Cold Few-Atom Systems With Large s-Wave Scattering Length

Doerte Blume, Qingze Guan
The University of Oklahoma

Supported by the NSF.

Experiment:
Maksim Kunitski,
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Frankfurt University

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Or Possibly Better: Low-Energy Few-Body Physics

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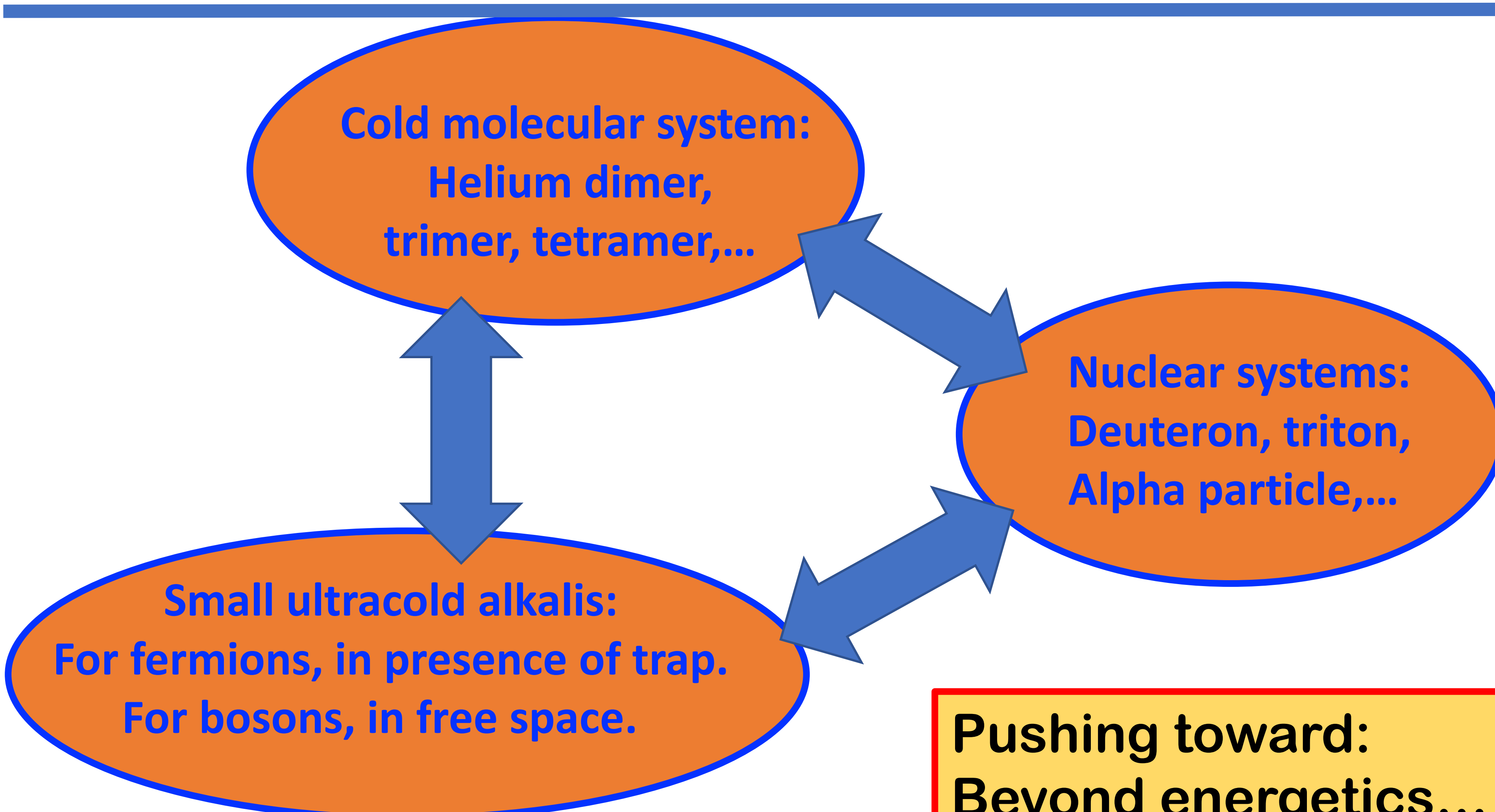
Three Few-Body Systems



Pushing toward:
Beyond energetics…
Beyond statics…

Common Characteristic?
Large s-Wave Scattering Length

1) Helium-helium (van der Waals length $r_{vdw} \approx 5.1a_0$):
   1) ^{3}He-^{4}He: $a_s = -34.15a_0$
   2) ^{4}He-^{4}He: $a_s = 170.86a_0$

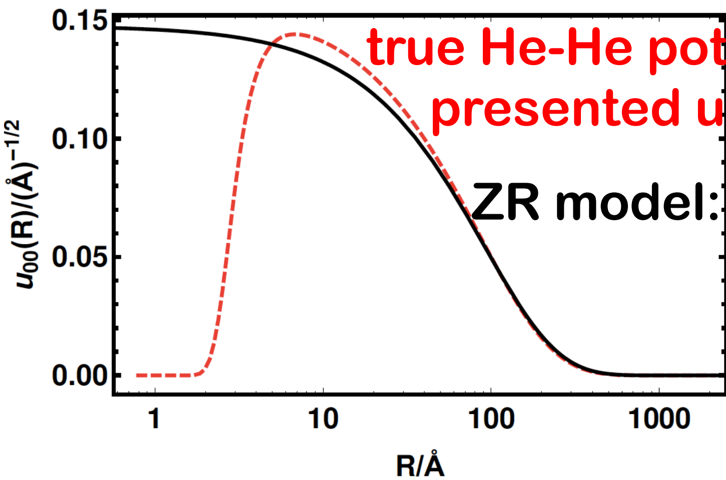
   "Naturally" large scattering length.
   No deep-lying bound states.
   Tunability?

2) Deuteron (nuclear force around 1 Fermi $= 10^{-15}$m):
   1) Singlet ($S = 0,\ T = 1$): $a_s^{S=0} = -23.7$Fermi
   2) Triplet ($S = 1,\ T = 0$): $a_s^{S=1} = 5.38$Fermi

3) Ultracold atoms: Feshbach resonances provide enormous tunability.

^{4}Helium-^{4}Helium Example: One Vibrational Bound State

Radial wave function:



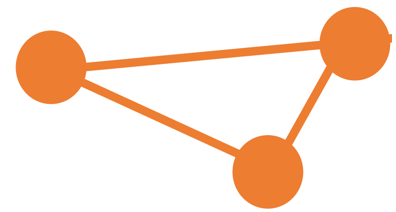
- ^{4}He-^{4}He bound state energy $E_{\text{dimer}} = -1.3$mK.
- No $J > 0$ bound states.
- Two-body s-wave scattering length $a_s = 171 a_0$.
- Two-body effective range $r_{\text{eff}} = 15.2 a_0$ (alternatively, two-body van der Waals length $r_{\text{vdw}} = 5.1 a_0$).

1K $= 8.6 \times 10^{-5}$ eV
1eV$= h \times 2.418 \times 10^{14}\text{Hz}$
$1 a_0 = 0.529\text{Angstrom}$

Three Bosons with Large $|a_s|$: Efimov Effect

Braaten, Hammer, Physics Reports 428, 259 (2006).

$$H = \frac{\vec{p}_{12}^{\,2}}{2\mu_{12}} + \frac{\vec{p}_{12,3}^{\,2}}{2\mu_{12,3}} + \sum_{j<k} g_2 \delta\left(\vec{r}_{jk}\right) + g_3 \delta(\vec{r}_1 - \vec{r}_2)\delta(\vec{r}_2 - \vec{r}_3).$$



$g_2 = \frac{4\pi\hbar^2 a_s}{m}$ and $g_3 = \frac{\#\,\hbar^2 \kappa_*^{-4}}{m}$, where $E_{unit} = \frac{\hbar^2 \kappa_*^2}{m}$.

Time-dependent SE for H possesses continuous scaling symmetry:

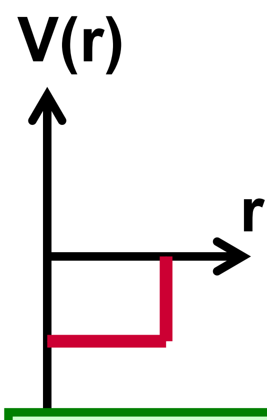
$$t \to \lambda^2 t;\ \vec{r} \to \lambda \vec{r};\ a_s \to \lambda a_s;\ E \to \lambda^{-2} E;\ \kappa_* \to \lambda^{-1} \kappa_*$$

Time-dependent SE for H also possesses discrete scaling symmetry:

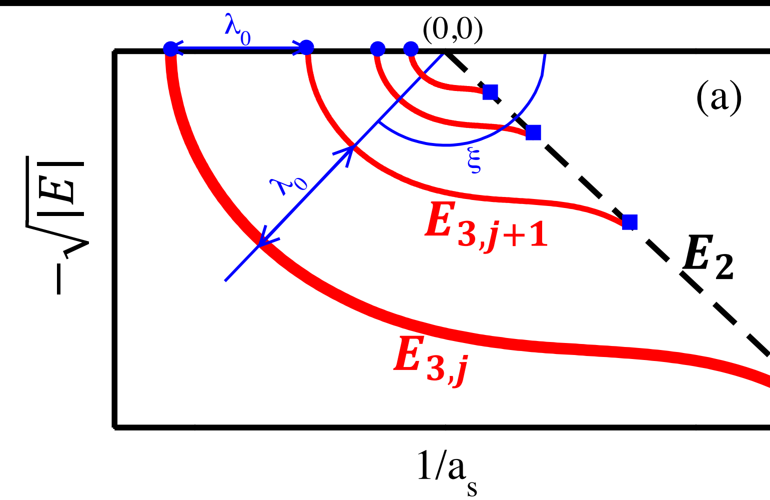
$$t \to \lambda_0^2 t;\ \vec{r} \to \lambda_0 \vec{r}\ a_s \to \lambda_0 a_s;\ E \to \lambda_0^{-2} E;\ \kappa_* \to \kappa_*;\ \lambda_0 \approx 22.7$$

Radial Scaling Law (Two Axes): Universally Linked States

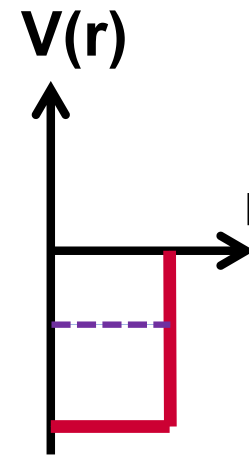
stronger attraction

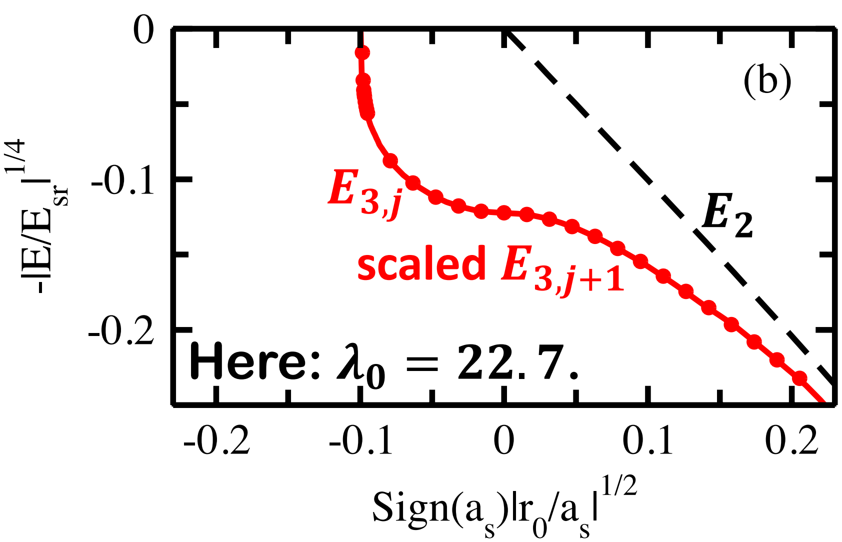


Spectrum is determined by a_s and three-body parameter κ_*.



Zero-range theory.
For plotting purposes, $\lambda_0 = 2$ instead of 22.7.



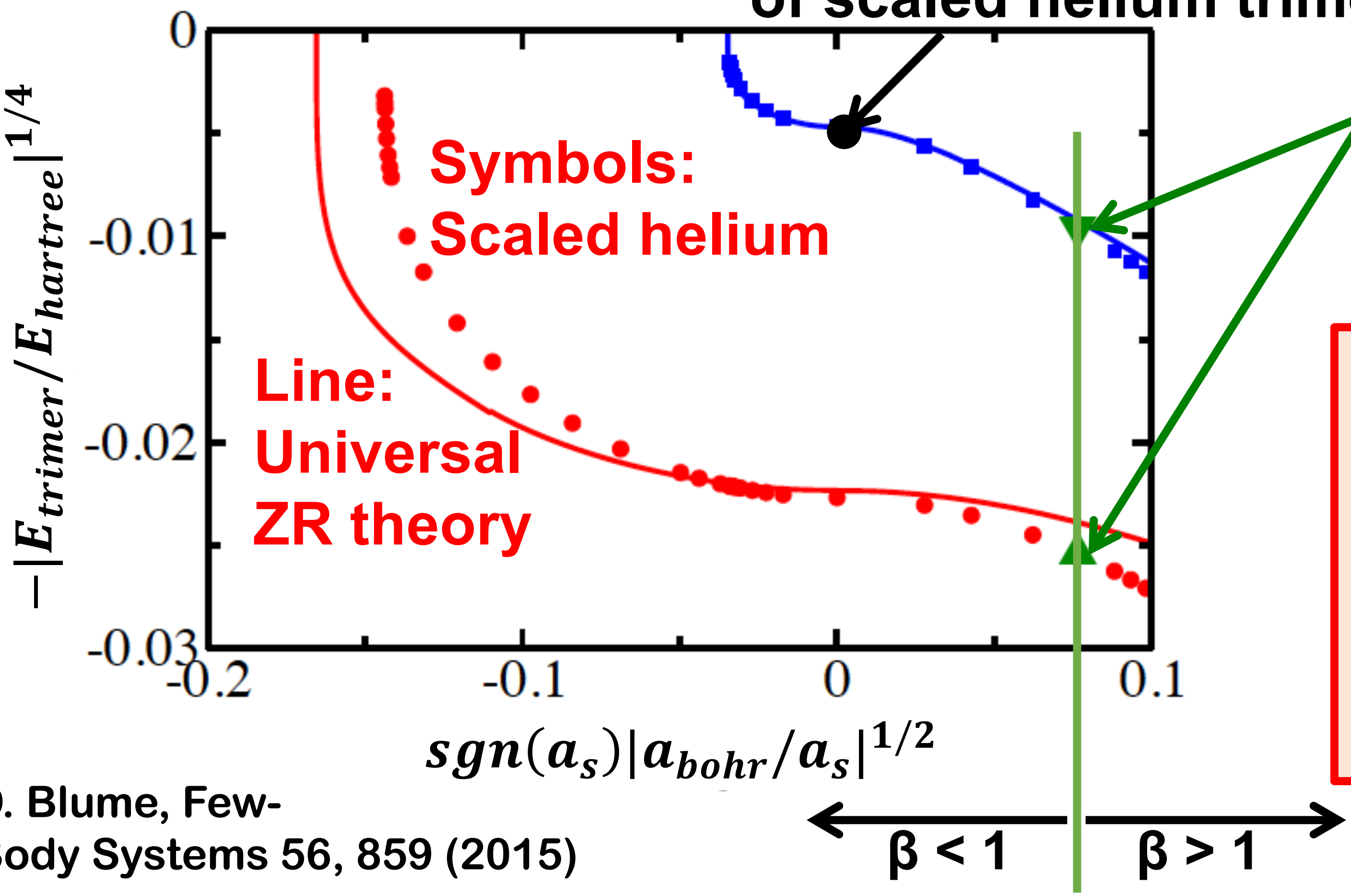


Numerical test for two-body plus three-body finite-range model potentials: Perfect "collapse" of neighboring energy levels.

Helium Trimer Excited State is an Efimov State

$\beta V_{He\text{-}He}(r_{12}) + \beta V_{He\text{-}He}(r_{23}) + \beta V_{He\text{-}He}(r_{31})$.

Three-body parameter is chosen such that ZR energy agrees with energy of scaled helium trimer excited state.

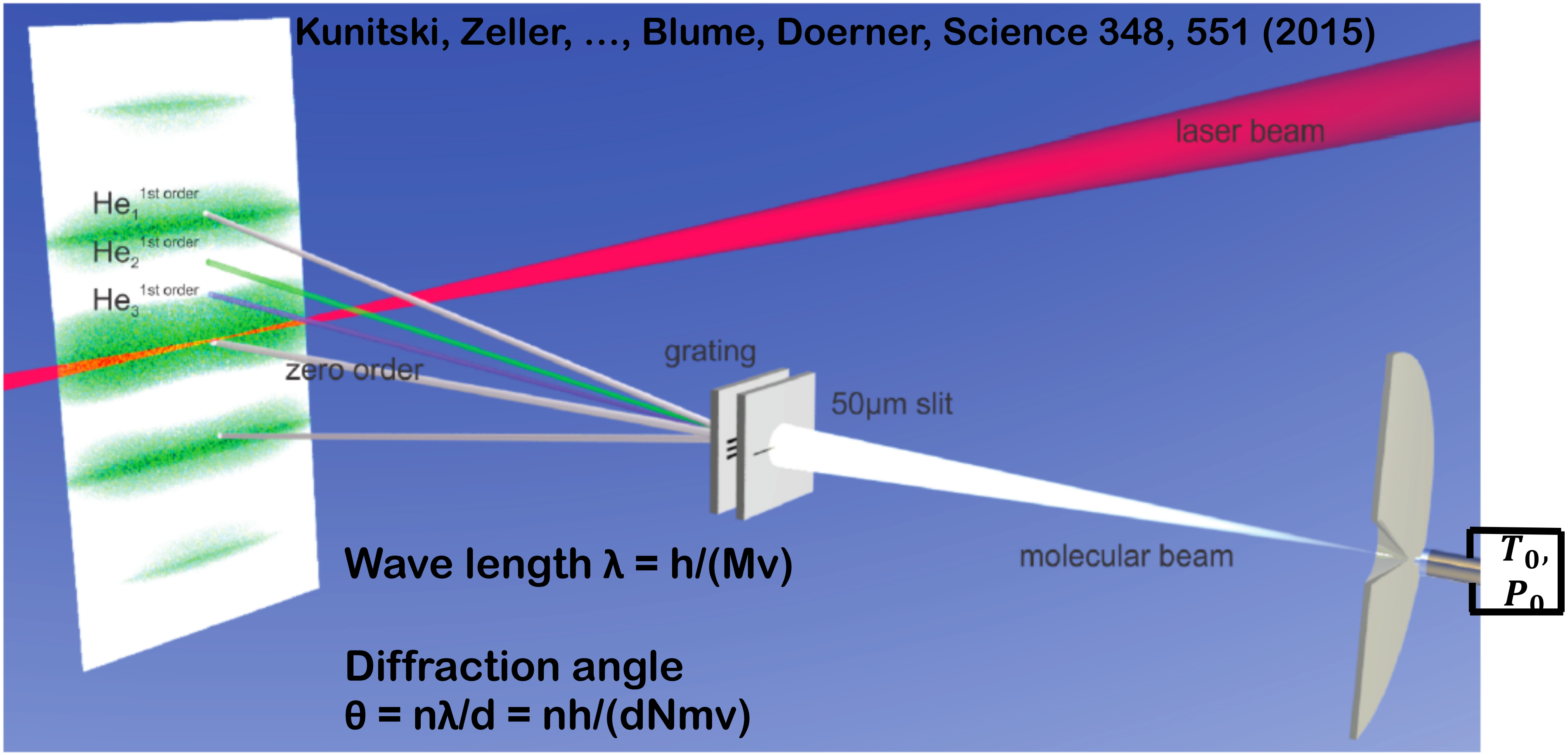


For the excited state, symbols agree with line! Molecular system follows predicted Efimov behavior.

D. Blume, Few-Body Systems 56, 859 (2015)

How to Prepare/Probe Helium Trimer Excited Efimov State?

Kunitski, Zeller, …, Blume, Doerner, Science 348, 551 (2015)



Wave length $\lambda = h/(Mv)$

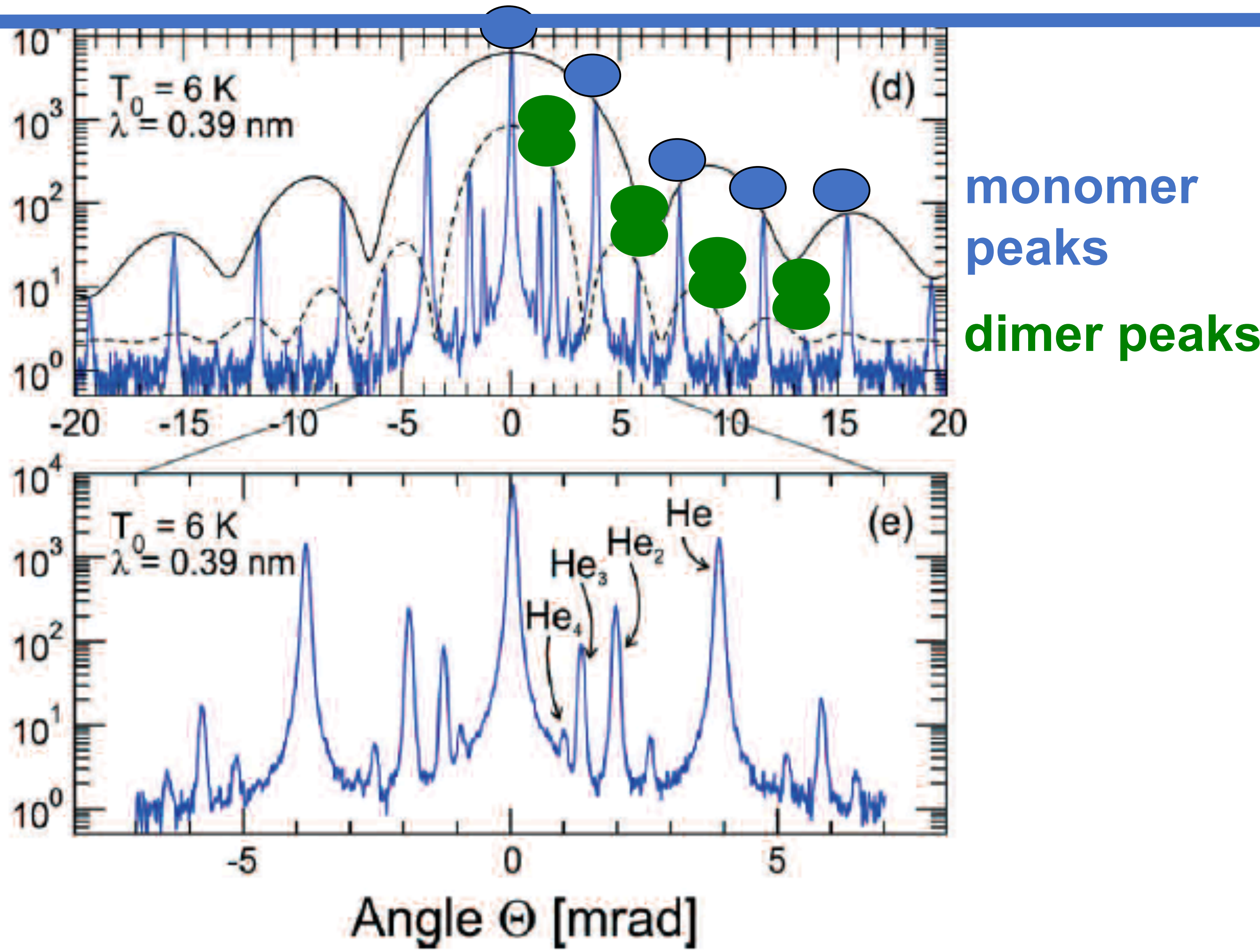
Diffraction angle
$\theta = n\lambda/d = nh/(dNmv)$

Grating serves as mass selector (N times atom mass m).

Matter Wave Diffraction Experiment

Kornilov, Toennies, 10.1051/ epn:2007003

Nozzle temperature and pressure can be adjusted.



How to Prepare/Probe Helium Trimer Excited Efimov State?

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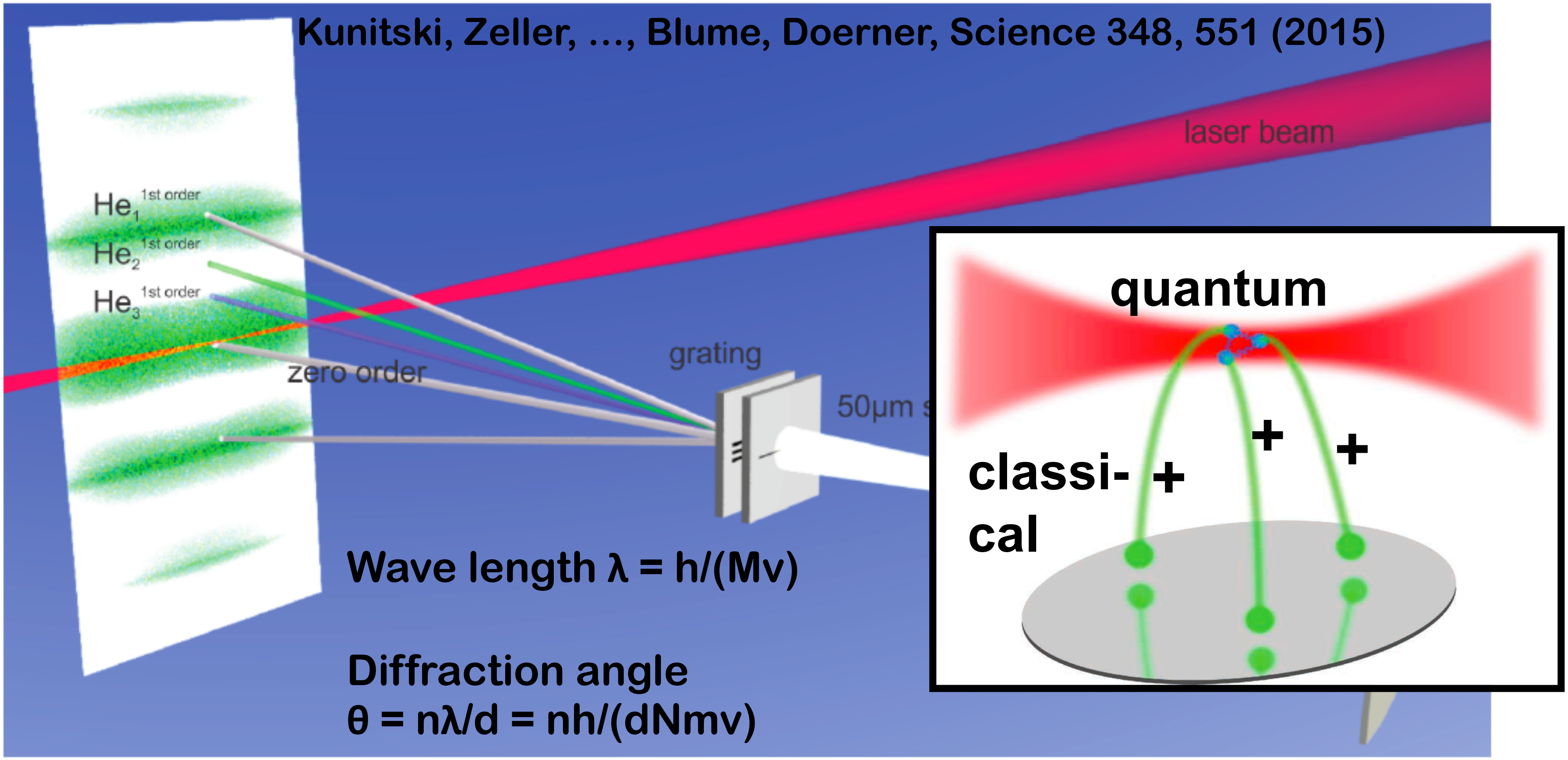


Wave length $\lambda = h/(Mv)$

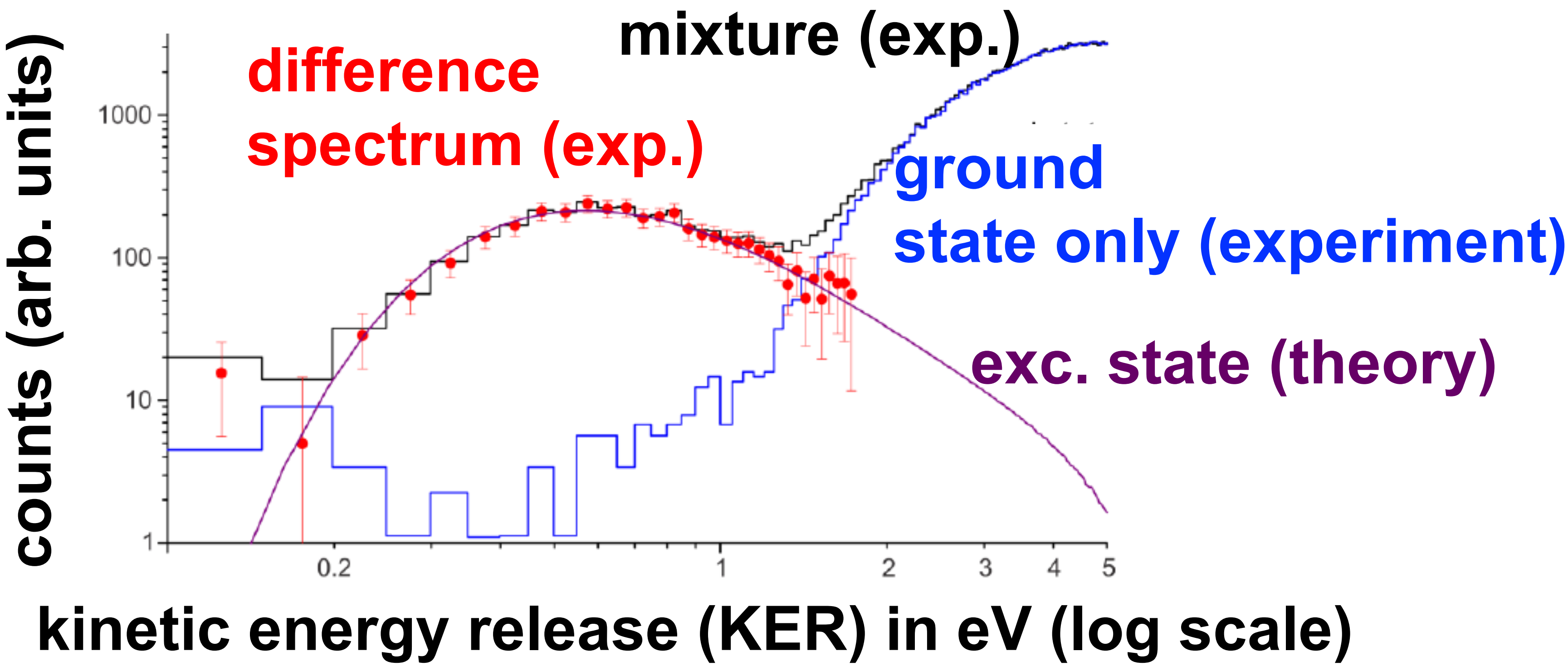
Diffraction angle
$\theta = n\lambda/d = nh/(dNmv)$

Grating serves as mass selector (N times atom mass m):

He$_3$ signal contains ground state trimer *and* excited state trimer.

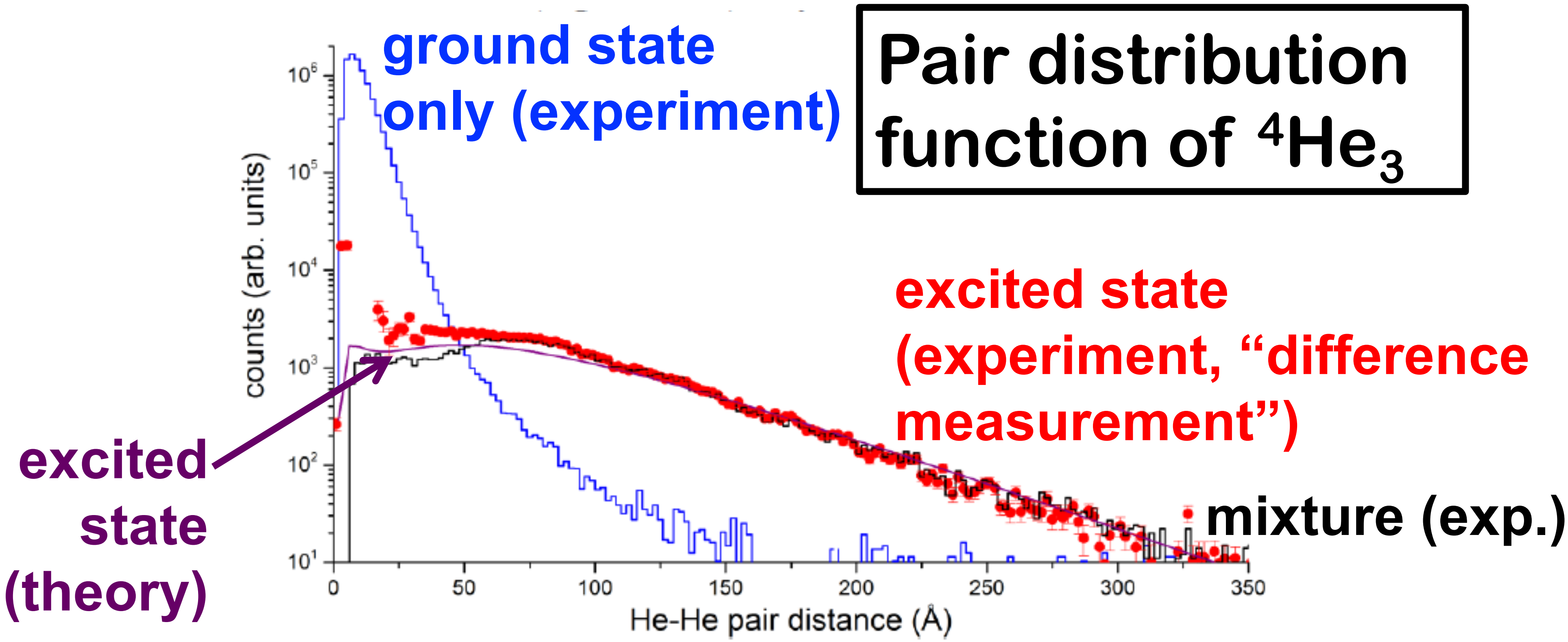
Laser beam ionizes trimer: Coulomb explosion of ^{4}He$_3$ (3 ions).

Kinetic Energy Release Measurement



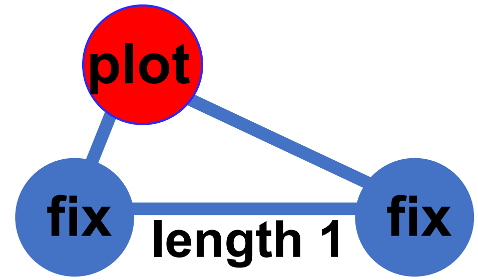
The ionization is instantaneous and the He-ions are distributed according to the quantum mechanical eigen states of the ground and excited helium trimers.
Large r_{12}, r_{23} and r_{31} correspond to small KER=$1/r_{12}+1/r_{23}+1/r_{31}$.

Reconstructing Real Space Properties

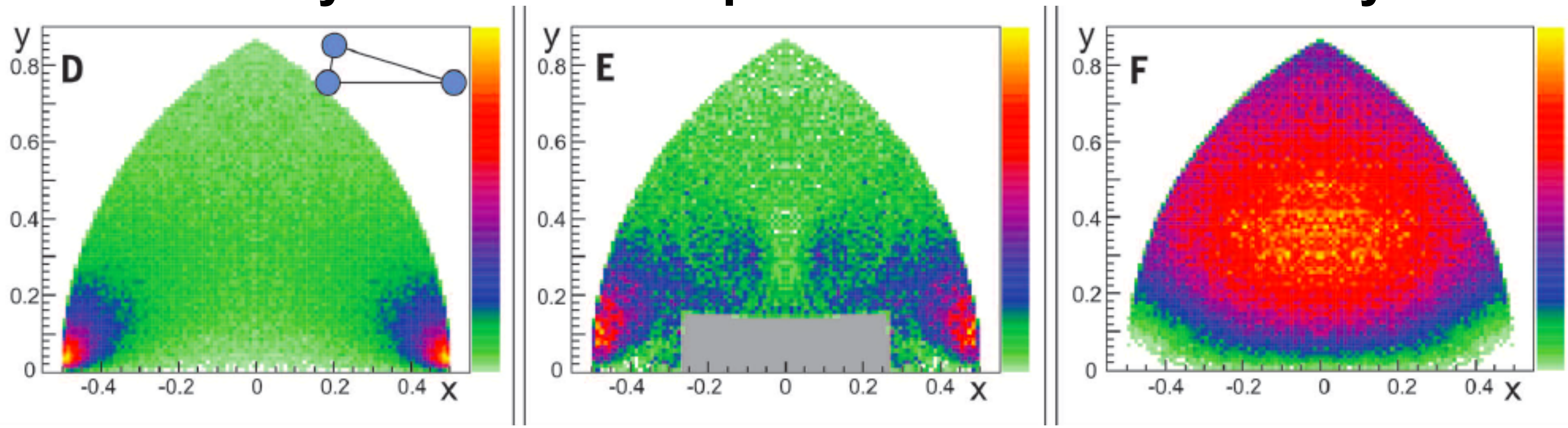


The excited state is eight times larger than the ground state. Assuming an "atom-dimer geometry", the tail can be fit to extract the binding energy of the excited helium trimer. Fit to experimental data yields 2.6(2)mK. Theory 2.65mK.

Normalized Structural Properties of $^4He_3$



excited state: theory	excited state: experiment	ground state: theory

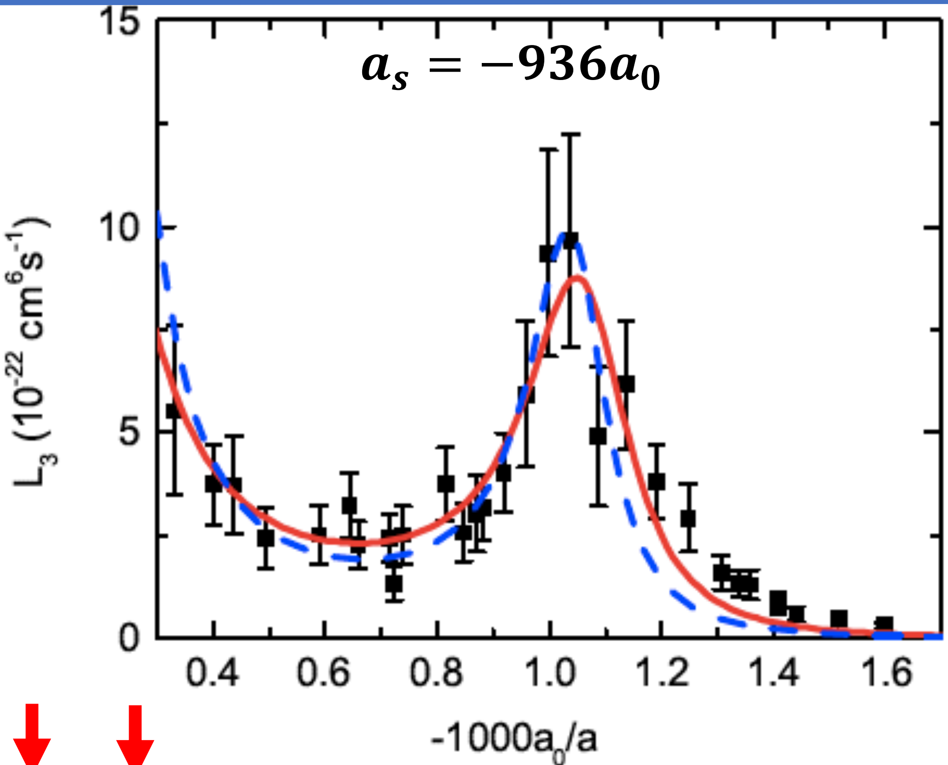


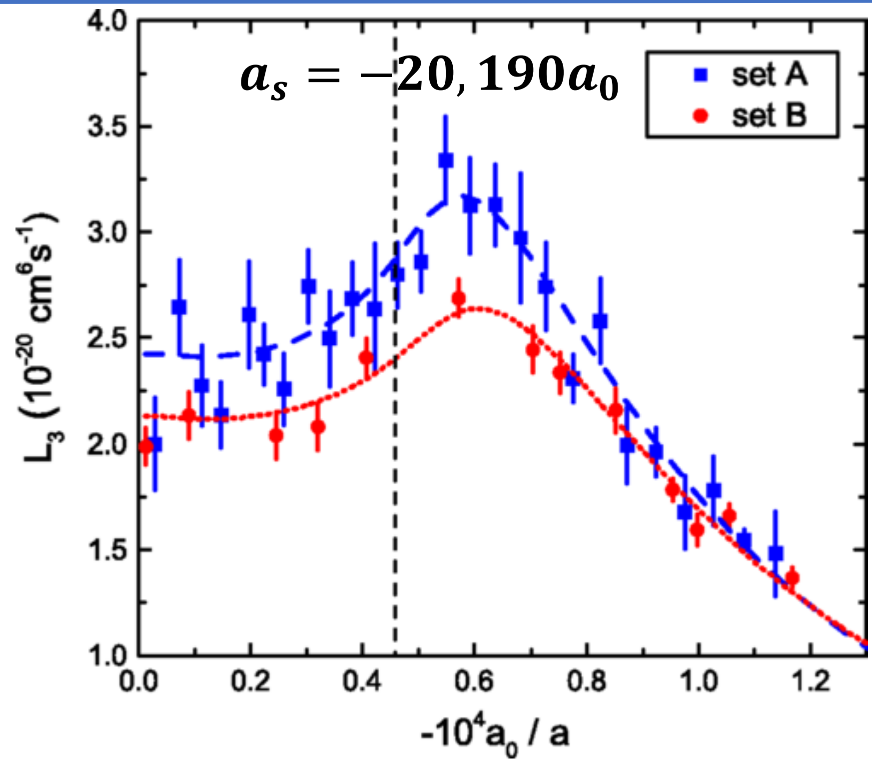
Divide all three interparticle distances by largest r_{ij} and plot k^{th} atom (positive y): Corresponds to placing atoms i and j at (−1/2,0) and (1/2,0).

Ground state and excited states have distinct characteristics!!!

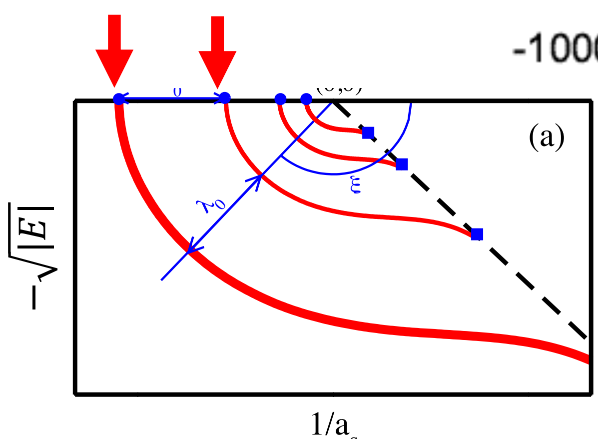
Message: Reconstruction of quantum mechanical trimer density.

Measurement of Loss Rate for Non-Degenerate ^{133}Cs Gas





Huang et al., PRL 112, 190401 (2014).



Enhanced losses when trimer is degenerate with three free atoms.

Ratio of $\lambda_0 = 21.6$ (compared to 22.7)! Confirmation of discrete scaling symmetry.

Relevance and Extensions of Three-Particle Efimov Scenario

Triton: Nuclear chart...

Nuclear Physics Around the Unitarity Limit

Sebastian König,[1,2,3,*] Harald W. Grießhammer,[4,†] H.-W. Hammer,[2,3,‡] and U. van Kolck[5,6,§]

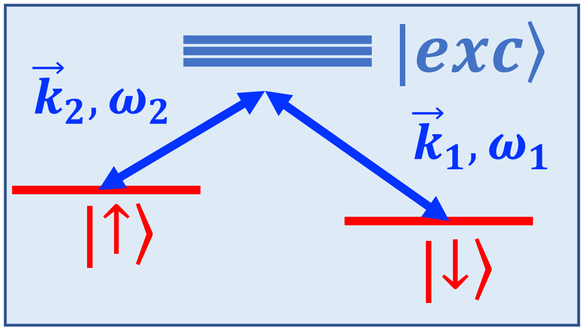
Extensions to three-magnon system (= Efimov trimer, Nishida et al.); halo nuclei; fermions and more than three particles; interplay between Efimov trimer and background (polaron);...

Modified single-particle dispersion...

Single-Particle Dispersion $\neq \frac{p_z^2}{2m}$

"Normally": Single-particle (SP) dispersion $= \frac{\vec{p}^2}{2m} = \frac{p_x^2+p_y^2+p_z^2}{2m}$

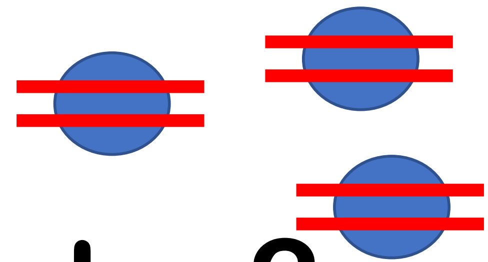
Now, single-particle $H = \left(\frac{p_x^2+p_y^2}{2m} + \frac{p_z^2}{2m}\right) I_2 + \frac{\hbar k_{so}}{m} p_z \sigma_z + \Omega \sigma_x + \delta \sigma_z$



Effect of modified single-particle dispersion on three identical bosons with large s-wave scattering length:
What happens to discrete scaling symmetry/Efimov physics?

Fermions w/ 3D SOC: Shi et al., PRL 112, 013201 (2014); PRA 91, 023618 (2015).

With SOC: Fate Of Three-Boson Efimov States?



$$H = \left(\frac{\vec{p}_{12}^{\,2}}{2\mu_{12}} + \frac{\vec{p}_{12,3}^{\,2}}{2\mu_{12,3}} + \sum_{j<k} g_2 \delta(\vec{r}_{jk}) + g_3 \delta(\vec{r}_1 - \vec{r}_2)\delta(\vec{r}_2 - \vec{r}_3)\right) I_8 + \frac{\hbar k_{so}}{m}(\dots) + \Omega(\dots) + \tilde{\delta}(\dots).$$

all three bosons feel 1D SOC

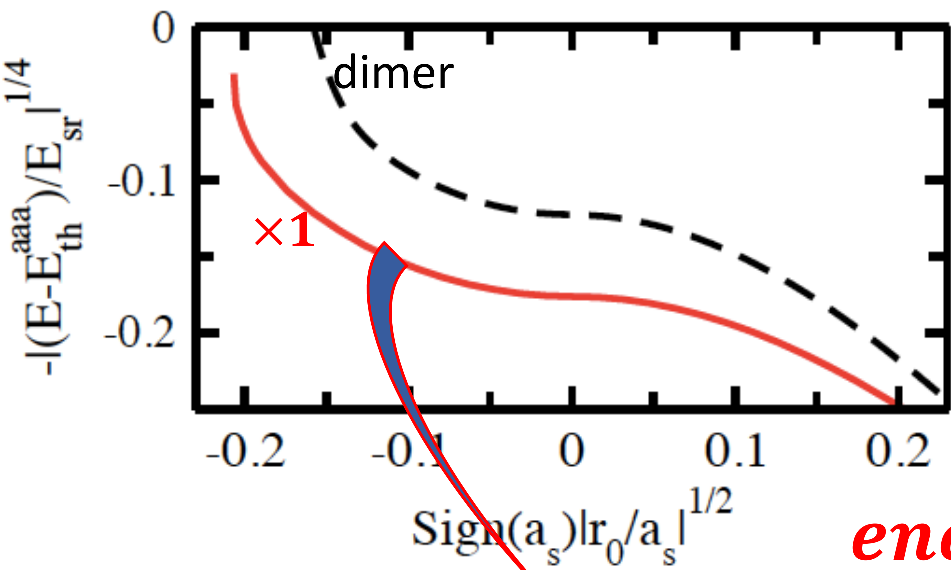
Easy to check: Continuous scaling symmetry!

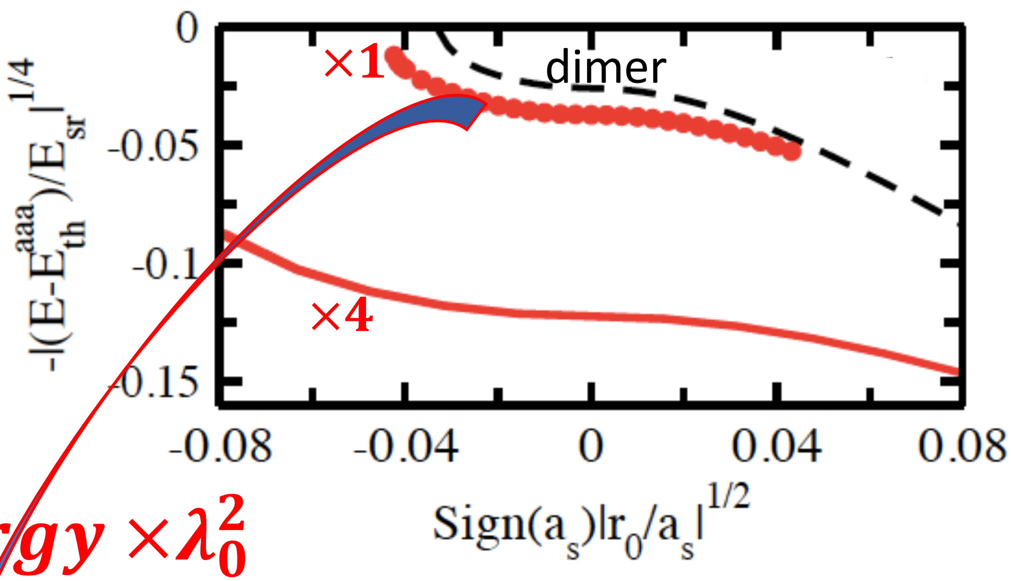
$t \to \lambda^2 t$; $\vec{r} \to \lambda \vec{r}$; $a_s \to \lambda a_s$; $k_{so} \to \lambda^{-1} k_{so}$; $\Omega \to \lambda^{-2}\Omega$;
$\tilde{\delta} \to \lambda^{-2}\tilde{\delta}$; $E \to \lambda^{-2} E$; $\kappa_* \to \lambda^{-1}\kappa_*$

Discrete scaling symmetry?

$t \to \lambda_0^2 t$; $\vec{r} \to \lambda_0 \vec{r}$ $a_s \to \lambda_0 a_s$; $k_{so} \to \lambda_0^{-1} k_{so}$; $\Omega \to \lambda_0^{-2}\Omega$;
$\tilde{\delta} \to \lambda_0^{-2}\tilde{\delta}$; $E \to \lambda_0^{-2} E$; $\kappa_* \to \kappa_*$; $\lambda_0 \approx 22.7$

Generalized Radial Scaling Law?
Example: $\widetilde{\delta} = 0$ And $(\kappa_*)^{-1} = 66 r_0$

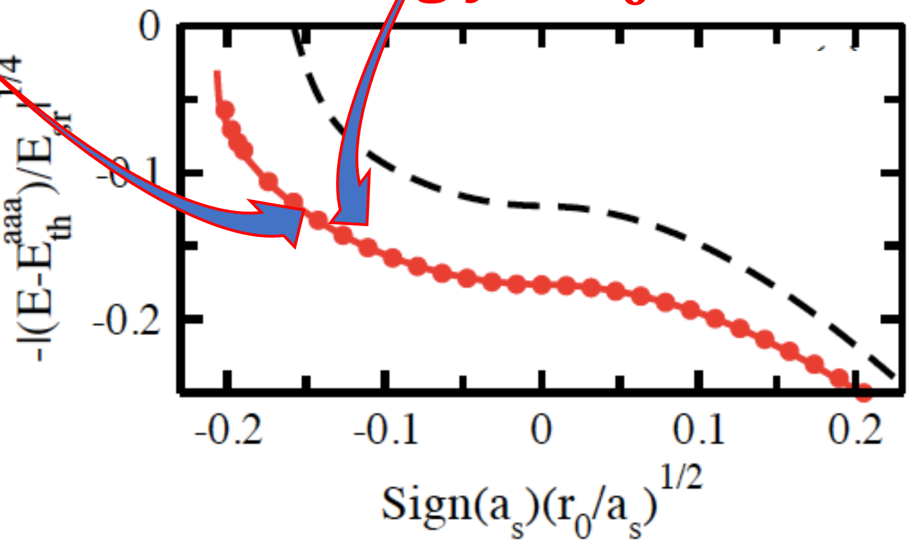




energy $\times \lambda_0^2$

$(k_{so})^{-1} = 25 r_0.$

$\Omega = 0.0016 \frac{\hbar^2}{m r_0^2}.$



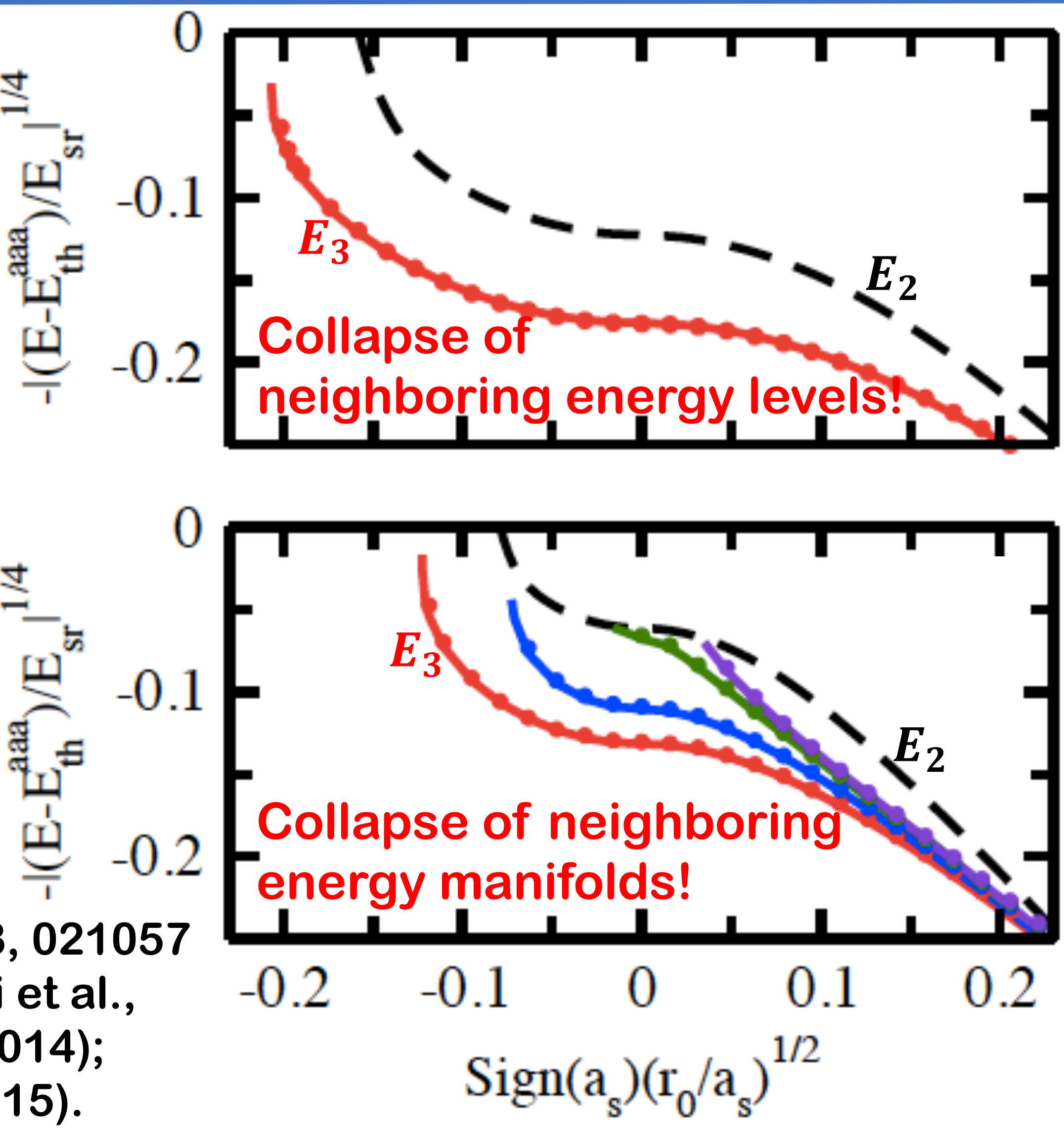
$(k_{so})^{-1} = \lambda_0 25 r_0.$

$\Omega = (0.0016/\lambda_0^2) \frac{\hbar^2}{m r_0^2}.$

Generalized Radial Scaling Law: Five Instead Of Two Axes

Discrete scaling symmetry ($\lambda_0 \approx 22.7$)!

$a_s \to \lambda_0 a_s$;
$k_{so} \to \lambda_0^{-1} k_{so}$;
$\Omega \to \lambda_0^{-2} \Omega$;
$\tilde{\delta} \to \lambda_0^{-2} \tilde{\delta}$;
$E \to \lambda_0^{-2} E$.
$\kappa_* \to \kappa_*$.



Solid line (gr. st.):
$(\kappa_*)^{-1} = 66 r_0$.
$(k_{so})^{-1} = 25 r_0$.
$\Omega = 2E_{so}$. $\tilde{\delta} = 0$.
Dots (exc. st. of H with scaled parameters).

Solid lines (gr. st. manifold):
$(\kappa_*)^{-1} = 66 r_0$.
$(k_{so})^{-1} = 100 r_0$.
$\Omega = 2E_{so}$. $\tilde{\delta} = 0$.
Dots (exc. st. manifold of H with scaled parameters).

Guan, Blume, PRX 8, 021057 (2018). See also Shi et al., PRL 112, 013201 (2014); PRA 91, 023618 (2015).

Why Does The Discrete Scaling Symmetry Survive?

We have no proof, "only" numerical evidence.

Three new finite length scales that are set by k_{so}, Ω, and $\tilde{\delta}$.
κ_* is determined by short-range, high-energy physics:
Spin-orbit coupling modifies low- but not high-energy portion of single-particle dispersion.

In effective-field theory language: scale separation.

Rotation approach (two-particle illustration): Does SOC modify two-body boundary condition?

- **Get rid of V_{soc} by rotating the Hamiltonian: $R^{-1}HR$, where rotation operator $R = \exp(-\iota k_{so}\Sigma z)$**
- $\Rightarrow R^{-1}HR = H_{no-soc} + \iota E_{so}[\Sigma z, \Sigma p_z]/\hbar + \mathcal{O}(\vec{r})$

Zhang et al.; PRA 86, 053608 (2012). Guan, Blume; PRA 95, 020702(R) (2017).

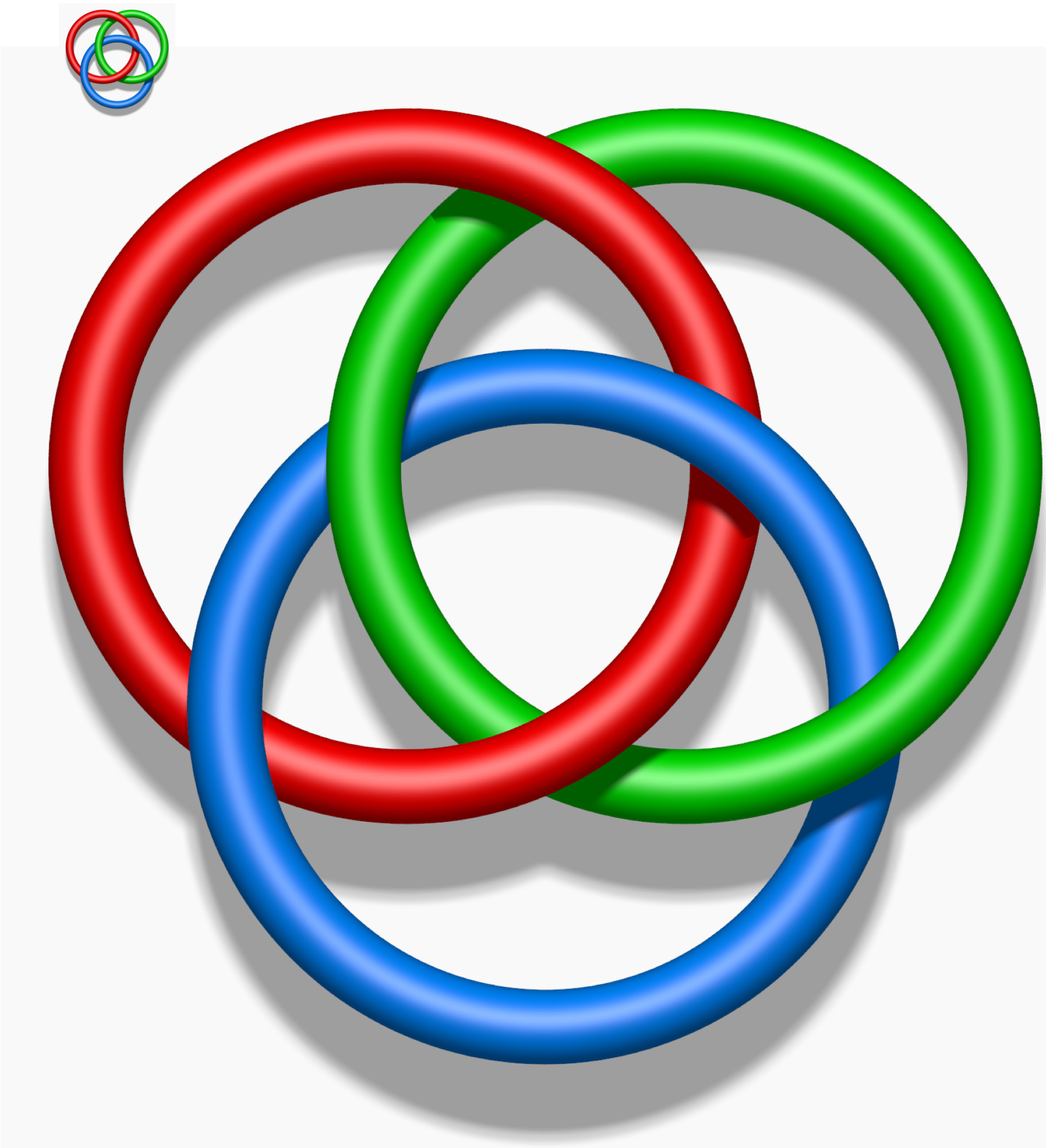
Summary

Three identical bosons with large s-wave scattering length:

Spatial correlations for excited helium trimer (Coulomb explosion).

In the presence of spin-orbit coupling, generalized radial Efimov scaling law (discrete scaling symmetry survives).

Spatio-temporal control of few-body system with large s-wave scattering length:
Helium dimer as an example.
Didn’t discuss: Single-atom imaging in Selim Jochim’s group.



Borromean rings:

The blue ring lies under the green ring (the "blue-green dimer" is unbound). If the red ring is cut open, the trimer flies apart.