Density Functional approach to cold atoms and neutron matter

Denis Lacroix

INSTITUT DE PHYSIQUE NUCLÉAIRE ORSAY

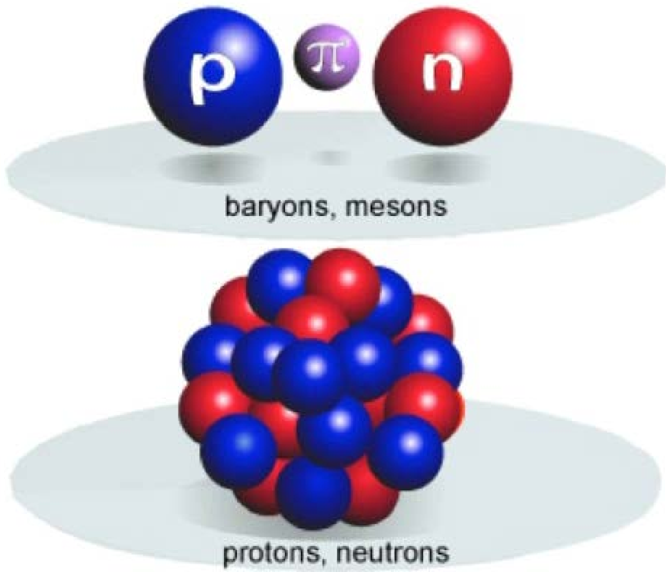
Outline:

- Brief discussion on DFT for nuclei
- EFT guiding the construction of DFT/EDF: resummation
- Unitary gas guidance: role of large but finite s-wave scattering length
- Applications: cold atoms and neutron matter (Ground state and excited states)

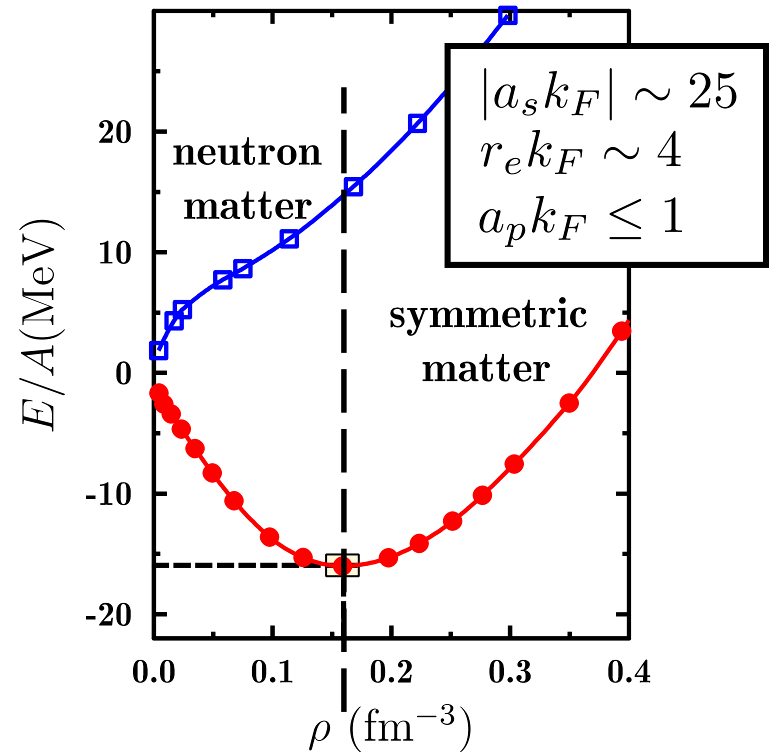
Coll: J. Bonnard, A. Boulet, M. Grasso and C.J. Yang

Short summary of the atomic nuclei properties

generalities



Some orders of magnitude and general aspects

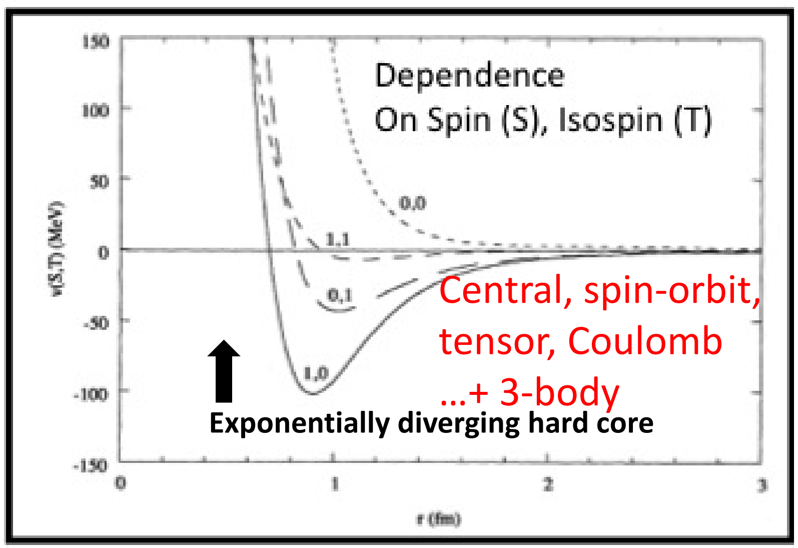


The nucleon-nucleon interaction is complex

$$\phi_{\text{nucleon}} \equiv \phi(\mathbf{r}, \sigma, \tau)$$

$\sigma = \uparrow, \downarrow$ spin

$\tau = n, p$ isospin



Wiringa, Rev. Mod. Phys. 1993

New generation of int.
Ex: pionless EFT

- -LO : 2 parameters
- -NLO + 7 parameters
- -N^{3}LO + 15 parameters
- -N^{5}LO +26 parameters
- ….

Machleidt and Sammaruca, Phys. Script. 2016

Direct link to low energy constant

$$a_s,\ r_e,\ a_p, \cdots$$

Dilemma: failure of the Hartree-Fock theory /success of DFT/EDF

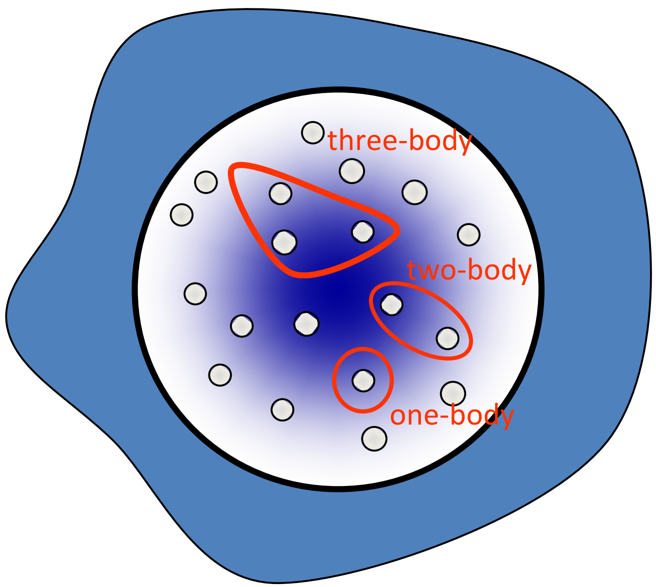
Simplicity

Many aspects of nuclei can be fairly well understood assuming that nucleons behaves like independent particles in an external one-body field

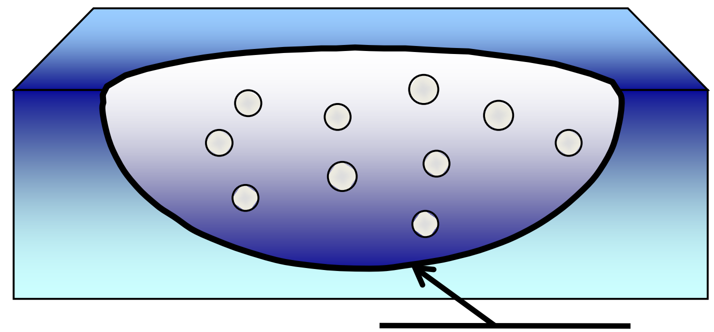
Complexity

The natural approach to map a many-boby problem into a one-body theory (HF) does not work in nuclear physics

➡ The Energy Density Functional approach



Mean-field:
(DFT/EDF)



Self-consistent
Mean-field

Complex many-body states:

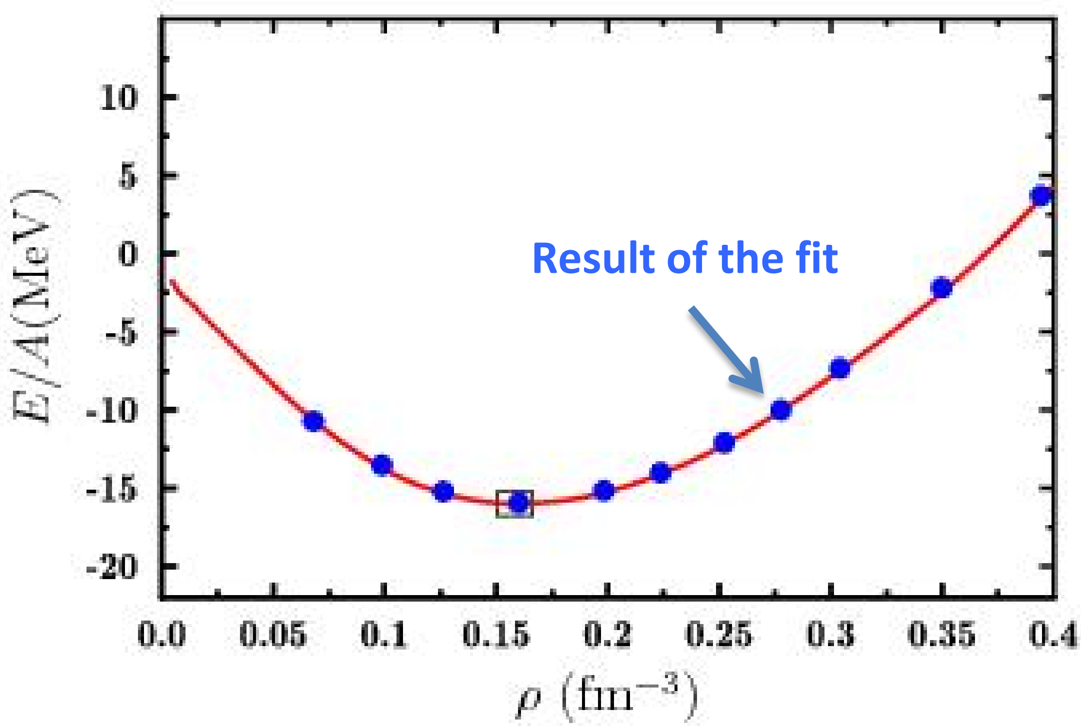
$$\Psi(r_1, \cdots, r_{12}, \cdots, r_{123}, \cdots)$$

Independent particles or quasi-particle states

Parameters of the functional are directly adjusted on data

Link to underlying bare Hamiltonian is lost

EDF from a simple perspective



Exercise : fit the curve with

$$E = \left\langle \frac{p^2}{2m} \right\rangle + U[\rho]$$

In nuclear matter:

$$\left\langle \frac{p^2}{2m} \right\rangle = \frac{3}{5} \left(\frac{3\pi^2}{2} \right)^{2/3} \rho^{5/3}$$

Fit with 5th order polynomial of the density (Local density approximation)

- **An excellent fit is obtained**
- **Coefficients contains many-body physics**
- **Contains resummation of many-body effects to all orders**

Basic aspects of Functional approach to nuclear systems

Nuclear takes different names:

Skyrme/Gogny Hartree-Fock,
Nuclear Density Functional Theory (nuclear DFT)
Energy Density Functional (EDF)
Relativistic Mean-Field ...

What type of Density Functional Theory we use ?

In its simplest form in DFT, the exact ground state of an N-body problem can be replaced by the minimization of an energy functional of the local one-body density $\rho(r)$.
At the minimum, the energy and the density corresponds to the exact energy

Hohenberg, Kohn, Phys. Rev. (1965)

$$\langle \Psi | \hat{H} | \Psi \rangle \Rightarrow \mathcal{E}[\rho(r)]$$

Extensions:

- Introduction of auxiliary state $\Psi = \mathcal{A}(\varphi_1(r_1), \cdots \varphi_N(r_N))$

  Kohn, Sham, Phys. Rev. (1965) $\rho(r) = \sum_i |\varphi_i(r)|^2$

  $\{\varphi_i\} \rightarrow \rho(r) \rightarrow \mathcal{E} \rightarrow U_{KS}(r) \rightarrow \{\varphi_i\}...$

  $h[\rho]|\varphi_i\rangle = \varepsilon_i|\varphi_i\rangle$ with $h[\rho] = \frac{\partial \mathcal{E}}{\partial \rho}$

- GGA, Meta GGA, ...

  A primer to DFT, Lectures notes in Physics 620 (2003)

  $\mathcal{E} = \mathcal{E}[\rho(r), \nabla\rho, \Delta\rho, \tau, ...]$

- DFT with pairing

  $\mathcal{E} = \mathcal{E}[\rho, \kappa...]$

  Oliveira et al, PRL (1988)

  (and much more ...)

- Time-dependent DFT

  Runge, Gross PRL (1984)

Nuclear Energy Density Functional based on effective interaction

Illustration with the Skyrme Functional

Vautherin, Brink, PRC (1972)

$$
\begin{aligned}
v(\mathbf{r}_1-\mathbf{r}_2) &= t_0\,(1+x_0\hat{P}_\sigma)\,\delta(\mathbf{r}) \\
&+\tfrac{1}{2}\,t_1\,(1+x_1\hat{P}_\sigma)\big[\mathbf{P}'^2\,\delta(\mathbf{r})+\delta(\mathbf{r})\,\mathbf{P}^2\big] \\
&+t_2\,(1+x_2\hat{P}_\sigma)\,\mathbf{P}'\cdot\delta(\mathbf{r})\,\mathbf{P} \\
&+iW_0\sigma.\big[\mathbf{P}'\times\delta(\mathbf{r})\,\mathbf{P}\big] \\
&+\tfrac{1}{6}\,t_3\,(1+x_3\hat{P}_\sigma)\,\rho^\alpha(\mathbf{R})\,\delta(\mathbf{r})
\end{aligned}
$$

$$
\mathcal{E} = \langle\Psi|H(\rho)|\Psi\rangle = \int \mathcal{H}(r)d^3\mathbf{r}
$$

$$
\mathcal{H} = \mathcal{K}+\mathcal{H}_0+\mathcal{H}_3+\mathcal{H}_{\text{eff}} + \mathcal{H}_{\text{fin}}+\mathcal{H}_{\text{so}}+\mathcal{H}_{\text{sg}}+\mathcal{H}_{\text{Coul}}
$$

$$
\begin{aligned}
\mathcal{H}_0 &= \tfrac{1}{4}t_0\big[(2+x_0)\rho^2-(2x_0+1)(\rho_p^2+\rho_n^2)\big] \\
\mathcal{H}_3 &= \tfrac{1}{24}t_3\rho^\alpha\big[(2+x_3)\rho^2-(2x_3+1)(\rho_p^2+\rho_n^2)\big] \\
\mathcal{H}_{\text{eff}} &= \tfrac{1}{8}\big[t_1(2+x_1)+t_2(2+x_2)\big]\tau\rho \\
&+ \tfrac{1}{8}\big[t_2(2x_2+1)-t_1(2x_2+1)\big](\tau_p\rho_p+\tau_n\rho_n) \\
\mathcal{H}_{\text{fin}} &= \tfrac{1}{32}\big[3t_1(2+x_1)-t_2(2+x_2)\big](\nabla\rho)^2 \\
&- \tfrac{1}{32}\big[3t_1(2x_1+1)+t_2(2x_2+1)\big]\big[(\nabla\rho_p)^2+(\nabla\rho_n)^2\big] \\
\mathcal{H}_{\text{so}} &= \tfrac{1}{2}W_0\big[\mathbf{J}.\nabla\rho+\mathbf{J}_p.\nabla\rho_p+\mathbf{J}_n.\nabla\rho_n\big] \\
\mathcal{H}_{\text{sg}} &= -\tfrac{1}{16}(t_1x_1+t_2x_2)\mathbf{J}^2+\tfrac{1}{16}(t_1-t_2)\big[\mathbf{J}_p^2+\mathbf{J}_n^2\big]
\end{aligned}
$$

Functional of $\rho, \rho_n, \rho_p, \tau, \tau_n, \tau_p, \mathbf{J}, \ldots$

Around 10-14 parameters to be adjusted

Nuclear Energy Density Functional based on effective interaction

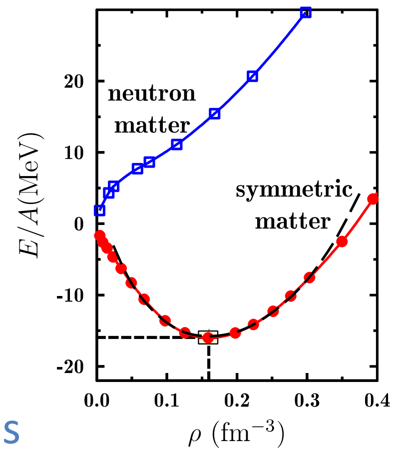
Constraining the functional

See for instance, Meyer EJC1997

Vautherin, Brink, PRC (1972)

$$
\begin{aligned}
v(\mathbf{r}_1-\mathbf{r}_2) &= t_0\,(1+x_0\hat{P}_\sigma)\,\delta(\mathbf{r}) \\
&+\tfrac{1}{2}t_1\,(1+x_1\hat{P}_\sigma)\left[\mathbf{P}'^2\,\delta(\mathbf{r})+\delta(\mathbf{r})\,\mathbf{P}^2\right] \\
&+t_2\,(1+x_2\hat{P}_\sigma)\,\mathbf{P}'\cdot\delta(\mathbf{r})\,\mathbf{P} \\
&+iW_0\sigma.\left[\mathbf{P}'\times\delta(\mathbf{r})\,\mathbf{P}\right] \\
&+\tfrac{1}{6}t_3\,(1+x_3\hat{P}_\sigma)\,\rho^\alpha(\mathbf{R})\,\delta(\mathbf{r})
\end{aligned}
$$

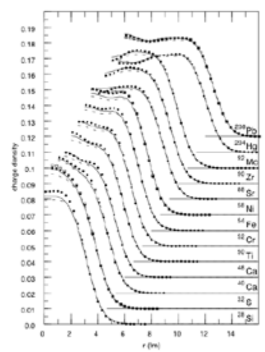
Infinite nuclear matter and Nuclear Masses



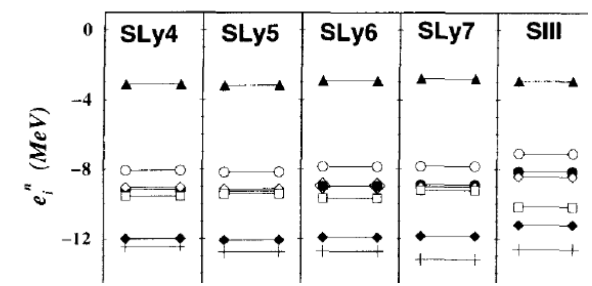
Constraints

$$E_0,\ \left.\frac{\partial E}{\partial \rho}\right|_{\rho_0},\ \left.\frac{\partial^2 E}{\partial \rho^2}\right|_{\rho_0}$$

Densities



Shell effect



Chabanat et al, NPA (1998)

Dynamics

Time (fm/c)

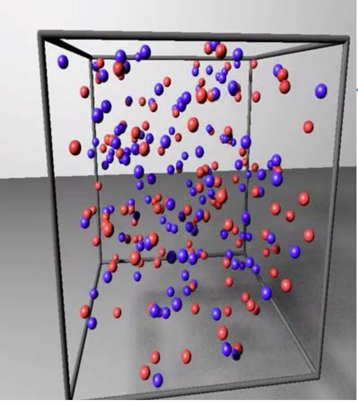
1000 2000 3000 4000 5000 5300 5500 5600

Scamps, Simenel, Lacroix, PRC 92 (2015)
Tanimura, Lacroix, Scamps, PRC 92 (2015)

Since we directly fit on experiments Complex correlation much beyond Hartree-Fock are included

Since we directly fit on experiments there is no more link with the interaction and associated low energy constants…

Can we link the energy density functional to the low energy constants of the bare interaction?
and render it less empirical?

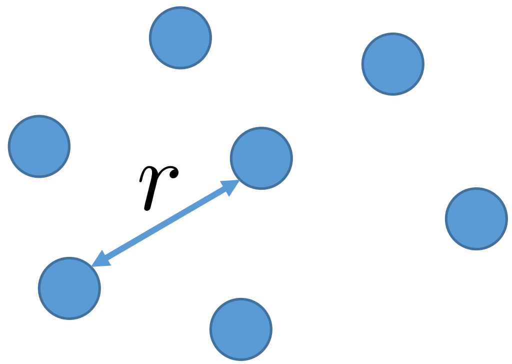


Towards a constructive approach for DFT

The low-density Fermi gas limit: the EFT guidance

EFT strategy

See for instance: R. J. Furnstahl, in *Renormalization Group and Effective Field Theory Approaches to Many-Body Systems*, edited by A. Schwenk and J. Polonyi, Lecture Notes in Physics, Vol. 852 (Springer, Berlin, 2012), Chap. 3.



At low density *r* is large

$$\Delta r \Delta k \sim 1$$

→ We only need a low-momentum expansion
Of the interaction

$$\langle \mathbf{k}|V_{\rm eft}|\mathbf{k}'\rangle = C_0 + \frac{1}{2}C_2(\mathbf{k}^2+\mathbf{k}'^2) + C_2'\,\mathbf{k}\cdot\mathbf{k}' + \cdots$$

Example of the s-wave

C_0, C_2, C'_2 are directly linked to low energy constant

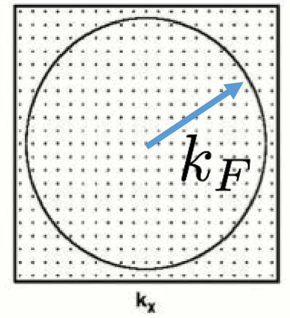
$$\sigma = \frac{4\pi}{k^2}\frac{1}{1+\cot^2\delta_0} = \frac{4\pi a^2}{ak^2+[1-ar_{\rm ef}k^2/2]^2}$$

$$C_0 = \frac{4\pi\hbar^2}{m}a_s,\ C_2 = \frac{2\pi\hbar^2}{m}r_e a_s^2,\ C_2' = \frac{4\pi\hbar^2}{m}a_p^3.$$

Constructive many-body perturbative approach

$$E = E^{\rm HF} + E^{2^{\rm nd}} + E^{3^{\rm rd}} + \ldots$$

H.W. Hammer and R.J. Furnstahl, NPA678 (2000)



General strategy: infinite systems

$$E = E^{\mathrm{HF}} + E^{2^{\mathrm{nd}}} + E^{3^{\mathrm{rd}}} + \ldots$$

$$\frac{E}{E_{\mathrm{FG}}} = 1 + \frac{10}{9\pi}(\nu - 1)(k_F a_s) + (\nu - 1)\frac{4}{21\pi^2}(11 - 2\ln 2)(k_F a_s)^2 + \cdots$$

$\rho = \frac{\nu}{6\pi^2} k_F^2$ with ν degeneracy

Many-body Perturbation Theory

E^{HF} $E^{2^{\mathrm{nd}}}$ $E^{3^{\mathrm{rd}}}$ $+\cdots$ [MBPT]

Expansion as polynomial of LEC: $(a_s k_F)$ $(r_e k_F)$ $\cdots$

Functionals of increasing complexity

$$E \equiv \mathcal{E}(\rho)$$

Difficulty

valid for $a_s k_F < 1$

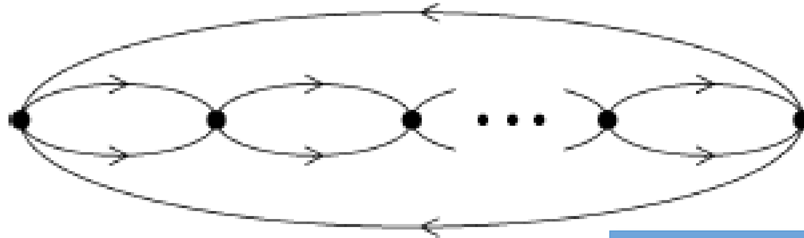
For neutron matter $a_s = -18.9$ fm, $r_e = 2.7$ fm → Valid for $\rho < 10^{-6}$ fm^{-3}

The “magic” technique: resummation

Highlighting work

Schaefer, Kao, Cotanch, NPA 762 (2005)

Resummation of particle-particle diagrams

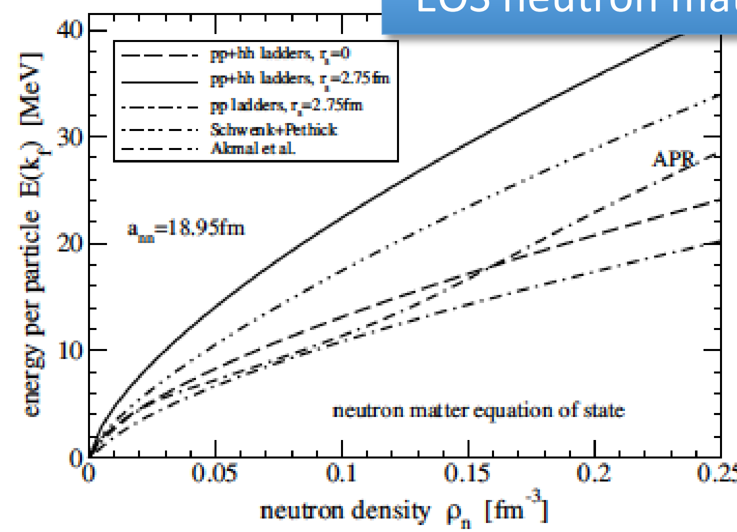


$$\frac{E_{\rm PP}}{A} = \frac{3(g-1)\pi^2}{k_{\rm F}^3} \int \frac{d^3P}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \theta_k^- \frac{4\pi a/M}{1 - \frac{k_{\rm F} a}{\pi} f_{\rm PP}(\kappa, s)}.$$

Contains terms to all order in $(a_s k_F)$

Results strongly depends on selected diagram

EOS neutron matter



Kaiser, EPJA 48 (2012)

The pragmatic approach

$$E \sim \frac{3}{5}\frac{\hbar^2 k_F^2}{2m} \frac{\frac{10}{9\pi}(a_s k_F)}{1 - \frac{6}{35\pi}(11 - 2\ln 2)(a_s k_F)} \sim \langle f_{\rm PP} \rangle$$

$$\frac{E}{A} = \left(\frac{3}{5} + \frac{2k_F a_s/3}{\pi - 2k_F a_s}\right)\frac{k_F^2}{2M} \qquad \langle f_{\rm PP} \rangle \underset{+\infty}{\rightarrow} 2$$

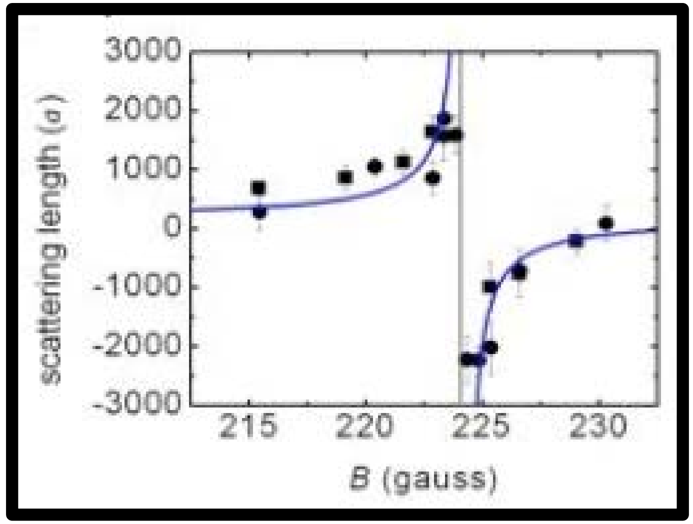
Steele, nucl-th-0010066v2

Interpretations:

- -Minimal Padé approximation
- -Phase-space average
- -asymptotic values
- -…

Resummed formula for Unitary gas

Great interest of resummed expression:
It has a finite limit for Unitary gas



For unitary gas:

- low density system
- $a_s \to +\infty$

$$\frac{3}{5}\frac{\hbar^2 k_F^2}{2m}\frac{\frac{10}{9\pi}(a_s k_F)}{1-\underbrace{\frac{6}{35\pi}(11-2\ln 2)}_{=\langle f\rangle}(a_s k_F)} \longrightarrow 0.32\,\frac{3}{5}\frac{\hbar^2 k_F^2}{2m}$$

$$\frac{E}{A}=\left(\frac{3}{5}+\frac{2k_F a_s/3}{\pi-2k_F a_s}\right)\frac{k_F^2}{2M} \longrightarrow 0.4\,\frac{3}{5}\frac{\hbar^2 k_F^2}{2m}$$

Not so far from the “admitted” value of the Bertsch parameter for unitary gas (0.37)

Important remark for us, unitary gas has the simplest DFT ever !

$$\mathcal{E}[\rho]=\xi\times\mathcal{E}_{\rm FG}[\rho]$$

$$\xi = 0.37$$

The interest for us is that in neutron matter a_s is very large

Density Functional Theory for system at or close to unitarity

A very pragmatic approach

Lacroix, PRA 94 (2016)

Minimal DFT for unitary gas

$$\frac{E}{E_{\rm FG}} = \left\{ 1 + \frac{(ak_F)A_0}{1 - A_1(ak_F)} \right\}$$

$|a_s k_F| \ll 1$ → $|a_s k_F| \gg 1$

$$\frac{E}{E_{\rm FG}} = 1 + \frac{10}{9\pi}(\nu - 1)(k_F a_s) + \sout{(\nu - 1)\frac{4}{21\pi^2}(11 - 2\ln 2)(k_F a_s)^2 + \cdots}$$

$$\frac{E}{E_{\rm FG}} = \xi_0$$

Adjusting only on low density

$$A_0 = \frac{10}{9\pi}(\nu - 1)$$

$$A_0 A_1 = (\nu - 1)\frac{4}{21\pi^2}(11 - 2\ln 2)$$

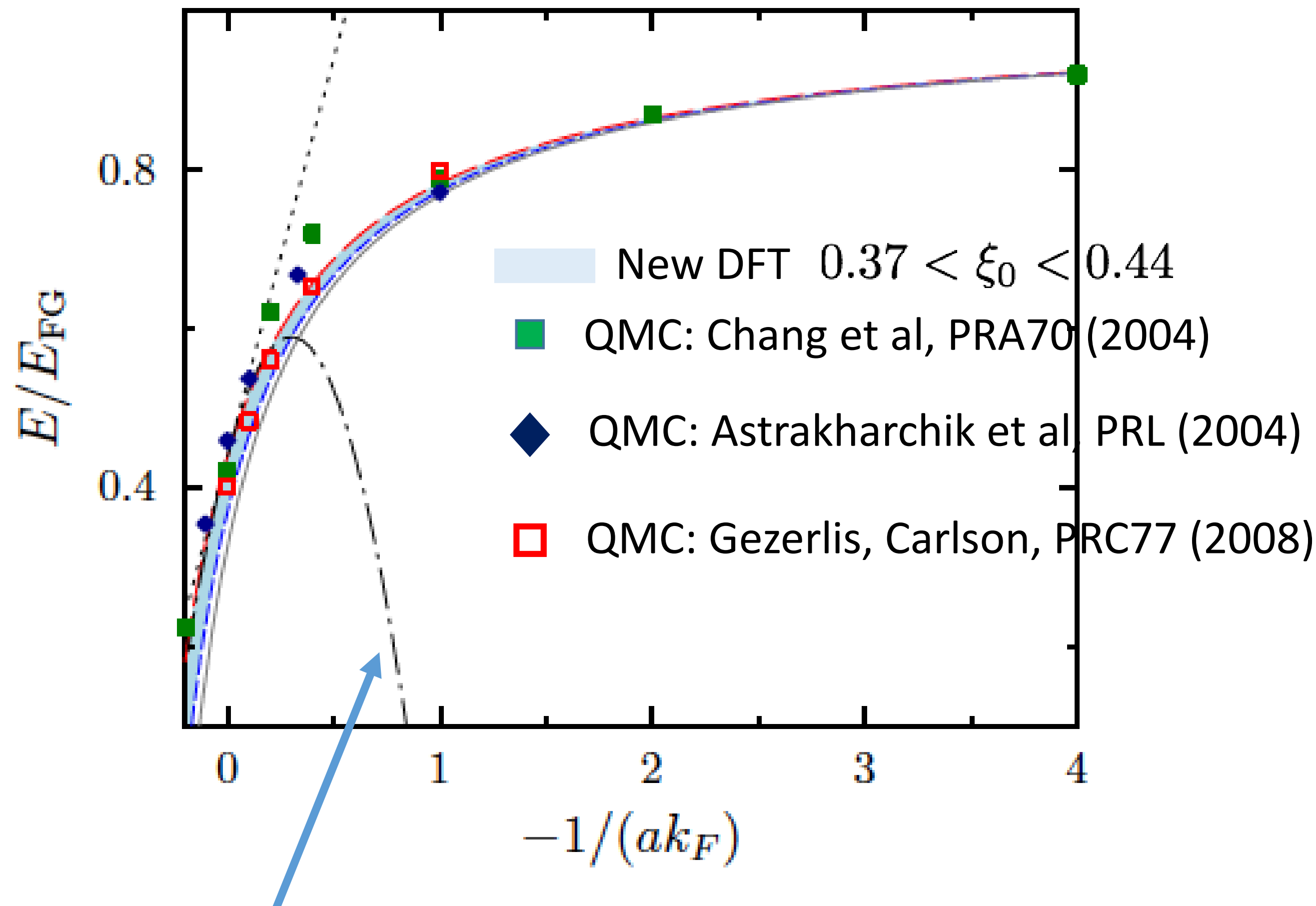
Adding the unitarity constraint

$$A_0 = \frac{10}{9\pi}(\nu - 1)$$

$$1 - \frac{A_0}{A_1} = \xi_0$$

Result of the DFT for at or close to unitarity

Lacroix, PRA 94 (2016)



$$\frac{E}{E_{\rm FG}} \simeq \xi_0 - \frac{\zeta}{(ak_F)} - \frac{5}{3}\frac{\nu}{(ak_F)^2} + \cdots \qquad \zeta \simeq \nu \simeq 1$$

Taylor expansion in $(a_s k_F)^{-1}$: Bulgac and Bertsch, PRL 94 (2005)

Example of applications

Lacroix, PRA 94 (2016)

$$\frac{E}{E_{\rm FG}} = \mathcal{F}(a_s, k_F) \equiv \mathcal{F}(a_s, \rho)$$

Any quantity that could be obtained through partial derivatives of the energy with respect to a_s or k_F or ρ is straightforward to obtain

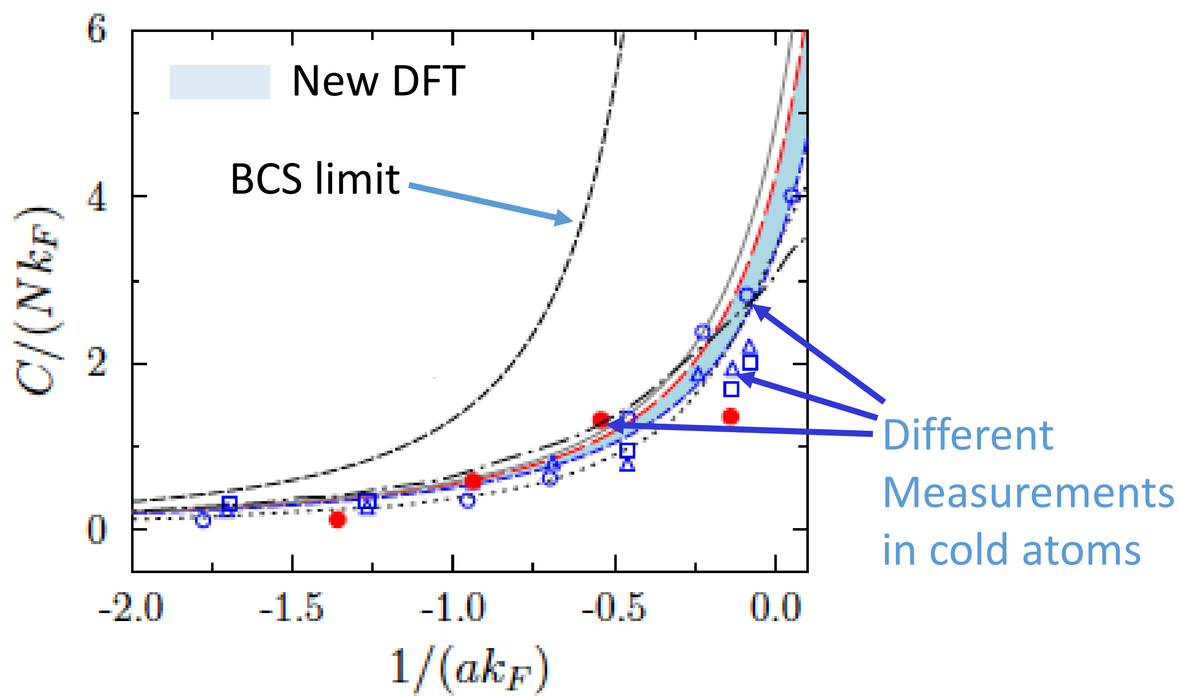
Estimate of the density dependence of the Tan contact parameter

E. Braaten, Lect. Not. Phys. 836 (2011).

$$\frac{C}{Nk_F} = \frac{(3\pi^2)}{k_F^4}\mathcal{C}$$

$$\mathcal{C} = \frac{4\pi m a_s^2}{\hbar^2}\left(\frac{d\mathcal{E}}{da_s}\right)$$

$$\mathcal{E} = \frac{k_F^3 E}{3\pi^2 N}$$



Example of applications: thermodynamical quantities around unitarity

Boulet, DL, Phys. Rev. C 97 (2018)

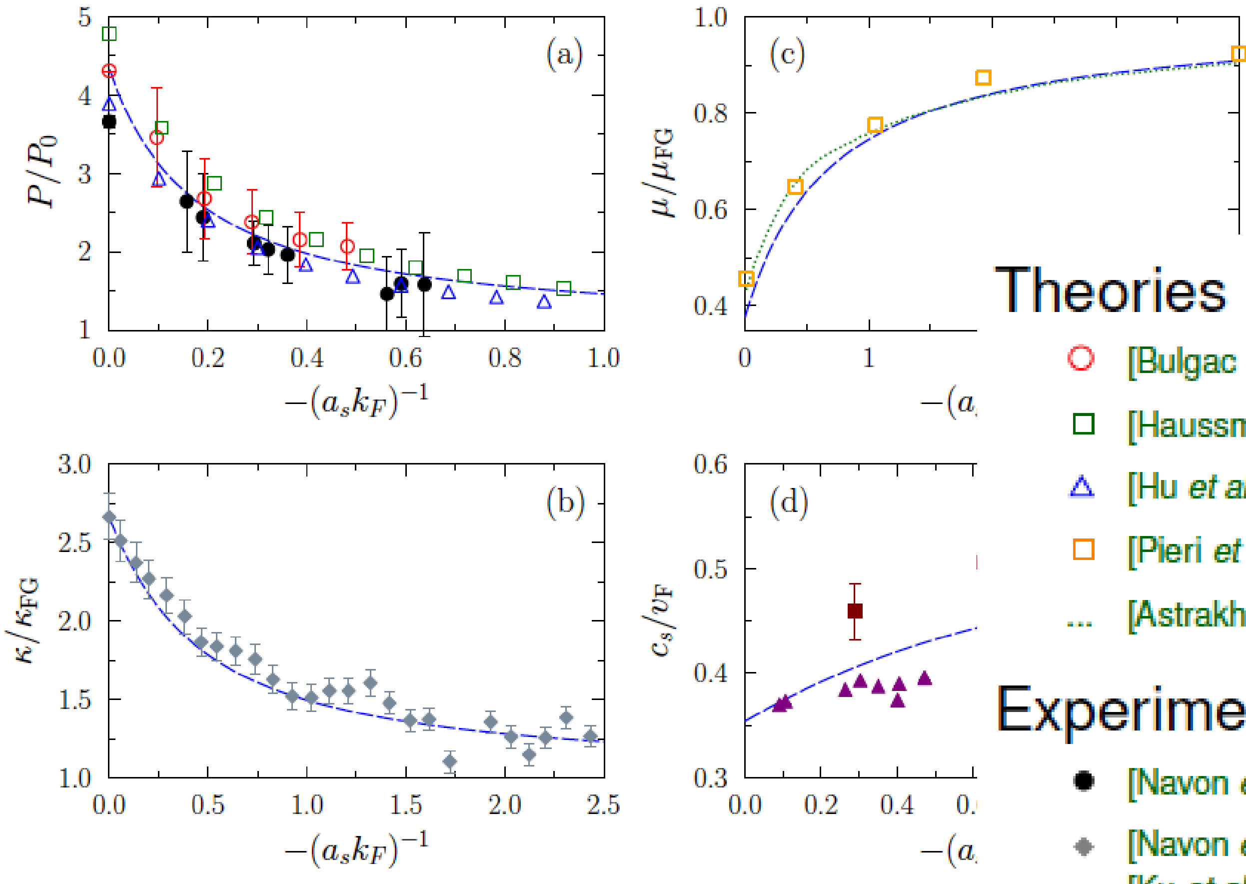
Pressure

Chemical potential

$$P = \rho_n^2 \frac{\partial E/N}{\partial \rho_n}\Big|_N$$

$$\mu = \frac{\partial E}{\partial N}\Big|_V = \frac{\partial \rho_n E/N}{\partial \rho_n}\Big|_V$$

$$\kappa = \frac{1}{\rho_n}\left(\frac{\partial P}{\partial \rho_n}\Big|_N\right)^{-1}$$



Theories

- ○ [Bulgac *et al.*, PRA **78** (2008)]
- □ [Haussmann *et al.*, PRA **75** (2007)]
- △ [Hu *et al.*, Europhys. Lett. **74** (2006)]
- □ [Pieri *et al.*, PRB **72** (2005)]
- ... [Astrakharchik *et al.*, PRL **93** (2004)]

Experiments

- ● [Navon *et al.*, Science **328** (2010)]
- ◆ [Navon *et al.*, Science **328** (2010)] [Ku *et al.*, Science **335** (2012)]
- ■ [Weimer *et al.*, PRL **114** (2015)]
- ▲ [Joseph *et al.*, PRL **98** (2007)]

Compressibility

Sound v

Example of applications: thermodynamical quantities around unitarity

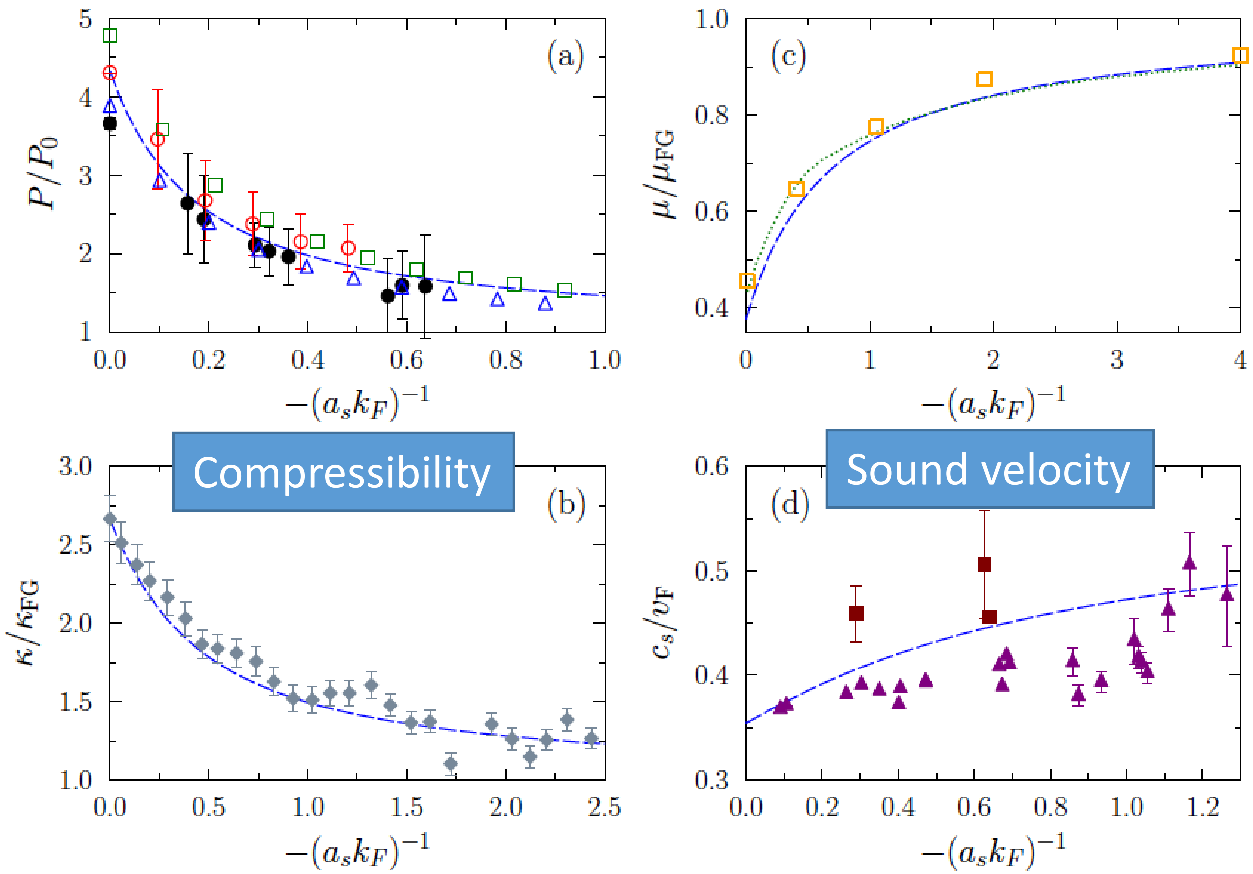
Boulet, DL, Phys. Rev. C 97 (2018)

Pressure

$$P = \rho_n^2 \frac{\partial E/N}{\partial \rho_n}\bigg|_N$$

Chemical potential

$$\mu = \frac{\partial E}{\partial N}\bigg|_V = \frac{\partial \rho_n E/N}{\partial \rho_n}\bigg|_V$$



$$\kappa = \frac{1}{\rho_n}\left(\frac{\partial P}{\partial \rho_n}\bigg|_N\right)^{-1}$$

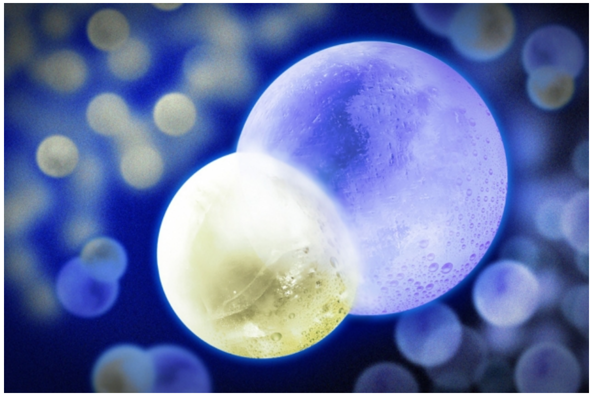
$$c_s^2 = \frac{1}{m\rho_n\kappa}$$

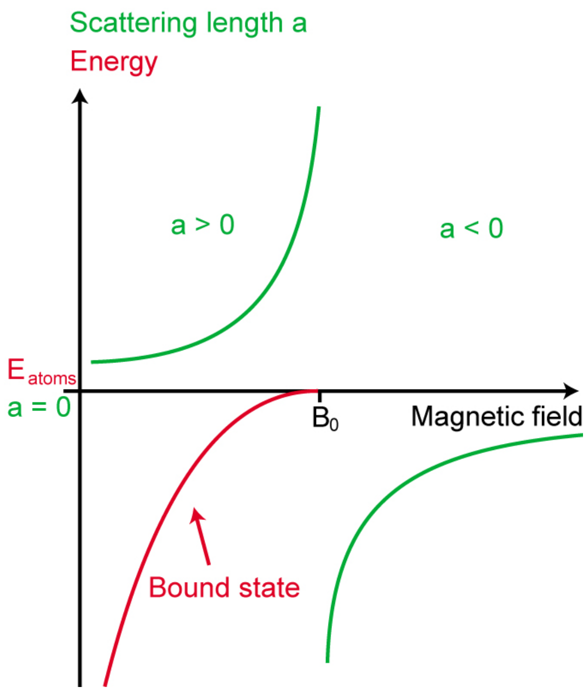
Theories

- ○ [Bulgac *et al.*, PRA **78** (2008)]
- □ [Haussmann *et al.*, PRA **75** (2007)]
- △ [Hu *et al.*, Europhys. Lett. 74 (2006)]
- □ [Pieri *et al.*, PRB **72** (2005)]
- ... [Astrakharchik *et al.*, PRL **93** (2004)]

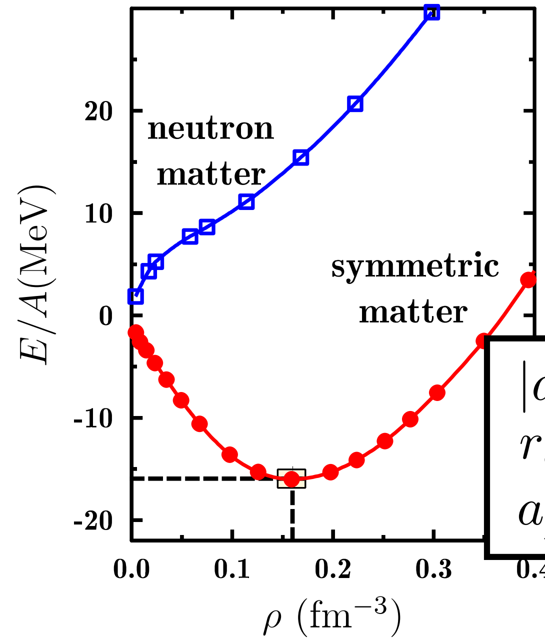
Experiments

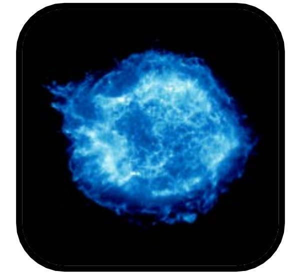
- ● [Navon *et al.*, Science **328** (2010)]
- ◆ [Navon *et al.*, Science **328** (2010)] [Ku *et al.*, Science **335** (2012)]
- ■ [Weimer *et al.*, PRL **114** (2015)]
- ▲ [Joseph *et al.*, PRL **98** (2007)]





Most often, only a_s matter





$$|a_s k_F| \sim 25$$
$$r_e k_F \sim 4$$
$$a_p k_F \leq 1$$

$$a_s = -18.9 \text{ fm}$$

$$r_e = 2.7 \text{ fm}$$

$$a_p^3 = 0.2 - 0.6 \text{ fm}^3$$

There is a hierarchy of scales $a_p \ll r_e \ll a_s$

but $r_e, \; a_p \cdots$ could not be neglected

and k_F is not small

From cold atom to neutron matter: inclusion of effective range

Lacroix, PRA 94 (2016)

$$\frac{E}{E_{\rm FG}} = 1 - \frac{U_0}{1-(a_s k_F)^{-1}U_1} + \frac{R_0(r_e k_F)}{[1-R_1(a_s k_F)^{-1}][1-R_1(a_s k_F)^{-1}+R_2(r_e k_F)]}$$

Effective range part
(form obtained by resumming
effective range effects
in HF theory)

New constraints

$|a_s k_F| \ll 1$ → $|a_s k_F| \gg 1$

$$\frac{E}{E_{\rm FG}} 1 + \frac{10}{9\pi}(\nu-1)(k_F a_s) + (\nu-1)\frac{1}{6\pi}(k_F r_e)(k_F a_s)^2 + \cdots$$

$$\xi(+\infty, r_e k_F) \equiv \xi_0 + (r_e k_F)\eta_e + (r_e k_F)^2 \delta_e$$

Forbes, Gandolfi, Gezerlis, PRA86 (2012)

$$\begin{cases} U_0 = (1-\xi_0) = 0.62400, \\ U_1 = \frac{9\pi}{10}(1-\xi_0) = 1.76432, \\ R_0 = \eta_e = 0.12700, \\ R_1 = \sqrt{\frac{6\pi\eta_e}{(\nu-1)}} = 1.54722, \\ R_2 = -\delta_e/\eta_e = 0.43307. \end{cases}$$

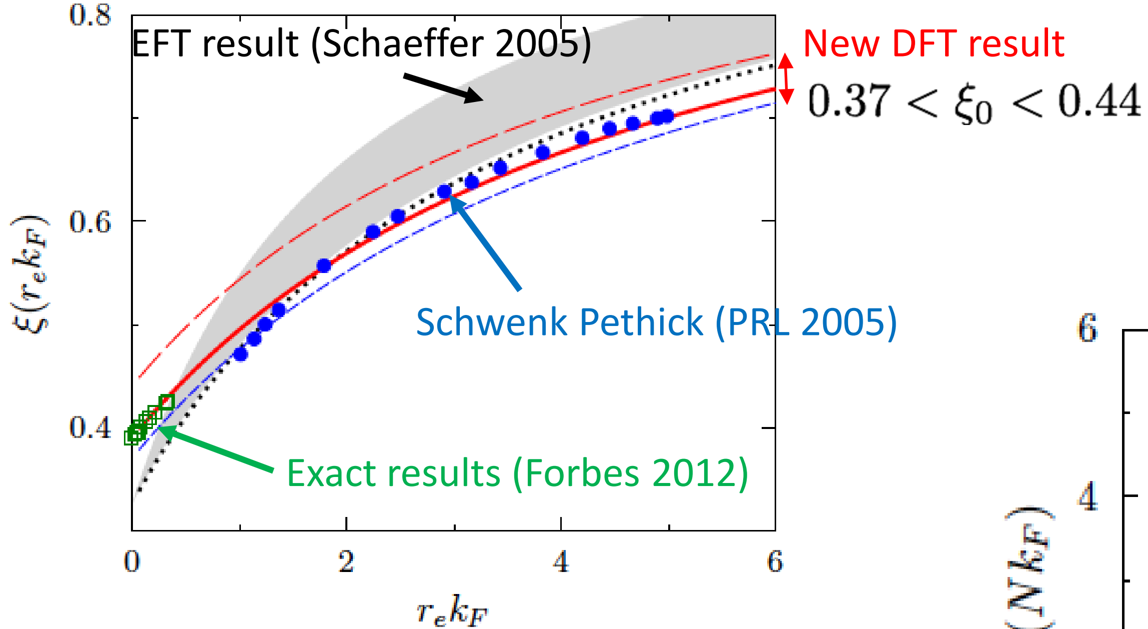
$\xi_0 = 0.376$,
$\eta_e = 0.127$
$\delta_e = -0.055$

Inclusion of effective range effects in cold atoms

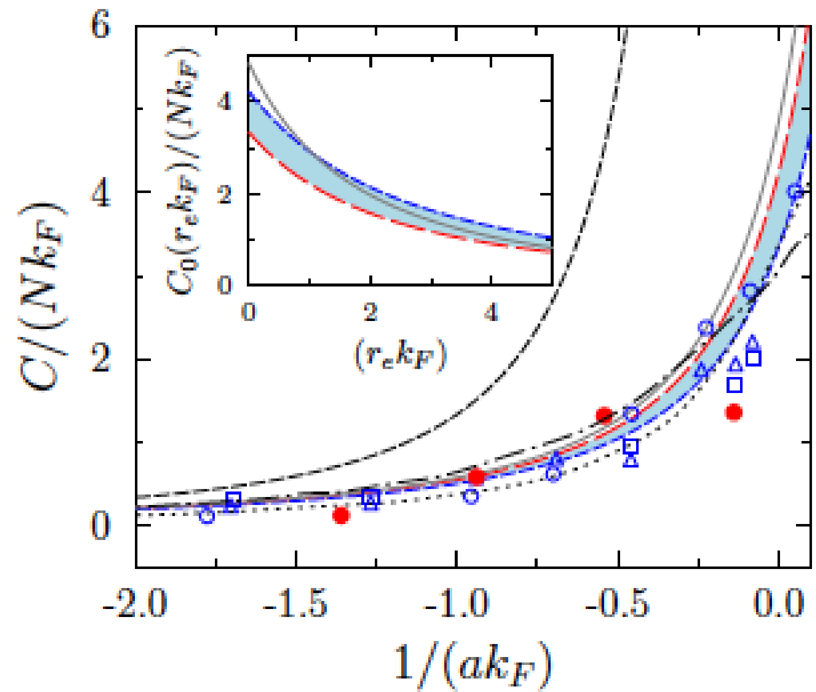
At unitarity

Lacroix, PRA 94 (2016)

$$\frac{E}{E_{\rm FG}} = \xi(r_e k_F)$$

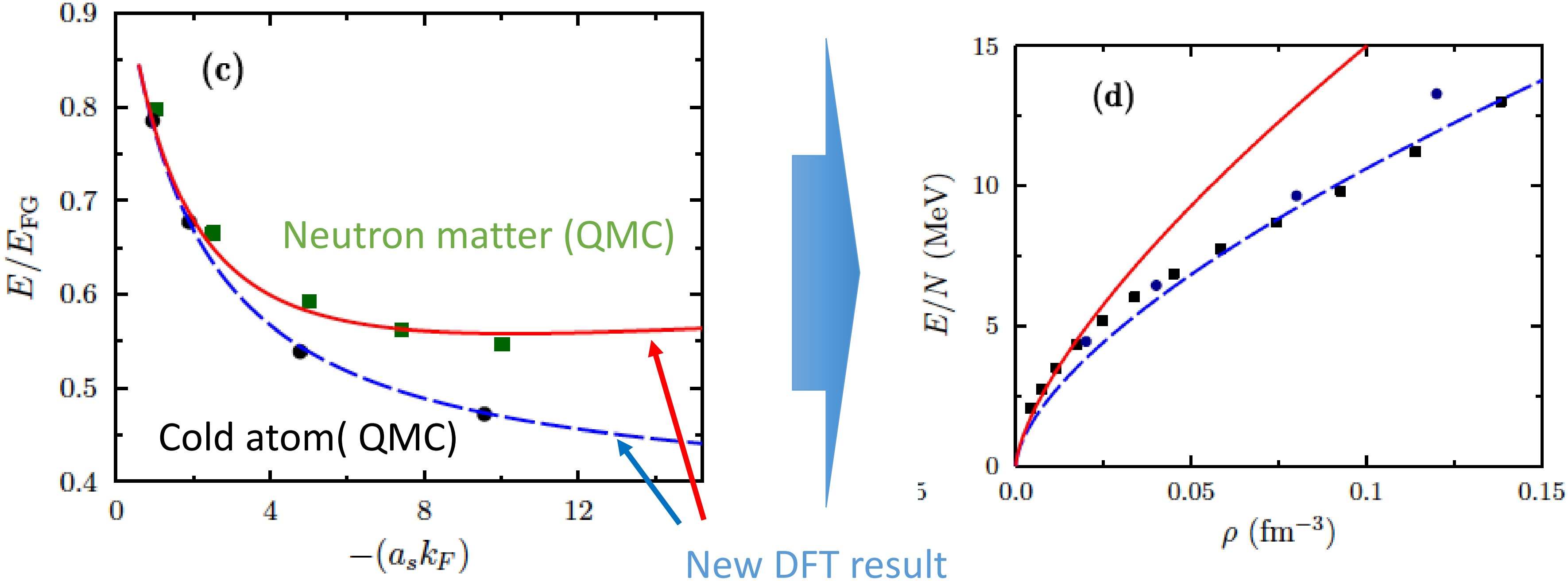


Contact with r_e effects



EDF with no-free parameters: Predictive power for neutron matter

Lacroix, Boulet, Grasso, Yang, PRC 95 (2017)



[QMC: Gezerlis, Carlson, PRC81 (2010)]

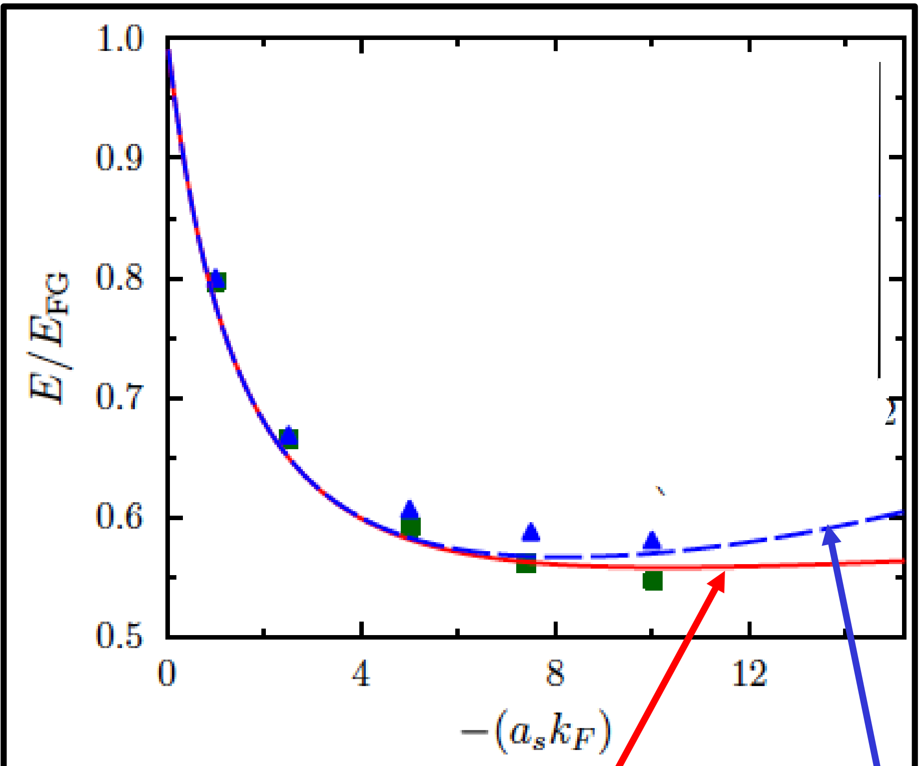
Range of validity

Lee-Yang $\rho < 10^{-6}$ fm^{-3}

New DFT $\rho < 0.01$ fm^{-3}

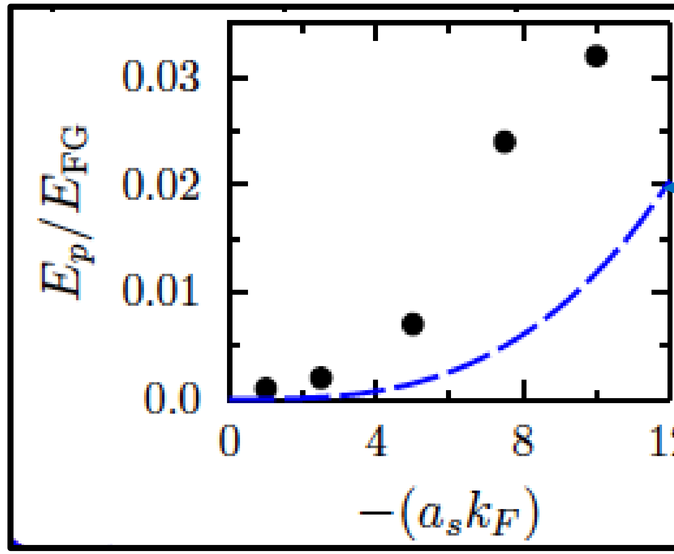
Including the p-wave ?

Lacroix, Boulet, Grasso, Yang, PRC 95 (2017)



Remember $a_p \ll r_e \ll a_s$

$$a_p^3 = 0.2 - 0.6 \text{ fm}^3$$



$a_p^3 = 0.2$

(AV4 interaction)

no p-wave

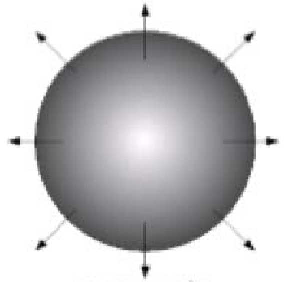
With p-wave (LO only)

$$(\nu + 1)\frac{1}{3\pi}(k_F a_p)^3$$

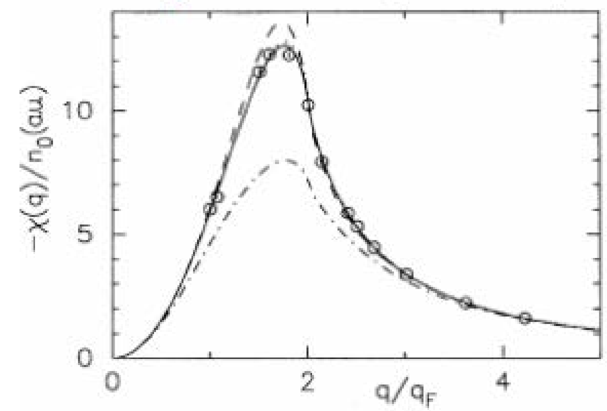
From static to dynamic

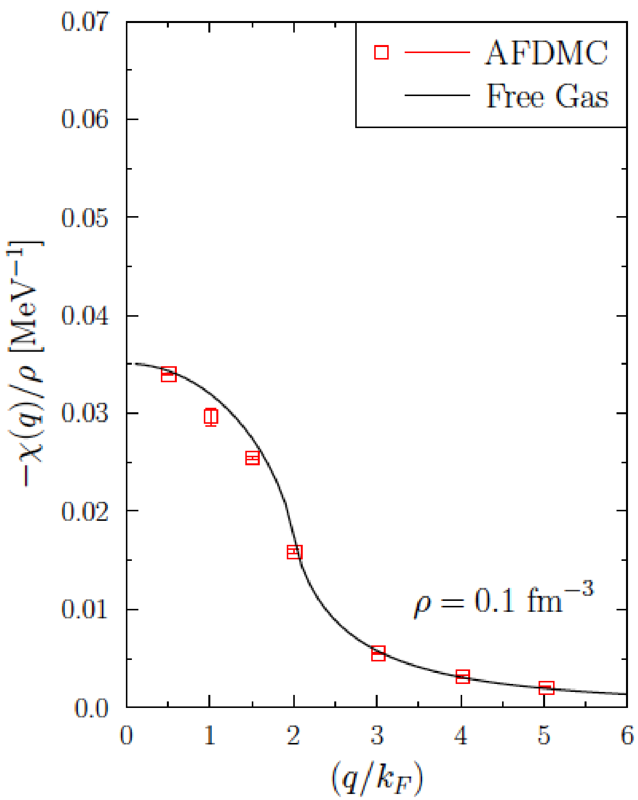
Static response in cold atoms and neutron matter

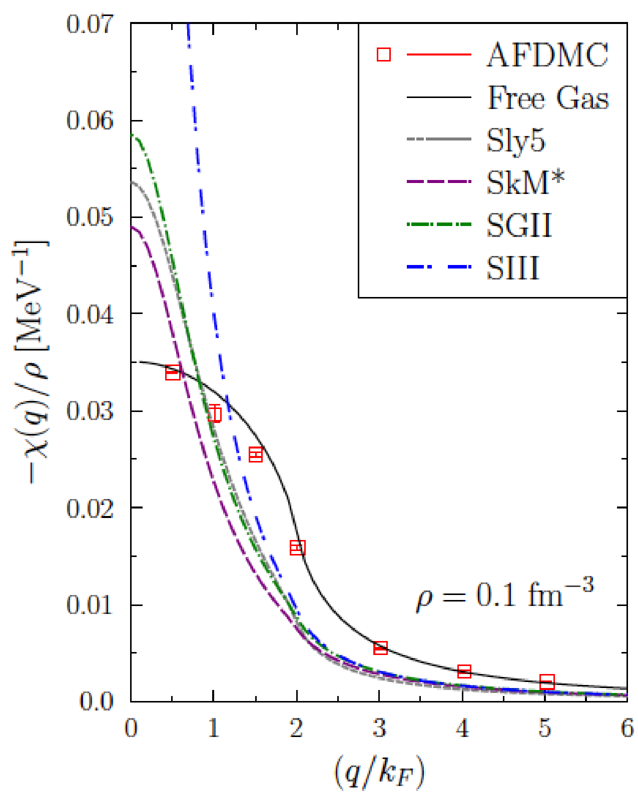
Boulet, DL, Phys. Rev. C 97 (2018)



For comparison: electron gas respon
[Moroni *et al.*, PRL **75** (1995)]







Empirical functional
Do not reproduce the neutron
Matter static response

This gives a strong constraint
On the functional

[Buraczynski and Gezerlis, PRL **116** (2016)]

Static response in cold atoms and neutron matter

Boulet, DL, Phys. Rev. C 97 (2018)

$$E = \int d\mathbf{r} \Big(\underbrace{\mathcal{K}[\rho(\mathbf{r})]}_{kinetic} + \underbrace{\mathcal{V}[\rho(\mathbf{r})]}_{interaction} \Big)$$

External field

$$\hat{V}_{\text{ext}} = \sum_j \phi(q,\omega) e^{i\mathbf{q}\cdot\mathbf{r_j} - i\omega t}$$

Assuming $m^* = m$

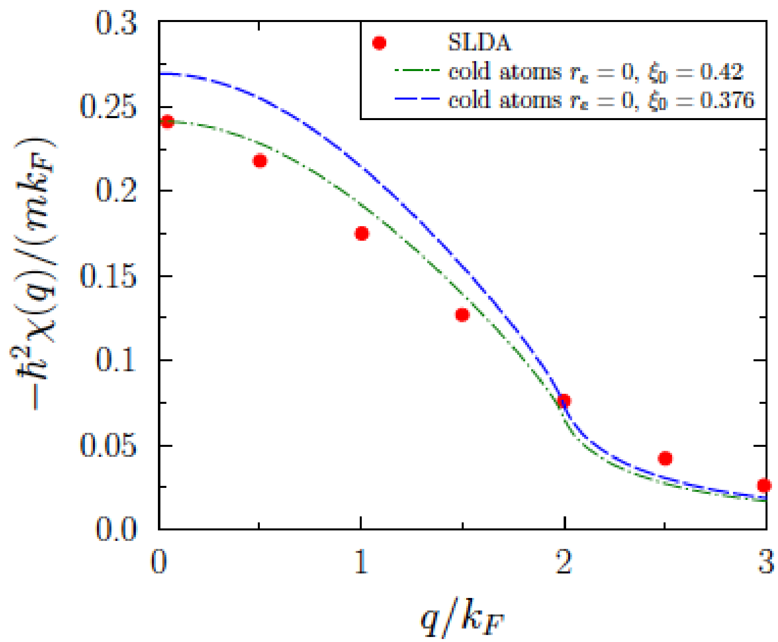
Comparison with Superfluid LDA (Bulgac et al) in cold atoms

Response function χ

$$\rho(\mathbf{r}) \equiv \rho \to \rho + \delta\rho$$

$$\delta\rho = -\chi(q,\omega)\phi(q,\omega)$$

$$\chi = \chi_0 \left[1 - \frac{\delta^2 \mathcal{V}}{\delta\rho^2} \chi_0 \right]^{-1}$$

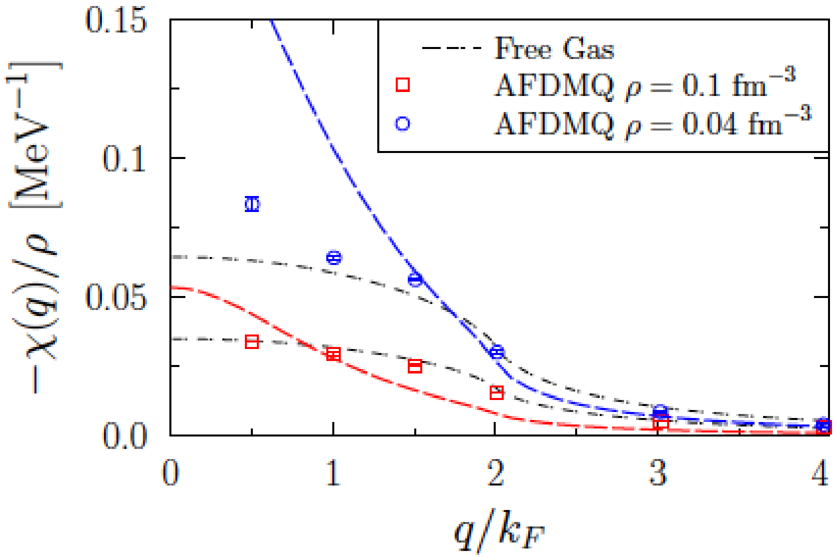


SLDA: [Forbes and Sharma, PRA **90** (2014)]

Static response in cold atoms and neutron matter

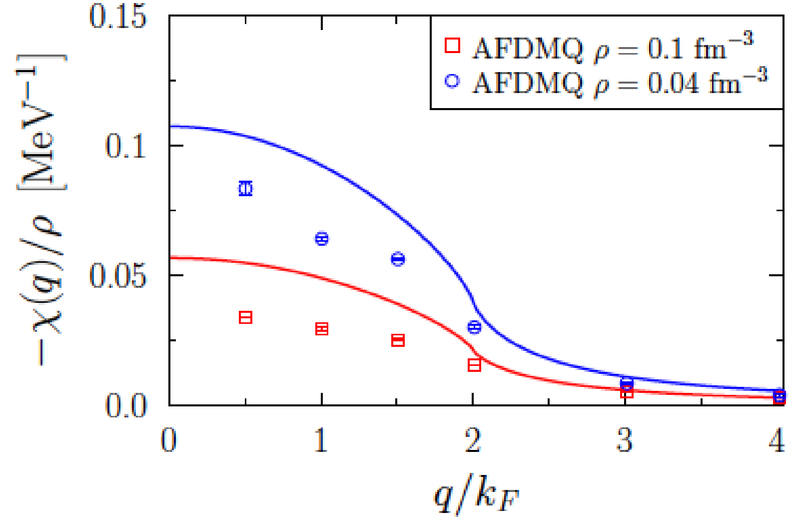
Boulet, DL, Phys. Rev. C 97 (2018)

Empirical functional (Sly5)

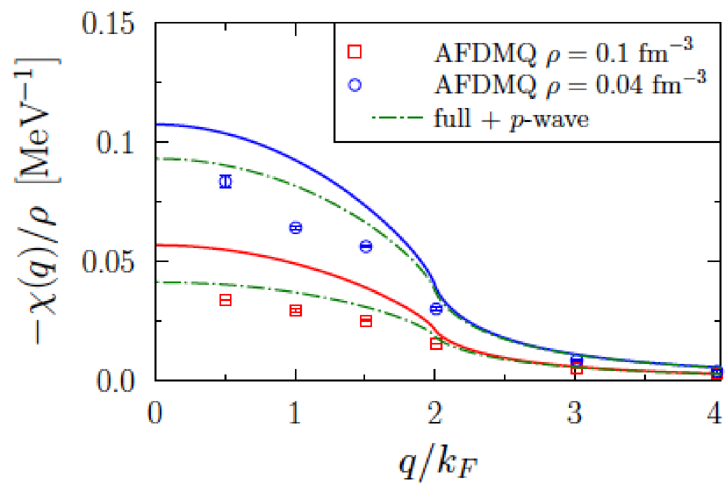


[Buraczynski and Gezerlis, PRL **116** (2016)]

Non-empirical functional

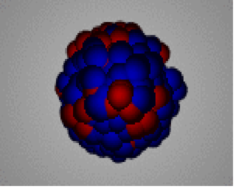


Non-empirical functional + *p*-wave



Adding *p*-wave
(leading order term only)

$$\frac{E_p}{E_{FG}} = \frac{1}{\pi}(a_p k_F)^3$$



Dynamical response in cold atoms and neutron matter

Boulet, DL, Phys. Rev. C 97 (2018)

Anisoptropic trap

$$U(\mathbf{r}) = \frac{m\omega_0^2}{2}\left(x^2 + y^2 + \lambda^2 z^2\right)$$

Hypothesis

➡ Hydrodynamical regime

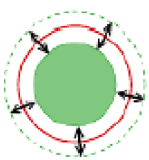
$$\nabla^2 P = -\frac{1}{m}\nabla \cdot [\rho \nabla U]$$

➡ Polytropic equation of state

[Heiselberg, PRL **93** (2004)]

$$P \propto \rho^\Gamma \quad \text{with} \quad \Gamma = \kappa P$$

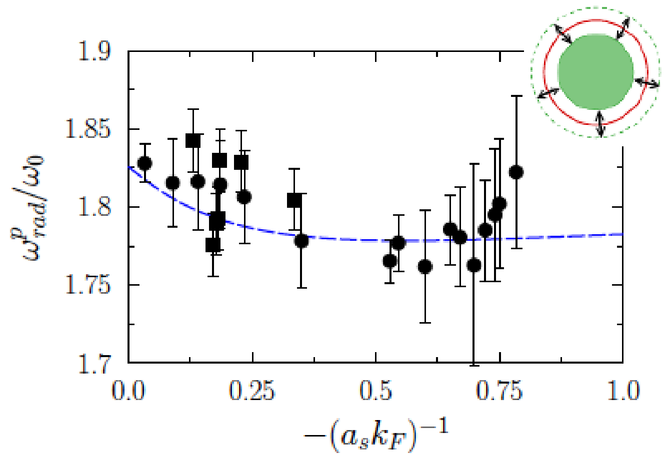
Solution of cigar-shaped / prolate ($\lambda \ll 1$):

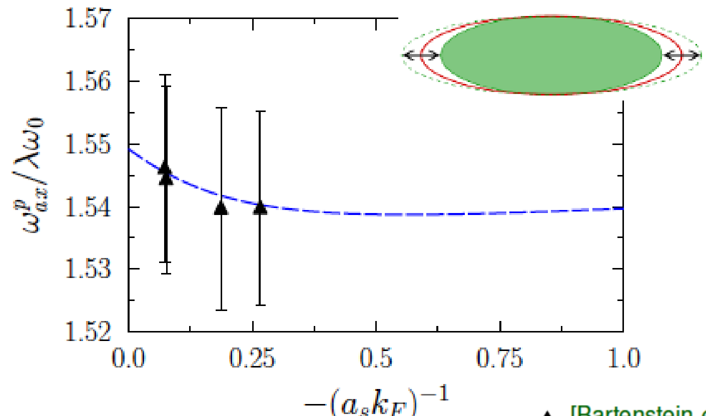


$$\frac{\omega_{ax}^p}{\lambda\omega_0} = \sqrt{3 - \Gamma^{-1}}$$

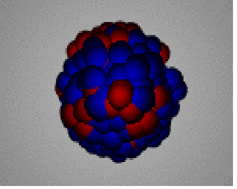
$$\frac{\omega_{rad}^p}{\omega_0} = \sqrt{2\Gamma}$$

Cold atoms results





- ▲ [Bartenstein *et al.*, PRL **92** (2004)]
- ● [Kinast, PRA **70** (2004)]
- ■ [Kinast, PRL **92** (2004)]



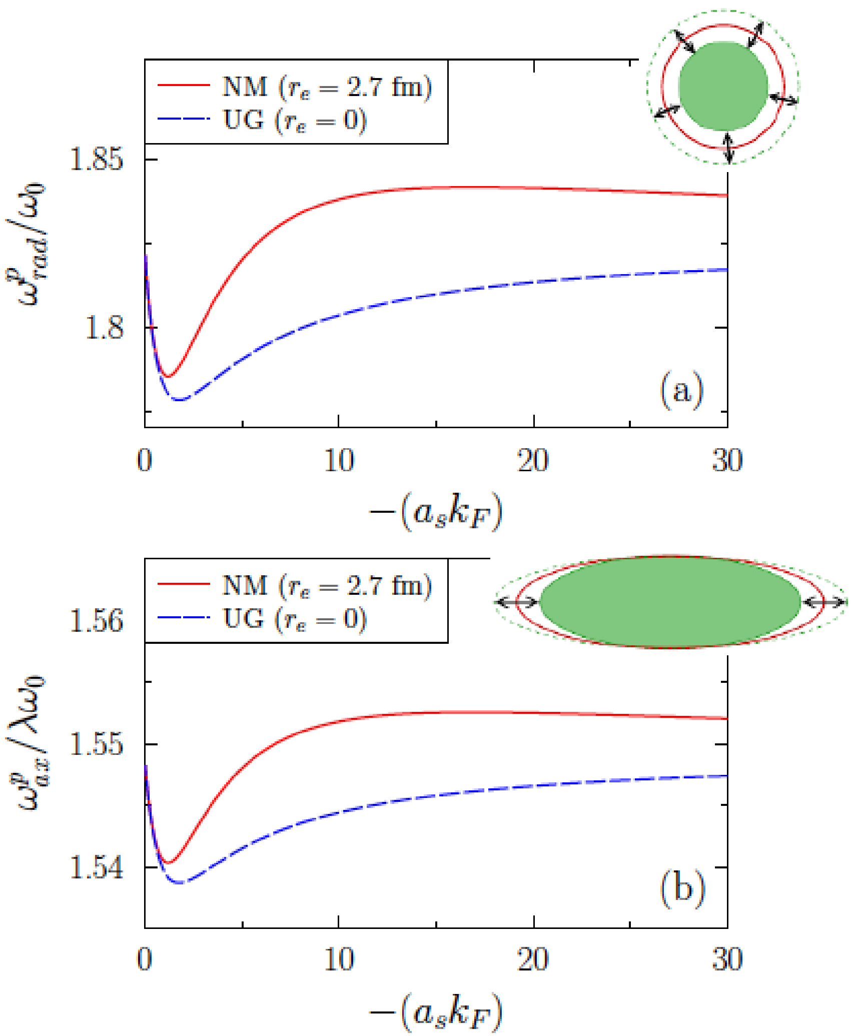
Dynamical response in cold atoms and neutron matter

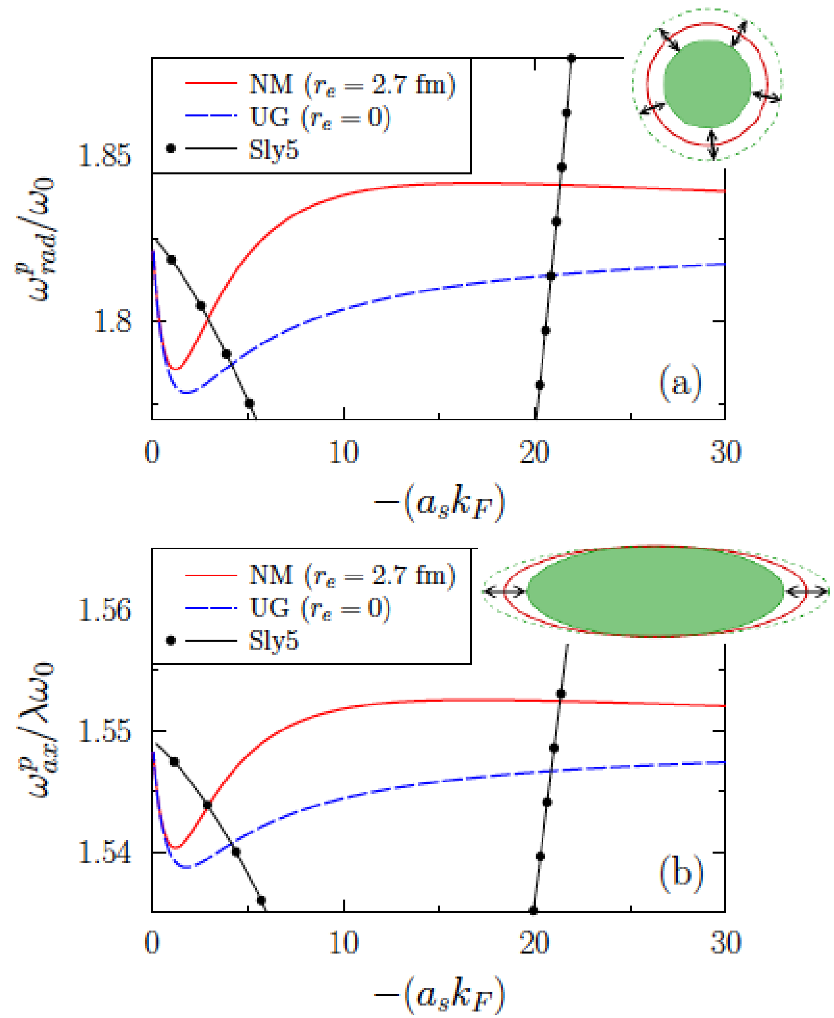
Neutron matter

Anisoptropic trap

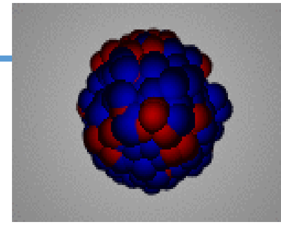
Boulet, DL, Phys. Rev. C 97 (2018)

$$U(\mathbf{r}) = \frac{m\omega_0^2}{2}\left(x^2 + y^2 + \lambda^2 z^2\right)$$



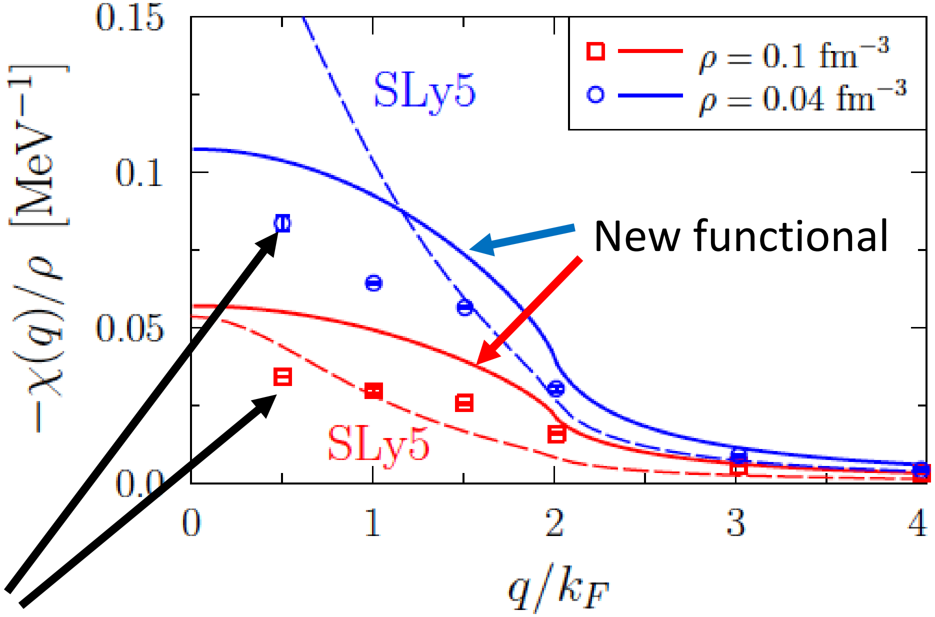


Superfluidity and effective mass

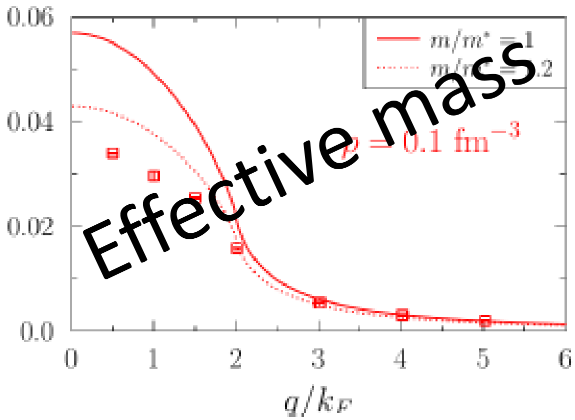


Boulet, DL, Phys. Rev. C 97 (2018)

Static response of neutron matter

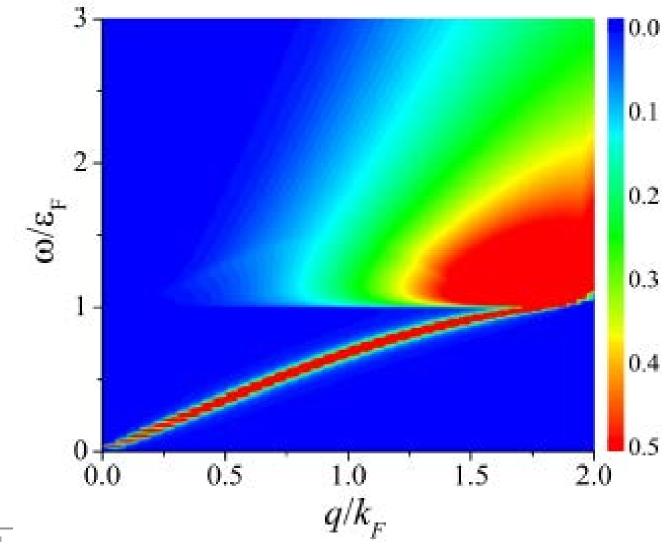


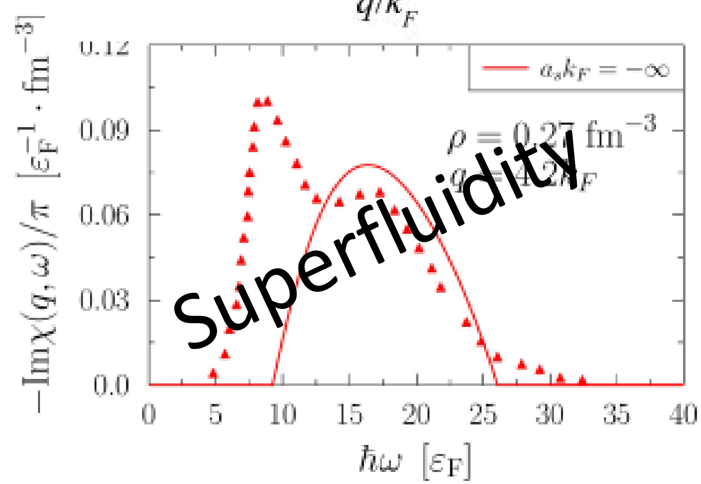
QMC: Buraczynski,Gezerlis, PRL 116 (2016)]



Dynamical response (atomic gases)

[S. Hoinka *et al.*, PRL **109**, 050403 (2012)]





[P. Zou *et al.*, New J. Phys. **18**, 113044 (2016)]

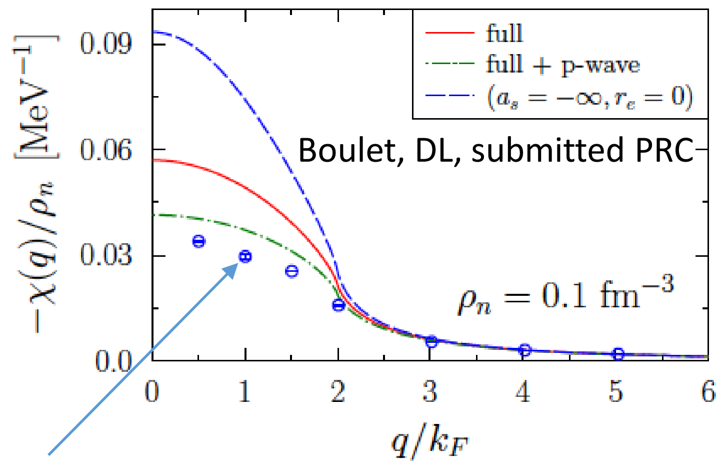
Conclusion and ongoing work

Conclusions

- We propose a new way design the nuclear (cold atom) DFT to parameters of the interaction
  - Low energy constants becomes the only "non-freely" adjustable parameters
  - Validity $\rho < 0.01 \text{ fm}^{-3}$
- The new DFT reproduces ab-initio results in cold atoms and neutron matter
- Transition from s-wave driven (low density) to unitary gas driven (Bertsch parameter) regime
- Link between non-empirical and empirical DFT

Applications and on-going work

Static and dynamical response in neutron matter

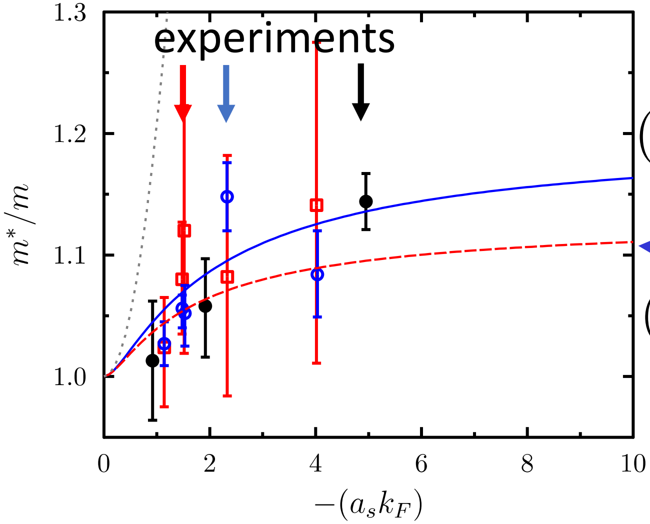


AFDMC: Buraczynski,Gezerlis, PRL 116 (2016)]

Some Opportunities:
cross-fertilization between atomic and nuclear physics

Some Fermi liquid properties are scarcely known: effective mass, …

Effective mass in cold atoms



$(m^*/m)_{\rm UG} = 1.2$

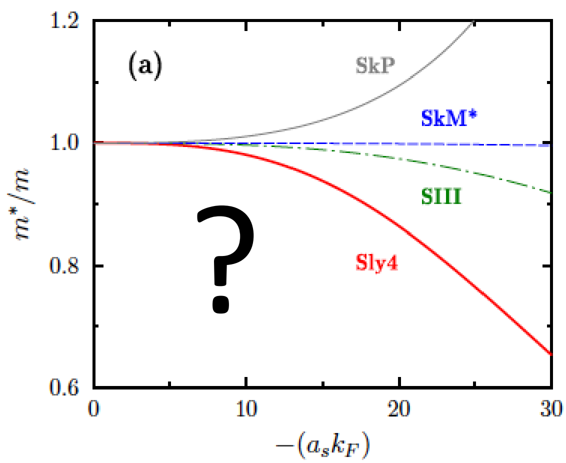
$(m^*/m)_{\rm UG} = 1.13$

Proposed functional:
(unpublished and eventually never published)

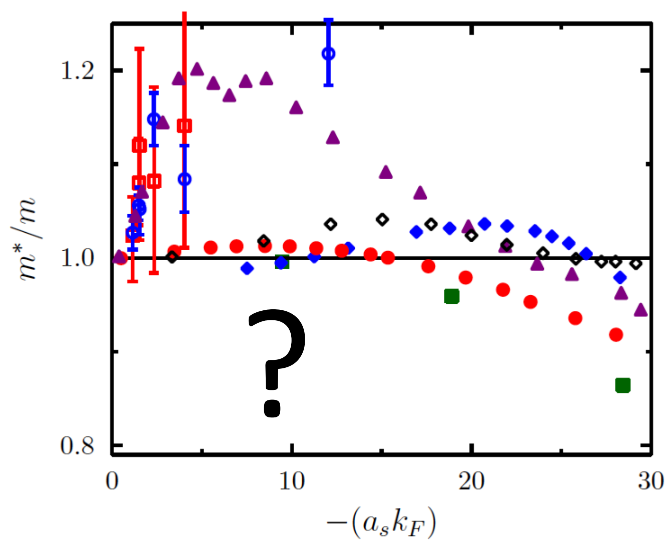
$$\left(\frac{m}{m^*}\right) = 1 - \frac{M_0}{\left[1-(a_s k_F)^{-1} M_1\right]^2}$$

Effective mass in neutron matter

EDF

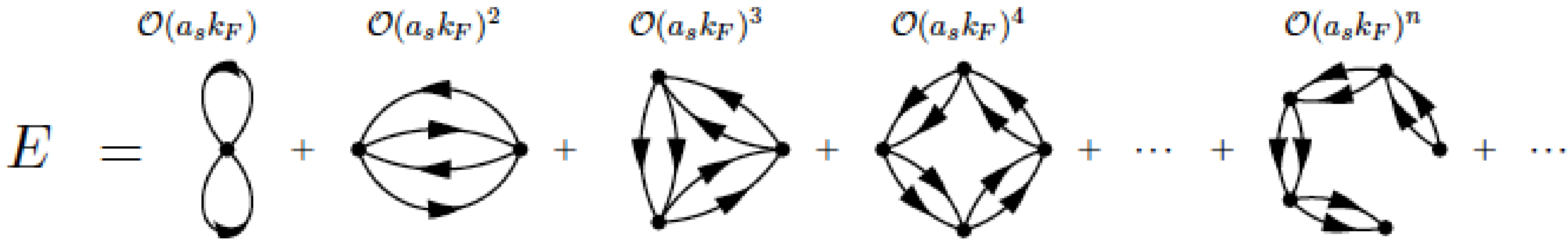


Ab-initio



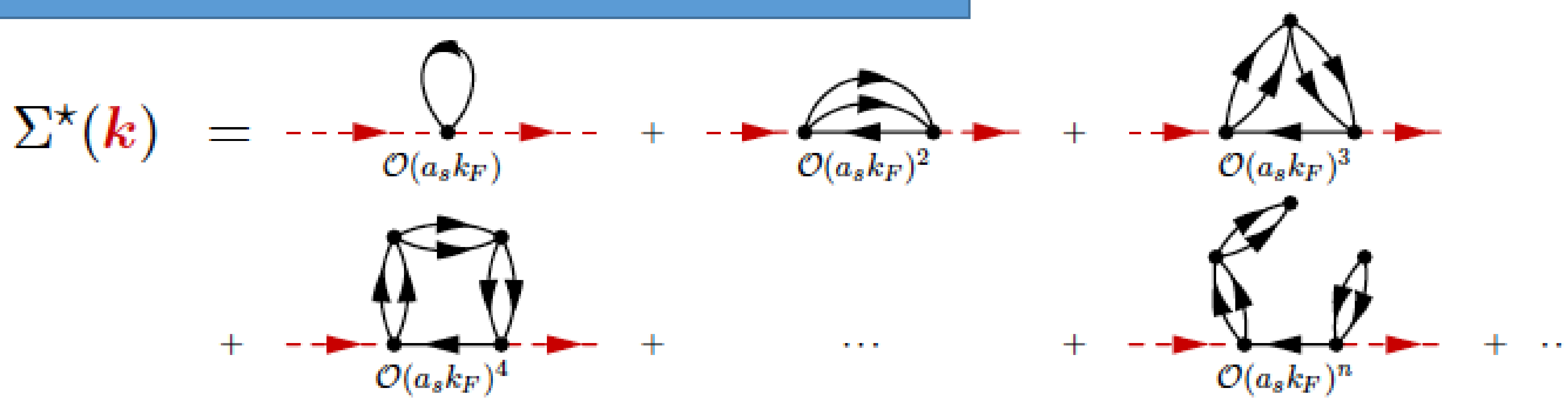
Some follow up: Fermi liquid or non-Fermi liquid theory of strongly interacting systems

Resummation technique for the energy



$$= \frac{g(g-1)}{2}\left(\frac{4\pi a_s}{m}\right)\iint \frac{d^3P}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\frac{\theta_{\boldsymbol{k}}^-}{1-(a_s k_F)F(P,k)}$$

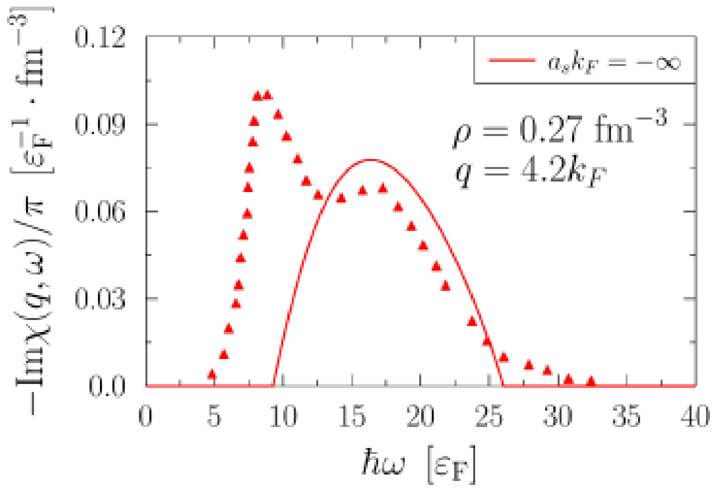
Equivalent for the (on-shell) self-energy: more difficult



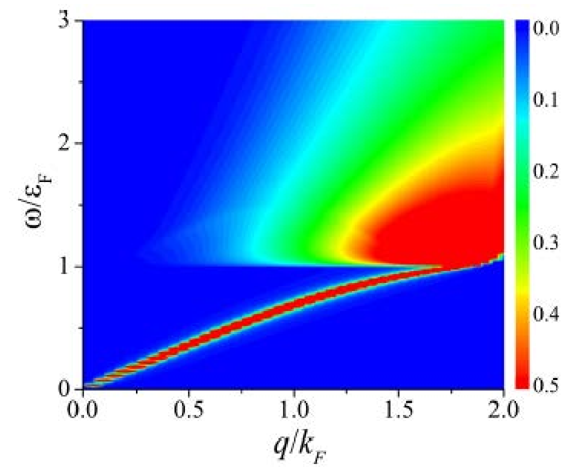
(courtesy A. Boulet)

Pairing effect should be included

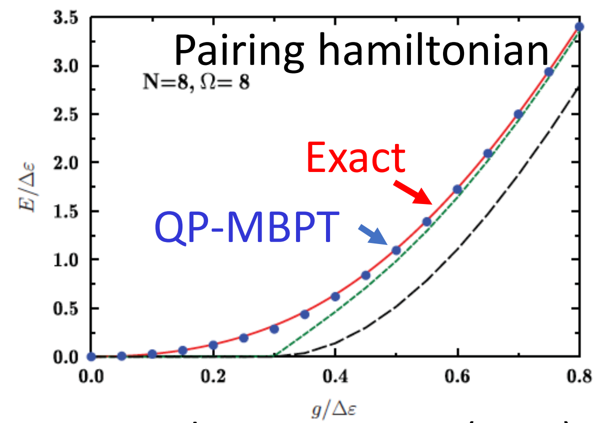
- Odd-even effects in nuclei
- Dynamical response in neutron matter and cold atoms, vortices, …



[P. Zou *et al.*, New J. Phys. **18**, 113044 (2016)]



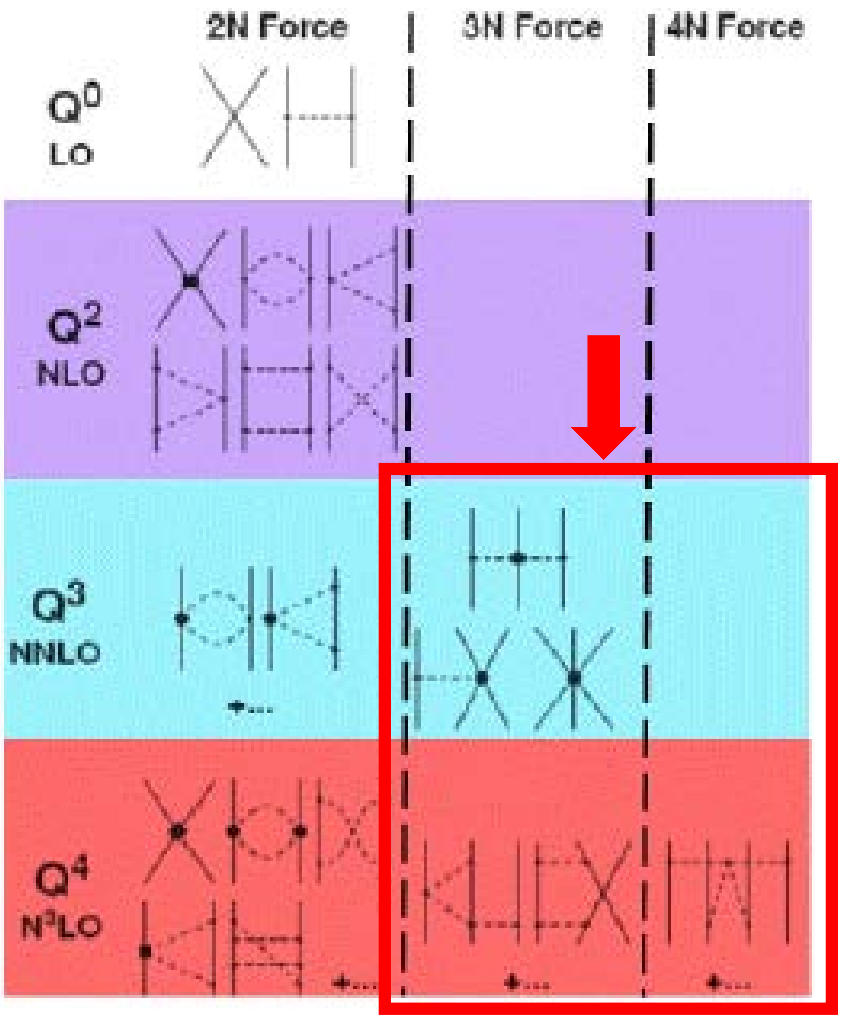
- Towards a *resummation technique with pairing* based on MBPT of quasi-particles? Green-Gorkov, …



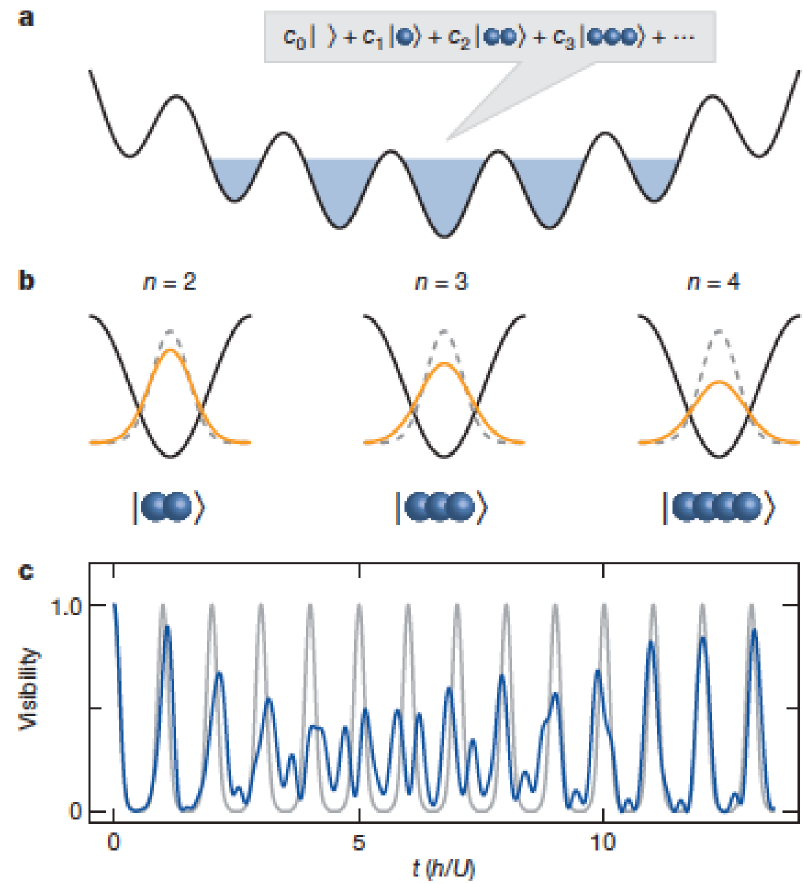
Lacroix, Gambacurta, PRC86 (2012)
Ripoche et al, PRC95 (2017)

Many-body physics with multi-body interactions

Nucleon-nucleon interaction from chiral EFT



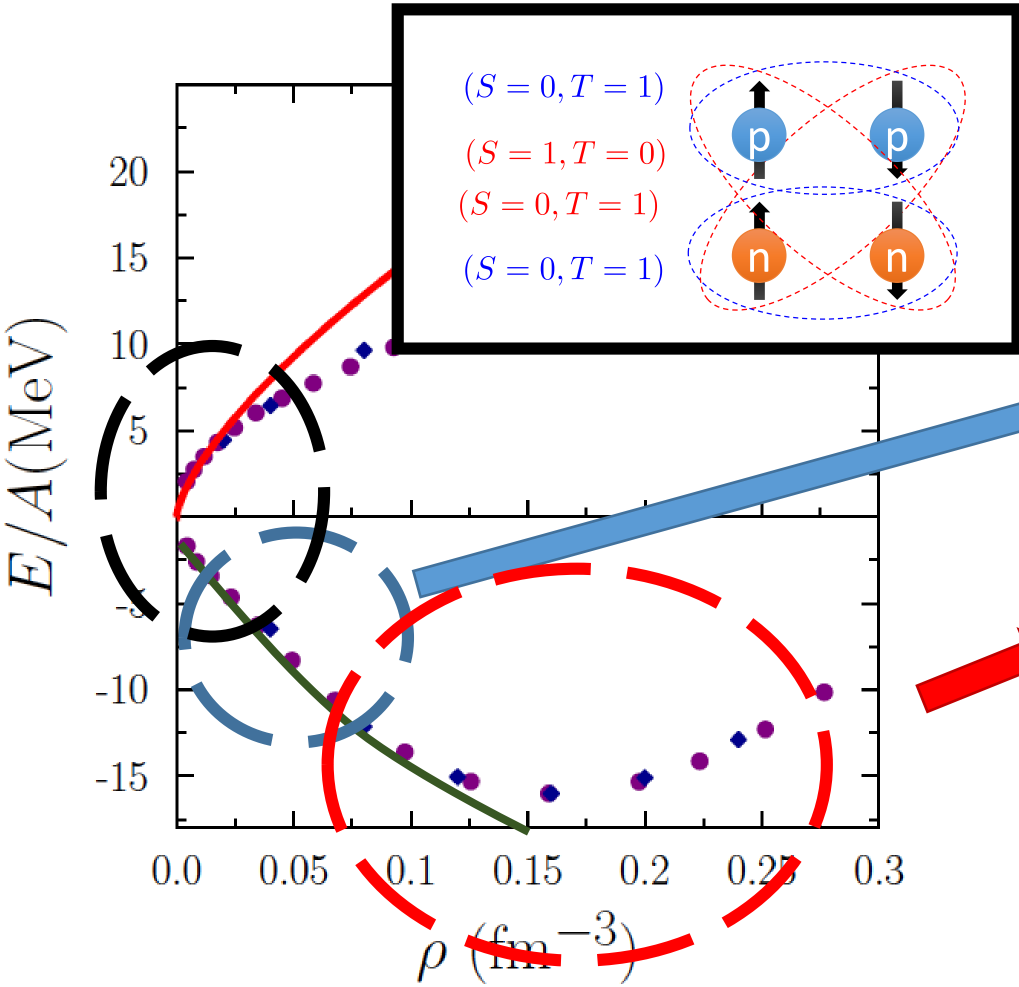
Systems with tunable multi-body interaction
Can now be formed on a lattice



S. Will et al, Nature (2010)

Some opportunities

Some common interest



Here, existence of bound dimers, Trimer, …

Do we expect the same functional?

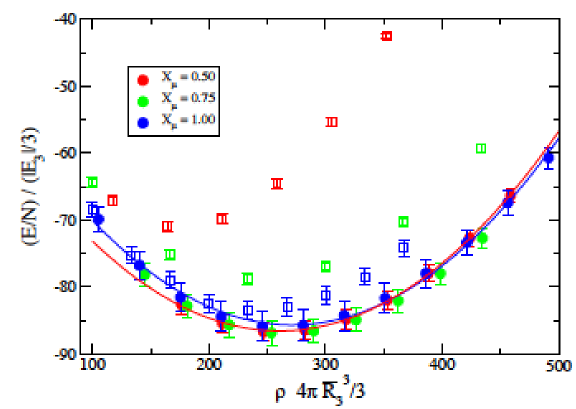
$$\frac{E}{N}+\frac{\varepsilon_B}{2}=\frac{3}{5}E_F\left[\frac{5(k_F a')}{18\pi}+\frac{64(k_F a')^{5/2}}{27\sqrt{6\pi^5}}+\cdots\right]$$

a' dimer-dimer scattering length

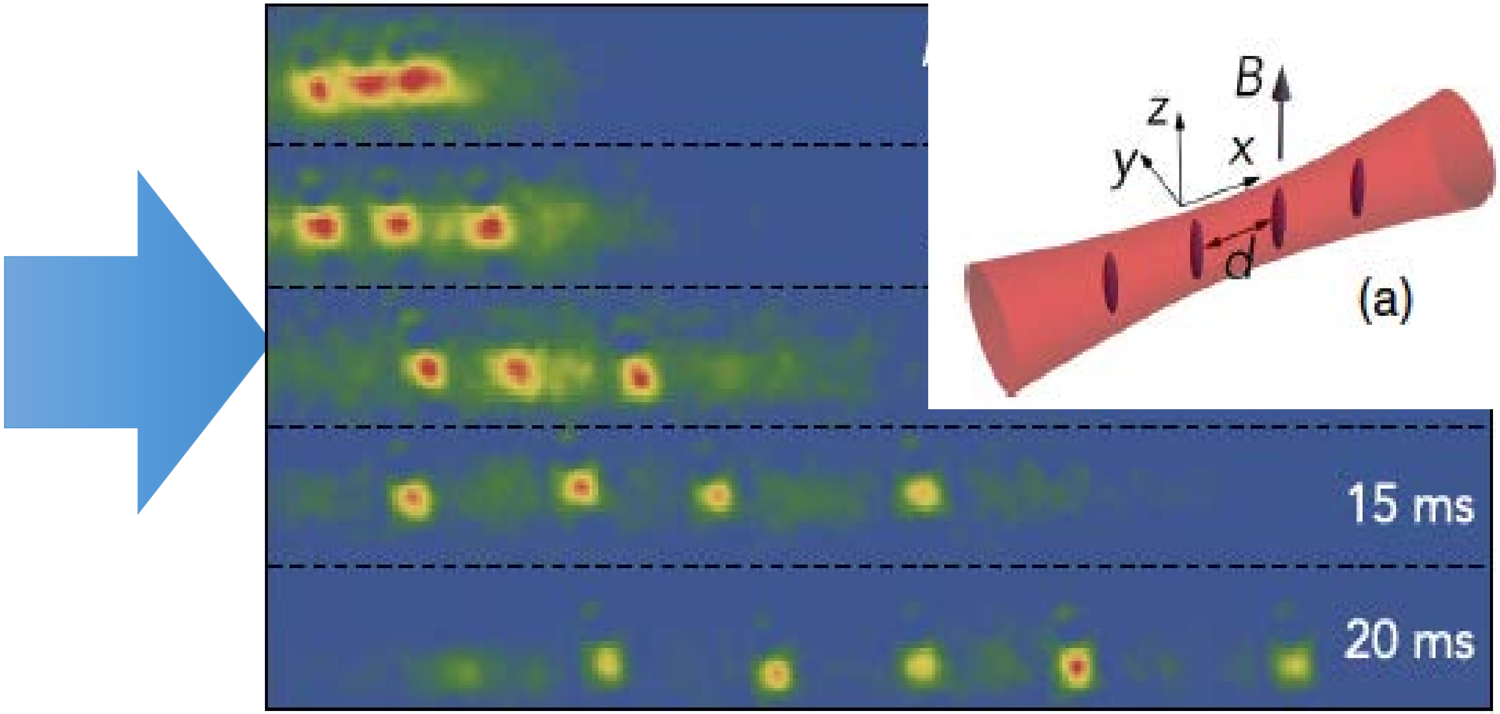
Incompressible systems with 3-nucleons forces

?

J. Carlson et al, PRL (2018)



Quantum droplet physics



Ferrier-Barbut, PRL 116, (2016)

Additional discussion

Difficulty in nuclear systems

Yang, Grasso, Lacroix PRC94 (2016)

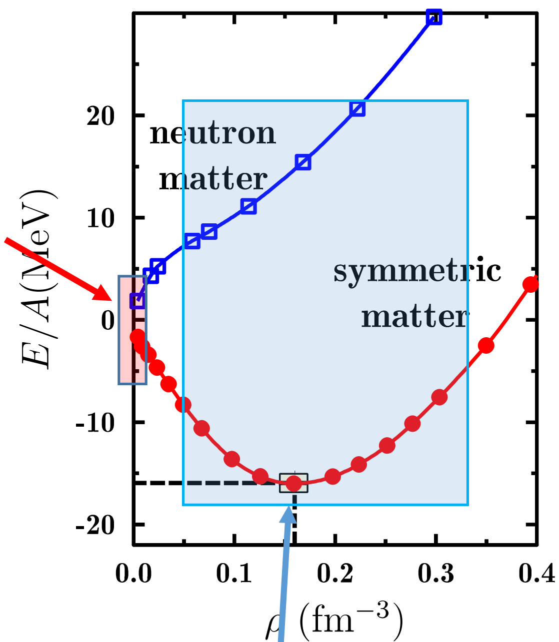
Skyrme functional

$$
\begin{aligned}
v(\mathbf{r}_1 - \mathbf{r}_2) &= t_0\,(1 + x_0 \hat{P}_\sigma)\,\delta(\mathbf{r}) \\
&+ \tfrac{1}{2} t_1\,(1 + x_1 \hat{P}_\sigma)\left[\mathbf{P}'^2\,\delta(\mathbf{r}) + \delta(\mathbf{r})\,\mathbf{P}^2\right] \\
&+ t_2\,(1 + x_2 \hat{P}_\sigma)\,\mathbf{P}' \cdot \delta(\mathbf{r})\,\mathbf{P}
\end{aligned}
$$

MBPT + expansion in LEC is valid here

is very close to the EFT starting point

$$
\langle \mathbf{k} | V_{\mathrm{eft}} | \mathbf{k}' \rangle = C_0 + \frac{1}{2} C_2 (\mathbf{k}^2 + \mathbf{k}'^2) + C_2' \, \mathbf{k} \cdot \mathbf{k}' + \cdots
$$



But Skyrme works because it has been adjusted here !!!

Additional remarks on traditional Skyrme

Lacroix, Boulet, Yang, Grasso, PRC94 (2016)

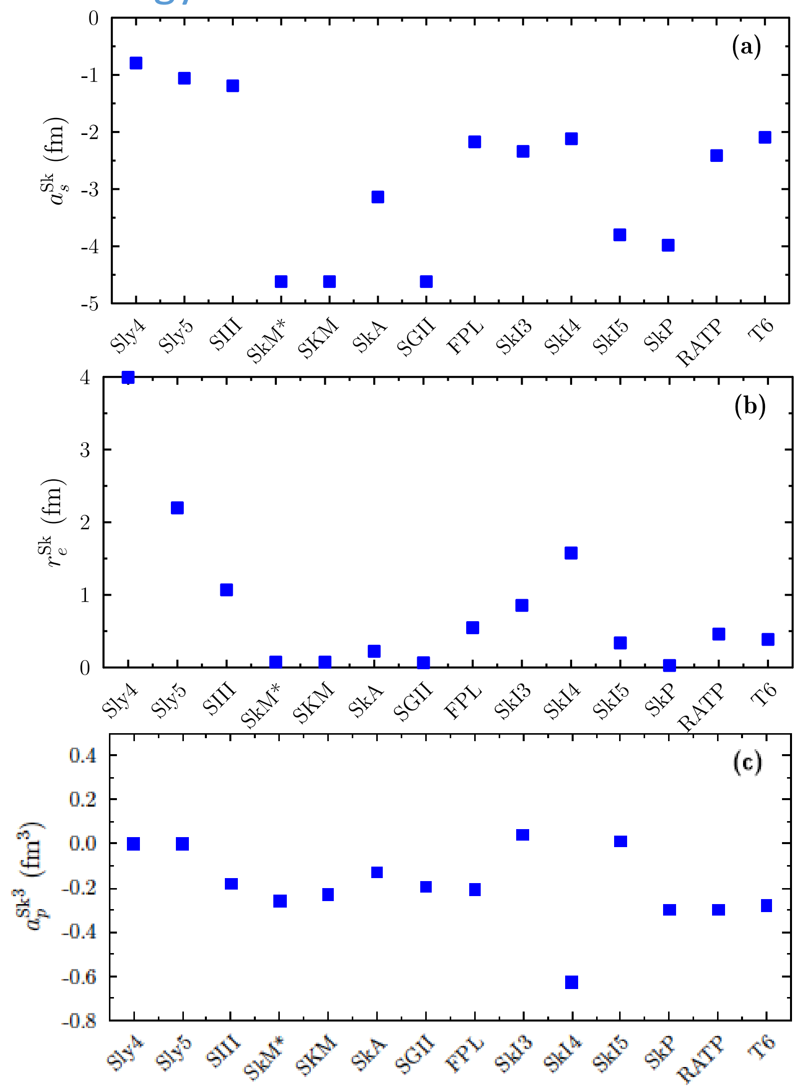
Due to the analogy, one can define equivalent low energy constant

$$C_0 = t_0(1 - x_0) = \frac{4\pi\hbar^2}{m} a_s,$$

$$C_2 = t_1(1 - x_1) = \frac{2\pi\hbar^2}{m} r_e a_s^2,$$

$$C_2' = t_2(1 + x_2) = \frac{4\pi\hbar^2}{m} a_p^3.$$

See discussion in Furnstahl, EFT for DFT (2007)



Very far from $a_s = -18.9$ fm

Can we make contact with Skyrme like empirical functional ?

Lacroix, Boulet, Grasso, Yang, PRC 95 (2017)

Starting point

$$\frac{E}{E_{\rm FG}} = 1 - \frac{U_0}{1-(a_s k_F)^{-1}U_1} + \frac{R_0(r_e k_F)}{[1-R_1(a_s k_F)^{-1}][1-R_1(a_s k_F)^{-1}+R_2(r_e k_F)]}$$

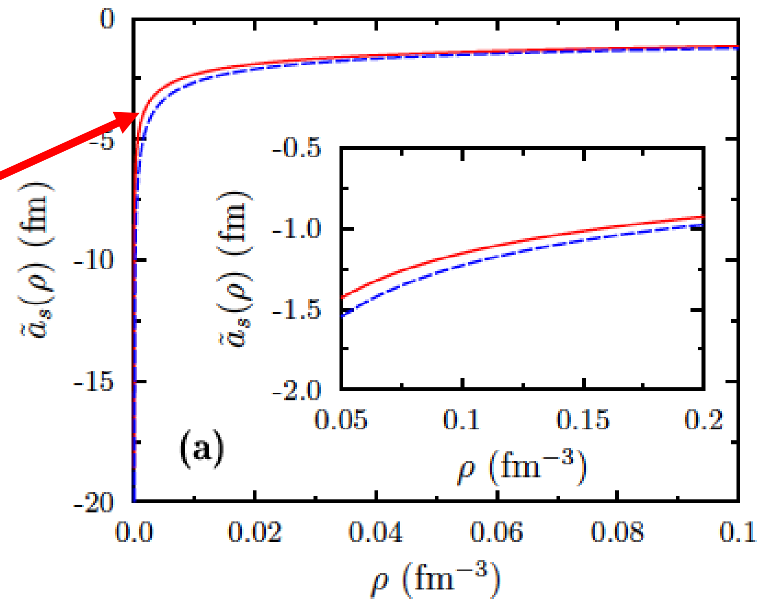
Rewrite it as

$$\frac{E}{E_{\rm FG}} = 1 + \frac{k_F^3}{4\pi^2 E_{\rm FG}}\left\{\frac{\widetilde{C}_0(k_F)}{3} + \frac{k_F^2}{10}[(\nu-1)\widetilde{C}_2(k_F) + (\nu+1)\widetilde{C}_2'(k_F)]\right\}$$

Define density dependent scattering length and range

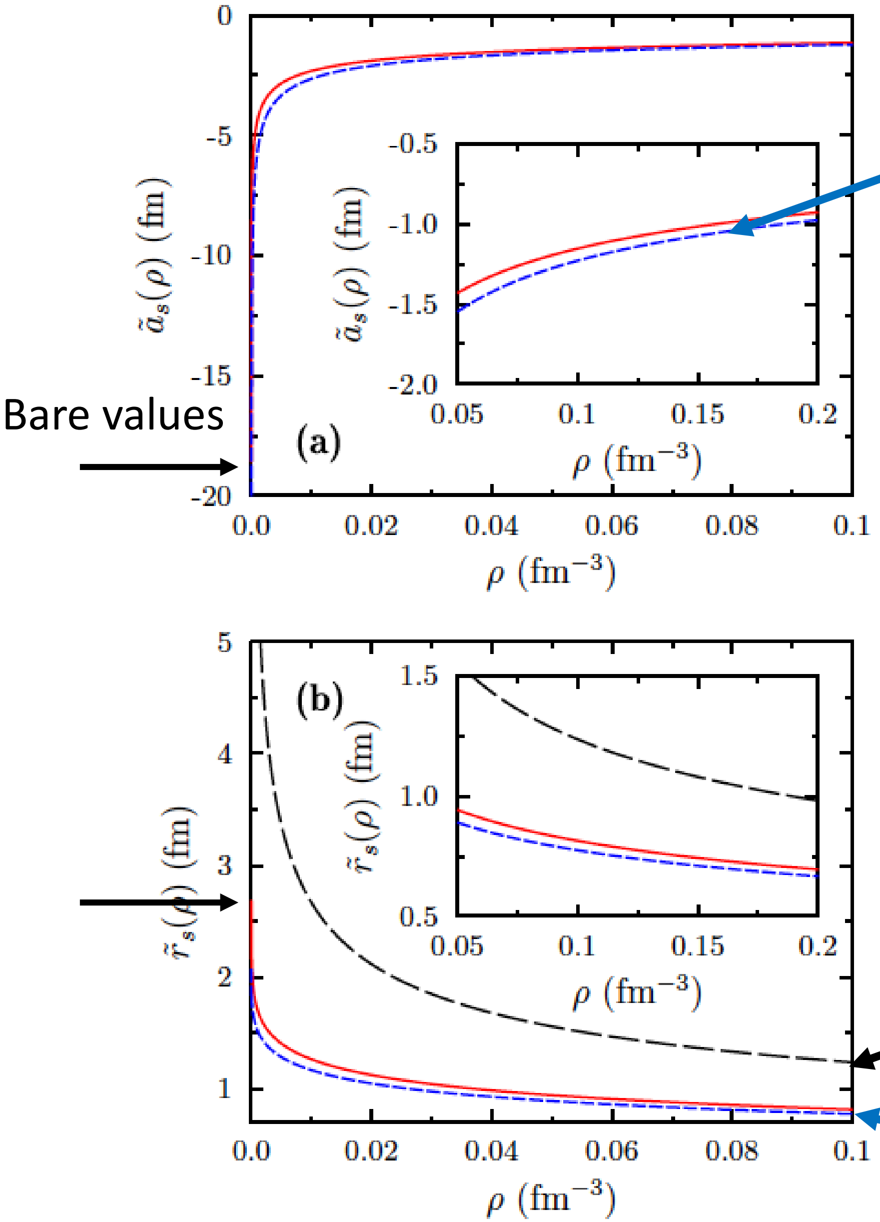
$$\tilde{C}_0(k_F) = \frac{4\pi\hbar^2}{m}\tilde{a}_s(k_F)$$

$$\tilde{C}_2(k_F) = \frac{2\pi\hbar^2}{m}\tilde{r}_e(k_F)\tilde{a}_s^2(k_F)$$



Can we make contact with empirical functional ?

Lacroix, Boulet, Grasso, Yang, PRC 95 (2017)



$$\tilde{a}_s(k_F) \simeq -\frac{9\pi}{10k_F}(1-\xi_0)$$

- Fast evolution at low density followed by a slower evolution around saturation density
- Around normal density, a_s dominated by the unitary constraint
- At normal density, a_s is washed out
- Finite r_s plays a non-negligible role

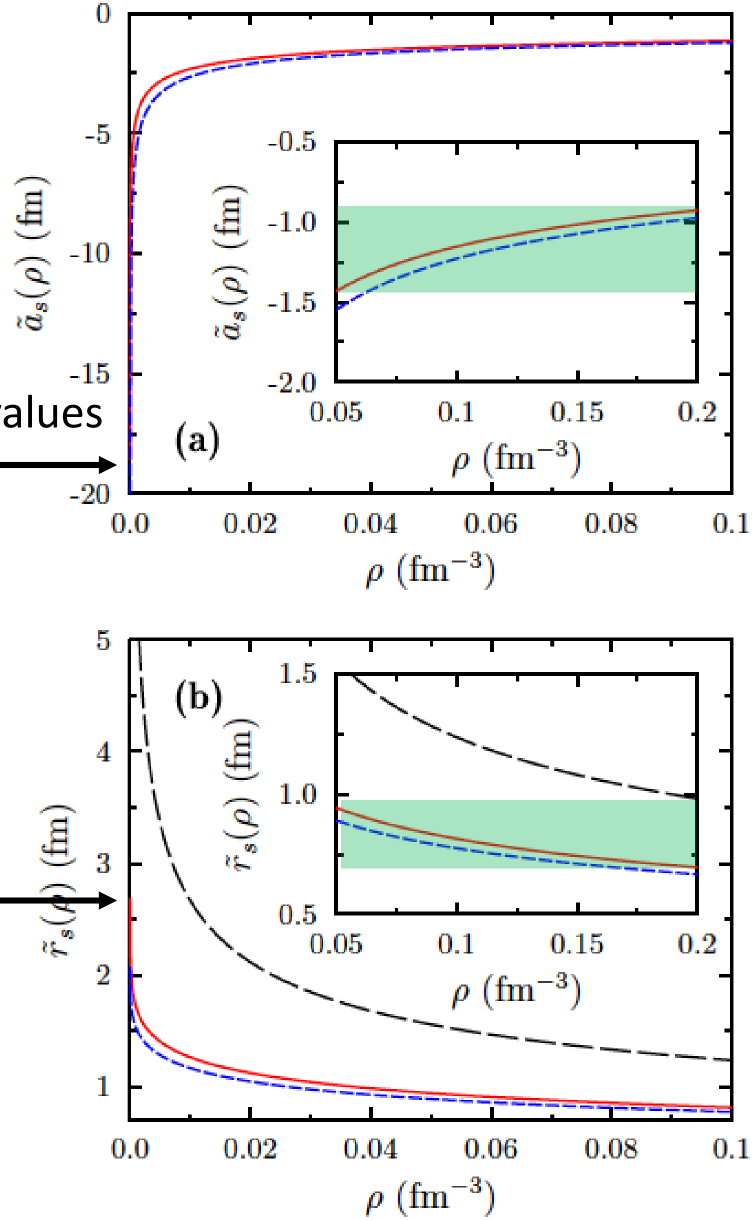
$$\tilde{r}_e(k_F) \simeq \frac{200}{27(\nu-1)} \frac{\eta_e^2}{(1-\xi_0)^2 \delta_e k_F}$$

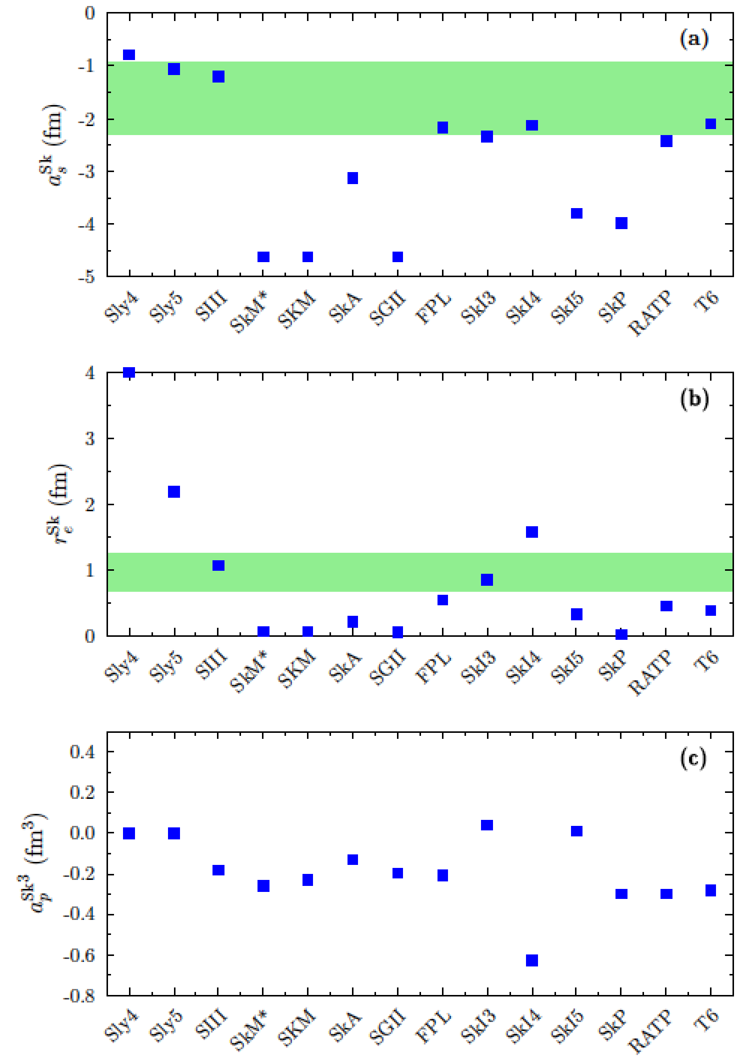
$$\tilde{r}_e(k_F) \simeq \frac{200}{27(\nu-1)} \frac{\eta_e}{(1-\xi_0)^2} \frac{r_e}{[1+\delta_e(r_e k_F)/\eta_e]}$$

Can we make contact with empirical functional ?

Lacroix, Boulet, Grasso, Yang, PRC 95 (2017)

Bare values





Gives an empirical explanation of the Skyrme success

Backup

From cold atom to neutron matter: inclusion of effective range

Resummation of effective range effect in the HF theory

Assume a gaussian interaction $$v(\mathbf{r}_1,\mathbf{r}_2) = \left\{v_0 + v_\sigma P_\sigma\right\}\frac{e^{-(r^2/\mu^2)}}{(\mu\sqrt{\pi})^3}$$

HF energy $$\frac{E_G^{(1)}}{N} = \frac{\rho}{2}\left[A - BF\left(\mu k_F\right)\right]$$

with

$$F(x) = \frac{12}{x^6}(1-\mathrm{e}^{-x^2}) + \frac{6}{x^4}(\mathrm{e}^{-x^2}-3) + \frac{6\sqrt{\pi}}{x^3}\mathrm{Erf}(x)$$

Low density expansion is recovered if:

$$A = -\frac{2\hbar^2\pi}{\nu m\mu^2}\left[(\nu-1)r_e a_s^2 - 2(\nu+1)a_p^3\right]$$

$$B = +\frac{2\hbar^2\pi}{\nu m\mu^2}\left[(\nu-1)r_e a_s^2 + 2(\nu+1)a_p^3\right]$$

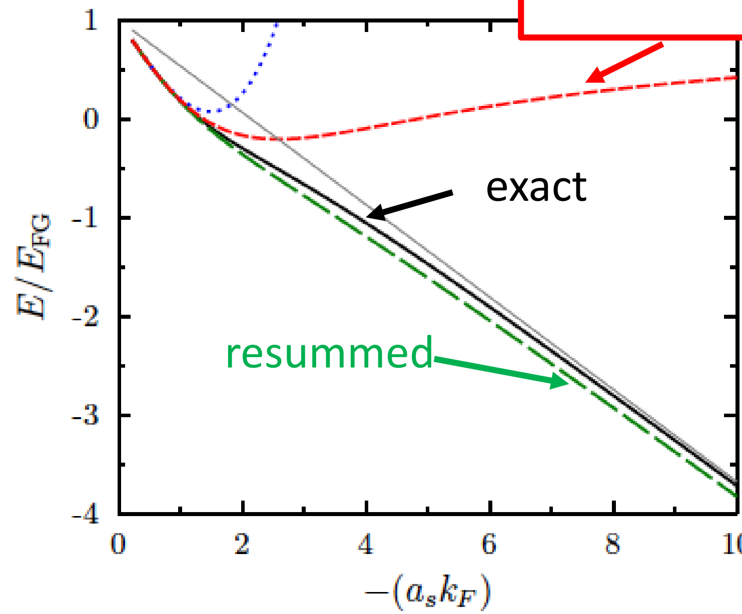
$$\mu^2 = -(r_e a_s)$$

Approximate resummation of r_e effects

$$\frac{E_G^{(1)}}{N} \simeq \frac{\rho}{2}\left[A - B\left\{1 - \frac{3}{10}(\mu k_F)^2\right\}\right]$$

$$\simeq \frac{\rho}{2}\left[A - \frac{B}{1+\frac{3}{10}(\mu k_F)^2}\right].$$

$$\frac{E_G^{(1)}}{N} \simeq \frac{\rho_0}{2}\frac{[A-B]}{1-\frac{3B}{10[A-B]}(\mu k_F)^2}$$



resummation With finite limit for $a_s k_F \to +\infty$