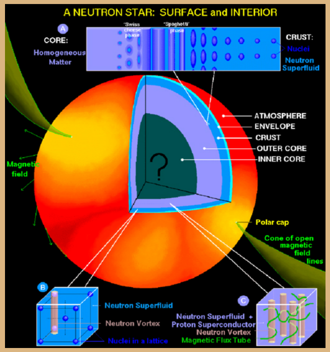
From neutrons to atoms (in 2D) and back

Alex Gezerlis

UNIVERSITY of GUELPH

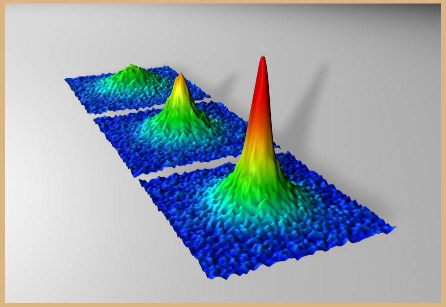
"Exploring nuclear physics with ultracold atoms" workshop
ECT*, Trento, Italy
June 18, 2018

Outline

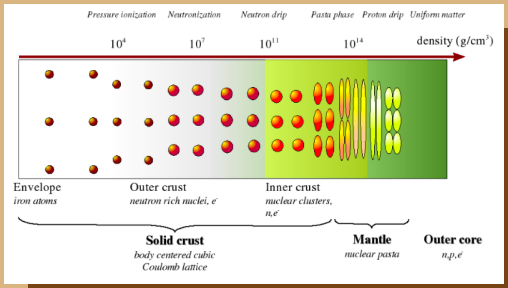


Credit: Dany Page

Motivation



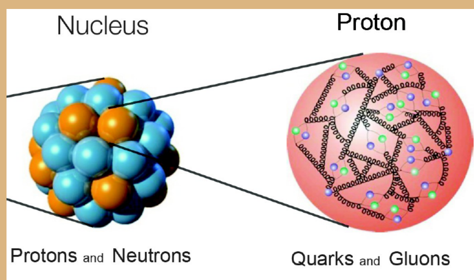
2D cold gases



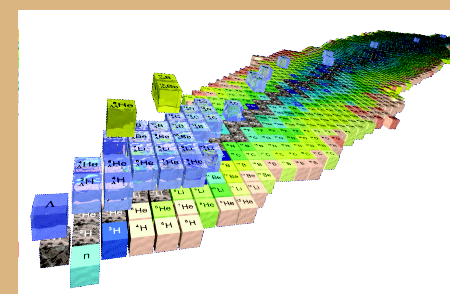
Static response

Physical systems studied

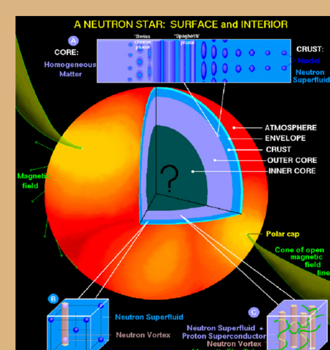
Nuclear forces

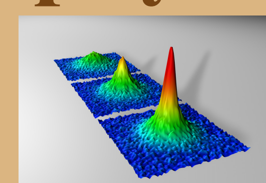


Nuclear structure



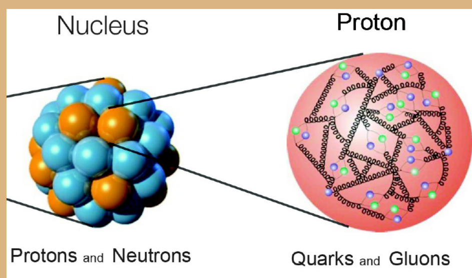
Nuclear astrophysics



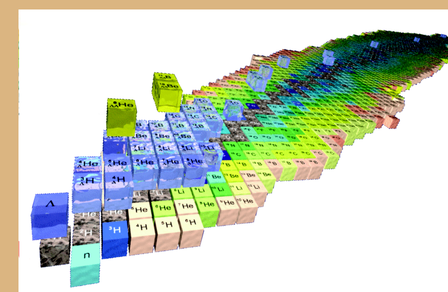


Physical systems studied

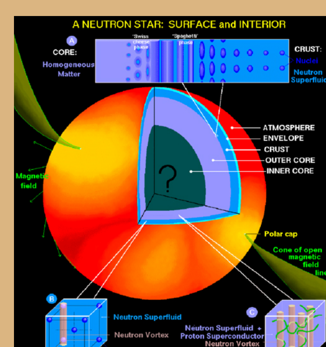
Few nucleons

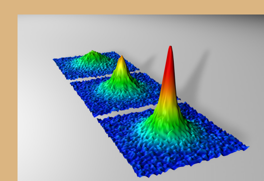


Many nucleons



Very many nucleons





Nuclear many-body problem

$$H\Psi = E\Psi$$

where $$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \cdots$$

so

$$H\Psi(\mathbf{r}_1, \cdots, \mathbf{r}_A; s_1, \cdots, s_A; t_1, \cdots, t_A) = E\Psi(\mathbf{r}_1, \cdots, \mathbf{r}_A; s_1, \cdots, s_A; t_1, \cdots, t_A)$$

i.e. $2^A \binom{A}{Z}$ complex coupled second-order differential equations

Nuclear many-body methods

Phenomenological (fit to A-body experiment)

Ab initio (fit to few-body experiment)

Nuclear many-body methods

Phenomenological (fit to A-body experiment)

- **Hartree-Fock/Hartree-Fock-Bogoliubov (HF/HFB)**
  mean-field theory, a priori inapplicable, unreasonably effective

- **Energy-density functionals (EDF)**
  like mean-field but with wider applicability

Nuclear many-body methods

Ab initio (fit to few-body experiment)

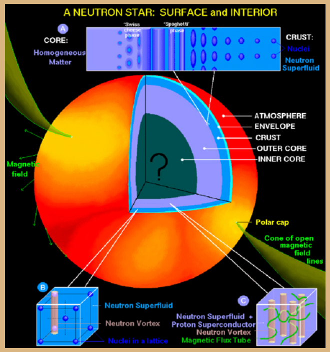
- **Quantum Monte Carlo (QMC)**
  stochastically solve the many-body problem “exactly”

- **Perturbative Theories (PT)**
  first few diagrams only (though no small expansion parameter present)

- **Resummation schemes (e.g. SCGF)**
  selected class of diagrams up to infinite order

- **Coupled cluster (CC)**
  generate np-nh excitations of a reference state

Outline



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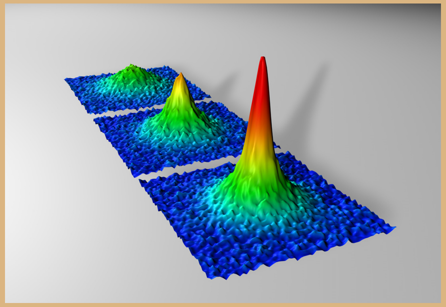
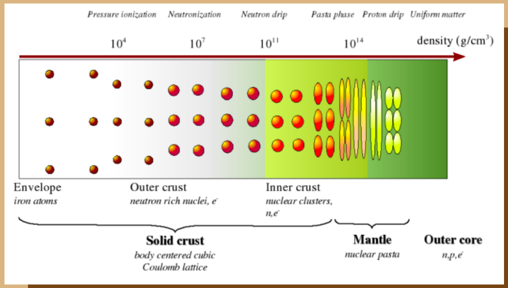
Motivation



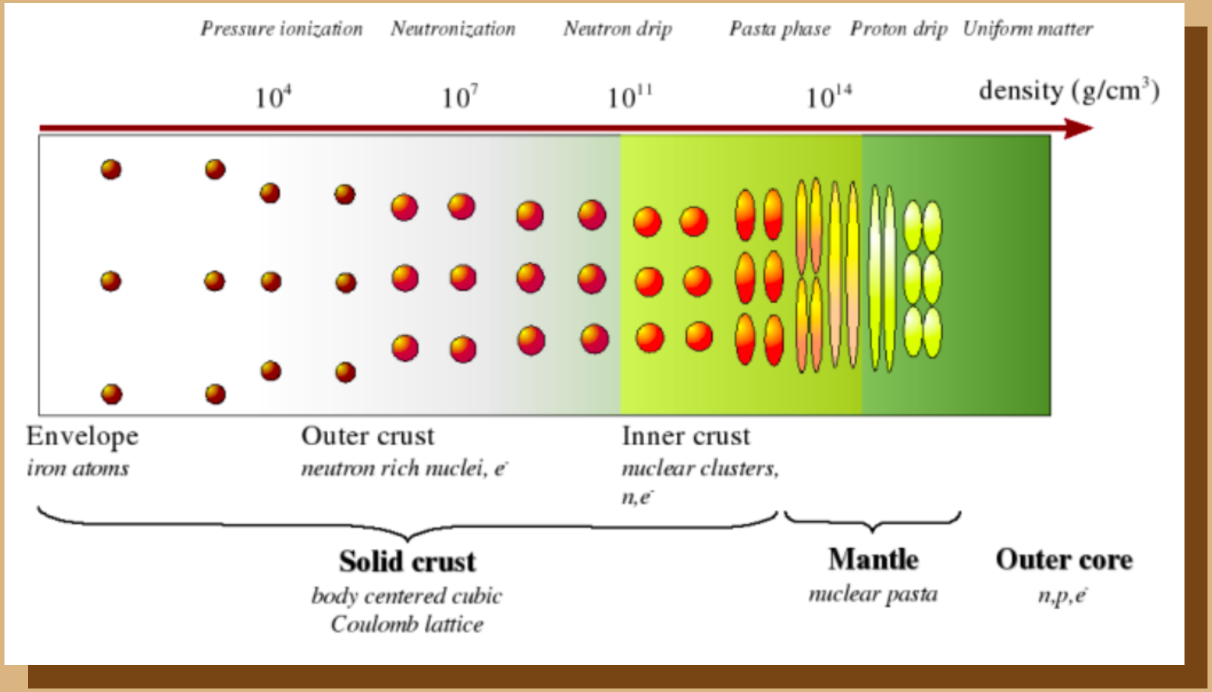
2D cold gases



Static response

Nuclear pasta provides further motivation to study 2D cold gases

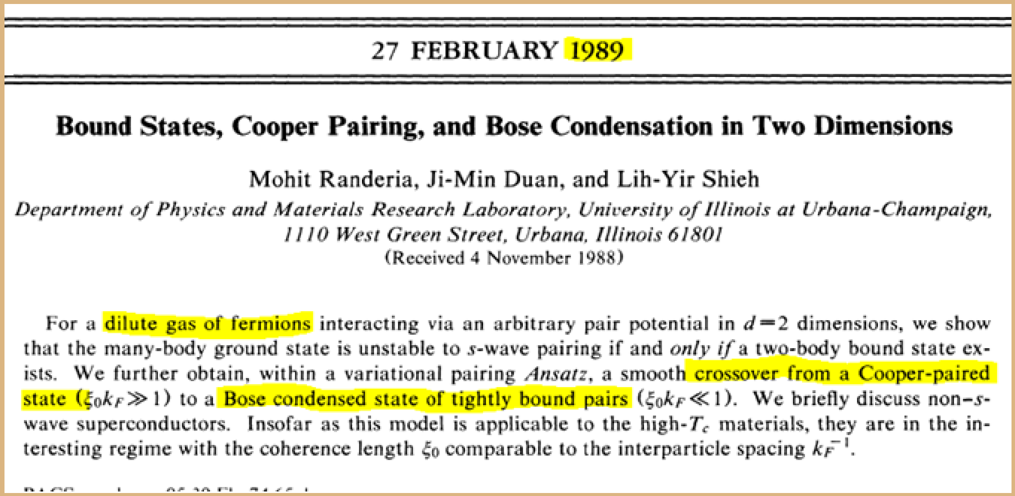
Neutron star crusts inhomogeneous



2D history

Theory

Mean-field BCS calculation for a 2D Fermi gas was done in the 1980s (extending earlier work on 3D systems done independently by Eagles and Leggett)



27 FEBRUARY 1989

Bound States, Cooper Pairing, and Bose Condensation in Two Dimensions

Mohit Randeria, Ji-Min Duan, and Lih-Yir Shieh

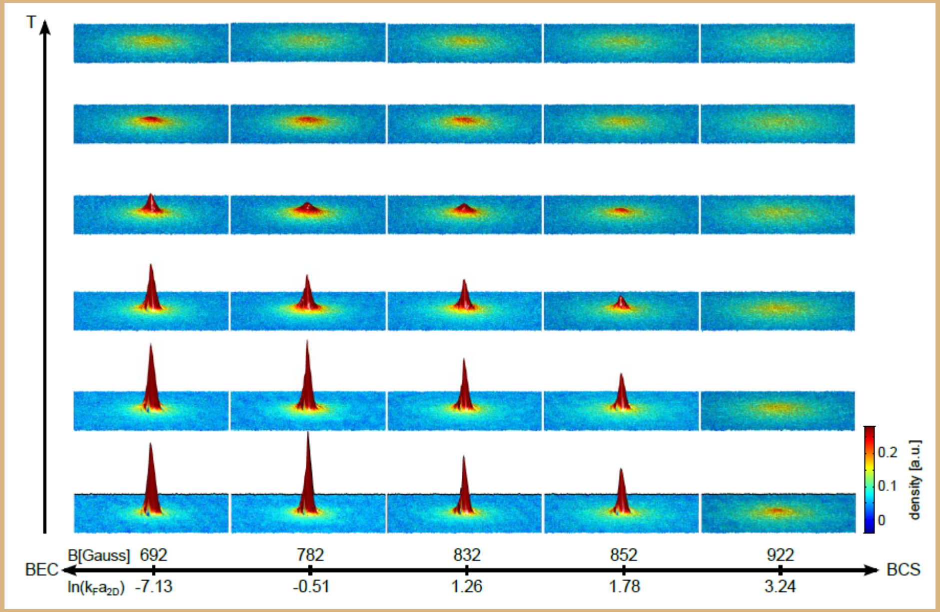
Department of Physics and Materials Research Laboratory, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois 61801

(Received 4 November 1988)

For a dilute gas of fermions interacting via an arbitrary pair potential in $d=2$ dimensions, we show that the many-body ground state is unstable to s-wave pairing if and *only if* a two-body bound state exists. We further obtain, within a variational pairing *Ansatz*, a smooth crossover from a Cooper-paired state ($\xi_0 k_F \gg 1$) to a Bose condensed state of tightly bound pairs ($\xi_0 k_F \ll 1$). We briefly discuss non-s-wave superconductors. Insofar as this model is applicable to the high-T_c materials, they are in the interesting regime with the coherence length ξ_0 comparable to the interparticle spacing k_F^{-1}.

Experiment

- The atomic Fermi gas can be restricted to quasi-2D geometries in the lab
- There has been a large amount of interest in these gases recently



M. Ries et al., Phys. Rev. Lett. **114,** 230401 (2015).

2D present

Several experimental groups probing homogeneous case, dynamics, etc.

See other talks at this workshop

Two-body Schrödinger equation

$$-\frac{\hbar^2}{2m_r}\nabla^2\psi(r,\theta,\phi)+V(r)\psi(r,\theta,\phi)=E\psi(r,\theta,\phi)$$

$$k^2=\frac{2m_rE}{\hbar^2}$$

3D

$$\psi(r,\theta,\phi)=\sum_{l,m}^{\infty}a_l\left[\frac{u_l(r)}{r}\right]Y_l^m(\theta,\phi) \longrightarrow -\frac{\partial^2 u_l(r)}{\partial r^2}=u_l(r)\left[k^2-\frac{2m_r}{\hbar^2}V(r)-\frac{l(l+1)}{r^2}\right]$$

$$\frac{\partial^2 y_0(r)}{\partial r^2}=0$$

$$y_0(r)=\beta\left(r-a_{3\mathrm{D}}\right)$$

2D

$$\psi(r,\theta)=\sum_{l=0}^{\infty}a_l\left[\frac{u_l(r)}{\sqrt{r}}\right]T_l(\theta) \longrightarrow -\frac{\partial^2 u_l(r)}{\partial r^2}=u_l(r)\left[k^2-\frac{2m_r}{\hbar^2}V(r)-\frac{l^2-1/4}{r^2}\right]$$

$$\frac{\partial^2 y_0(r)}{\partial r^2}=-\frac{1}{4r^2}y_0(r)$$

$$y_0(r)=\beta\sqrt{r}\log(r/a_{2\mathrm{D}})$$

Scattering length and eff. range

3D

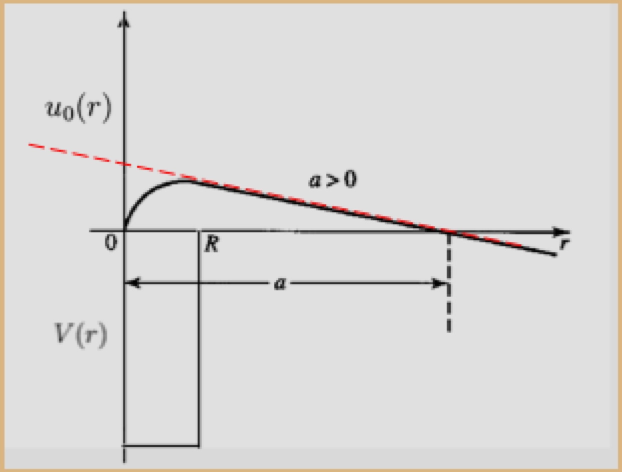
$$-\frac{\partial^2 u_0(r)}{\partial r^2} = u_0(r)\left[k^2 - \frac{2m_r}{\hbar^2}V(r)\right]$$

Scattering length: $y_0(r) = \beta\,(r - a_{3\text{D}})$

Effective range: $r_e = 2\int_0^\infty \left[y_0^2(r) - u_0^2(r)\right]_{k\to 0} dr$

Asymptotic form (red)

Wave function (black)

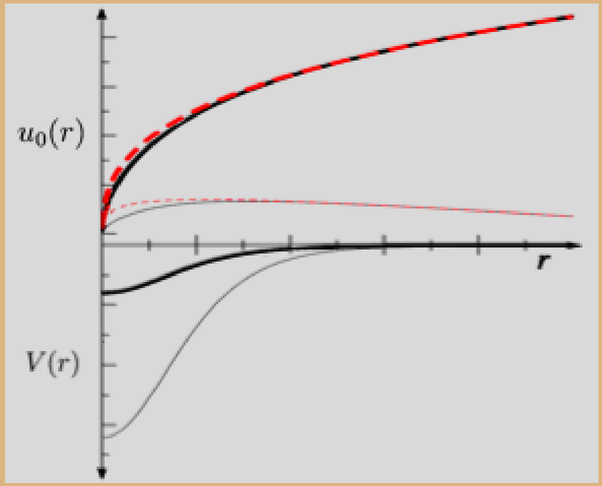


2D

$$-\frac{\partial^2 u_0(r)}{\partial r^2} = u_0(r)\left[k^2 - \frac{2m_r}{\hbar^2}V(r) + \frac{1}{4r^2}\right]$$

Scattering length: $y_0(r) = \beta\sqrt{r}\log(r/a_{2\text{D}})$

Effective range: $r_e^2 = 4\int_0^\infty \left[y_0^2(r) - u_0^2(r)\right]_{k\to 0} dr$



2D notation

scattering length

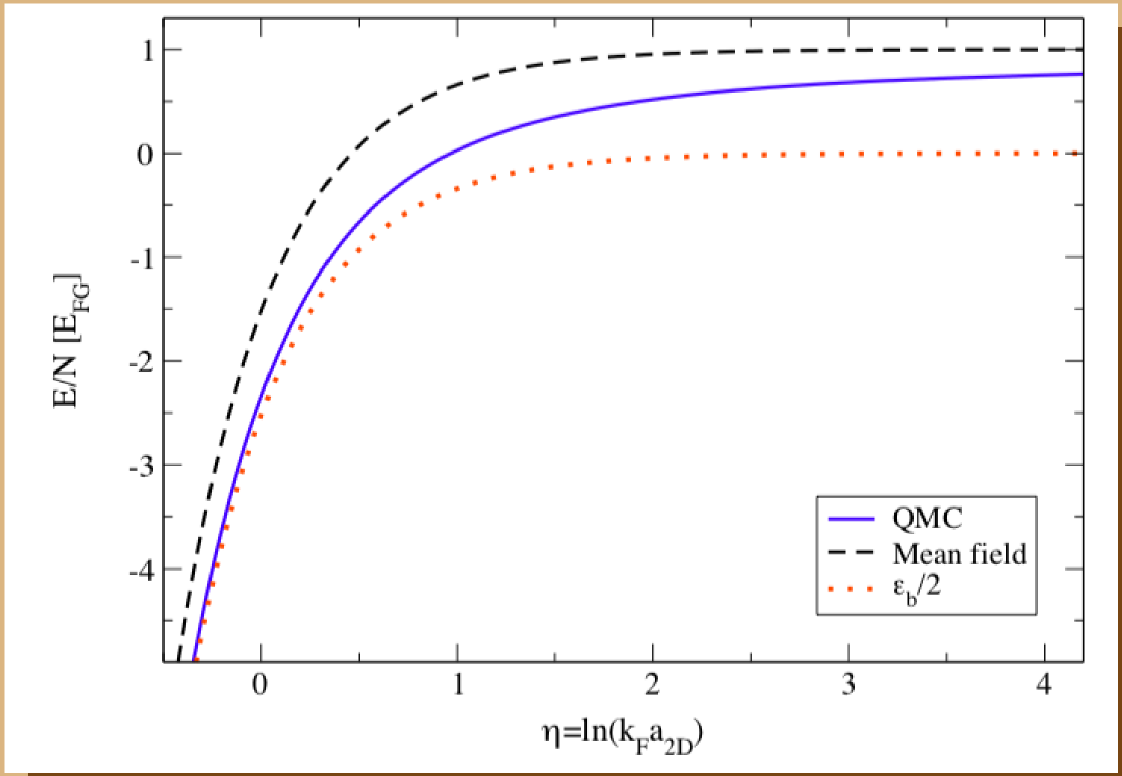
$$\eta = \ln(k_F a)$$

coupling

"density"

2D results

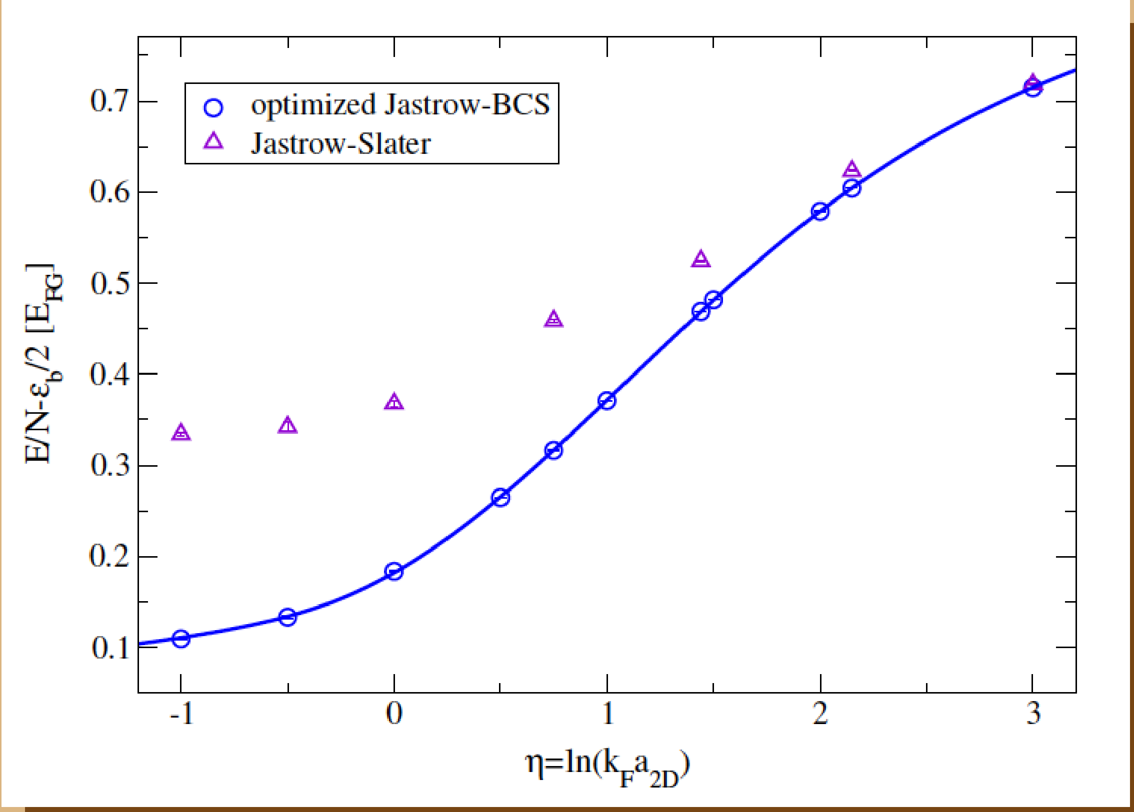
Equation of State



A. Galea, H. Dawkins, S. Gandolfi, and A. Gezerlis, Phys. Rev. A **93**, 023602 (2016)

ATOMS

2D results



Equation of State

- We subtract the binding energy per particle εb/2 (which is a two- body quantity)
- Clear signature of pairing effects

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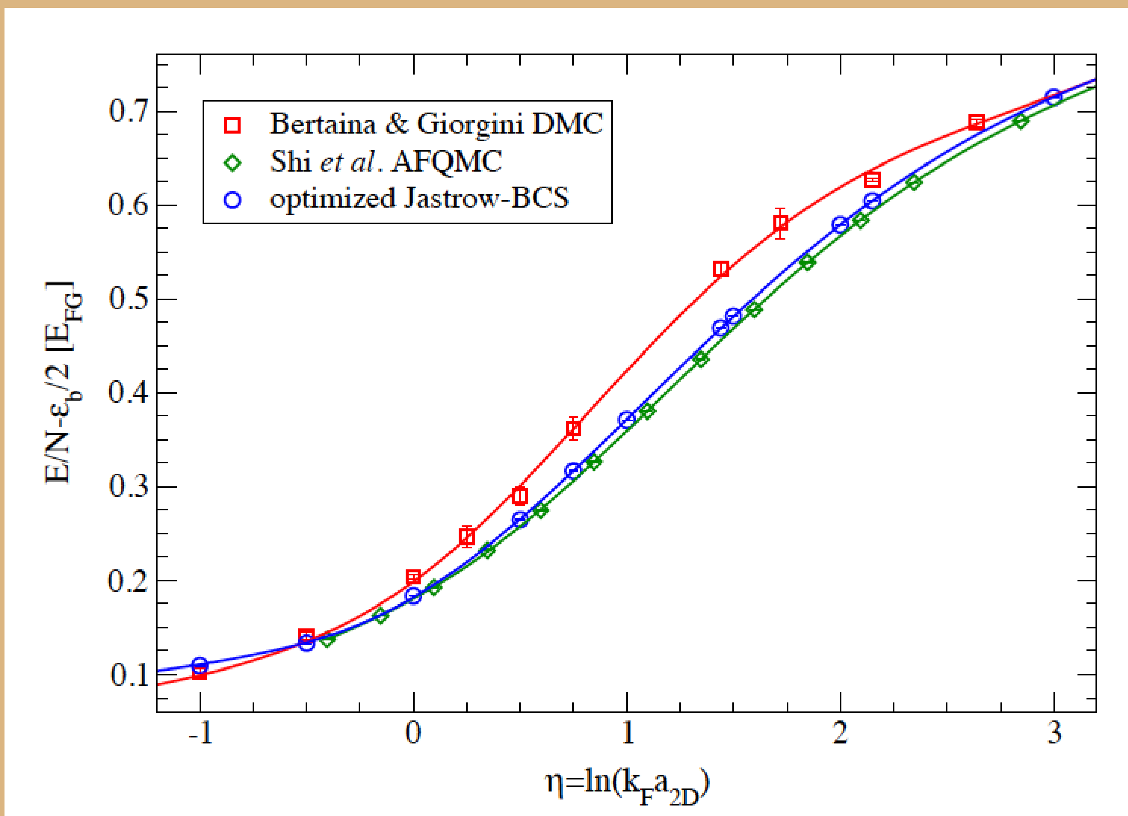
A. Galea, H. Dawkins, S. Gandolfi, and A. Gezerlis, Phys. Rev. A **93**, 023602 (2016)

A. Galea, T. Zielinski, S. Gandolfi, and A. Gezerlis, J Low Temp Phys **189**, 451 (2017)

2D results



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A. Galea, T. Zielinski, S. Gandolfi, and A. Gezerlis, J Low Temp Phys **189**, 451 (2017)

Equation of State

- We subtract the binding energy per particle εb/2 (which is a two- body quantity)
- Red – first DMC calculations of the strongly interacting Fermi gas (2011)
- Blue – our DMC results, giving a tighter upper bound to the true ground state energy
- Green – exact AFQMC calculations (2015)

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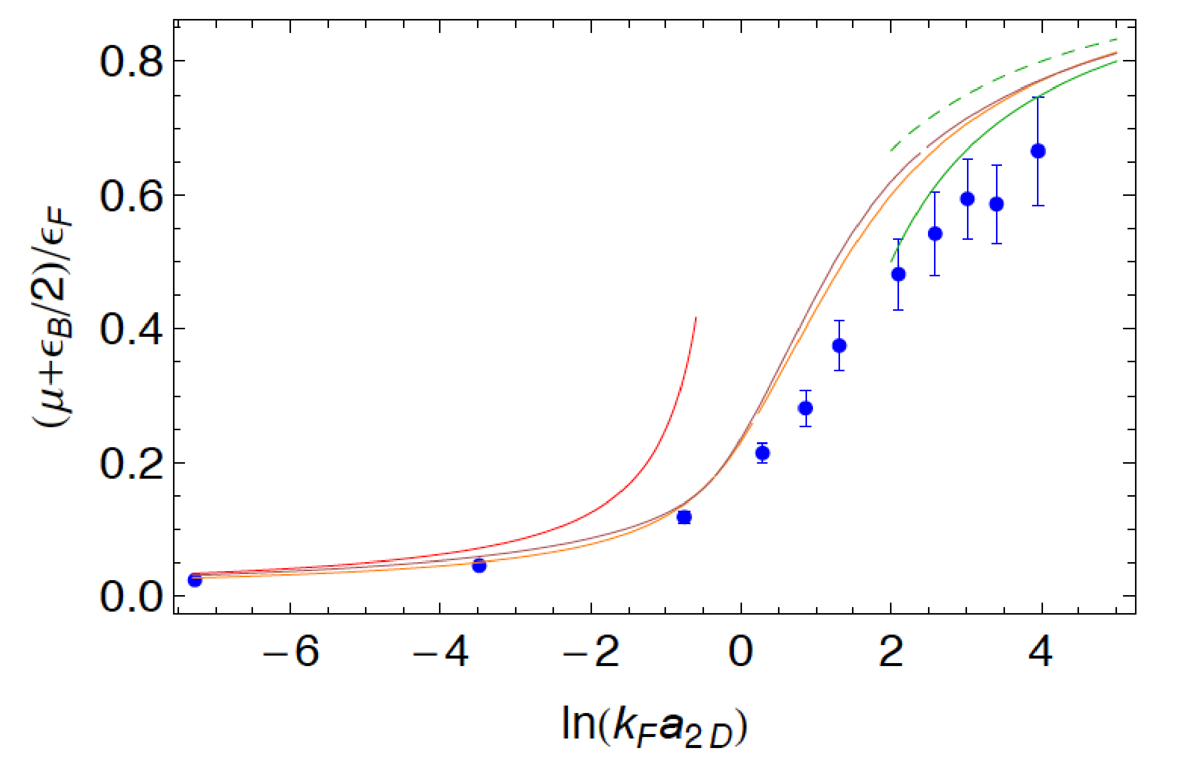
2D results: comparison to experiment



Chemical Potential

- Our DMC result (maroon) is very similar to AFQMC (orange)
- Experimental results (and figure) provided by Tilman Enss (Universität Heidelberg)

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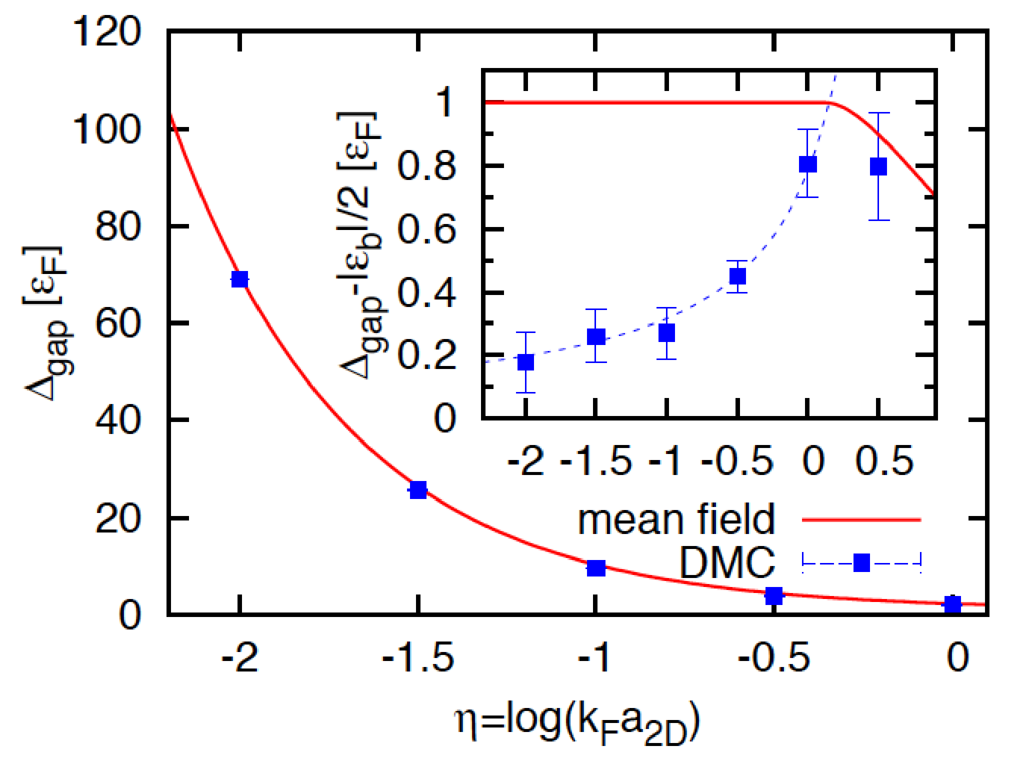
2D results



Pairing gap

- We calculate the pairing gap based on odd-even energy staggering
- The gap becomes very large when pairs are tightly bound
- A suppression compared to the mean-field result can be seen when adding the binding energy per particle εb/2

Bertaina & Giorgini, Phys. Rev. Lett. **106**, 110403 (2011)

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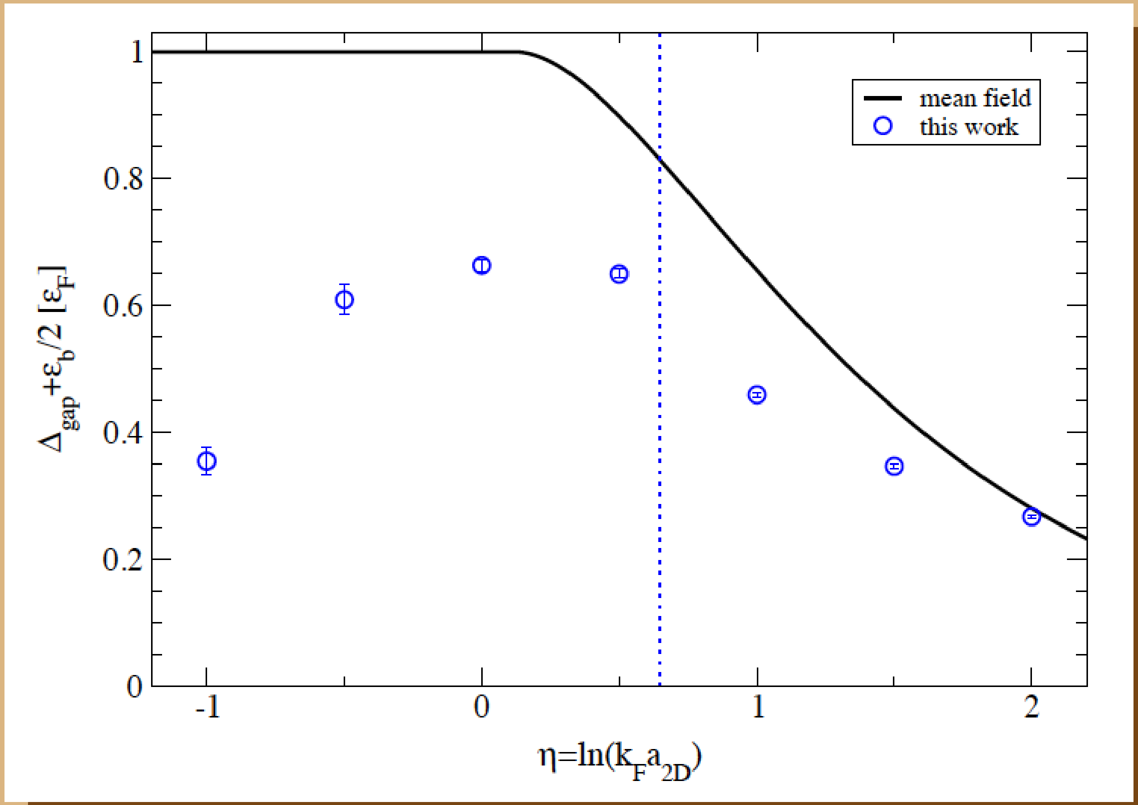
2D results



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ATOMS

Conclusions

- Rich connections between physics of nuclei, compact stars, and cold atoms
- Exciting time in terms of interplay between nuclear interactions, QCD, and many-body approaches
- Ab initio and phenomenology are mutually beneficial

Acknowledgments

Funding

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MINISTÈRE DE LA RECHERCHE ET DE L'INNOVATION

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- Achim Schwenk

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- Ingo Tews

IPN Orsay

- Denis Lacroix