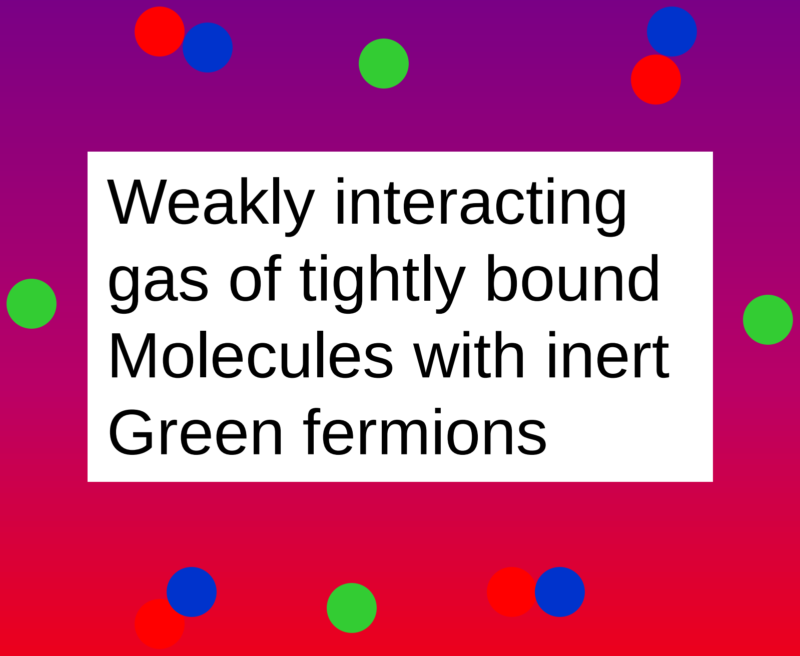
BCS Pairing ($g \ll E_F$ or $k_F a_s \rightarrow 0^-$)

$\mu = E_F > 0$



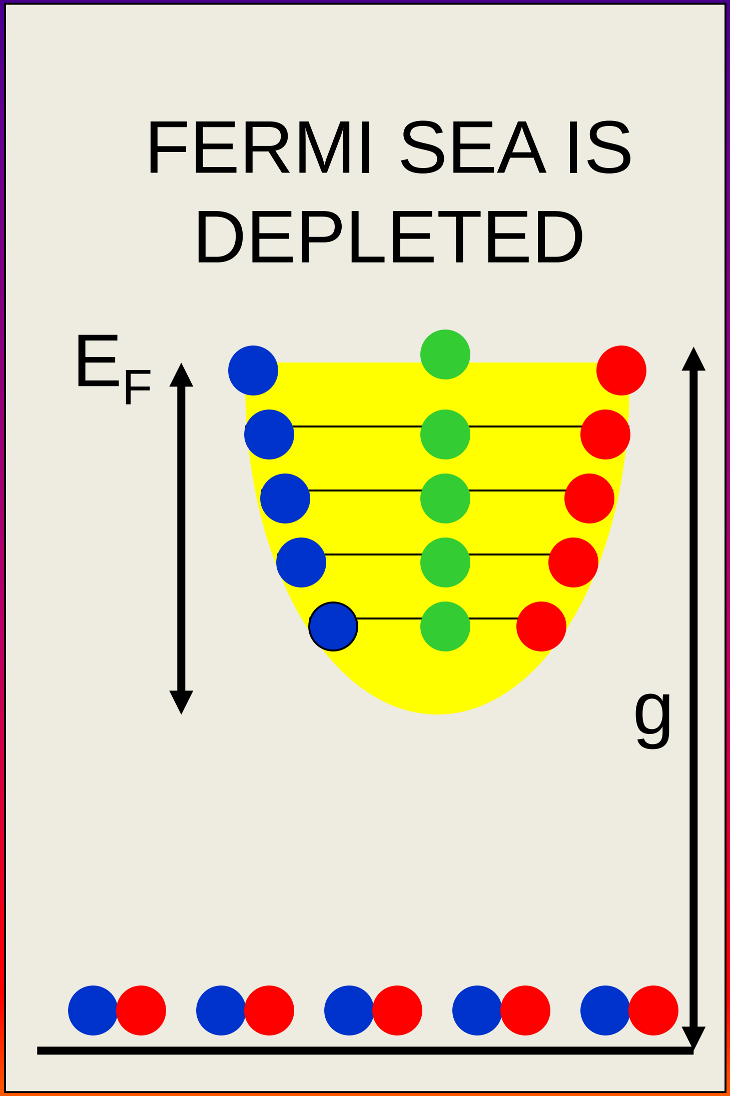


BEC Pairing ($g \gg E_F$ or $k_F a_s \rightarrow 0^+$)



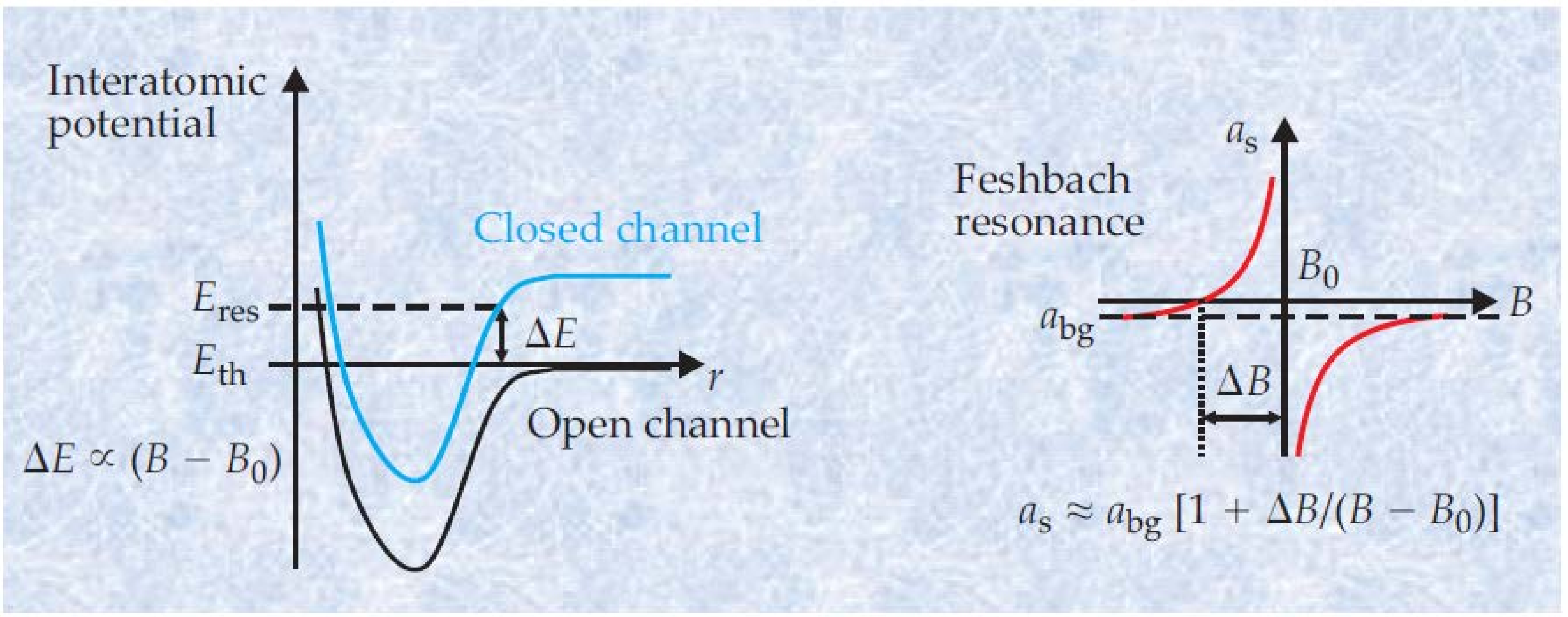
Weakly interacting gas of tightly bound Molecules with inert Green fermions

$$2\mu = -E_b < 0$$



FERMI SEA IS DEPLETED

Feshbach Resonances

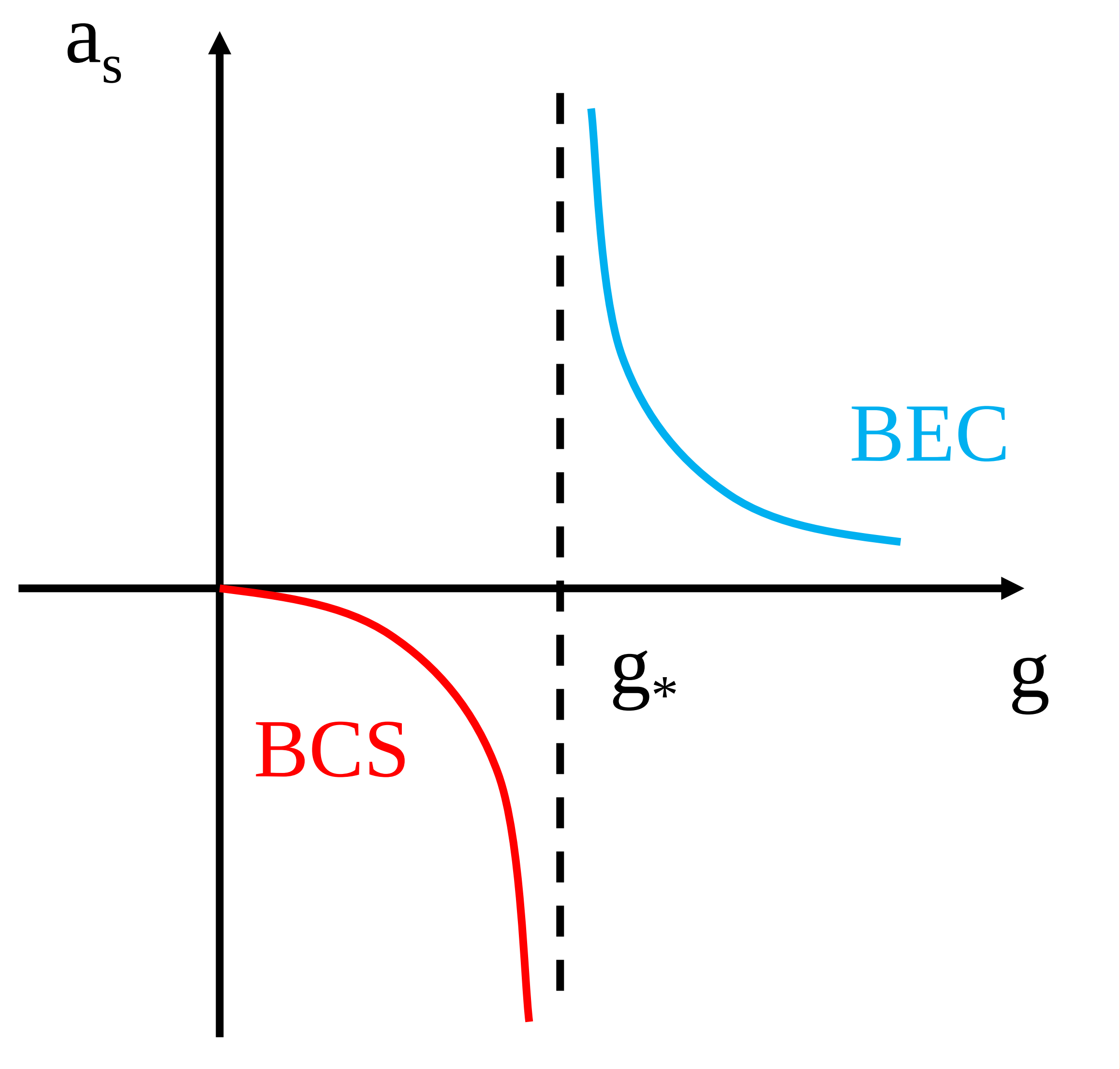


Contact interaction

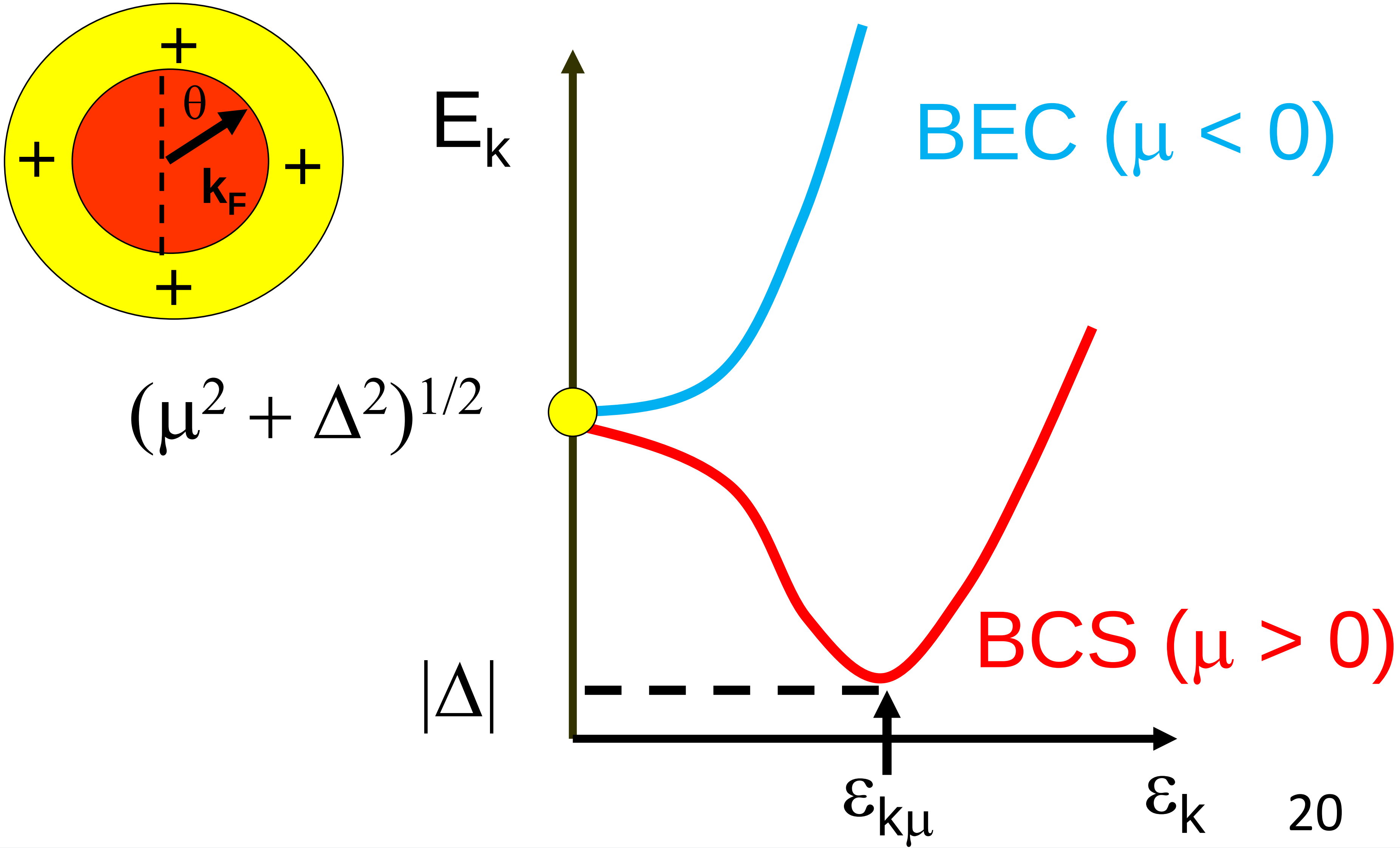
$$g \to a_s \to a_S(B)$$

B-dependent scattering length

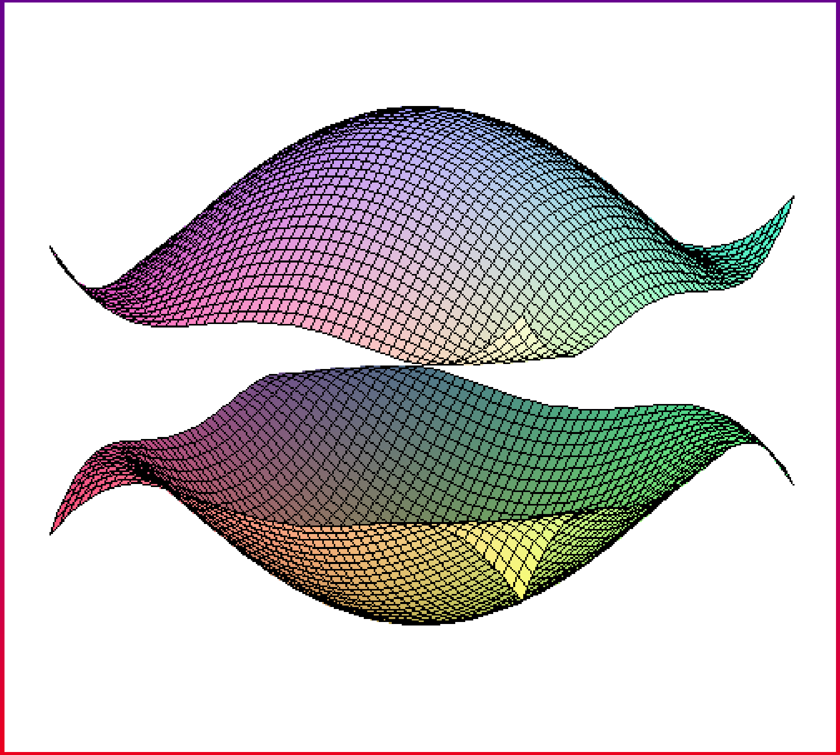
Scattering Length



$E(k) = [(\varepsilon_k - \mu)^2 + \Delta^2]^{1/2}$

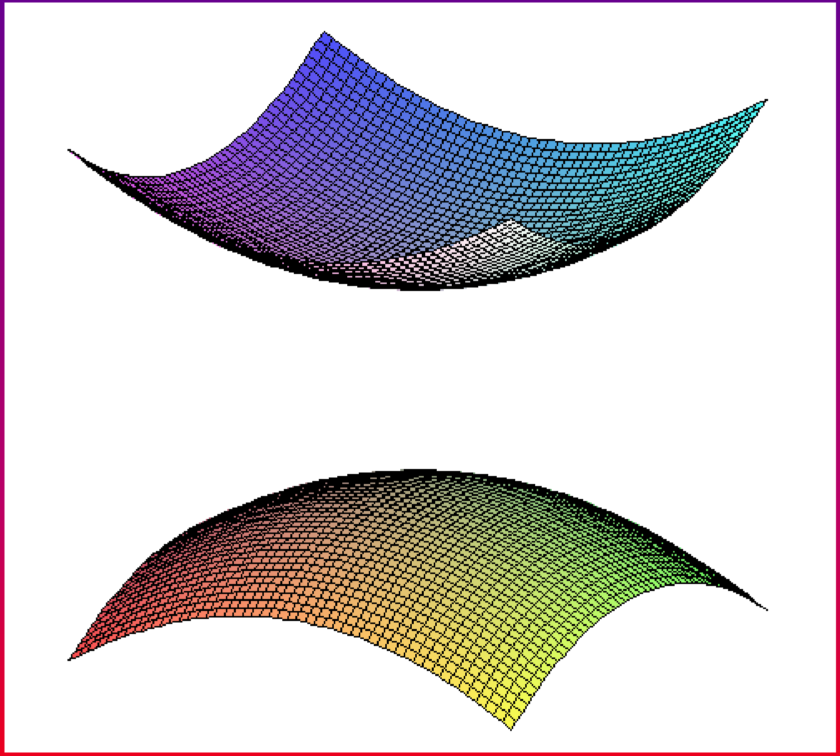


E(k) at T = 0 and k_x = 0 (S-wave)



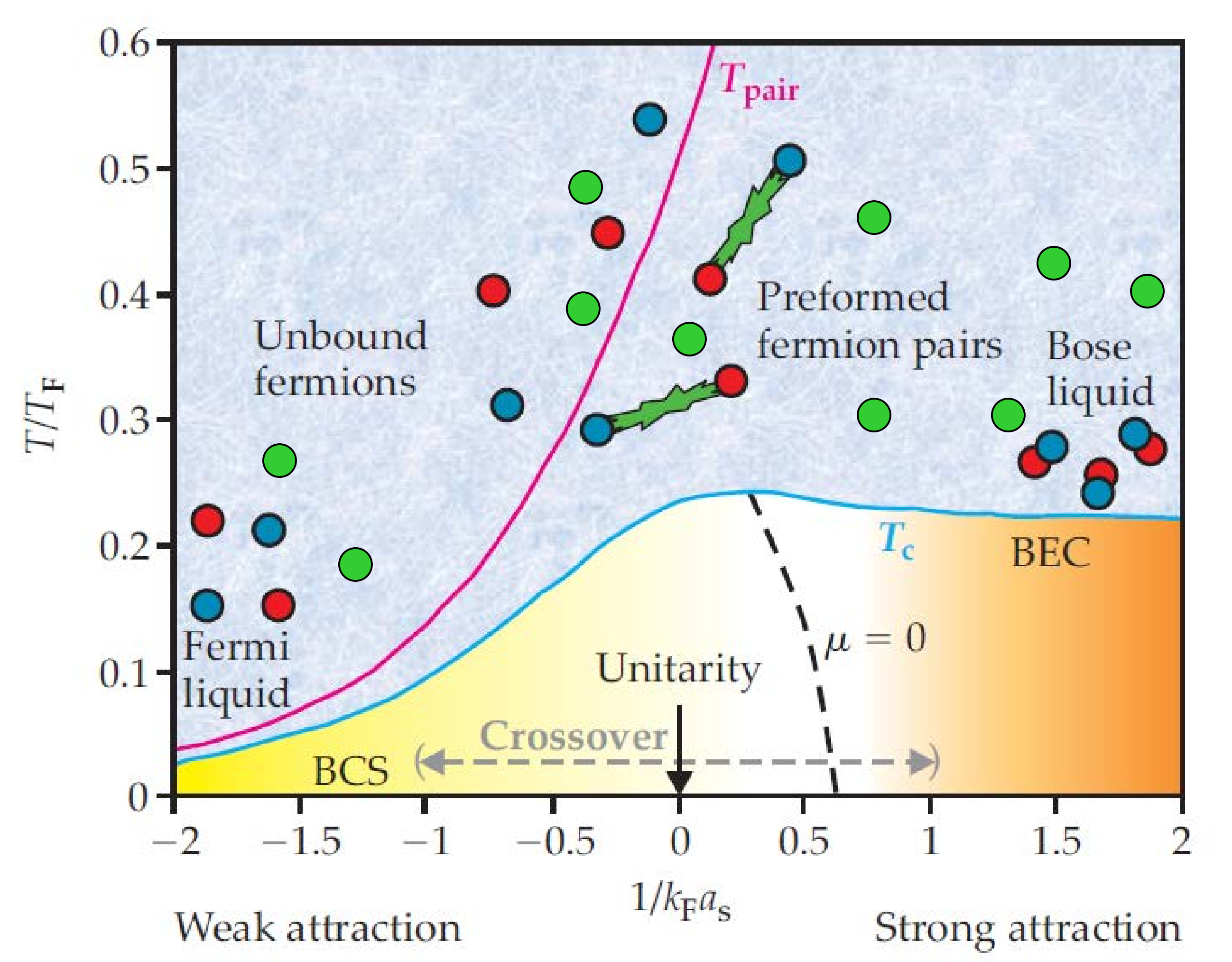
$\mu > 0$

Same Topology



$\mu < 0$

QCD-like color superfluidity nearly identical to BCS-BEC crossover of SU(2) case



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Inert fermions

Change of scale

$$n_{2c} = k_{F2c}^3 / 3\pi^2$$

$$n_{3c} = k_{F3c}^3 / 2\pi^2$$

$$n_{2c} = n_{3c}$$

$$k_{F2c} = (3/2)^{1/3} k_{F3c}$$

$$T_{F2c} = k_{F2c}^2 / 2m$$

$$T_{F3c} = k_{F3c}^2 / 2m$$

$$T_{F2c} = (3/2)^{2/3} T_{F3c}$$

SdM, Physics Today (2008) + inert Green fermions

Outline of Talk

1) Motivation: color superfluidity and ultracold fermions

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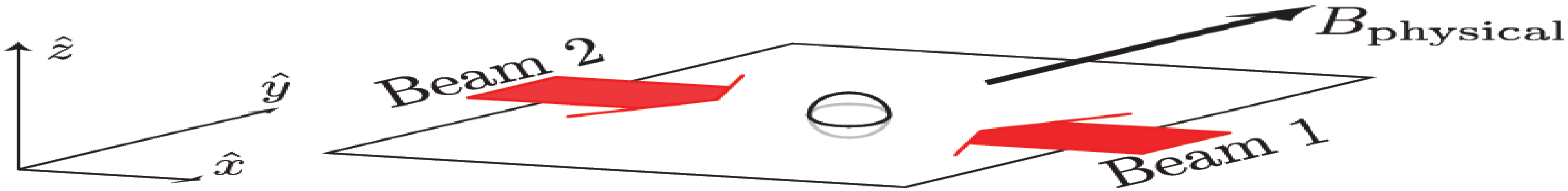
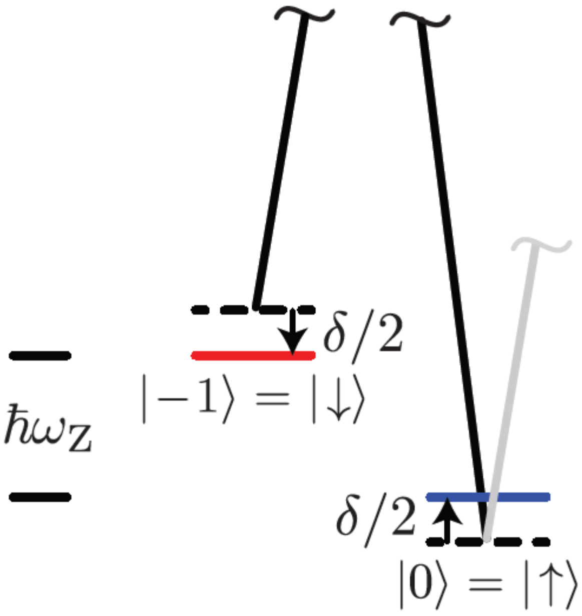
Raman process and spin-orbit coupling

Geometry



$$\begin{pmatrix} \dfrac{(\mathbf{k}-\mathbf{k}_R)^2}{2m}+\dfrac{\delta}{2} & \dfrac{\Omega}{2} \\ \dfrac{\Omega}{2} & \dfrac{(\mathbf{k}+\mathbf{k}_R)^2}{2m}-\dfrac{\delta}{2} \end{pmatrix}$$



SU(2) rotation to new spin basis:

$\sigma_x \rightarrow \sigma_z\,;\ \sigma_z \rightarrow \sigma_y\,;\ \sigma_y \rightarrow \sigma_x$

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doi:10.1038/nature09887

Spin–orbit-coupled Bose–Einstein condensates

Y.-J. Lin[1], K. Jiménez-García[1,2] & I. B. Spielman[1]

^{87}Rb

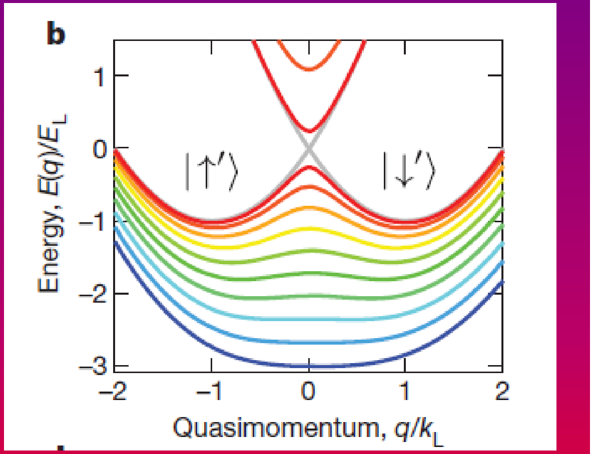
$$\begin{pmatrix} \frac{\mathbf{k}^2+k_R^2}{2m}+\frac{\Omega}{2} & -i\left(\frac{\delta}{2}-\frac{k_R}{m}k_x\right) \\ i\left(\frac{\delta}{2}-\frac{k_R}{m}k_x\right) & \frac{\mathbf{k}^2+k_R^2}{2m}-\frac{\Omega}{2} \end{pmatrix}$$

spin-orbit

detuning

Raman coupling

Experimental phase diagram for ^{87}Rb: bosons with two internal states (spin-1/2)



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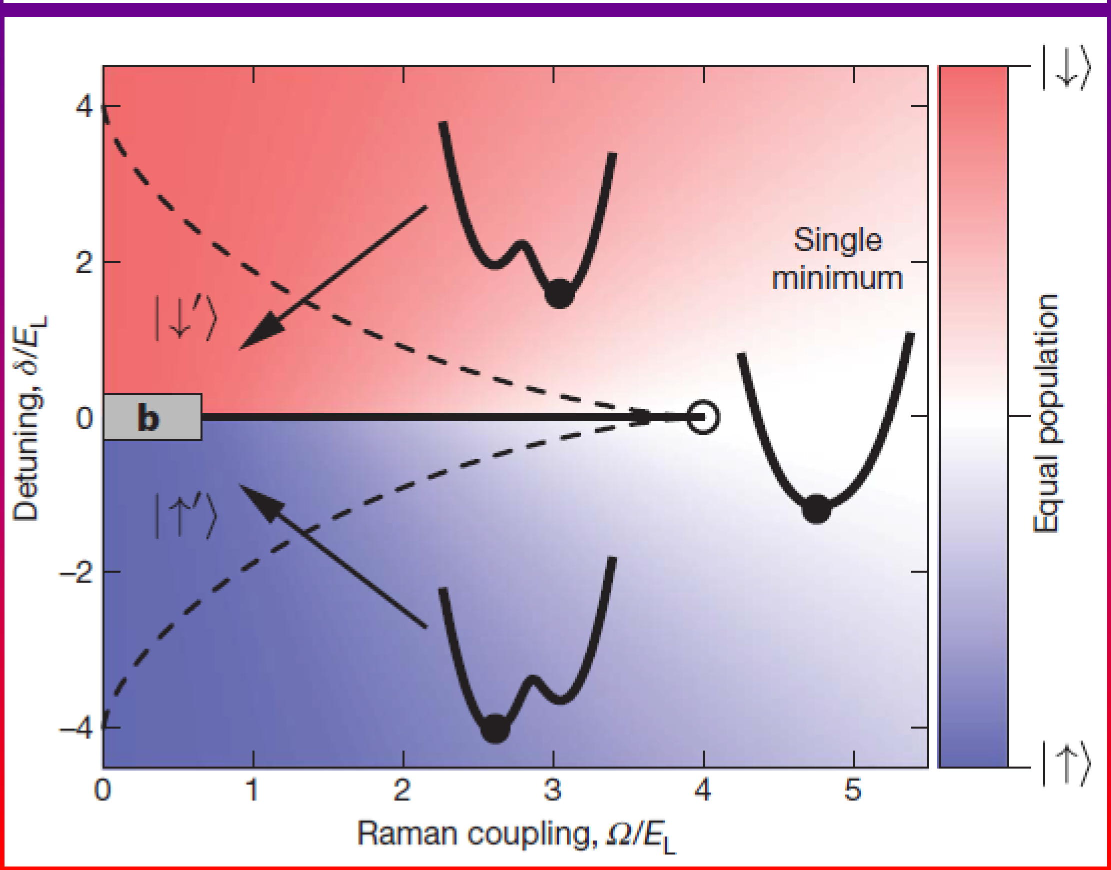


Y.-J. Lin[1], K. Jiménez-García[1,2] & I. B. Spielman[1]

Case with three internal states: color-orbit and color flip fields

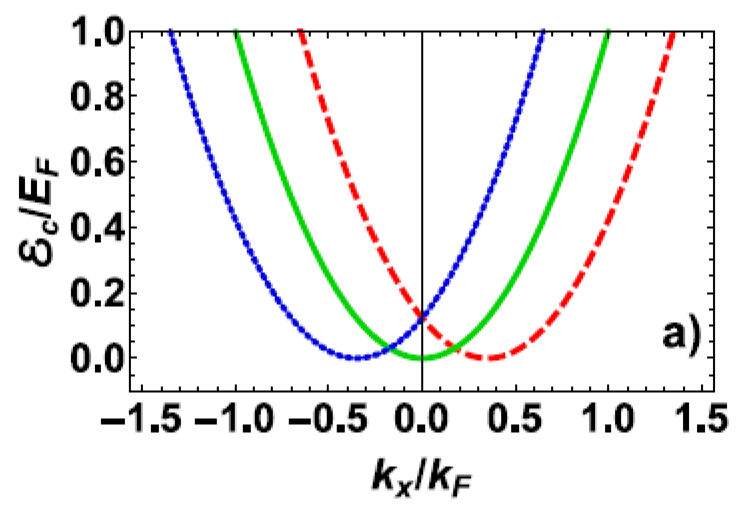
$$\mathbf{H}_0(\mathbf{k}) = \begin{pmatrix} \varepsilon_R(\mathbf{k}) & \Omega_{RG} & \cancel{\Omega_{RB}} \\ \Omega^*_{RG} & \varepsilon_G(\mathbf{k}) & \Omega_{GB} \\ \cancel{\Omega^*_{RB}} & \Omega^*_{GB} & \varepsilon_B(\mathbf{k}) \end{pmatrix}$$

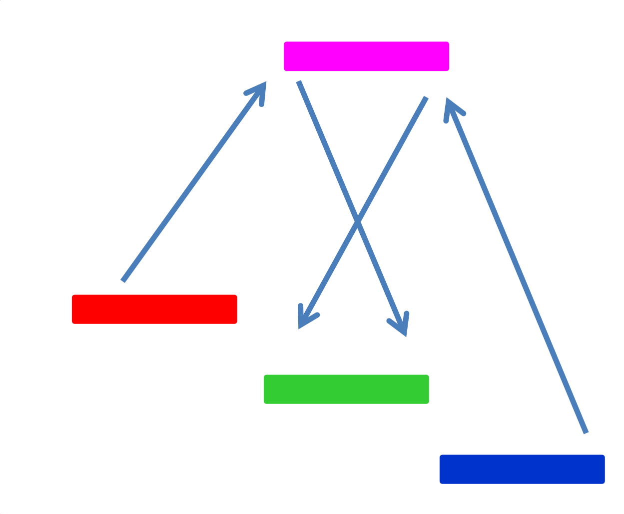
$$c = \{R, G, B\}$$

^{6}Li, ^{40}K, ^{173}Yb

$$\varepsilon_c(\mathbf{k}) = (\mathbf{k} - \mathbf{k}_c)^2/(2m) + \eta_c$$

Raman Process





Case with three internal states color-orbit and color-flip fields

$$\mathbf{H}_{\mathrm{IP}}(\mathbf{k}) = \begin{pmatrix} \varepsilon_{\mathrm{R}}(\mathbf{k}) & -h_x(\mathbf{k})/\sqrt{2} & 0 \\ -h_x(\mathbf{k})/\sqrt{2} & \varepsilon_{\mathrm{G}}(\mathbf{k}) & -h_x(\mathbf{k})/\sqrt{2} \\ 0 & -h_x(\mathbf{k})/\sqrt{2} & \varepsilon_{\mathrm{B}}(\mathbf{k}) \end{pmatrix}$$

$$\varepsilon_{\mathrm{R}}(\mathbf{k}) = \varepsilon(\mathbf{k}) - h_z(\mathbf{k}) + b_z$$

$$\varepsilon_{\mathrm{G}}(\mathbf{k}) = \varepsilon(\mathbf{k})$$

$$\varepsilon_{\mathrm{B}}(\mathbf{k}) = \varepsilon(\mathbf{k}) + h_z(\mathbf{k}) + b_z$$

Kinetic energies of
Red, Green and Blue fermions

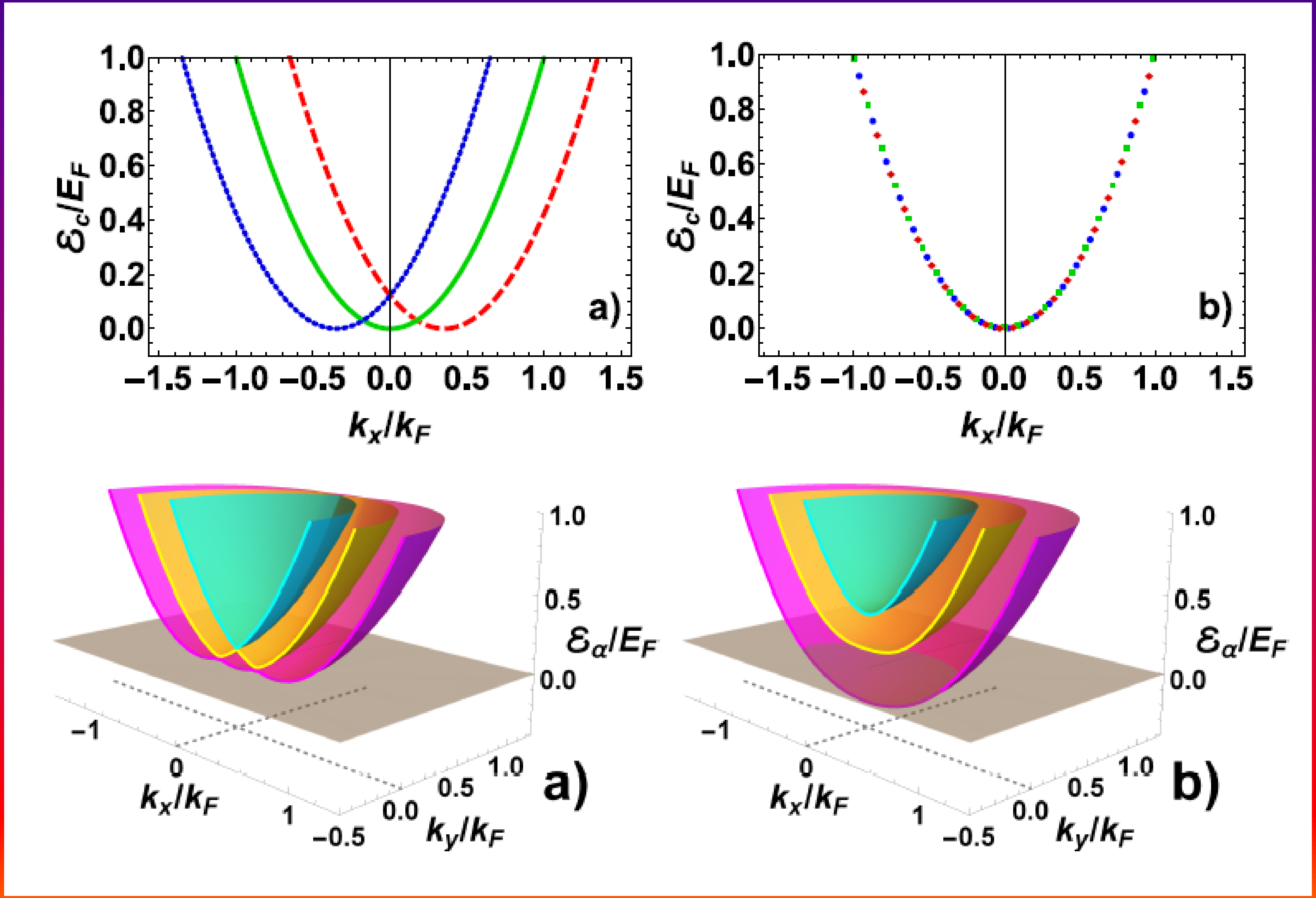
$$h_z(\mathbf{k}) = 2k_T k_x/(2m) + \delta$$

Color-orbit and
Color-Zeeman fields

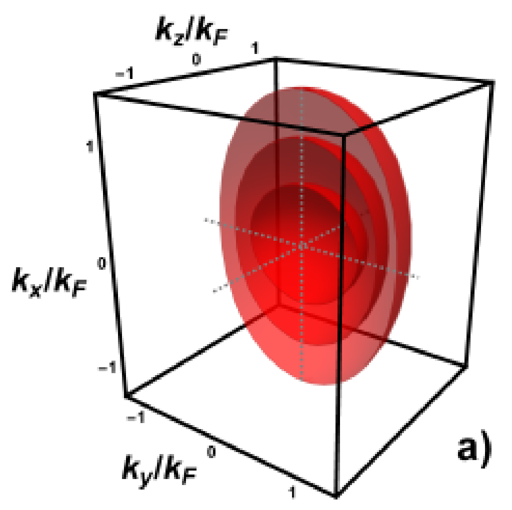
$$h_x(\mathbf{k}) = -\sqrt{2}\Omega$$

Color-flip field

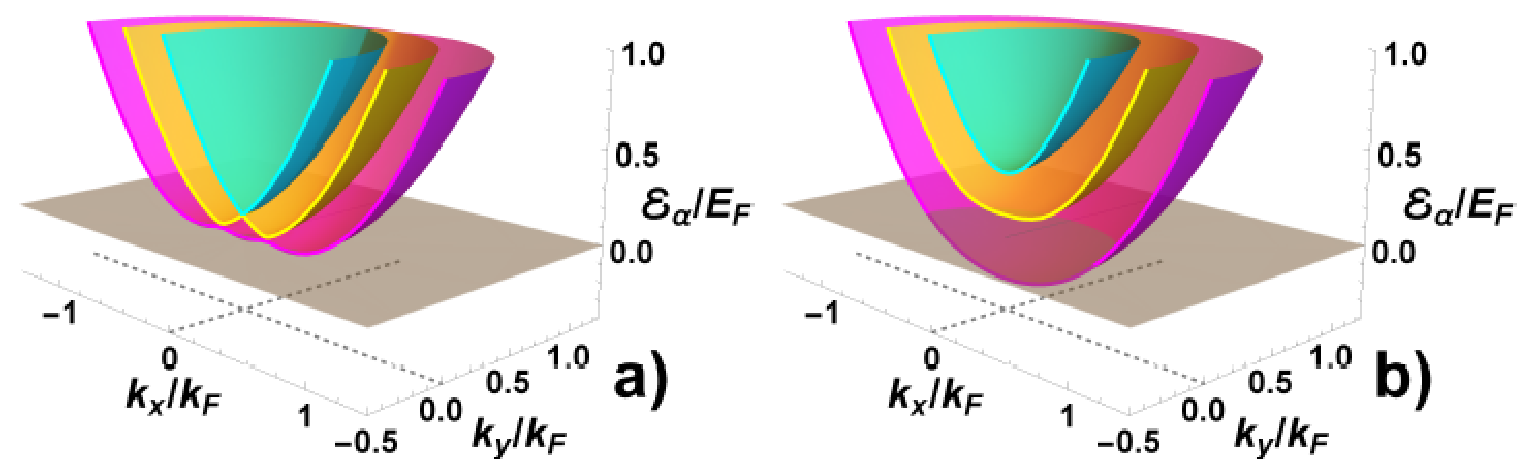
Case with three internal states: color-orbit and color flip fields



Colored fermions are a correlated three band system



Example of Fermi Surface



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1) Motivation: color superfluidity and ultracold fermions

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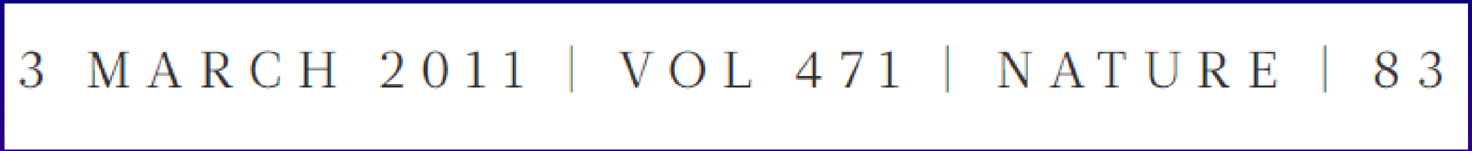
Start with SU(2) case

- For simplicity and to gain insight let me start first with the SU(2) case: two colors or simple peudospin-1/2 fermions.

- How spin-orbit and Zeeman fields change the crossover from BCS to BEC as interactions are tuned?



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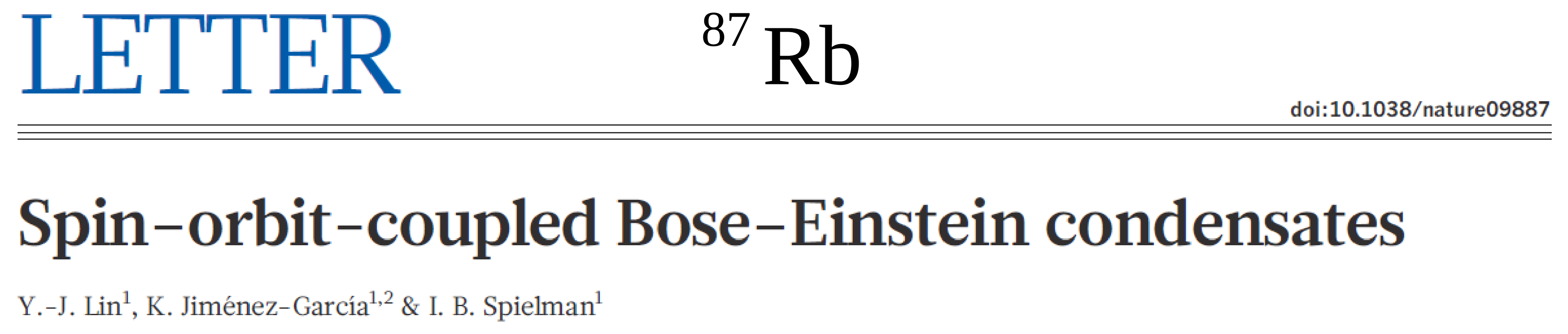


LETTER

^{87}Rb

doi:10.1038/nature09887

Spin–orbit-coupled Bose–Einstein condensates

Y.-J. Lin[1], K. Jiménez-García[1,2] & I. B. Spielman[1]

$$\begin{pmatrix} \dfrac{\mathbf{k}^2+k_R^2}{2m}+\dfrac{\Omega}{2} & -i\left(\dfrac{\delta}{2}-\dfrac{k_R}{m}k_x\right) \\ i\left(\dfrac{\delta}{2}-\dfrac{k_R}{m}k_x\right) & \dfrac{\mathbf{k}^2+k_R^2}{2m}-\dfrac{\Omega}{2} \end{pmatrix}$$

spin-orbit

detuning

Raman coupling

Zeeman and Spin-Orbit Hamiltonian

Hamiltonian Matrix

$$\mathbf{H}_0(\mathbf{k}) = \varepsilon(\mathbf{k})\mathbf{1} - h_x(\mathbf{k})\boldsymbol{\sigma}_x - h_y(\mathbf{k})\boldsymbol{\sigma}_y - h_z(\mathbf{k})\boldsymbol{\sigma}_z$$

$$\varepsilon_{\Uparrow}(\mathbf{k}) = \varepsilon(\mathbf{k}) - \left|h_{\text{eff}}(\mathbf{k})\right|$$

$$\varepsilon_{\Downarrow}(\mathbf{k}) = \varepsilon(\mathbf{k}) + \left|h_{\text{eff}}(\mathbf{k})\right|$$

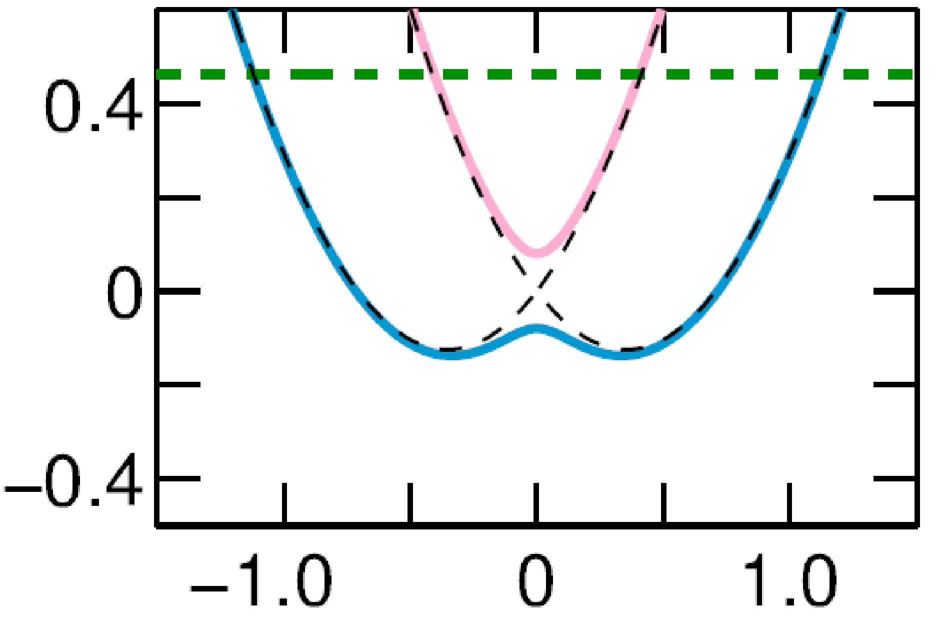
$$\left|h_{\text{eff}}(\mathbf{k})\right| = \sqrt{\left|h_x(\mathbf{k})\right|^2 + \left|h_y(\mathbf{k})\right|^2 + \left|h_z(\mathbf{k})\right|^2}$$

Energy Dispersions in the ERD case

$$h_x(\mathbf{k}) = 0$$

$$\frac{h_y(\mathbf{k})}{\varepsilon_F} = 0.71\frac{k_x}{k_F}$$

$$\frac{h_z(\mathbf{k})}{\varepsilon_F} = 0.05$$



$$\varepsilon_{\Uparrow}(\mathbf{k}) = \varepsilon(\mathbf{k}) - \sqrt{h_z^2 + |vk_x|^2}$$

$$\varepsilon_{\Downarrow}(\mathbf{k}) = \varepsilon(\mathbf{k}) + \sqrt{h_z^2 + |vk_x|^2}$$

Can have intra- and inter-helicity pairing.

Bring Interactions Back (real space)

$$\mathcal{H}(\mathbf{r}) = \mathcal{H}_0(\mathbf{r}) + \mathcal{H}_I(\mathbf{r})$$

$$\mathcal{H}_0(\mathbf{r}) = \sum_{\alpha\beta} \psi^\dagger_\alpha(\mathbf{r}) \left[\hat{K}_\alpha \delta_{\alpha\beta} - h_i(\mathbf{r}) \sigma_{i,\alpha\beta} \right] \psi_\beta(\mathbf{r})$$

Kinetic Energy

Spin-orbit and Zeeman

$$\mathcal{H}_I(\mathbf{r}) \;=\; -g\psi^\dagger_\uparrow(\mathbf{r})\psi^\dagger_\downarrow(\mathbf{r})\psi_\downarrow(\mathbf{r})\psi_\uparrow(\mathbf{r})$$

Contact Interaction

Bring interactions back:
Hamiltonian in initial spin basis

$$
\begin{array}{cc}
 & \begin{array}{cccc} \psi_{k\uparrow} & \psi_{k\downarrow} & \psi^{+}_{-k\uparrow} & \psi^{+}_{-k\downarrow} \end{array} \\
\begin{array}{c} \psi^{+}_{k\uparrow} \\ \psi^{+}_{k\downarrow} \\ \psi_{-k\uparrow} \\ \psi_{-k\downarrow} \end{array} & \mathbf{H}_0 = \begin{pmatrix} \tilde{K}_\uparrow(\mathbf{k}) & -h_\perp(\mathbf{k}) & 0 & -\Delta_0 \\ -h^*_\perp(\mathbf{k}) & \tilde{K}_\downarrow(\mathbf{k}) & \Delta_0 & 0 \\ 0 & \Delta_0^\dagger & -\tilde{K}_\uparrow(-\mathbf{k}) & h^*_\perp(-\mathbf{k}) \\ -\Delta_0^\dagger & 0 & h_\perp(-\mathbf{k}) & -\tilde{K}_\downarrow(-\mathbf{k}) \end{pmatrix}
\end{array}
$$

$$\tilde{K}_\uparrow(\mathbf{k}) = \xi(\mathbf{k}) - h_z$$

$$\tilde{K}_\downarrow(\mathbf{k}) = \xi(\mathbf{k}) + h_z$$

Bring interactions back: Hamiltonian in the helicity basis

$$\tilde{\mathrm{H}}_0 = \begin{array}{c|cccc} & \Phi_{k\Uparrow} & \Phi_{k\Downarrow} & \Phi^{+}_{-k\Uparrow} & \Phi^{+}_{-k\Downarrow} \\ \hline \Phi^{+}_{k\Uparrow} & \xi_{\Uparrow}(\mathbf{k}) & 0 & \boxed{\Delta_T(\mathbf{k})e^{-i\varphi_{\mathbf{k}}}} & -\Delta_S(\mathbf{k}) \\ \Phi^{+}_{k\Downarrow} & 0 & \xi_{\Downarrow}(\mathbf{k}) & \Delta_S(\mathbf{k}) & \boxed{-\Delta_T e^{i\varphi_{\mathbf{k}}}} \\ \Phi_{-k\Uparrow} & \boxed{\Delta_T^*(\mathbf{k})e^{i\varphi_{\mathbf{k}}}} & -\Delta_S^*(\mathbf{k}) & -\xi_{\Uparrow}(\mathbf{k}) & 0 \\ \Phi_{-k\Downarrow} & \Delta_S^*(\mathbf{k}) & \boxed{-\Delta_T^*(\mathbf{k})e^{-i\varphi_{\mathbf{k}}}} & 0 & -\xi_{\Downarrow}(\mathbf{k}) \end{array}$$

$$\varphi_{\mathbf{k}} = \mathrm{Arg}\left[h_{\perp}(\mathbf{k})\right]$$

Excitation Spectrum

$$E_1(\mathbf{k}) = \sqrt{\left[\left(\frac{\xi_{\Uparrow} - \xi_{\Downarrow}}{2}\right) - \sqrt{\left(\frac{\xi_{\Uparrow} + \xi_{\Downarrow}}{2}\right)^2 + |\Delta_S(\mathbf{k})|^2}\right]^2 + |\Delta_T(\mathbf{k})|^2},$$

★

$$E_2(\mathbf{k}) = \sqrt{\left[\left(\frac{\xi_{\Uparrow} - \xi_{\Downarrow}}{2}\right) + \sqrt{\left(\frac{\xi_{\Uparrow} + \xi_{\Downarrow}}{2}\right)^2 + |\Delta_S(\mathbf{k})|^2}\right]^2 + |\Delta_T(\mathbf{k})|^2},$$

Can be zero

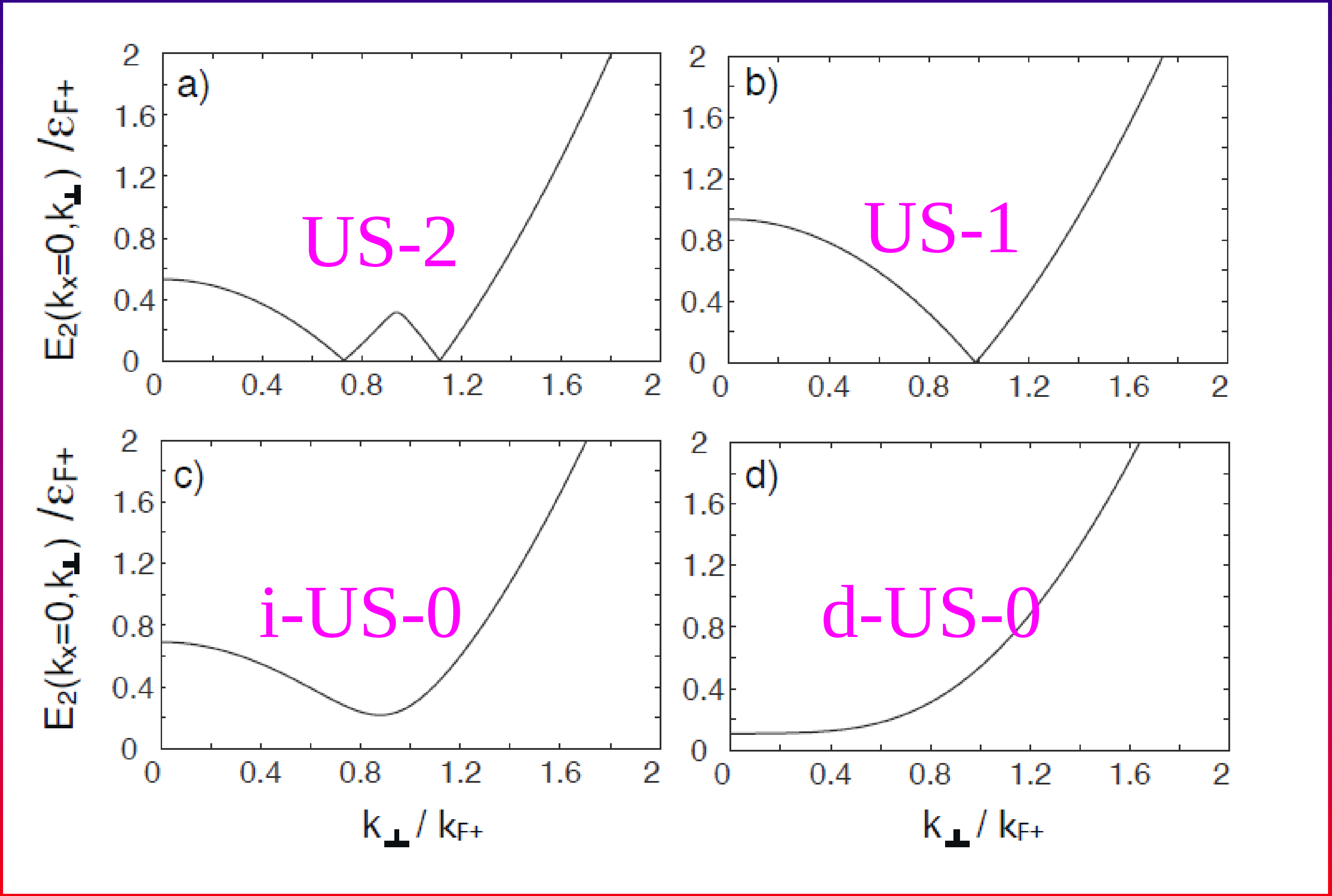
$$E_3(\mathbf{k}) = -E_2(\mathbf{k})$$

$$\xi_{\Uparrow}(\mathbf{k}) = \xi(\mathbf{k}) - \left|h_{\text{eff}}(\mathbf{k})\right|$$

$$E_4(\mathbf{k}) = -E_1(\mathbf{k})$$

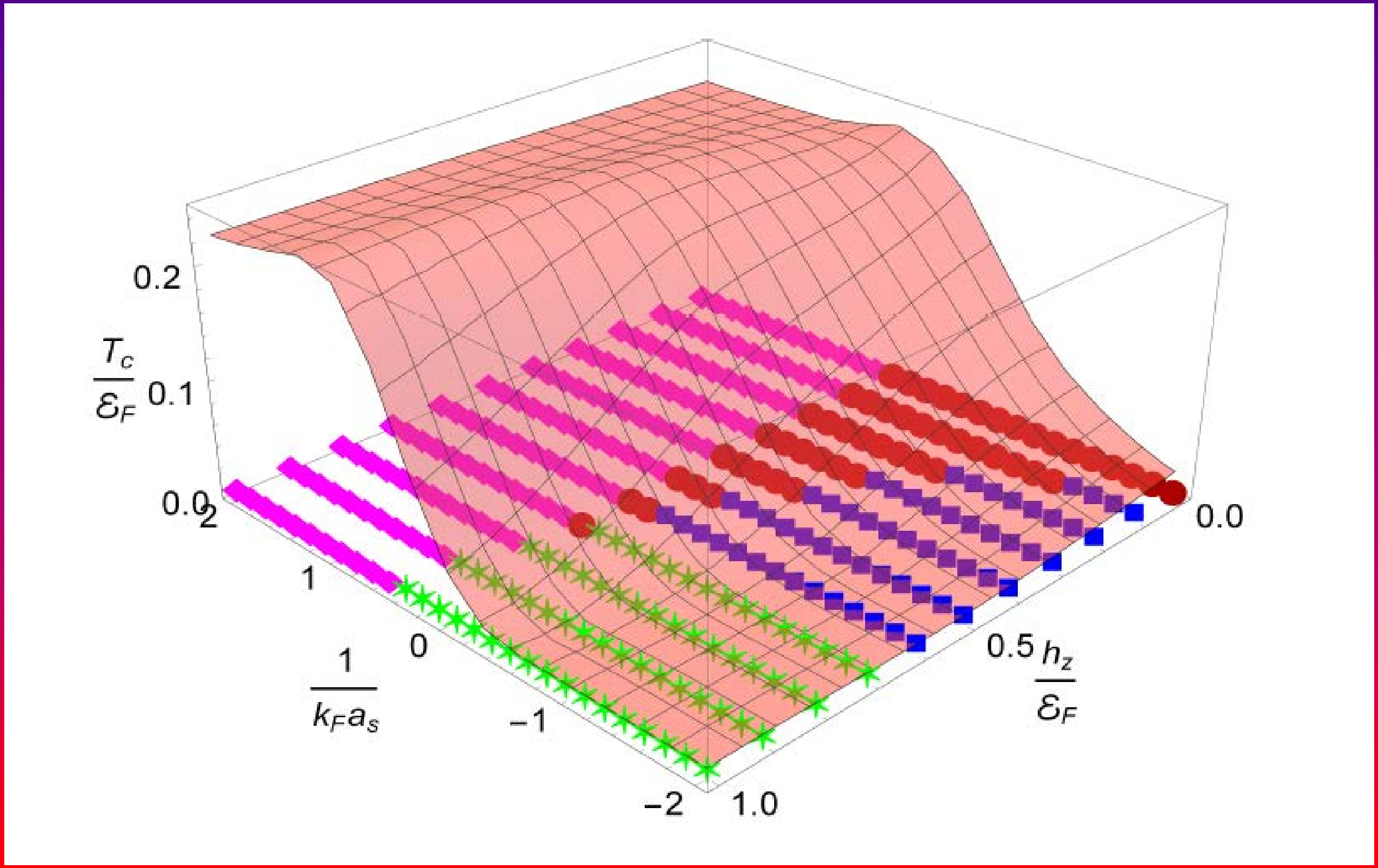
$$\xi_{\Downarrow}(\mathbf{k}) = \xi(\mathbf{k}) + \left|h_{\text{eff}}(\mathbf{k})\right|$$

Excitation Spectrum (ERD)



$$\Delta_T(\mathbf{k}) = \Delta_0 |h_\perp(\mathbf{k})| / |\mathbf{h}_{\text{eff}}(\mathbf{k})| \quad = 0$$

Phase diagram for finite spin-orbit coupling and changing Zeeman field



gapped

● d-US-0

● i-US-0

gapless

● US-2

● US-1

Triple-point: US-0/US-1/US-2

Now look at SU(3) case

- Let me analyze the SU(3) case: three colors or pseudo-spin-1 fermions.

- How color-orbit and color-flip fields change the crossover from BCS to BEC as interactions are tuned?

SU(3) invariant kinetic energy and three identical interaction channels

$$\hat{H}_{\mathrm{IP}} = \sum_{\mathbf{k}} \mathbf{F}^{\dagger}(\mathbf{k}) \mathbf{H}_{\mathrm{IP}}(\mathbf{k}) \mathbf{F}(\mathbf{k})$$

$$\varepsilon_R(\mathbf{k}) = \varepsilon_G(\mathbf{k}) = \varepsilon_B(\mathbf{k}) = \varepsilon(\mathbf{k})$$

$$\mathbf{F}^{\dagger}(\mathbf{k}) \quad = \quad \left[f_R^{\dagger}(\mathbf{k}), f_G^{\dagger}(\mathbf{k}), f_B^{\dagger}(\mathbf{k}) \right]$$

$$\hat{H}_{\mathrm{INT}} = -\frac{1}{V} \sum_{\mathbf{Q}, \{c \neq c'\}} g_{cc'} a_{cc'}^{\dagger}(\mathbf{Q}) a_{cc'}(\mathbf{Q}),$$

$$a_{cc'}^{\dagger}(\mathbf{Q}) = \sum_{\mathbf{k}} f_c^{\dagger}(\mathbf{k} + \mathbf{Q}/2) f_{c'}^{\dagger}(-\mathbf{k} - \mathbf{Q}/2)$$

Pair operator

$$g_{RG} = g_{RB} = g_{GB} = g \quad \Rightarrow \quad a_{RG} = a_{RB} = a_{GB} = a_s$$

No color-orbit and no color-flip fields

$$\hat{H}_{\mathrm{IP}} = \sum_{\mathbf{k}} \mathbf{F}^{\dagger}(\mathbf{k})\mathbf{H}_{\mathrm{IP}}(\mathbf{k})\mathbf{F}(\mathbf{k})$$

$$\mathbf{H}_{\mathrm{IP}}(\mathbf{k}) = \varepsilon(\mathbf{k})\mathbf{1}$$

KE is SU(3) invariant

$$\hat{H}_{\mathrm{INT}} = -\frac{1}{V} \sum_{\mathbf{Q},\{c \neq c'\}} g_{cc'} a^{\dagger}_{cc'}(\mathbf{Q}) a_{cc'}(\mathbf{Q}),$$

$$g_{RG} = g_{RB} = g_{GB} = g$$

NOT VERY INTERESTING, JUST CROSSOVER!

Can go to a mixed color basis where only two mixed colors pair and the third one is inert as a result of SU(3) invariance!

Add color-orbit and color-flip fields (near zero temperature)

$$\hat{H}_0 = \frac{1}{2}\sum_{\mathbf{k}} \mathbf{f}_N^\dagger(\mathbf{k})\mathbf{H}_0(\mathbf{k})\mathbf{f}_N(\mathbf{k}) + V\sum_{c\neq c'}\frac{|\Delta_{cc'}|^2}{g_{cc'}} + \mathcal{C}(\mu),$$

$$\mathbf{f}_N^\dagger(\mathbf{k}) \quad = \left[f_{\mathrm{R}}^\dagger(\mathbf{k}), f_{\mathrm{G}}^\dagger(\mathbf{k}), f_{\mathrm{B}}^\dagger(\mathbf{k}), f_{\mathrm{R}}(-\mathbf{k}), f_{\mathrm{G}}(-\mathbf{k}), f_{\mathrm{B}}(-\mathbf{k})\right]$$

$$\mathbf{H}_0(\mathbf{k}) = \begin{pmatrix} \overline{\mathbf{H}}_{\mathrm{IP}}(\mathbf{k}) & \boldsymbol{\Delta} \\ \boldsymbol{\Delta}^\dagger & -\overline{\mathbf{H}}^*_{\mathrm{IP}}(-\mathbf{k}) \end{pmatrix},$$

Spectrum has 3 quasiparticle and 3 quasihole bands

Hamiltonian Blocks

$$\varepsilon_c(\mathbf{k}) \; = \; (\mathbf{k} - \mathbf{k}_c)^2/(2m) + \eta_c$$

$$h_x(\mathbf{k}) = -\sqrt{2}\Omega$$

$$\mathbf{H}_{\mathrm{IP}}(\mathbf{k}) = \begin{pmatrix} \varepsilon_{\mathrm{R}}(\mathbf{k}) & -h_x(\mathbf{k})/\sqrt{2} & 0 \\ -h_x(\mathbf{k})/\sqrt{2} & \varepsilon_{\mathrm{G}}(\mathbf{k}) & -h_x(\mathbf{k})/\sqrt{2} \\ 0 & -h_x(\mathbf{k})/\sqrt{2} & \varepsilon_{\mathrm{B}}(\mathbf{k}) \end{pmatrix},$$

$$\boldsymbol{\Delta} = \begin{pmatrix} 0 & \Delta_{RG} & \Delta_{RB} \\ -\Delta_{RG} & 0 & \Delta_{GB} \\ -\Delta_{RB} & -\Delta_{GB} & 0 \end{pmatrix}$$

Mixed (rotated) color basis

$$\widetilde{\mathbf{H}}_0(\mathbf{k}) = \begin{pmatrix} \mathbf{H}_M(\mathbf{k}) & \boldsymbol{\Delta}_M \\ \boldsymbol{\Delta}_M^\dagger & -\mathbf{H}_M^*(-\mathbf{k}) \end{pmatrix}$$

$$\mathbf{H}_{M,\alpha\beta}(\mathbf{k}) = \xi_\alpha(\mathbf{k})\delta_{\alpha\beta} \qquad \Delta_{M,\alpha\beta}(\mathbf{k}) = \Delta_{\alpha\beta}(\mathbf{k})$$

$$\mathbf{R}(\mathbf{k}) = \begin{pmatrix} R_{\Uparrow R}(\mathbf{k}) & R_{\Uparrow G}(\mathbf{k}) & R_{\Uparrow B}(\mathbf{k}) \\ R_{0R}(\mathbf{k}) & R_{0G}(\mathbf{k}) & R_{0B}(\mathbf{k}) \\ R_{\Downarrow R}(\mathbf{k}) & R_{\Downarrow G}(\mathbf{k}) & R_{\Downarrow B}(\mathbf{k}) \end{pmatrix}$$

$$\Delta_{\alpha\beta}(\mathbf{k}) = R_{\alpha c}(\mathbf{k})\Delta_{cc'}R_{c'\beta}(-\mathbf{k}),$$

Order parameter tensor (mixed color basis)

$$\Delta_{\alpha\beta}(\mathbf{k}) = R_{\alpha c}(\mathbf{k})\Delta_{cc'}R_{c'\beta}(-\mathbf{k}),$$

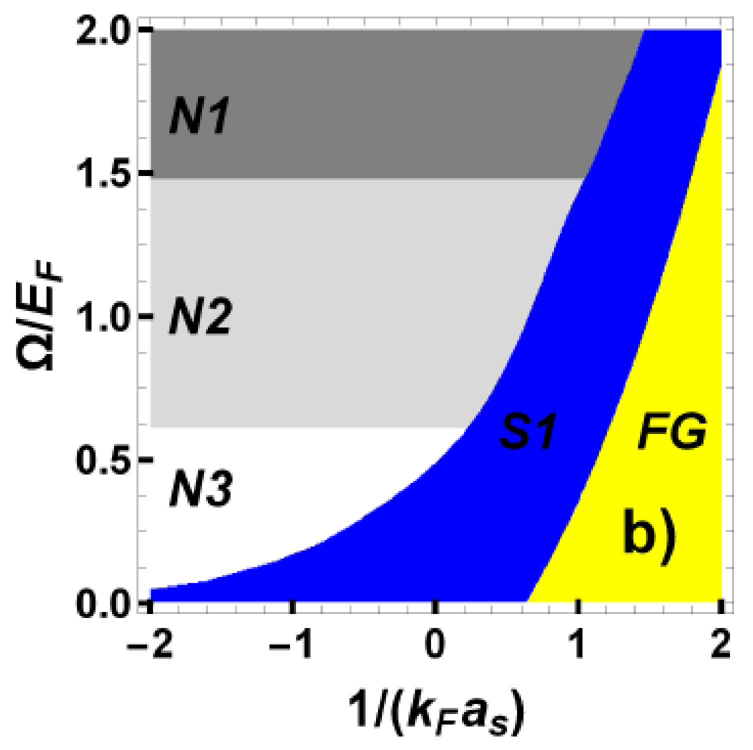
$$\begin{pmatrix} \Delta_{\Uparrow\Uparrow}(\mathbf{k}) & \Delta_{\Uparrow 0}(\mathbf{k}) & \Delta_{\Uparrow\Downarrow}(\mathbf{k}) \\ \Delta_{0\Uparrow}(\mathbf{k}) & \Delta_{00}(\mathbf{k}) & \Delta_{0\Downarrow}(\mathbf{k}) \\ \Delta_{\Downarrow\Uparrow}(\mathbf{k}) & \Delta_{\Downarrow 0}(\mathbf{k}) & \Delta_{\Downarrow\Downarrow}(\mathbf{k}) \end{pmatrix}$$

Zero color-orbit coupling

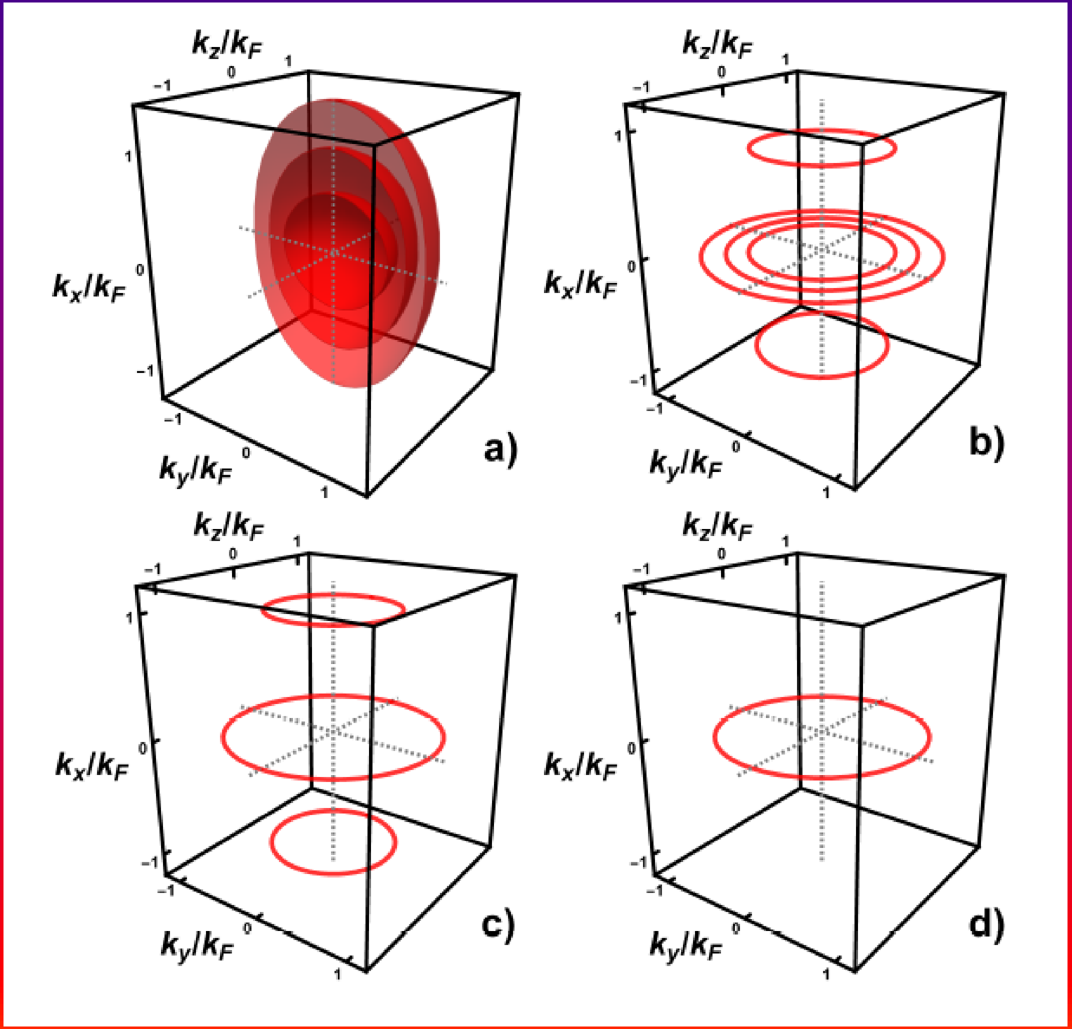
Color - orbit coupling is zero, but color - flip field Ω is not!

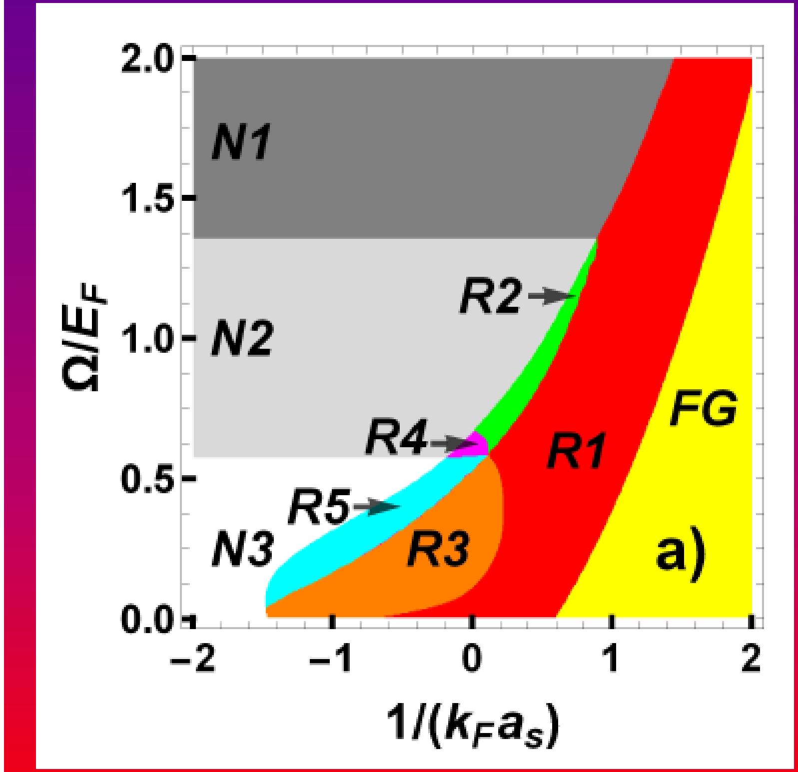
One of the three quasiparticle bands has a surface of nodes, the other two are fully gapped.

When color - flip field Ω is zero one mixed color band is completely inert.



Non-zero color-orbit coupling





Outline of Talk

1) Motivation: color superfluidity and ultracold fermions

2) Introduction to spin-orbit and color-orbit coupling

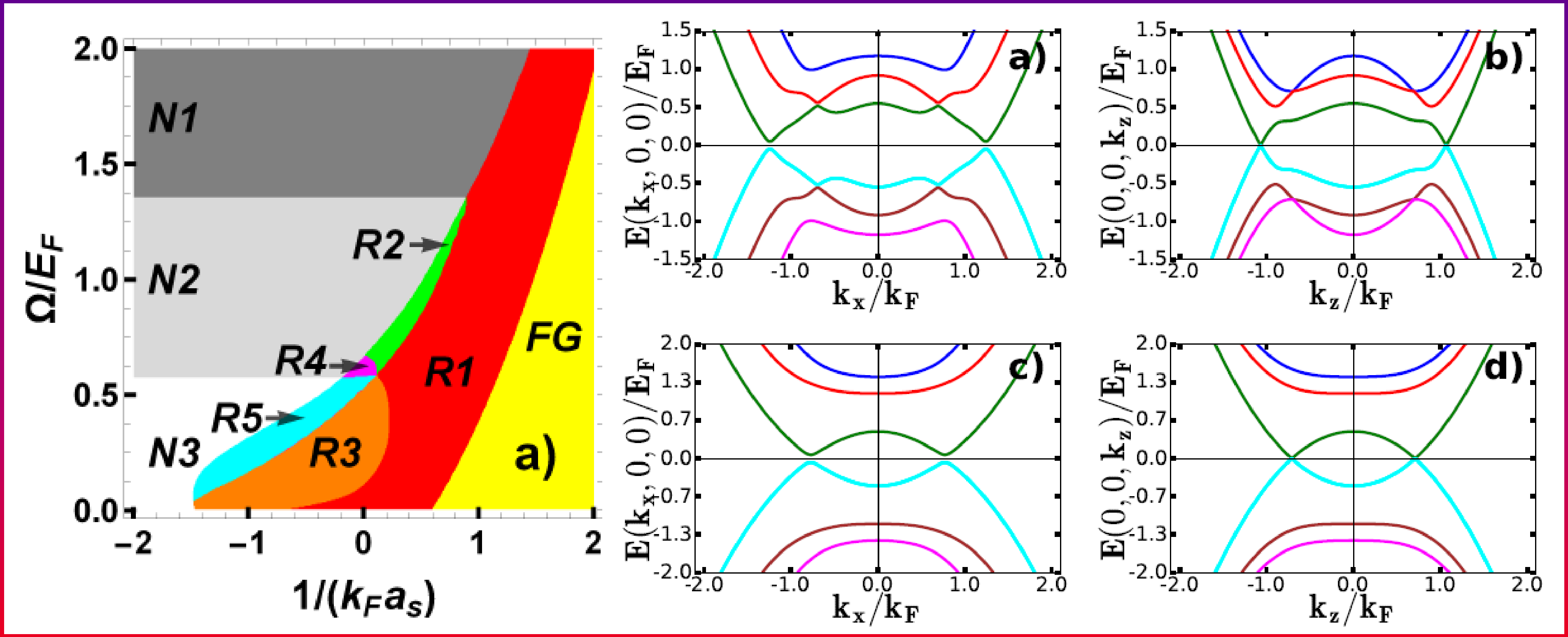
3) Interacting fermions with color-orbit and color-flip fields

4) Spectroscopic and thermodynamic properties

5) Conclusions

Non-zero color-orbit coupling

R3



Quintuple and pentacritical point

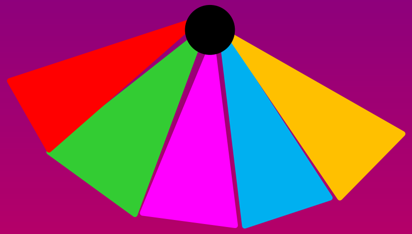
R1

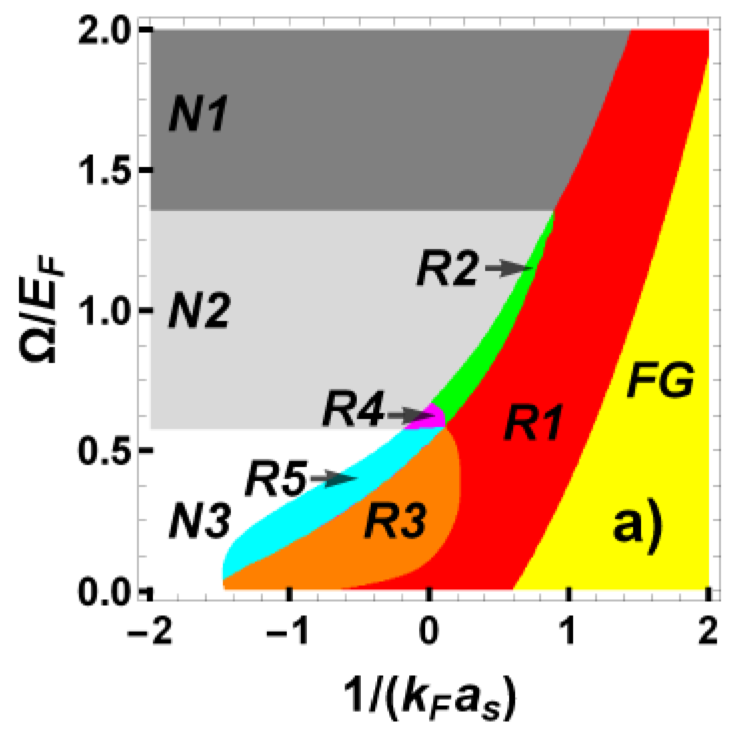
Color compressibility near quintuple point

🔴 R1 🟢 R2

🟣 R4 🔵 R5

🟠 R3





$$\kappa = n^{-2}[\partial n / \partial \mu]_{T,V}$$

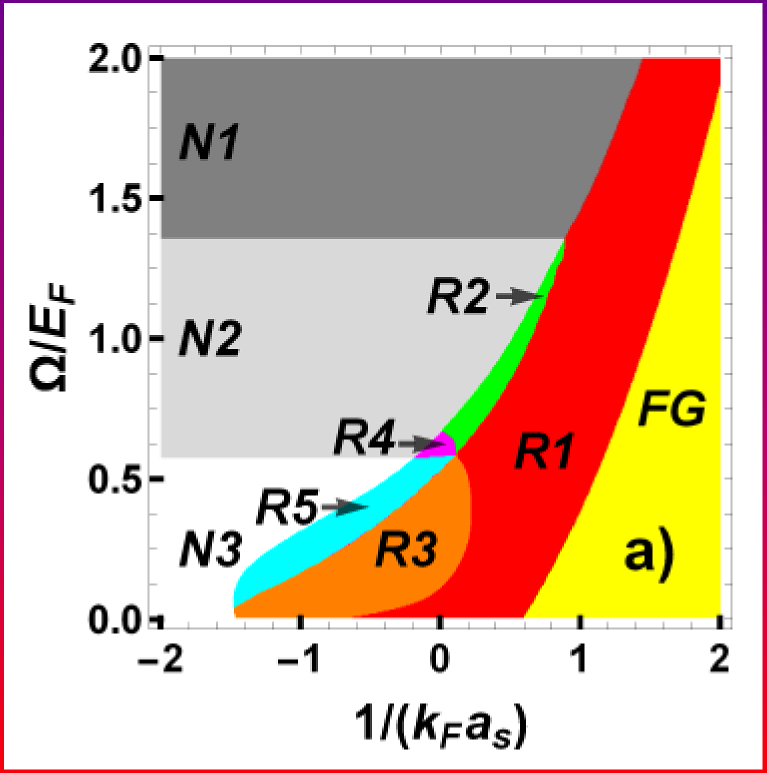
$$\kappa = \kappa_R - \alpha_{Rm} \left| \Omega - \Omega_c(\lambda) \right|^{1/2}$$

Color compressibility near gapless R1 to fully gapped FG line

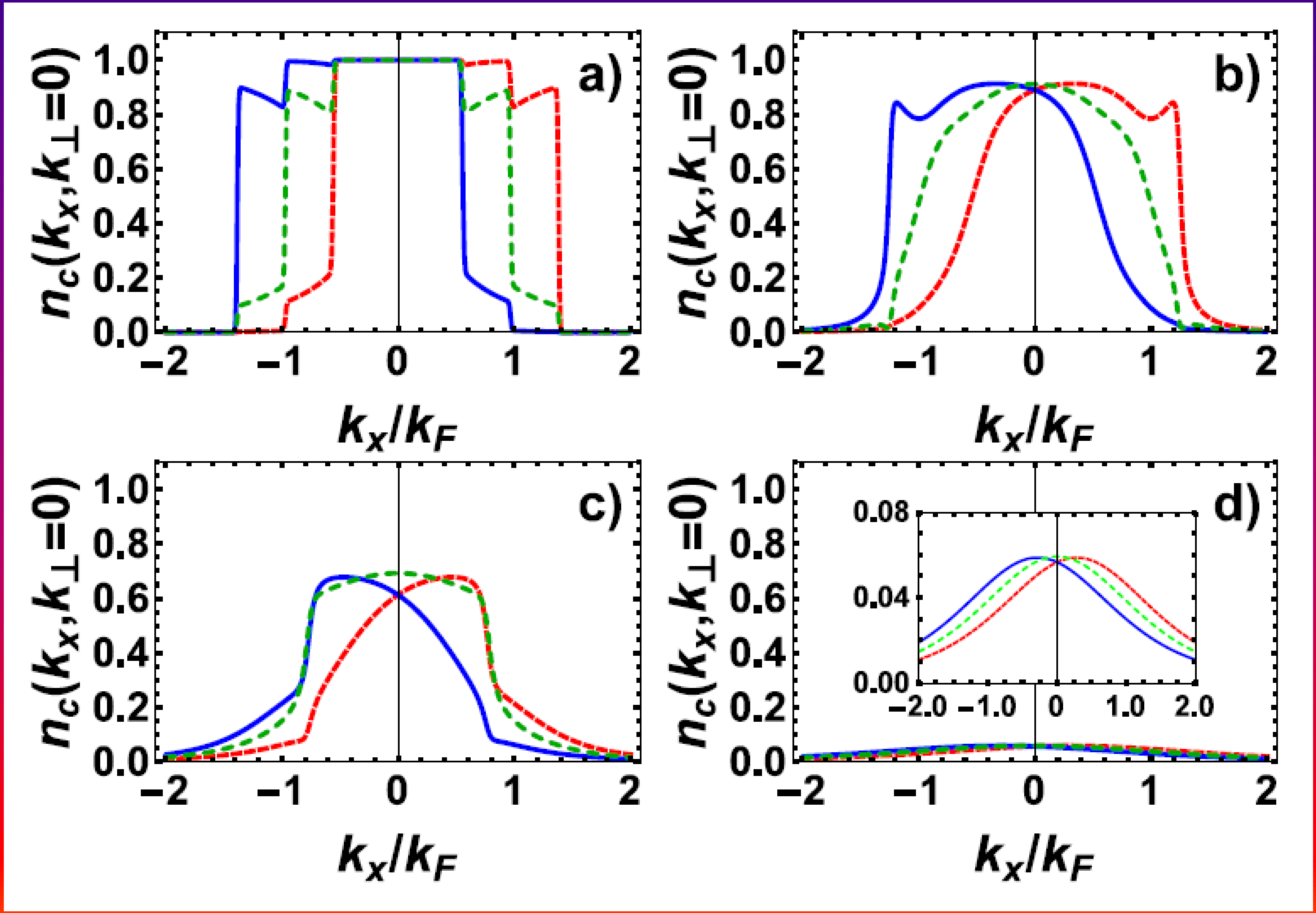
● R1 ● FG

$$\kappa = n^{-2}[\partial n / \partial \mu]_{T,V}$$

$$\kappa = \kappa_R \left| \ln \left| \Omega - \Omega_c(\lambda) \right| \right|$$



Momentum distributions of original colors



N3

R3

R1

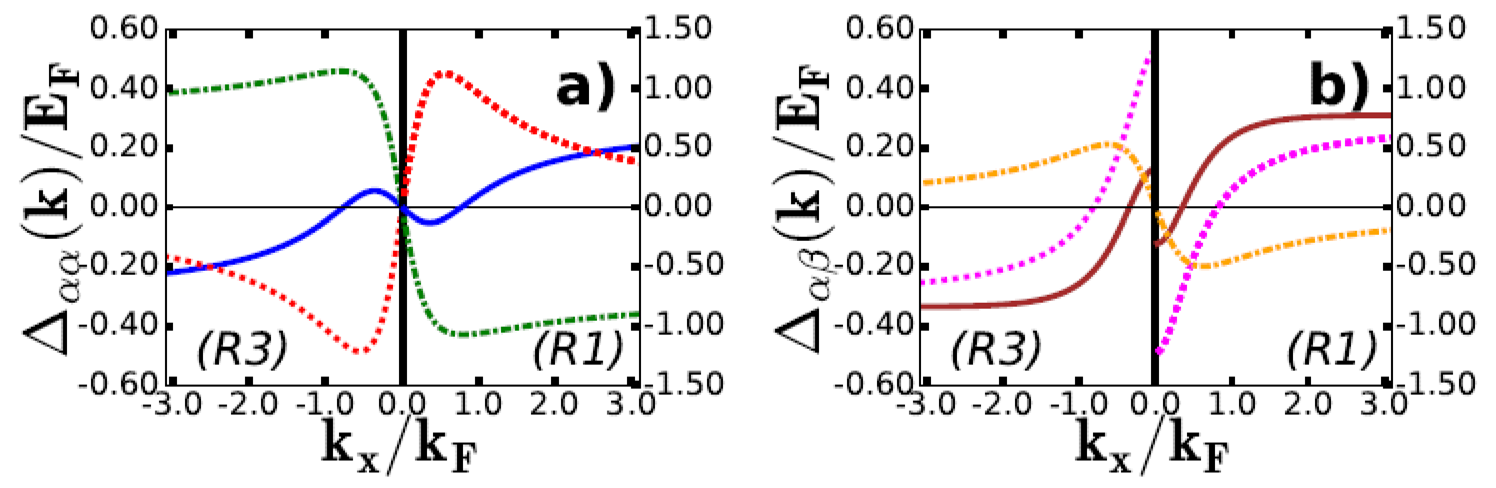
FG

Order parameter tensor (mixed color basis)

$$\Delta_{\alpha\beta}(\mathbf{k}) = R_{\alpha c}(\mathbf{k})\Delta_{cc'}R_{c'\beta}(-\mathbf{k}),$$

$$\begin{pmatrix} \Delta_{\Uparrow\Uparrow}(\mathbf{k}) & \Delta_{\Uparrow 0}(\mathbf{k}) & \Delta_{\Uparrow\Downarrow}(\mathbf{k}) \\ \Delta_{0\Uparrow}(\mathbf{k}) & \Delta_{00}(\mathbf{k}) & \Delta_{0\Downarrow}(\mathbf{k}) \\ \Delta_{\Downarrow\Uparrow}(\mathbf{k}) & \Delta_{\Downarrow 0}(\mathbf{k}) & \Delta_{\Downarrow\Downarrow}(\mathbf{k}) \end{pmatrix}$$

Order parameter tensor (mixed color basis)



In a) the solid blue curve describes $\Delta_{\Uparrow\Uparrow}(\mathbf{k})$, the dashed red curve describes $\Delta_{00}(\mathbf{k})$, and the dot-dashed green curve describes $\Delta_{\Downarrow\Downarrow}(\mathbf{k})$. In b) the solid brown line represents $\Delta_{\Uparrow 0}(\mathbf{k})$, the dashed magenta line represents $\Delta_{\Downarrow 0}(\mathbf{k})$, the dot-dashed orange line represents $\Delta_{\Uparrow\Downarrow}(\mathbf{k})$.

Order parameter tensor (total pseudo-spin basis)

$$\tilde{\Delta}_{Sm_s}(\mathbf{k}) = M^{Sm_s}_{\alpha\beta}\,\Delta_{\alpha\beta(\mathbf{k})}$$

SINGLET CHANNEL

$$\tilde{\Delta}_{00}(\mathbf{k})$$

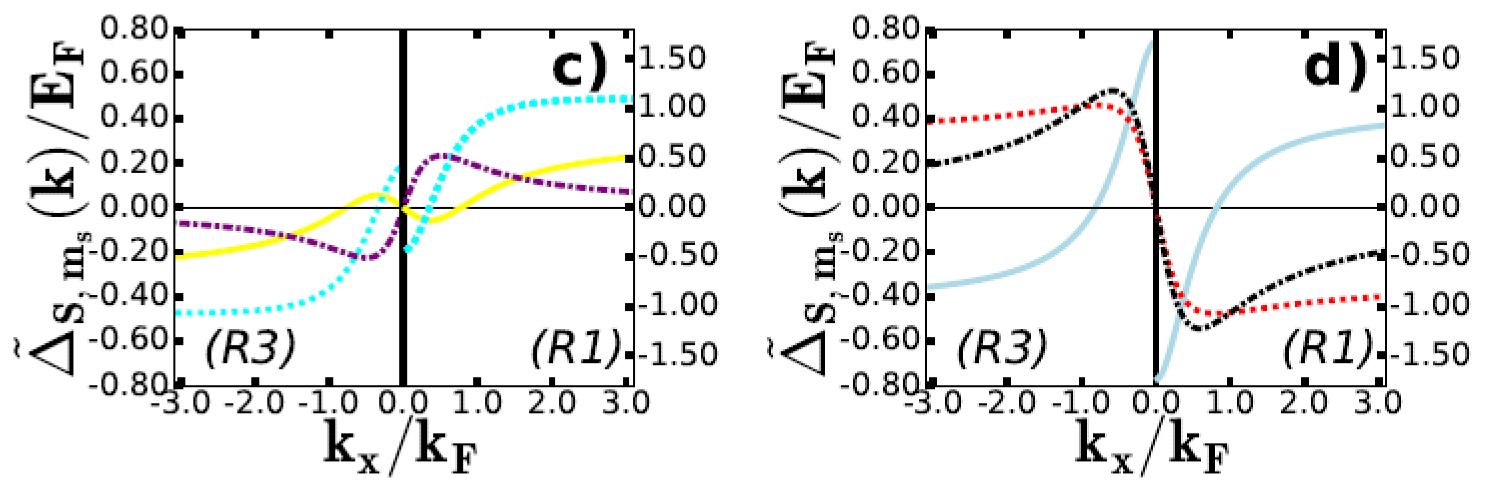
TRIPLET CHANNEL

$$\tilde{\Delta}_{11}(\mathbf{k}), \tilde{\Delta}_{10}(\mathbf{k}), \tilde{\Delta}_{1\bar{1}}(\mathbf{k})$$

QUINTUPLET CHANNEL

$$\tilde{\Delta}_{22}(\mathbf{k}), \tilde{\Delta}_{21}(\mathbf{k}), \tilde{\Delta}_{20}(\mathbf{k}), \tilde{\Delta}_{2\bar{1}}(\mathbf{k}), \tilde{\Delta}_{2\bar{2}}(\mathbf{k})$$

Order parameter tensor (total pseudo-spin basis)



In c) the solid yellow curve corresponds to $\widetilde{\Delta}_{22}(\mathbf{k})$, the dashed cyan curve corresponds to $\widetilde{\Delta}_{21}(\mathbf{k})$, the dot-dashed purple curve corresponds to $\widetilde{\Delta}_{20}(\mathbf{k})$. In d) the solid light-blue line indicates $\widetilde{\Delta}_{2\bar{1}}(\mathbf{k})$, the dashed red line indicates $\widetilde{\Delta}_{2\bar{2}}(\mathbf{k})$, the dot-dashed black line indicates $\widetilde{\Delta}_{00}(\mathbf{k})$. **SINGLET AND QUINTUPLET PAIRING**

Outline of talk

1) Motivation: color superfluidity and ultracold fermions

2) Introduction to spin-orbit and color-orbit coupling

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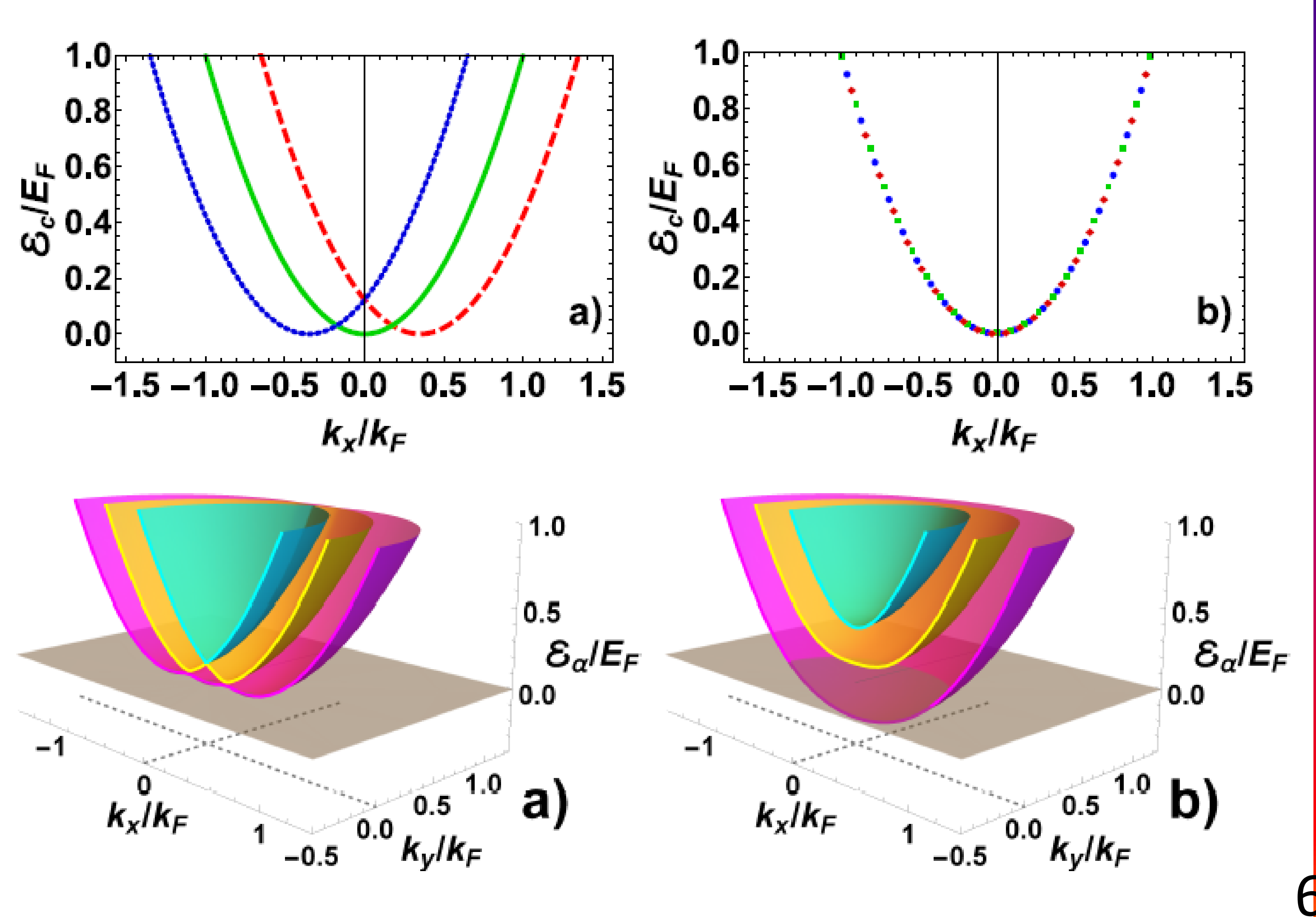
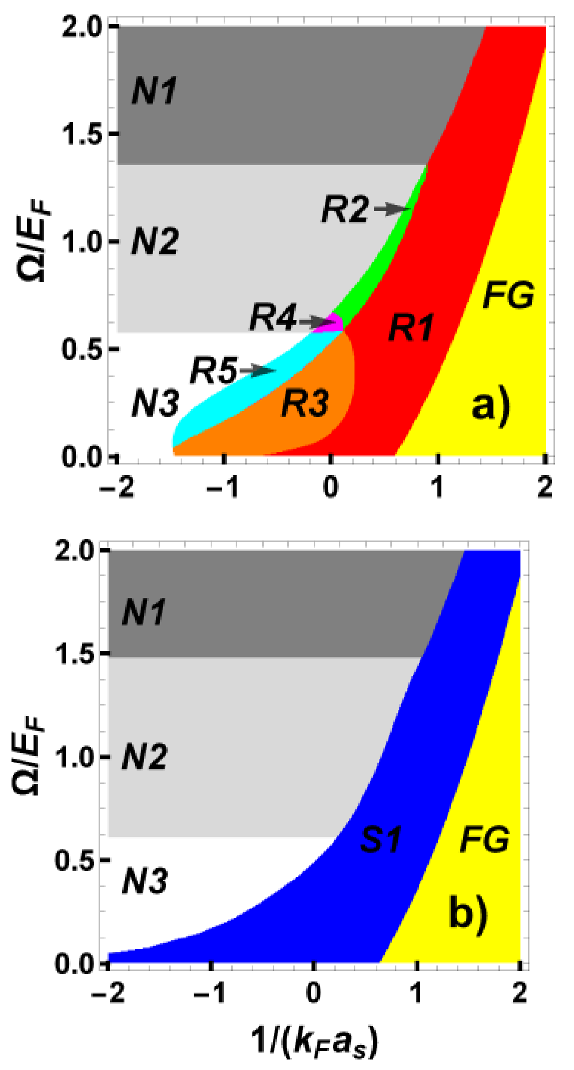
4) Spectroscopic and thermodynamic properties

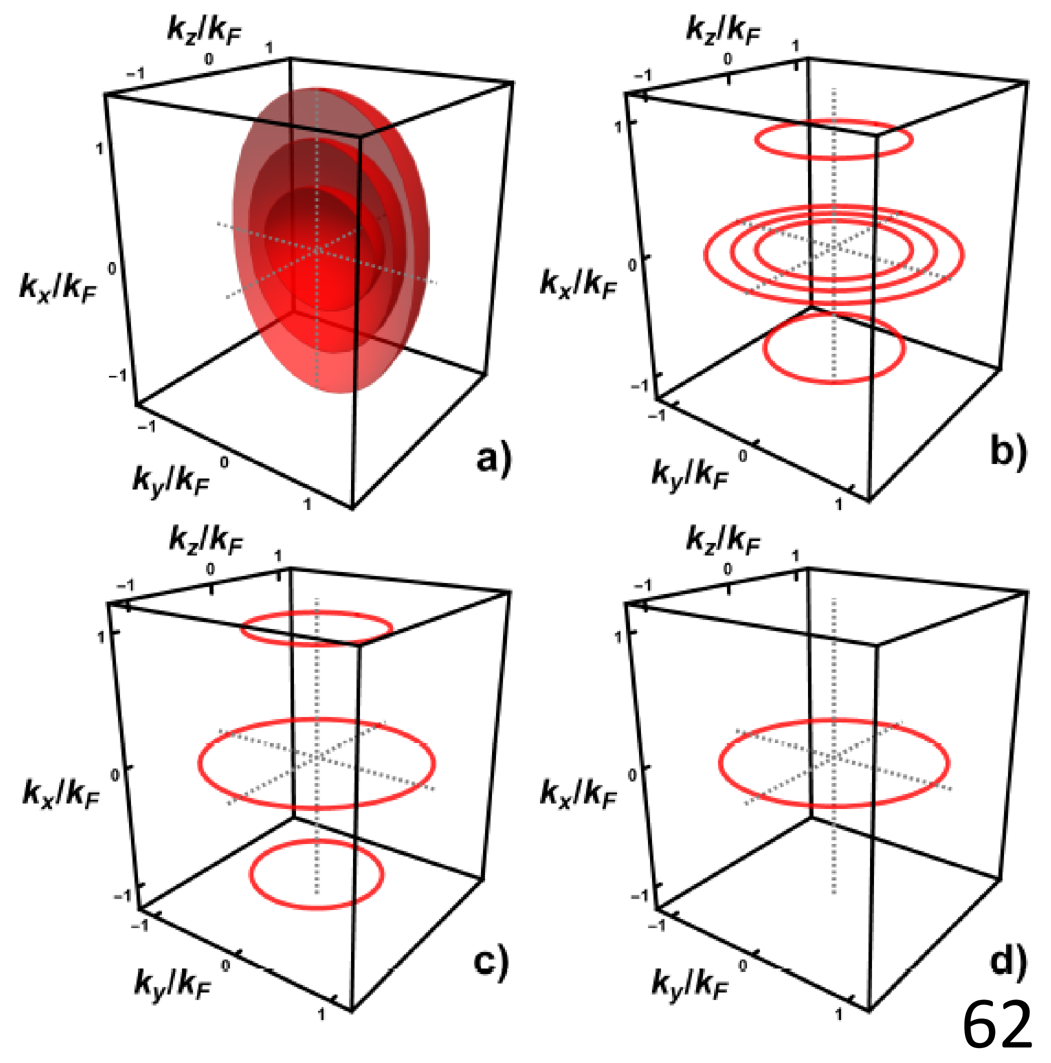
5) Conclusions

Conclusions in pictures: color-orbit and color flip fields



Conclusions in pictures: color-orbit and color flip fields





Conclusions in words

Ultracold fermions with three internal states can exhibit very unusual color superfluidity in the presence of color-orbit and color-flip fields, where SU(3) symmetry is explicitly broken.

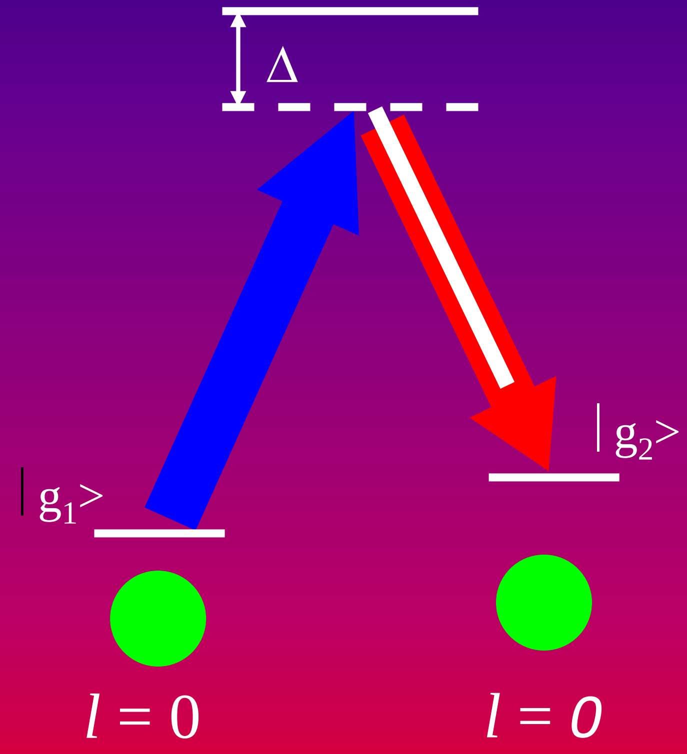
The phase diagram of color-flip versus interaction parameter for fixed color-orbit coupling exhibits several topological phases associated with the nodal structure of the quasiparticle excitation spectrum. The phase diagram exhibits a pentacritical point where five nodal superfluid phases merge.

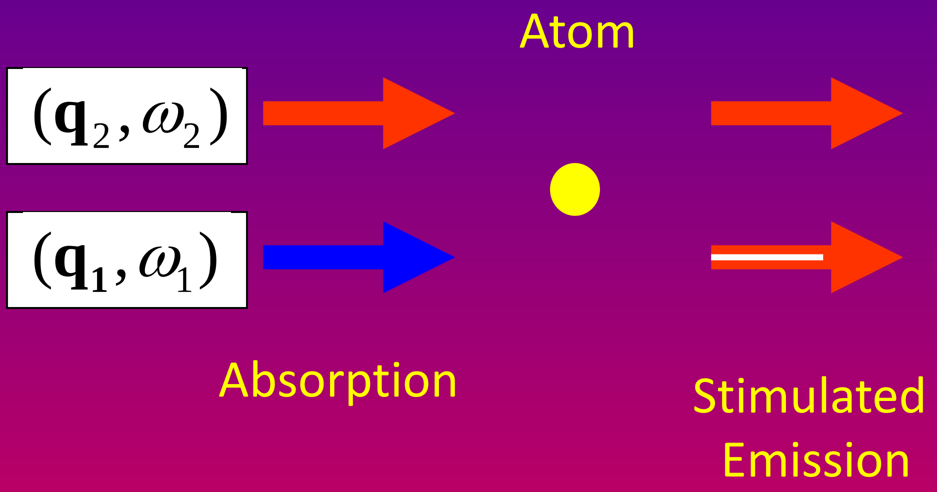
Even for interactions that occur only in the color s-wave channel, the order parameter for superfluidity exhibits singlet, triplet and quintuplet components due to the presence of color-orbit and color-flip fields.

These topological phases can be probed through measurements of spectroscopic properties such as excitation spectra, momentum distributions and density of states.

THE END

Two-photon Raman process - I





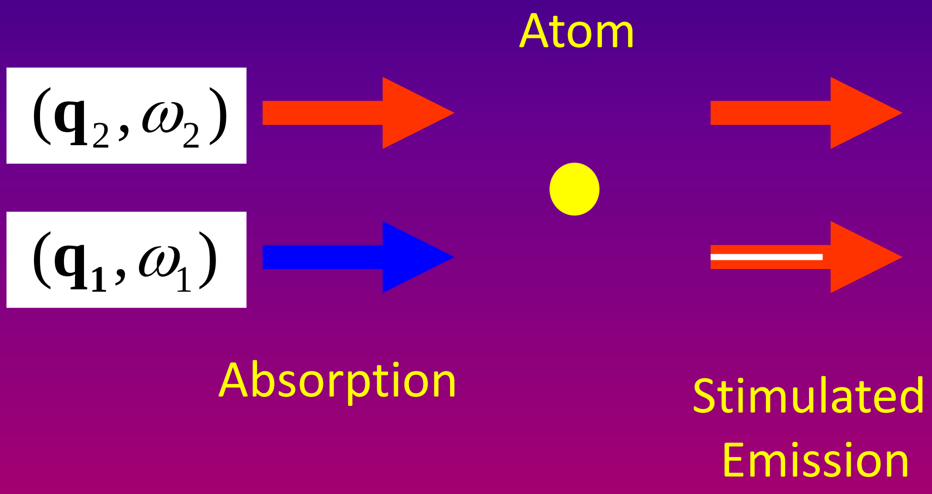
Momentum conservation:

$$\hbar \mathbf{k}_\mathbf{i} + \hbar \mathbf{q}_\mathbf{1} = \hbar \mathbf{k}_\mathbf{f} + \hbar \mathbf{q}_\mathbf{2}$$

Energy conservation:

$$\varepsilon_i + \frac{(\hbar \mathbf{k}_\mathbf{i})^2}{2m} + \hbar\omega_1 = \varepsilon_f + \frac{(\hbar \mathbf{k}_\mathbf{f})^2}{2m} + \hbar\omega_2$$

Two-photon Raman process - II



Atom

$(\mathbf{q}_2, \omega_2)$

$(\mathbf{q}_1, \omega_1)$

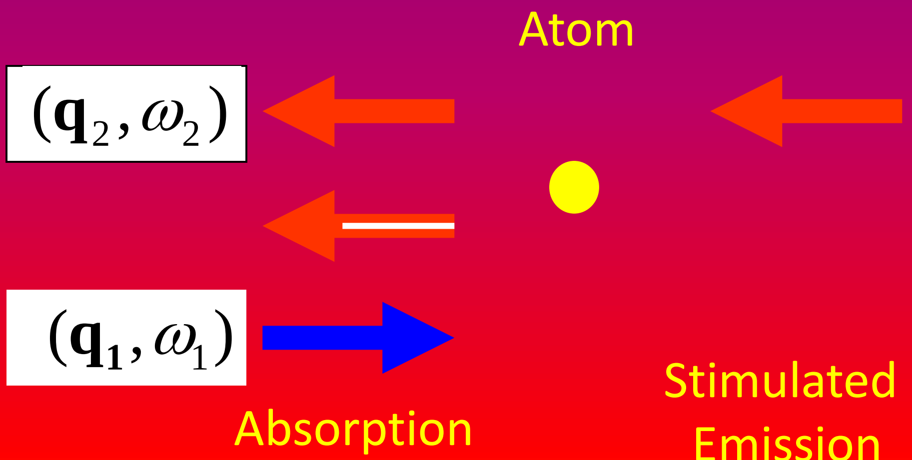
Absorption

Stimulated Emission

$$\hbar \mathbf{k}_\mathbf{f} = \hbar \mathbf{k}_\mathbf{i} + \hbar(\mathbf{q}_1 - \mathbf{q}_2)$$

$$\hbar \delta \mathbf{k} \approx \mathbf{0}$$

Momentum transfer to atom



Atom

$(\mathbf{q}_2, \omega_2)$

$(\mathbf{q}_1, \omega_1)$

Absorption

Stimulated Emission

$$\hbar \mathbf{k}_\mathbf{f} = \hbar \mathbf{k}_\mathbf{i} + \hbar(\mathbf{q}_1 - \mathbf{q}_2)$$

$$\hbar \delta \mathbf{k} \approx 2\hbar \mathbf{q}_1$$

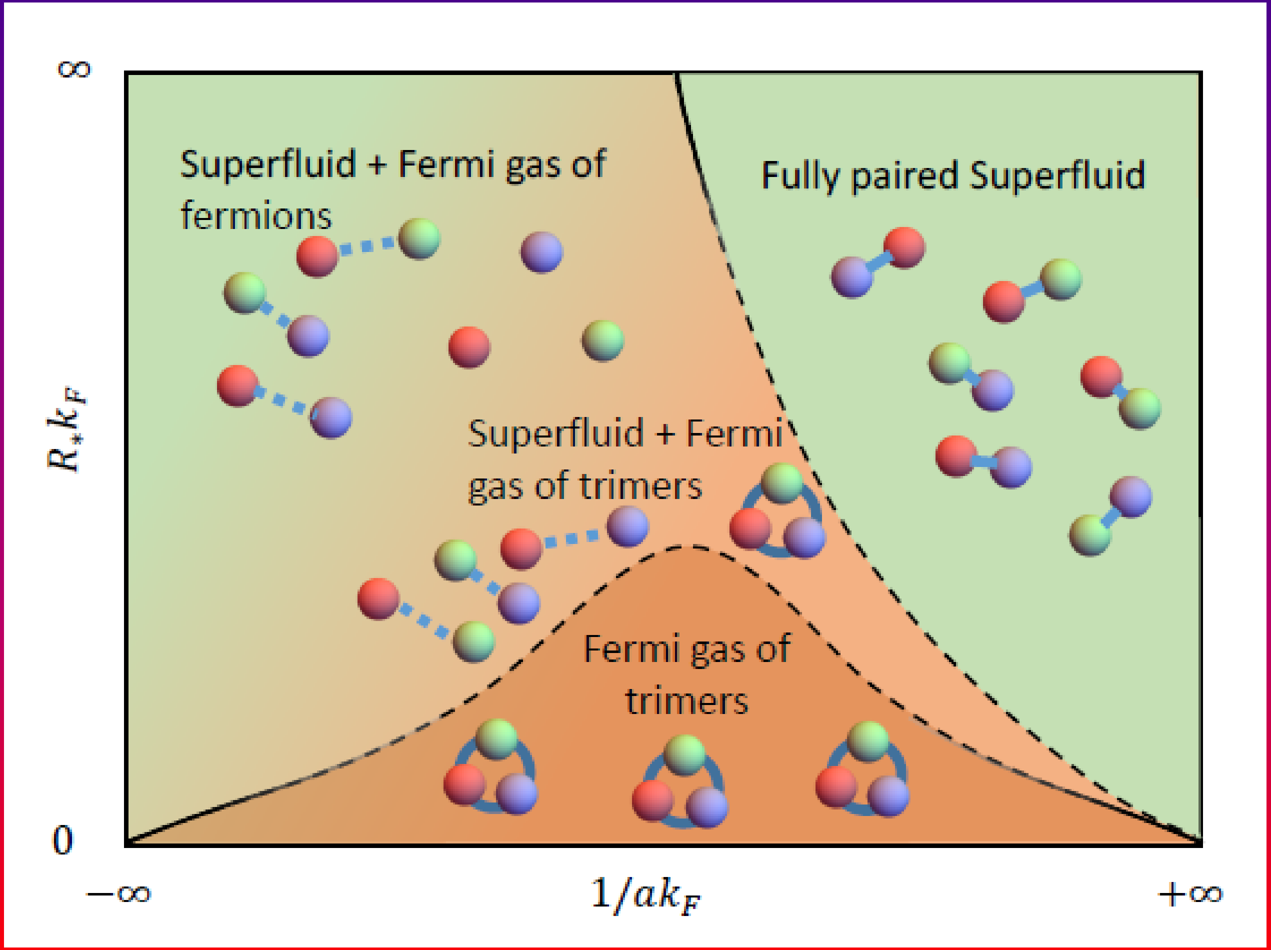
Momentum transfer to atom

Efimov States

High density

Intermediate density

Low density

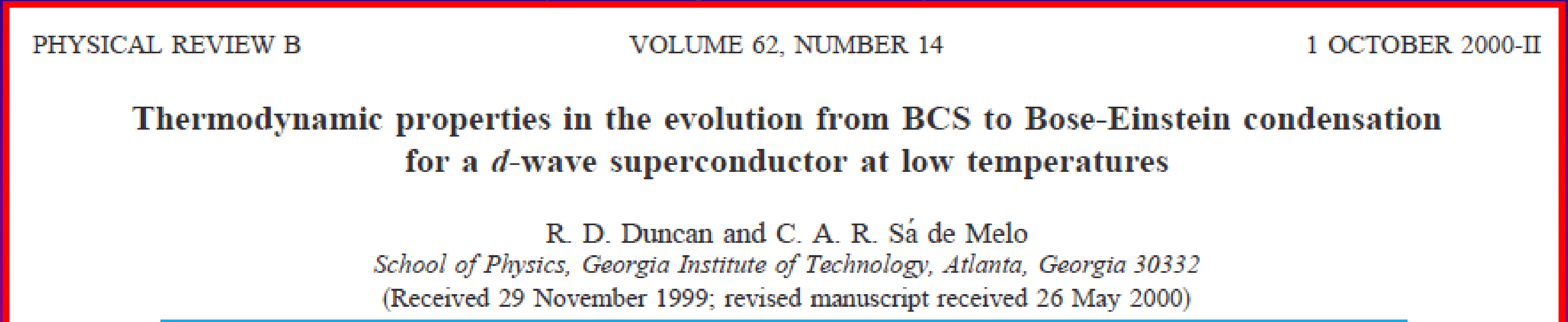


Shaji

Kono subarashii shinpojiumu no \ shusaisha no minasama, \ happyou sasete itadakimashite \ arigatou gozaimasu.

Watakushi wa nihongo hanasemasen shi, \ kyou takusan no gaikokujin no kata ga\ irasshaimasu shi, \ moushiwake gozaimasen ga \ eigo de happyou wo sasete itadakimasu.

Some of our earlier work on topological superfluids



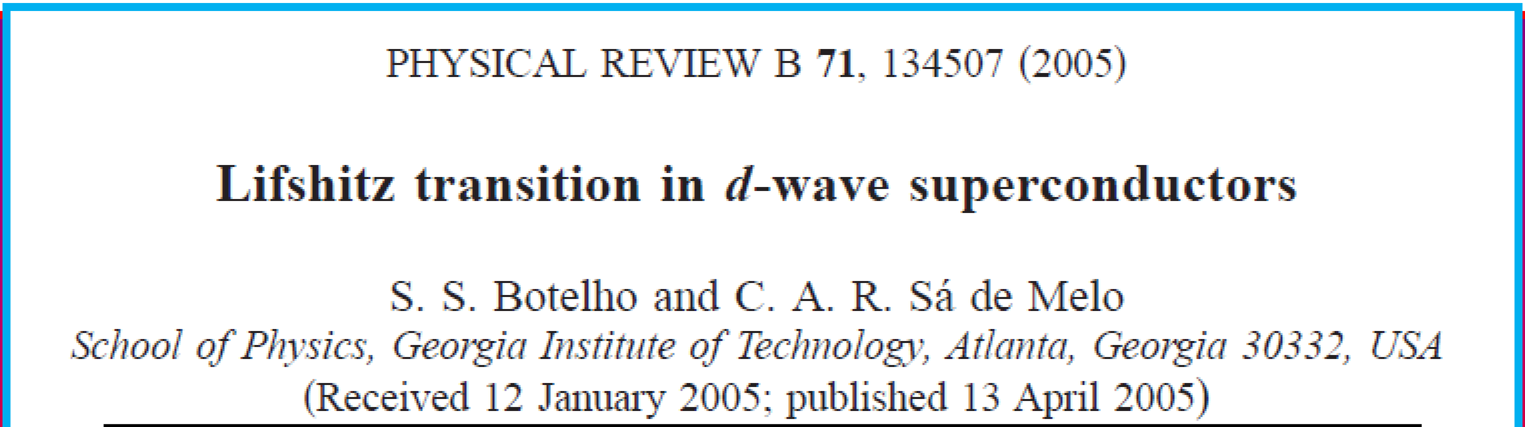
PHYSICAL REVIEW B VOLUME 62, NUMBER 14 1 OCTOBER 2000-II

Thermodynamic properties in the evolution from BCS to Bose-Einstein condensation for a *d*-wave superconductor at low temperatures

R. D. Duncan and C. A. R. Sá de Melo

School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332

(Received 29 November 1999; revised manuscript received 26 May 2000)



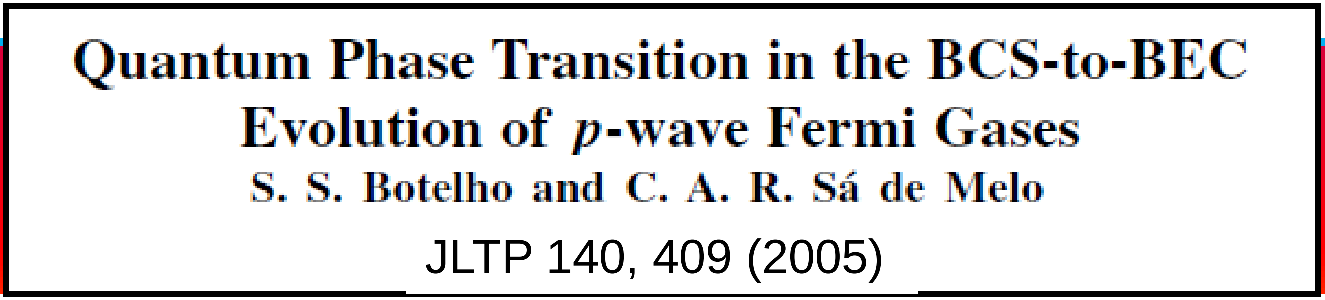
PHYSICAL REVIEW B **71**, 134507 (2005)

Lifshitz transition in *d*-wave superconductors

S. S. Botelho and C. A. R. Sá de Melo

School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332, USA

(Received 12 January 2005; published 13 April 2005)



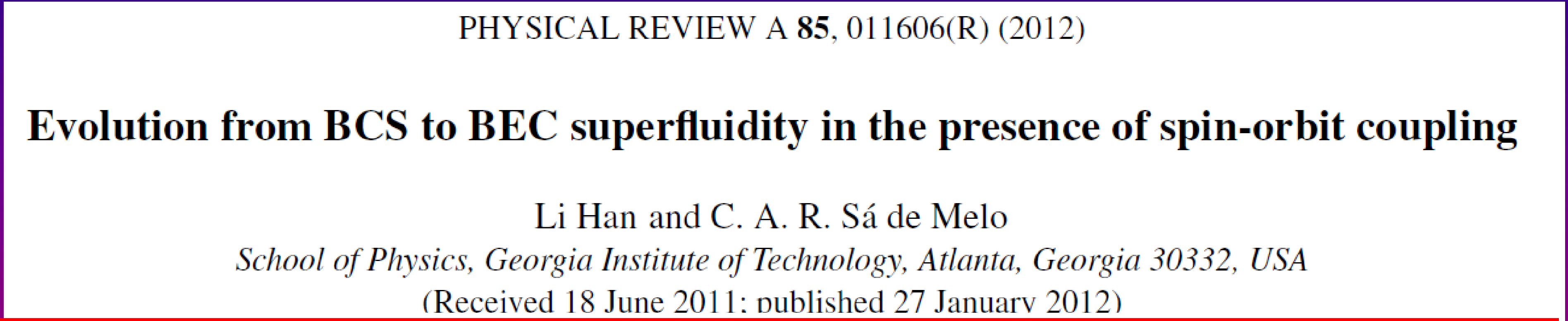
Quantum Phase Transition in the BCS-to-BEC Evolution of *p*-wave Fermi Gases

S. S. Botelho and C. A. R. Sá de Melo

JLTP 140, 409 (2005)

G. E. Volovik's "Exotic Properties of Superfluid ^{3}He" (1992).

Topological superfluids with spin-orbit coupling for SU(2) fermions

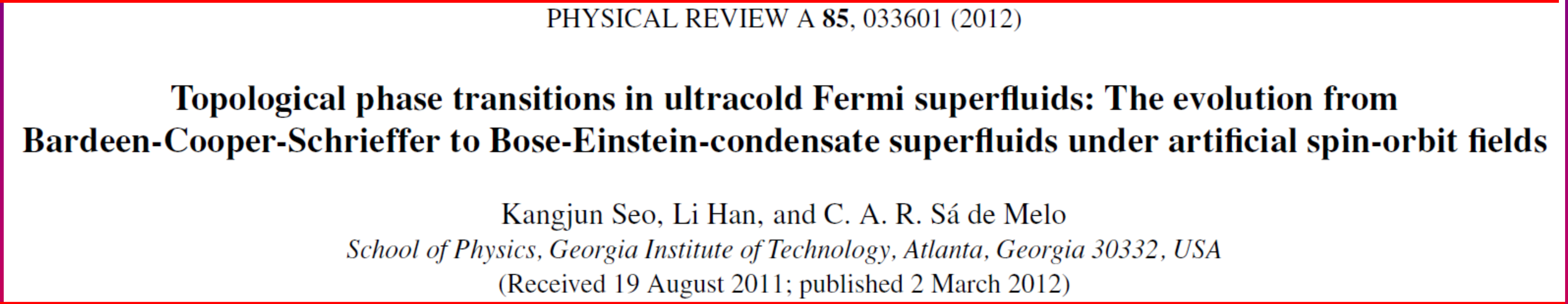


PHYSICAL REVIEW A **85**, 011606(R) (2012)

Evolution from BCS to BEC superfluidity in the presence of spin-orbit coupling

Li Han and C. A. R. Sá de Melo
School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332, USA

(Received 18 June 2011; published 27 January 2012)

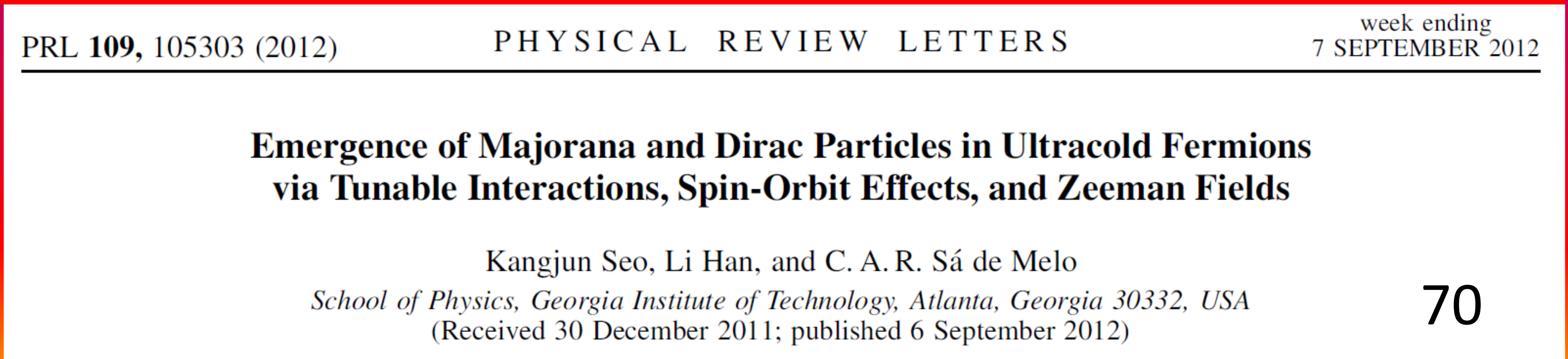


PHYSICAL REVIEW A **85**, 033601 (2012)

Topological phase transitions in ultracold Fermi superfluids: The evolution from Bardeen-Cooper-Schrieffer to Bose-Einstein-condensate superfluids under artificial spin-orbit fields

Kangjun Seo, Li Han, and C. A. R. Sá de Melo
School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332, USA

(Received 19 August 2011; published 2 March 2012)

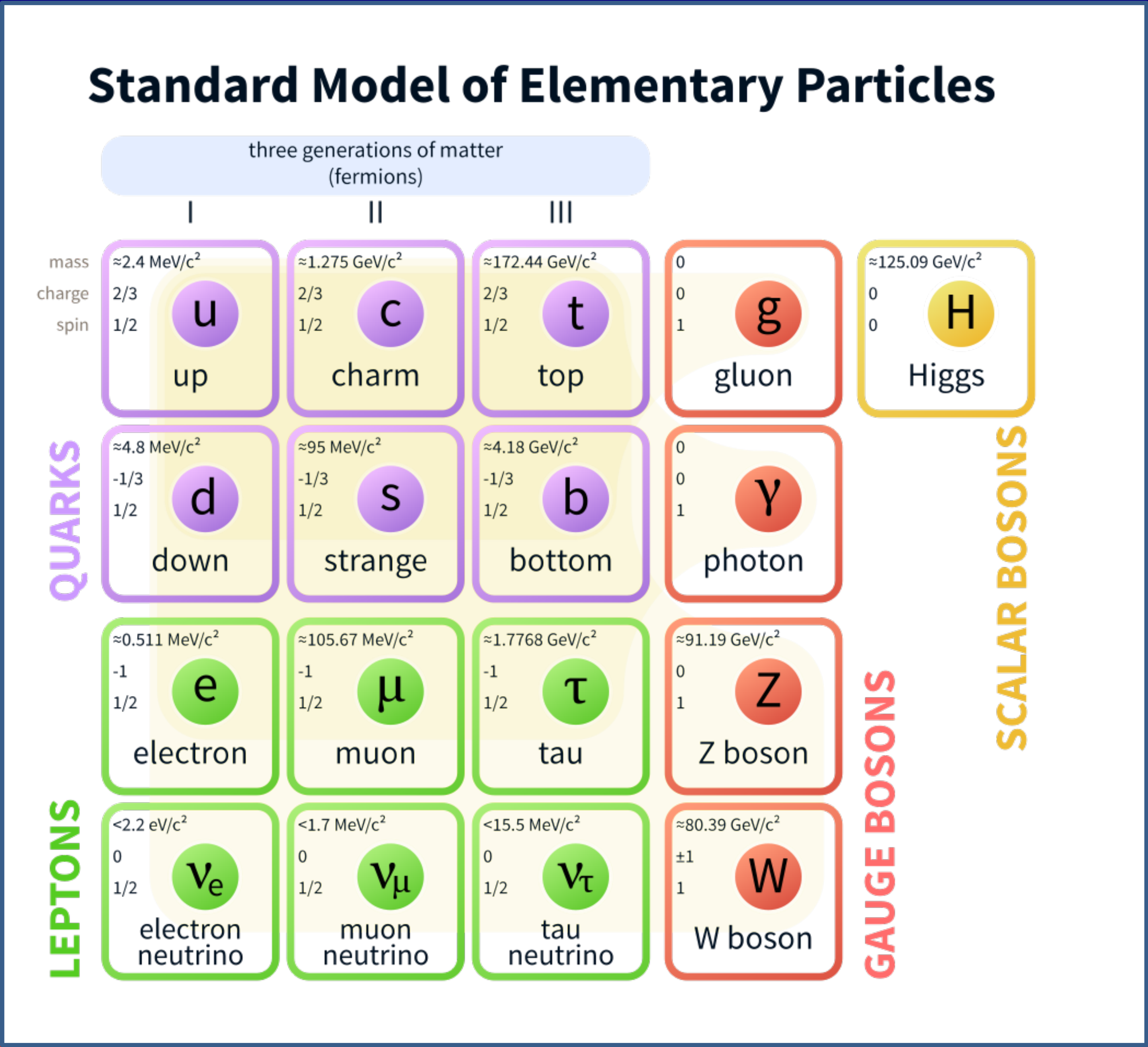


PRL **109,** 105303 (2012) PHYSICAL REVIEW LETTERS week ending 7 SEPTEMBER 2012

Emergence of Majorana and Dirac Particles in Ultracold Fermions via Tunable Interactions, Spin-Orbit Effects, and Zeeman Fields

Kangjun Seo, Li Han, and C. A. R. Sá de Melo
School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332, USA

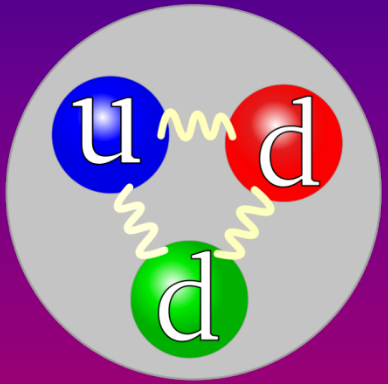
(Received 30 December 2011; published 6 September 2012)

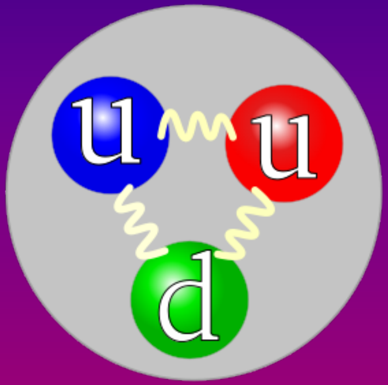


Standard Model of Elementary Particles

		three generations of matter (fermions)				
		I	II	III		
QUARKS	mass charge spin	≈2.4 MeV/c^2 2/3 1/2 u up	≈1.275 GeV/c^2 2/3 1/2 c charm	≈172.44 GeV/c^2 2/3 1/2 t top	0 0 1 g gluon	≈125.09 GeV/c^2 0 0 H Higgs
		≈4.8 MeV/c^2 -1/3 1/2 d down	≈95 MeV/c^2 -1/3 1/2 s strange	≈4.18 GeV/c^2 -1/3 1/2 b bottom	0 0 1 γ photon	
LEPTONS		≈0.511 MeV/c^2 -1 1/2 e electron	≈105.67 MeV/c^2 -1 1/2 μ muon	≈1.7768 GeV/c^2 -1 1/2 τ tau	≈91.19 GeV/c^2 0 1 Z Z boson	
		<2.2 eV/c^2 0 1/2 ν_e electron neutrino	<1.7 MeV/c^2 0 1/2 ν_μ muon neutrino	<15.5 MeV/c^2 0 1/2 ν_τ tau neutrino	≈80.39 GeV/c^2 ±1 1 W W boson	
					GAUGE BOSONS	SCALAR BOSONS

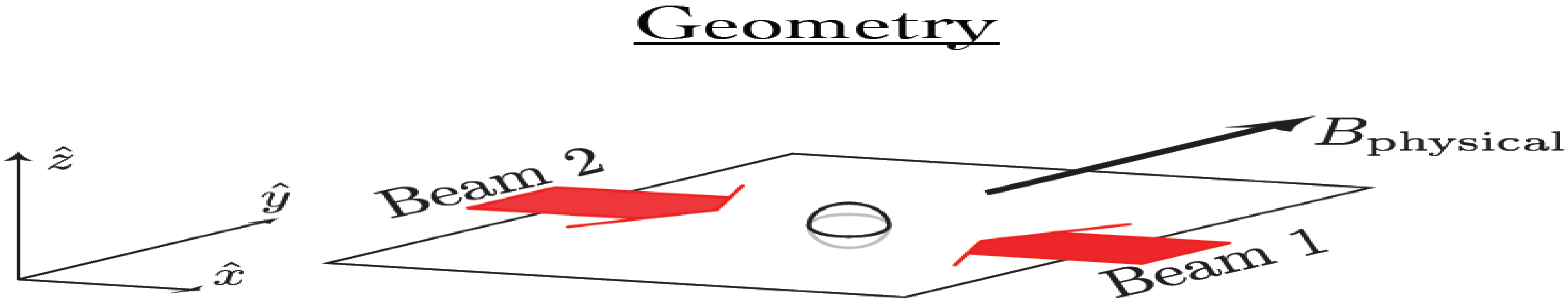
Structure of Neutron and Proton



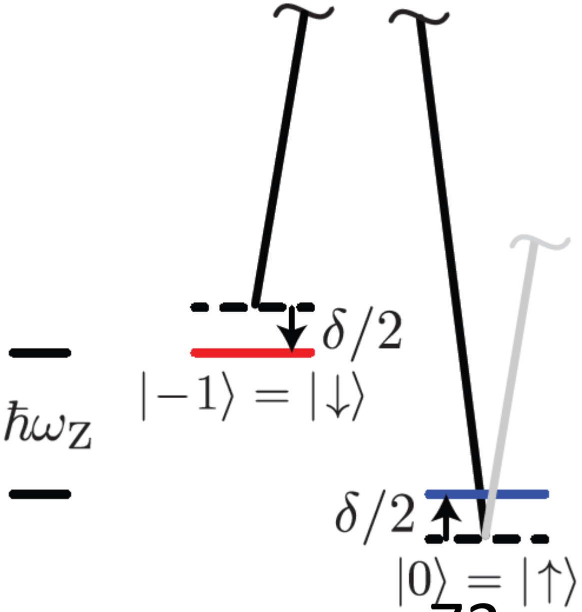


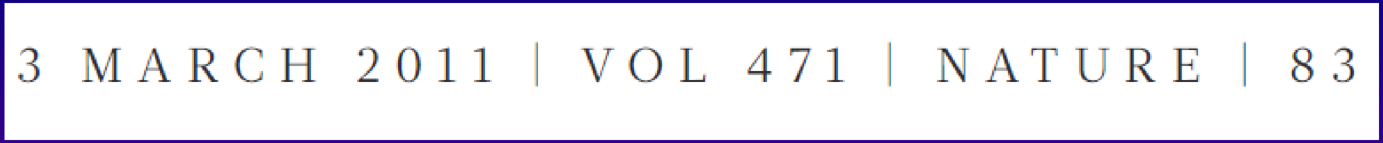
Having electric charge, mass, color charge and flavor, quarks are the only known elementary particles that engage in all four fundamental interactions of contemporary physics: electromagnetism, gravitation, strong interaction, and weak interaction.

SU(2) rotation to new spin basis: $\sigma_x \rightarrow \sigma_z$; $\sigma_z \rightarrow \sigma_y$; $\sigma_y \rightarrow \sigma_x$



$$\begin{pmatrix} \frac{\mathbf{k}^2+k_R^2}{2m}+\frac{\Omega}{2} & -i\left(\frac{\delta}{2}-\frac{k_R}{m}k_x\right) \\ i\left(\frac{\delta}{2}-\frac{k_R}{m}k_x\right) & \frac{\mathbf{k}^2+k_R^2}{2m}-\frac{\Omega}{2} \end{pmatrix}$$





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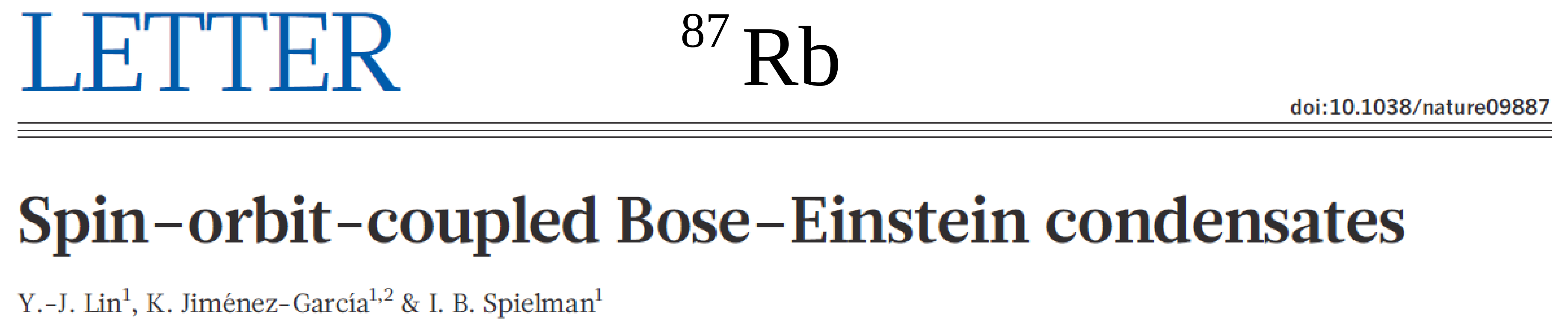


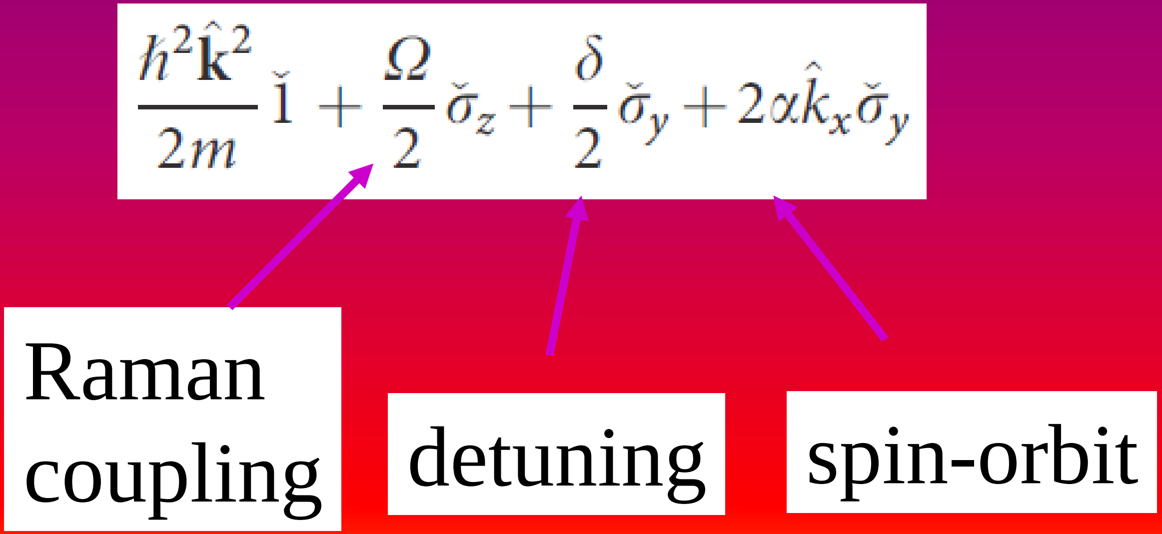
LETTER

^{87}Rb

doi:10.1038/nature09887

Spin–orbit-coupled Bose–Einstein condensates

Y.-J. Lin[1], K. Jiménez-García[1,2] & I. B. Spielman[1]



$$\frac{\hbar^2\hat{\mathbf{k}}^2}{2m}\breve{1}+\frac{\Omega}{2}\breve{\sigma}_z+\frac{\delta}{2}\breve{\sigma}_y+2\alpha\hat{k}_x\breve{\sigma}_y$$

Raman coupling

detuning

spin-orbit

Eigenvectors

$$\Phi(\mathbf{k}) = \mathbf{U}^{\dagger}_{\mathbf{k}}\Psi(\mathbf{k})$$

$$\mathbf{U}_{\mathbf{k}} = \begin{pmatrix} u_{\mathbf{k}} & v_{\mathbf{k}} \\ -v^{*}_{\mathbf{k}} & u_{\mathbf{k}} \end{pmatrix}$$

$$\phi_{\Uparrow}(\mathbf{k}) = u_{\mathbf{k}}\psi_{\mathbf{k}\uparrow} - v_{\mathbf{k}}\psi_{\mathbf{k}\downarrow}$$

$$\phi_{\Downarrow}(\mathbf{k}) = v^{*}_{\mathbf{k}}\psi_{\mathbf{k}\uparrow} + u_{\mathbf{k}}\psi_{\mathbf{k}\downarrow}$$

$$\xi_{\Uparrow}(\mathbf{k}) = \varepsilon_{\Uparrow}(\mathbf{k}) - \mu$$
$$\xi_{\Downarrow}(\mathbf{k}) = \varepsilon_{\Downarrow}(\mathbf{k}) - \mu$$

$$\mathbf{E}(\mathbf{k}) = \begin{pmatrix} \xi_{\Uparrow}(\mathbf{k}) & 0 \\ 0 & \xi_{\Downarrow}(\mathbf{k}) \end{pmatrix}$$

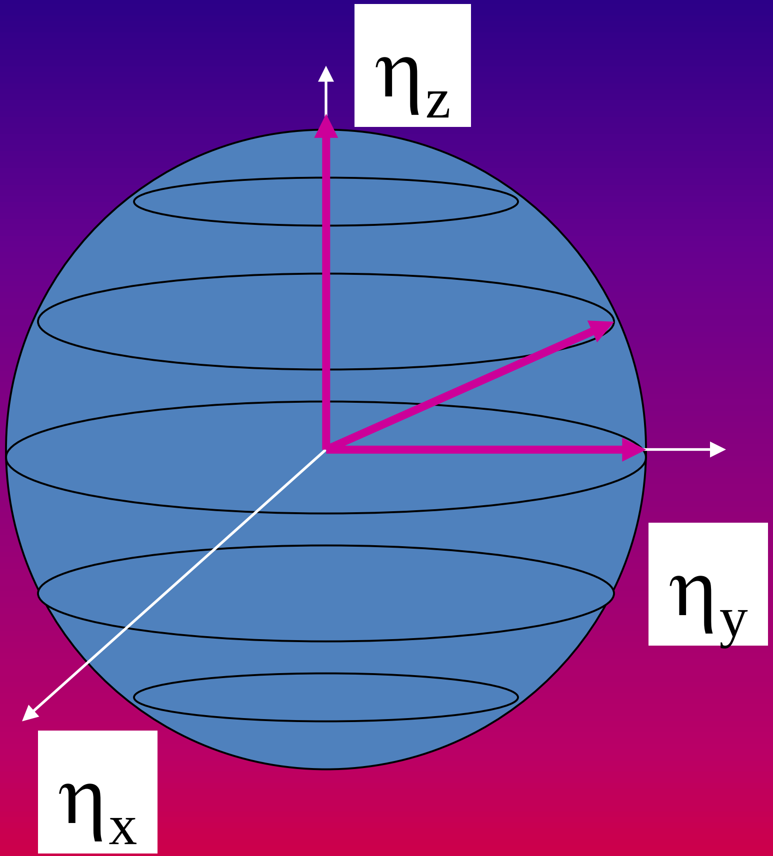
$$\mathbf{E}(\mathbf{k}) = \mathbf{U}^{\dagger}_{\mathbf{k}}\mathbf{H}_0(\mathbf{k})\mathbf{U}_{\mathbf{k}}$$

Order Parameter: Singlet & Triplet

$$\Delta_S(\mathbf{k}) = \Delta_0 h_{||}(\mathbf{k})/|\mathbf{h}_{\text{eff}}(\mathbf{k})|$$

$$\Delta_T(\mathbf{k}) = \Delta_0 |h_{\perp}(\mathbf{k})|/|\mathbf{h}_{\text{eff}}(\mathbf{k})|$$

$$|\Delta_T(\mathbf{k})|^2 + |\Delta_S(\mathbf{k})|^2 = |\Delta_0|^2$$



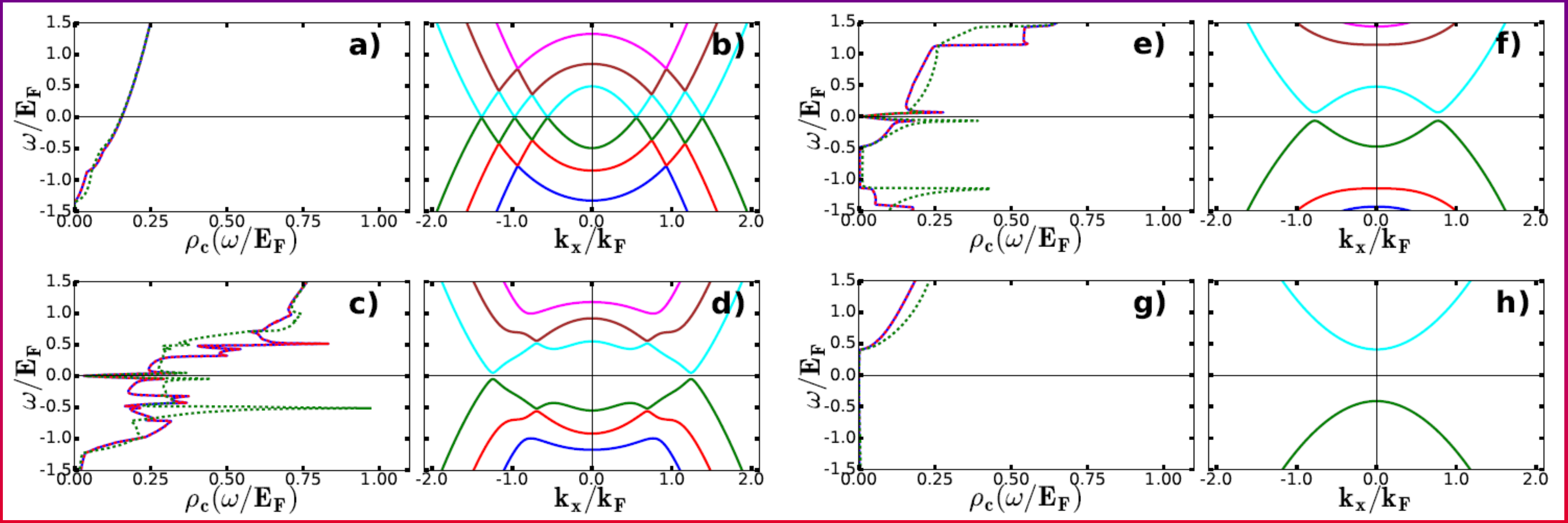
$$h_{\perp}(\mathbf{k}) = vk_x \quad h_z(\mathbf{k}) = h_z$$

$$h_{\text{eff}}(\mathbf{k}) = (0, vk_x, h_z)$$

Color density of states

N3

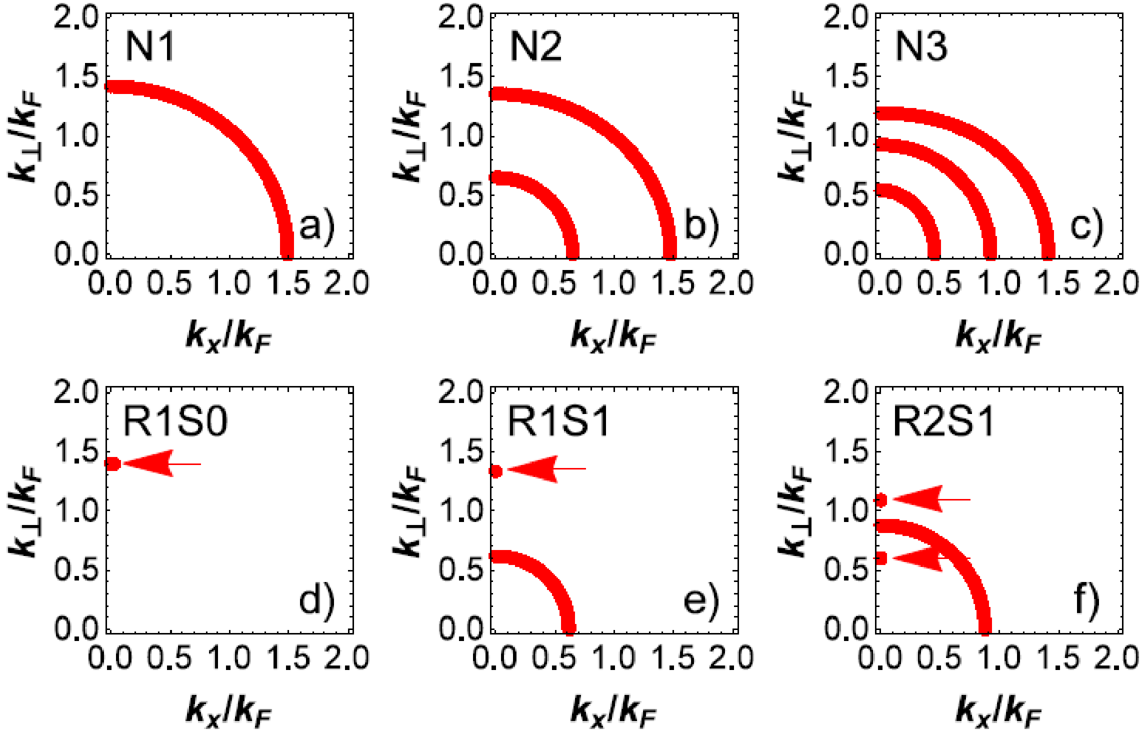
R1



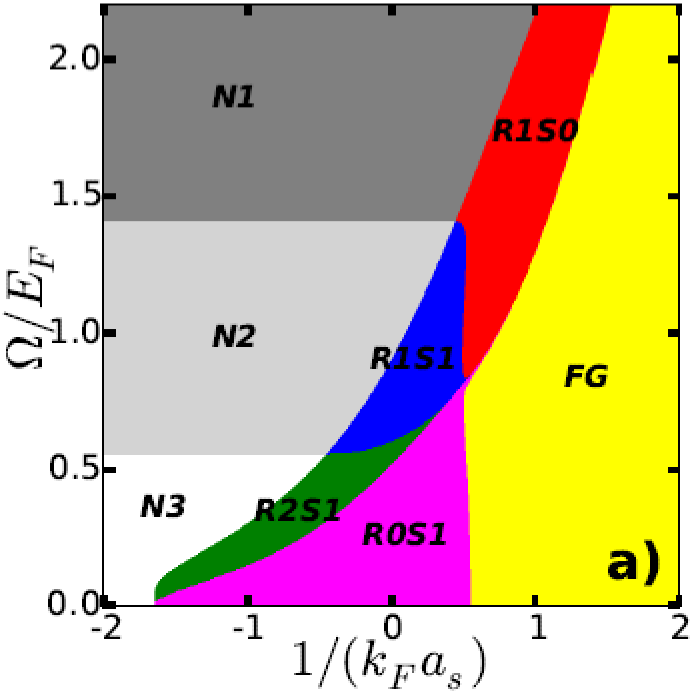
R3

FG

Single interaction channel

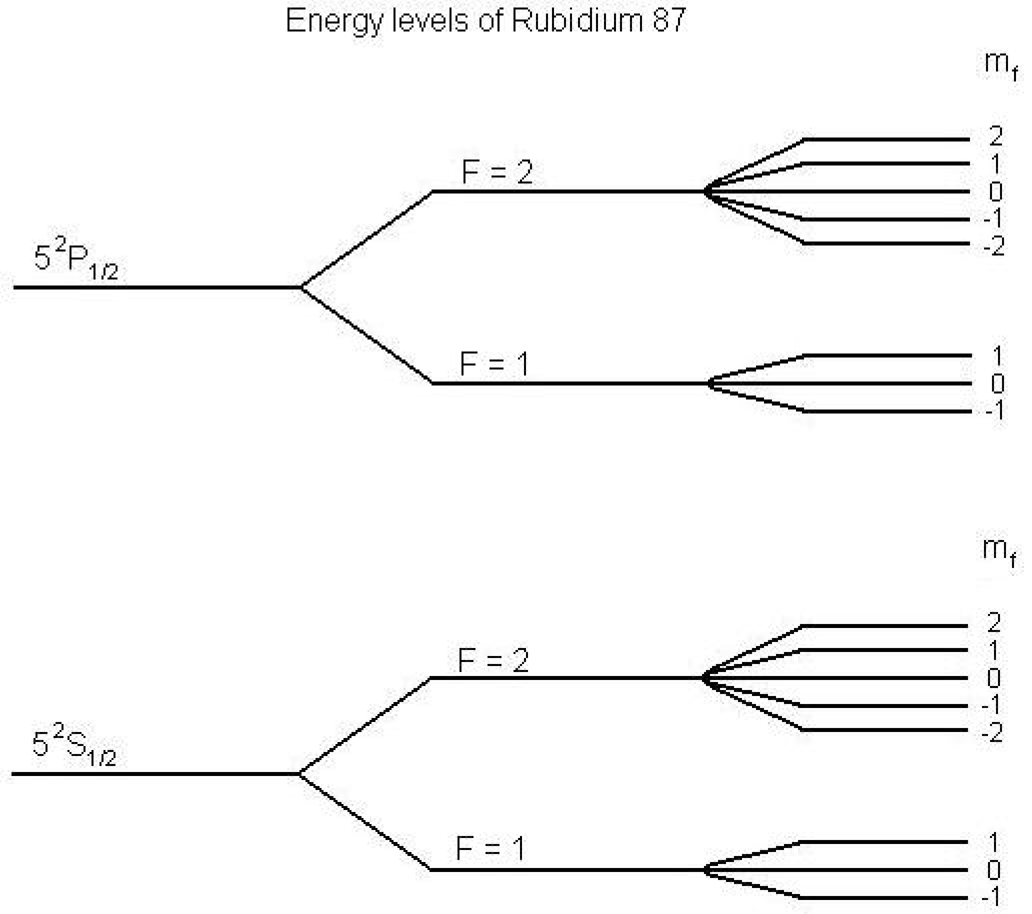


NODAL STRUCTURE OF QUASIPARTICLE SPECTRUM



PHASE DIAGRAM

Hyperfine Structure of ^{87}Rb



Hyperfine Structure of ^{6}Li

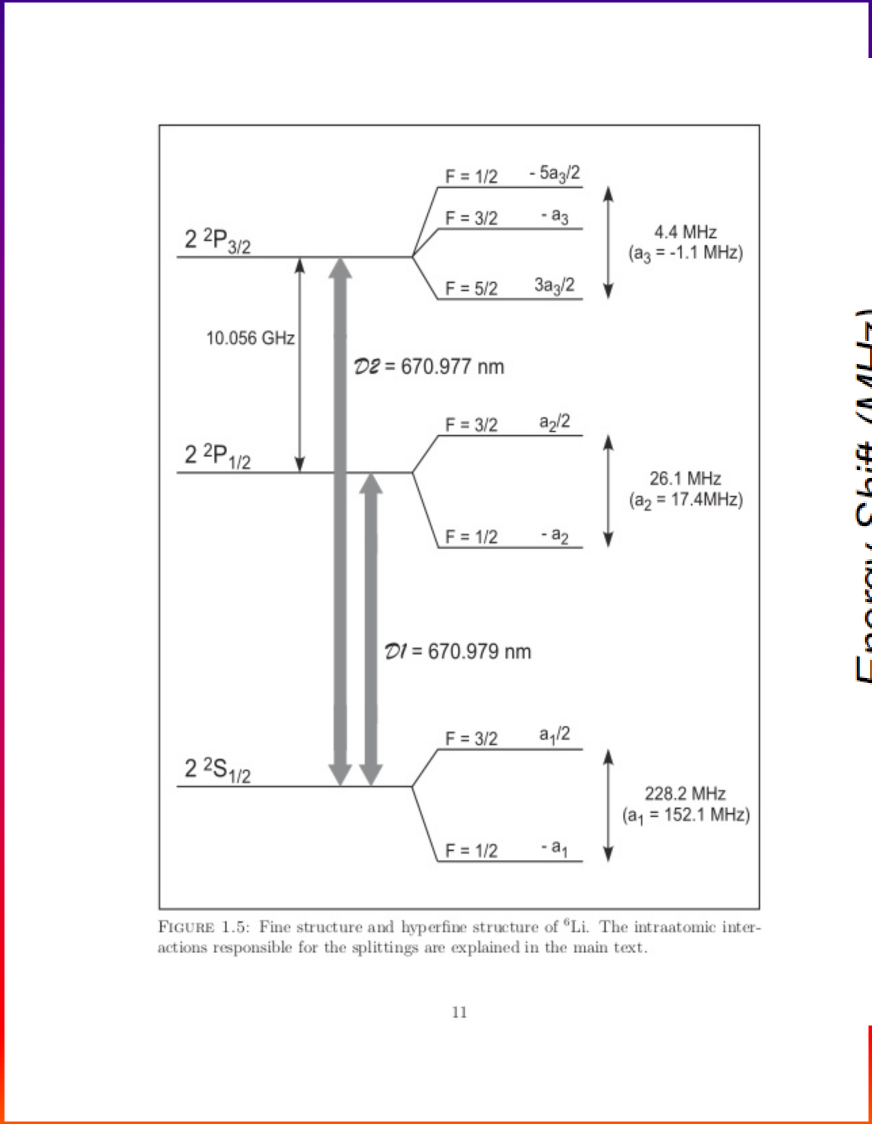


FIGURE 1.5: Fine structure and hyperfine structure of ^{6}Li. The intraatomic interactions responsible for the splittings are explained in the main text.

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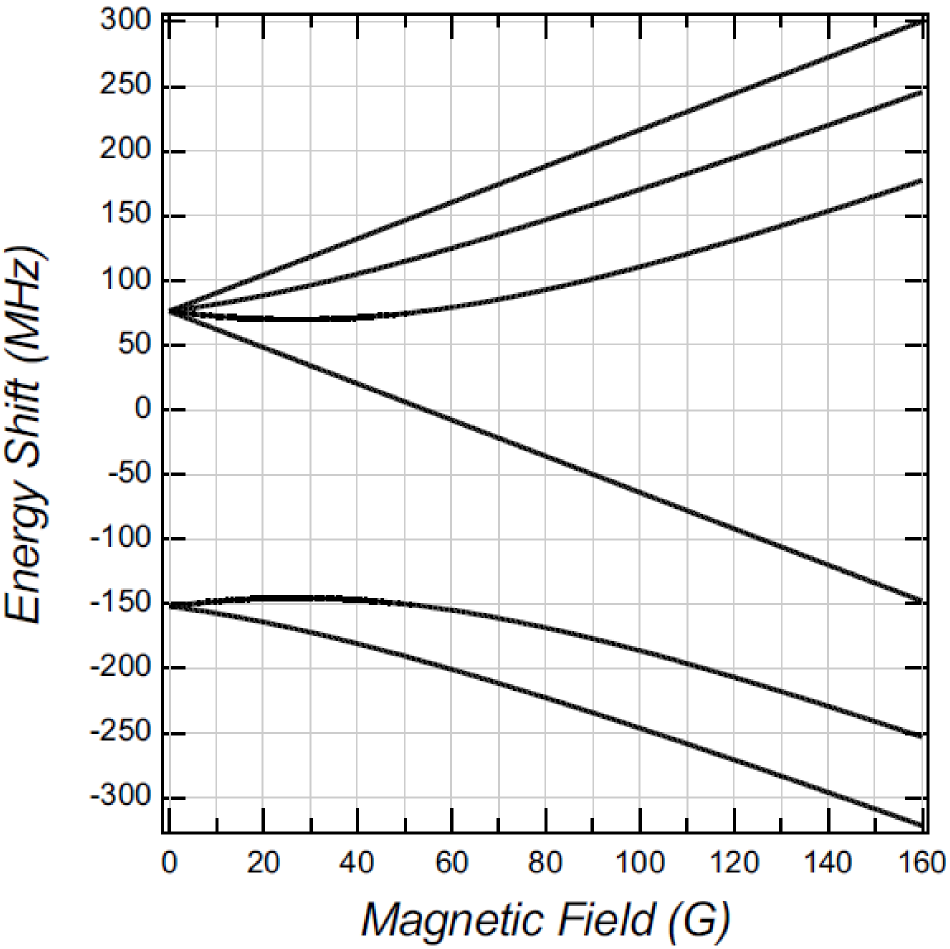
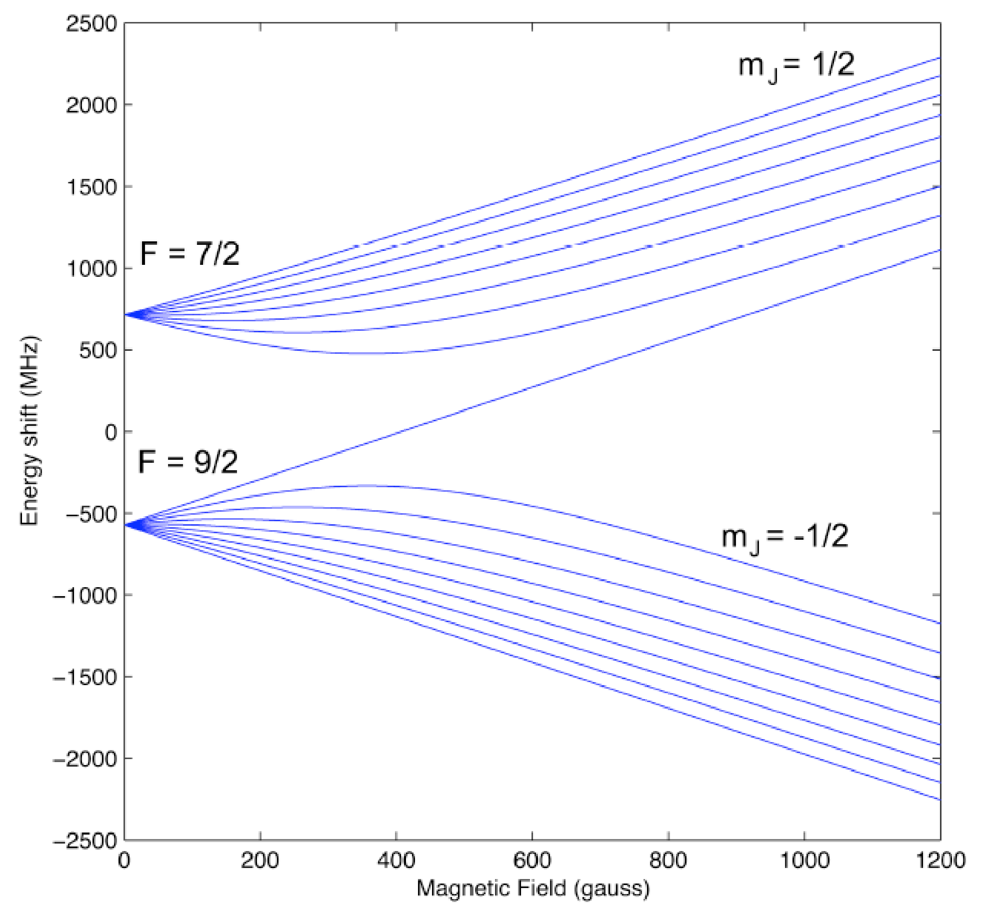


Hyperfine Structure of ^{40}K



Hyperfine Structure of ^{173}Yb