Overview and Outlook on the theoretical calculations of the π^0 radiative decay width

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Outline:

- π^0 decay modes
- Theoretical overview
- $\pi^0 \to \gamma\gamma$ (in col. with Bachir Moussallam)
- Discussion – Outlook
- Summary

Decays of π^0

neutral pion: the lightest hadron
rich experimental activity: Hades (see I.Ciepal), KLOE-2 (see S. Giovannella), A2 (E. Mornacchi), JLab (see L. Gan), NA62, ...
accessible also at lattice (see A. Gérardin)

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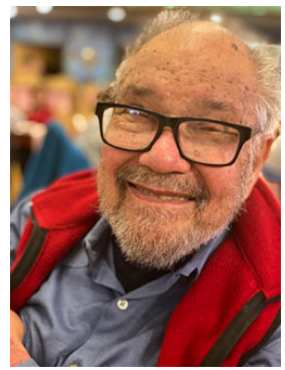
[Bernstein, Holstein: Neutral Pion Lifetime Measurements and the QCD Chiral Anomaly]
[Meißner: Theory and Experiment – A Tribute to Aron Bernstein]



1931-2020 RIP

Decays of π^0 within SM

Five measured decays (with corresponding Γ_i/Γ)

- $\pi^0 \to \gamma\gamma$ (98.8%)
- $\pi^0 \to e^+e^-\gamma$ (1.17%) – *see talk by T.Husek*
  - $\pi^0 \to$ positronium γ (2×10^{-9})
- $\pi^0 \to e^+e^-e^+e^-$ (3×10^{-5})
- $\pi^0 \to e^+e^-$ (6×10^{-8}) – *see talk by Bai-Long Hoid*

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other possible decay modes

- $\pi^0 \to 4\gamma$ (theory vs. exp: 2×10^{-11} vs $< 2 \times 10^{-8}$)
- $\pi^0 \to \nu\bar{\nu}$
  helicity supression – limit: $< 4 \times 10^{-9}$ (NA62 '21, 60x improvement), theory: $< 5 \times 10^{-10}$ (direct) or $< 10^{-24}$ (cosmology)
- $\pi^0 \to \nu\bar{\nu}\gamma$ (Wolfram mode)
  SM: $\sim 10^{-18}$, exp.limit $< 2 \times 10^{-7}$ (NA62 '19)

Theoretical background

Prehistory of $\pi^0 \to \gamma\gamma$

- one of the most "important" process in particle physics
- connected with gauge principle (VVP vs VVA) and chiral anomaly
- J. Steinberger 1949: first calculation
- D. Sutherland 1967, M. Veltman 1967 theorem
- S. Adler 1969; J. Bell, R. Jackiw 1969 anomaly
- no quantum corrections to anomaly
- chiral correction to $\pi^0 \to \gamma\gamma$ (this talk)

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1939-2023 RIP

On behalf of the MIT Physics community, Department Head Deepto Chakrabarty is saddened to announce the death late yesterday, June 14, 2023, of eminent theoretical physicist and longtime Center for Theoretical Physics faculty member Roman Jackiw.

A detailed obituary is in preparation, and will be published in MIT News online (news.mit.edu).

EFT $\rightarrow$ ChPT

EFT

- separated degrees of freedom (simplification)
- building the most general Lagrangian
- ordering principle (powercounting)

example: ChPT

- goldstone bosons (spontaneous symmetry breakdown of chiral symmetry)
- momentum power counting
- Lagrangian up to NNNLO

ChPT: effective theory of low-energy QCD

[Coleman, WZ, Callan '69]: nonlinear realization of SSB

$$u(\phi) \to g_R u h^\dagger = h u g_L^\dagger$$

construction up to NNNLO: [Weinberg '79], [Gasser, Leutwyler '84], [Bijnens, Colangelo, Ecker '99], [Bijnens, Hermansson-Truedsson, Wang '19]

$$\mathcal{L}_{\chi PT} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \mathcal{L}^{(8)}$$

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$$\mathcal{L}_{\chi PT} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \mathcal{L}^{(8)}$$

$$\mathcal{L}^{(2)} = \frac{F_0^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle$$

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$$\mathcal{L}_{\chi PT} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \mathcal{L}^{(8)}$$

$$\begin{aligned}
\mathcal{L}^{(4)} = {} & L_1 \langle u_\mu u^\mu \rangle^2 + L_2 \langle u_\mu u_\nu \rangle \langle u^\mu u^\nu \rangle + L_3 \langle u^\mu u_\mu u^\nu u_\nu \rangle + L_4 \langle u^\mu u_\mu \rangle \langle \chi_+ \rangle \\
& + L_5 \langle u^\mu u_\mu \chi_+ \rangle + L_6 \langle \chi_+ \rangle^2 + L_7 \langle \chi_- \rangle^2 + L_8 \langle \chi_+ \chi_- \rangle \\
& - i L_9 \langle F^R_{\mu\nu} u u^\mu u^\nu u^\dagger + F^L_{\mu\nu} u^\dagger u^\mu u^\nu u \rangle + L_{10} \langle F^R_{\mu\nu} U F^{L\mu\nu} U^\dagger \rangle \\
& + H_1 \langle F^R_{\mu\nu} F^{\mu\nu R} + F^L_{\mu\nu} F^{\mu\nu L} \rangle + H_2 \langle \chi_+^2 - \chi_-^2 \rangle / 4
\end{aligned}$$

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$$\mathcal{L}_{\chi PT} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \mathcal{L}^{(8)}$$

$\mathcal{L}_\chi^{(6)} = C_1 \langle u \cdot u \nabla_\mu u_\nu \nabla^\mu u^\nu \rangle + \ldots$ (together 94 terms!)

ChPT: effective theory of low-energy QCD

[Coleman, WZ, Callan '69]: nonlinear realization of SSB

$$u(\phi) \to g_R u h^\dagger = h u g_L^\dagger$$

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$$\mathcal{L}_{\chi PT} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \mathcal{L}^{(8)}$$

$\mathcal{L}_\chi^{(8)} = C_1^8 \langle \nabla_\mu u^\nu \nabla_\rho u^\sigma \rangle \langle \nabla_\nu u^\mu \nabla_\sigma u^\rho \rangle + \ldots$ (together 1254 terms!)

ChPT: effective theory of low-energy QCD

[Coleman, WZ, Callan '69]: nonlinear realization of SSB

$$u(\phi) \to g_R u h^\dagger = h u g_L^\dagger$$

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$$\mathcal{L}_{\chi PT} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \mathcal{L}^{(8)}$$

fun fact: one type (n-pions starting at $O(p^n)$) possible to all orders in a closed form:

$$\mathcal{L}^n_{\chi\text{PT}} = \sum_{j=1}^{d_n} c_j \langle u_{\mu_{j_1}} \dots u^{\mu_{j_1}} \dots u_{\mu_{j_{n/2}}} \dots u^{\mu_{j_{n/2}}} \rangle$$

see e.g. [Brown, KK, Oktem, Paranjape, Trnka'23]

Odd sector

So far, the effective Lagrangian has a larger symmetry than QCD.

$\phi \leftrightarrow -\phi$ in this Lagrangian: only even number of GB

odd intrinsic parity is however not symmetry of original QCD

from phenomenology we know it exists: $K^+K^- \to 3\pi$

$\Rightarrow$ symmetry pattern of QCD must be studied more carefully

Odd sector

- first we need to add EM interaction:
  $\partial_\mu U \to D_\mu U = \partial_\mu U + i[U, v_\mu], \qquad v_\mu \sim Q A_\mu$
- and add by hands monomial to $\mathcal{L}$:

$$U F_{\mu\nu} \tilde{F}^{\mu\nu}$$

  U can be transformed out: we have to add (at least two) derivatives on U – vanishes in chiral limit (Sutherland theorem)

way out: anomaly, in fact two anomalies

$$\partial^\mu A^i_\mu \neq 0$$

(non-trivial for $i = 0, 3, 8$, or for π^0, η, η' states)

incorporated to the action by Wess, Zumino and Witten (WZW)
two-flavour case: [Kaiser'01]

Calculation within 2-flavour ChPT: Anomaly

The form of anomaly is determined by 'gauging' external fields, where v and a are gauge fields of

$$L = R = 1 + i\alpha, \quad \text{and} \quad L^\dagger = R = 1 + i\beta$$

Bardeen '69:

$$\delta S\{v, a, s, p, \theta\} = -\frac{N_C}{16\pi^2} \int d^4x\, \epsilon^{\mu\nu\rho\sigma} \langle \hat{\beta}(\hat{v}_{\mu\nu} + i[\hat{a}_\mu, \hat{a}_\nu])\rangle \langle v_{\rho\sigma}\rangle$$

$$\mathcal{L}_{\text{WZW}} = -\frac{N_C}{32\pi^2}\, \epsilon^{\mu\nu\rho\sigma} \left\{ \langle U^\dagger \hat{r}_\mu U \hat{l}_\nu - \hat{r}_\mu \hat{l}_\nu + i\Sigma_\mu (U^\dagger \hat{r}_\nu U + \hat{l}_\nu)\rangle \langle v_{\rho\sigma}\rangle + \tfrac{2}{3} \langle \Sigma_\mu \Sigma_\nu \Sigma_\rho \rangle \langle v_\sigma \rangle \right\}$$

with $\Sigma_\mu = U^\dagger \partial_\mu U, \quad \hat{r}_\mu = \hat{v}_\mu + \hat{a}_\mu, \quad \hat{l}_\mu = \hat{v}_\mu - \hat{a}_\mu.$

Calculation within 2-flavour ChPT

NLO odd-intrinsic Lagrangian is given by [Bijnens, Girlanda, Talavera '02]

$$\mathcal{L}_6^W = \sum_{i=1}^{13} c_i^W o_i^W \,, \qquad c_i^W = c_i^{Wr} + \eta_i (c\mu)^{d-4} \Lambda \,,$$

monomial (o_i^W)	i 2-flavour	$384\pi^2 F^2 \eta_i$
$\epsilon^{\mu\nu\alpha\beta}\langle\chi_+ [f_{-\mu\nu}, u_\alpha u_\beta]\rangle$	1	0
$\epsilon^{\mu\nu\alpha\beta}\langle\chi_- \{f_{+\mu\nu}, u_\alpha u_\beta\}\rangle$	2	0
$i\epsilon^{\mu\nu\alpha\beta}\langle\chi_- f_{+\mu\nu} f_{+\alpha\beta}\rangle$	3	0
$i\epsilon^{\mu\nu\alpha\beta}\langle\chi_- f_{-\mu\nu} f_{-\alpha\beta}\rangle$	4	0
$i\epsilon^{\mu\nu\alpha\beta}\langle\chi_+ [f_{+\mu\nu}, f_{-\alpha\beta}]\rangle$	5	0
$\epsilon^{\mu\nu\alpha\beta}\langle f_{+\mu\nu}\rangle\langle\chi_- u_\alpha u_\beta\rangle$	6	$-5N_C$
$i\epsilon^{\mu\nu\alpha\beta}\langle f_{+\mu\nu}\rangle\langle f_{+\alpha\beta}\chi_-\rangle$	7	$4N_C$
$i\epsilon^{\mu\nu\alpha\beta}\langle f_{+\mu\nu}\rangle\langle f_{+\alpha\beta}\rangle\langle\chi_-\rangle$	8	$-2N_C$
$i\epsilon^{\mu\nu\alpha\beta}\langle f_{+\gamma\mu}\rangle\langle h_{\gamma\nu} u_\alpha u_\beta\rangle$	9	$2N_C$
$i\epsilon^{\mu\nu\alpha\beta}\langle f_{+\gamma\mu}\rangle\langle f_{-\gamma\nu} u_\alpha u_\beta\rangle$	10	$-6N_C$
$\epsilon^{\mu\nu\alpha\beta}\langle f_{+\mu\nu}\rangle\langle f_{+\gamma\alpha} h_{\gamma\beta}\rangle$	11	$4N_C$
$\epsilon^{\mu\nu\alpha\beta}\langle f_{+\mu\nu}\rangle\langle f_{+\gamma\alpha} f_{-\gamma\beta}\rangle$	12	0
$\epsilon^{\mu\nu\alpha\beta}\langle\nabla_\gamma f_{+\gamma\mu}\rangle\langle f_{+\nu\alpha} u_\beta\rangle$	13	$-4N_C$

n.b. it depends on the form of $\mathcal{L}_4$ [KK, Novotny 02], [Ananth.,Moussallam 02]

$\pi^0 \to \gamma\gamma$ in the formal ChPT expansion

$\pi^0 \to \gamma\gamma$: LO

- π^0 lightest hadron

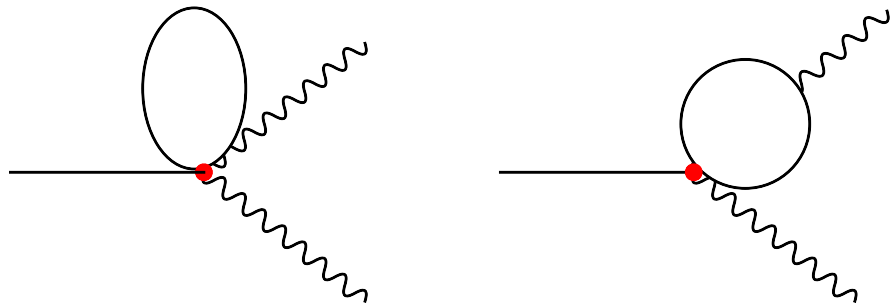
  $\Rightarrow$ primary decay mode $\pi^0 \to \gamma\gamma$

- in chiral limit exact due to QCD axial anomaly:

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{m_{\pi^0}^3}{64\pi}\left(\frac{\alpha N_C}{3\pi F_\pi}\right)^2 = 7.73\,eV$$

$\pi^0 \to \gamma\gamma$: Correction to the current algebra prediction

- using [Pagels and Zepeda '72] sum rules in [Kitazawa '85]
- NLO corrections are hidden in $F \to F_\pi$ and $O(p^6)$ LECs [Donoghue, Holstein, Lin '85] [Bijnens, Bramon, Cornet '88]



- in 3-flavour case we can study π^0, η, η' mixing, resulting to [Goity, Bernstein, Holstein '02]:

$$\Gamma^{\text{NLO}} = 8.1 \pm 0.08\,\text{eV}$$

in 2-flavour case EM corrections [Ananth., Moussallam '02]:

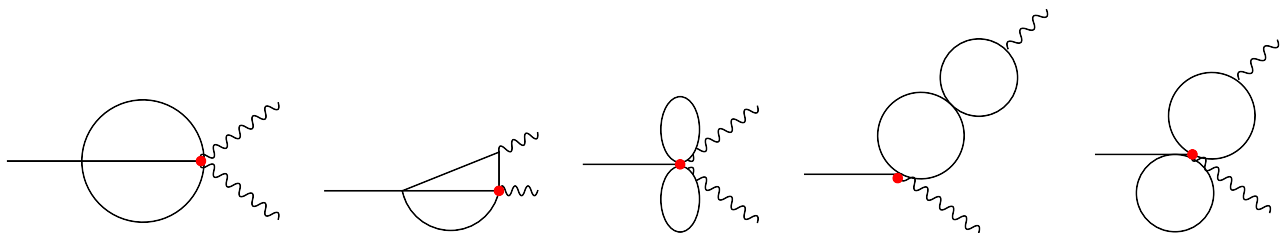
$$\Gamma^{\text{NLO}} = 8.06 \pm 0.02 \pm 0.06\,\text{eV}$$

- study based on dispersion relations [Ioffe, Oganesian '07]:

$$\Gamma^{\text{NLO}} = 7.93 \pm 0.11\,\text{eV}$$

$\pi^0 \to \gamma\gamma$ at NNLO in 2 flavour ChPT [KK, Moussallam'09]

- NLO: a) One-loop diagrams with one vertex from $\mathcal{L}^{WZ}$, b) tree diagrams with one vertex from $\mathcal{L}^{WZ}$ and one vertex from $O(p^4)$ Lagrangian, c) tree diagrams with one vertex from $O(p^6)$ anomalous-parity sector
- $O(p^6)$ anomalous-parity sector from [Bijnens, Girlanda, Talavera '02]
- representation of chiral field: $U = \sigma + i\frac{\tau\cdot\pi}{F}$, $\sigma = \sqrt{1-\vec{\pi}^2/F^2}$ (no $\gamma 4\pi$ vertex at LO)
- two-loop



- verification of Z-factor, F_π/F [Bürgi '96], [Bijnens, Colangelo, Ecker, Gasser, Sainio '02]
- double log checked by Weinberg consistency relations

$\pi^0 \to \gamma\gamma$ at NNLO, result

$$A_{NNLO} = \frac{e^2}{F_\pi}\Big\{\frac{1}{4\pi^2}$$
$$+\frac{16}{3}m_\pi^2(-4c_3^{Wr}-4c_7^{Wr}+c_{11}^{Wr})+\frac{64}{9}B(m_d-m_u)(5c_3^{Wr}+c_7^{Wr}+2c_8^{Wr})$$
$$-\frac{M^4}{24\pi^2F^4}\left(\frac{1}{16\pi^2}L_\pi\right)^2+\frac{M^4}{16\pi^2F^4}L_\pi\left[\frac{3}{256\pi^4}+\frac{32F^2}{3}\left(2c_2^{Wr}+4c_3^{Wr}+2c_6^{Wr}+4c_7^{Wr}-c_{11}^{Wr}\right)\right]$$
$$+\frac{32M^2B(m_d-m_u)}{48\pi^2F^4}L_\pi\left[-6c_2^{Wr}-11c_3^{Wr}+6c_4^{Wr}-12c_5^{Wr}-c_7^{Wr}-2c_8^{Wr}\right]$$
$$+\frac{M^4}{F^4}\lambda_+ +\frac{M^2B(m_d-m_u)}{F^4}\lambda_- +\frac{B^2(m_d-m_u)^2}{F^4}\lambda_{--}\Big\}$$

$$\lambda_+ = \frac{1}{\pi^2}\left[-\frac{2}{3}d_+^{Wr}(\mu)-8c_6^r-\frac{1}{4}(l_4^r)^2+\frac{1}{512\pi^4}\left(-\frac{983}{288}-\frac{4}{3}\zeta(3)+3\sqrt{3}\,\mathrm{Cl}_2(\pi/3)\right)\right]$$
$$+\frac{16}{3}F^2\left[8l_3^r(c_3^{Wr}+c_7^{Wr})+l_4^r(-4c_3^{Wr}-4c_7^{Wr}+c_{11}^{Wr})\right]$$
$$\lambda_- = \frac{64}{9}\left[d_-^{Wr}(\mu)+F^2l_4^r\,(5c_3^{Wr}+c_7^{Wr}+2c_8^{Wr})\right]$$
$$\lambda_{--} = d_{--}^{Wr}(\mu)-128F^2l_7(c_3^{Wr}+c_7^{Wr})\ .$$

- 4 LECs in 2 combinations of NLO
- additional 4 LECs in 3 combinations of NNLO

Is it at all possible to make some reliable prediction?

$\pi^0 \to \gamma\gamma$: modified counting

- Use of $SU(3)$ phenomenology via $c_i^{Wr} \leftrightarrow C_i^{Wr}$ connection (based on [Gasser, Haefeli, Ivanov, Schmid '07,'08])

$$c_i^{Wr} = \frac{\alpha_i}{m_s} + \left(\beta_i + \gamma_{ij} C_j^{Wr} + \delta_i \ln \frac{B_0 m_s}{\mu^2}\right) + O(m_s)$$

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- implementation of modified counting

$$m_u, m_d \sim O(p^2) \quad \text{and} \quad m_s \sim O(p)$$

Result:

$$A_{NNLO}^{mod} = \frac{e^2}{F_\pi}\left\{\frac{1}{4\pi^2} - \frac{64}{3} m_\pi^2 C_7^{Wr} + \frac{1}{16\pi^2}\frac{m_d - m_u}{m_s}\left[1 - \frac{3}{2}\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\right]\right.$$
$$+ 32B(m_d - m_u)\left[\frac{4}{3} C_7^{Wr} + 4C_8^{Wr}\left(1 - 3\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\right)\right.$$
$$\left.\left. - \frac{1}{16\pi^2 F_\pi^2}\left(3L_7^r + L_8^r - \frac{1}{512\pi^2}(L_K + \frac{2}{3} L_\eta)\right)\right] - \frac{1}{24\pi^2}\left(\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\right)^2\right\}$$

$\pi \to \gamma\gamma$: Phenomenology inputs

- $F_\pi = 92.22 \pm 0.07$ MeV (using updated value of V_{ud} [Towner, Hardy'08]). using quark mass ratio (from lattice), pseudo-scalar meson masses, R from $\eta \to 3\pi$
- $\frac{m_d - m_u}{m_s} = (2.29 \pm 0.23)\, 10^{-2}$
- $B(m_d - m_u) = (0.32 \pm 0.03)\, M_{\pi^0}^2$
- $3L_7 + L_8^r(\mu) = (0.10 \pm 0.06)\, 10^{-3} \quad (\mu = M_\eta)$ (from pseudo-scalar meson masses formula [Gasser, Leutwyler '85])
- $C_7^W = 0$ (more precisely $C_7^W \ll C_8^W$, motivated by simple resonance saturation)
- $C_8^W = (0.58 \pm 0.2) 10^{-3} \text{GeV}^{-2}$ (from $\eta \to 2\gamma$)

result [KK, Moussallam'09]: $\Gamma_{\pi^0 \to 2\gamma} = (8.09 \pm 0.11)$ eV

Theoretical contemplations

$\pi^0 \to \gamma\gamma$: comments

1) F_π is a crucial ingredient

- F_π vs $\hat{F}_\pi$ [Bernard, Oertel, Passemar, Stern '08]
- using $\pi^0 \to \gamma\gamma$:

$$F_\pi = 93.85 \pm 1.4\,\text{MeV}$$

$$\text{cf with } \hat{F}_\pi = 92.22(7)$$

$$(1.2\sigma \text{ difference})$$

- our F_π from PDG is based on π_{l2} and SM using [Marciano, Sirlin'93]
- important input V_{ud}: new update by [Hardy,Towner '20]

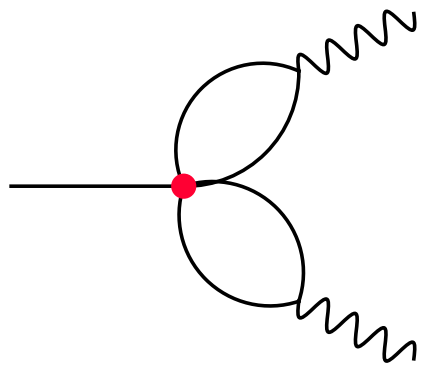
$$0.97418(26) \to 0.97373(31)$$

we should change $F_\pi \to 92.3 \ldots$

$\pi^0 \to \gamma\gamma$: comments

2) $\eta \to \gamma\gamma$:

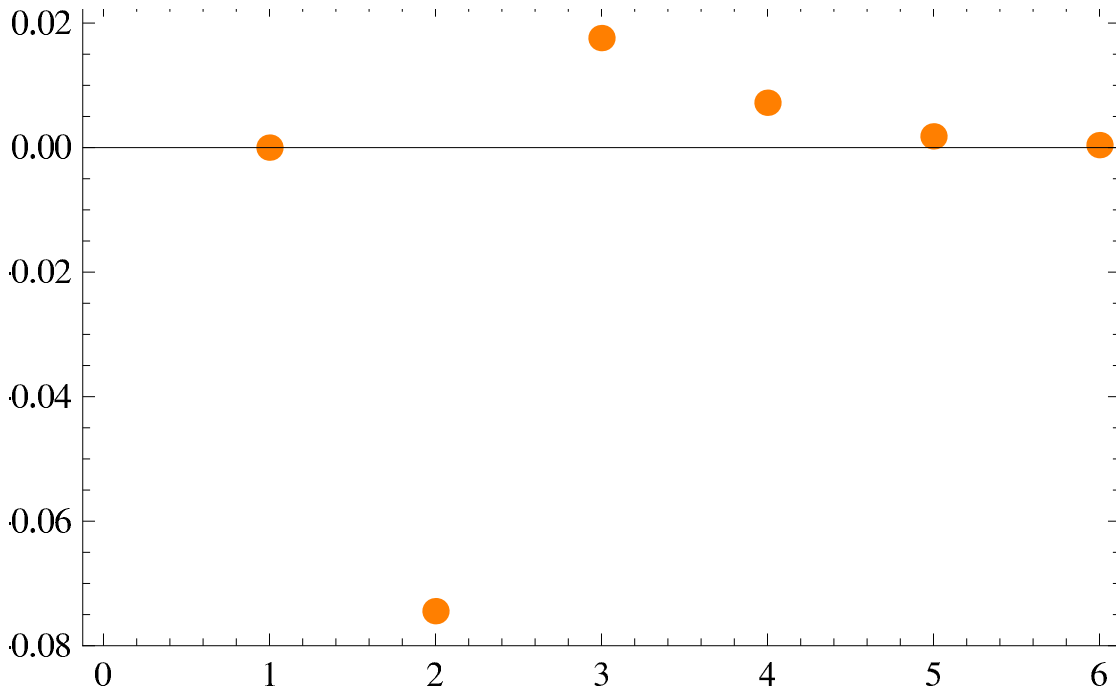
- important ingredient ($\to$ constant C_8)
- plan to calculate it also at two loop
- ongoing project with J.Bijnens...



$\pi^0 \to \gamma\gamma$: comments

3) deeper theoretical thoughts

a) Leading logarithm contribution of individual orders in percent of the leading order: [Bijnens,KK,Lanz'12]



$\pi^0 \to \gamma\gamma$: comments

3) deeper theoretical thoughts
b) surprising connections [Bijnens,KK,Lanz'12] (see also Bai-Long Hoid)

Adler-Lee-Treiman-Zee-Terentev theorem on triangle and box anomaly

$$F^{3\pi}(0,0,0) = \frac{1}{eF_\pi^2} F_{\pi\gamma\gamma}(0,0)$$

is valid up to two loops for leading logs beyond the soft-photon limit

$\pi^0 \to \gamma\gamma$: comments

4) deeper theoretical thoughts
c) new theoretical activity on ChPT

- amplitude methods – cf. [KK '21 – The ChPT: top-down and bottom-up]
- huge activity on positivity bounds (restarted recently by: [Adams, Arkani-Hamed, Dubovsky, Nicolis, and Rattazzi '06: Causality, analyticity and an IR obstruction to UV completion])
- another footprint of QCD at low energy, we might have missed?

Summary

of $\Gamma(\pi^0 \to \gamma\gamma)$

$$\text{theory: } (8.09 \pm 0.11) \text{ eV}$$
$$\text{PrimEx I+II: } (7.80 \pm 0.12) \text{ eV}$$

or equivalently π^0 lifetime:

$$\text{theory: } 8.04 \pm 0.11 \times 10^{-17} \text{ s}$$
$$\text{PrimEx I+II: } 8.34 \pm 0.13 \times 10^{-17} \text{ s}$$
$$\longrightarrow 1.8\ \sigma \text{ discrepancy}$$

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Thank you for your attention!

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and pushing us to understand it better