ECT* Workshop “Exploring Nuclear Physics With Ultracold Atoms”

Dilute pure neutron matter investigated by cold atom experiments

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Institute for Photon Science and Technology, The University of Tokyo

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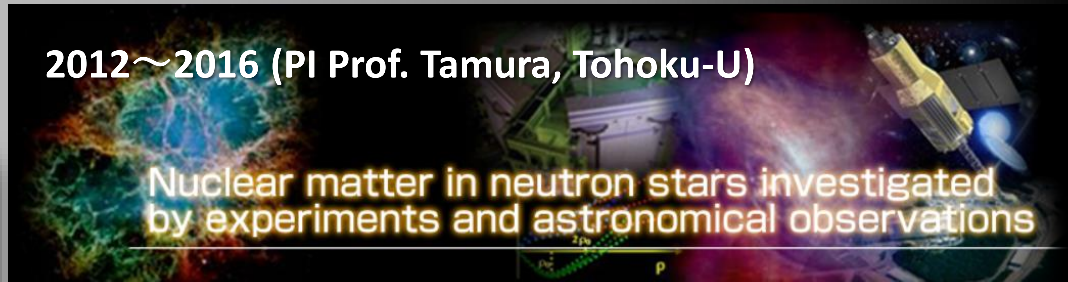
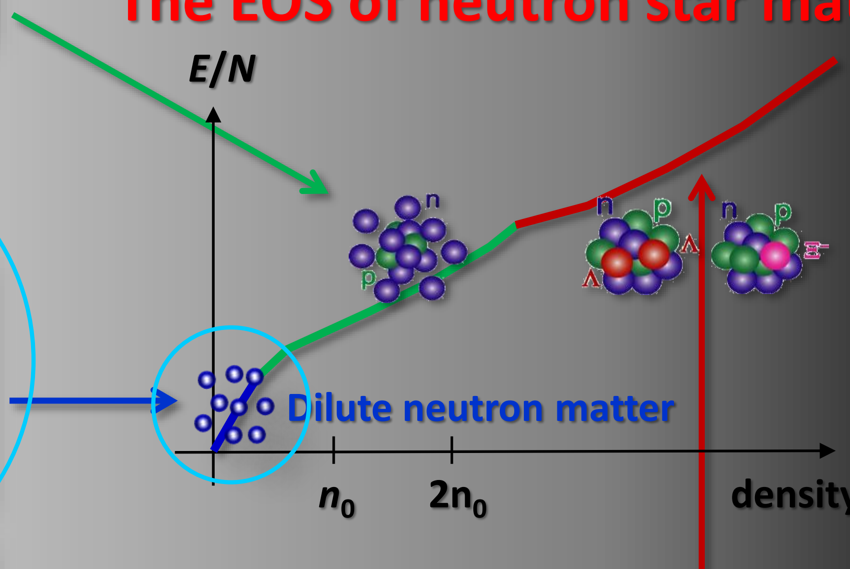
Acknowledgement

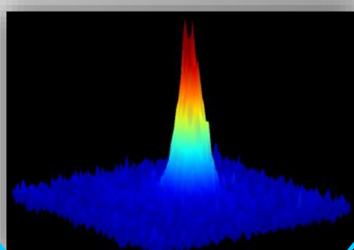


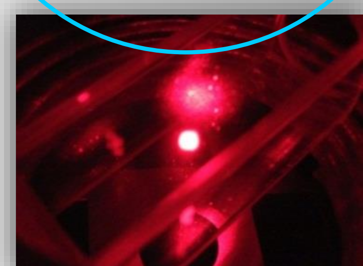
RIKEN RIBF

The EOS of neutron star matter



Fermi superfluid

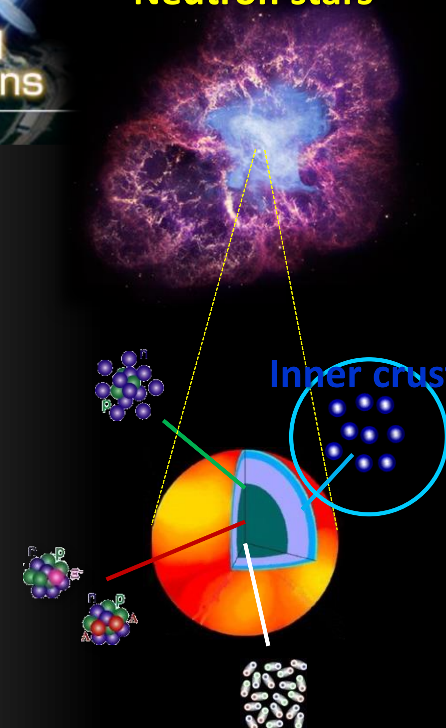


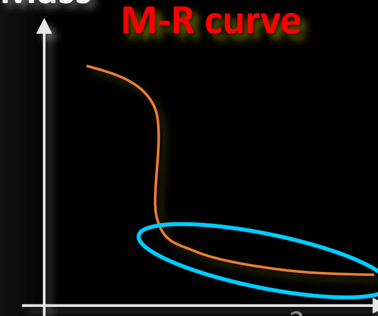


Cold Fermi gas

KEK J-PARK

Neutron stars

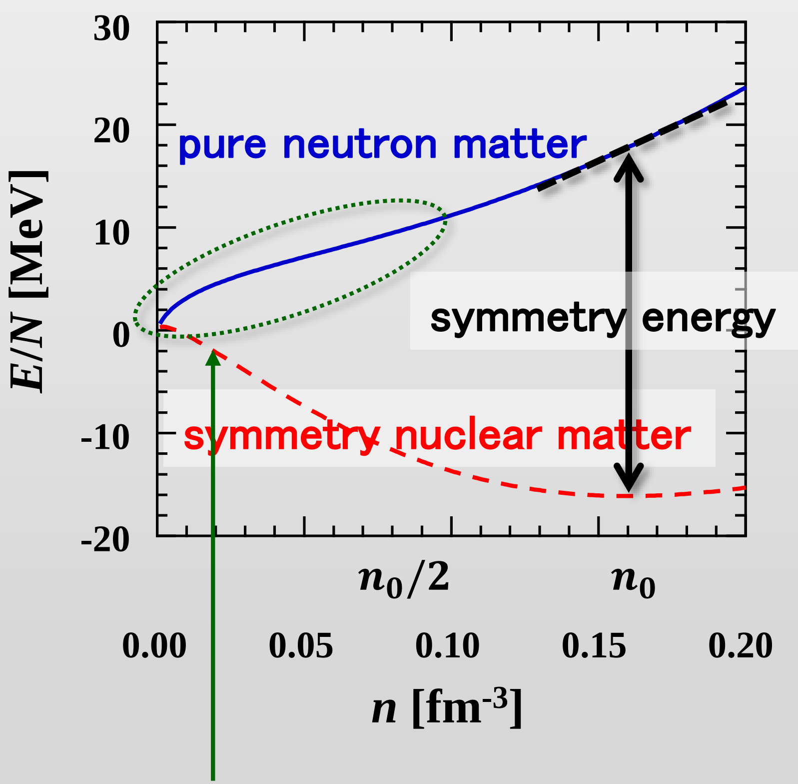




Contents

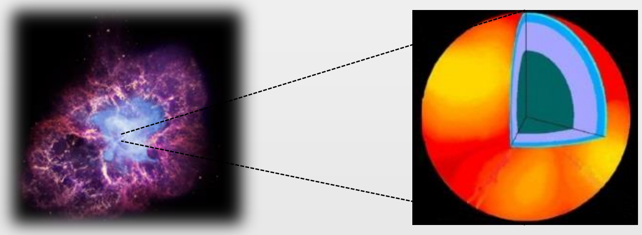
EOS of nuclear matter

nuclear matter EOS

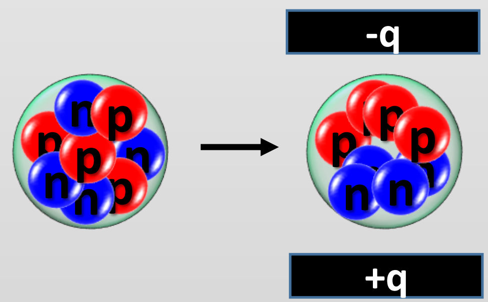


Dilute region is difficult for nuclear experiments

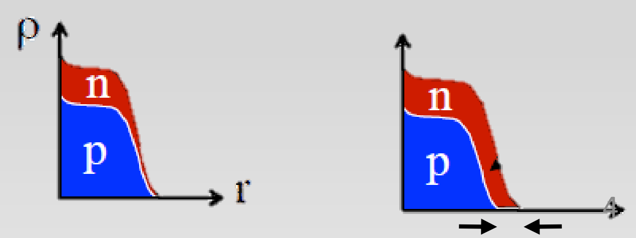
- Neutron stars
- Proto–neutron stars



- Optical response (Dipole polarizability)

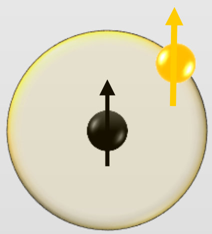


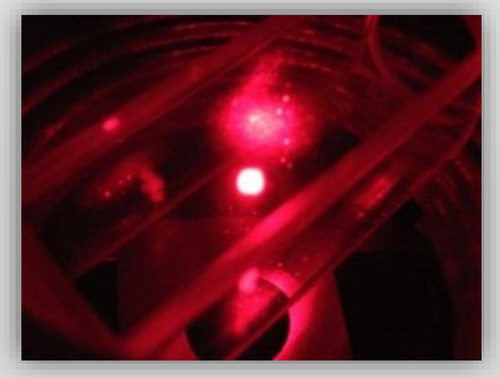
- Neutron skin of neutron-rich nuclei



Quantum simulation of neutron matter using ultracold atomic gases

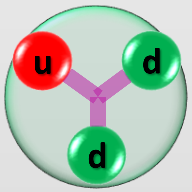
Atomic gas (^{6}Li)





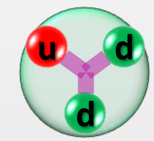
Laser cooling

Neutron matter



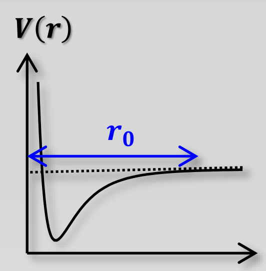
Neutron star

Cold atom and neutron matter

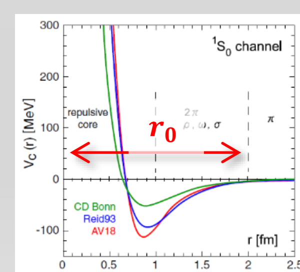
	Atomic gas (^{6}Li)	Neutron matter
		
Particle	Spin-1/2 fermion	Spin-1/2 fermion
Mass (mc^2)	$5600\ \mathrm{MeV}$	$940\ \mathrm{MeV}$
Force	Electro-magnetic force	Nuclear force
Potential type	Short range	Short range
Interaction region (r_0)	$\sim 2\ \mathrm{nm}$	$\sim 2\ \mathrm{fm}$
Inter particle distance (d)	$\sim 250\ \mathrm{nm}$	$> 2.3\ \mathrm{fm}\ (n = n_0/2)$
Temperature (T)	$\sim 100\ \mathrm{nK}$	$10^5 \sim 10^7\ \mathrm{K}$
Thermal length ($\lambda_T = \frac{\hbar}{\sqrt{2\pi m k_B T}}$)	$\sim 350\ \mathrm{nm}$	$100 \sim 900\ \mathrm{fm}$

$$r_0 < d, \lambda_T$$

Dilute and Low energy

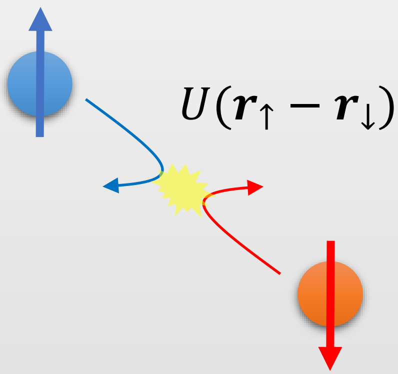


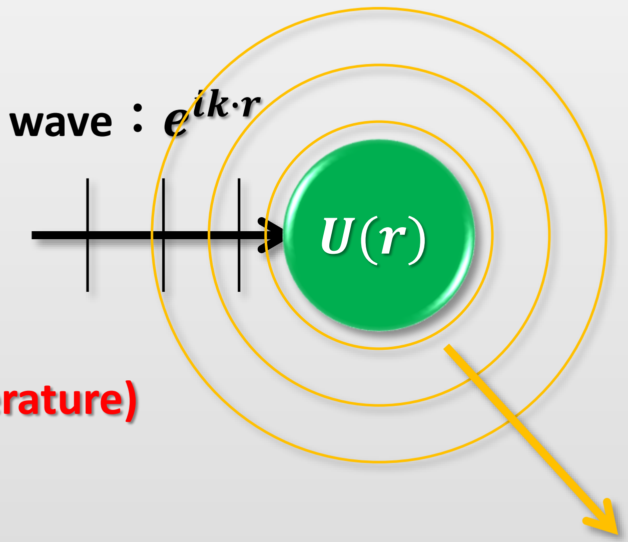
Van der Waals potential



NN potential (spin singlet)
[N.Ishii, Phys. Rev. Lett. 99, 022001 (2007)]

Collisional phase shift





- **Low energy (low temperature)**
- **Dilute**
- **Short-range potential**
- **Two-body elastic collision**

Scattering wave : $\psi(\boldsymbol{r}) \xrightarrow{|\boldsymbol{r}_{\uparrow}-\boldsymbol{r}_{\downarrow}| \gg r_0} \frac{C}{kr}\sin(kr + \delta_0)$

Phase shift : $\cot\delta_0 = -\frac{1}{ak} + \frac{1}{2} r_{\mathrm{e}} k$

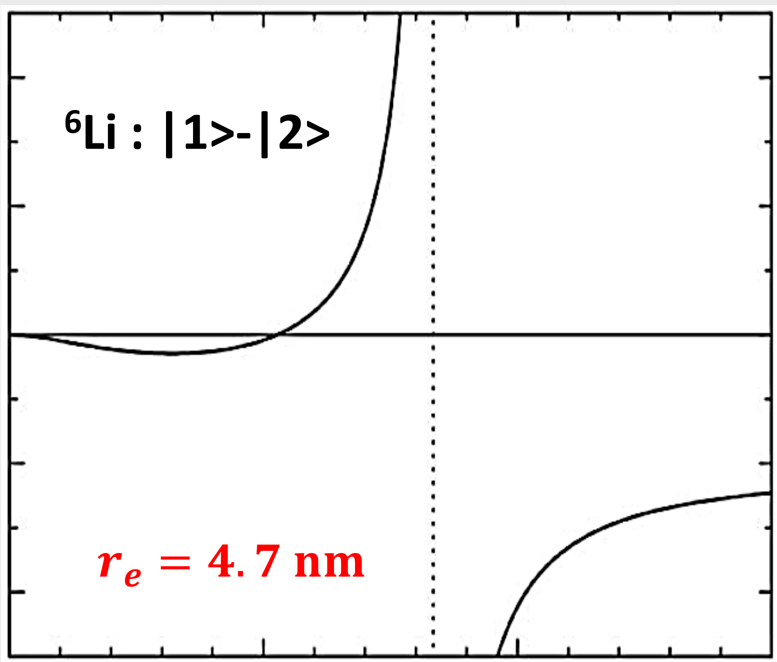
S-wave scattering

- **Scattering length of 6Li atoms**

Feshbach resonance

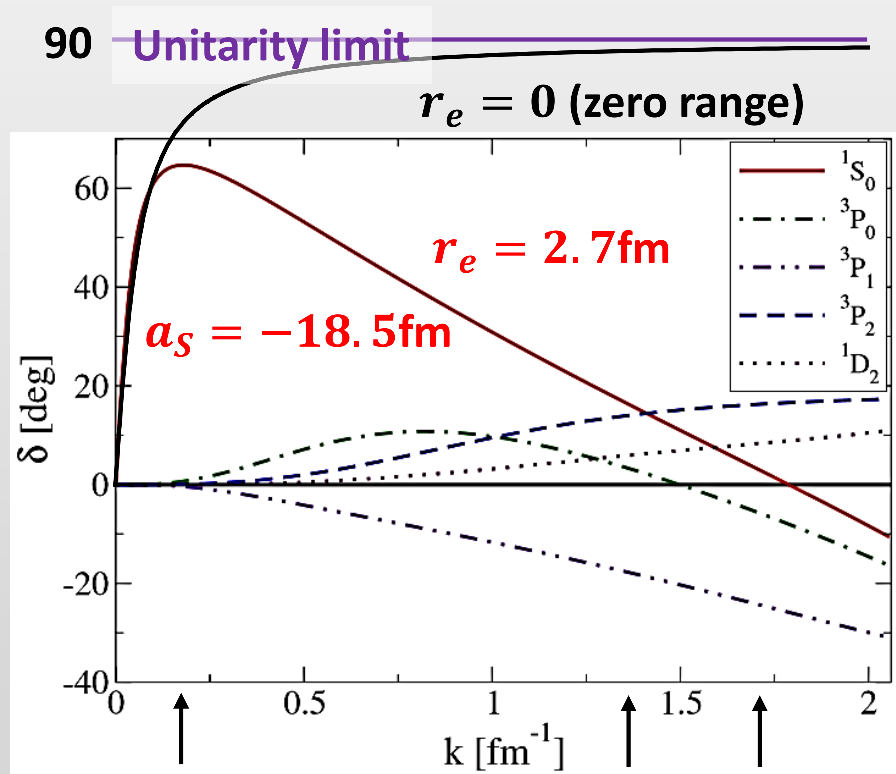
$-\infty < a_S < +\infty$

Scattering length : $a(B)$



832Gauss Magnetic field

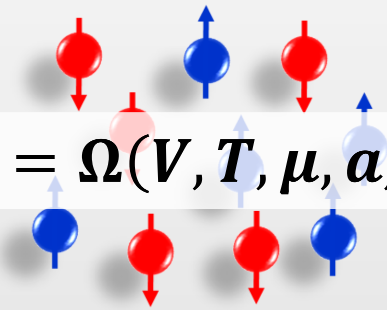
- **Collisional phase shift of neutrons**



$n_0/100$ $n_0/2$ n_0

[A. Gezerlis, C. J. Pethick, A. Schwenk, arXiv:1406.6109]

Universal Equation of state of the many-body system



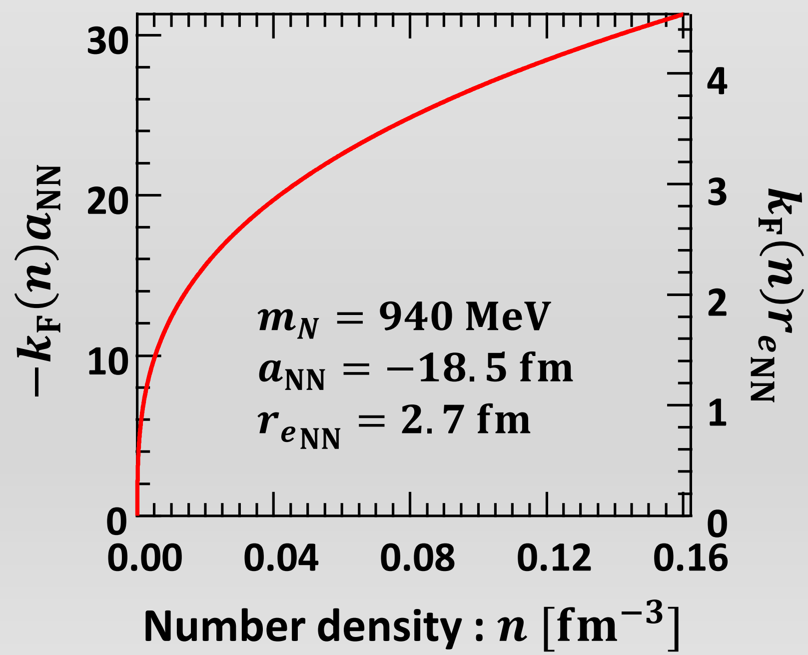
$$\Omega = \Omega(V, T, \mu, a, r_e)$$

$$\Rightarrow \quad \frac{\mathcal{F}}{\frac{3}{5} n \varepsilon_F} = f_{\mathcal{F}}\left(\frac{T}{T_F}, k_F a, k_F r_e\right)$$

Density dependent dimensionless parameters for neutron matter

Unitarity limit

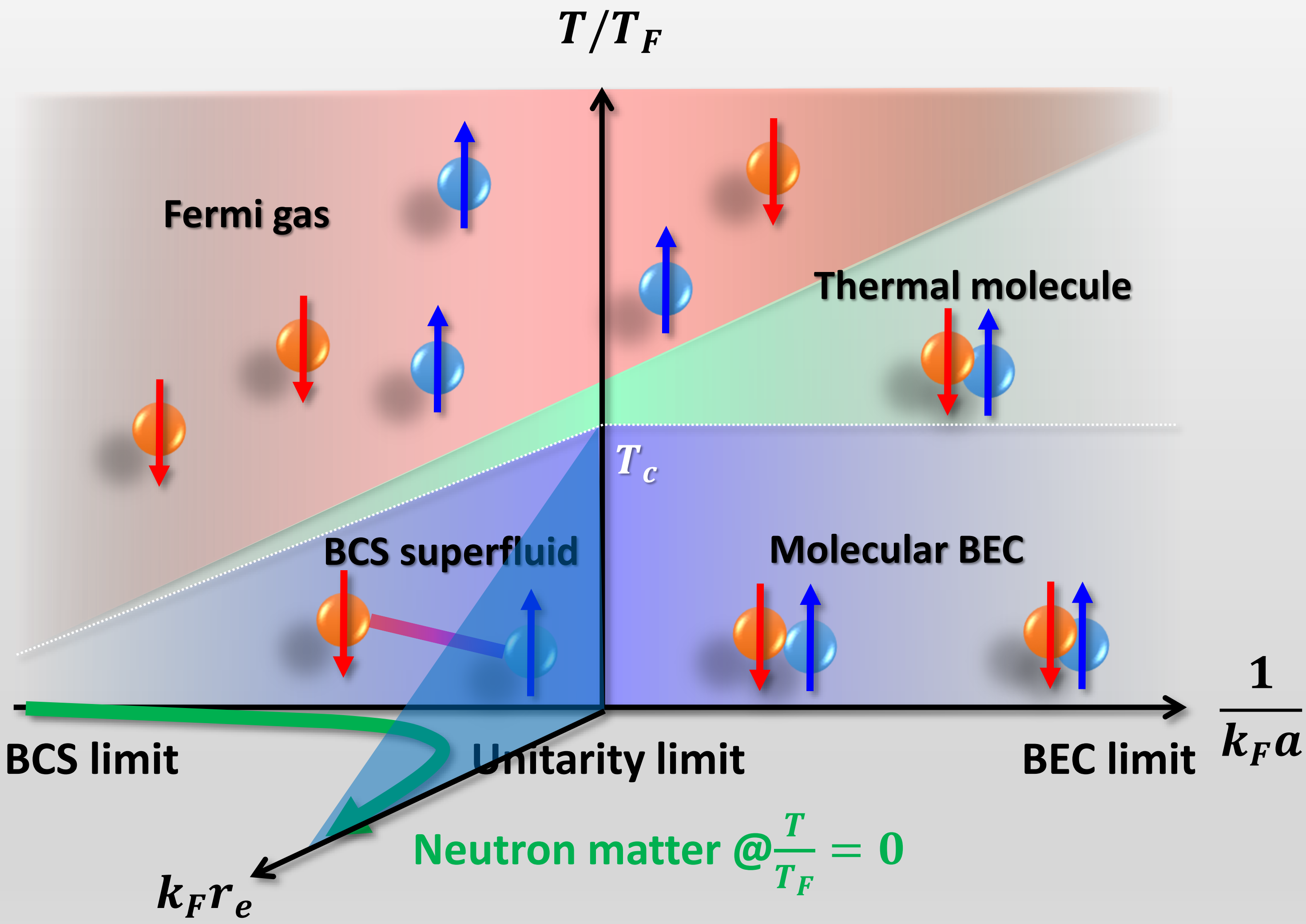
BCS limit



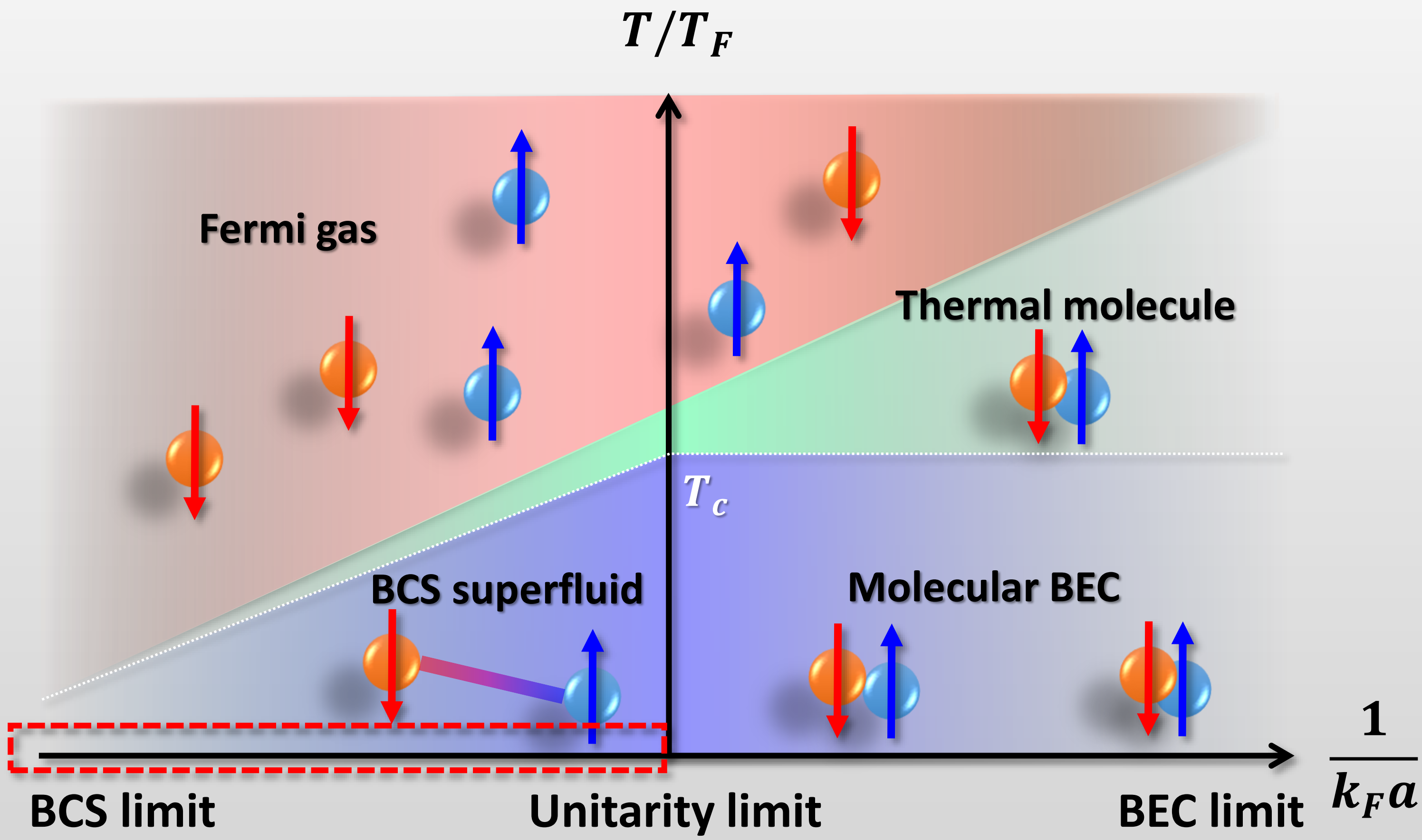
Finite range

Zero range

Phase diagram of cold atom and neutron matter



The first step to neutron matter EOS



Zero-range & Zero- T/T_F EOS : $$\frac{\mathcal{E}}{\frac{3}{5} n \varepsilon_F} = f_{\mathcal{E}}\left(\frac{1}{k_F a}\right)$$

Preceding experiment

The Equation of State of a Low-Temperature Fermi Gas with Tunable Interactions

N. Navon[*,†], S. Nascimbène[*], F. Chevy, C. Salomon

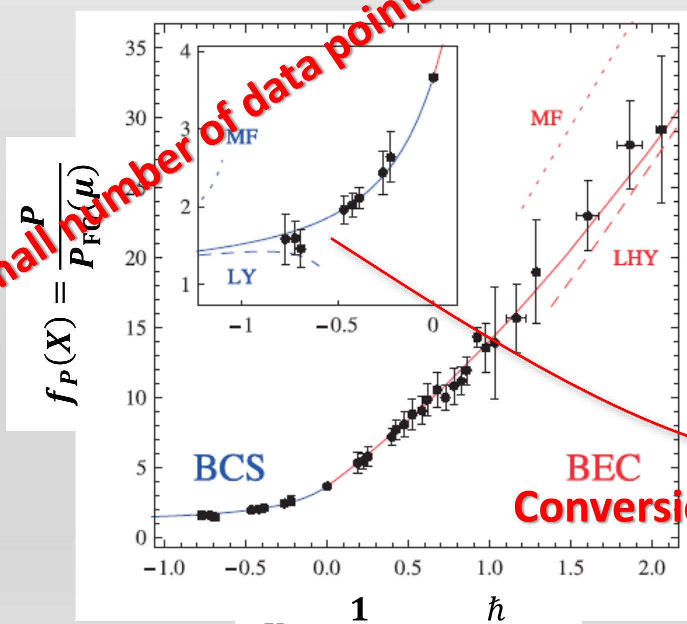
+ Author Affiliations

[†]To whom correspondence should be addressed. E-mail: navon@ens.fr

[*] These authors contributed equally to this work.

Science 07 May 2010:
Vol. 328, Issue 5979, pp. 729-732
DOI: 10.1126/science.1187582

Pressure

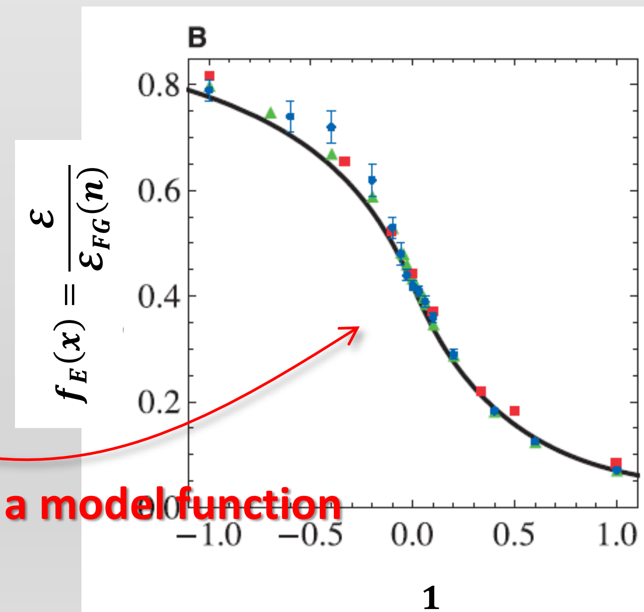


$$f_P(X) = \frac{P}{P_{FG}(\mu)}$$

$$X = \frac{1}{k_\mu a} = \frac{\hbar}{\sqrt{2m\mu a}}$$

Small number of data points

Internal energy



$$f_E(x) = \frac{\mathcal{E}}{\mathcal{E}_{FG}(n)}$$

$$x = \frac{1}{k_F a}$$

Conversion by a model function

Motivation from the point of view of physics

Validity of BCS mean-field model at unitary regime

Hamiltonian :

$$\hat{\mathcal{H}} - \mu\hat{\mathcal{N}} = \sum_{\sigma}\int\left(\frac{\hbar^2}{2m}\nabla\hat{\Psi}_{\sigma}^{\dagger}(\boldsymbol{r})\nabla\hat{\Psi}_{\sigma}(\boldsymbol{r}) - \mu\right)d\boldsymbol{r} - \frac{\hbar^2}{m}g(a)\int \hat{\Psi}_{\uparrow}^{\dagger}(\boldsymbol{r})\hat{\Psi}_{\downarrow}^{\dagger}(\boldsymbol{r})\hat{\Psi}_{\downarrow}(\boldsymbol{r})\hat{\Psi}_{\uparrow}(\boldsymbol{r})d\boldsymbol{r}$$

BCS mean field @ T=0

Renormalized coupling constant : $\frac{1}{g(a)} = -\frac{1}{4\pi a} + \frac{1}{V}\sum_{k}\frac{1}{k^2} > 0$

[Marini, M., Pistolesi, F. & Strinati, G. Eur. Phys. J. B (1998) 1: 151]

Gap equation : $-\frac{1}{a} = \frac{2}{\pi}\sqrt{2m\Delta}I_1(\mu/\Delta)$ ⟺ $x = -\frac{2}{\pi}\sqrt{f_{\Delta}(x)}I_1\left(\frac{f_{\mu}(x)}{f_{\Delta}(x)}\right)$

EOS

$$\left(x \equiv \frac{1}{k_F a}, f_{\mu} \equiv \frac{\mu}{\varepsilon_F}, f_{\Delta} \equiv \frac{\Delta}{\varepsilon_F}\right)$$

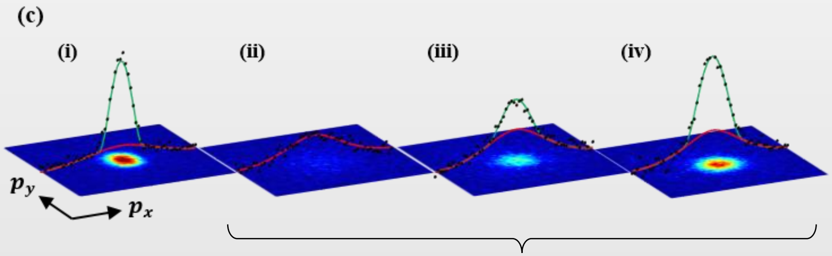
Pairing gap

Contents

Recent results

- Dual BECs of paired-fermions and bosons

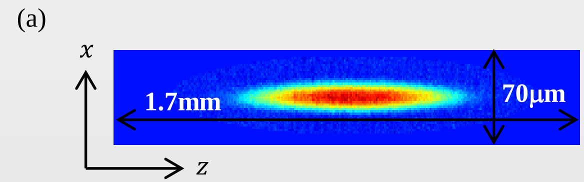
[*J. Phys. B: At. Mol. Opt. Phys.* **50** 01LT01 (2017)]



6Li(fermion) **7Li(boson)**

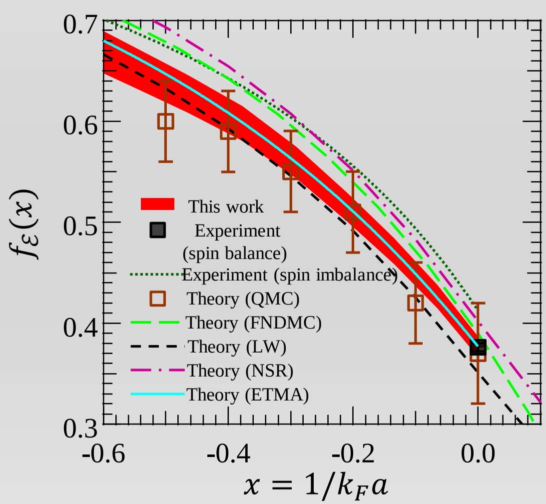
- Probe condition for Li atom imaging

[J. Phys. Soc. Jpn. **86**, 104301 (2017)]



- Thermodynamic quantities from the BCS region to the unitarity limit

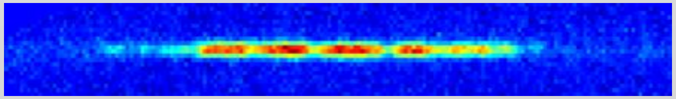
[Phys. Rev. X **7**, 041004 (2017)]



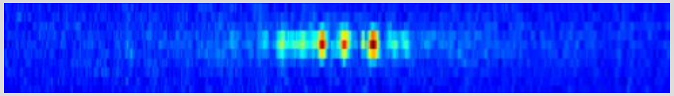
- Dynamical phase transition for attractive bosons

[submitted]

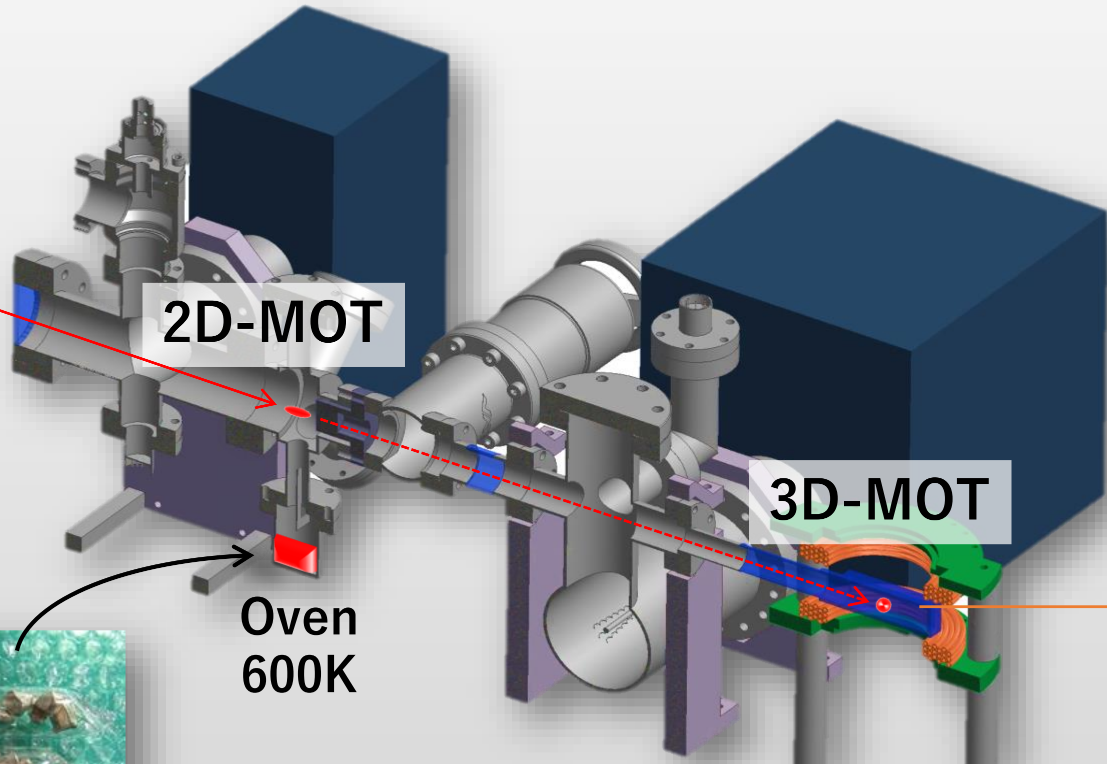
Repulsive BEC (Dark solitons)



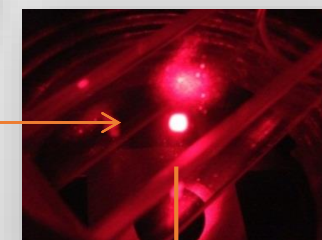
Attractive BEC (Bright solitons)



Our experimental apparatus

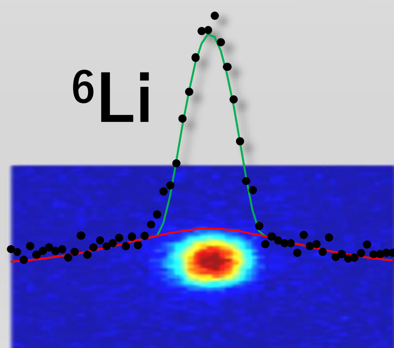


Cold atoms
1mK

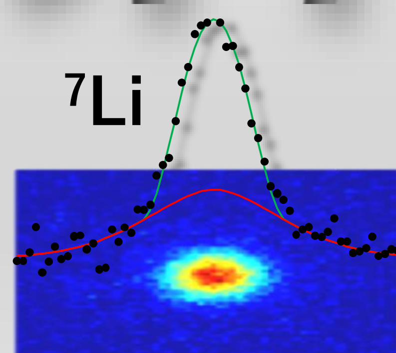


Lithium

^{6}Li



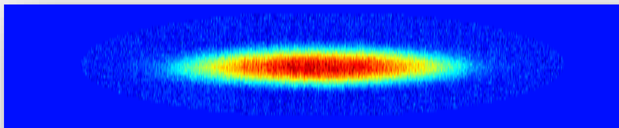
^{7}Li



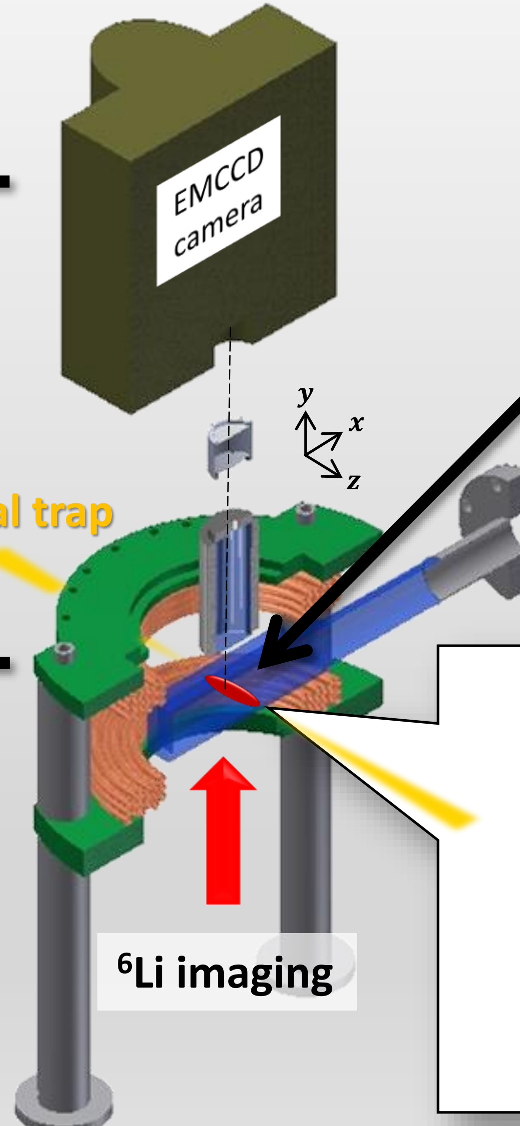
Evaporative cooling
< 100nK

Experimental setup

Density distribution of ^{6}Li (Fermion)

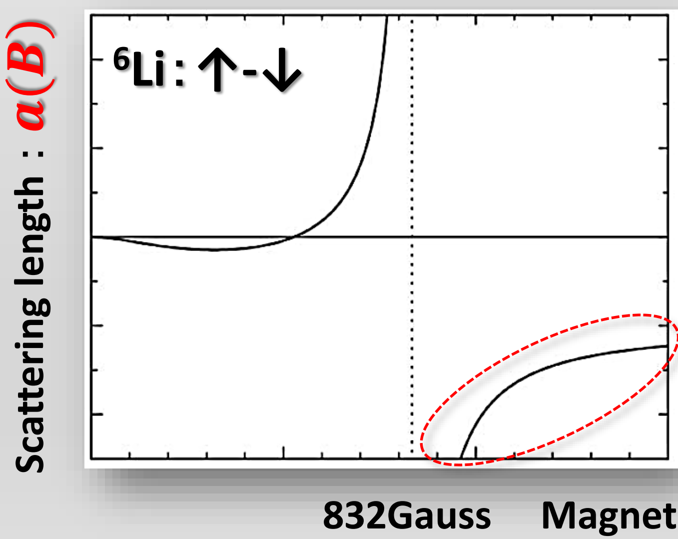


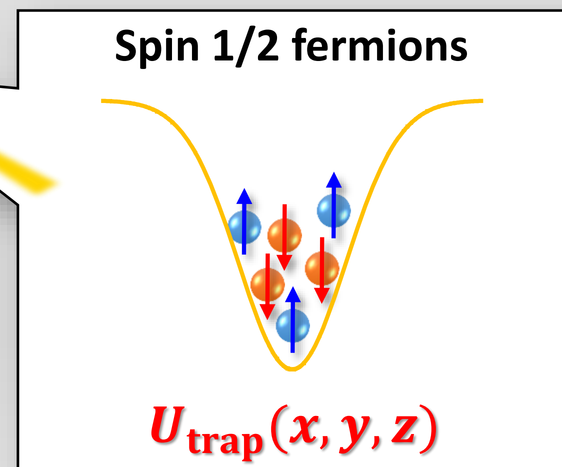
$$\bar{n}(x,z) = \int_{-\infty}^{+\infty} n(x,y,z)\,dy$$



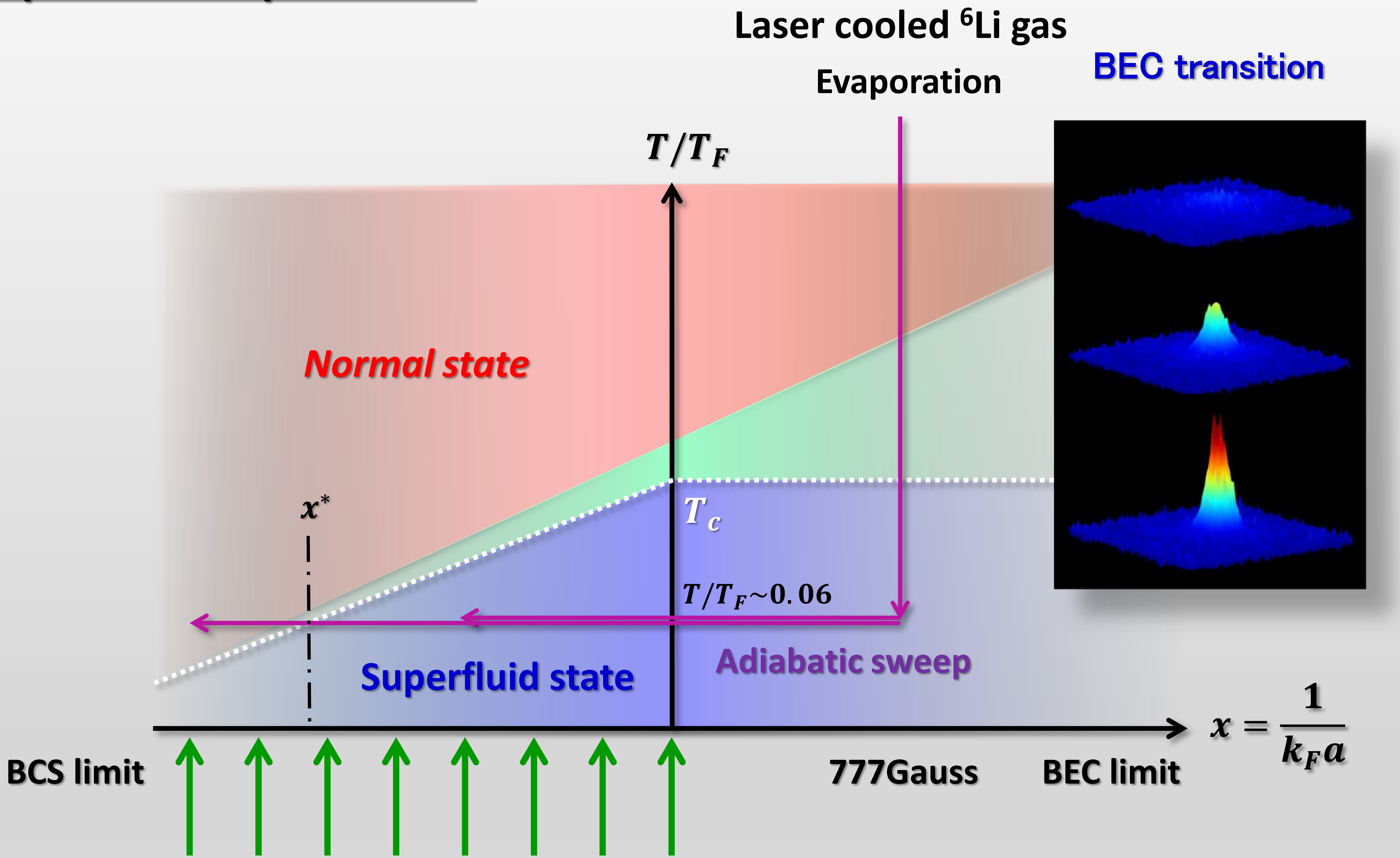
Laser cooling

Feshbach resonance





Experimental procedure

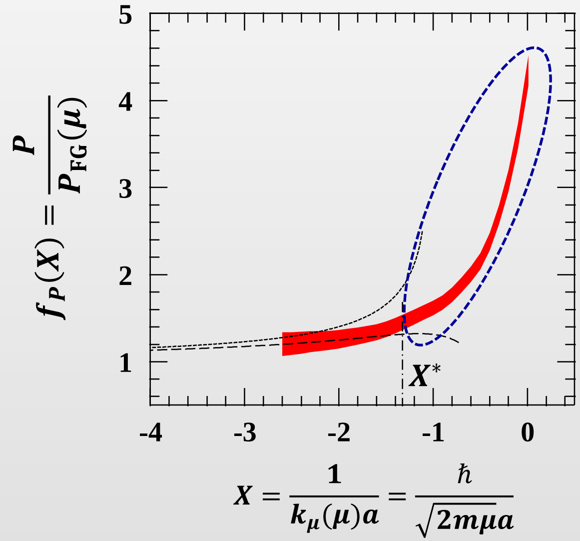


Take *in-situ* images from the BCS limit to the unitarity limit

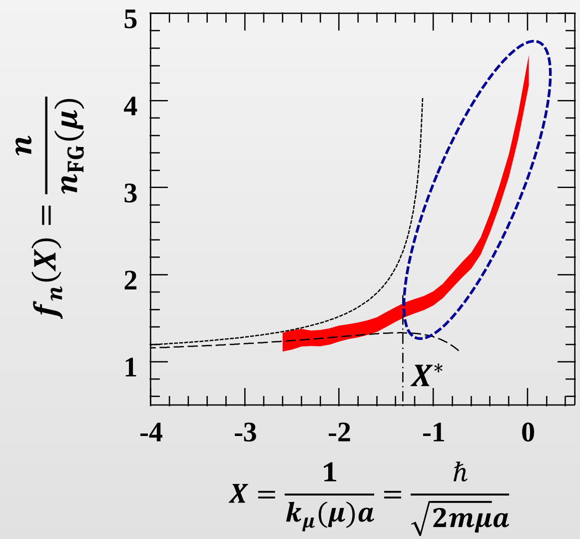
[Phys. Rev. X **7**, 041004 (2017)]

Result : The dimensionless EOS

Pressure

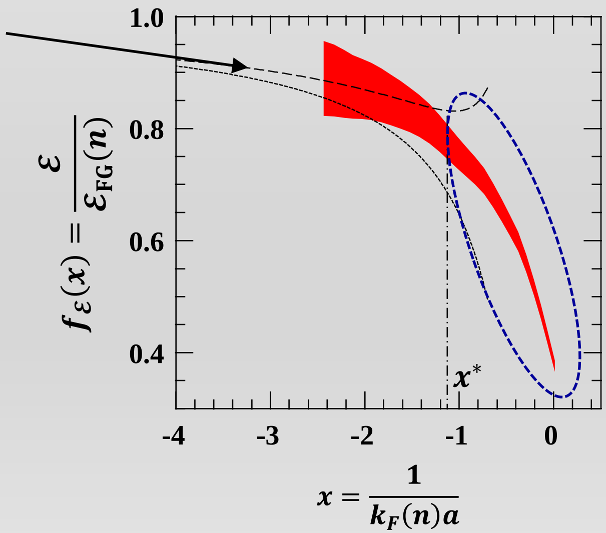


Number density

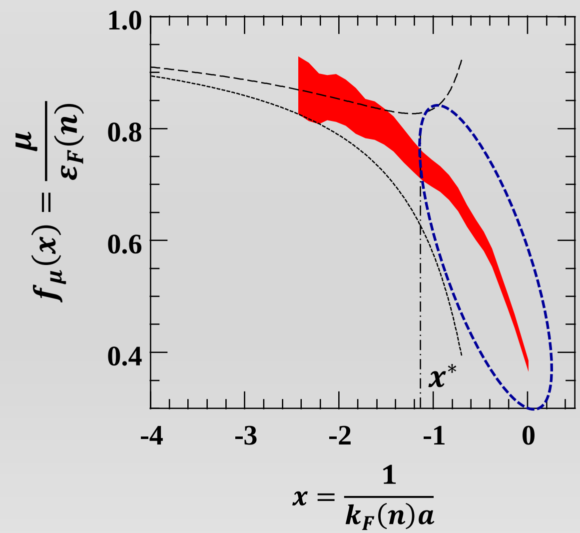


$$f_E^{MF+LY}(x) = 1 + \frac{10}{9\pi}x^{-1} + \frac{4(11-2\ln 2)}{21\pi^2}x^{-2}$$

Internal energy density

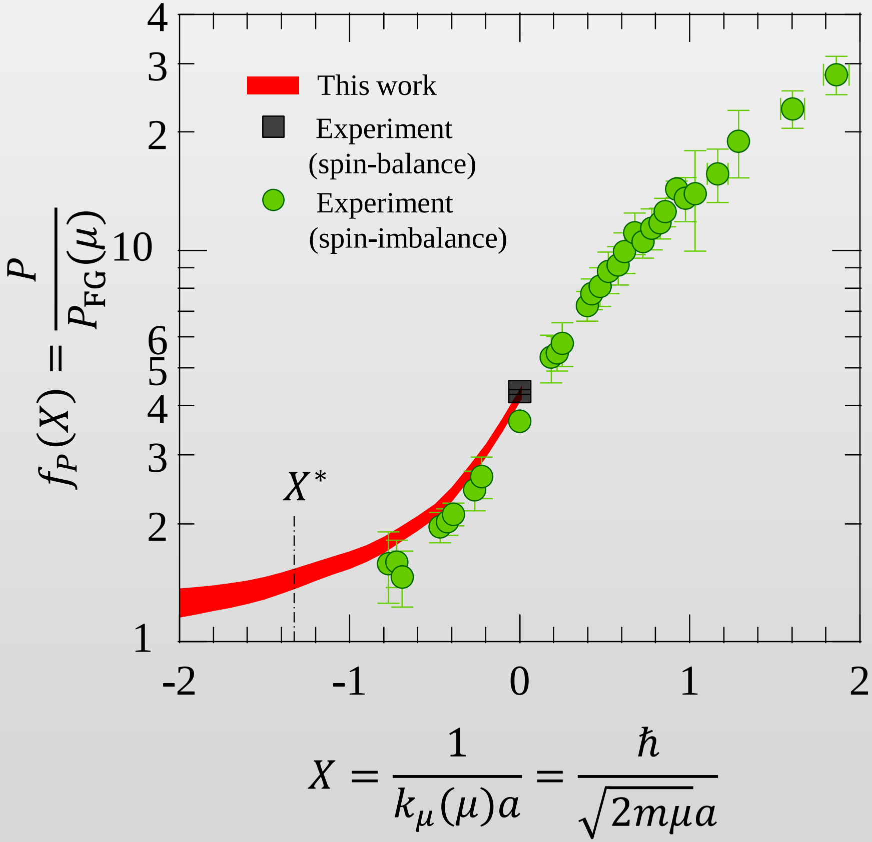


Chemical potential

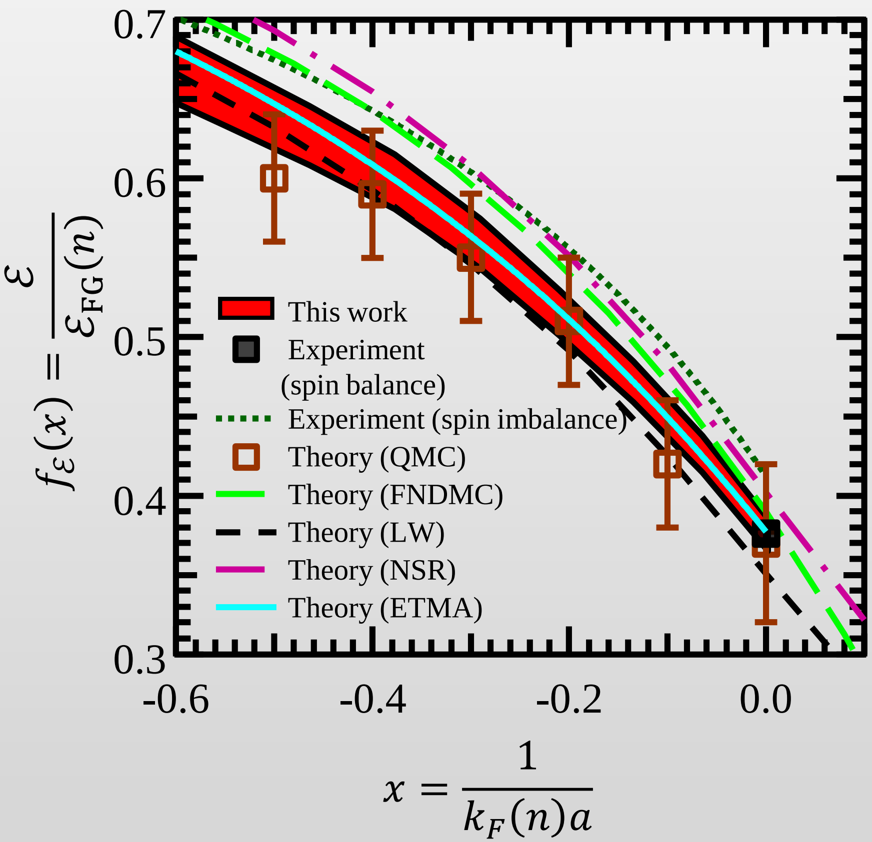


Comparisons

Pressure



Internal energy density



- Experiment (ENS) : N. Navon, Science **328**, 729 (2010)
- Experiment (MIT) : M. Ku, Science **335**, 563 (2012)

- Theory (QMC) : A Bulgac, Phys. Rev. A **78**, 023625 (2008)
- Theory (FNDMC) : S Gandolfi, Phys. Rev. A **83**, 041601(R) (2011)
- Theory (LW) : R. Haussmann, Phys. Rev. A **75**, 023610 (2007)
- Theory (NSR) : H. Hu, EPL **74**, 574 (2006)
- Theory (ETMA) : H. Tajima, Phys. Rev. A **93**, 013610 (2016)

Contact density (short range correlation) : C

Differential form of internal energy density :

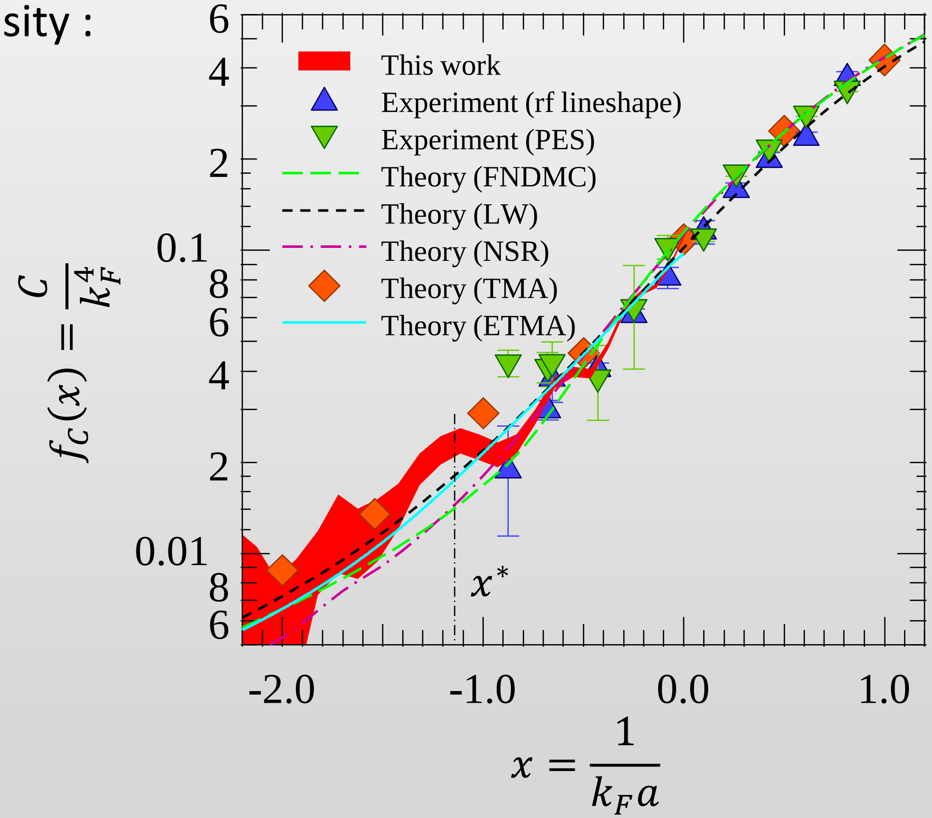
$$d\mathcal{E} = \mu dn - \left(\frac{\hbar^2}{4\pi m} C\right) da^{-1}$$

Dimensionless contact :

$$f_C(x) = \frac{C}{k_F(n)^4}$$

Universal relation :

$$f_C(x) = -\frac{2}{5\pi} \frac{df_{\mathcal{E}}(x)}{dx}$$



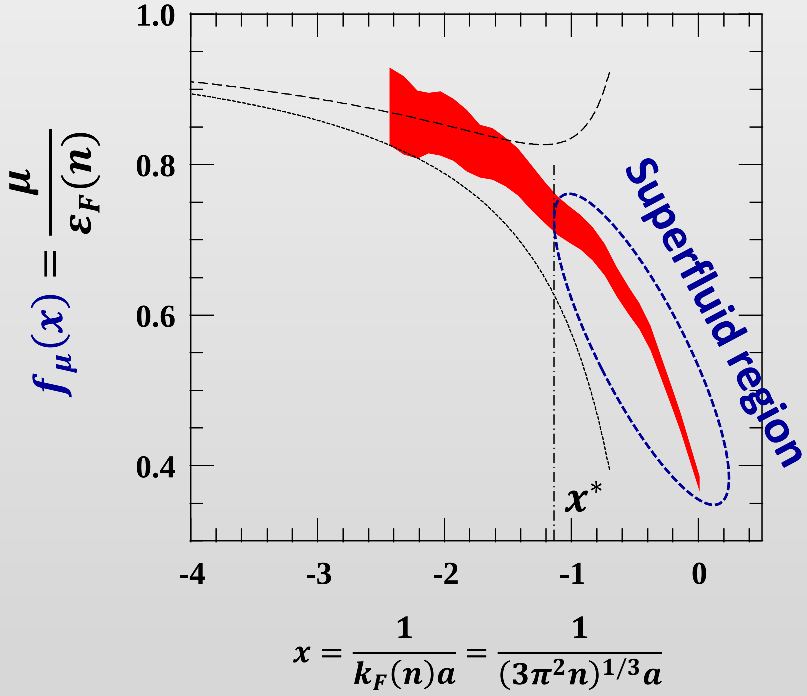
Experiment (PES)

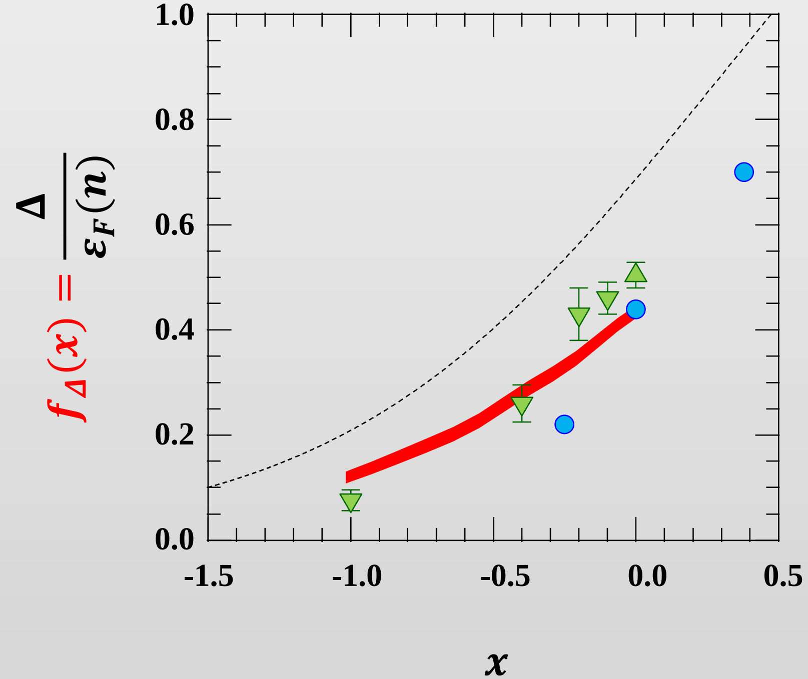
Experiment (rf lineshape)

Yoav Sagi, Phys. Rev. Lett. 114, 075301 (2015)

Validity of BCS mean-field model at unitary regime

Gap equation : $x = -\frac{2}{\pi}\sqrt{f_\Delta(x)}\, I_1\left(\frac{f_\mu(x)}{f_\Delta(x)}\right)$ [Marini, Eur. Phys. J. B **1**, 151-159 (1998)]





- ● : Experiment (rf spectroscopy) [A. Schirotzek, Phys. Rev. Lett. 101, 140403 (2008)]
- ▲ : Theory (QMC) [J. Carlson, Phys. Rev. Lett. 95, 060401 (2005)]
- ▼ : Theory (QMC) [A. Gezerlis, Phys. Rev. C 77, 032801 (2008)]

$$\longrightarrow \Delta = E(N+1) - \frac{E(N) + E(N+2)}{2}$$

Neutron matter EOS from the cold atom experiment

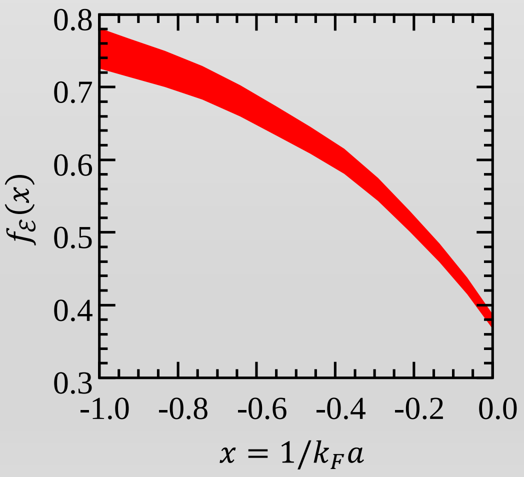
Internal energy per neutron at $T/T_F = 0$ limit :

$$E/N = \frac{3}{5}\varepsilon_F(n) \times f_\mathcal{E}(k_F(n)a_{NN})$$

$$E/N = \frac{3}{5}\varepsilon_F(n)\{f_\mathcal{E}(k_F(n)a_{NN}) + 0.127k_F(n)r_{eNN}\} \text{ for } k_F r_e < 0.3$$

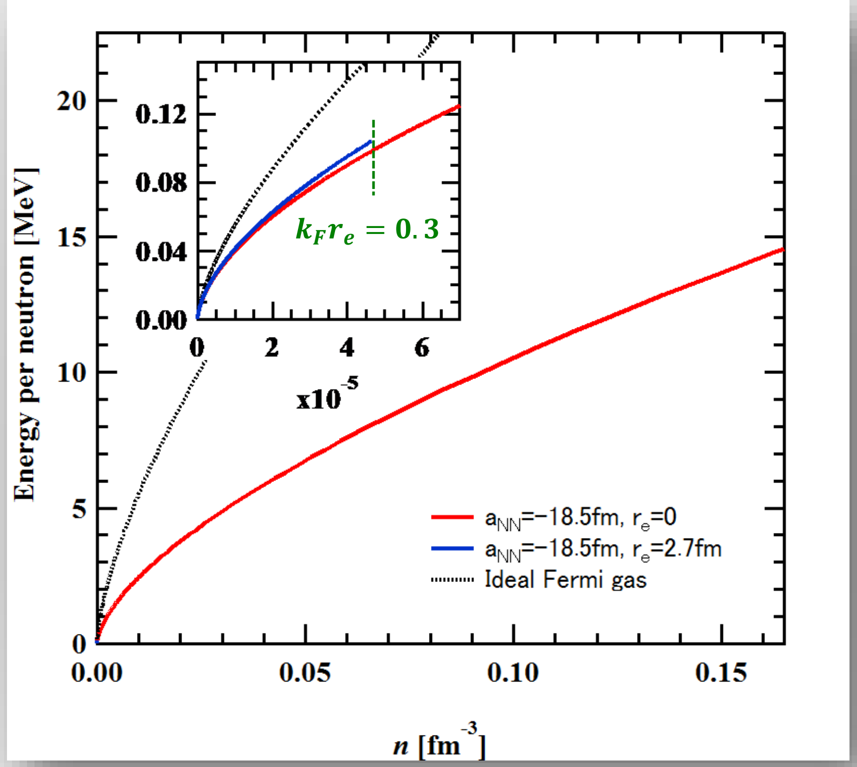
[M.M.Forbes, *et al.*, PRA **86**, 053603 (2012)]

Our result



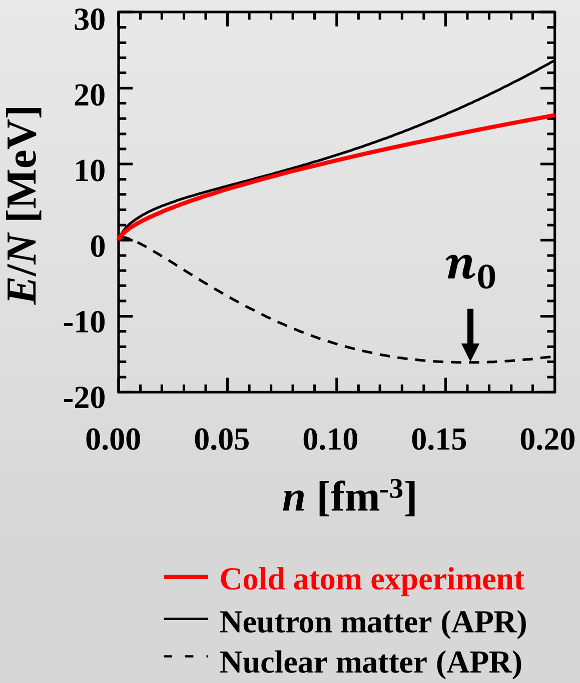
$m_N = 940$ MeV

$a_{NN} = -18.5$ fm

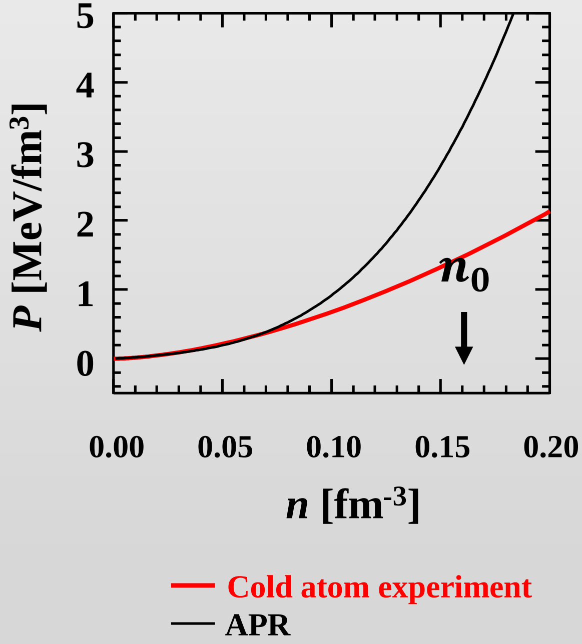


Compasison with a nuclear theory

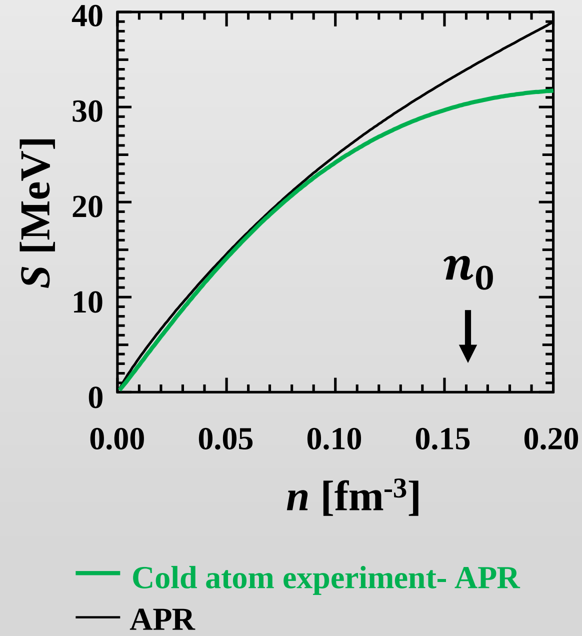
Energy



Pressure



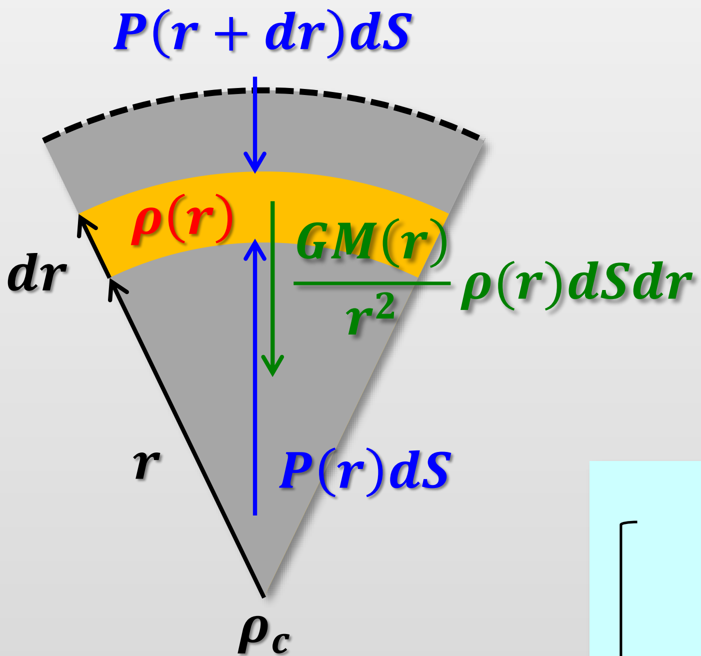
Symmetry Energy

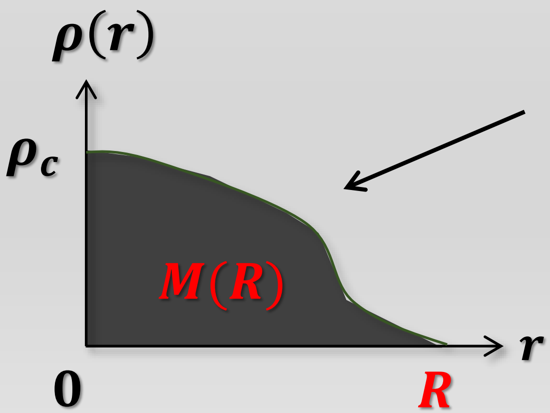


APR : [A. Akmal, Phys. Rev. C 58, 1804 (1998)]

M-R curve for neutron stars

Density distributions $\rho(r)$ in neutron stars are determined by the EOS of neutron star matter





Force balance : $\frac{dP(r)}{dr} = -\frac{G\rho(r)M(r)}{r^2}$

Mass : $M(r) = \int_o^r 4\pi r^2 \rho(r) dr$

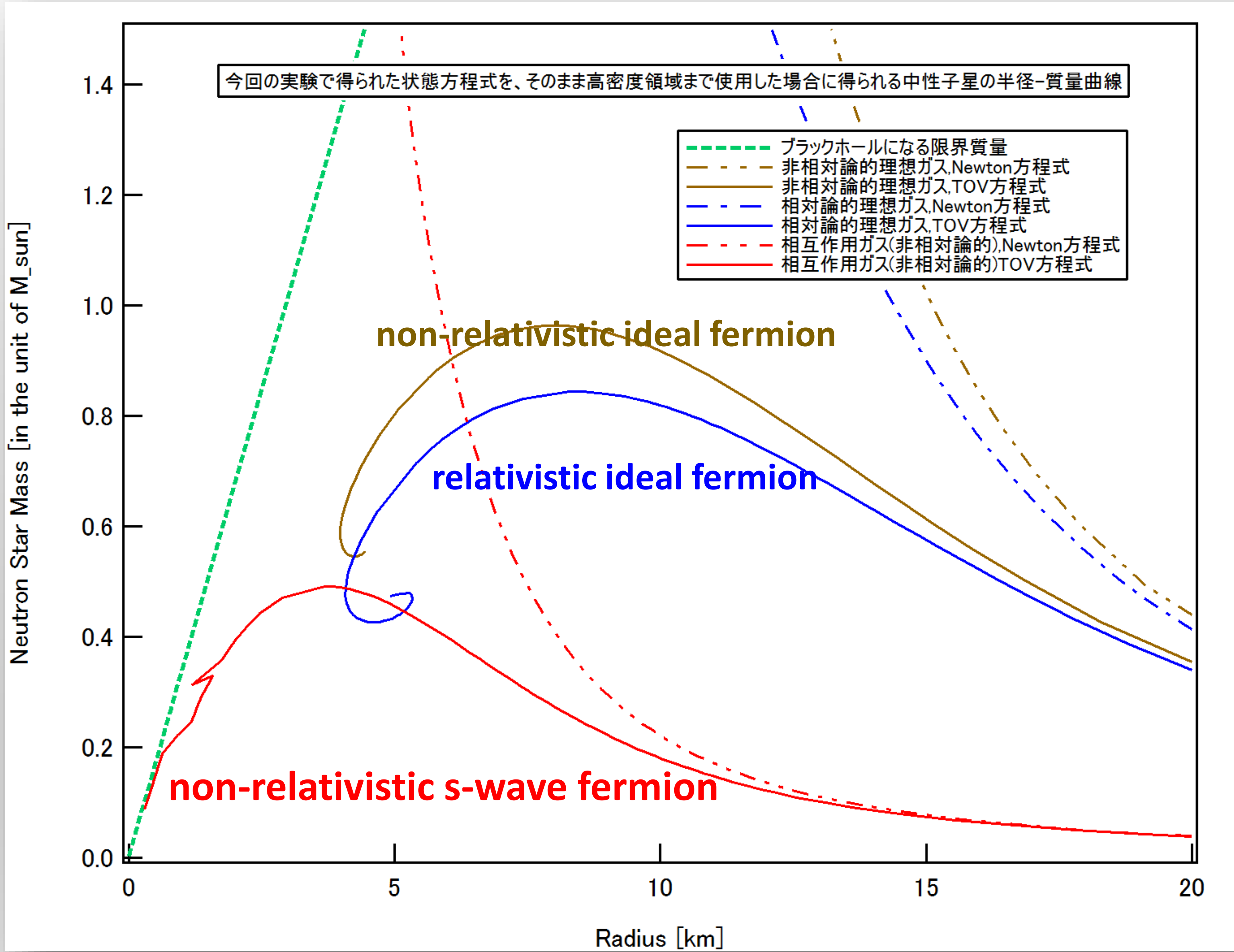
General relativity correction

TOV : $\frac{dP}{dr} = -\frac{G\{m(r) + 4\pi P r^3/c^2\}\{\rho(r) + P/c^2\}}{r^2\{1 - 2Gm(r)/c^2 r\}}$

EOS : $P = \mathcal{F}_P(\rho, (n_\mathrm{N}, n_\mathrm{P}, n_\Lambda, \cdots)) = \rho^2 \frac{\partial \mathcal{F}_E(\rho)}{\partial \rho}$

Mass : $m(r) = \int_o^r 4\pi r^2 \rho(r) dr$

M-R curve from the s-wave EOS



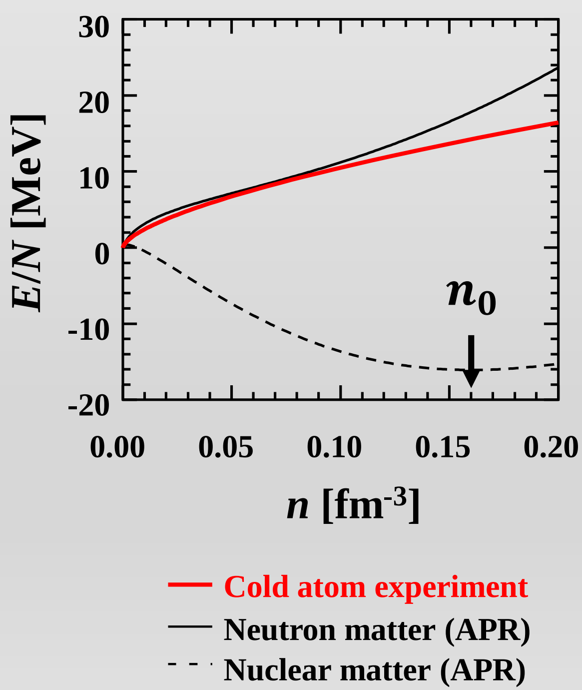
Contents

Discussions

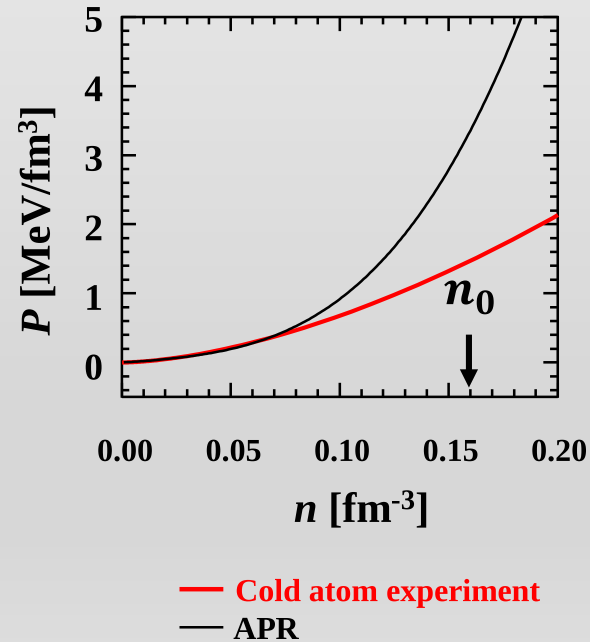
1. Where is the reliable region?

2. Can this EOS constrain the neutron matter EOS?

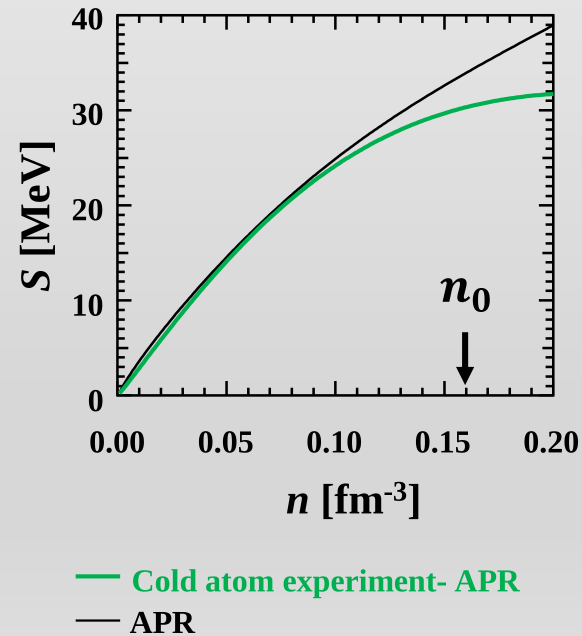
Energy



Pressure



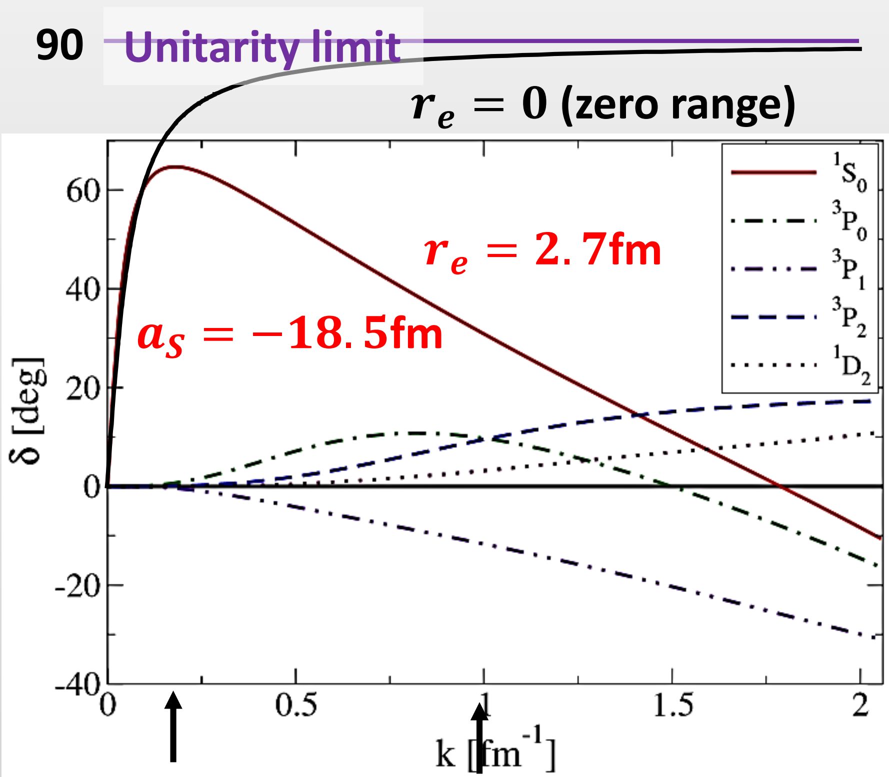
Symmetry Energy



APR : [A. Akmal, Phys. Rev. C 58, 1804 (1998)]

1. Where is the reliable region? → Dilute limit only

We must consider finite range effect

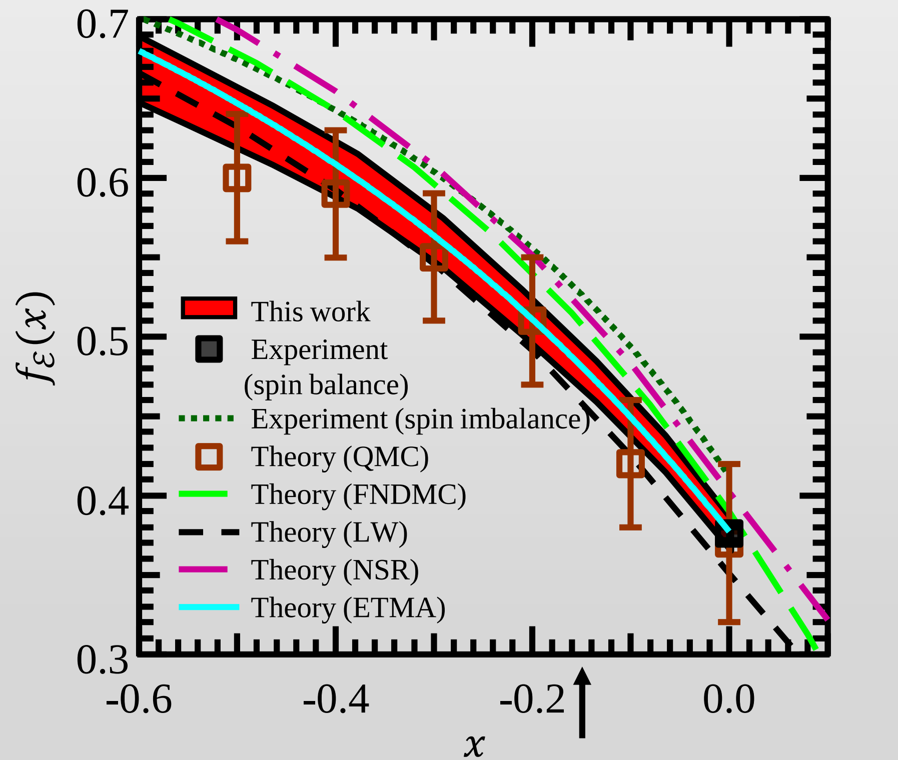


$n_0/100$ **$n_0/5$**

($1/k_F a \sim -0.15$) ($1/k_F a \sim -0.055$)

($k_F r_e \sim 1$) ($k_F r_e \sim 2.7$)

Internal energy density

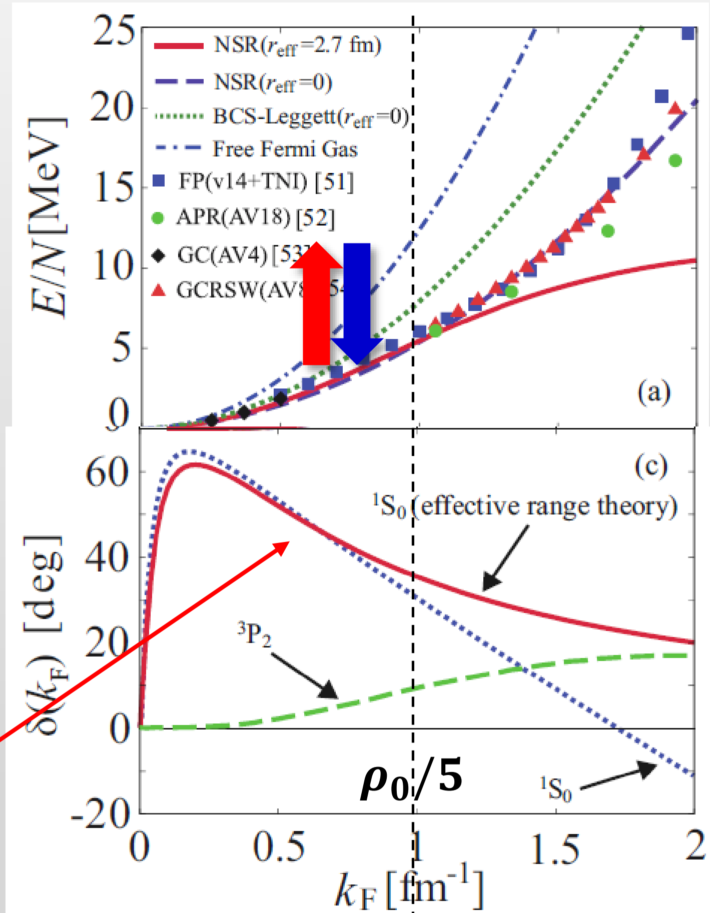


$n_0/100$

2. Can this EOS constrain the neutron matter EOS?

NSR approach

$$\cot\delta_0 = -\frac{1}{ak} + \frac{1}{2}r_e k$$



Pieter van Wyk, Phys. Rev. A **97**, 013601 (2018)

$E(r_e = 2.7\text{fm}) > E(r_e = 0)$ for $\rho < \rho_0/5$

Two key effects of effective range

Cutoff momentum : $p_c = \frac{1}{r_e}\left[1 + \sqrt{1 - \frac{2r_e}{a}}\right]$

Energy increase due to decrease of pairing gap

Pairing interaction is restricted to

$$0 \le p \lesssim p_c$$

Energy decrease due to Hartree energy

$$E_{\rm Hartree} = -U n_\uparrow n_\downarrow = -U n^2$$

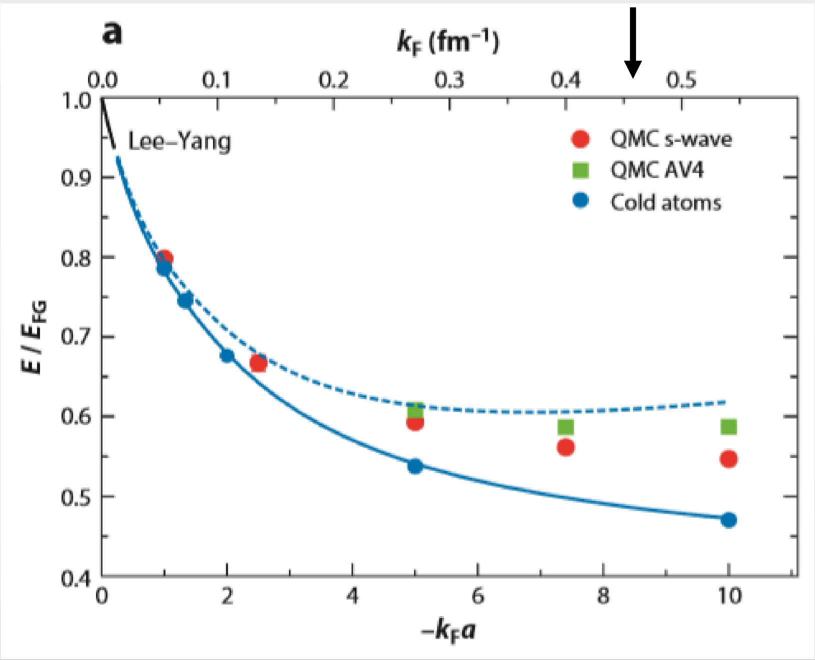
$$U = \frac{4\pi a}{m}\frac{1}{1 - p_c a}$$

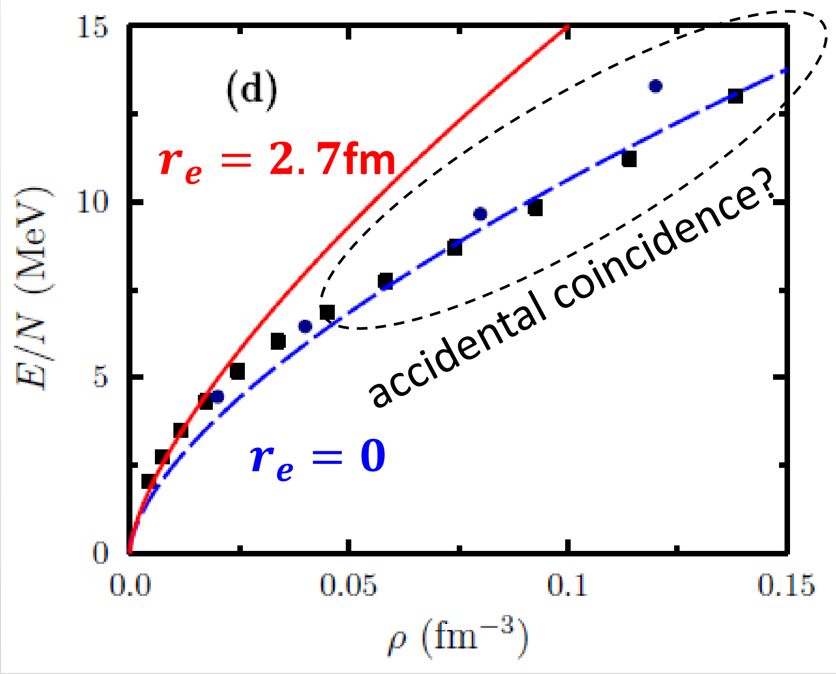
2. Can this EOS constrain the neutron matter EOS?

QMC approach

Density-functional approach

$\rho_0/50$





Stefano Gandolfi, Annual Review of Nuclear and Particle Science 65 (2015): 303-328.

$E(r_e = 2.7\text{fm}) > E(r_e = 0)$ for $\rho < \rho_0/50$

Denis Lacroix, Phys. Rev. C **95**, 054306 (2017)

$E(r_e = 2.7\text{fm}) > E(r_e = 0)$ for $\rho < \rho_0$

Conclusion

1. The determined EOS provides the asymptotic curve of neutron matter at the dilute limit

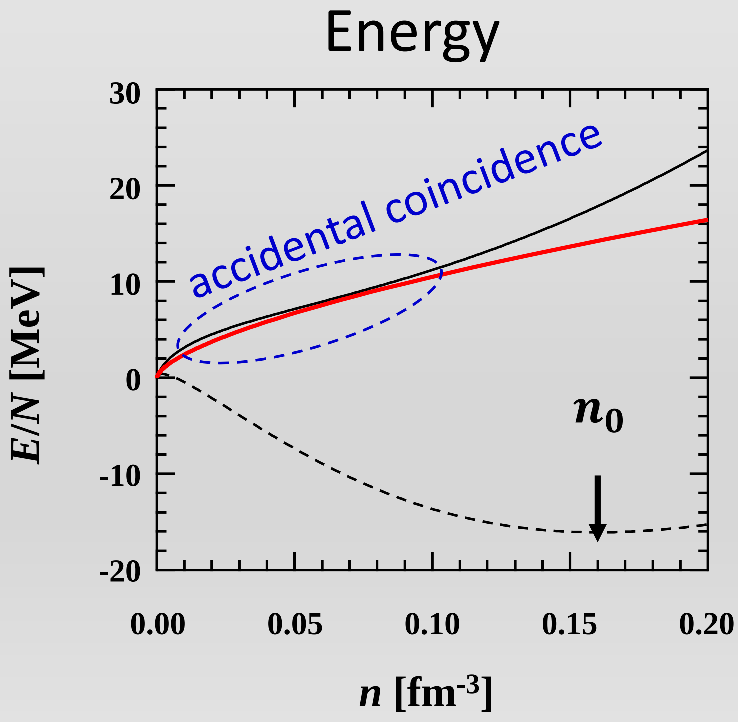
2. The small deviation at low density can be caused by finite range effect

3. Coincidence between the zero-range EOS and the nuclear theory up to $n_0/2$ can be accidental.

4. The EOS gives the lower bound to the neutron matter EOS up to $n_0/5$

5. Future experiments will draw the correct EOS curve

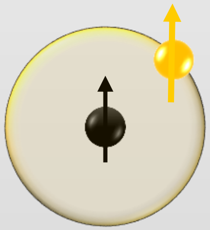
"Optical control of the scattering length and effective range for magnetically tunable Feshbach resonances in ultracold gases", Haibin Wu and J. E. Thomas, Phys. Rev. A **86**, 063625 (2012)



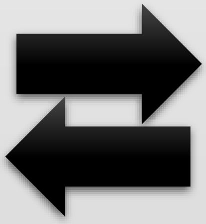
Summary

This is one of the nice examples of cold atom quantum simulator to progress nuclear physics

Atomic gas (^{6}Li)

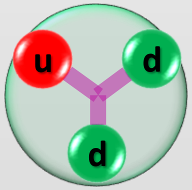


Experiment
Theory



New physics

Neutron matter



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2. J. Phys. Soc. Jpn. 86, 104301 (2017).
3. Journal of Physics B: Atomic, Molecular and Optical Physics, 50(1), 01LT01 (2017).
4. Phys. Rev. A 95, 043625 (2017).
5. Journal of Low Temperature Physics, 1-8 (2016).