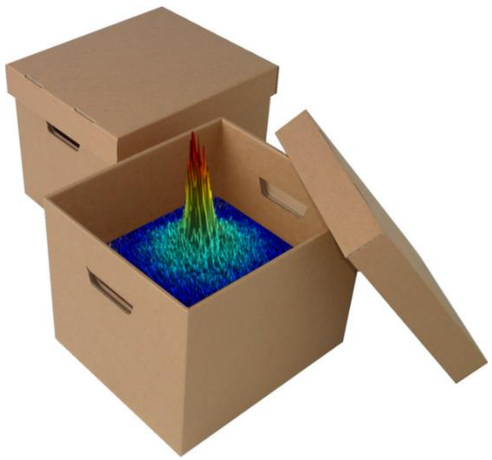
Yale University

UNIVERSITY OF CAMBRIDGE

Turbulence in a Quantum Gas



Nir Navon

Experimentalists: Christoph Eigen, Jinyi Zhang, Alex Gaunt, Raphael Lopes, Sam Garratt, Patrik Turzak, Rob Smith, Zoran Hadzibabic
Theorists: Kazuya Fujimoto, Makoto Tsubota

EPSRC Engineering and Physical Sciences Research Council

erc

the David & Lucile Packard FOUNDATION

Trento, 21st June 2018

Outline

- What is turbulence, and why is it interesting?
- The optical box trap: a new tool for ultracold atoms
- Low-energy excitations of a box-trapped BEC
- A turbulent steady-state
- A synthetic dissipation scale and turbulent-cascade fluxes

Hierarchy in complexity

Equilibrium

Overarching principle: maximum entropy
Many methods to efficiently calculate static quantities
Phase transitions, etc.

Near equilibrium

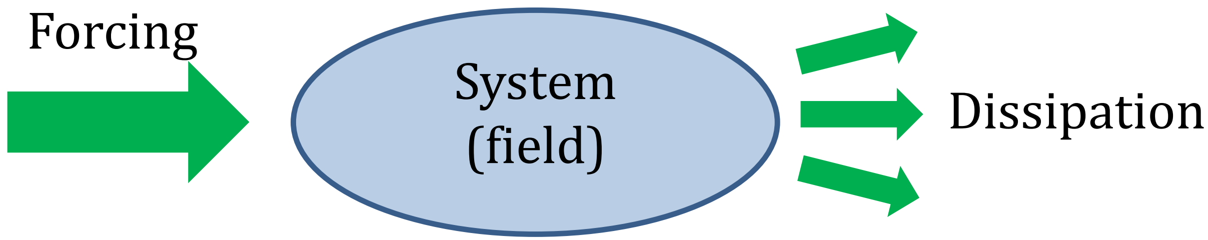
Linear response theory
Universality of low-energy excitations
Transport coefficients, etc.

Far from equilibrium

Nothing like equilibrium states
Existence of an overarching principle?
Even classical problem intractable

What is Turbulence ?

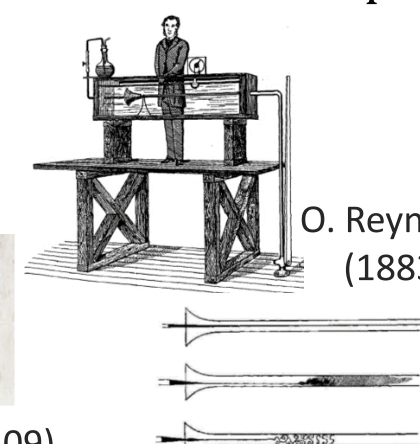
Quintessential phenomenon of out-of-equilibrium physics



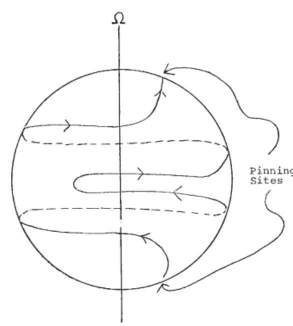
Far-from-equilibrium states that (usually) involves

- drive + dissipation
- steady in a statistical sense
- local restoration of symmetries (isotropy/homogeneity)
- many interacting degrees of freedom
- chaotic properties

Leonardo da Vinci (1509)

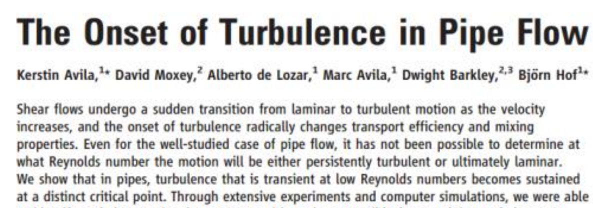


O. Reynolds (1883)



SF turbulence neutron stars / glitches? (1969)

And ongoing…



The Onset of Turbulence in Pipe Flow

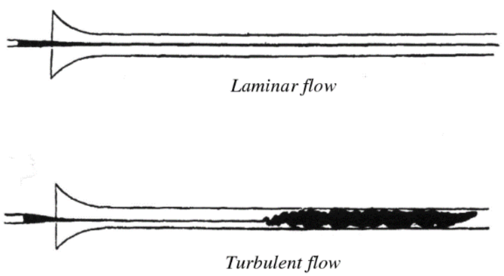
Kerstin Avila,[1*] David Moxey,[2] Alberto de Lozar,[1] Marc Avila,[1] Dwight Barkley,[2,3] Björn Hof[1*]

Shear flows undergo a sudden transition from laminar to turbulent motion as the velocity increases, and the onset of turbulence radically changes transport efficiency and mixing properties. Even for the well-studied case of pipe flow, it has not been possible to determine at what Reynolds number the motion will be either persistently turbulent or ultimately laminar. We show that in pipes, turbulence that is transient at low Reynolds numbers becomes sustained at a distinct critical point. Through extensive experiments and computer simulations, we were able to identify and characterize the processes ultimately responsible for sustaining turbulence. In

Science (2011)

Hydrodynamic vs. Wave Turbulence

Hydrodynamic (HD) equations **v**(**r**,t)
- Static solutions (e.g. Poiseuille)
- Static solutions can be unstable
    → time invariance is broken
    → chaotic, turbulent state can emerge



Eq of motion for ***wave*** occupations
e.g. **n**(**k**,t)
Nonlinearity: waves ~~eigenmodes~~
Analytic insights for weak interactions
(weak wave turbulence)

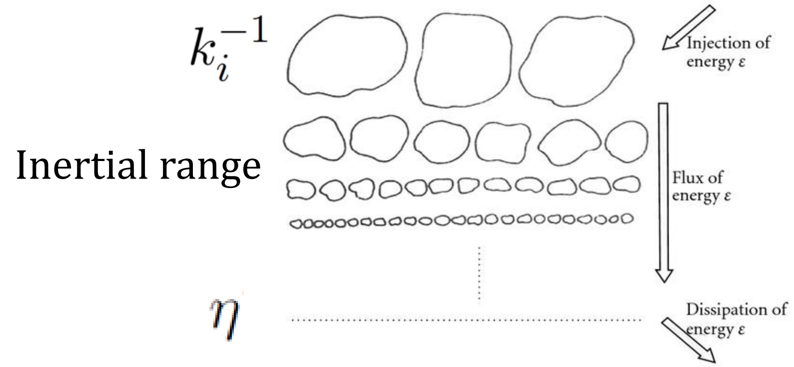
Turbulence is complicated because:
- nonlinearity (e.g. inertial term in NS)
- no perturbation theory, far beyond linear response
- many length and time scales simultaneously involved

Even the fully classical problem is numerically untractable for many realistic flows

How can one say anything general?

Turbulent cascades

A central phenomenology of turbulence: cascades of *something*, *gradually* transported across different length scales

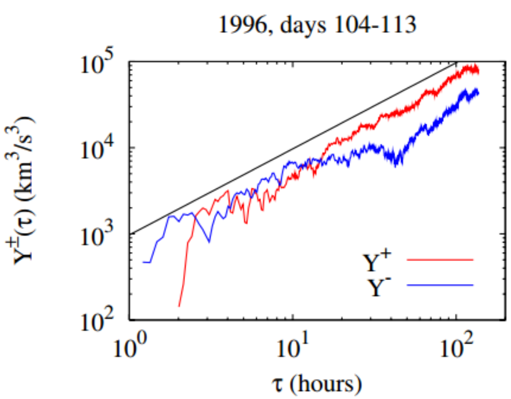


The most famous example: "Kolmogorov 5/3" (hydro)

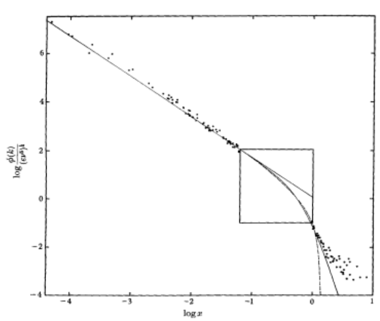
kinetic energy cascade of incompressible flow

$$E_k = C_K \epsilon^{2/3} k^{-5/3}$$

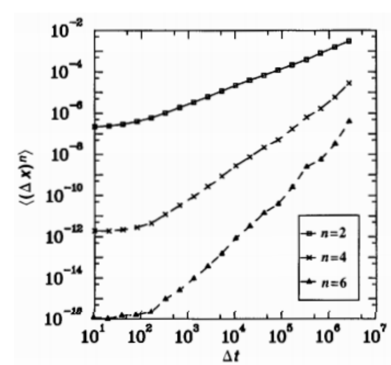
Turbulent cascades are everywhere



Interplanetary Plasma
Sorriso-Valvo et al, PRL 2007



Water in a tidal channel
Grant et al, J. Fluid Mech. 1962



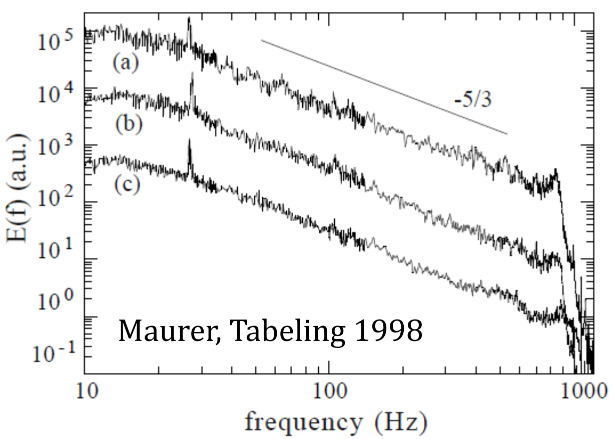
Foreign exchange markets
Ghashgale et al, Nature 1996

Turbulence in quantum fluids

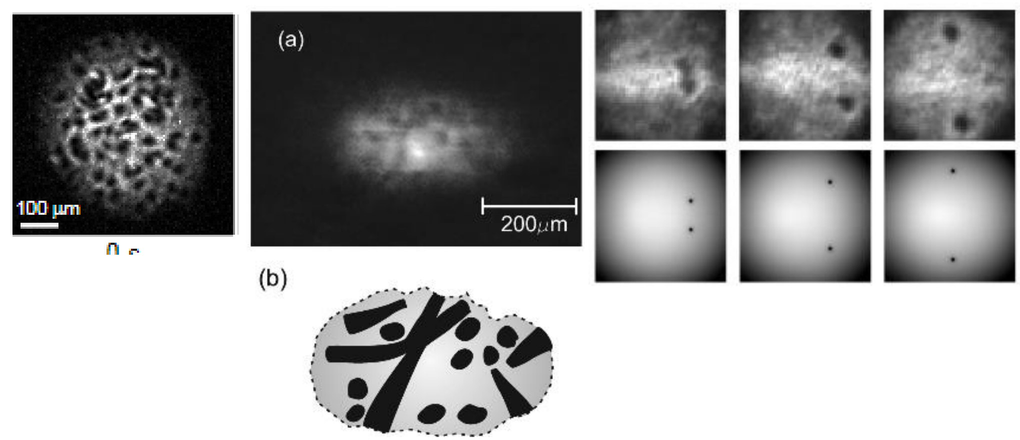
Interesting twists: - no (intrinsic) dissipation scale (at low T)
- strongly restricted flow (irrotational)

For a long time, only SF helium, but no 1st-principle description

(neutron stars are conjecured to be in a SF turbulent state, possible



Groups in Maryland, Lancaster, Saclay, Lyon, Grenoble, …

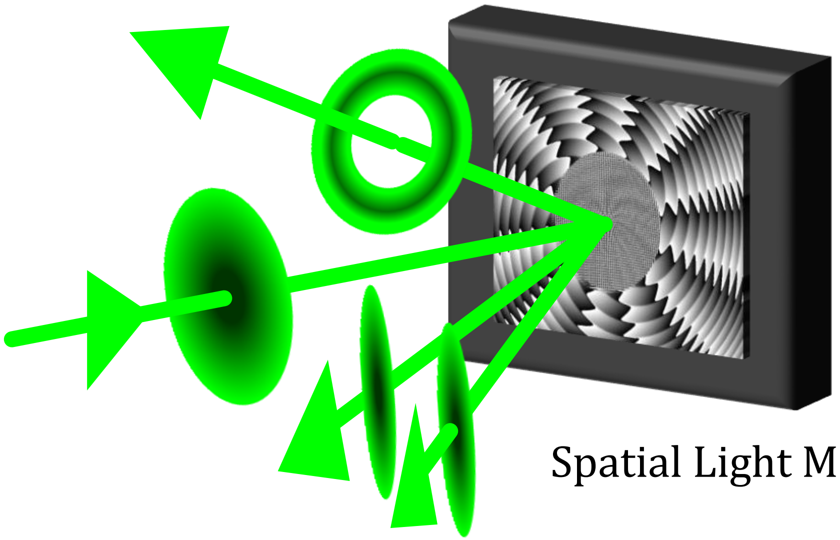


V. Bagnato (Brazil), B. P. Anderson (US), Y. Shin (South Korea), P. Engels (Washington), K. Helmerson and T. Neely (Australia)

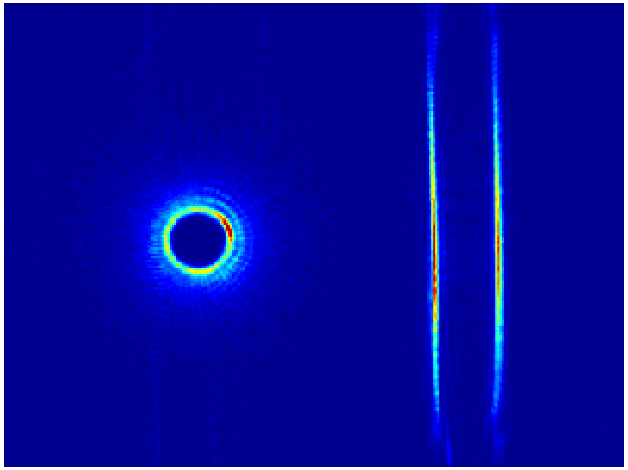
Nonlinear Schrodinger Equation $$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\Delta + g|\psi|^2\right)\psi$$

Quantum-gas fluid than can be simulated on all relevant scales, and we have new knobs!

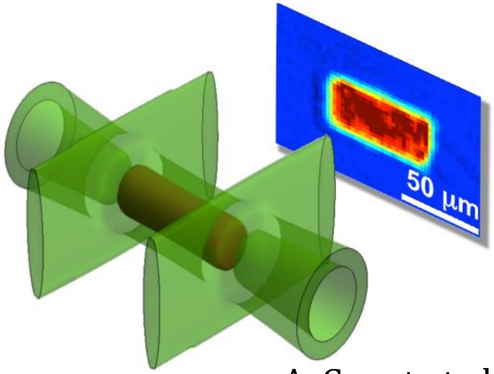
The Box Potential



Spatial Light Modulator



With (bosonic) ^{87}Rb atoms in :



A. Gaunt et al., PRL 110, 200406 (2013)

Hierarchy in complexity

Equilibrium

Overarching principle: maximum entropy
Many methods to efficiently calculate static quantities
Phase transitions, etc.

Near equilibrium

Linear response theory
Universality of low-energy excitations
Transport coefficients, etc.

Drive strength

Far from equilibrium

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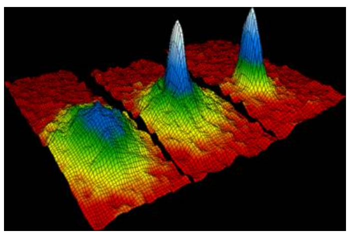
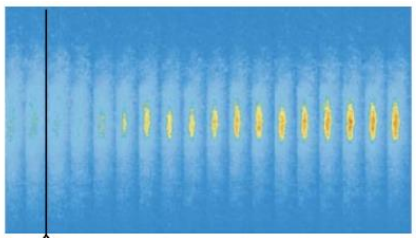
Harmonic vs. Uniform BEC

BEC occurs both in real and momentum space

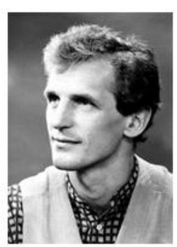


Time of Flight (JILA)



In-situ (MIT)

2001

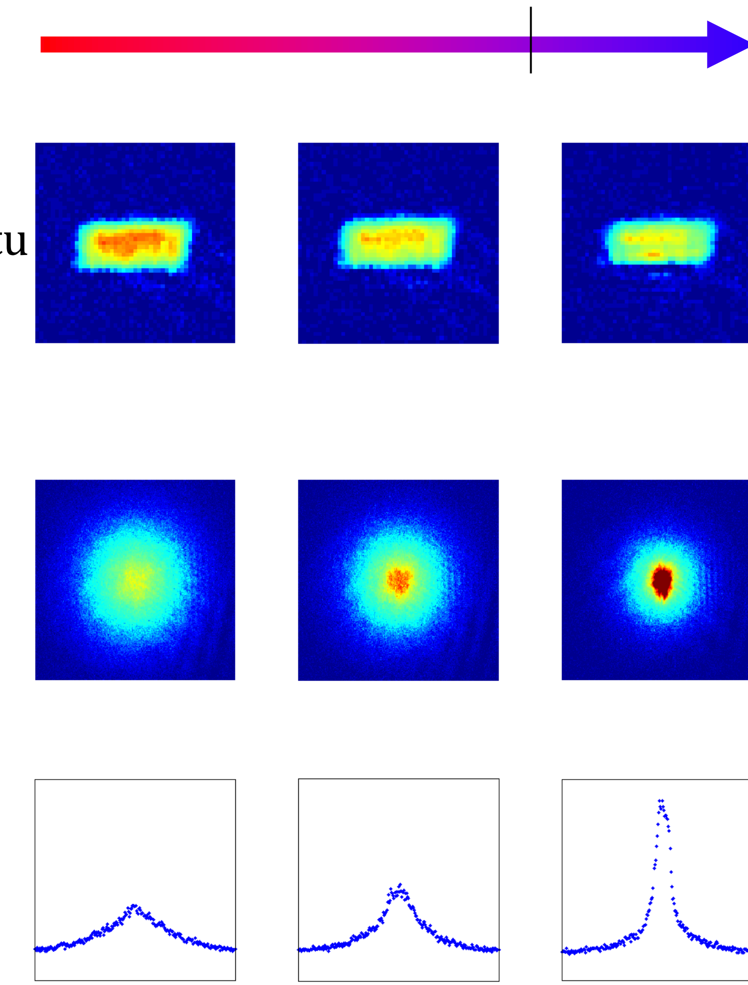


C. Wieman, E. Cornell

W. Ketterle

Reduce Temperature T_c

In-situ

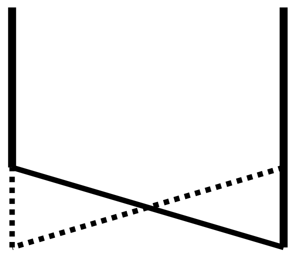


Only in momentum space !

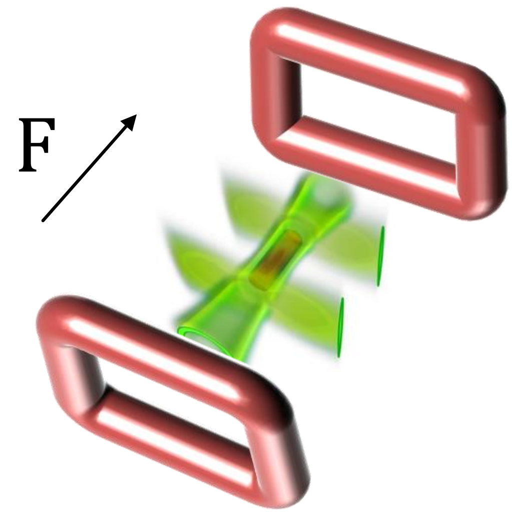
Shaken, not stirred

Pump energy at the largest scale, see what happens…

Easily realisable perturbation potential : time-dependent gradient



$$V(\mathbf{r},t) = \alpha \sin(\Omega t) x$$



4 Stages :

- Linear response
- Nonlinear response
- Turbulence
- Relaxation (ongoing)

?

(and back?)

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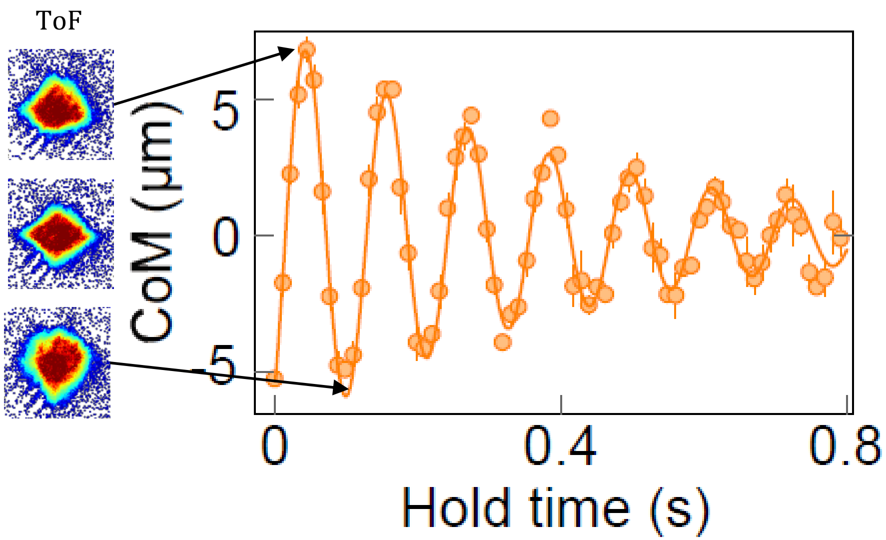
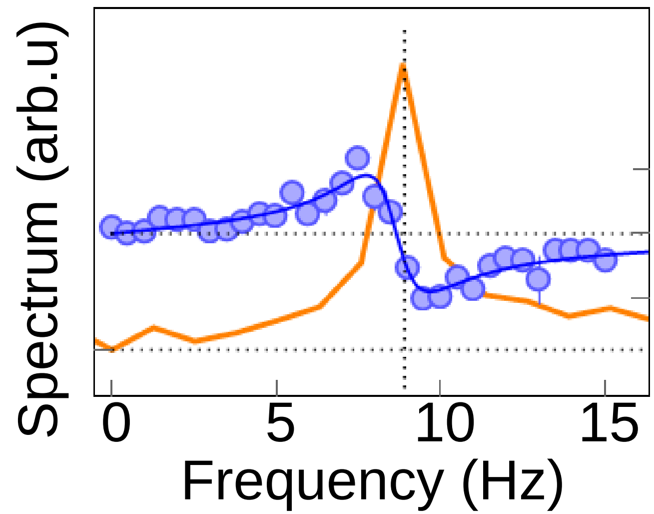
Linear regime: collective modes

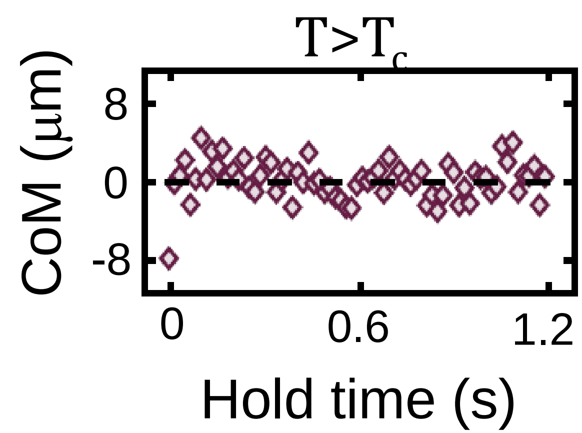
Single kick



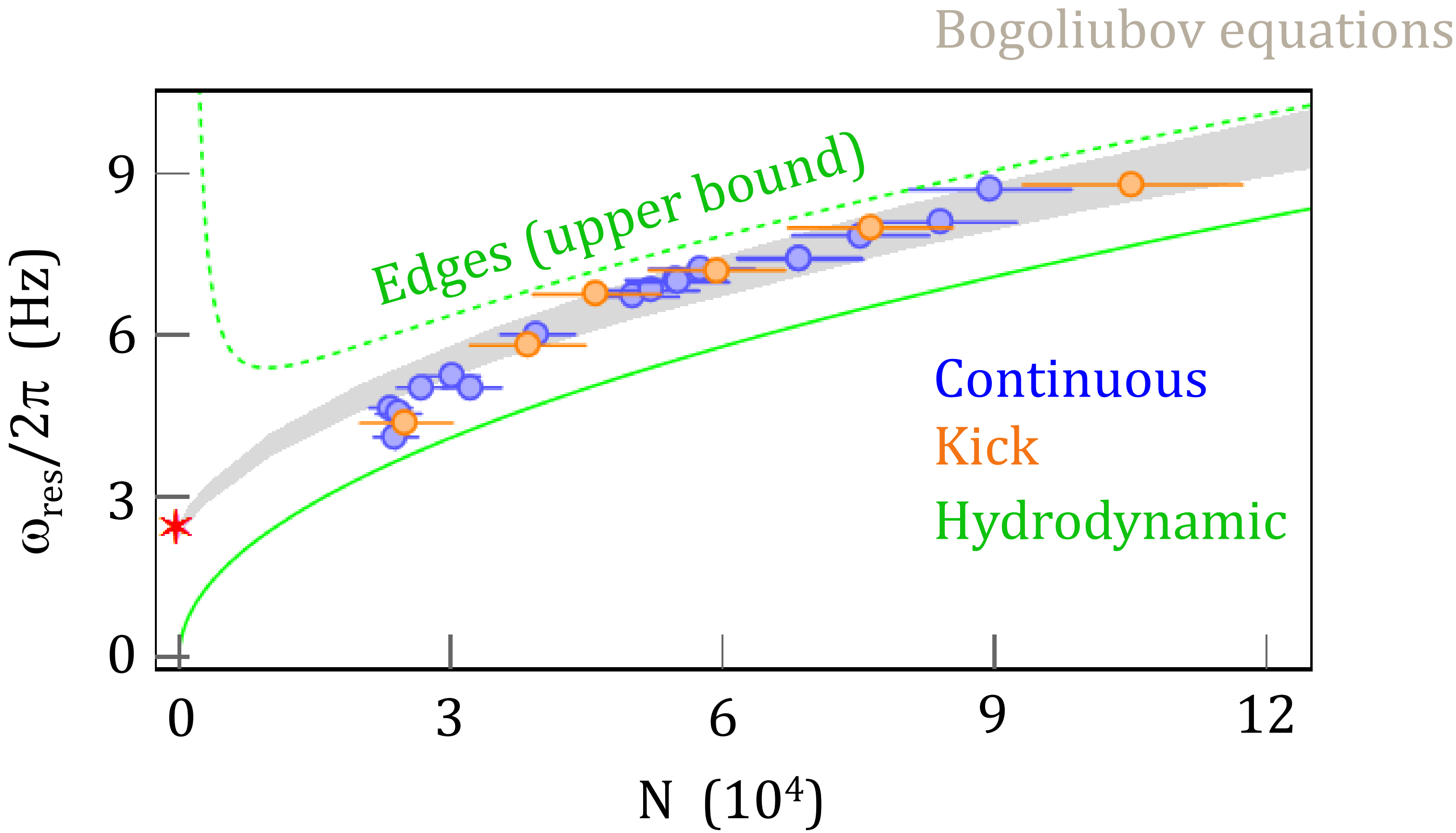


Or response to a continuous drive

Fun note: this mode exists only below T_c



Linear regime: collective modes



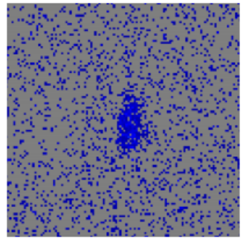
Weak perturbation : Bogoliubov equations from linearised GPE

$$\psi(\mathbf{r},t) = e^{-i\mu t/\hbar}(\psi_0(\mathbf{r}) + u(\mathbf{r})e^{-i\omega t} - v^*(\mathbf{r})e^{i\omega t})$$

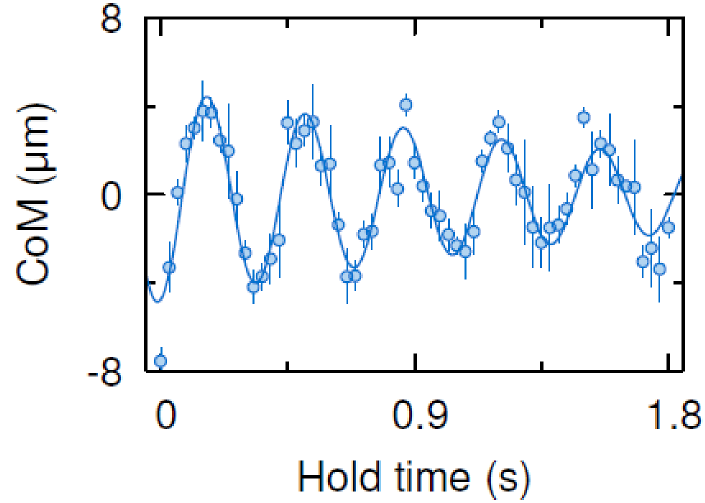
Non-trivial modes in the box (with fixed boundary conditions)

Linear regime: collective modes

The single-particle limit



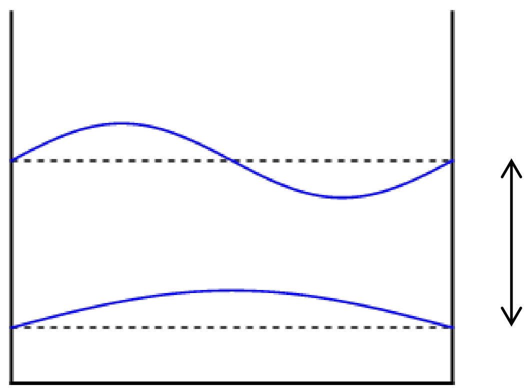
$N \approx 10^3$ (with 140ms ToF)



We measure $\frac{\hbar\omega_{\text{res}}}{2\pi} \approx 2.6 \text{ Hz}$

n=2

n=1



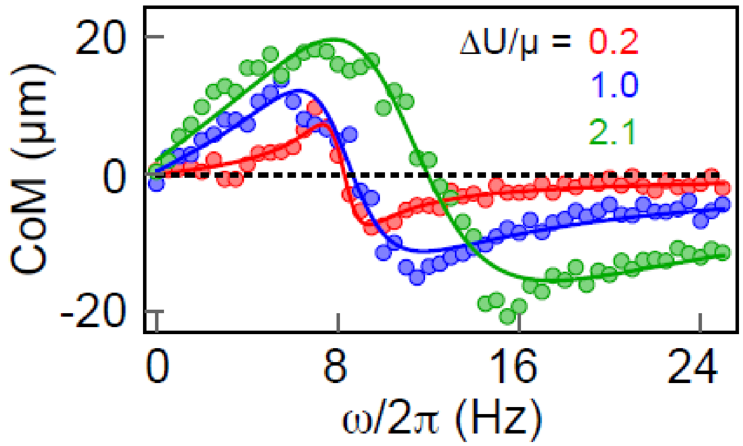
$$\Delta E = 3\pi^2 \frac{\hbar^2}{2mL^2}$$

(gives 2.7 Hz)

Good thing we have a BEC… $E_{n=1} \approx k_B \times 40 \text{ pK}$

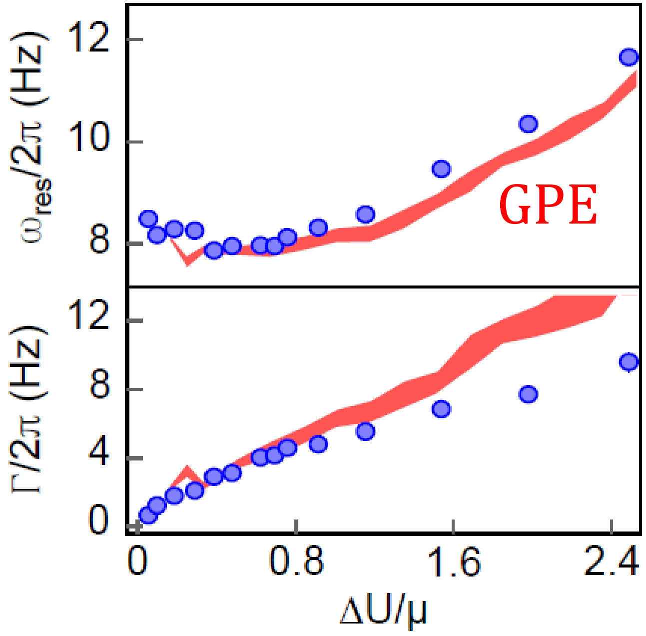
but $\mu \ll E_{n=1}$ (0.3 Hz $\ll$ 0.8 Hz)

Nonlinear Response



Beyond Bogoliubov necessary

No “leader-order” frequency shift
Significant “leading-order” broadening
Energy has to go somewhere… everywhere?



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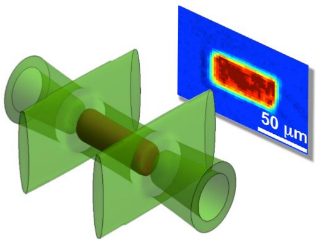
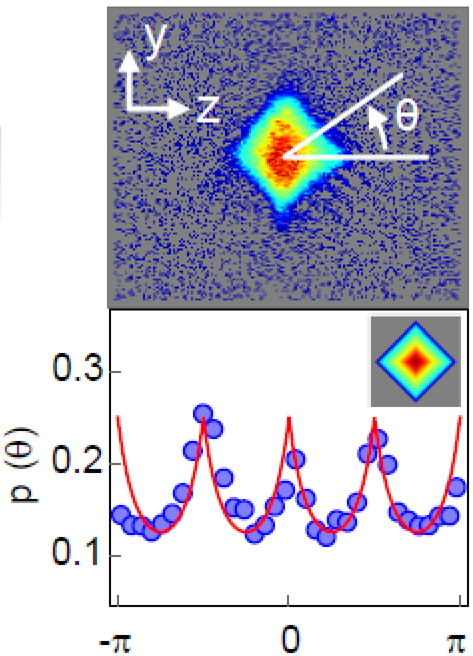
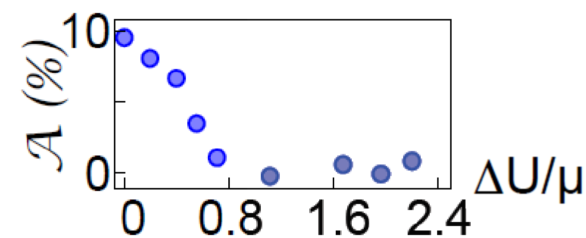
Emergence of Turbulence
(in the experiment)

Initial BEC





Anistropy parameter $\mathcal{A} = \frac{1}{2} \int \mathrm{d}\theta \left| p(\theta) - \frac{1}{2\pi} \right|$



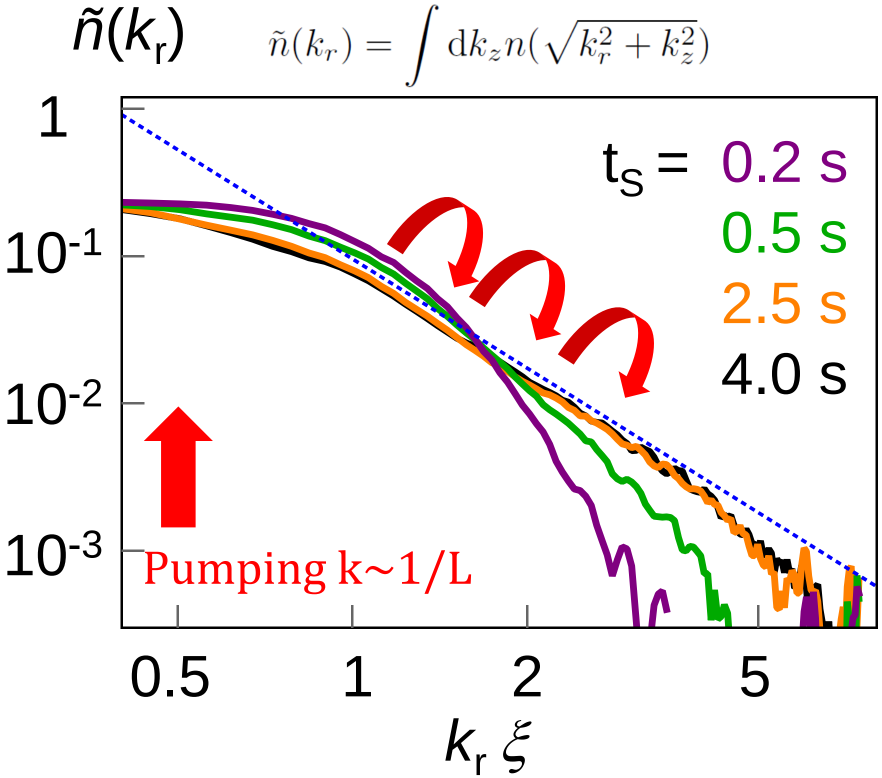
Isotropic state yet highly degenerate (not thermal!)

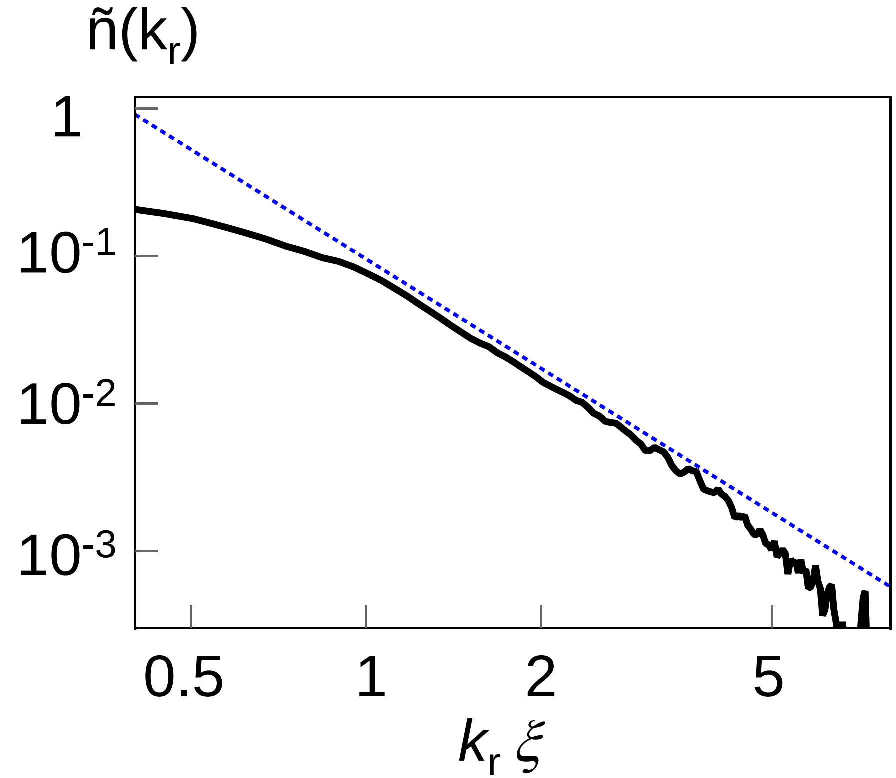
Is ToF a sane technique to use for a highly degenerate gas ?

Turbulent Cascade

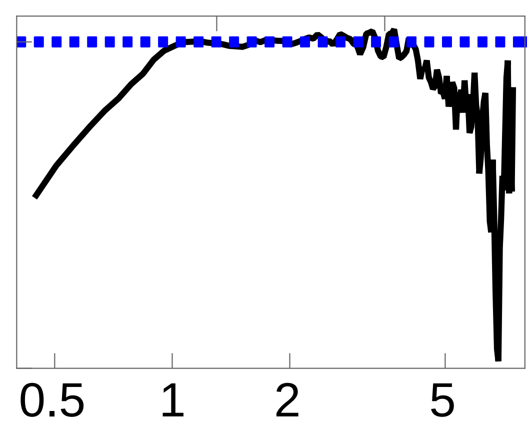
N. Navon *et al*, *Nature* **539** 72 (2016)

$$\tilde{n}(k_r) = \int \mathrm{d}k_z n(\sqrt{k_r^2 + k_z^2})$$

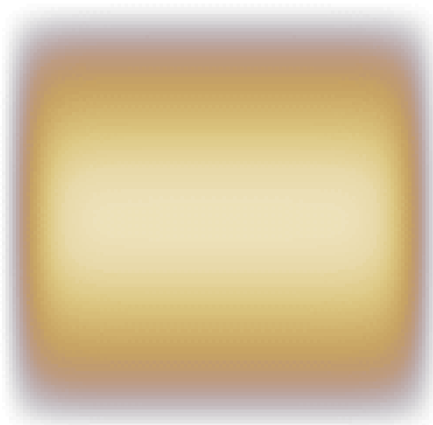




Compensated spectrum $k_r^{\gamma-1}\,\tilde{n}(k_r)$



Emergence of Turbulence
(numerical simulations)

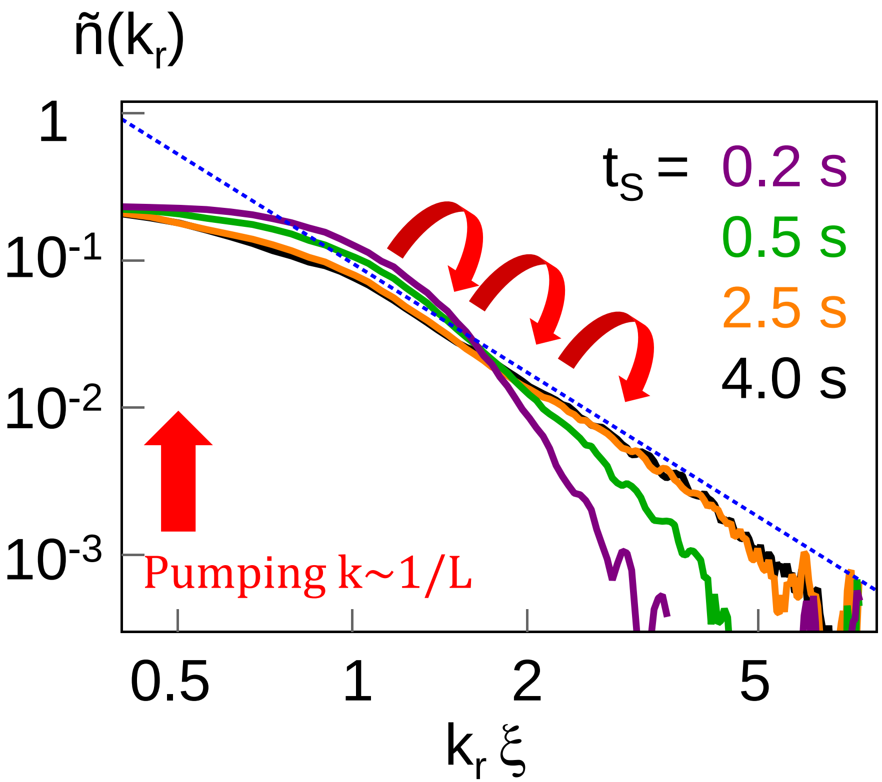


In situ density

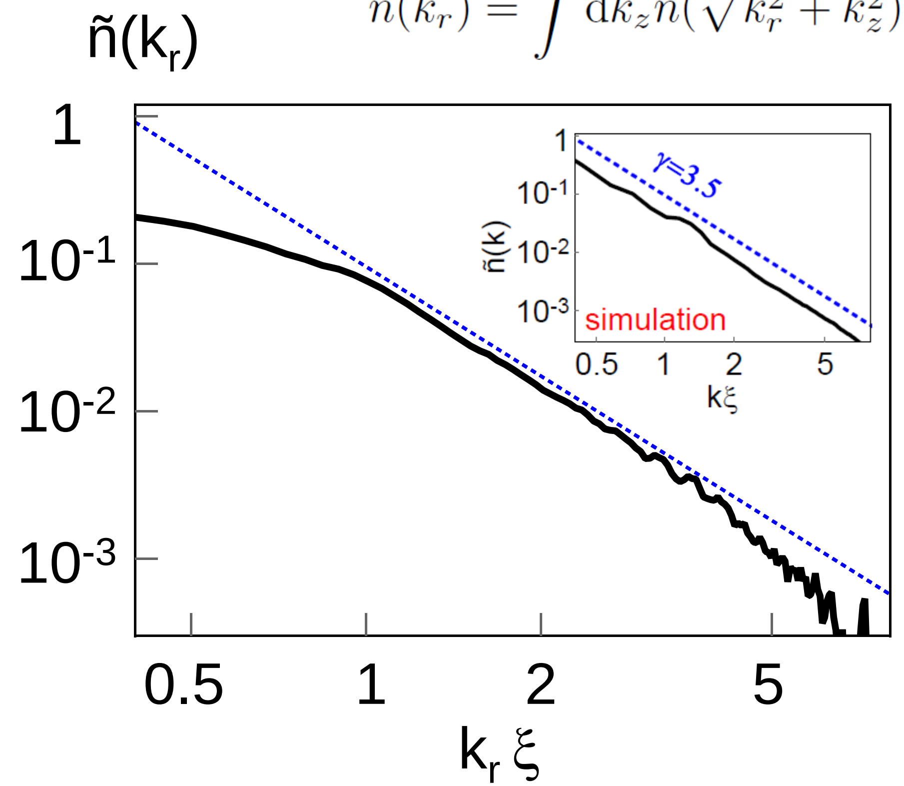
GPE in 256^3 grid with 10 μs steps for up to 4s (GPU)

30 min computation time per 1 sec experimental time

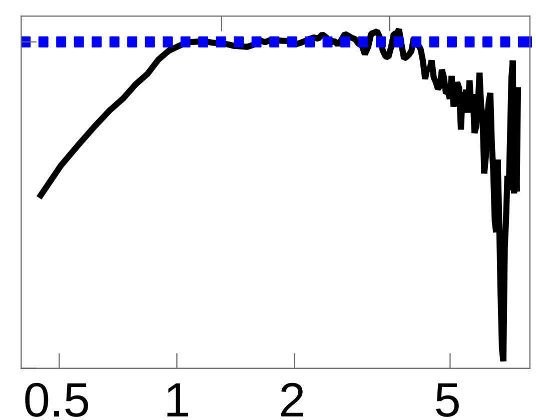
Turbulent Cascade



$$\tilde{n}(k_r) = \int \mathrm{d}k_z n(\sqrt{k_r^2 + k_z^2})$$



Compensated spectrum $k_r^{\gamma-1}\tilde{n}(k_r)$



Value of γ? $n(\mathbf{k}) \propto k^{-\gamma}$

Summary/Outlook

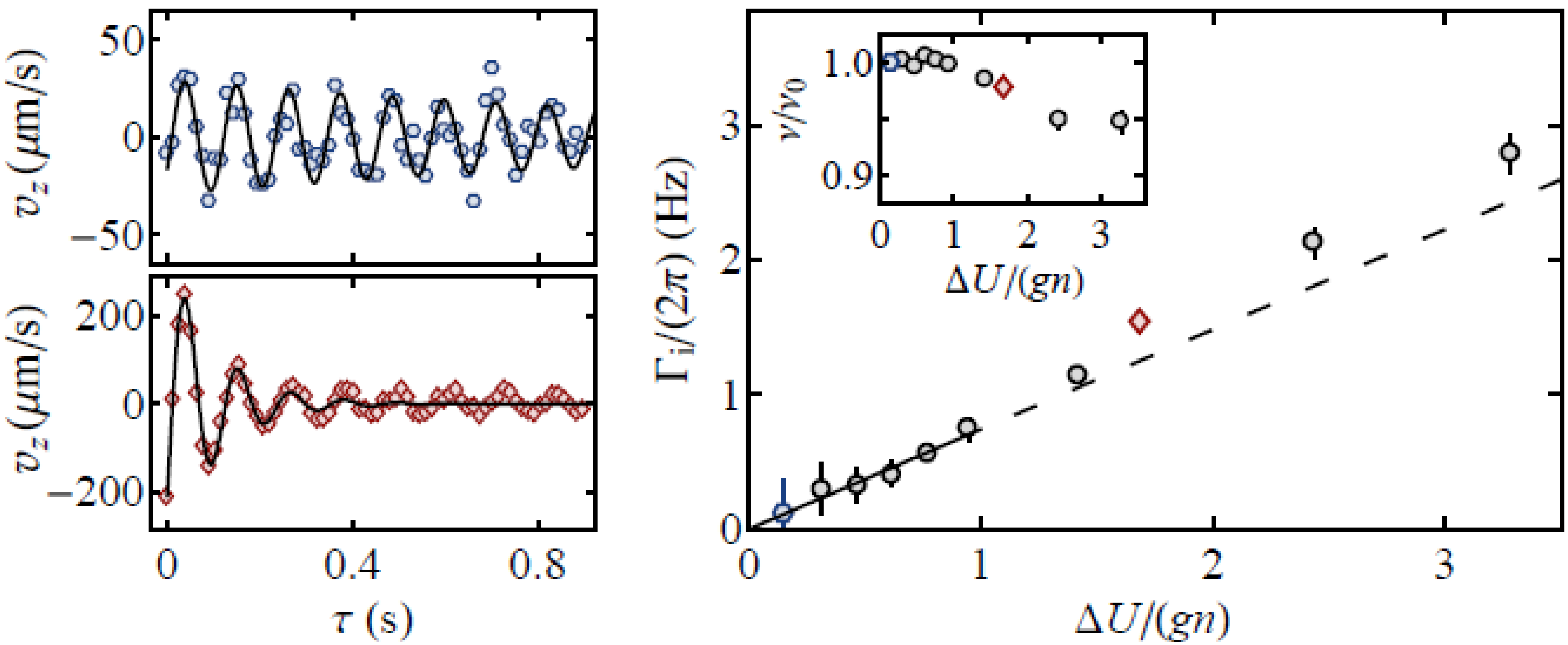
- Even one of the simplest quantum system imaginable exhibits spectacularly rich physics

- New system and new (AMO-specific) tools to study turbulence

- GPE simulations confirm experiment (a surprise in and of itself)

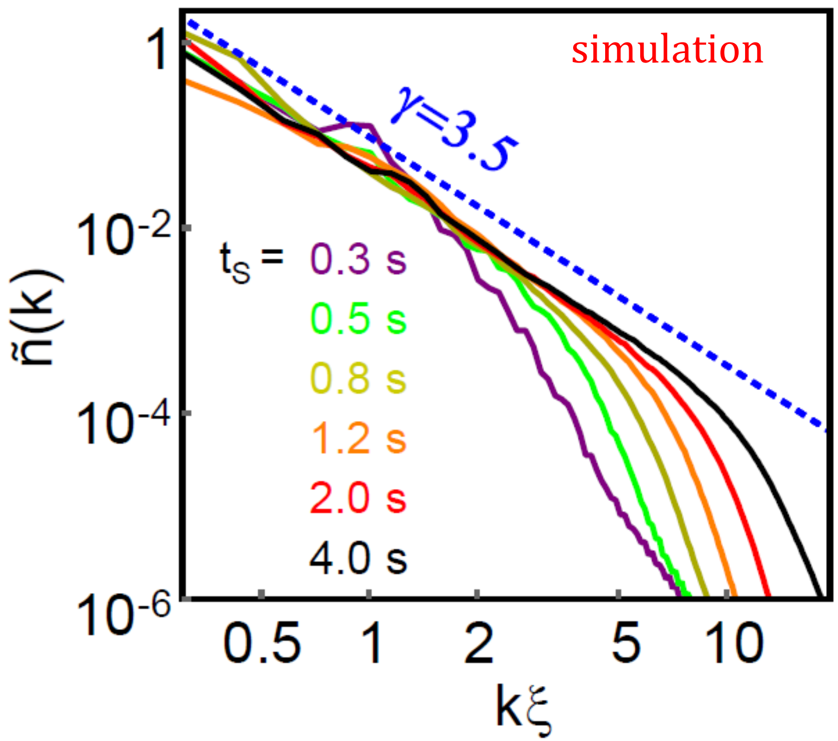
- Many questions: (i) Cascade exponent not understood yet
(related: Quantify the deviation from weak wave turbulence theory)
(ii) Establish the validity of classical field methods for this system
(iii) Interplay play of compressible and incompressible-fluid turbulence

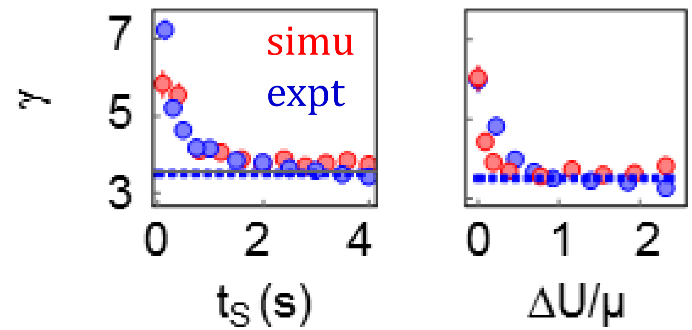
Nonlinear behavior in real time

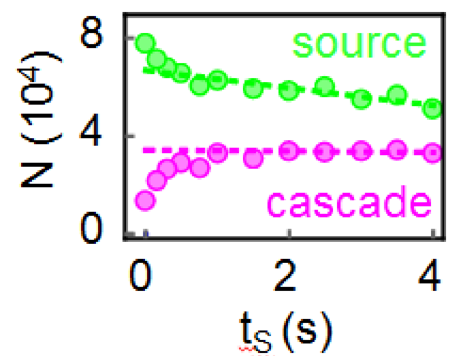


Dynamics of the Cascade

$$n(\mathbf{k}) \propto k^{-\gamma} \qquad \tilde{n}(k_r) = \int \mathrm{d}k_z n(\sqrt{k_r^2 + k_z^2})$$







What type of cascade is it ?

Two broad classes : - vortex turbulence

(quantized circulation / incompressible flow)

- wave turbulence

(density fluctuations / compressible flow)

Energy spectra

(with K. Fujimoto and M. Tsubota)

$$E[\psi] = \int d\mathbf{r}\, \psi^* \left(-\nabla^2 + \frac{g}{2}|\psi|^2\right)\psi$$

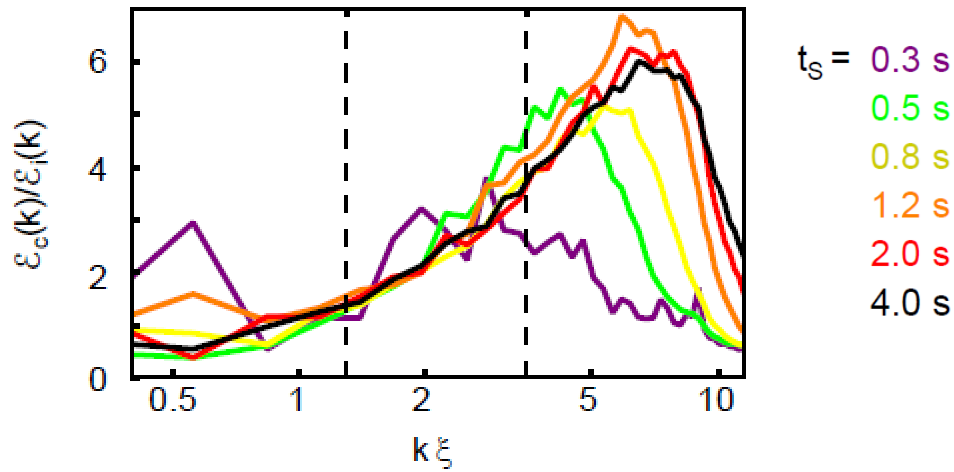
$$E_q = \int d\mathbf{r}(\nabla\sqrt{n})^2$$
$$E_{\rm kin} = \int d\mathbf{r}\, n(\nabla\phi)^2$$
$$E_{\rm int} = \frac{g}{2}\int d\mathbf{r}|\psi|^4$$

Define flow field $\mathbf{w}(\mathbf{r}) = \sqrt{n}\nabla\phi$

Helmholtz decomposition $\mathbf{w}(\mathbf{r}) = \mathbf{w}^c(\mathbf{r}) + \mathbf{w}^i(\mathbf{r})$

$\nabla \times \mathbf{w}^c = 0$ and $\nabla . \mathbf{w}^i = 0$

Then (Fourier) $E^{i,c}_{\rm kin}(\mathbf{k}) = |\tilde{\mathbf{w}}^{i,c}(\mathbf{k})|^2$



Compressible-flow energy dominates over incompressible-flow one