Pairing in dilute neutron matter

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Outline

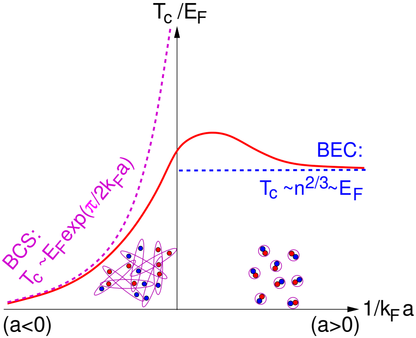
- Introduction
  - BCS-BEC crossover in cold atoms
  - Neutron matter in the crossover regime
  - Relevance for astrophysics (neutron stars)
  - Situation in the literature
- Screening (or antiscreening) corrections
- Low-density limit and the Gor'kov-Melik-Barkhudarov (GMB) result
- Nozières-Schmitt-Rink (NSR) correction to T_c
- Summary and outlook

References

1. S. Ramanan and M. U., Phys. Rev. C 88, 054315 (2013) (NSR only)
2. S. Ramanan and M. U., arXiv:1804.04332 (2018) (screening + NSR)

Introduction: BCS-BEC crossover in ultracold atoms

- consider Fermionic atoms, two spin states ($\uparrow$, $\downarrow$), contact interaction
- scattering length a can be tuned (Feshbach resonance)
- on resonance: unitary limit $a \to \infty$
- fermionic atoms $\leftrightarrow$ molecules
- at zero temperature: crossover from BCS superfluid (Cooper pairs) to BEC (molecules)



- Nozières-Schmitt-Rink (NSR) theory includes non-condensed pairs above T_c $\to$ interpolates between BCS and BEC limits
- at unitarity ($1/k_F a = 0$):

	BCS	NSR	exp.
T_c/E_F	0.5	0.22	0.17

- discrepancy NSR vs. exp. explained by screening [e.g. Pisani et al. PRB 97 (2018)]

BCS-BEC crossover in neutron matter?

- No *nn* bound state $\longrightarrow$ BEC side of crossover cannot be realized
- Unitary limit in cold atoms: $\boxed{1/|a| \ll k_F \ll 1/r}$ (r = interaction range)
- *nn* scattering in the $^1S_0$ channel: $a \approx -18$ fm, $r \approx 2.7$ fm

  $\longrightarrow$ one can realize $\boxed{1/|a| \lesssim k_F \lesssim 1/r}$
- Neutron matter with $k_F \sim 0.05 - 0.4$ fm^{-1} similar to unitary Fermi gas
- Crossover physics (e.g., NSR) important for $n \sim 10^{-5} - 0.002$ fm^{-3}
- Weak-coupling regime $k_F|a| < 1$ at densities below 10^{-5} fm^{-3} is only of academic interest
- At densities above 0.002 fm^{-3}, Δ/E_F gets smaller again because of finite range (momentum dependence) of the *nn* interaction (Δ = pairing gap)

Digression: Neutron stars

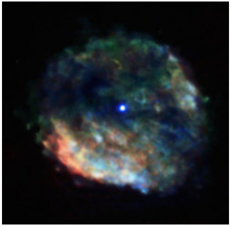
- Neutron star formed at the end of the "life" of an intermediate-mass star (supernova)
- $M \sim 1 - 2\ M_\odot$ in a radius of $R \sim 10 - 15$ km
  $\rightarrow$ average density $\sim 5 \times 10^{14}$ g/cm^3
  ($\sim 2\times$ nuclear matter saturation density)
- Cools down rapidly by neutrino emission within ~ 1 month: $T \lesssim 10^9$ K ~ 100 keV
- Internal structure of a neutron star:

  outer crust: Coulomb lattice of neutron rich nuclei in a degenerate electron gas

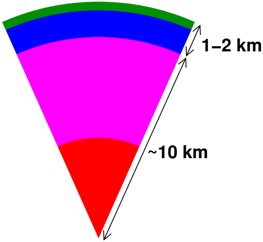
  inner crust: unbound neutrons form a neutron gas between the nuclei (clusters)

  outer core: homogeneous matter (n, p, e^-)

  inner core: new degrees of freedom: hyperons? quark matter?

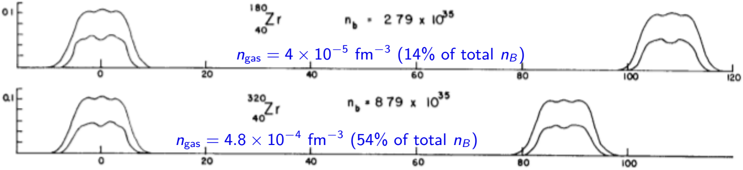


RCW103 [Chandra X-ray telescope]



Relevance of superfluidity in dilute neutron matter

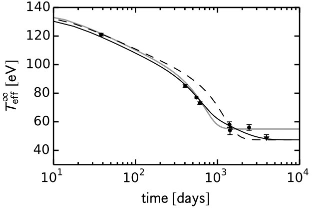
- Upper layers of the inner crust (close to neutron-drip density $\sim 2.5 \times 10^{-4}$ fm^{-3})
  [Negele & Vautherin, NPA 207 (1973); similar results by Baldo et al., PRC 76 (2007)]



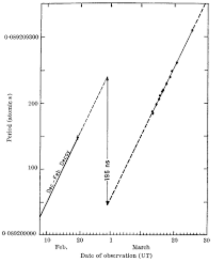
$\rightarrow$ In spite of its low density, the neutron gas is relevant because it occupies a much larger volume than the clusters

- Deeper in the crust: n_{gas} increases up to $\sim n_0/2 = 0.08$ fm^{-3}
- Examples for observable manifestations of superfluidity in the crust:

Cooling of accreting neutron stars
[Deibel et al., ApJ 839 (2017)]

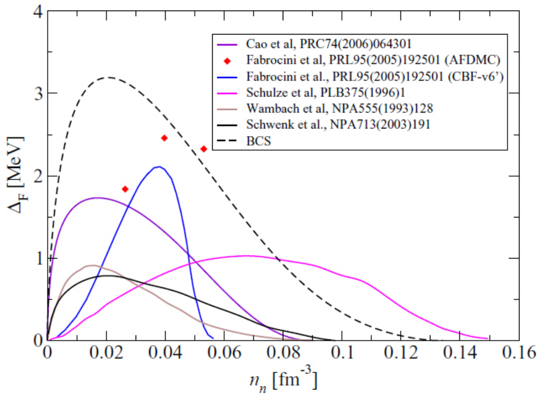


Pulsar glitches
[Radhakrishnan & Manchester, Nature 222 (1969)]



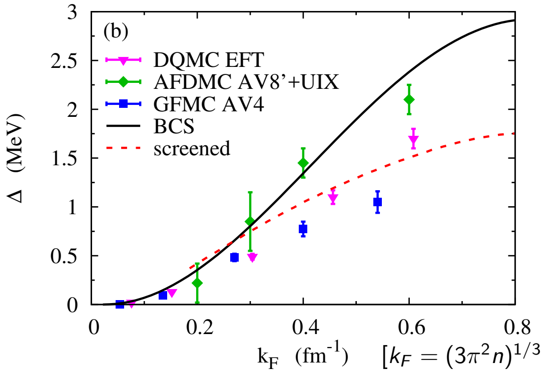
Pairing in neutron matter: existing literature

- Throughout this talk: consider <u>uniform</u> neutron matter
- Compilation of results [Chamel and Haensel, Liv. Rev. Relativity (2008)]



- Literature more or less agrees on the BCS result (except moderate uncertainty due to density of states $\propto m^*$)
- Large corrections beyond BCS, but no consensus

Zoom on low densities: QMC vs. screening



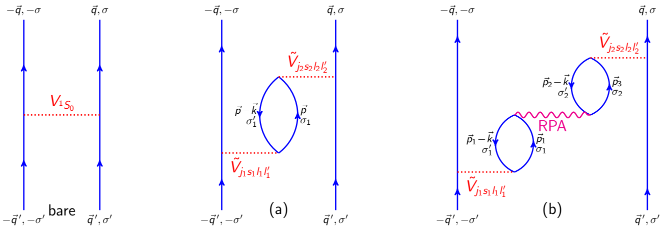
$[k_F = (3\pi^2 n)^{1/3}]$

DQMC: Abe and Seki, PRC 79 (2009)
AFDMC: Gandolfi et al., PRL 101 (2008)
GFMC: Gezerlis and Carlson, PRC 81 (2010)
screened: Cao, Lombardo, and Schuck, PRC 74 (2006)

- No screening at low density? $\longrightarrow$ Third part of the talk

Screening of the pairing interaction

- Diagrams (analogous to screening of Coulomb interaction)



($\tilde{V}$ = anti-symmetrized matrix element)

- As interaction V, we use $V_{\text{low-}k}$ derived from AV_{18}, with cutoff $\Lambda = 2\ \text{fm}^{-1}$

- Approximation: neglect energy transfer
  $\rightarrow$ (a) and (b) can be treated as a corrections to pairing interaction V

- Contact interaction at very weak coupling [diagram (a) only]:
  repulsive exchange of spin fluctuations ($S = 1$) should reduce T_c by $\sim 50\%$
  [Gor'kov and Melik-Barkhudarov (1961)]

- Away from weak-coupling limit: necessary to include RPA [diagram (b)]

RPA effect on the $S = 0$ and $S = 1$ contributions

- diagram (a): $S = 0$ contribution attractive,
  $S = 1$ repulsive and about $3\times$ stronger than $S = 0$

- RPA in Landau approximation: (Π_0 = Lindhard function)

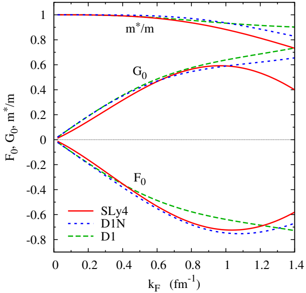
$$V_{ph}^{RPA} = \frac{f_0}{1 - f_0\Pi_0} + \frac{g_0}{1 - g_0\Pi_0}\vec{\sigma}_1 \cdot \vec{\sigma}_2$$

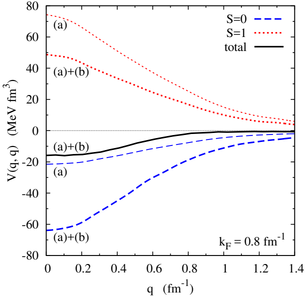
- generally $f_0 < 0$, $g_0 > 0$ (at least at low density)
  (contact interaction: $g_0 = -f_0$)

- $f_0 < 0 \;\rightarrow\;$ RPA enhances $S = 0$ contribution
  $g_0 > 0 \;\rightarrow\;$ RPA reduces $S = 1$ contribution

- RPA effect (diagram (b)) gets more important with increasing density

- example: at $k_F = 0.8\ \mathrm{fm}^{-1}$, net result is attractive
  $\rightarrow$ antiscreening instead of screening!





Gap equation and critical temperature

- gap equation: $$\Delta(k) = -\frac{2}{\pi}\int_0^\infty dq\, q^2\, V(k,q)\,\frac{\tanh(\frac{E(q)}{2T})}{2E(q)}\,\Delta(q)$$

  with $E(q) = \sqrt{\xi(q)^2 + \Delta(q)^2}$, $\quad \xi(q) = \epsilon(q) - \mu$

- $\Delta \to 0$ for $T \to T_c$: linearize gap equation

- T_c can be obtained from the eigenvalue equation

  $$-\frac{2}{\pi}\int_0^\infty dq\, q^2\, V(k,q)\,\frac{\tanh(\frac{\xi(q)}{2T})}{2\xi(q)}\,\phi(q) = \eta(T)\phi(k)$$

  as the temperature where the largest eigenvalue satisfies $\eta(T_c) = 1$

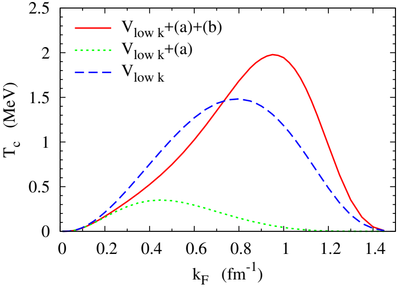
- T_c and the gap at $T = 0$ are related by the BCS relation

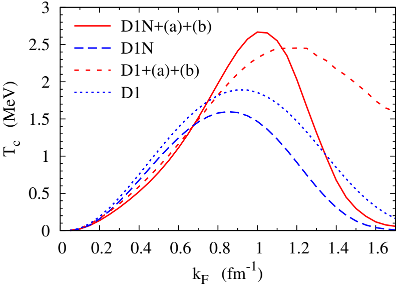
  $$\Delta(k_F; T = 0) \approx 1.76\, T_c$$

  (exact at weak coupling and very good approximation at all densities)

Critical temperature

- $V_{\text{low-}k}+$ Landau parameters from SLy4:
- diagram (a) results in dramatic screening
- from $k_F \sim 0.7\ \text{fm}^{-1}$ ($n \sim 0.01\ \text{fm}^{-3}$), screening turns into antiscreening $\to T_c$ is increased, not reduced!
- repeat calculation with Gogny D1 and D1N:
- T_c depends on the choice of the interaction
- again, screening turns into antiscreening at $k_F \approx 0.7 - 0.8\ \text{fm}^{-1}$
- NSR effect not included here
- additional reduction of T_c from quasiparticle residue (Z factor < 1)? [Cao, Lombardo, and Schuck]





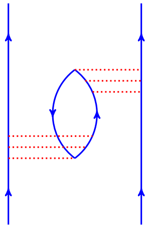
The low-density limit: breakdown of "perturbativeness"

- So far, screening seems to disappear at low density
  $\rightarrow$ contradiction to GMB result (reduction of T_c by a factor $(4e)^{-1/3} \approx 0.45$)

- In their original paper, GMB use contact interaction

- But: as vertices they use the scattering length a (i.e., the full T matrix) instead of the coupling constant ($\simeq V$)

- This corresponds to an (approximate) resummation of ladder diagrams in the vertices



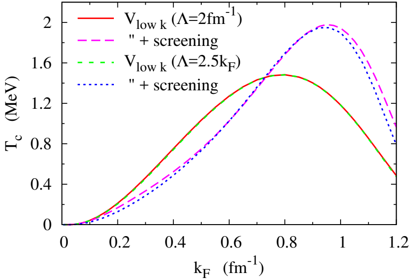
- Back to the nn interaction: scattering length a very large
  $\rightarrow$ even soft potential $V_{\text{low-}k}$ needs to be resummed at low momenta (small k_F)

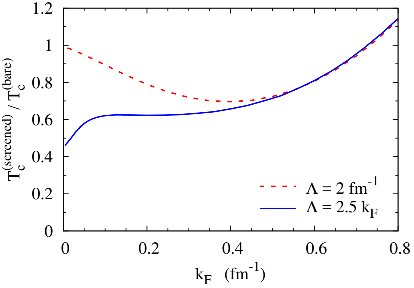
- But at low density, the $V_{\text{low-}k}$-cutoff Λ (typically $\Lambda = 2\ \text{fm}^{-1}$) can be lowered further (down to $\Lambda \sim 2.5 k_F$) without changing the BCS gap or T_c

- Using a density-dependent cutoff $\Lambda = 2.5 k_F$, we have $V_{^1S_0} \xrightarrow{k_F \to 0} a$
  $\rightarrow$ Can we recover the GMB result in this way?

Results with density dependent cutoff

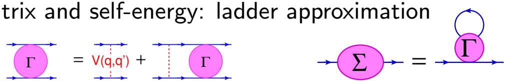
- Without screening: $\Lambda = 2 \text{ fm}^{-3}$ and $\Lambda = 2.5\, k_F$ give the same T_c at all densities
- With screening: T_c with $\Lambda = 2 \text{ fm}^{-3}$ and $\Lambda = 2.5\, k_F$ agree over large range of k_F but cutoff dependence at large and small densities
- Ratio $T_c^{(\text{screened})}/T_c^{(\text{bare})}$ tends towards the GMB result 0.45 but only at very low densities
- Notice: GMB derived for $|k_F a| \ll 1$, i.e., $k_F \ll 0.05 \text{ fm}^{-1}$ ($n \ll 4 \cdot 10^{-6} \text{ fm}^{-3}$)
- At the lowest relevant densities, the ratio is $T_c^{(\text{screened})}/T_c^{(\text{bare})} \approx 0.6$





Nozières-Schmitt-Rink (NSR) correction to the density

- In-medium T matrix and self-energy: ladder approximation



- density from s.-p. Green's function: $n = \frac{2}{\beta} \sum_{\vec{k},\omega_n} \mathcal{G}(\vec{k},\omega_n)$ $\quad$ (ω_n =Matsubara frequency)

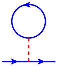
- BCS: $\mathcal{G} = \mathcal{G}_0 \quad \rightarrow \quad n = n_0 = 2 \sum_{\vec{k}} f(\xi_{\vec{k}})$ $\quad$ (for $T \geq T_c$)

- NSR: truncate Dyson equation: $\mathcal{G} = \mathcal{G}_0 + \mathcal{G}_0^2 \Sigma \quad \rightarrow \quad n = n_0 + n_{corr}$

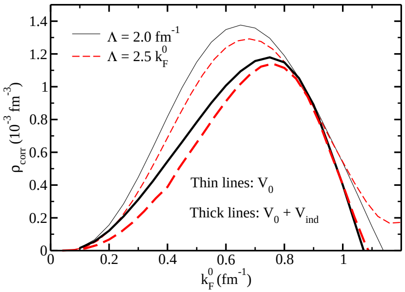
- mean-field shift $U_k = \Sigma(k,\xi_k)$ already included in s.-p. energy ξ_k
  [Zimmermann and Stolz (1985)]

$$\Sigma(k,i\omega_n) \rightarrow \Sigma(k,i\omega_n) - U_k$$

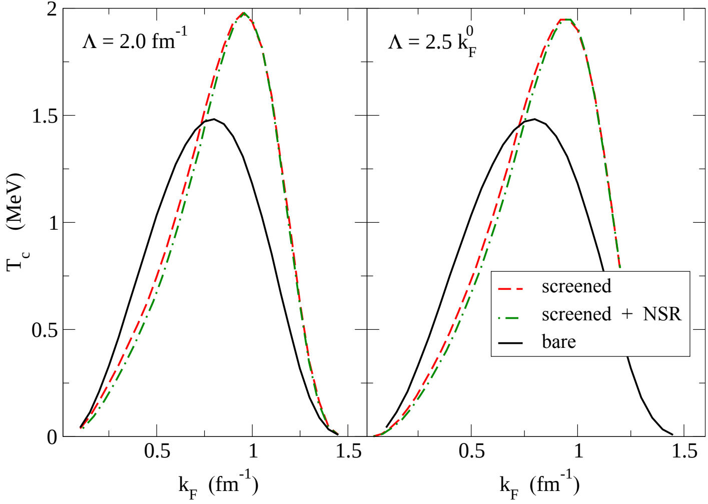
- approximate U_k by HF self-energy



- $n_{corr}/n \rightarrow 0$ at large n
  but slightly cutoff dependent



Critical temperature including screening and NSR



- NSR effect visible but much weaker than screening (antiscreening)
- Screening suppresses NSR effect: partial compensation of cutoff dependence [constant (left) vs. density dependent (right) cutoff]

Summary

- Dilute neutron matter $\approx$ cold atoms in the BCS-BEC crossover regime
- T_c of dilute neutron matter relevant for neutron stars (cooling, glitches)
- Screening corrections: very important, but large theoretical uncertainties
- RPA bubble exchange: calculation with realistic Landau parameters suggests that screening turns into antiscreening beyond $0.01 - 0.02\ \mathrm{fm}^{-3}$
- To retrieve the GMB result in the low-density limit, the $V_{\text{low-}k}$cutoff must be scaled with k_F
- Reduction of T_c (for given density) at $n \lesssim 0.01\ \mathrm{fm}^{-3}$ due to non-condensed pairs (NSR theory) is much weaker than screening effect

Outlook

- reduction of T_c due to quasiparticle residue $Z < 1$
- derive Fermi-liquid parameters and pairing from one interaction: in-medium similarity renormalization group (IMSRG) instead of $V_{\text{low-}k}$
- Soft interactions (RG evolved to small cutoffs) may be useful also in cold atoms

Appendix

- More details on the low-density limit
- More details on the NSR correction to the density

Low-density limit and (wrong) derivation of the GMB result

- For $k_F \to 0$ diagram (b) vanishes
- For diagram (a) we may consider $q, q' \simeq k_F \to 0$

$\to$ replace each $\tilde{V}$ by $2V_{^1S_0}(0,0) \equiv 2V_0$: $\quad V_{(a)}(q,q') \approx -2\pi V_0^2 \langle \Pi_0 \rangle$

with $\quad \langle \Pi_0 \rangle = \int \frac{d\Omega_{\vec{q}\vec{q}'}}{4\pi} \Pi_0(|\vec{q} - \vec{q}'|) =$ angle averaged Lindhard function

- Special case: $q = q' = k_F$: $\quad V_{(a)}(k_F, k_F) \approx 2\pi V_0^2 N_0 \frac{1}{3} \ln 4e$

with $\quad N_0 = \frac{m^* k_F}{\pi^2} =$ density of states

- Weak-coupling formula: $\quad T_c \propto \exp\left(\frac{1}{2\pi N_0 V(k_F, k_F)}\right)$
- Replace $V \to V_0 + V_{(a)}$:

$$\frac{T_c^{\text{screened}}}{T_c^{bare}} \approx \exp\left(\frac{1}{2\pi N_0 [V_0 + 2\pi V_0^2 N_0 \frac{1}{3} \ln 4e]} - \frac{1}{2\pi N_0 V_0}\right)$$

$$= \exp\left(\frac{-\frac{1}{3} \ln 4e}{1 + 2\pi V_0 N_0 \frac{1}{3} \ln 4e}\right) \approx (4e)^{-1/3} \approx 0.45$$

Failure of the weak-coupling formula at low density

- At low T, the kernel

$$\frac{\tanh(\xi(q)/2T)}{2\xi(q)}$$

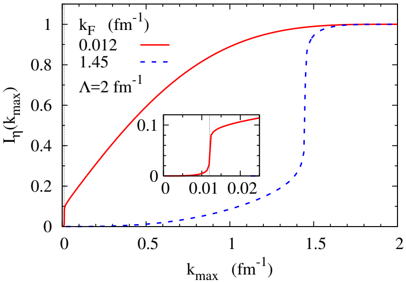
  is strongly peaked at $q = k_F$

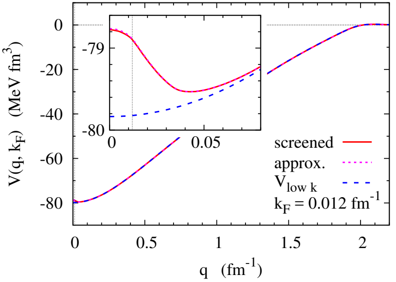
- Weak-coupling formula assumes that the integral in the gap equation is dominated by this peak

- But: e.g. at $k_F = 0.012\ \mathrm{fm}^{-1}$, peak contributes $< 10\%$ to the integral

- $V_{(a)}$ limited to much smaller range of q ($\lesssim$ a few times k_F) than V_0

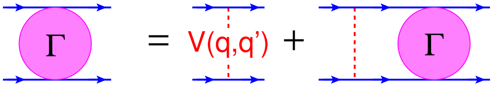
  $\rightarrow$ T_c almost unaffected by $V_{(a)}$.





T matrix with low-momentum interaction $V_{\text{low-}k}$

- $V_{\text{low-}k}$: low-momentum interaction generated from a realistic NN interaction by renormalization group methods (cutoff Λ)
- difficulty: numerical matrix elements $V(q, q')$, not separable
- T matrix:



$$\Gamma(K, q, q', \omega) = V(q, q') + \frac{2}{\pi} \int dq'' q''^2 V(q, q'') \bar{G}_0^{(2)}(K, q'', \omega) \Gamma(K, q'', q', \omega)$$

$$\bar{G}_0^{(2)}(K, q, \omega) = \text{angle average of } G_0^{(2)} = \frac{1 - f(\frac{\vec{K}}{2} + \vec{q}) - f(\frac{\vec{K}}{2} - \vec{q})}{\omega - \frac{K^2}{4m} - \frac{q^2}{m} + i\varepsilon}$$

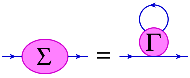
- solve this integral equation by diagonalizing $V \bar{G}_0^{(2)}$:

$$\frac{2}{\pi} \int dq' q'^2 V(q, q') \bar{G}_0^{(2)}(K, q', \omega) \phi_\nu(q', K, \omega) = \eta_\nu(K, \omega) \phi_\nu(q, K, \omega)$$

η_ν: Weinberg eigenvalues [Weinberg (1963)]

Contribution of non-condensed pairs to the density

- density from s.-p. Green's function: $n = \frac{2}{\beta} \sum_{\vec{k},\omega_n} \mathcal{G}(\vec{k},\omega_n)$ (ω_n =Matsubara frequency)
- BCS: $\mathcal{G} = \mathcal{G}_0 \quad \rightarrow \quad n = n_{\text{free}} = 2 \sum_{\vec{k}} f(\xi_{\vec{k}})$ (for $T \geq T_c$)
- NSR: truncate Dyson equation at 1st order in Σ:



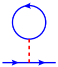
$$\mathcal{G} = \mathcal{G}_0 + \mathcal{G}_0^2 \Sigma \quad \rightarrow \quad n = n_{\text{free}} + n_{\text{corr}}$$

$$n_{\text{corr}} = -\frac{\partial}{\partial\mu} \int \frac{K^2 dK}{2\pi^2} \int \frac{d\omega}{\pi} g(\omega) \,\text{Im} \sum_{\nu} \log(1 - \eta_\nu(K,\omega))$$ (g =Bose function)

- mean-field shift $U_k = \Sigma(k,\xi_k)$ already included in s.-p. energy ξ_k
  [Zimmermann and Stolz (1985)]

$$\Sigma(k, i\omega_n) \rightarrow \Sigma(k, i\omega_n) - U_k$$

- approximate U_k by HF self-energy



- $n_{\text{corr}} \ll n$ at higher density
  but HF subtraction is cutoff dependent