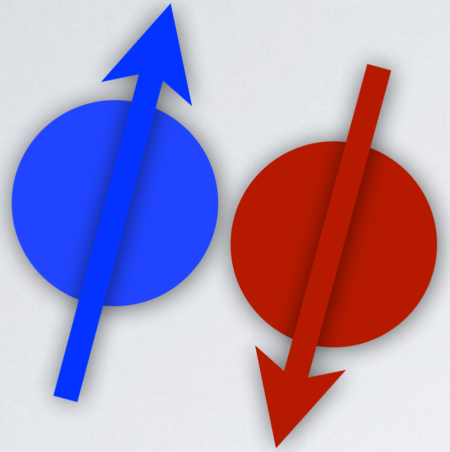
Cold Atoms and Nuclei

J. Carlson - LANL



Cold Atoms:

- Tunable interactions
- Bosons, SU(2) fermions
- Amazing control
- Density, Temperature, Dimensionality, Particle #, …

Nuclei:

- Fixed, complicated interactions, ~ SU(4)
- Nuclei: Fixed density
- Matter: Density, temperature variable Shell structure / superfluid pairing

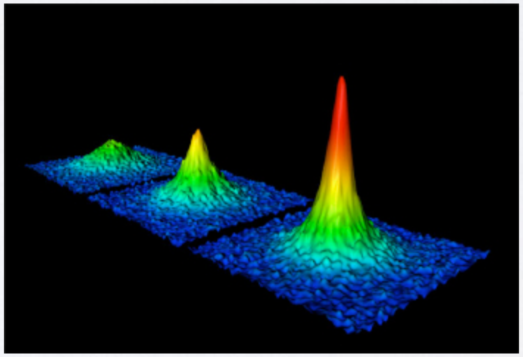
Cold Atoms and Nuclei:
(some) scientific questions

Nuclear Physics

- Limits of existence (# of n, p), clustering
- Equation of state (vs. density, T, proton fraction, …
- Weak interactions and Nuclear Response
- Exotic phases: e.g. high-density transitions to quark matter

Cold Atom Physics

- Equation of State
- Finite vs. Bulk Systems
- Density, Spin Response
- `Exotic' Superfluid Phases
- Dimensionality



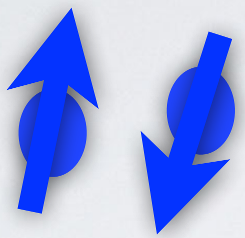
C. Regal et al. PRL 2004

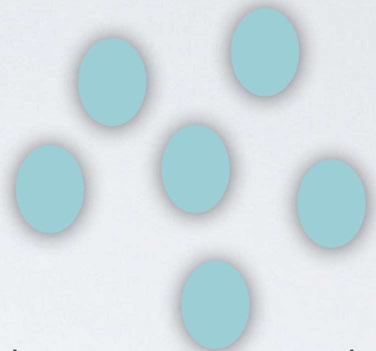
Recent cold atom work
In collaboration with

Stefano Gandolfi
Alex Gezerlis
Bira van Kolck
Kevin Schmidt
Silvio Vitiello
Bira v. Kolck
Shiwei Zhang

From few to many in Cold Atoms

SU(2) Fermions





Bosons
[or SU(N) fermions
at T=0]

Similarities:

Homogeneous bulk properties incorporated into DFT
gradient expansion, ...

Differences:

Nuclei and Bosons are self-bound,
electrons and SU(2) fermions are not

Nuclei are superfluid - pairing in finite systems

Exploring the transition from few to many in cold atoms?

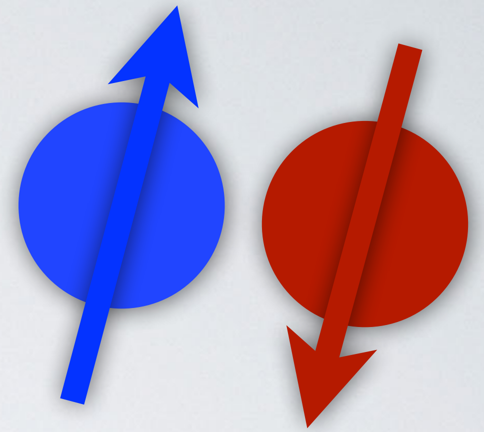
Different challenges

Scale invariance simplifies the density functional

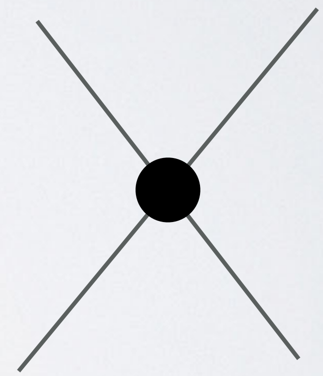
Study problems that are 'exactly' solvable

Homogeneous Unitary Fermi Gas

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} V_0 \, \delta(\mathbf{r_{ij}})$$



V_0 can be tuned across BCS ($|V_0| \sim 0$) to BEC ($-V_0 >> E_F$)
Concentrate on unitarity : zero energy bound state
infinite scattering length



$$E = \xi \; E_{FG} = \xi \; \frac{3}{5} \; \frac{\hbar^2 k_F^2}{2m}$$

$$\Delta = \delta \; \frac{\hbar^2 k_F^2}{2m}$$

$$T_c = t \; \frac{\hbar^2 k_F^2}{2m}$$

Values of ξ, δ, t are independent of ρ

`Exact' T=0 Algorithm: Auxiliary Field Monte Carlo

exact for unpolarized systems

$$H = \frac{1}{N_k^3} \sum_{\boldsymbol{k},\boldsymbol{j},\boldsymbol{m},s} \psi^\dagger_{\boldsymbol{j}s} \psi_{\boldsymbol{m}s} \epsilon_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot(\boldsymbol{r_j}-\boldsymbol{r_m})} + U \sum_{\boldsymbol{i}} n_{\boldsymbol{i}\uparrow} n_{\boldsymbol{i}\downarrow}\,.$$

$$\epsilon_k^{(2)} = \frac{\hbar^2 k^2}{2m}\,,\quad \epsilon_k^{(4)} = \frac{\hbar^2 k^2}{2m}\left[1-\beta^2 k^2 \alpha^2\right]$$

$$\epsilon_{\boldsymbol{k}}^{(h)} = \frac{\hbar^2}{m\alpha^2}\left[3-\cos(k_x\alpha)-\cos(k_y\alpha)-\cos(k_z\alpha)\right]$$

Different effective ranges
No sign problem for
attractive interactions

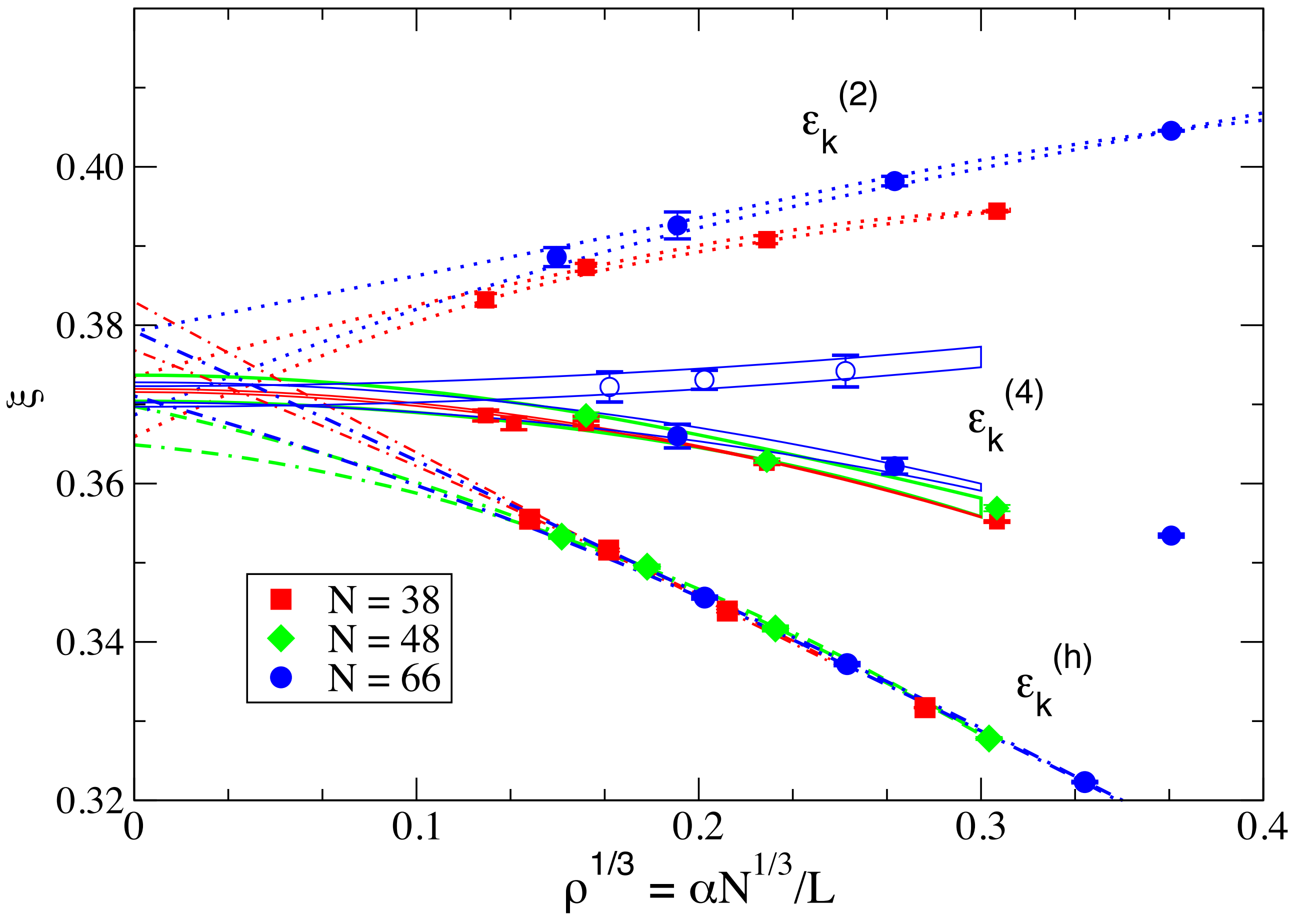
One step of the algorithm:
multiply by exp [- T dt / 2]
momentum space
Auxiliary field for exp [-V dt]
coordinate space
multiply by exp [- T dt /2]
momentum space

$$|BCS\rangle = \left[\sum_{\boldsymbol{k}} f_k c^\dagger_{\boldsymbol{k}\uparrow} c^\dagger_{-\boldsymbol{k}\downarrow}\right]^{N/2} |0\rangle$$

$$\langle W|BCS\rangle = \det A\,,$$

Use importance sampling with
BCS wave function

Homogeneous Gas: ξ and effective range $\mathcal{S}$



MIT expt

$$\xi \ = 0.376(0.005)$$

arXiv:1110.3309 (Hu, et al)

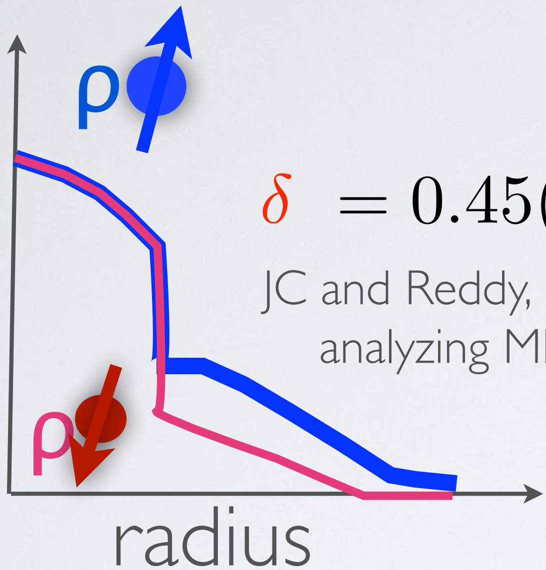
$$E(k_F r_e)/E_{FG} \ = \ \xi \ + \ \mathcal{S} k_F r_e + ...$$

$$\xi \ = \ 0.372 \pm 0.005$$
$$\mathcal{S} \ = \ 0.12 \pm 0.01$$

Carlson, Gandolfi, Schmidt, and Zhang, PRA 2011
Carlson, Gandolfi, and Gezerlis, PTEP 2012

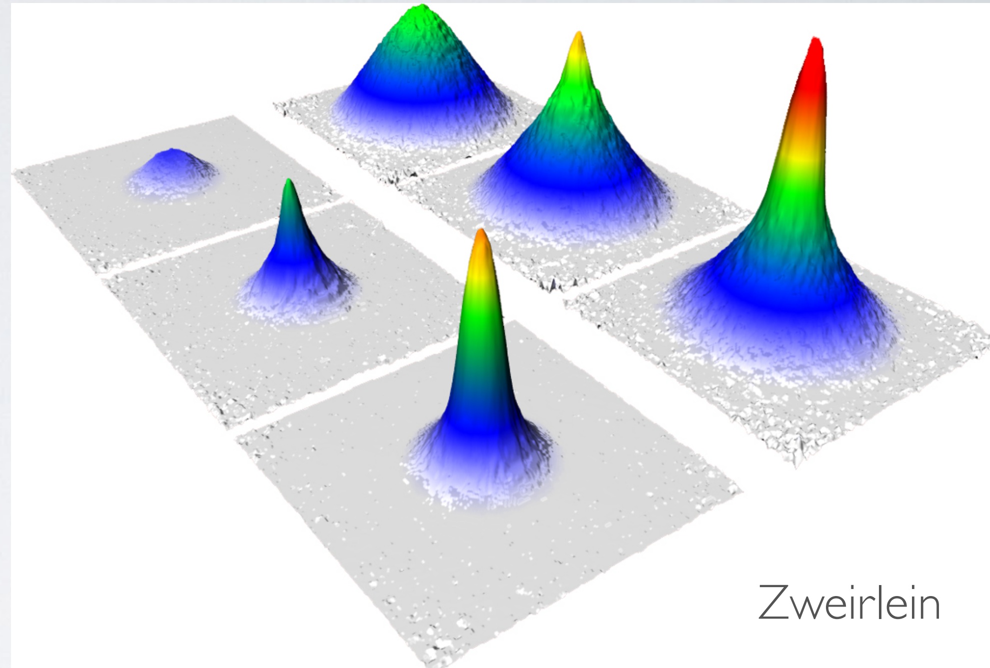
Spin excitations are high energy

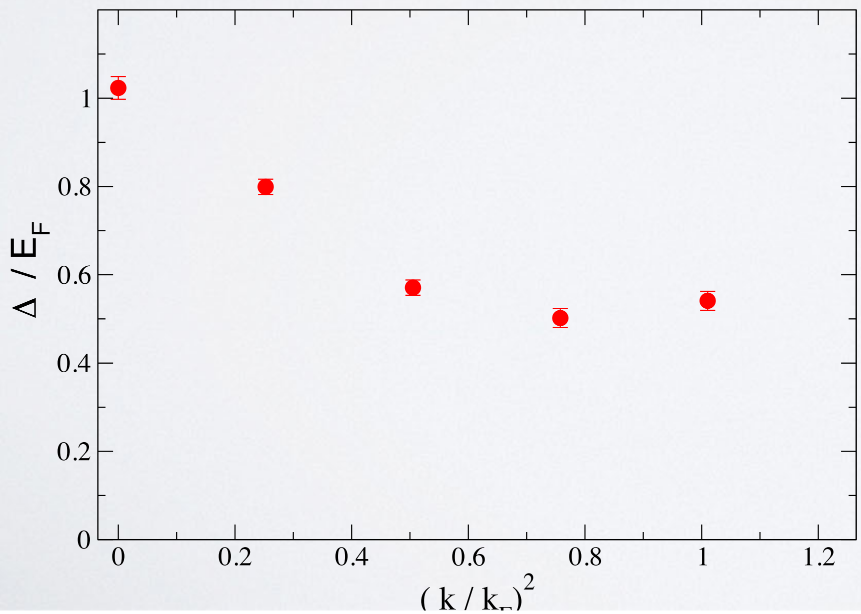
Spin up, down densities in a trap



$\delta \; = 0.45(05)$

JC and Reddy, PRL 2007
analyzing MIT data





$$\Delta \; = \; \delta \; \frac{\hbar^2 k_F^2}{2m}$$

$\delta \; = 0.50 \; (03)$

$(k_{min}/k_f)^2 \; = 0.80(10)$

JC and Reddy, PRL 2005

Density Functional for unpolarized systems

$$\mathcal{E}(x) = n(x)V(x) + \frac{3\cdot 2^{2/3}}{5^{5/3} m c_0^{2/3}} n(x)^{5/3} - \frac{4}{45}\frac{2c_1 - 9c_2}{mc_0}\frac{(\nabla n(x))^2}{n(x)} - \frac{12}{5}\frac{c_2}{mc_0}\nabla^2 n(x)\,.$$

Rupak and Schaefer Nucl.Phys.A816:52-64,2009
arXiv:0804.2678

Epsilon expansion at unitarity $c_2 \approx 0$

$$\mathcal{E}(x) = n(x)V(x) + 1.364\frac{n(x)^{5/3}}{m} + 0.022\frac{(\nabla n(x))^2}{mn(x)} + O(\nabla^4 n)$$

compare to free fermions

$$\mathcal{E}_{ETF}(x) = n(x)V(x) + 2.871\frac{n(x)^{5/3}}{m} + 0.014\frac{(\nabla n(x))^2}{mn(x)} + 0.167\frac{\nabla^2 n(x)}{m} + O(\nabla^4 n)$$

Note increase in coefficient of gradient term at unitarity compared to free Fermi gas

Change notation:

$$\mathcal{E} \;=\; V(r)\rho(r) \;+\; \xi\;(3\pi^2)^{2/3}\rho^{5/3} \;+ c_2\;\nabla\rho^{1/2}\cdot\nabla\rho^{1/2} + \ldots$$

$\hbar^2/(2m) \rightarrow 1$

Free fermions (BCS limit) $\quad c_2 \;=\; 0.111$

Free bosons (BEC limit M = 2m) $\quad c_2 \;=\; 0.5$

The gradient term is exactly like
the kinetic term in the Gross–Pitaevskii equation (BECs).
The density functional is scale invariant: $\quad$ 1/length^5

see also M. Forbes arXiv:1211.3779
for treatment with Superfluid Local Density Approximation

We use only bosonic degrees of freedom
no single-particle orbital summation for the density.

Computing the static response from weak external potentials

$$V(r) = V_0 \; E_F \; \cos(\mathbf{k} \cdot \mathbf{r})$$

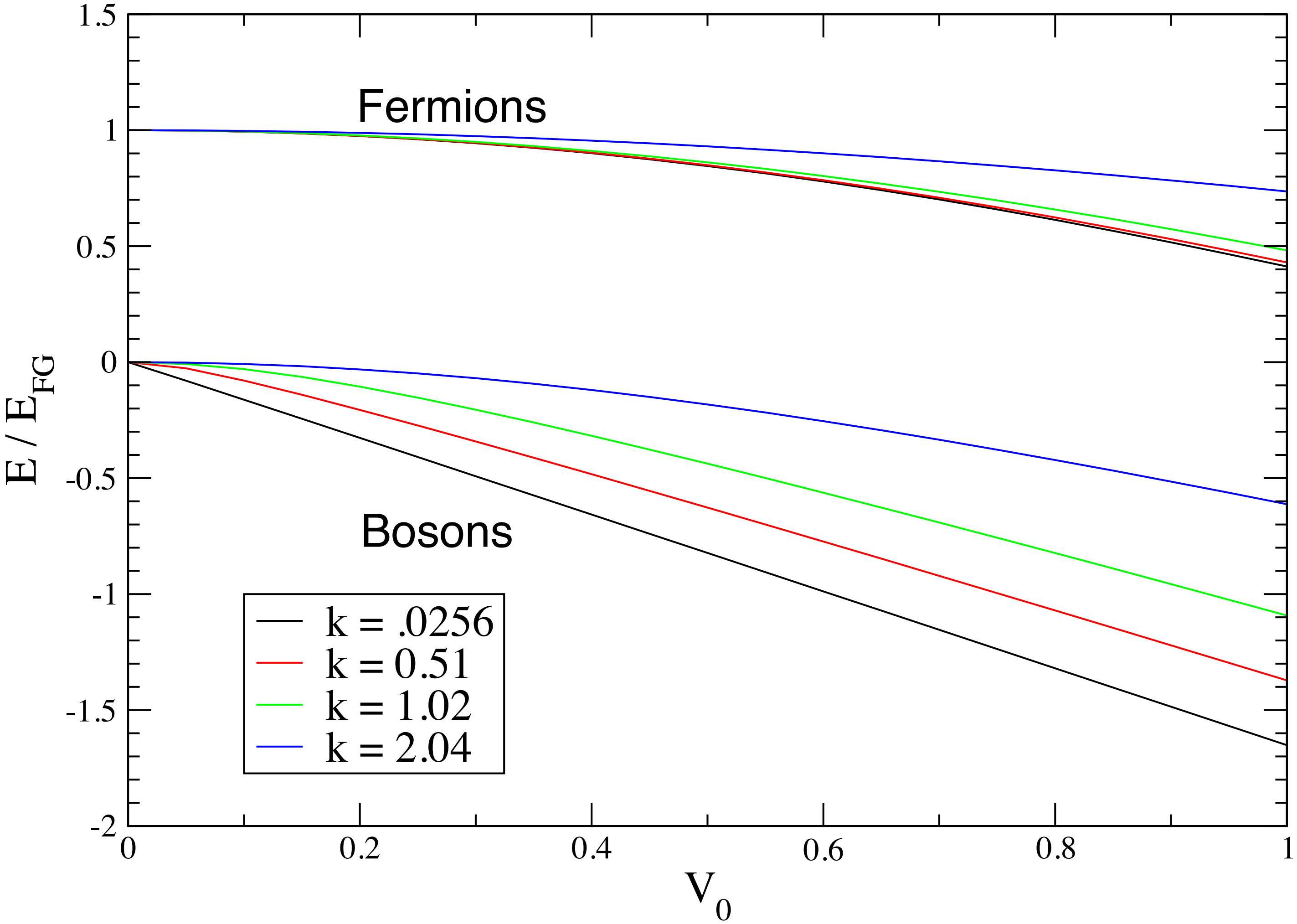
$$E(V_0) = E_0 - \frac{\sum_f \langle 0|V(r)|f\rangle \langle f|V(r)|0\rangle}{E_f - E_0}$$

$$E(V_0) = E_0 - \int d\omega \; S(k,\omega)/\omega$$

At low q, $E(V_0)$ determined by compressibility (ξ)
Next order in q determined by c_g

Use AFMC to compute the energy for weak/moderate external potentials

Static Response for BCS and BEC limits free Fermions and Bosons



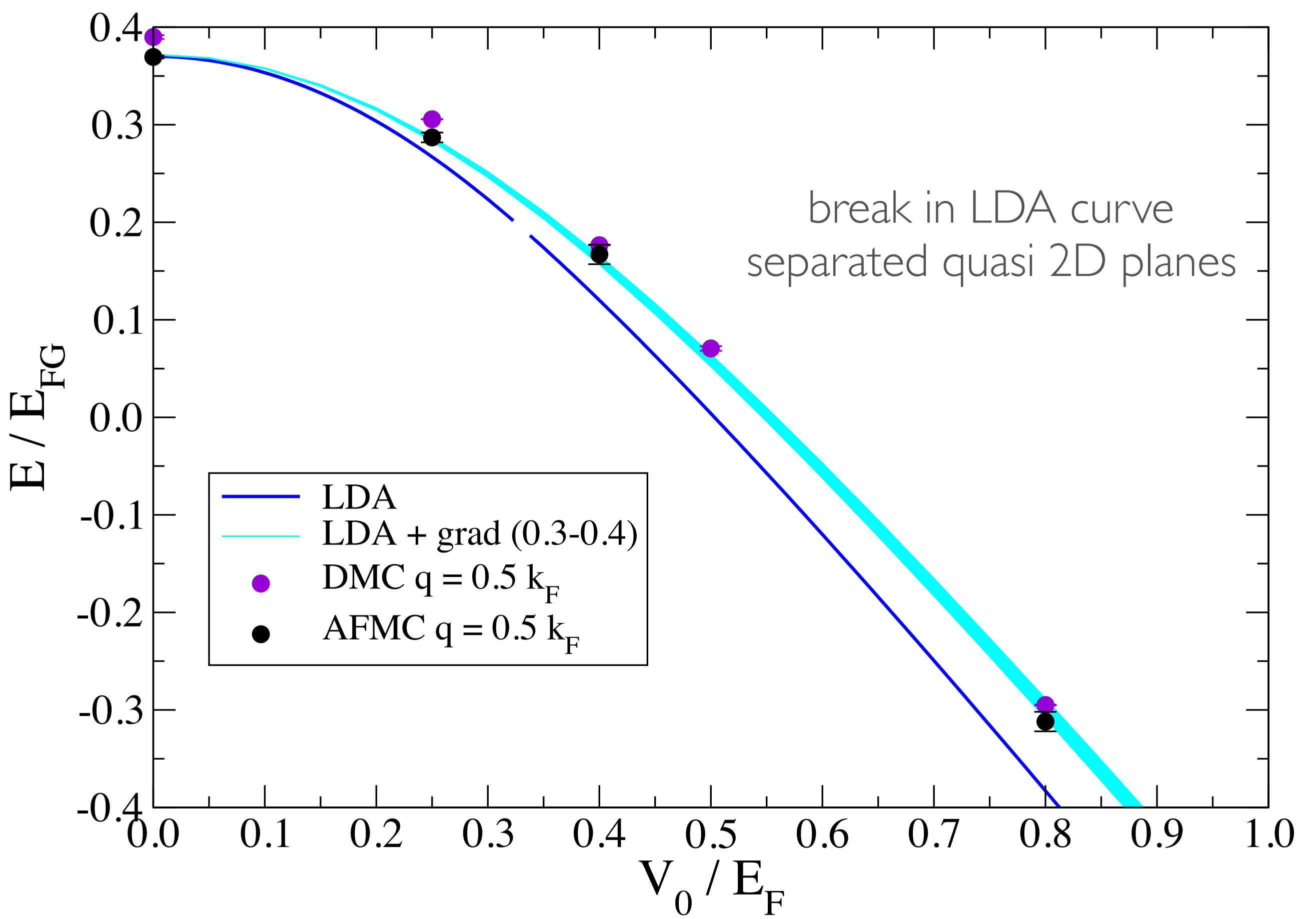
Calculation of c_g from weak external potential

N=66, $k/k_F = 0.5$, $V_0 = 0.25$

AFMC E = 0.291(4) $\Rightarrow$ c_g = 0.37 (0.07)

DMC E = 0.307(1) $\Rightarrow$ c_g = 0.33 (0.02)

Larger external potentials at $q = k_F / 2$

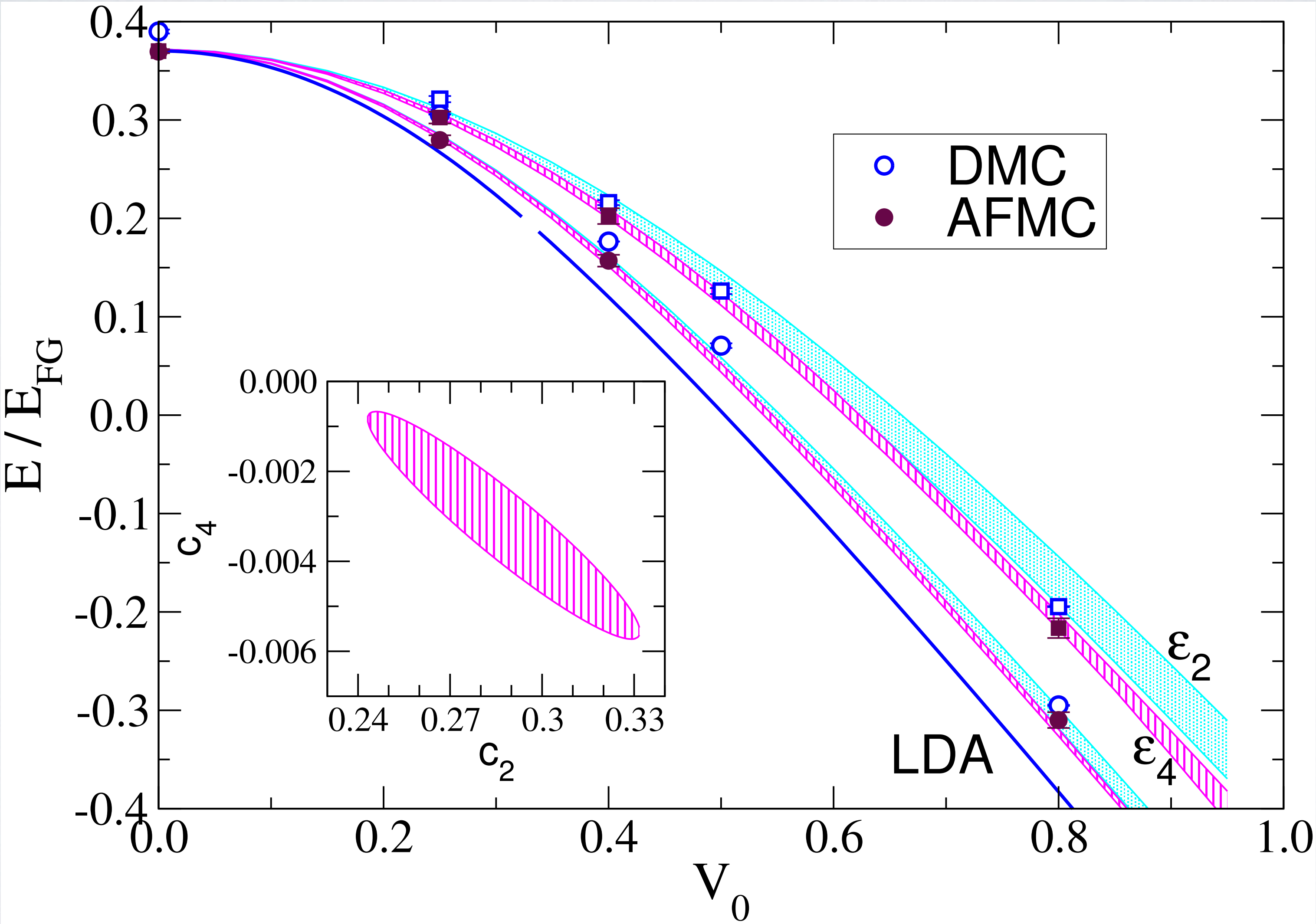


Carlson and Gandolfi
PRA, 2014

Higher-order gradients

Include calculations at higher q: $q/k_F \sim 1$

Lowest order gradient correction no longer sufficient



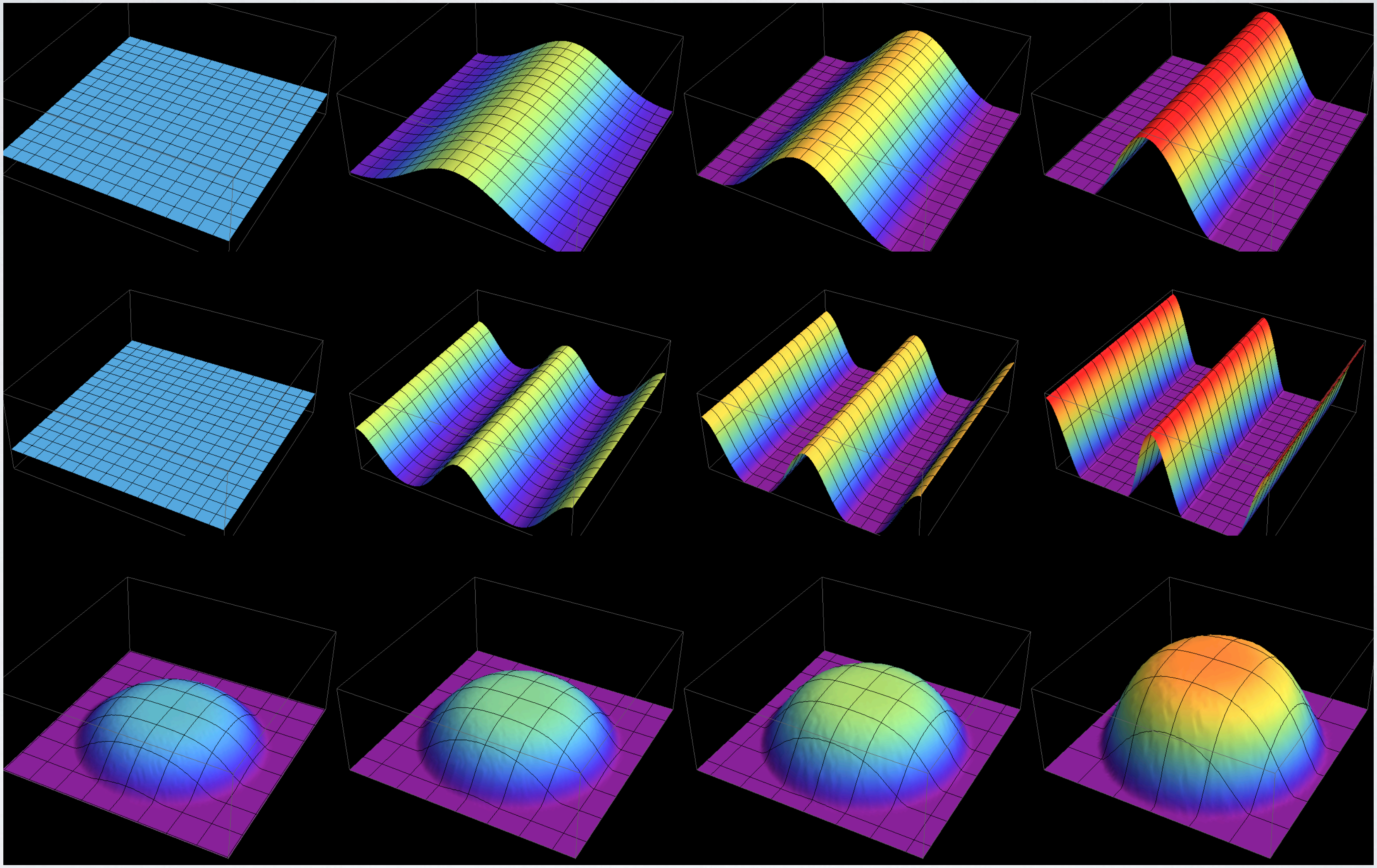
$$\mathcal{E}_4 \; = \; \mathcal{E}_2 \; + \; c_4 \frac{\nabla^2 \rho^{1/2} \nabla^2 \rho^{1/2}}{\rho^{2/3}}$$

Can apply density functional to arbitrary external potentials:

$V_0/E_F=0$ 0.25 0.40 0.80

$q=k_F/2$

$q=k_F$



N=8 30 40 50

What about finite systems?

Consider a small number of particles trapped in a harmonic oscillator:

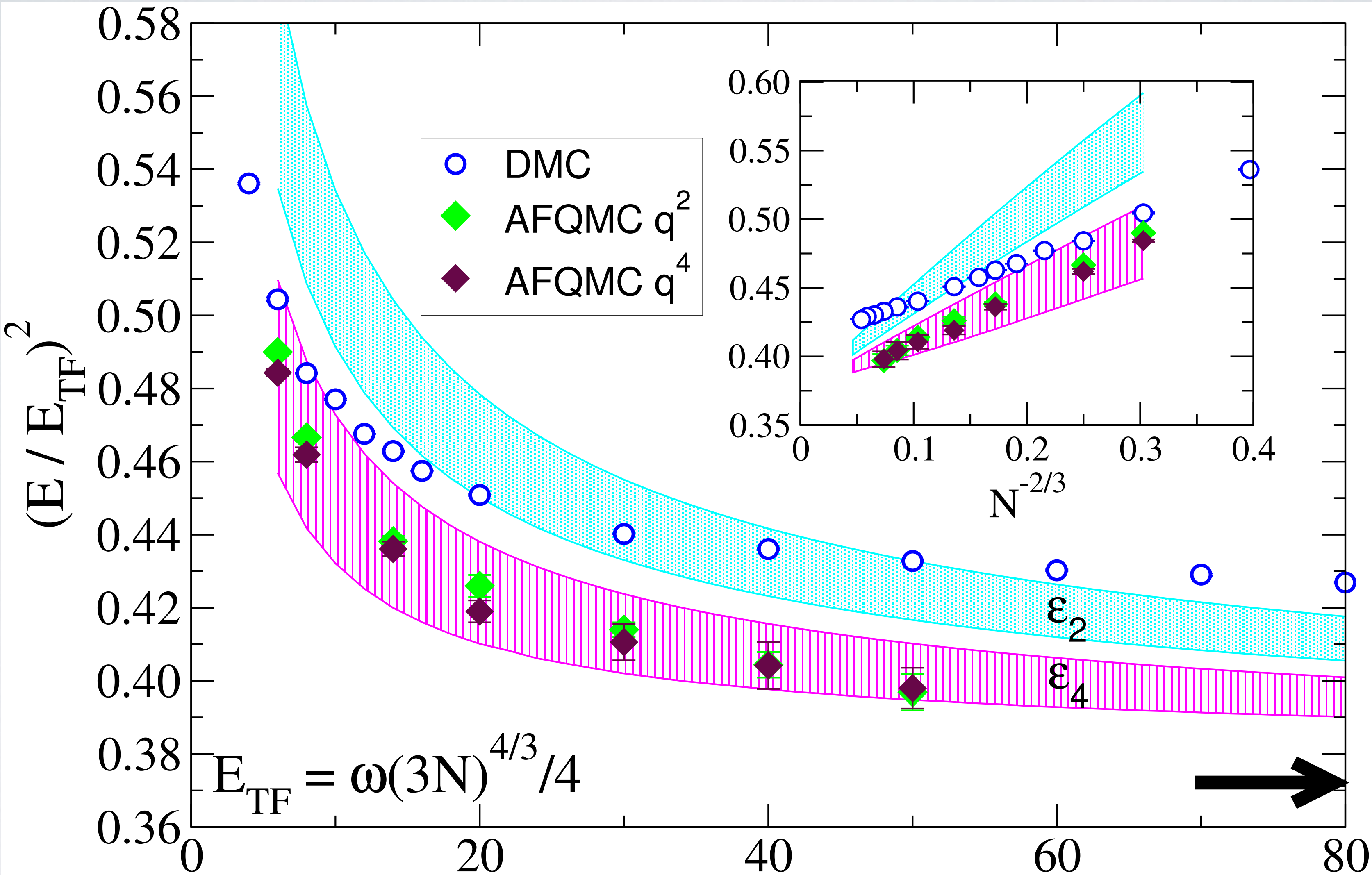
The density functional makes a unique prediction:

No knowledge of (fermionic) shell closures.
Pairing dominates - effectively bosonic DOF only.
Clear approach to the bulk limit.

Does this work and for what N?

Compare DFT prediction to AFMC calculations.
Simple dimensional analysis for large N: $(E/E_{TF})^2 \rightarrow \xi$

AFMC results for trapped fermions



Fourth order density functional gives excellent predictions for N ~ 10 and larger.
Correct approach to bulk ξ.
No evidence of shell gaps - isolated fermions cannot propagate across the system.
Works for much smaller N than typical nuclear density functionals.

Summary of Fermions at Unitarity

Low-Energy degrees of freedom are phonons in UFG

Scale invariance ties linear response to complete functional

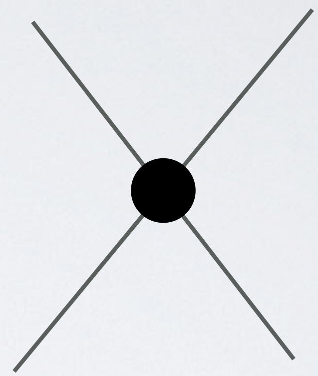
c_g = 0.3-0.4
compared to 0.111 for BCS (free fermions)
0.50 for BEC (free bosons of mass 2m)

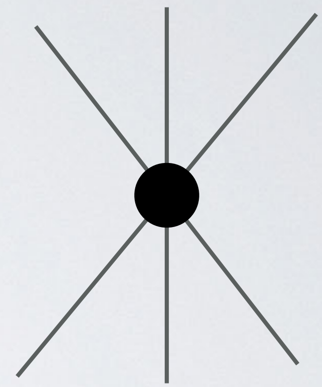
Quadratic corrections important for trapped fermions

No evidence for significant shell structure (large pairing gap) in the unitary Fermi Gas, even for small systems

Unitary Bosons

2-body attractive interaction tuned to unitarity





3-body repulsive interaction tuned to very weakly bound (Efimov) trimer: binding energy E_3

Ground state can be solved for exactly with DMC

Hamiltonian for Bosons

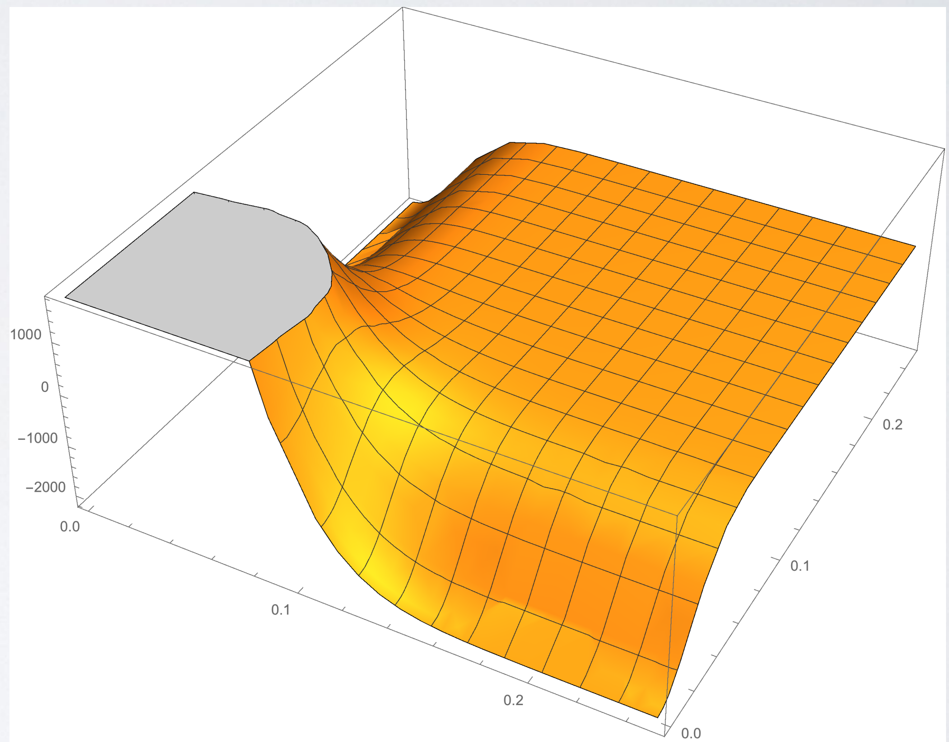
$$H = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk},$$

$$V_{ij} = V_2^0 \frac{\hbar^2}{m}\mu_2^2 \exp[-(\mu_2 r_{ij})^2/2],$$

$$V_{ijk} = V_3^0 \frac{\hbar^2}{m}\left(\frac{\mu_3}{2}\right)^2 \exp[-(\mu_3 R_{ijk}/2)^2/2],$$

$$R_{ijk} = (r_{ij}^2 + r_{ik}^2 + r_{jk}^2)^{1/2}.$$

$$X_\mu \equiv \mu_3/\mu_2 = 0.5,\ 0.75 \text{ and } 1.0.$$



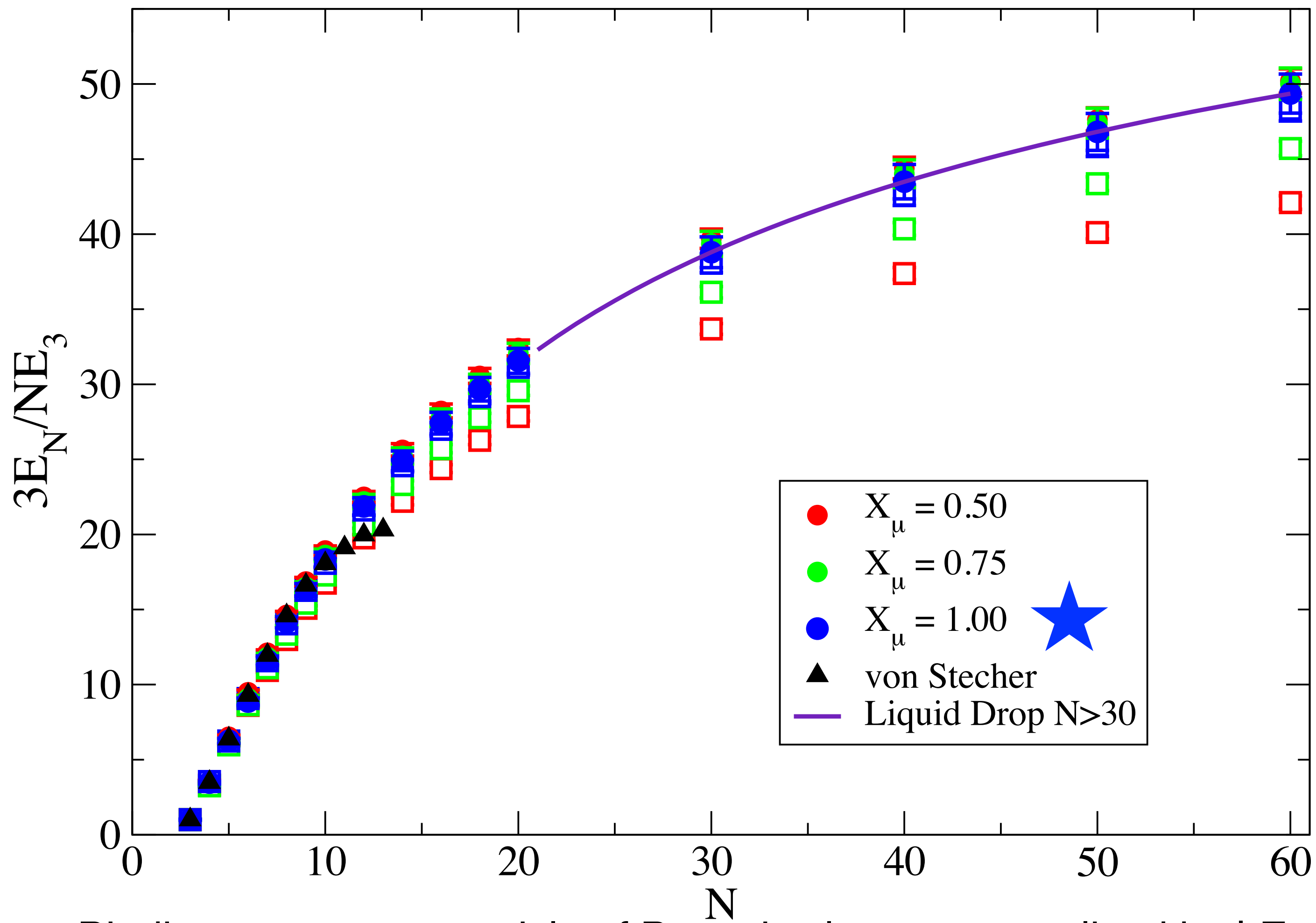
Potential for right angle vs. r_{12} and r_{13}

Many previous calculations use a zero-range 2-body interaction plus a hard-core 3-body binding energy: this fixes the trimer binding for a given radius.

The above interaction can be tuned to arbitrarily small 3-body binding energies with very small ranges.

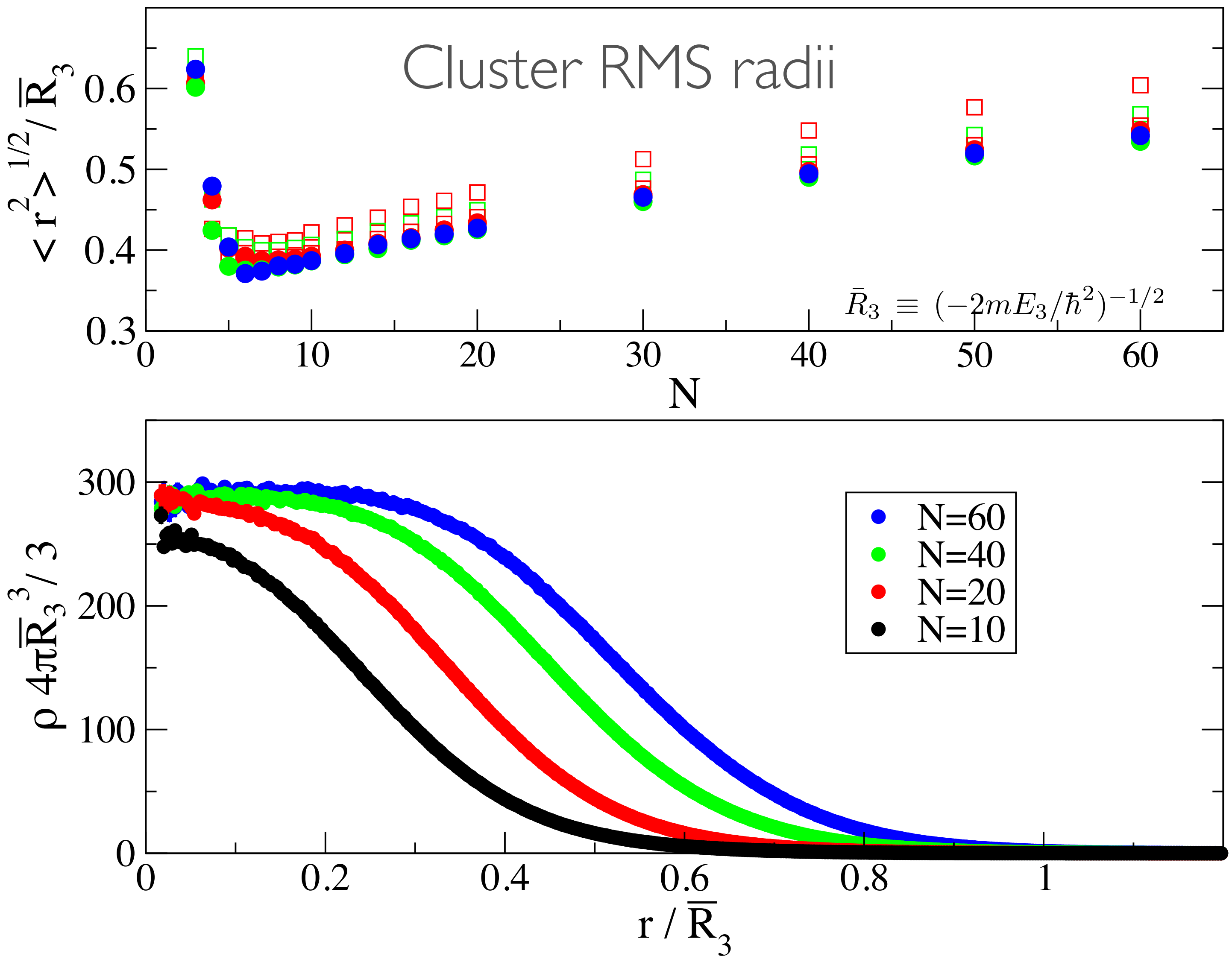
$R_3 = (2\ m\ |E_3| / \hbar^2)^{1/2}$

Cluster Binding Energy vs. # of Bosons



Binding energy per particle of Bosonic clusters normalized by | E_3 |
Filled symbols are more loosely bound trimers $\mu_2 \underline{R_3} \sim 65$

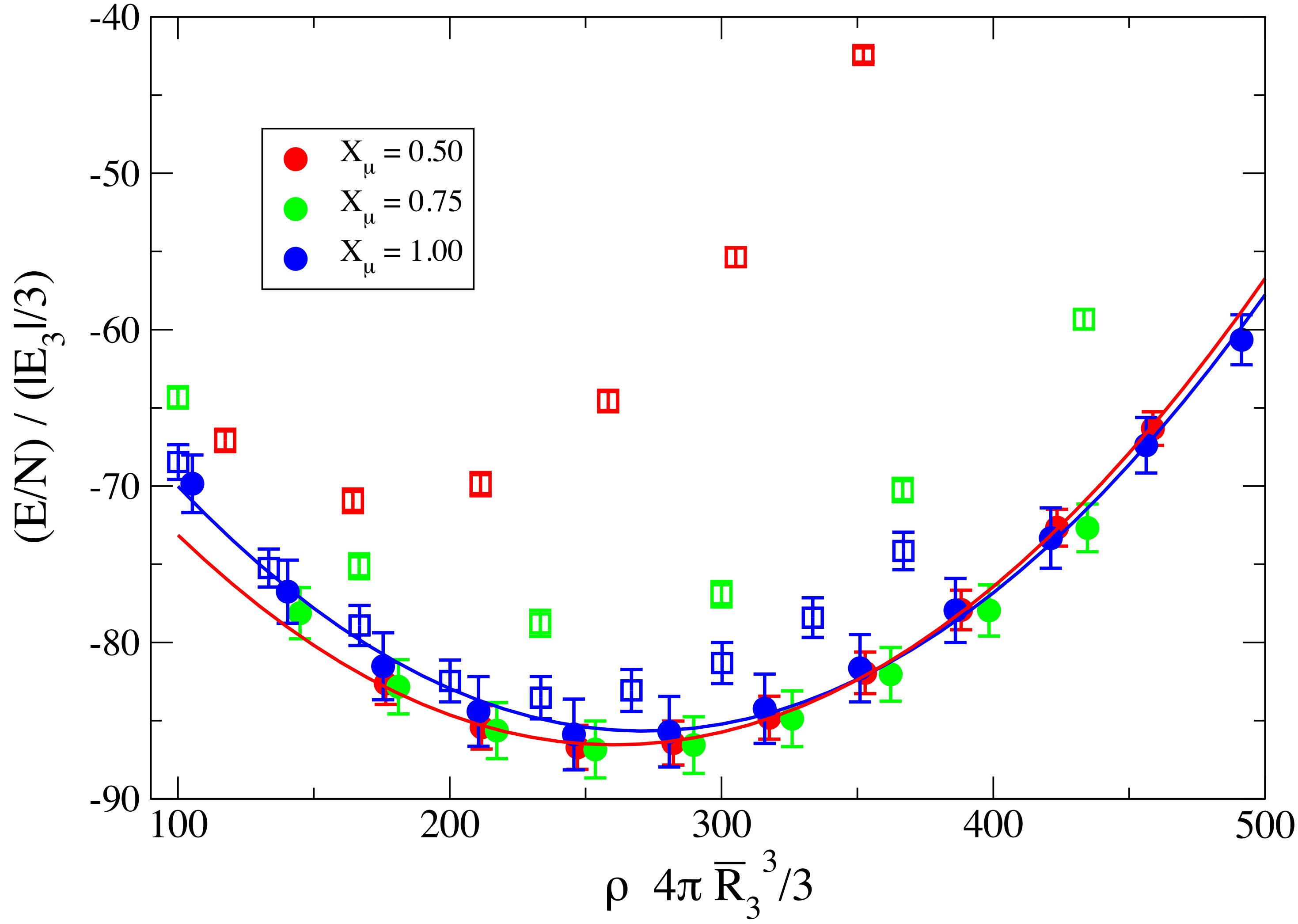
The solid line is a fit to the liquid drop formula for N > 30



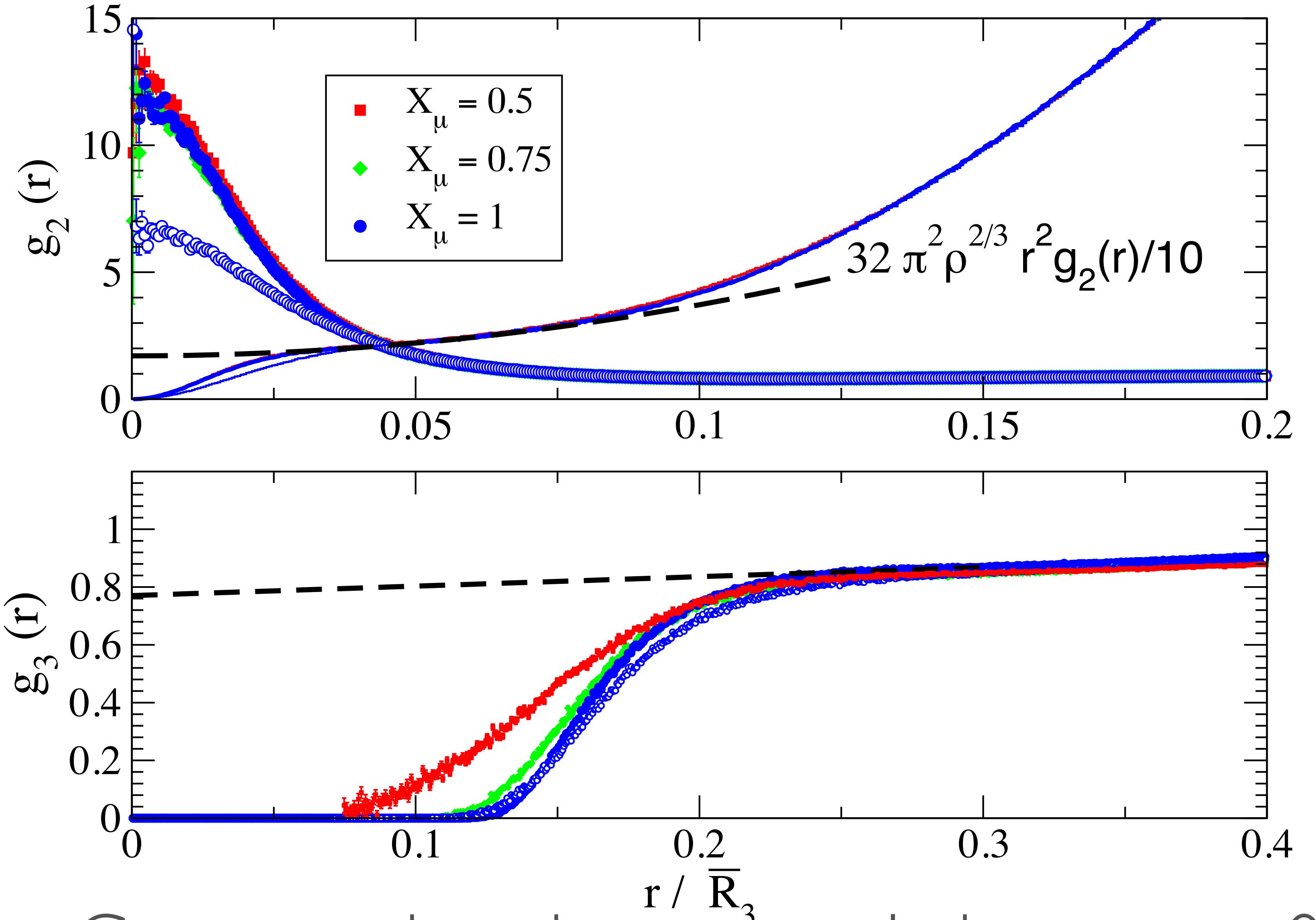
Cluster single-particle densities

Saturation at a very high density compared to N=3

Homogeneous Matter Equation of State



2- and 3-body distribution functions



Contacts given by extrapolation to r = 0

Contacts:

QMC contacts

$$\alpha_2 = 17(3)$$

$$\beta_3 = 0.9(1)$$

analysis of rapid quench experiments:

$$\alpha_2 = 22(1)$$

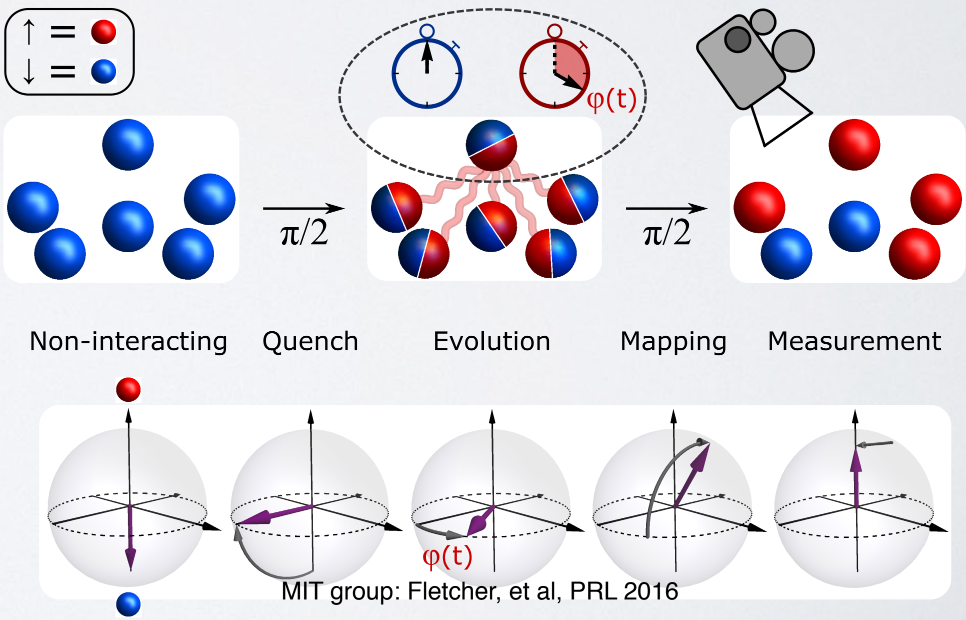
$$\beta_3 = 2.1(1)$$

Smith, Braaten, Kang, Platter PRL 2014
analysis of Jin experiment

Condensate Fraction

$$\eta = 0.92(1)$$

Cluster binding vs. N roughly similar to liquid 4He, but 4He has only 7% condensate



MIT group: Fletcher, et al, PRL 2016

Conclusions:
SU(2) Fermions and SU(N=∞) Bosons

- Unitary Bosons and Fermions are scale-invariant
- SU(2) Fermions are a superfluid gas
- SU(∞) 'Bosons' are self-bound into clusters
- Comparatively simple DFTs
- Can predict properties of small finite systems from calculations of inhomogeneous matter
- Experimentally testable

Outlook: what about SU(N) for N = 3,4, 5…

- 2- and 3-body interactions will stabilize all systems
- Transition from gas to self-bound clusters
- When are clusters of size > N bound for SU(N) at unitarity (Born-Oppenheimer arguments) ?
- What about finite range - eg. SU(4) EFTs for nuclei
- Can we learn about resonances / phase structure of matter from simulations with small N

Beyond unitary gases: systems of a few nucleons

Low density: 4 neutron resonances
High density: phase structure of QCD

Outlook: can we test dynamics ?

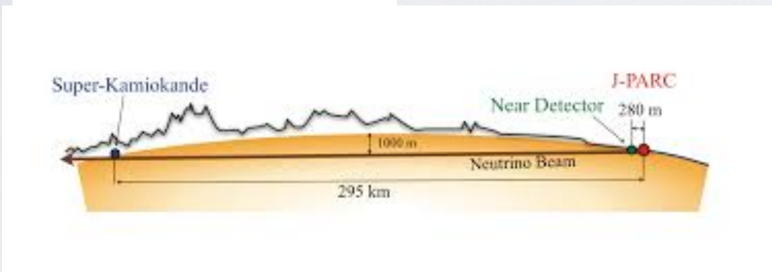
- Significant information on dynamics can be obtained through path integral simulations:
  - density, spin response
  - low-lying collective excitations
  - In nuclear physics neutrino and electron scattering

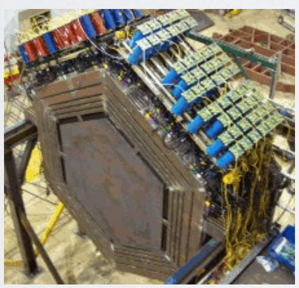
- Contacts are interesting, relate EOS to high-momentum tails: EOS can be obtained from a DFT, but high momentum tails?

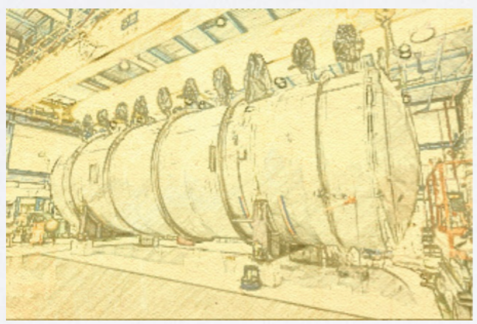
- At what energies and momenta does DFT start to break down?

Important Problems in Nuclear Dynamics

Neutrino Scattering (FNAL, J-PARK, Kamiokande)

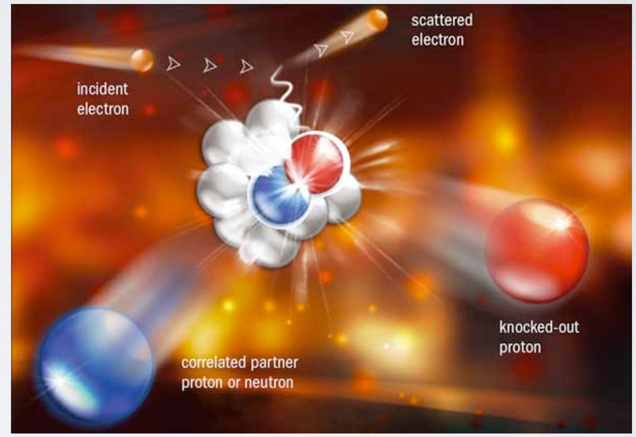






MINERva MicroBooNE

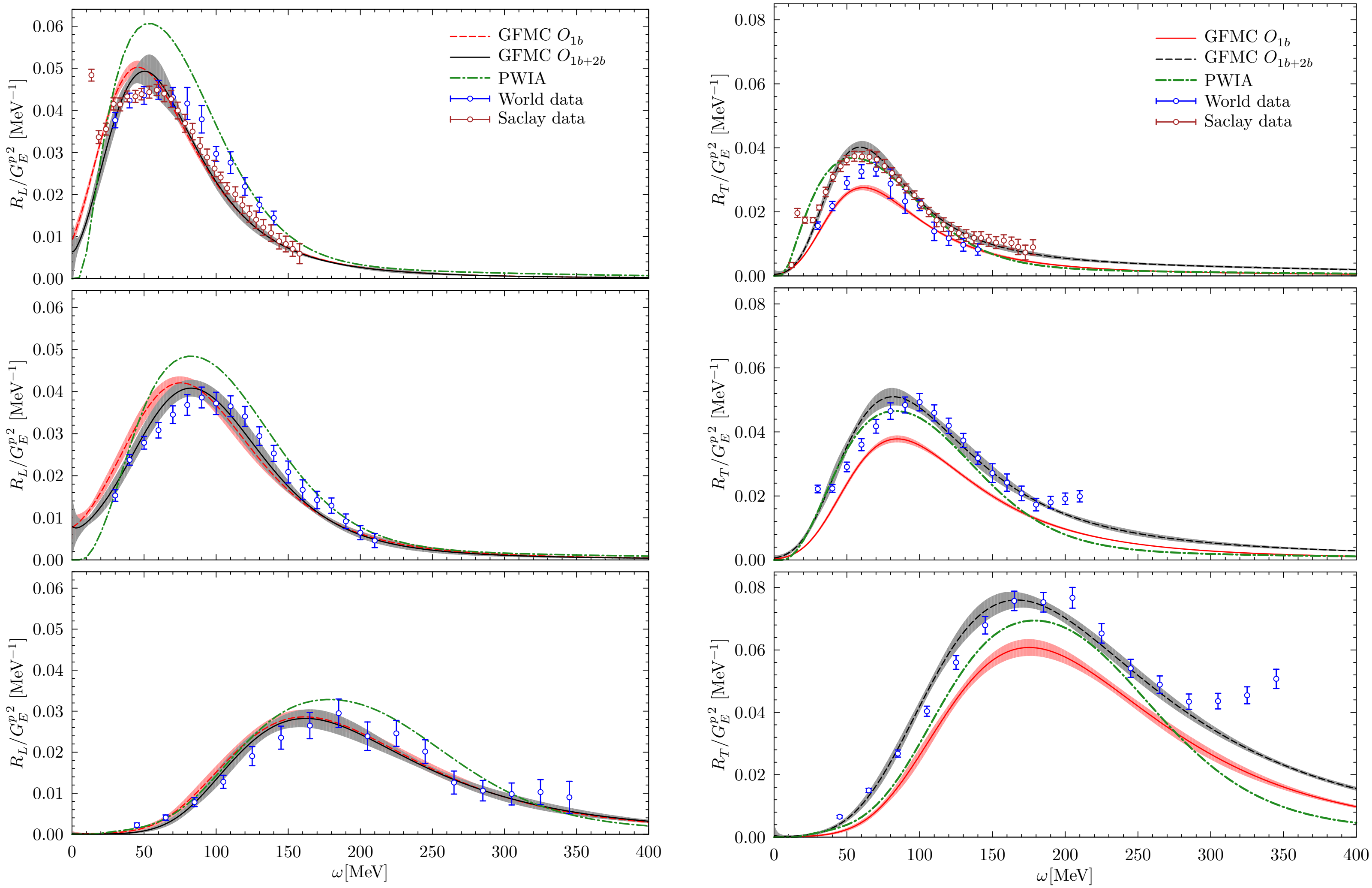
Electron Scattering (JLAB)



Back-to-Back neutron-proton pairs

^{12}C EM response

Longitudinal | Transverse

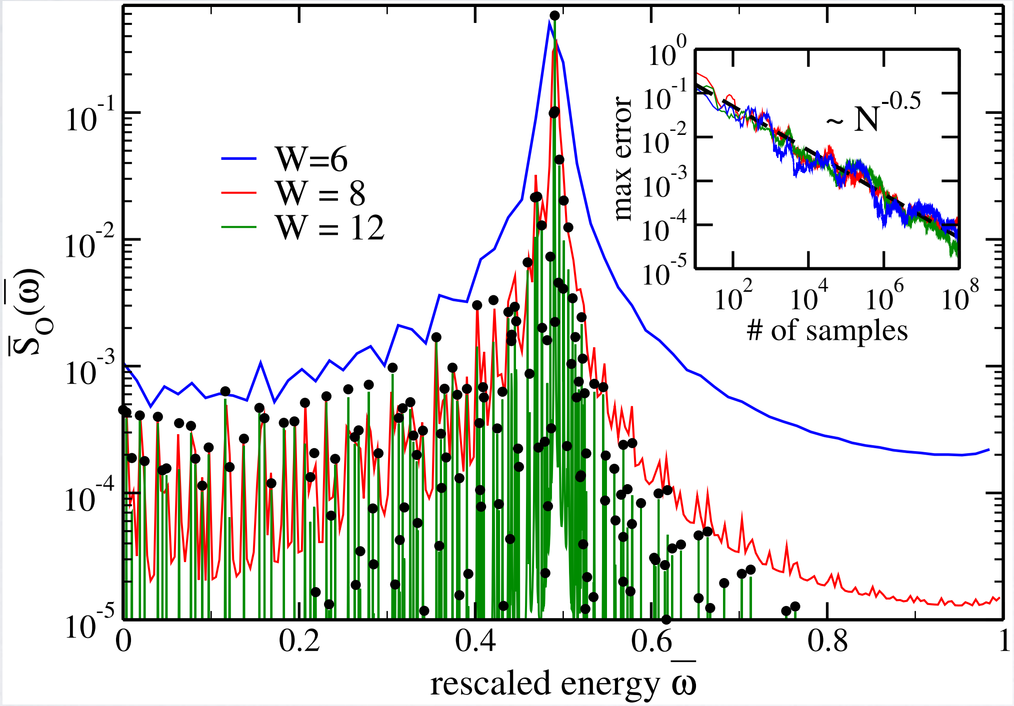


Lovato, et al, PRL, 2016

No enhancement without NN correlations and currents

Longer Term: Quantum Computing (?)
Alessandro Roggero; arXiV 1804.01505 (2018)

Simple Toy problem on 3D lattice



- Algorithms exist to calculate ground state
- QCs can implement exp [- I H t]
- Implemement linear response with Unitary operators

Similar ideas may be useful for High-energy scattering (short Real time propagation) on Standard (classical) computers

Summary and Outlook

- Many similarities and synergies between cold atom physics and physics of nuclei
- Great opportunity for nuclear physicists to expand their outlook and (hopefully) contribute across fields
- I look forward to an exciting and diverse program.

Thank you