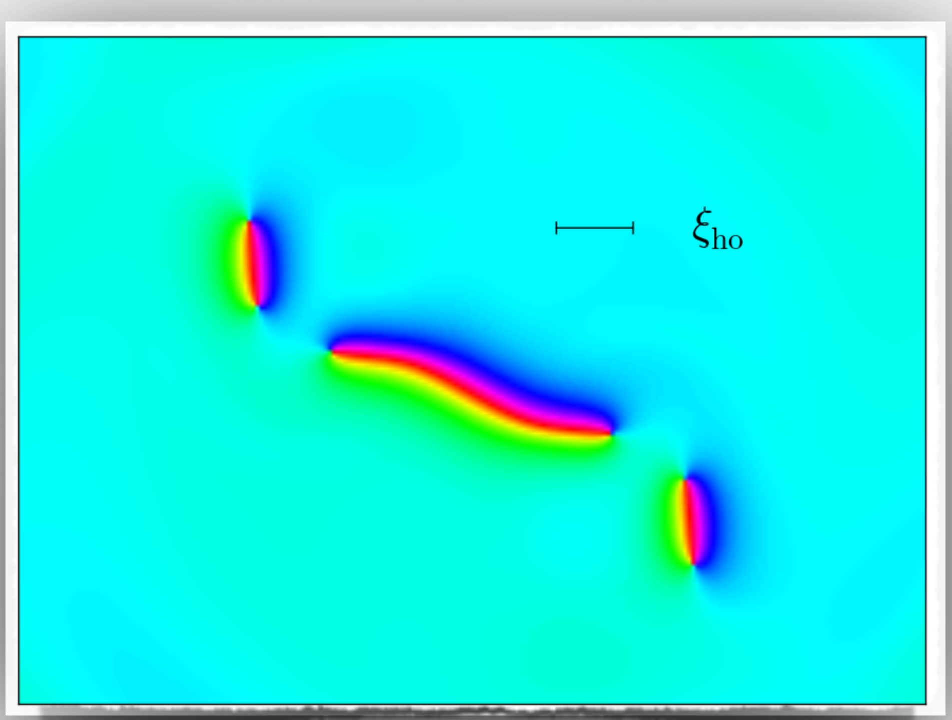
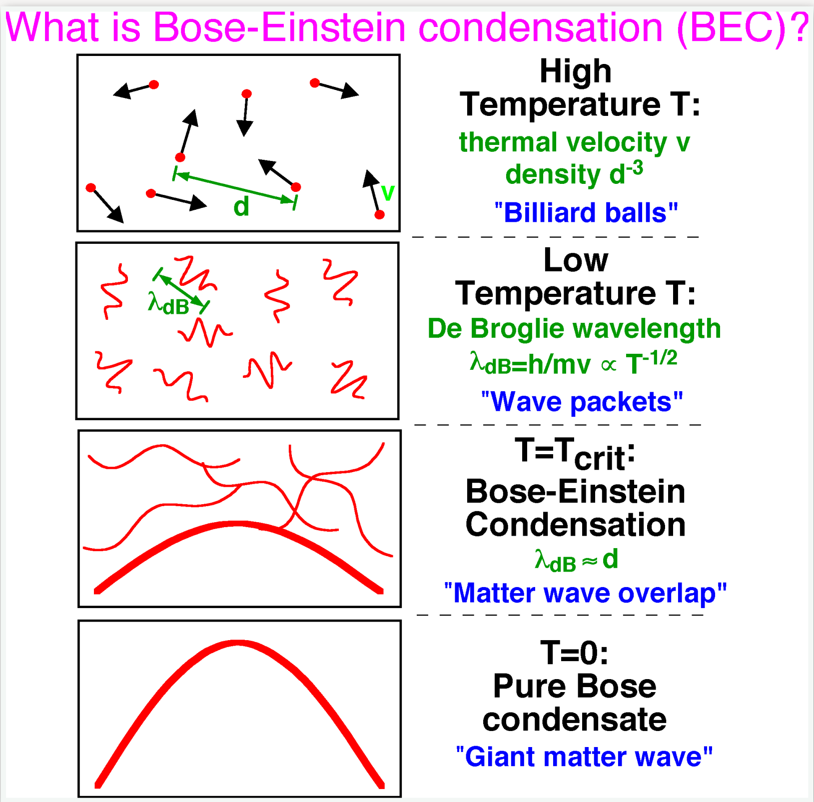
Bose-Bose Mixtures

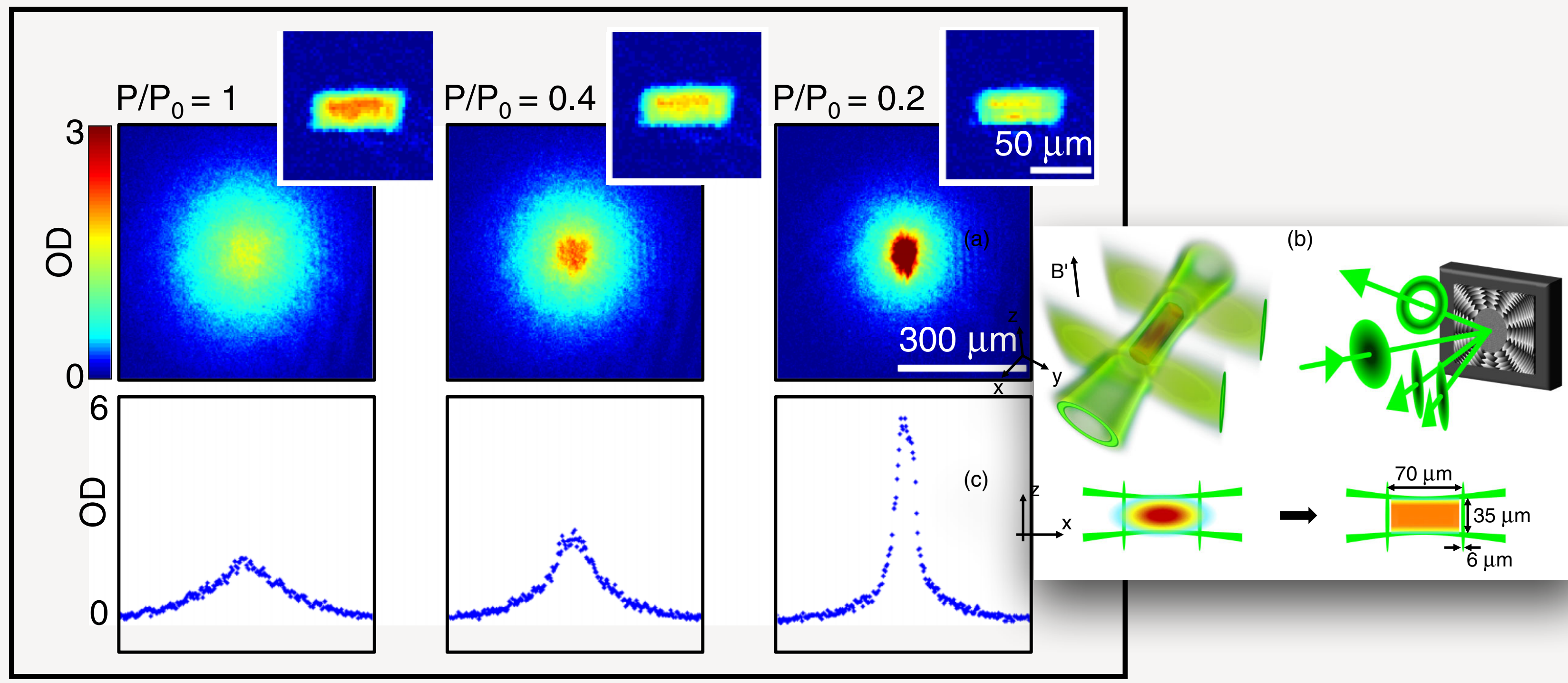


Alessio Recati

INO-CNR BEC Center, Trento University

Ultra-cold Bose Gases: Condensate





Hadzibabic PRL 2013

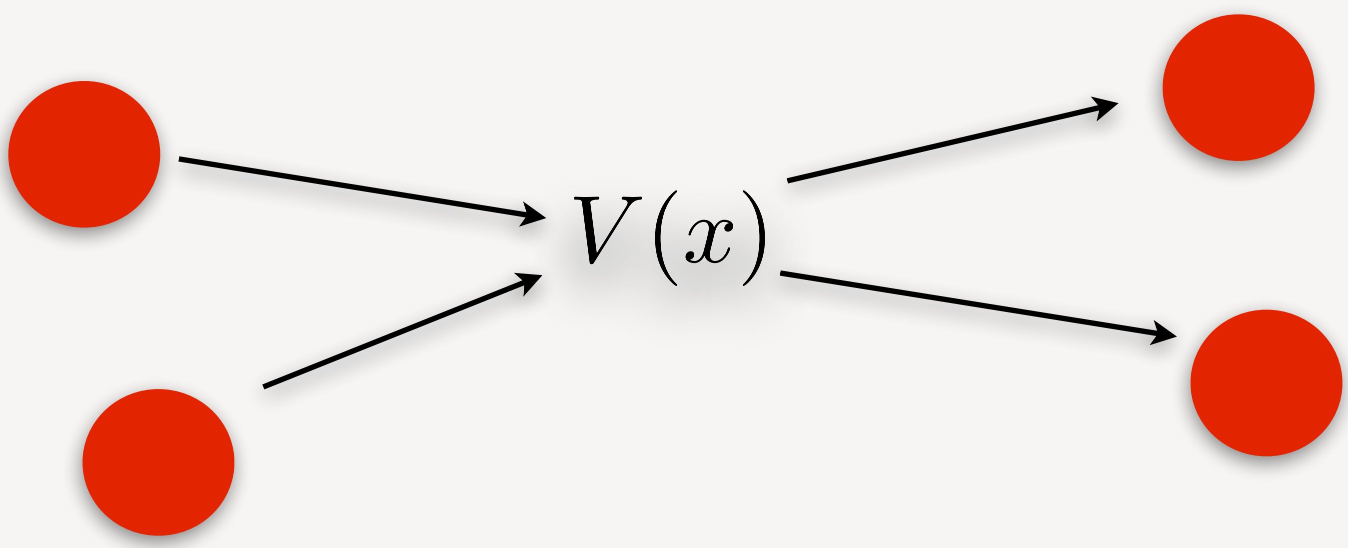
homogeneous:
$$\hat{a}_{p=0} = a_0$$
$$\hat{\psi} = a_0/\sqrt{V} + \ldots$$
$$e(n) = \frac{1}{2} g n^2$$

Dynamics and non-homogeneity- Gross-Pitaevskii equation: $\hat{\psi}(x) = \Psi(x) + \ldots$

$$i\hbar \frac{\partial}{\partial t}\Psi = \left[-\frac{\hbar^2 \nabla^2}{2m} + V(x) + g|\Psi|^2 \right] \Psi$$ with $$|\Psi(x)|^2 = n(x)$$

Ultra-cold Gases: s-wave scattering

The 2-body interaction potential ***W(x)*** can be replaced at **low-density and low-energy** by an effective contact (pseudo) potential which reproduces the low-energy behaviour of the microscopic potential



$$W(x) \rightarrow g\delta(x)(+reg.)$$

$$g = \frac{4\pi\hbar^2 a_s}{2m_r}$$

s-wave scattering length

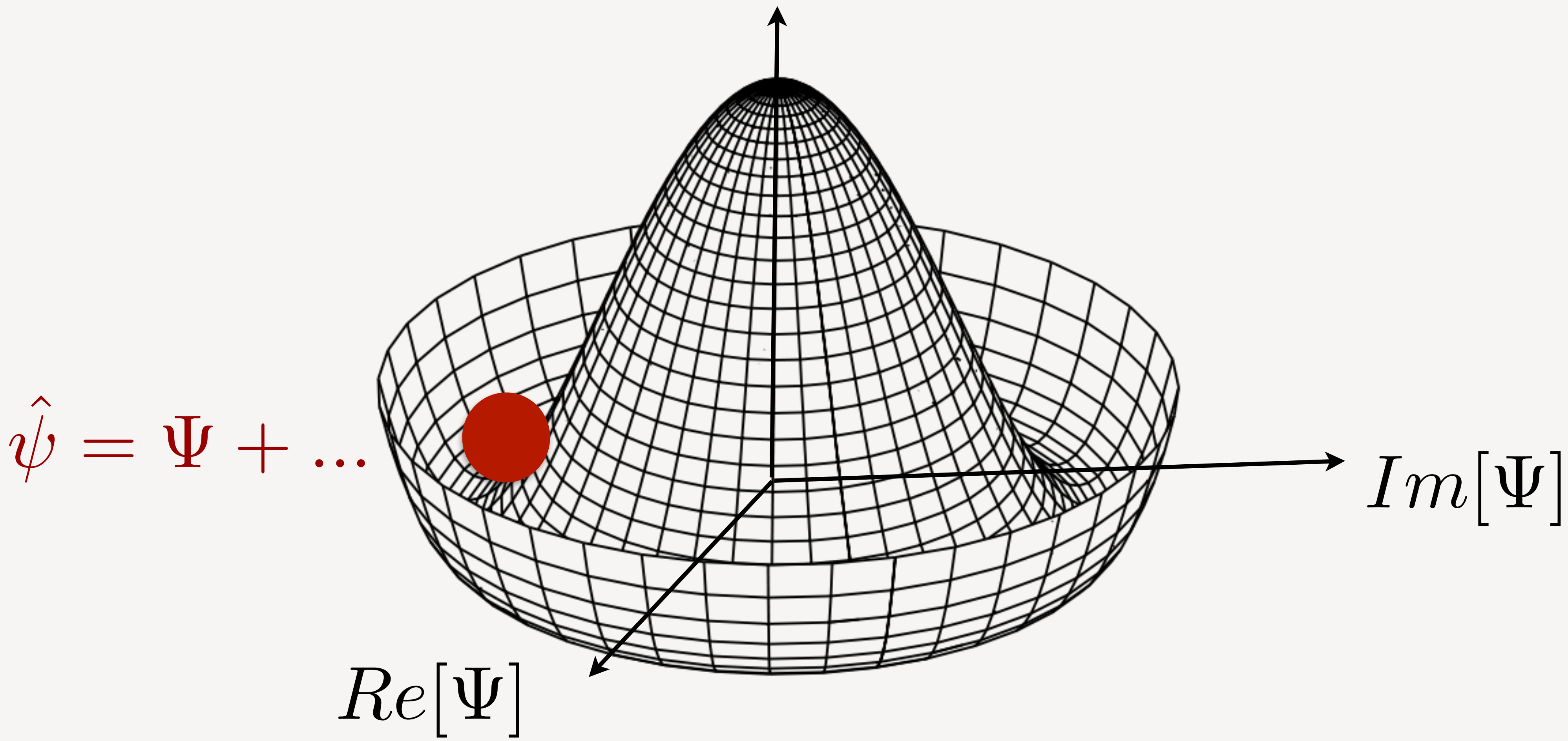
- Very low temperature (<1mK):
only l=0 collisions are relevant (due to centrifugal barrier)

$$f(k) \rightarrow (-1/a_s + r^* k^2/2 - ik)^{-1}$$

Fano-Feschbach: in “many” cases s-wave tunable

Ultra-cold Bose Gases: Condensate

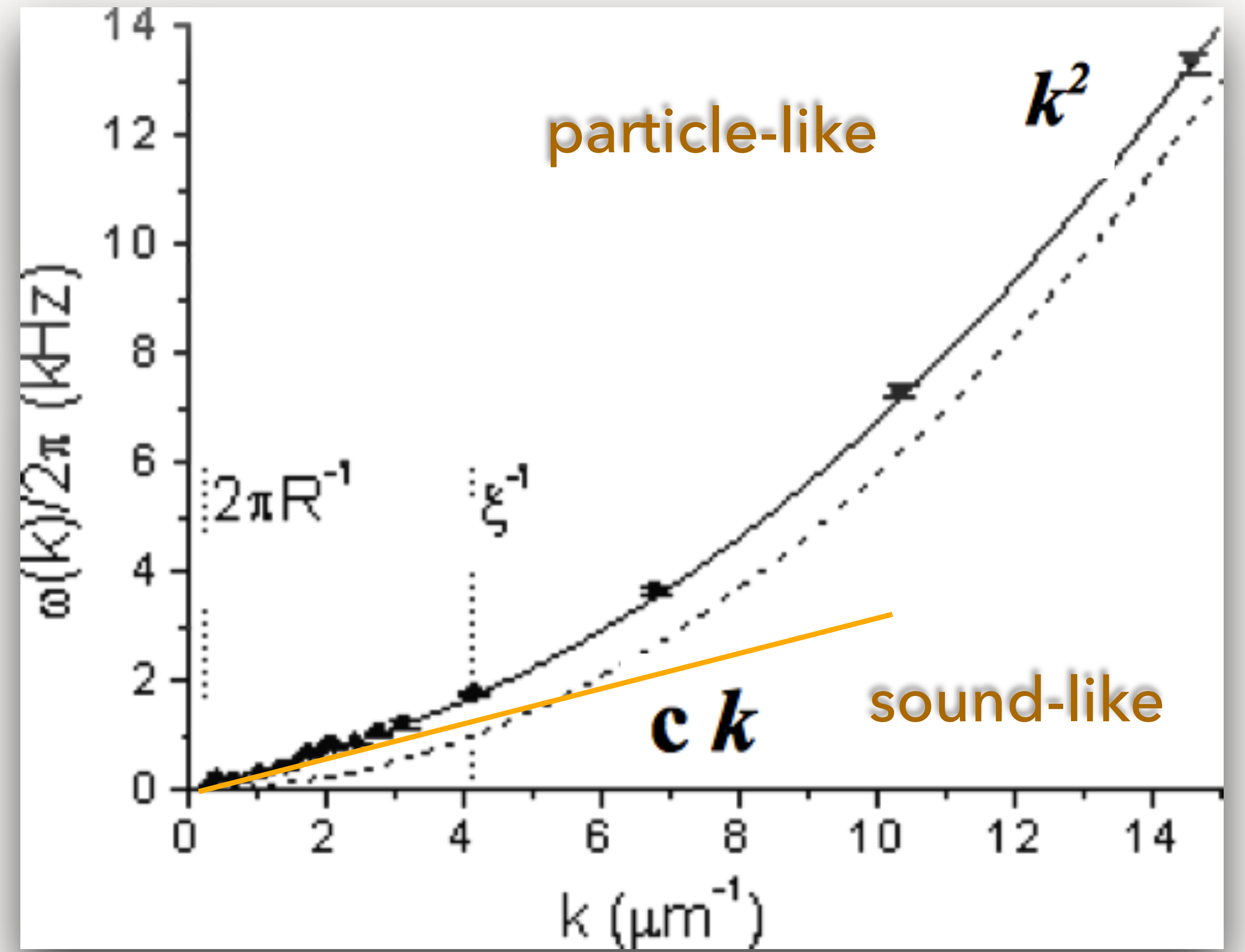
Ground state breaks U(1) symmetry - Number Conservation:
Goldstone mode -
no cost to change the global phase of the wave function
(gapless spectrum)



T=0 Bose gases: Elementary excitations

Uniform system (Mean-Field): $$e(n) = \frac{1}{2} g n^2$$

Bogoliubov Spectrum



[MIT 1999]

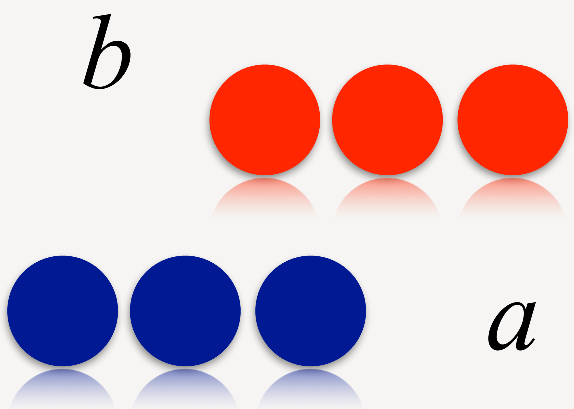
$$\omega_k = \sqrt{\frac{q^2}{2m}\left(2mc^2 + \frac{q^2}{2m}\right)}$$

where the speed of sound is:

$$c^2 = gn/m$$

and the healing length

$$\xi = \frac{\hbar}{\sqrt{2}mc}$$



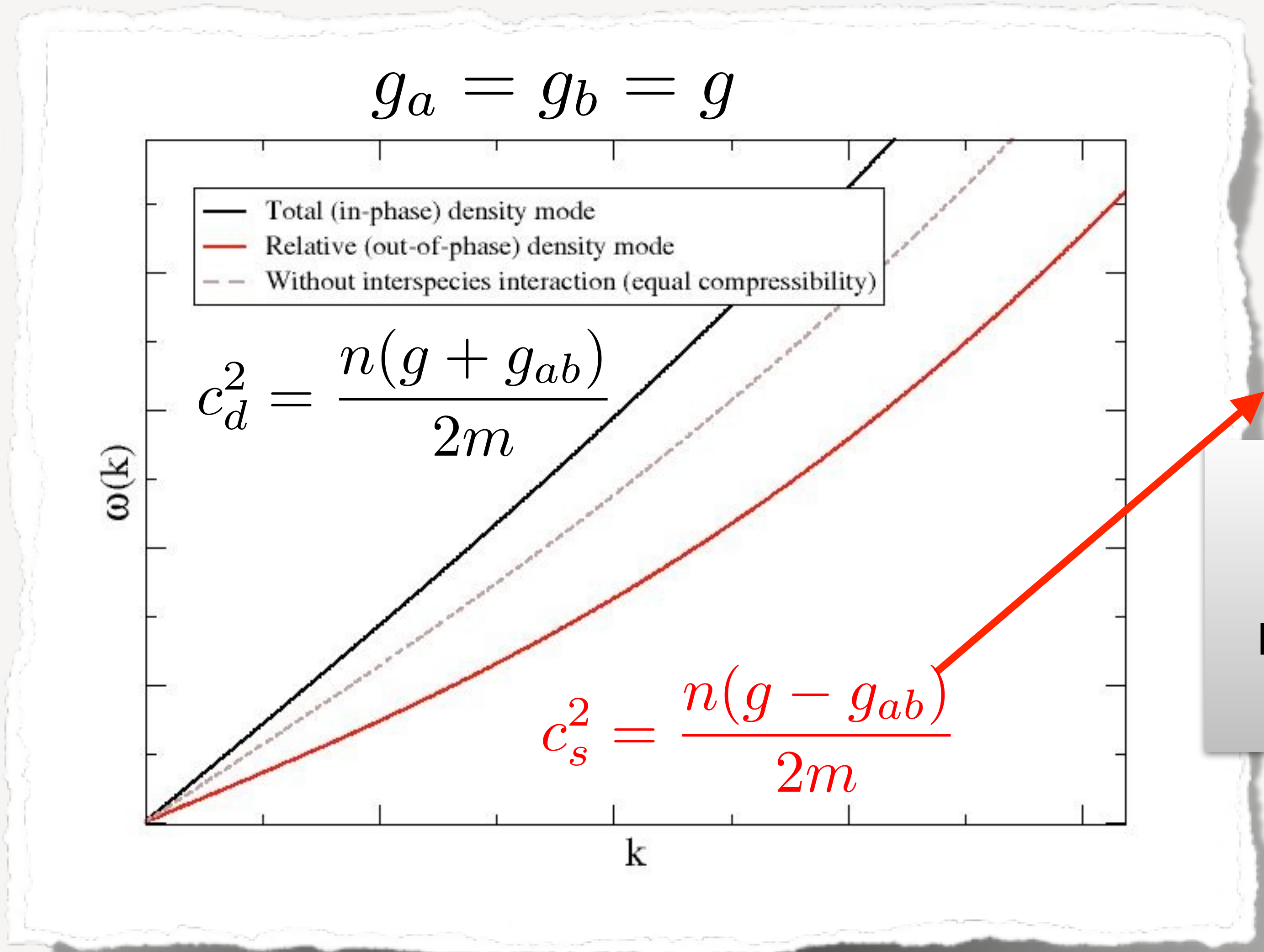
T=0 Bose mixtures

$$e(n_a, n_b) = \frac{1}{2} g_a n_a^2 + \frac{1}{2} g_b n_b^2 + g_{ab} n_a n_b$$

Both Na and Nb are conserved

Elementary excitations

Ground state breaks U(1)xU(1) symmetry: 2 Goldstone modes - coming from no cost to change the global and relative phase of the 2 order parameters



$g = g_{ab}$

Spin mode soft: unstable with respect to phase separation

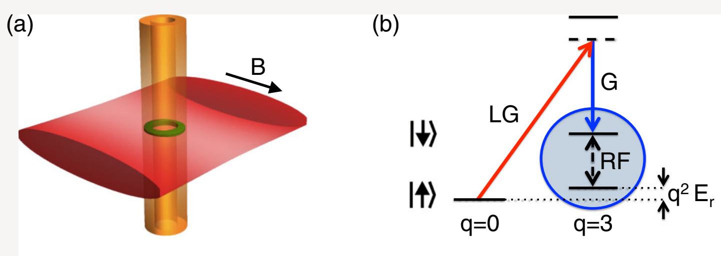
Supercurrent stability

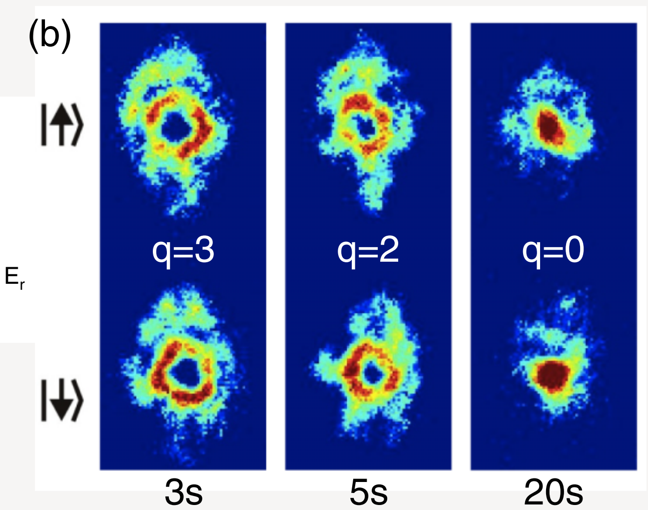
PRL 110, 025301 (2013) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2013

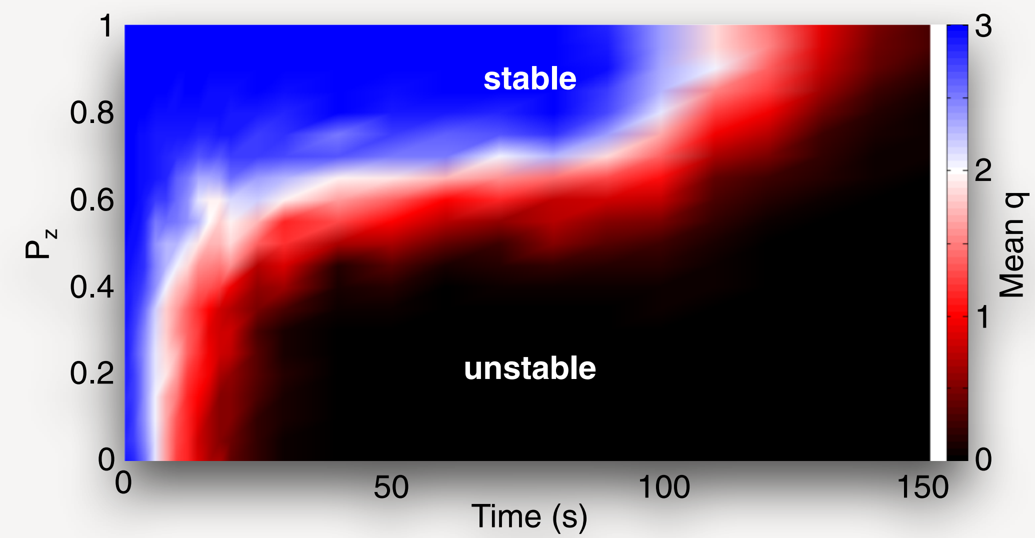
Persistent Currents in Spinor Condensates

Scott Beattie, Stuart Moulder, Richard J. Fletcher, and Zoran Hadzibabic

Experiment

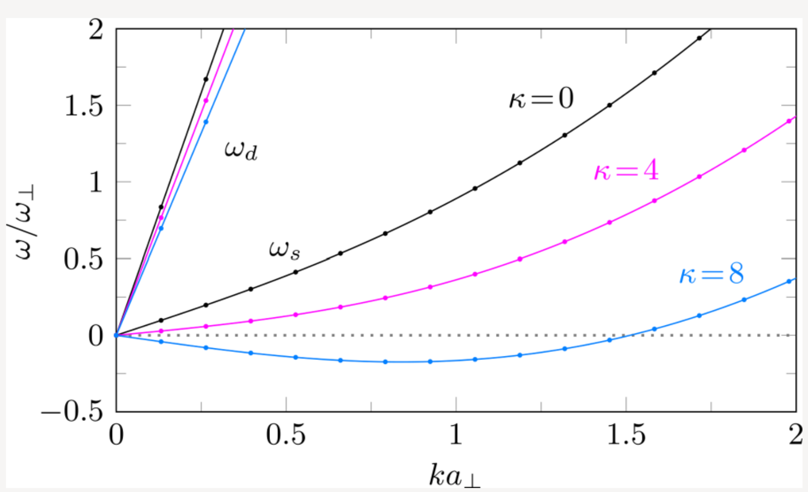




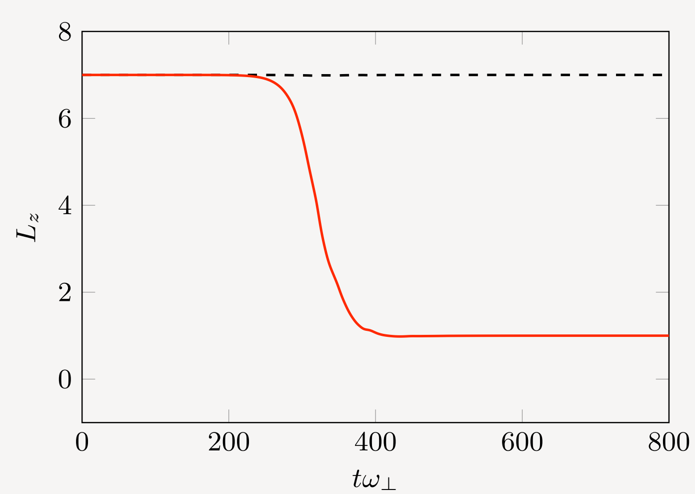


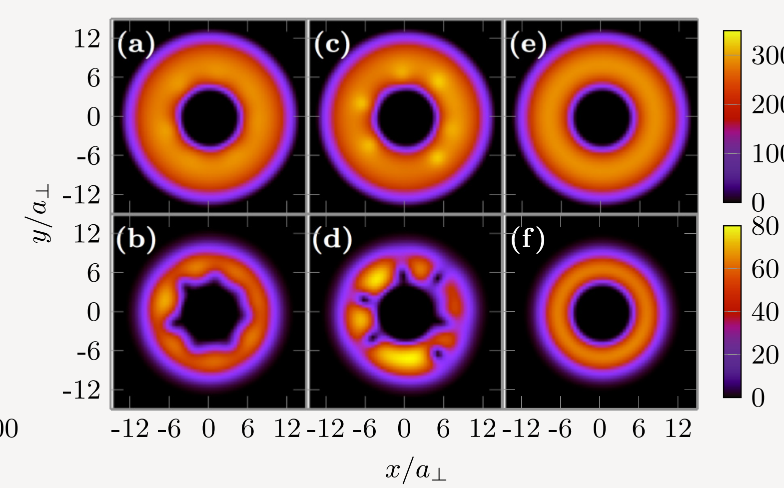
Theory

Bogolyubov dispersion relations



GP real time dynamics (Pz=0.8, κ=7)





phase slips due to vortices in the minority component (spin dominated)

[M. Abad, A. Sartori, S. Finazzi,, and AR, PRA (2014)].

Magnetic Topological Defect

It is obvious that if a defect (vortex, soliton) is created in one component the other component tries to fill it due to the repulsive mean-field interspecies interaction.

For the case of very soft spin excitations the defect does not (essentially) coupled to the total density since it can be considered almost incompressible: magnetic defect

$$\varepsilon_{\rm GP} = \sum_{i=1,2}\left[\frac{\hbar^2}{2m}|\nabla\Psi_i|^2 + (V_{\rm ext}-\mu_i)|\Psi_i|^2 + \frac{g_{ii}}{2}|\Psi_i|^4\right] + g_{12}|\Psi_1|^2|\Psi_2|^2,$$

$\Psi_i = \sqrt{n_{i,0}}\, f_i(\mathbf{r}) e^{i\phi_i(\mathbf{r})}$

$n = n_1(\mathbf{r}) + n_2(\mathbf{r})$

incompressibility

$$\frac{\varepsilon_{\rm GP}}{4\delta g n_{1,0}^2} = \frac{1}{4} f_1^2(f_1^2-2) + \frac{1}{2}\left\{\frac{n(\nabla_\eta f_1)^2}{n-n_{1,0}f_1^2} + f_1^2[(\nabla_\eta\phi_1)^2 - (\nabla_\eta\phi_2)^2] + \frac{n}{n_{1,0}}(\nabla_\eta\phi_2)^2\right\}$$

$$\mathbf{r}\to\boldsymbol{\eta} = \mathbf{r}/\xi_s \qquad \xi_s = \frac{\hbar}{\sqrt{4m\,\delta g\, n_{1,0}}}$$

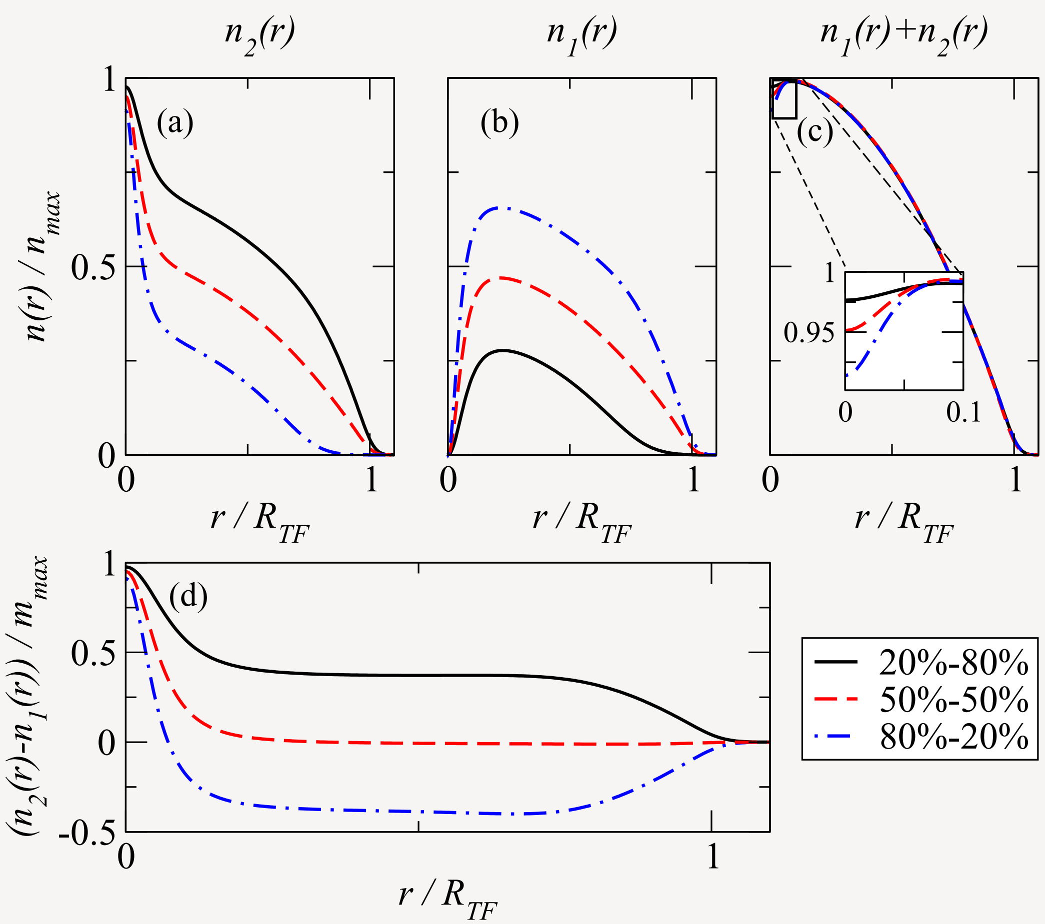
For a **vortex configuration** the equation for the *f's* reads:

$$\partial_\eta^2 f_1 + \left(1 - \frac{1}{\eta^2}\right) f_1 - f_1^3 + \frac{n_{1,0} f_1}{n - n_{1,0} f_1^2}\left[f_1 \partial_\eta^2 f_1 + \frac{n}{n - n_{1,0} f_1^2}(\partial_\eta f_1)^2\right] = 0$$

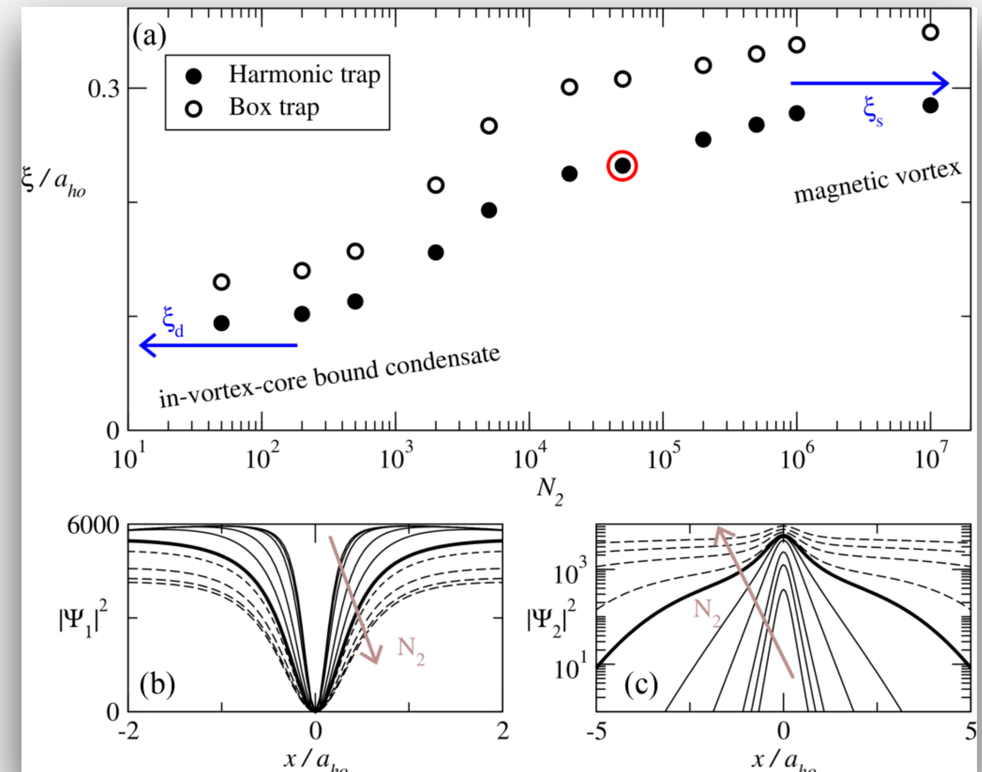
Pitaevskii vortex equation with renormalised healing length (i.e an *in-medium vortex*)

[A. Gallemí, L. P. Pitaevskii, S. Stringari, and AR, PRA (2018)].

Magnetic Topological Defect



Notice that at very large imbalance the vortex in the majority component is not affected creates a binding potential for the minority component



The very same argument applies to solitons. :

$$\sqrt{n_{1,0}}\, f_1(z) = \sqrt{n} \cos\left[\theta(z)/2\right] \Longrightarrow \partial^2_\eta \theta + \sin(\theta)\frac{\cos(\theta)+p}{1-p} = 0$$

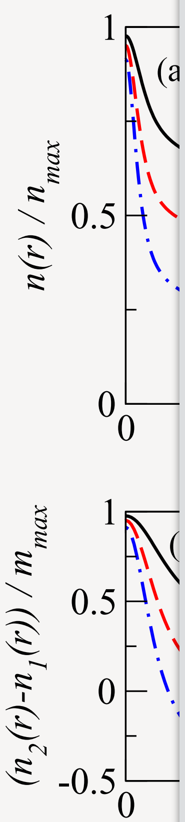
which can be solved analytically for any polarisation:

$$n_1(z) = n_{1,0}\frac{\cosh(\sqrt{1+p}\, z/\xi_s) - 1}{\cosh(\sqrt{1+p}\, z/\xi_s) + p}$$

For p close to 1 it reduces to the well-known Tsuzuki solution but with a renormalised healing length.

[A. Gallemí, L. P. Pitaevskii, S. Stringari, and AR, PRA (2018)].

Magnetic Topological Defect

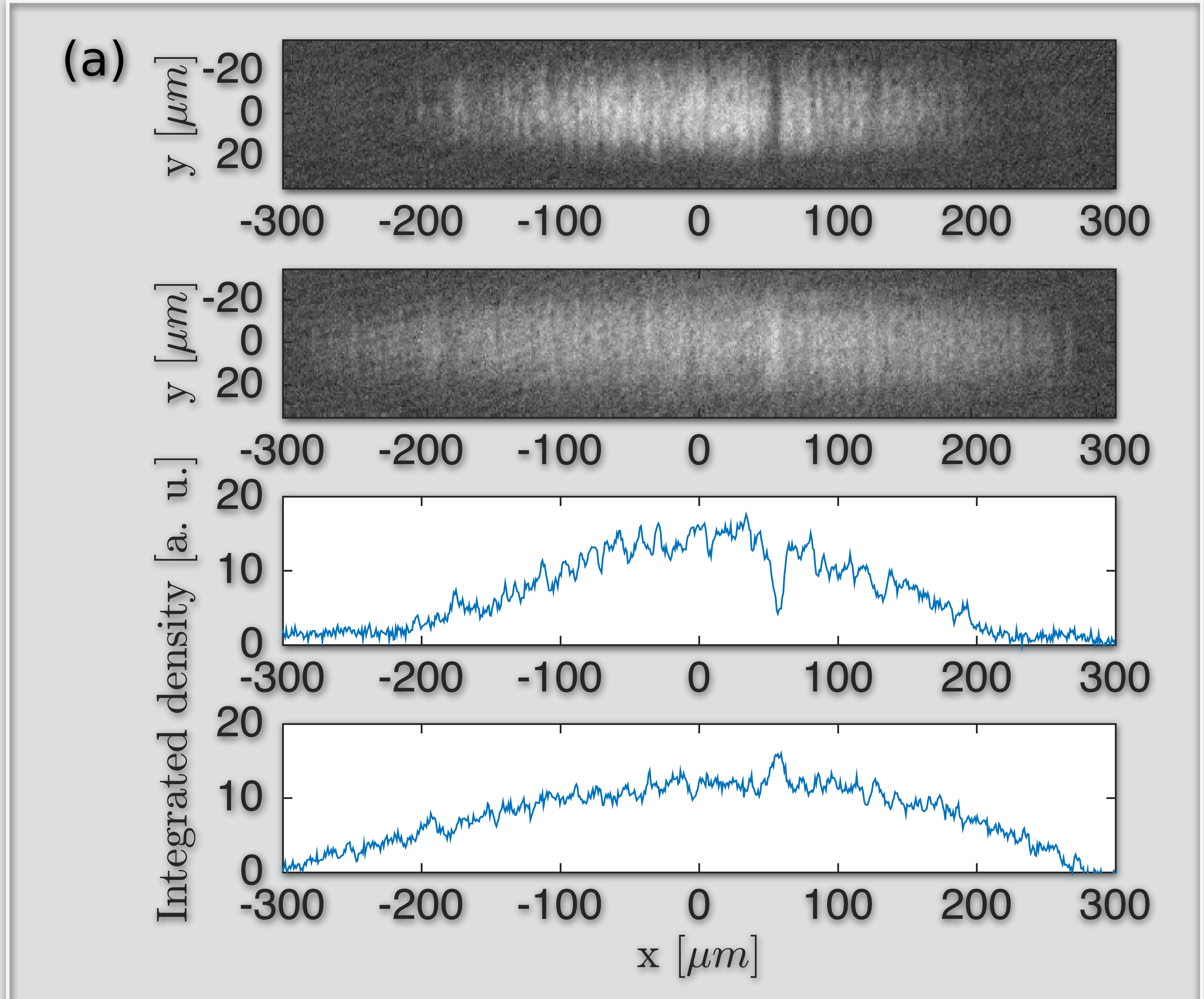


$\sqrt{n_{1,0}}\, f_1(z) = \sqrt{n}$

$n_1(z) = n_{1,0}$

n
a

P=0 magnetic soliton in the lab



Engels, PRA 94, 053617 (2016)

[A. Gallemí, L. P. Pitaevskii, S. Stringari, and AR, PRA (2018)].

Superfluid Drag

At the mean-field level the kinetic energy associated to a superfluid flow is simply

$$\varepsilon = \sum_{1,2} \frac{\hbar^2}{2m} n_i (\nabla\phi_i)^2$$

with a superfluid density which coincides with the total density and superfluid currents

$$\mathbf{j}_i = m n_i \nabla\phi_i$$

If quantum fluctuations are important also the so called superfluid drag (a.k.a. Andreev-Bashkin effect, a.k.a. entrainment) must be considered. In this case the superfluid current is related to the order parameter phase via a matrix $\mathbf{j}_i = \rho_{ij} \nabla\phi_j$

and for a Galilean invariant system (T=0): $mn = \sum_{ij} \rho_{ij}$

Such an effect has been introduced to study He-3/He-4 superfluid mixtures (1975), has been studied for mesoscopic rings, superconducting systems, neutron stars…

For a weakly interacting Bose-Bose mixtures the drag can be easily computed and for symmetric mixtures it reads

$$\rho_{12} \simeq mn\sqrt{na^3}\frac{g_{12}^2}{g^2}$$

[D. V. Fil and S. I. Schevchenko, PRA (2005)]

quantum depletion

Superfluid Drag & spin channel

$$E_{AB} = \sum_{ij} \frac{\hbar^2}{2m^2} \rho_{ij} \nabla\phi_i \cdot \nabla\phi_j \xrightarrow{\text{hydrodynamics}}$$

$$c_d^2 = \frac{n(g+g_{ab})}{2m} = \frac{n}{2m\kappa}$$

$$c_s^2 = \frac{n(g-g_{ab})}{2m}\left(1 - \frac{2\rho_{12}}{nm}\right) = \frac{nm/2 - \rho_{12}}{m^2\chi}$$

In other word the sum-rule for spin channel is not exhausted by the phonons and the Bijl-Feynman's relation does not hold [1].

- *Therefore an independent measure of the susceptibility and of the spin speed of sound could provide a direct measurement of the superfluid drag.*

On the other hand numerically one has access to all the six properties and check hydrodynamic prediction.

Susceptibility: $\frac{1}{\chi} = \frac{\partial^2 E/L}{n^2 \partial P^2}$

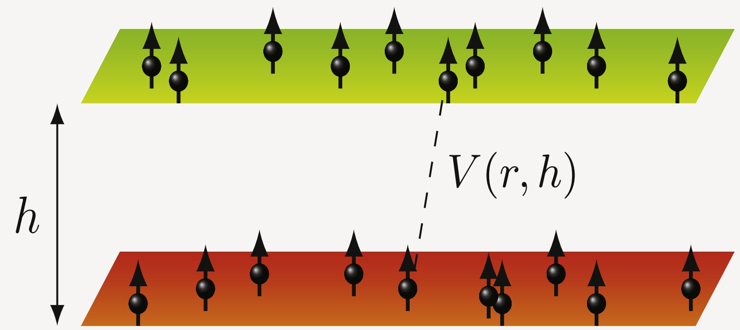
Superfluid densities [1]: $\rho_1 + \rho_2 \pm 2\rho_{12} = \lim_{\tau\to\infty} \frac{\langle [\mathbf{W}_1(\tau) \pm \mathbf{W}_2(\tau)]^2 \rangle}{2N\tau}$

Static Structure factors: $S_{d(s)}(k) = S_{11}(k) \pm S_{12}(k) \xrightarrow[k\to 0]{\lim} \begin{matrix} v_d \kappa k \\ v_s \chi k \end{matrix}$

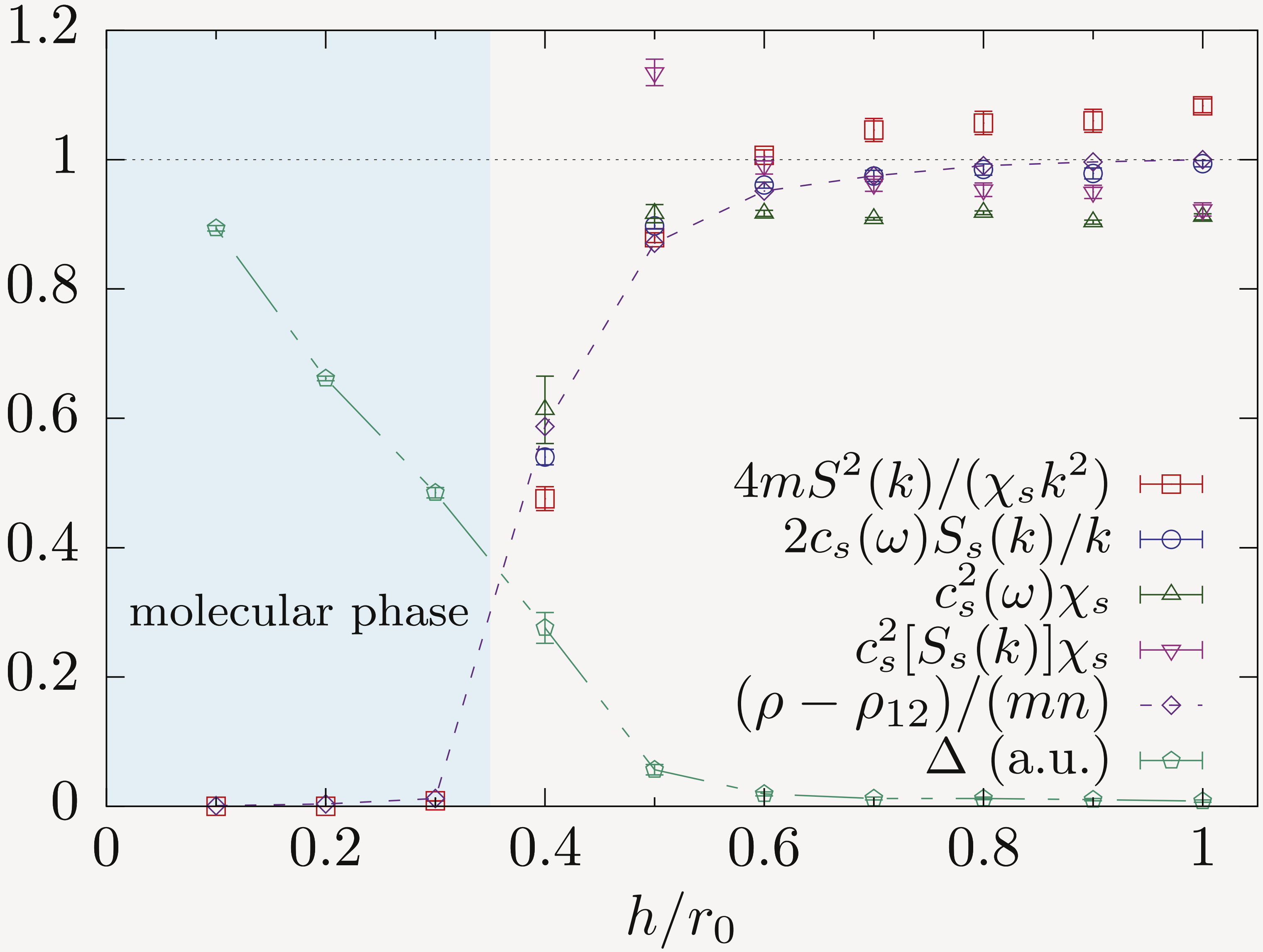
[1] J. Nespolo, G. Astracharchik and AR, NJP (2017)

Superfluid Drag & spin channel

1) Bi-layer dipolar gases [1]



$$V(r,h) = d^2 \frac{r^2 - h^2}{(r^2 + h^2)^{5/2}}$$

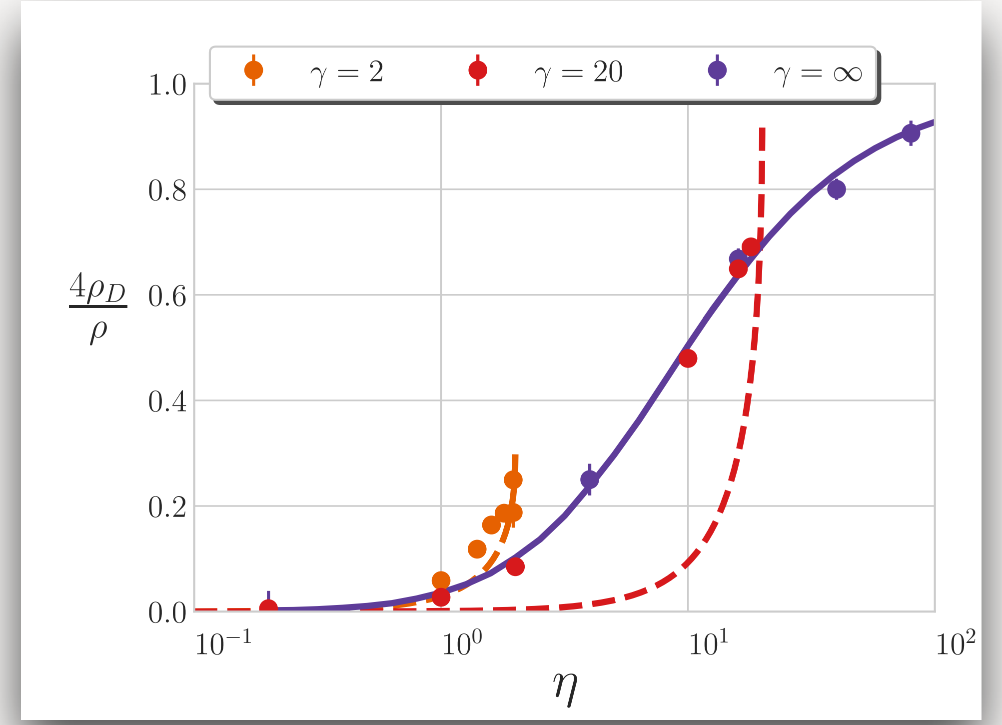


[1] J. Nespolo, G. Astracharchik and AR, NJP (2017)

Superfluid Drag & spin channel

2) 1D mixture [3]

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{N_a}\frac{\partial^2}{\partial x_i^2} + g\sum_{i<j}\delta(x_i - x_j)$$
$$- \frac{\hbar^2}{2m}\sum_{\alpha=1}^{N_b}\frac{\partial^2}{\partial x_\alpha^2} + g\sum_{\alpha<\beta}\delta(x_\alpha - x_\beta) + \tilde{g}\sum_{i,\alpha}\delta(x_i - x_\alpha)$$



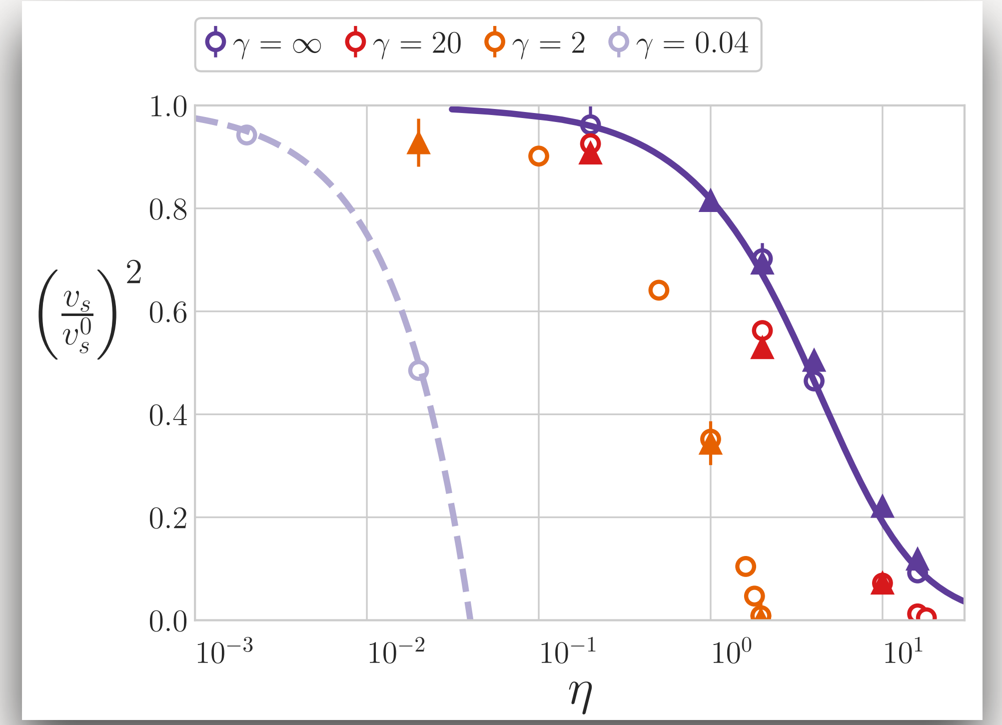


FIG. 4: (color online). Spin-wave velocity v_s as a function of η for different values of γ. The units are provided by $v_s^0 = \sqrt{\rho/m^2\chi_0}$, the spin-wave velocity in the absence of inter-species interactions. Open symbols refer to $\sqrt{(m_1)_{\rm sw}/m_{-1}}$ and solid symbols to m_0/m_{-1}. The dashed line corresponds to the mean-field prediction $v_s = \sqrt{n(g-\tilde{g})/2m}$ and the solid line to the exact solution in the Yang-Gaudin model [36].

[2] L. Parisi and S. Giorgini, arXiv:2018

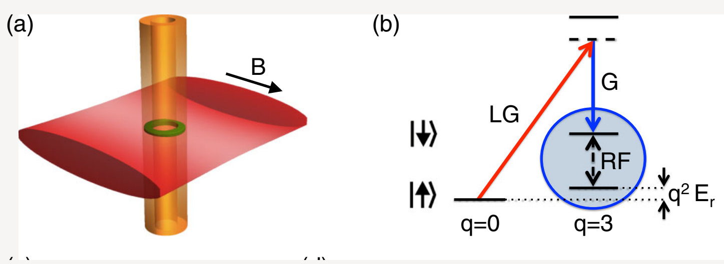
Supercurrent stability

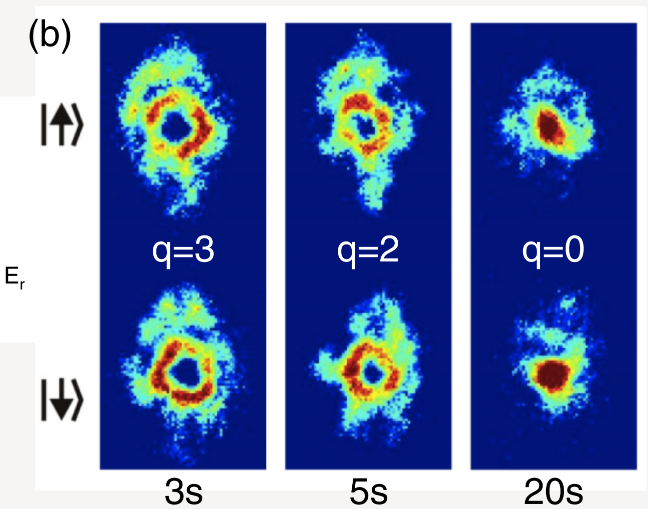
Experiment

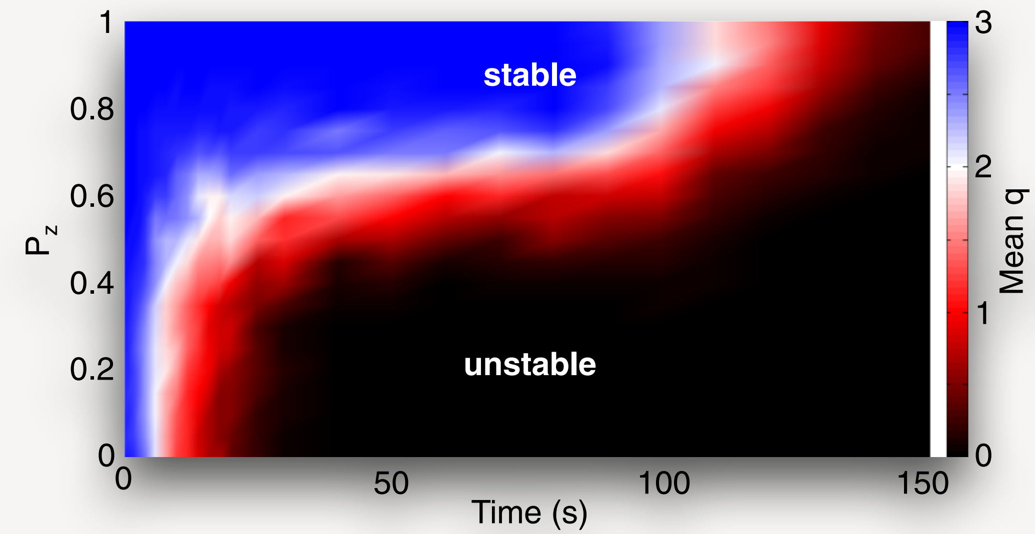
PRL 110, 025301 (2013) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2013

Persistent Currents in Spinor Condensates

Scott Beattie, Stuart Moulder, Richard J. Fletcher, and Zoran Hadzibabic

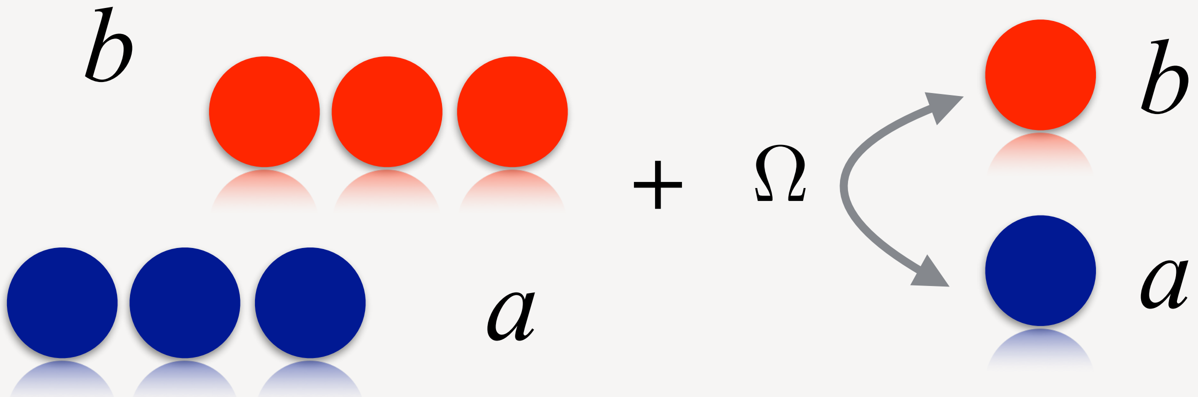






BUT if the RF is kept on the current is stable for over 1 minute

T=0 coherently coupled Bose gases



$$\Omega(|a\rangle\langle b| + |b\rangle\langle a|) = \Omega\sigma_x$$

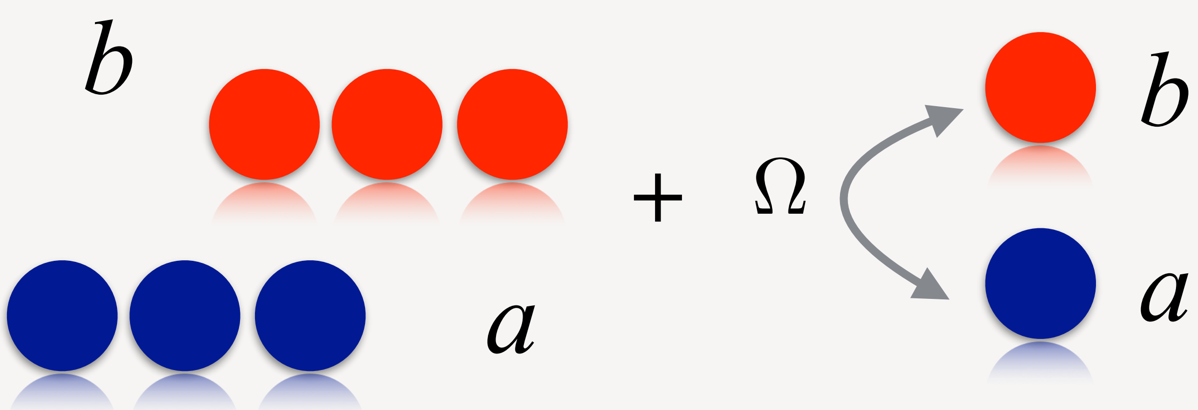
$$|\pm\rangle = (|a\rangle \pm |b\rangle)/\sqrt{2}$$

We consider a two component Bose gas with an interconversion term (Rabi coupling)

$$H = \int_{\mathbf{r}} \Bigg[\sum_{\sigma} \psi^{\dagger}_{\sigma\mathbf{r}} (-\nabla^2/2) \psi_{\sigma\mathbf{r}} - \frac{\Omega}{2} (\psi^{\dagger}_{\uparrow\mathbf{r}} \psi_{\downarrow\mathbf{r}} + \psi^{\dagger}_{\downarrow\mathbf{r}} \psi_{\uparrow\mathbf{r}})$$

$$\sum_{\sigma} V_{\sigma\mathbf{r}} \psi^{\dagger}_{\sigma\mathbf{r}} \psi_{\sigma\mathbf{r}} + \frac{1}{2} \sum_{\sigma\sigma'} g_{\sigma\sigma'} \psi^{\dagger}_{\sigma\mathbf{r}} \psi^{\dagger}_{\sigma'\mathbf{r}} \psi_{\sigma'\mathbf{r}} \psi_{\sigma\mathbf{r}} \Bigg]$$

T=0 coherently coupled Bose gases



$$\Omega(|a\rangle\langle b| + |b\rangle\langle a|) = \Omega\sigma_x$$

$$|\pm\rangle = (|a\rangle \pm |b\rangle)/\sqrt{2}$$

Assuming the gas condense in a ground state :

$$(\Psi_a = \sqrt{n_a}e^{i\phi_a}, \Psi_b = \sqrt{n_b}e^{i\phi_b})$$

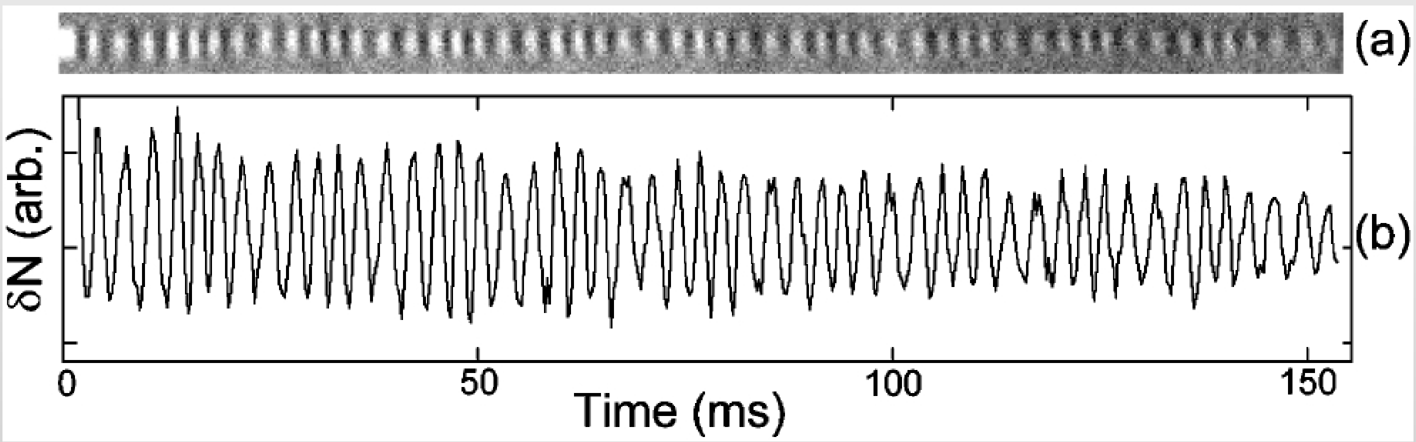
$$i\hbar\frac{\partial}{\partial t}\Psi_a = \left[-\frac{\hbar^2\nabla^2}{2m} + V_a + g_a|\Psi_a|^2 + g_{ab}|\Psi_b|^2\right]\Psi_a + \Omega\Psi_b \tag{1}$$

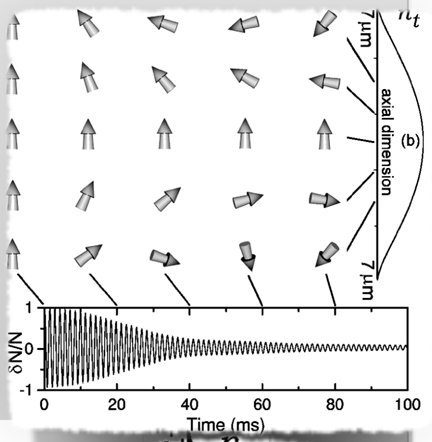
$$i\hbar\frac{\partial}{\partial t}\Psi_b = \left[-\frac{\hbar^2\nabla^2}{2m} + V_b + g_b|\Psi_b|^2 + g_{ab}|\Psi_a|^2\right]\Psi_b + \Omega^*\Psi_a, \tag{2}$$

T=0 coherently coupled Bose gases

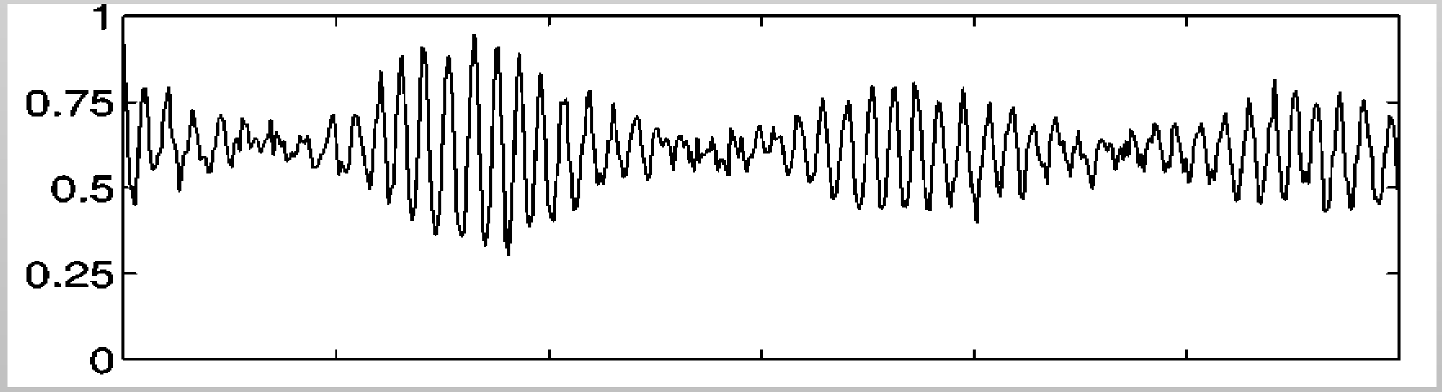
Seminal works on the order parameter twisting,
i.e. coherence of Rabi oscillation (& revival) for BECs, by Cornell/Holland ('98-'99)

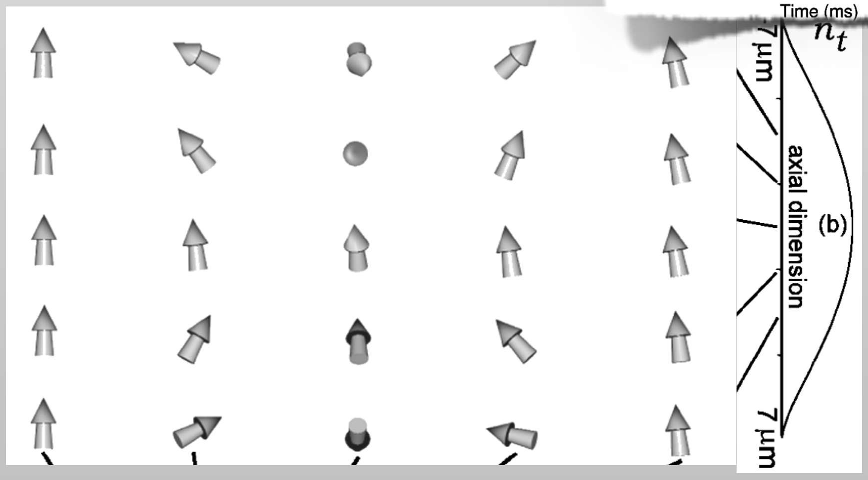
$$\Omega(z) = \Omega_0$$



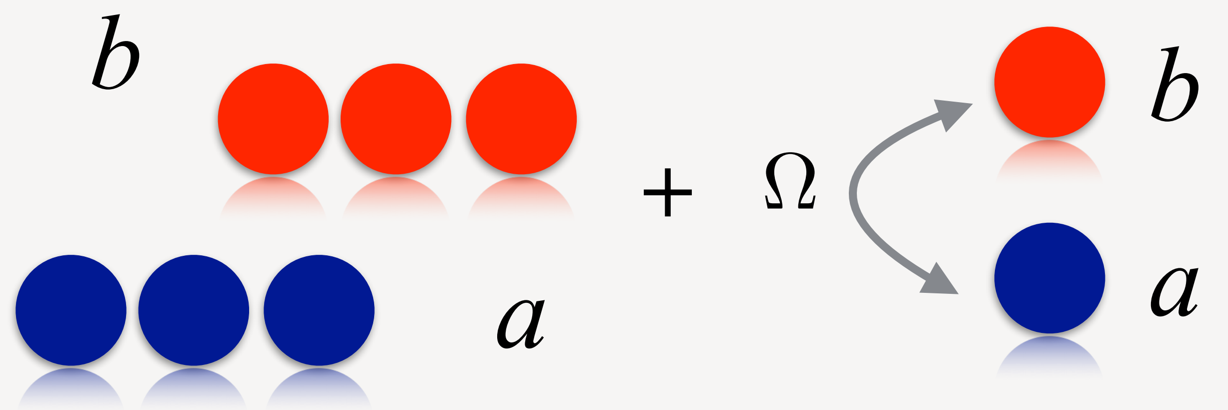


$$\Omega(z) = \Omega_0 + Bz$$





T=0 coherently coupled Bose gases



$$e(n_a, n_b) = \frac{1}{2} g_a n_a^2 + \frac{1}{2} g_b n_b^2 + g_{ab} n_a n_b$$
$$+2|\Omega| \cos(\phi_a - \phi_b) \sqrt{n_a n_b}$$

Only
Na+Nb
is conserved

Indeed the system is a single condensate
with a 2-component wave function

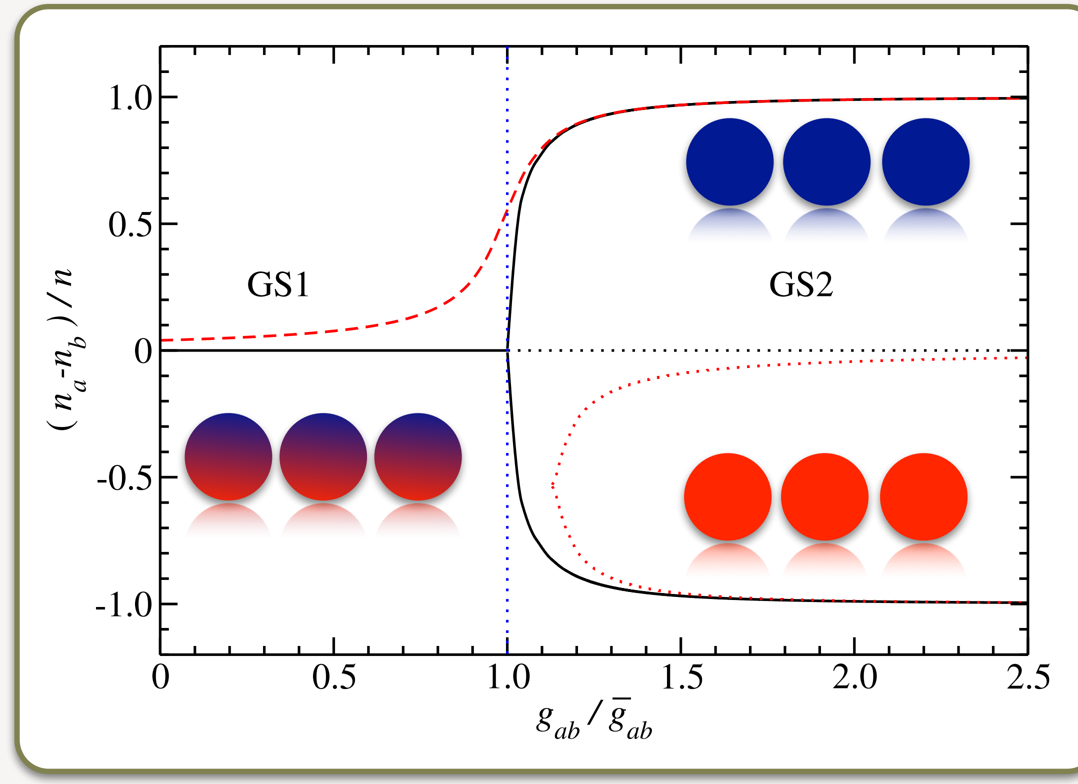
Elementary excitations

Ground state breaks U(1) symmetry:

1. Goldstone mode - coming from no cost to change the global total phase.
2. A gapped mode - due to the cost of changing the relative phase

Ground State

$$e(n_a, n_b) = \frac{1}{2} g_a n_a^2 + \frac{1}{2} g_b n_b^2 + g_{ab} n_a n_b$$
$$+2|\Omega| \cos(\phi_a - \phi_b) \sqrt{n_a n_b}$$



$$g_a = g_b = g$$

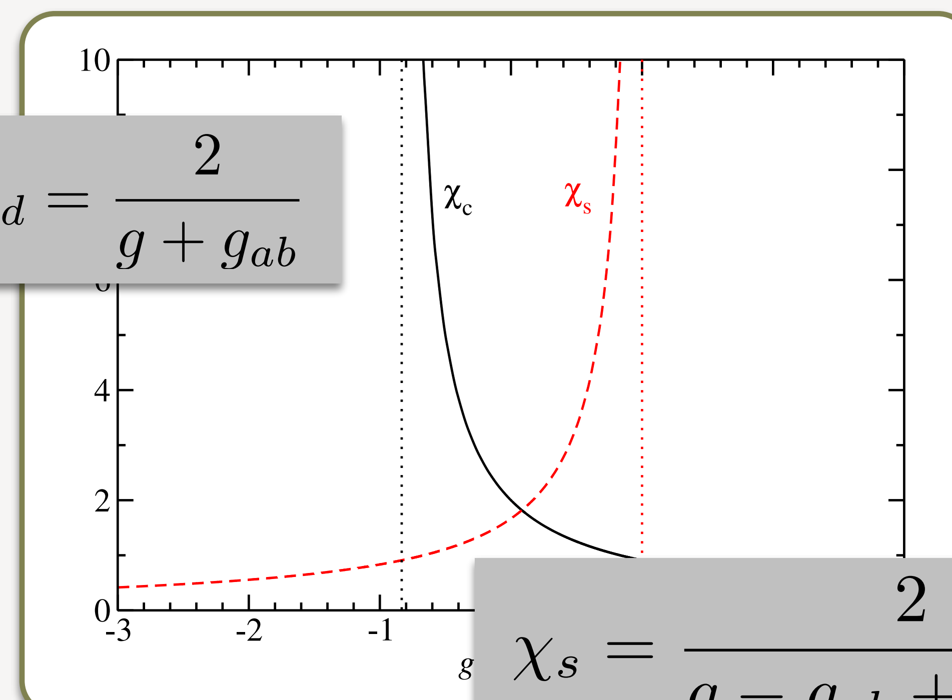
$$\phi_a - \phi_b = \pi$$

para $n_a - n_b = 0$

ferro $(n_a - n_b)_\pm = \pm n \sqrt{1 - \left(\frac{2|\Omega|}{(g - g_{ab}) n} \right)^2}$

Critical condition
for the II order phase transition

$$g - g_{ab} + \frac{2\Omega}{n} = 0$$

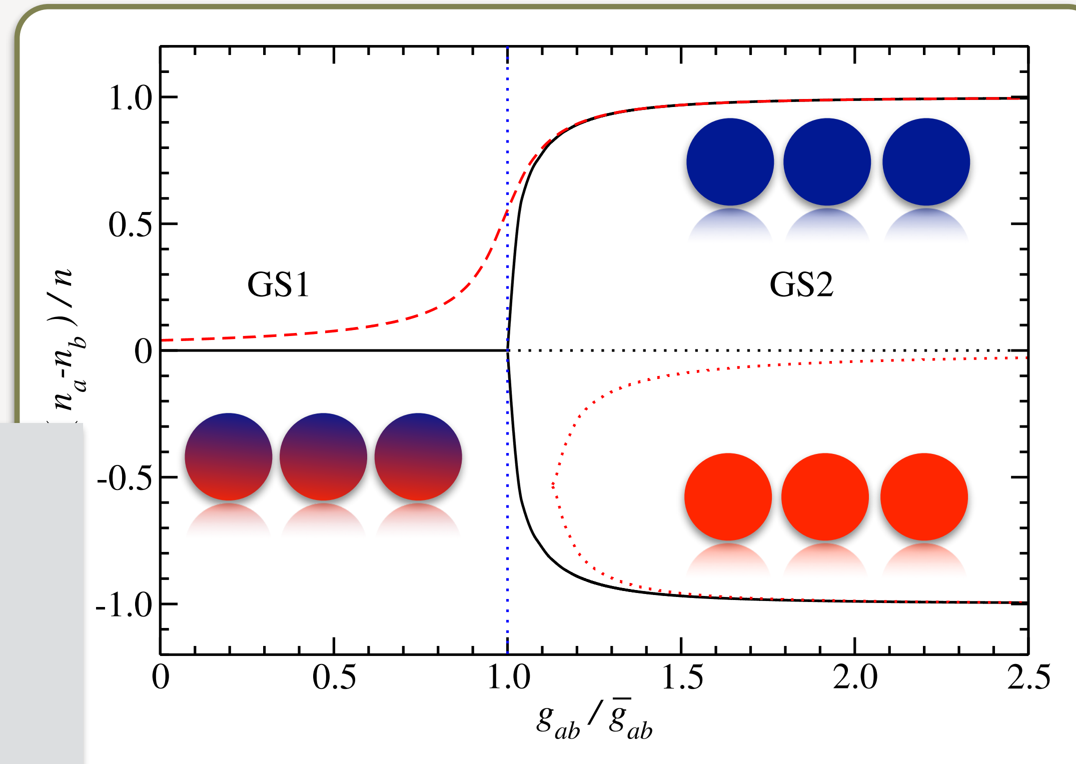


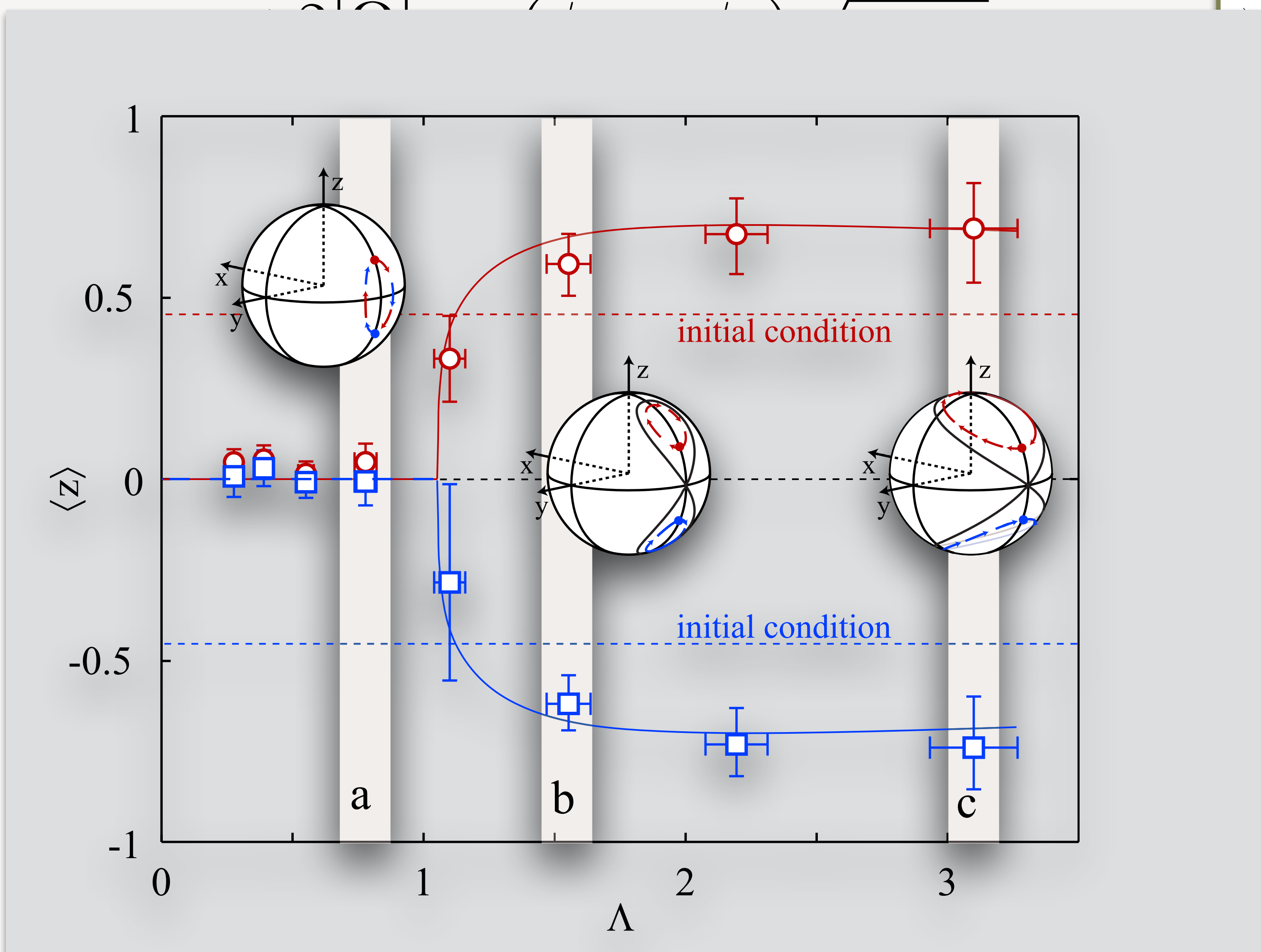
$$\chi_d = \frac{2}{g + g_{ab}}$$

$$\chi_s = \frac{2}{g - g_{ab} + 2\Omega/n}$$

Ground State

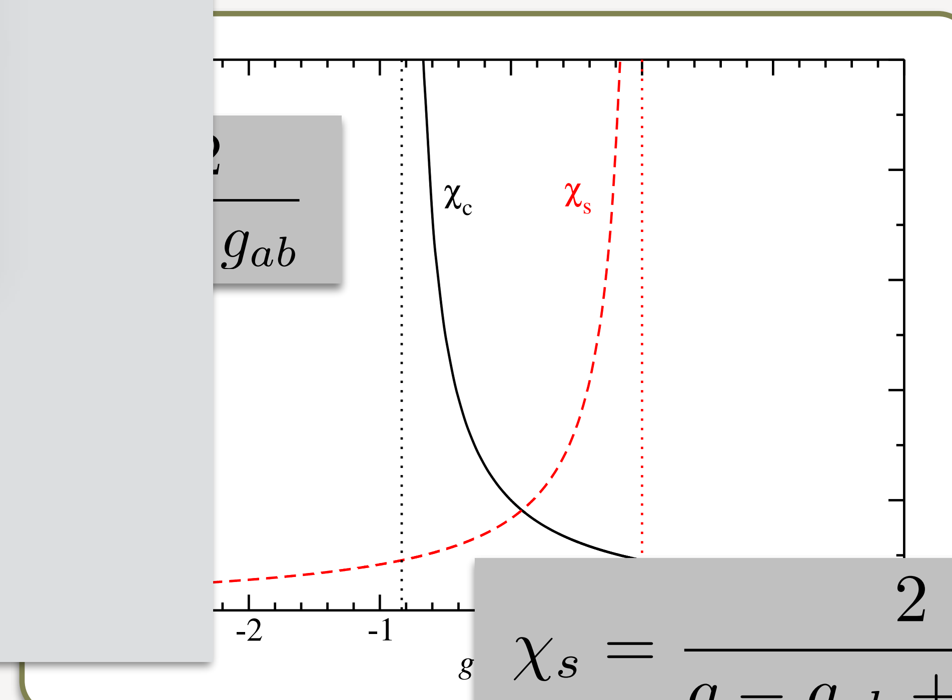
$$e(n_a, n_b) = \frac{1}{2} g_a n_a^2 + \frac{1}{2} g_b n_b^2 + g_{ab} n_a n_b$$





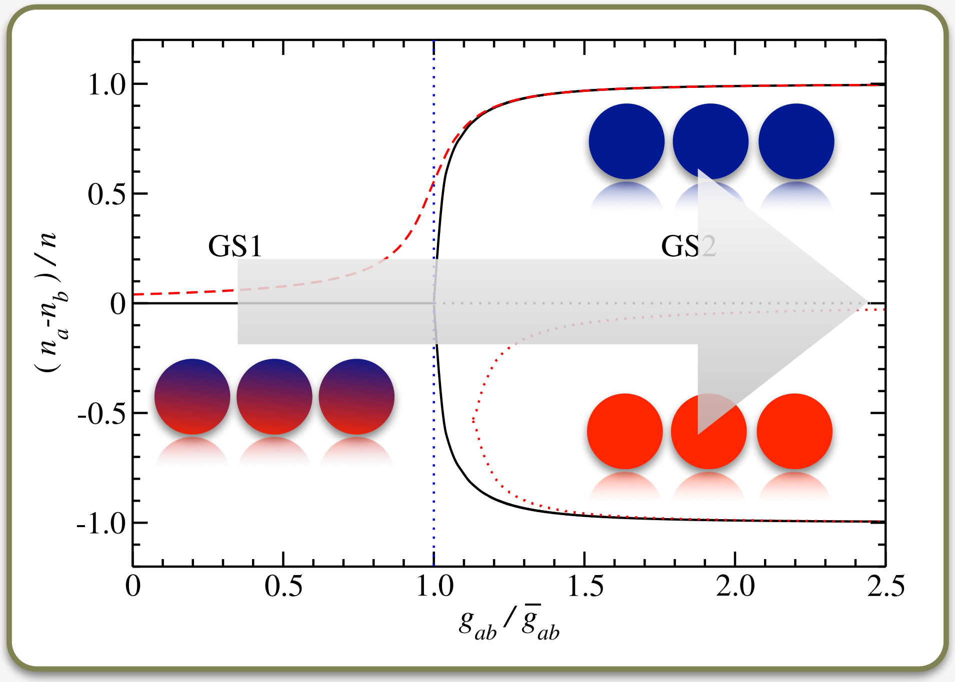
T. Zibold et al. PRL (2010)

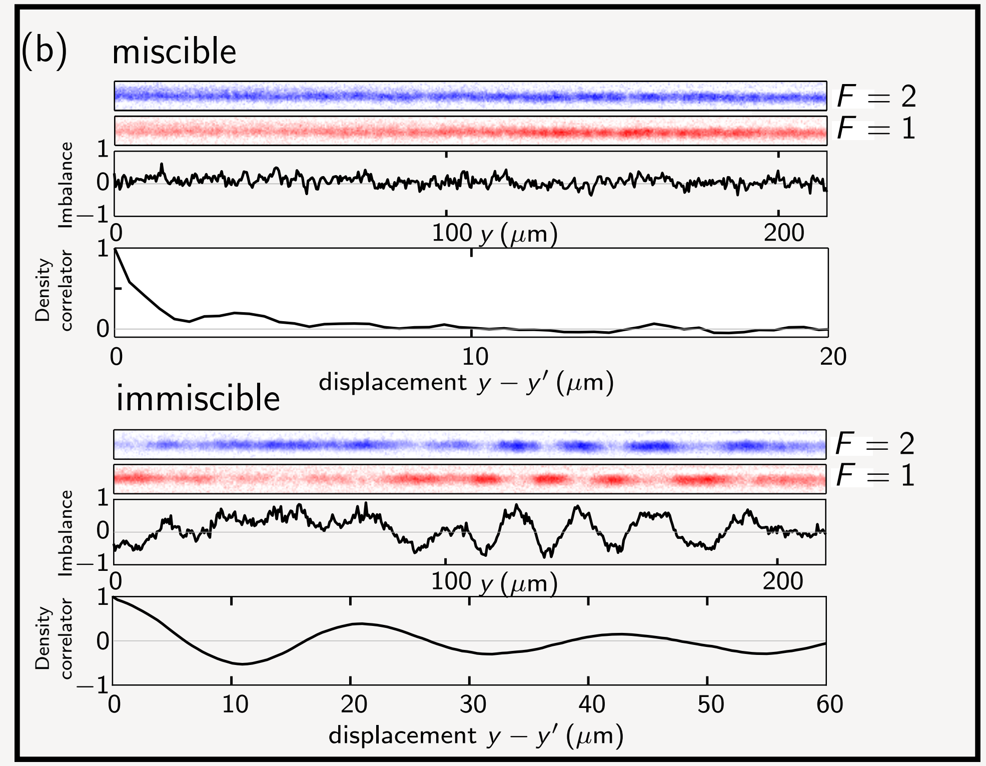
$$n\sqrt{1 - \left(\frac{2|\Omega|}{(g - g_{ab})n}\right)^2}$$

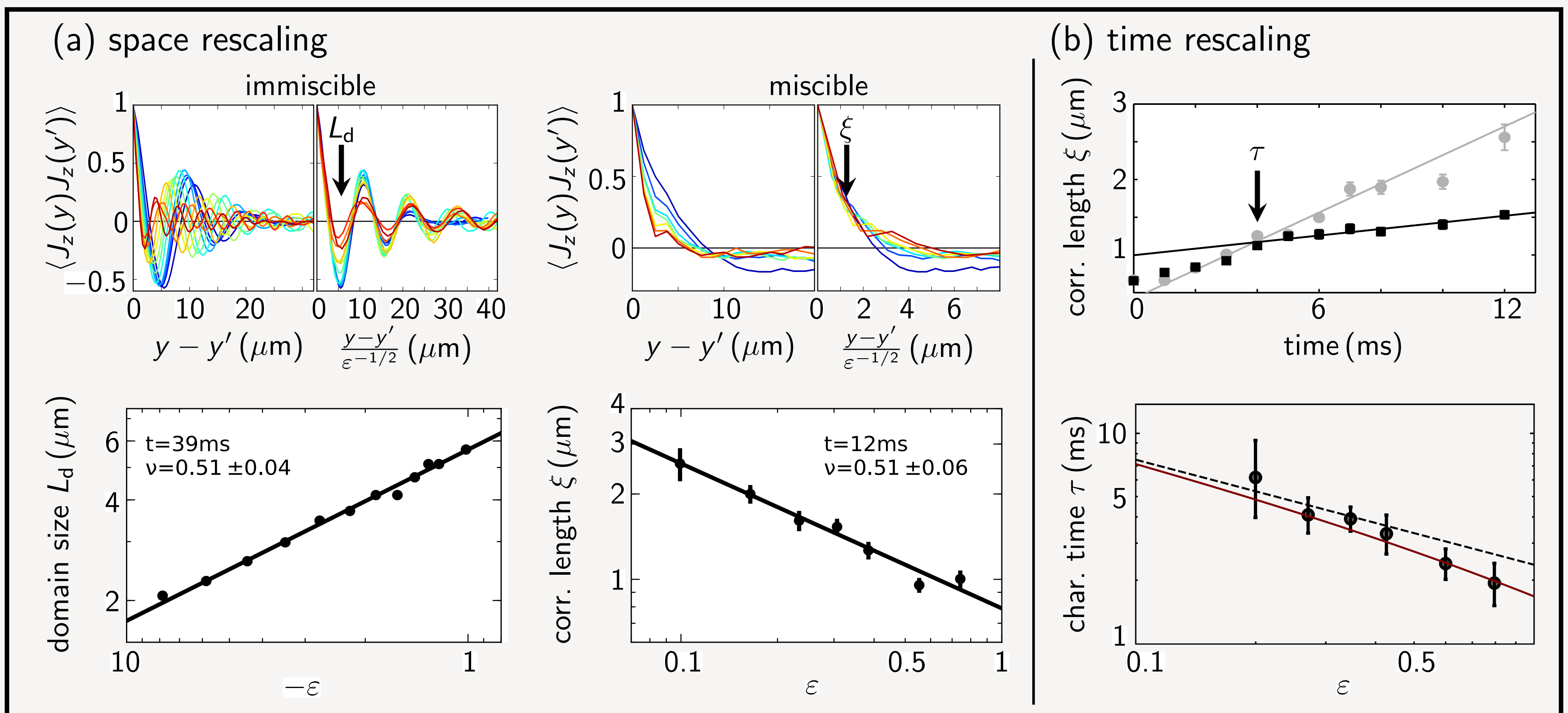


$$\chi_s = \frac{2}{g - g_{ab} + 2\Omega/n}$$

Z_2 Phase Transition

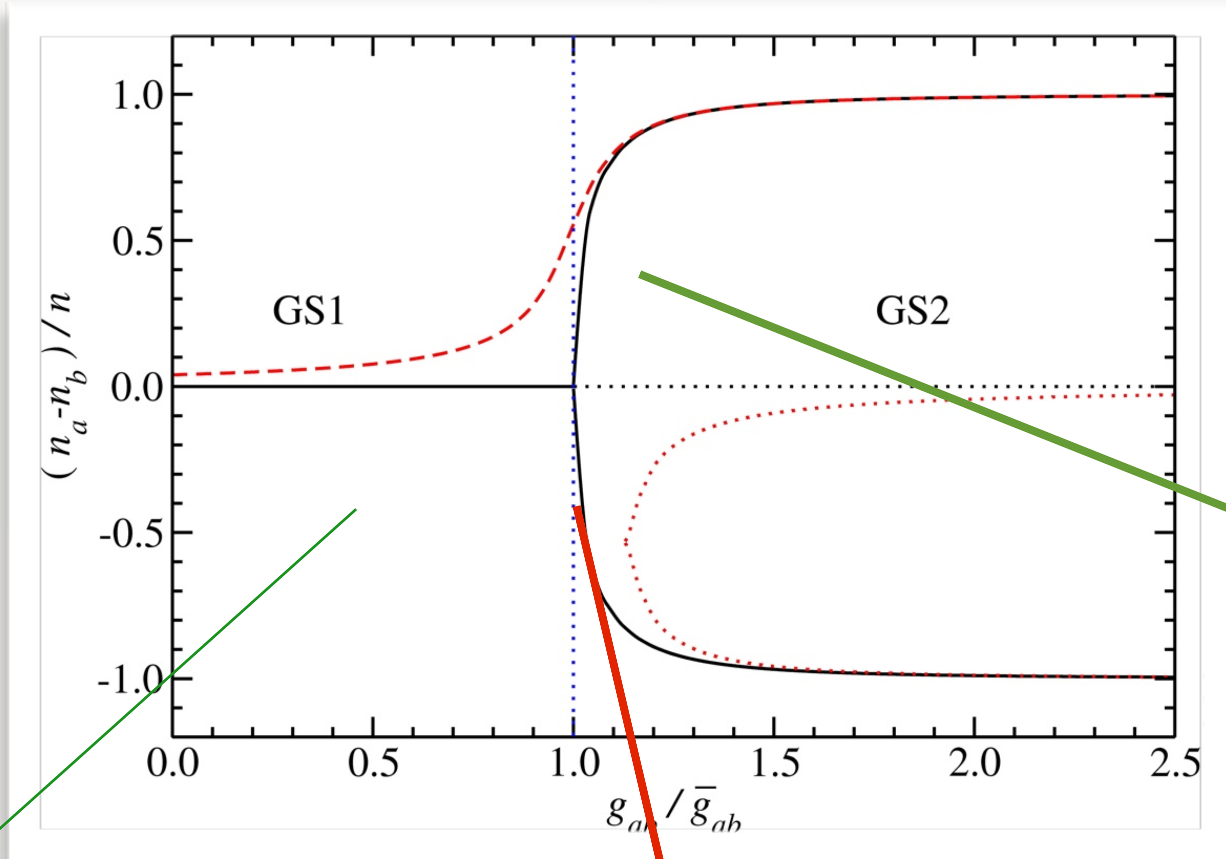






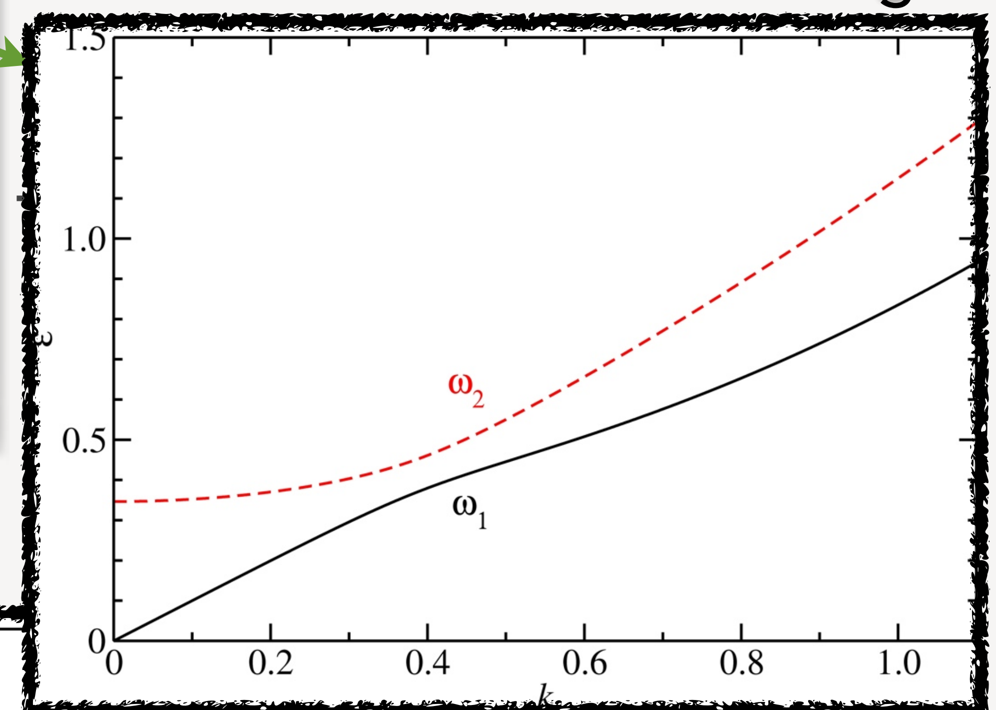
Observation of Scaling in the Dynamics of a Strongly Quenched Quantum Gas
Oberthaler group PRL (2015)

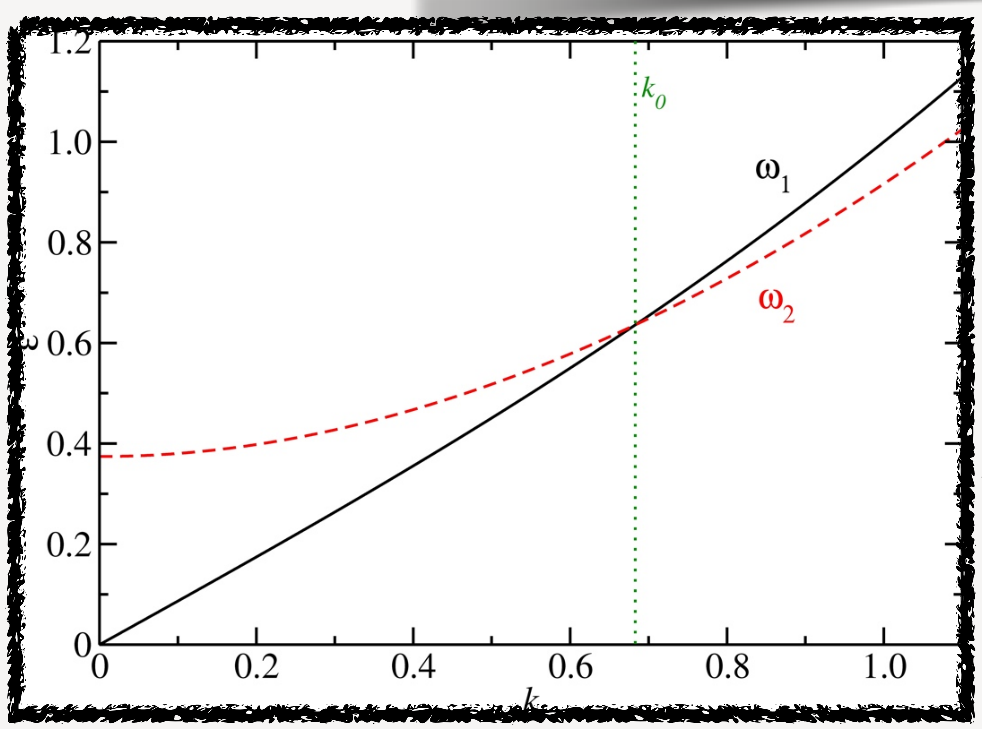
Excitations: Bogoliubov modes across the transition

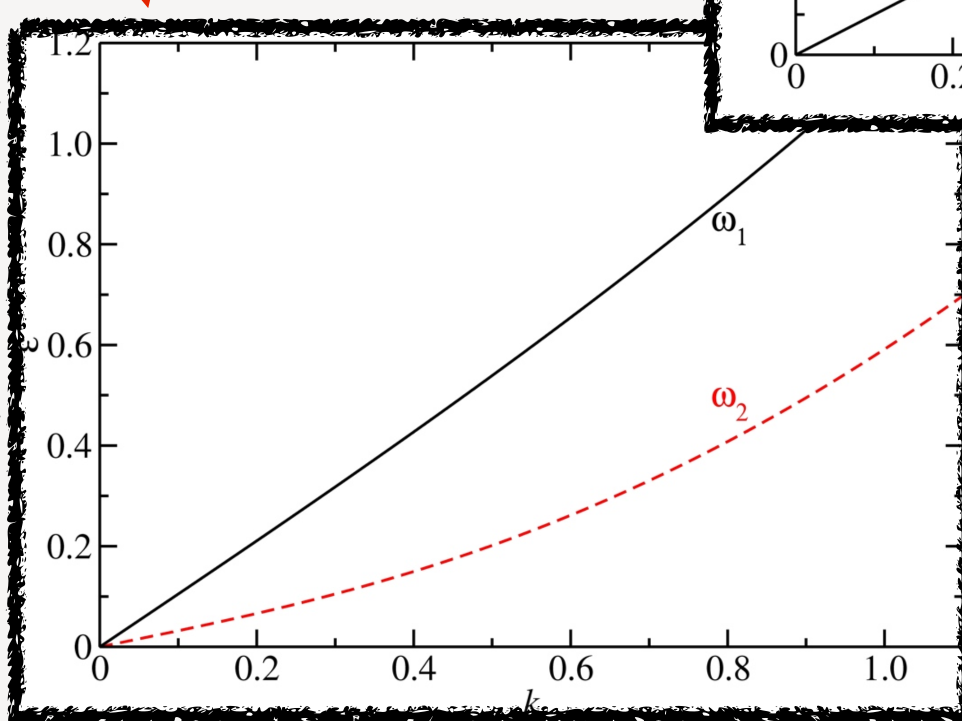


hybridization ⇒

avoided crossing



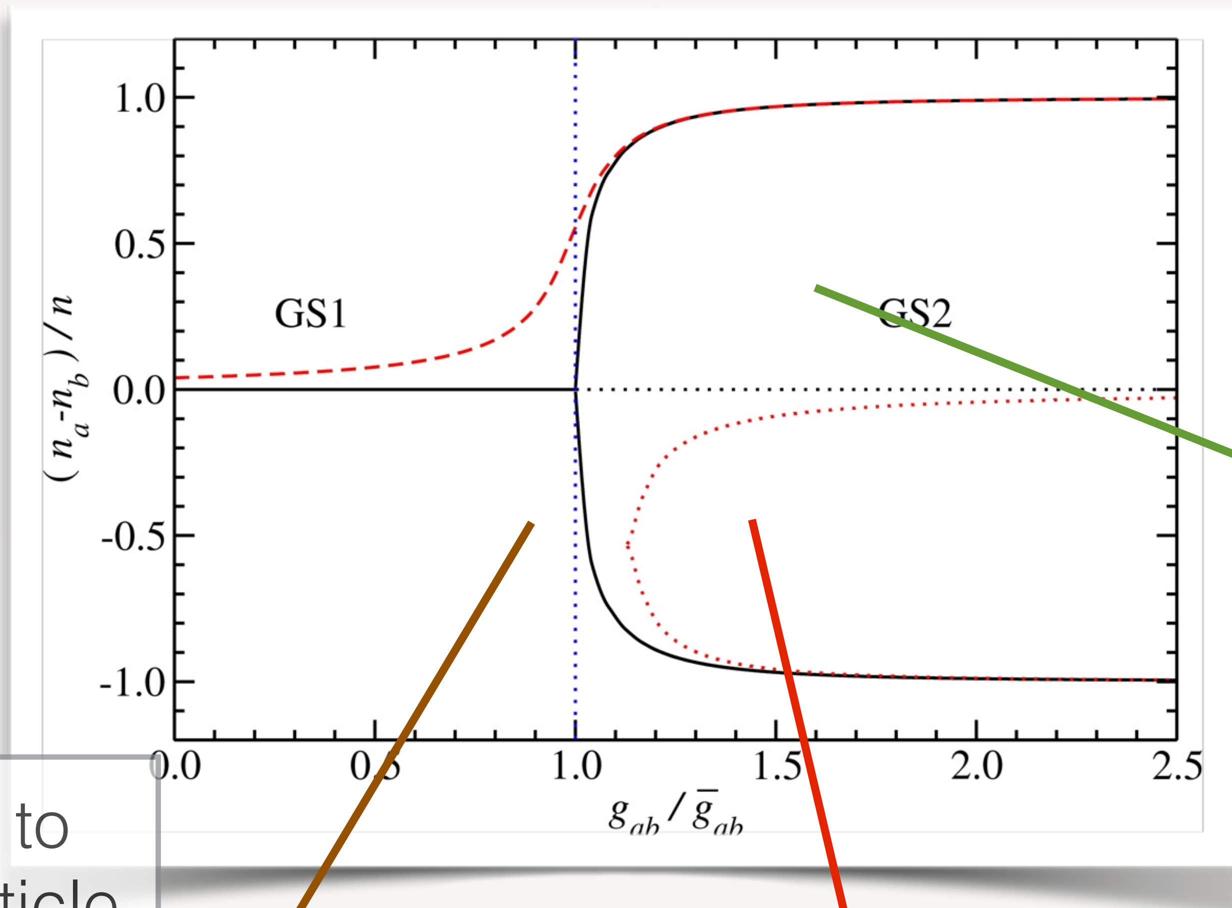


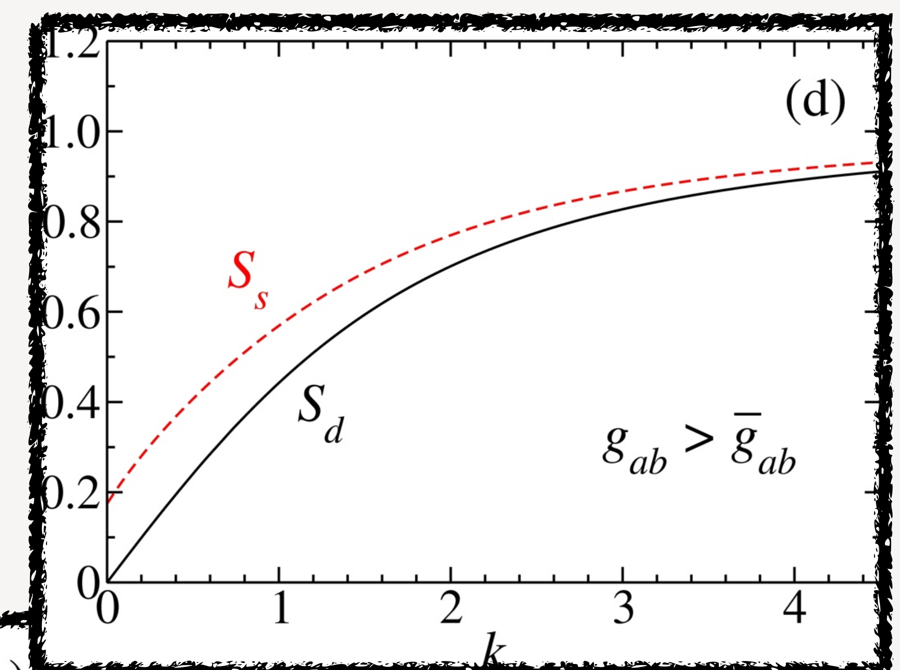


linear gapless modes (different behaviour in mixtures, HD)

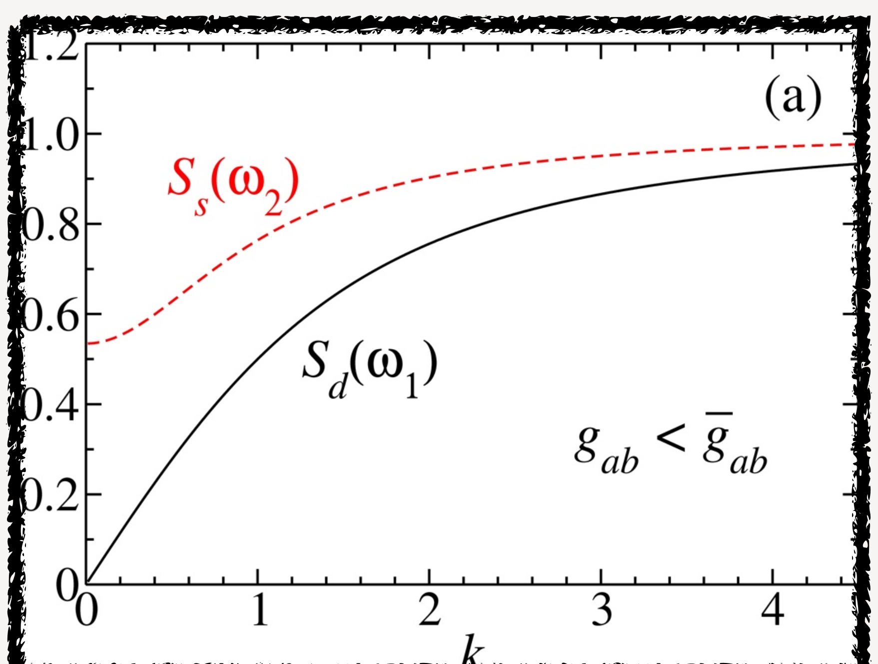
Static structure factor across the transition

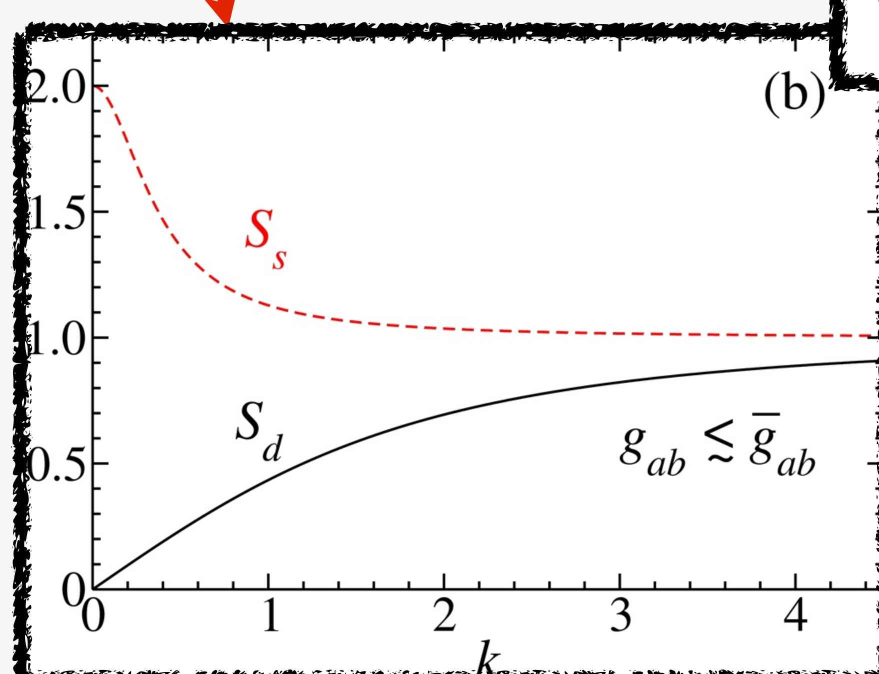
$$S_{d(s)}(k) = S^1_{d(s)}(k) + S^2_{d(s)}(k)$$





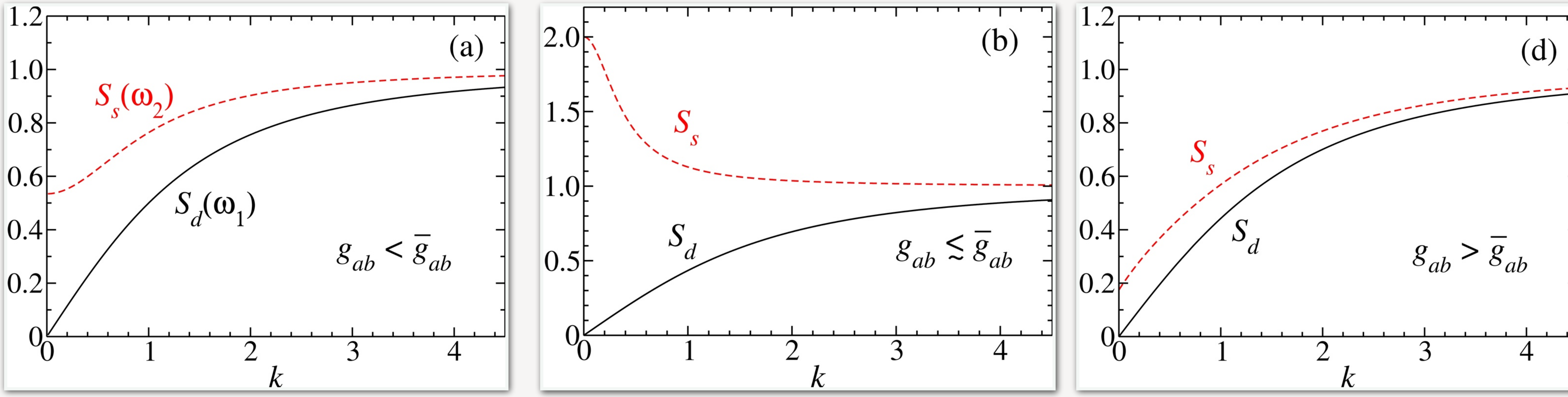
gap due to single-particle





Feynman criterion for the density response

Static structure factor and density/spin fluctuations



S(k) is the Fourier Transform of the density-density correlation function and one can write in particular the FLUCTUATIONS IN A REGION as:

$$\Delta N^2 = n \int S_d(\mathbf{k}, T) H(\mathbf{k}) \frac{d\mathbf{k}}{(2\pi)^D} \simeq N S_d(1/R, T)$$

geometrical factor

$$\Delta M^2 = n \int S_s(\mathbf{k}, T) H(\mathbf{k}) \frac{d\mathbf{k}}{(2\pi)^D} \simeq N S_s(1/R, T)$$

Close to the phase transition the fluctuations in the polarization grow ⇒ structure factor at k=0 grows (diverges for infinite system)

Vortices in coherently coupled BECs

Phase domain walls: simple picture [Son & Stephanov PRA '02]

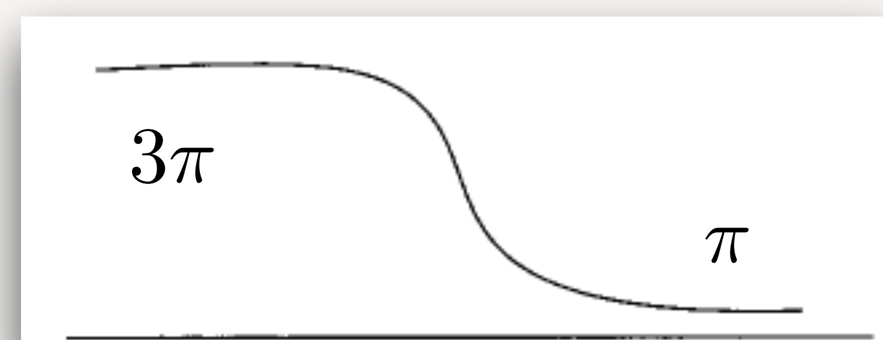
$$e(n_a, n_b) = \frac{1}{2} g_a n_a^2 + \frac{1}{2} g_b n_b^2 + g_{ab} n_a n_b + 2|\Omega| \cos(\phi_a - \phi_b)\sqrt{n_a n_b}$$

For fixed (equal) densities the functional energy of the relative phase reads:

$$E_{spin} = \int \left[\frac{\hbar^2 n}{4m} (\nabla \phi_s)^2 + 2\Omega n \cos(\phi_s) \right]$$

Global minimum for $\phi_s = (2n+1)\pi$

Domain wall or kink is a local minimum solution which connects 2 global minima

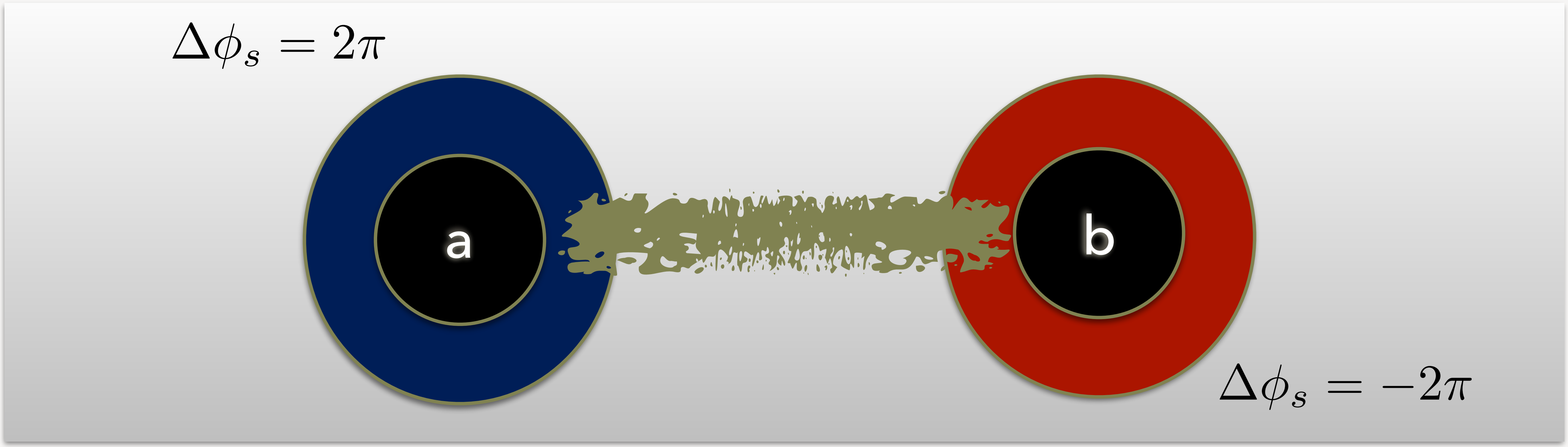


The kink surface tension reads

$$\sigma = \sqrt{\frac{8\hbar^3}{m} n \sqrt{\Omega}}$$

Vortices in coherently coupled BECs

Phase domain walls: simple picture [Son & Stephanov PRA '02]



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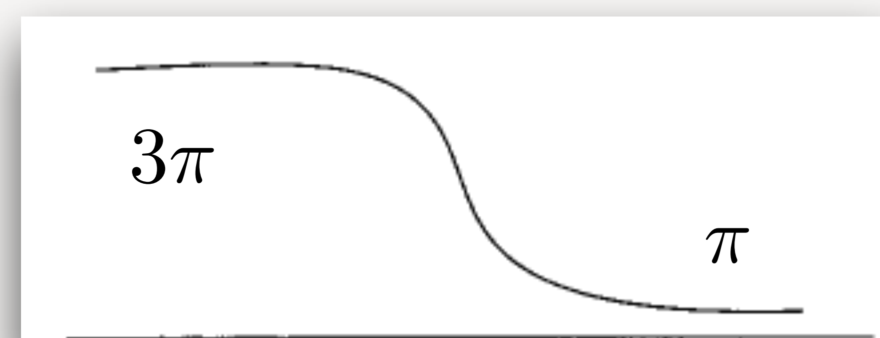

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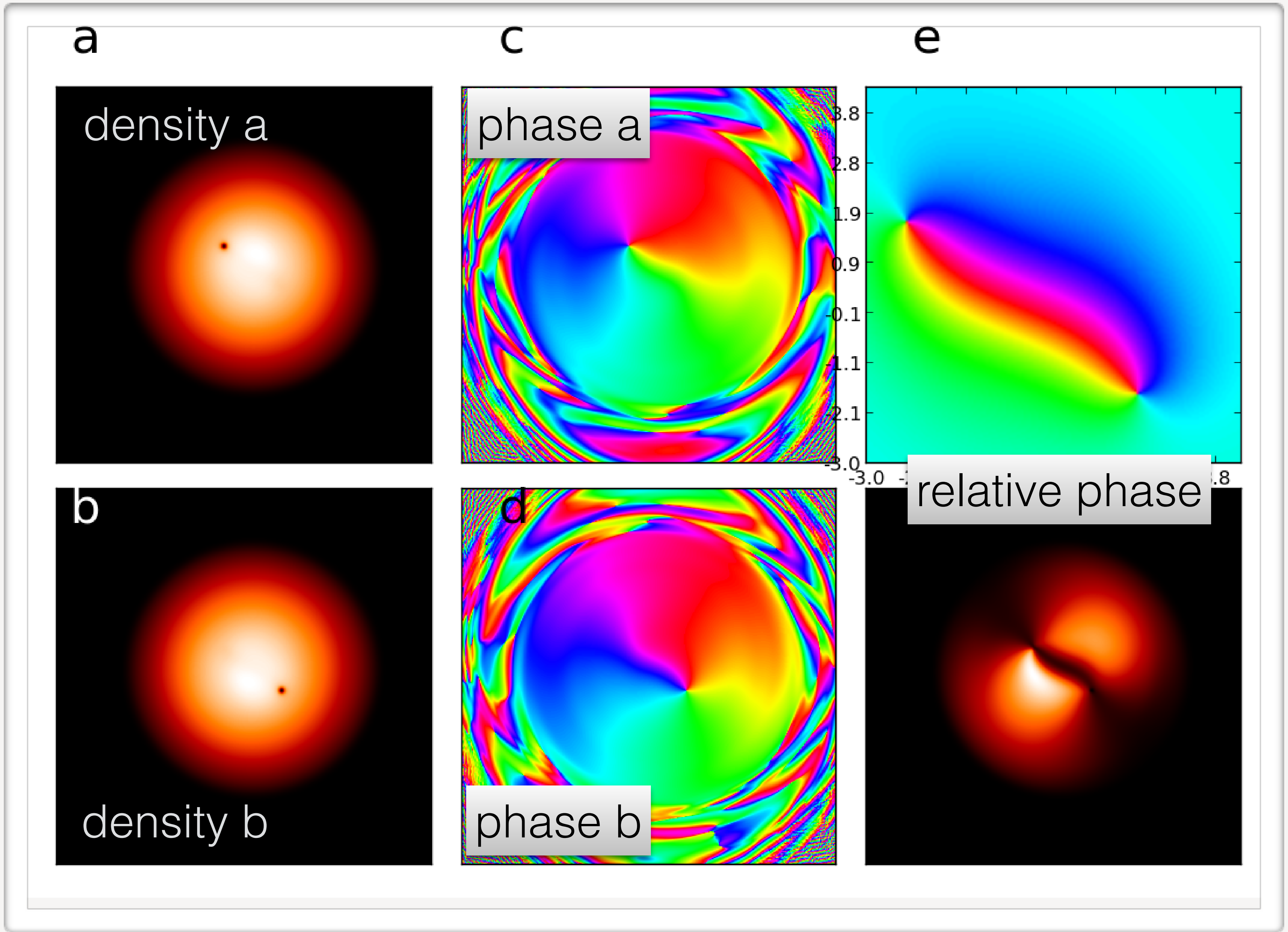
Vortices in coherently coupled BECs: vortex dimers



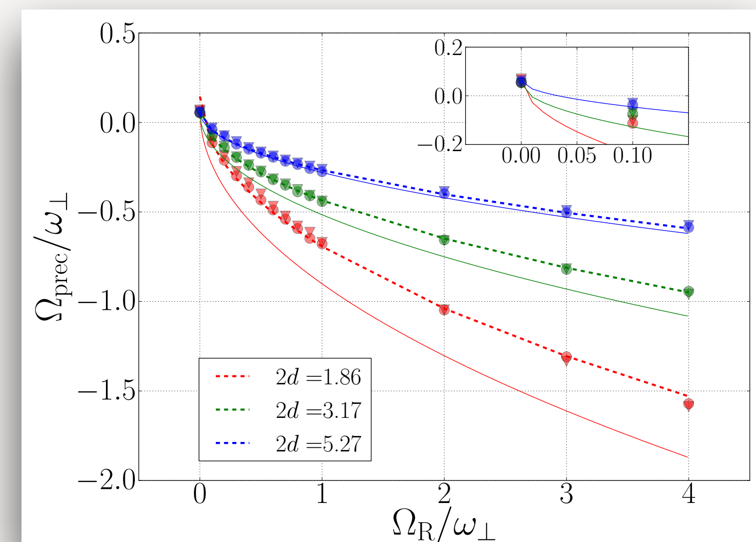
Tylutky, AR, Pitaevskii, Stringari, PRA (2016) -
see also K. Kasamatsu, M. Tsubota, and M. Ueda, PRL 93, 250406 (2004).

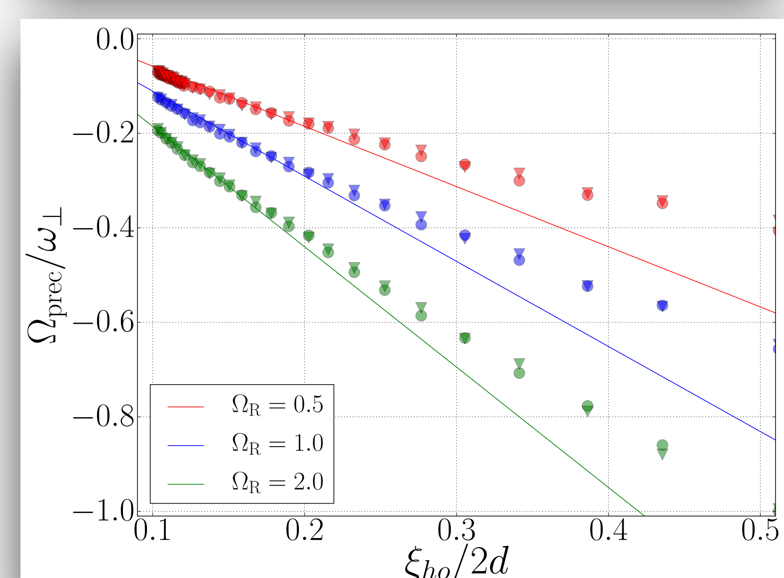
Vortices in coherently coupled BECs: vortex dimers

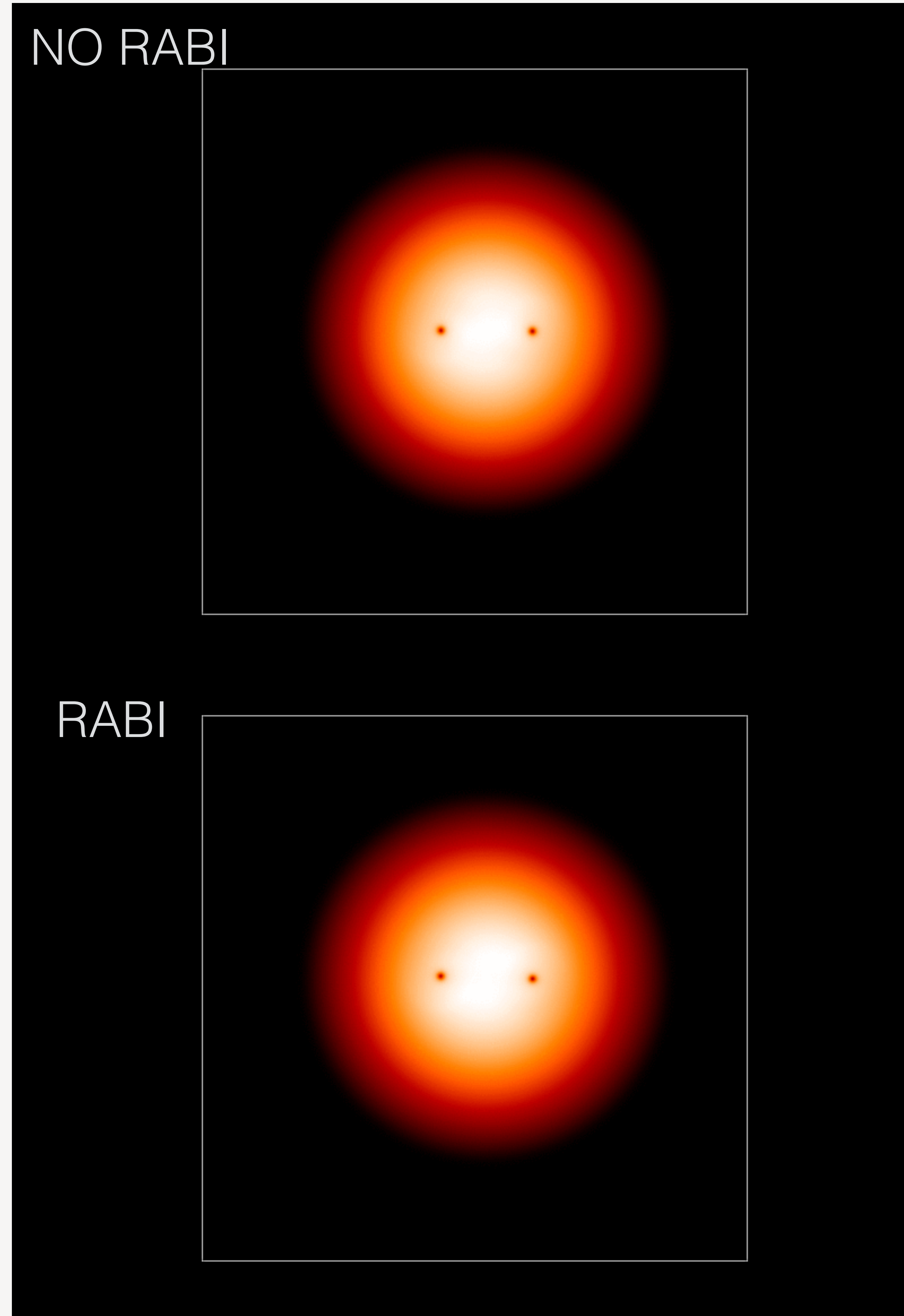
Magnus force = surface tension

$$n_s \mathbf{v}_l \times \kappa = -\sigma \propto -\sqrt{\Omega}$$

$$\omega_{rot} \propto \frac{\sqrt{\Omega}}{d}$$

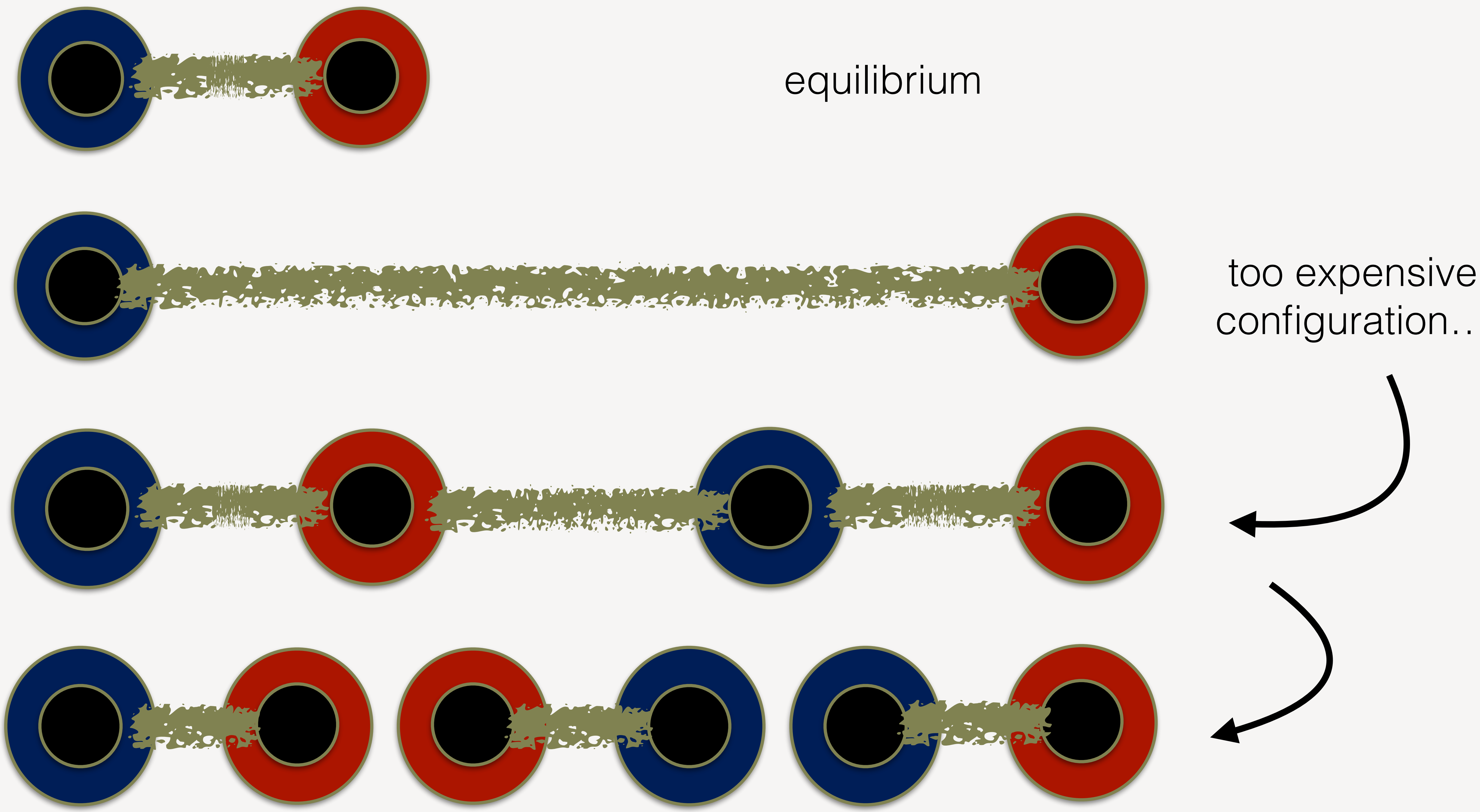




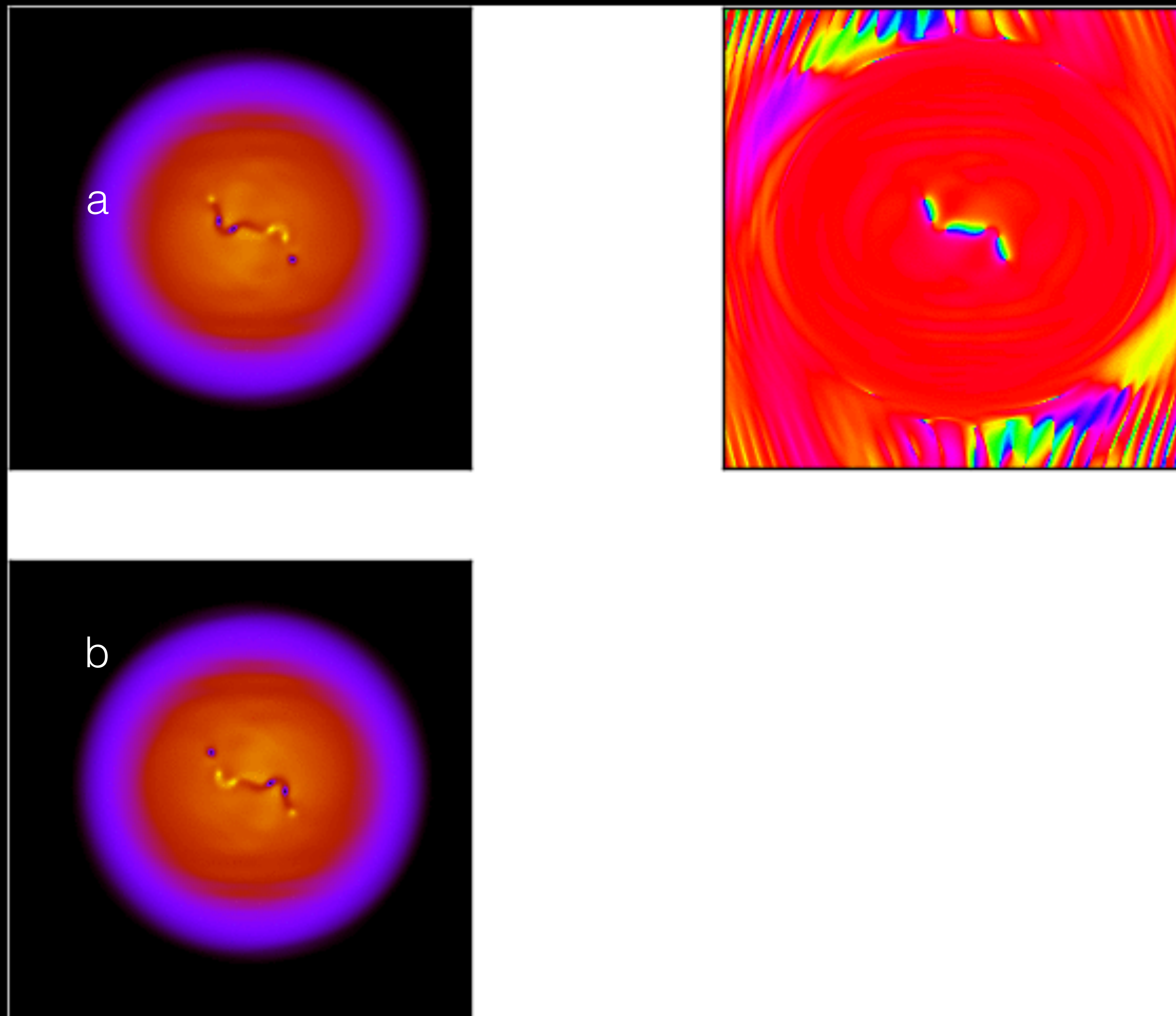


Tylutky, AR, Pitaevskii, Stringari, PRA (2016)

Vortices in coherently coupled BECs: vortex dimers & string breaking



Vortices in coherently coupled BECs:
vortex dimers & string breaking



The Rabi coupling strongly modified the physics of two component Bose gas at the few and many body level.

1. ITF-like (or phi^4) Ferromagnetic Transition *(2D not-at-all MF)*
2. Vortex dimer and string breaking
3. LHY corrections from 2.5 to 3-body corrections
4. Goldstone mode decay at the FM transition
5. Effective Resonances in the 2-body scattering interactions
6. Peculiar Repulsive Bound Pairs with an internal spin which depends on its motion (lattice)

….3-body (Petrov), new vortex lattices (Cipriani),
Persistent current (Abad), spin-dipole mode and sum-rules (AR),
Hawking radiation (Carusotto)….