Probing Low-Energy QCD, Fundamental Symmetry and BSM Physics with π^0, η and η' mesons

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Workshop on Precision Tests of Fundamental Physics with Light Mesons
ECT*,Trento, Italy, June 12 -16, 2023

Based on *Phys. Rept. 945 (2022), 1-105,*
L. Gan, B. Kubis, E.P., S. Tulin

Outline

1. Introduction and Motivation

2. $\eta \to 3\pi$: light quark mass extraction and
   test of C & CP violation

3. Test of CP violation (P & CP violation) in $\eta \to \mu^+\mu^-$

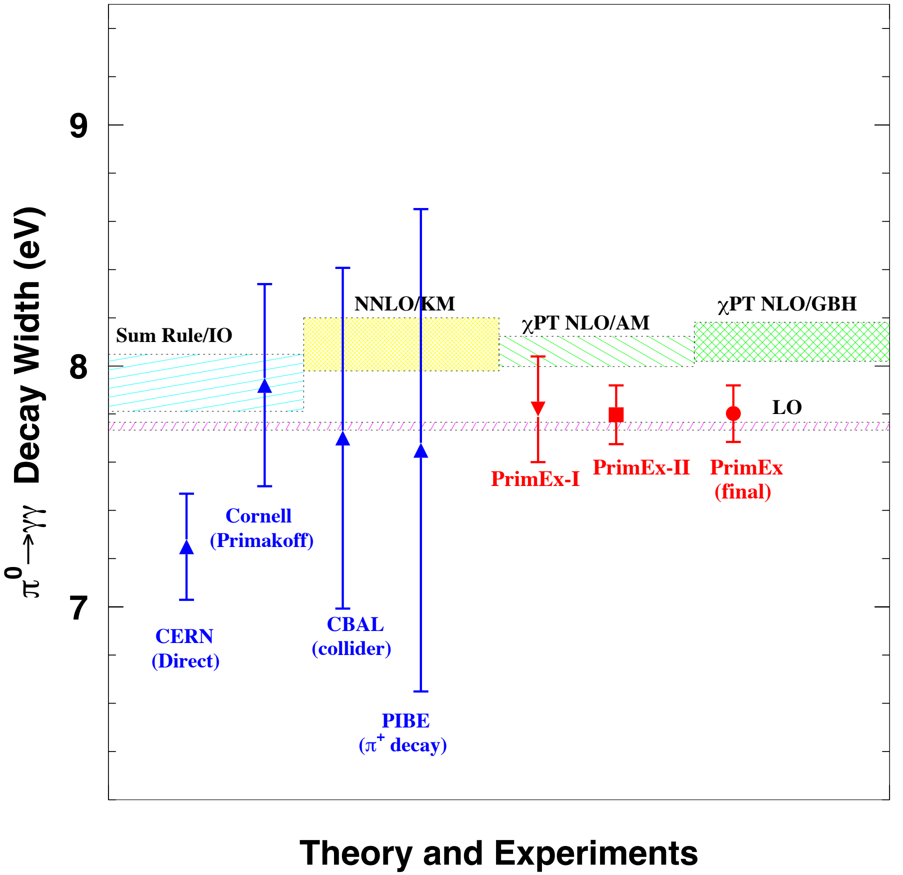
4. $\eta' \to \eta\pi\pi$ and chiral dynamics

5. Conclusion and Outlook

1. Introduction and Motivation

1.1 Why is it interesting to study π^0, η and η' physics?

- π^0 is the pseudo-Goldstone boson of chiral perturbation theory
- It is one of the most fundamental degree of freedom
- There are still some puzzles about this particle:



Gan, Kubis, E. P., Tulin'22

See talks by *K. Kampf*
L. Gan

From L.Gan

1.1 Why is it interesting to study η and η' physics?

- Quantum numbers $I^G J^{PC} = 0^+ 0^{-+}$
  - *C*, *P* eigenstates, all additive quantum numbers are zero
  - flavour-conserving laboratory for symmetry tests

- η: pseudo-Goldstone boson, $\boldsymbol{M_\eta = 547.862(17)\ \mathrm{MeV}}$, $\Gamma_\eta = 1.31$ keV

  All decay modes forbidden at leading order by *symmetries* (C, P, angular momentum, isospin/G-parity. . .)

- η': not a Goldstone boson due to $U(1)_A$ anomaly $\boldsymbol{M_{\eta'} = 957.78(6)\ \mathrm{MeV}}$
  $\Gamma_{\eta'} = 196$ keV

- Theoretical methods:
  - (large-N_c) chiral perturbation theory, RChPT
  - dispersion relations to resum final state interactions
  - Vector-meson dominance

1.1 Why is it interesting to study η and η' physics?

- In the study of η and η' physics, large amount of data have been collected:

  ⇨ *CBall, WASA, KLOE & KLOEII, BESIII, A2@MAMI, CLAS, GlueX*

  More to come: *JEF, REDTOP (Elam et al'22), LHCb?, JLab@22GeV*

  ⇨ See talk by *S. Taylor*

- Unique opportunity:
  - Test chiral dynamics at low energy
  - Extract fundamental parameters of the Standard Model:
    ex: light quark masses
  - Study of fundamental symmetries: P & CP and C & CP violation
  - Looking for beyond Standard Model Physics ⇨ Dark Sector

Rich physics program at η,η' factories

Standard Model highlights
- Theory input for light-by-light scattering for $(g\text{-}2)_\mu$
- Extraction of light quark masses
- QCD scalar dynamics

Fundamental symmetry tests
- P,CP violation
- C,CP violation

[Kobzarev & Okun (1964), Prentki & Veltman (1965), Lee (1965), Lee & Wolfenstein (1965), Bernstein et al (1965)]

Dark sectors (MeV—GeV)
- Vector bosons
- Scalars
- Pseudoscalars (ALPs)

(Plus other channels that have not been searched for to date)

From S.Tulin

Channel	Expt. branching ratio	Discussion
$\eta \to 2\gamma$	39.41(20)%	chiral anomaly, η–η' mixing
$\eta \to 3\pi^0$	32.68(23)%	$m_u - m_d$
$\eta \to \pi^0\gamma\gamma$	$2.56(22) \times 10^{-4}$	χPT at $O(p^6)$, leptophobic B boson, light Higgs scalars
$\eta \to \pi^0\pi^0\gamma\gamma$	$< 1.2 \times 10^{-3}$	χPT, axion-like particles (ALPs)
$\eta \to 4\gamma$	$< 2.8 \times 10^{-4}$	$< 10^{-11}$[52]
$\eta \to \pi^+\pi^-\pi^0$	22.92(28)%	$m_u - m_d$, C/CP violation, light Higgs scalars
$\eta \to \pi^+\pi^-\gamma$	4.22(8)%	chiral anomaly, theory input for singly-virtual TFF and $(g-2)_\mu$, P/CP violation
$\eta \to \pi^+\pi^-\gamma\gamma$	$< 2.1 \times 10^{-3}$	χPT, ALPs
$\eta \to e^+e^-\gamma$	$6.9(4) \times 10^{-3}$	theory input for $(g-2)_\mu$, dark photon, protophobic X boson
$\eta \to \mu^+\mu^-\gamma$	$3.1(4) \times 10^{-4}$	theory input for $(g-2)_\mu$, dark photon
$\eta \to e^+e^-$	$< 7 \times 10^{-7}$	theory input for $(g-2)_\mu$, BSM weak decays
$\eta \to \mu^+\mu^-$	$5.8(8) \times 10^{-6}$	theory input for $(g-2)_\mu$, BSM weak decays, P/CP violation
$\eta \to \pi^0\pi^0\ell^+\ell^-$		C/CP violation, ALPs
$\eta \to \pi^+\pi^-e^+e^-$	$2.68(11) \times 10^{-4}$	theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, ALPs
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$\eta \to e^+e^-e^+e^-$	$2.40(22) \times 10^{-5}$	theory input for $(g-2)_\mu$
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$\eta \to \mu^+\mu^-\mu^+\mu^-$	$< 3.6 \times 10^{-4}$	theory input for $(g-2)_\mu$
$\eta \to \pi^+\pi^-\pi^0\gamma$	$< 5 \times 10^{-4}$	direct emission only
$\eta \to \pi^\pm e^\mp \nu_e$	$< 1.7 \times 10^{-4}$	second-class current
$\eta \to \pi^+\pi^-$	$< 4.4 \times 10^{-6}$	P/CP violation
$\eta \to 2\pi^0$	$< 3.5 \times 10^{-4}$	P/CP violation
$\eta \to 4\pi^0$	$< 6.9 \times 10^{-7}$	P/CP violation

Gan, Kubis, E. P., Tulin'22

Rich physics program at η,η' factories

From S.Tulin

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Gan, Kubis, E. P., Tulin'22

2. $\eta \to 3\pi$: light quark mass extraction and test of C & CP violation

In collaboration with G. Colangelo, S. Lanz and H. Leutwyler (ITP-Bern)

Phys. Rev. Lett. 118 (2017) no.2, 022001
Eur.Phys.J. C78 (2018) no.11, 947

⇨ See talks by *I. Danilkin* and *M. Albaladejo*

2.1 Decays of η

$M_\eta = 547.862(17)$ MeV

- η decay from PDG:

η DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
	Neutral modes		
Γ_1	neutral modes	(72.12 ± 0.34) %	S=1.2
Γ_2	2γ	(39.41 ± 0.20) %	S=1.1
Γ_3	$3\pi^0$	(32.68 ± 0.23) %	S=1.1
	Charged modes		
Γ_8	charged modes	(28.10 ± 0.34) %	S=1.2
Γ_9	$\pi^+\pi^-\pi^0$	(22.92 ± 0.28) %	S=1.2
Γ_{10}	$\pi^+\pi^-\gamma$	$(\ 4.22\pm0.08)$ %	S=1.1

2.1 Why is it interesting to study $\eta \to 3\pi$?

- Decay forbidden by isospin symmetry $\eta(I^G = 0^+) \to 3\pi(I^G = 1^-)$

$$A = (m_u - m_d) A_1 + \alpha_{em} A_2$$

- α_{em} effects are small

  Sutherland'66, Bell & Sutherland'68
  Baur, Kambor, Wyler'96, Ditsche, Kubis, Meissner'09

- Decay rate measures the size of isospin breaking ($m_u - m_d$) in the SM:

$$L_{QCD} \to L_{IB} = -\frac{m_u - m_d}{2}\left(\bar{u}u - \bar{d}d\right)$$

Unique access to ($m_u - m_d$)

2.2 Quark mass ratio

- In the following, extraction of Q from $\eta \to \pi^+ \pi^- \pi^0$

$$\Gamma_{\eta\to\pi^+\pi^-\pi^0} = \frac{1}{Q^4}\frac{M_K^4}{M_\pi^4}\frac{\left(M_K^2 - M_\pi^2\right)^2}{6912\pi^3 F_\pi^4 M_\eta^3}\int_{s_{\min}}^{s_{\max}} ds \int_{u_-(s)}^{u_+(s)} du \, \left|M(s,t,u)\right|^2$$

Determined from experiment

Determined from:
- Dispersive calculation
- ChPT

Fit to Dalitz distr.

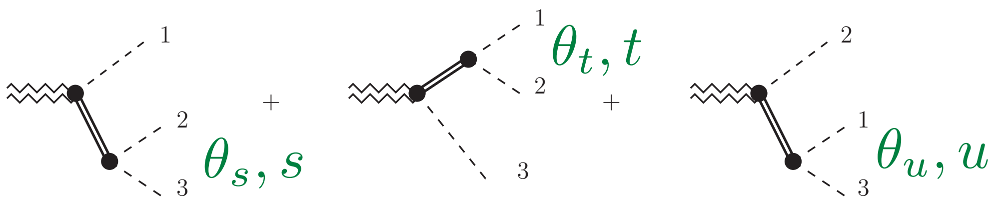
$$\left[Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} \right]$$

$$\left[\hat{m} \equiv \frac{m_d + m_u}{2} \right]$$

- Aim: Compute M(s,t,u) with the *best accuracy*

2.3 Method

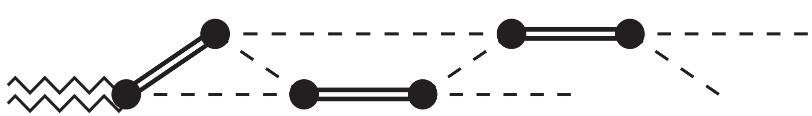
- Decompose amplitide in partial waves:



- Usual assumption: 3 BWs (ρ^+, ρ^-, ρ^0) + background term

⇨ Improve to include final states interactions + restore unitarity

- Use a *Khuri-Treiman* approach or *dispersive* approach
  Restore 3 body unitarity and take into account the final state interactions in a systematic way



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2.3 Representation of the amplitude

- Decomposition of the amplitude as a function of isospin states

$$M(s,t,u) = M_0(s) + (s-u)M_1(t) + (s-t)M_1(u) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)$$

Fuchs, Sazdjian & Stern'93
Anisovich & Leutwyler'96

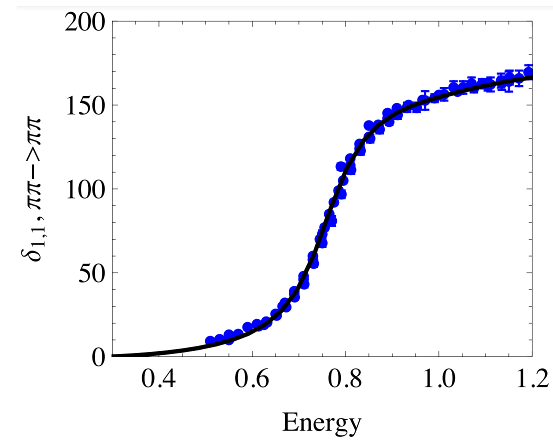
  - M_I isospin *I* rescattering in two particles
  - Amplitude in terms of S and P waves ⇨ exact up to NNLO ($\mathcal{O}(p^6)$)
  - Main two body rescattering corrections inside M_I

2.3 Representation of the amplitude

- Decomposition of the amplitude as a function of isospin states

$$M(s,t,u) = M_0^0(s) + (s-u)M_1^1(t) + (s-t)M_1^1(u) + M_0^2(t) + M_0^2(u) - \frac{2}{3}M_0^2(s)$$

Roy analysis
Colangelo et al.'01



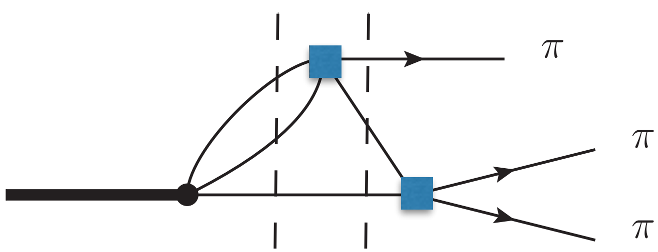
input

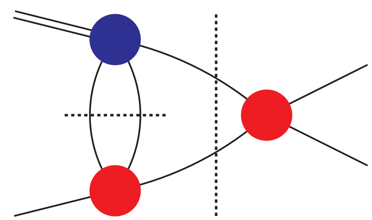
- Unitarity relation:

$\pi\pi \to \pi\pi$

$$disc\left[M_\ell^I(s)\right] = \rho(s)t_\ell^*(s)\left(M_\ell^I(s) + \hat{M}_\ell^I(s)\right)$$

right-hand cut   left-hand cut





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- Unitarity relation:

$$disc\left[M_\ell^I(s)\right] = \rho(s)t_\ell^*(s)\left(M_\ell^I(s) + \hat{M}_\ell^I(s)\right)$$

- Relation of dispersion to reconstruct the amplitude everywhere:

$$M_I(s) = \Omega_I(s)\left(P_I(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'^n}\frac{\sin\delta_I(s')\,\hat{M}_I(s')}{|\Omega_I(s')|\left(s'-s-i\varepsilon\right)}\right)$$

$$\left[\Omega_I(s) = \exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}ds'\frac{\delta_I(s')}{s'(s'-s-i\varepsilon)}\right)\right]$$

Omnès function

Gasser & Rusetsky'18

- $P_I(s)$ determined from a fit to NLO ChPT + experimental Dalitz plot

Subtraction constants

- Extension of the numbers of parameters compared to *Anisovich & Leutwyler'96*

$$P_0(s) = \alpha_0 + \beta_0 s + \gamma_0 s^2 + \delta_0 s^3$$
$$P_1(s) = \alpha_1 + \beta_1 s + \gamma_1 s^2$$
$$P_2(s) = \alpha_2 + \beta_2 s + \gamma_2 s^2$$

- In the work of *Anisovich & Leutwyler'96* matching to one loop ChPT
  Use of the SU(2) x SU(2) chiral theorem
  ⇨ The amplitude has an *Adler zero* along the line s=u

- Now data on the Dalitz plot exist from KLOE, WASA, MAMI and BES III
  ⇨ Use the data to directly fit the subtraction constants

- However normalization to be fixed to ChPT!

Subtraction constants

- The subtraction constants are

$$P_0(s) = \alpha_0 + \beta_0 s + \gamma_0 s^2 + \delta_0 s^3$$
$$P_1(s) = \alpha_1 + \beta_1 s + \gamma_1 s^2$$
$$P_2(s) = \alpha_2 + \beta_2 s + \gamma_2 s^2 + \delta_0 s^3$$

Only *6 coefficients* are of physical relevance

- They are determined from combining ChPT with a fit to KLOE Dalitz plot

- Taylor expand the dispersive M_I
  Subtraction constants ⇔ Taylor coefficients

$$M_0(s) = A_0 + B_0 s + C_0 s^2 + D_0 s^3 + \ldots$$
$$M_1(s) = A_1 + B_1 s + C_1 s^2 + \ldots$$
$$M_2(s) = A_2 + B_2 s + C_2 s^2 + D_2 s^3 +$$

- Gauge freedom in the decomposition of M(s,t,u)

Subtraction constants

- Build some gauge independent combinations of Taylor coefficients

$$
\begin{aligned}
H_0 &= A_0 + \frac{4}{3}A_2 + s_0\left(B_0 + \frac{4}{3}B_2\right) \\
H_1 &= A_1 + \frac{1}{9}\left(3B_0 - 5B_2\right) - 3C_2 s_0 \\
H_2 &= C_0 + \frac{4}{3}C_2, \qquad H_3 = B_1 + C_2 \\
H_4 &= D_0 + \frac{4}{3}D_2, \qquad H_5 = C_1 - 3D_2
\end{aligned}
$$

⇨

$$H_0^{ChPT} = 1 + 0.176 + O\left(p^4\right)$$

$$h_1^{ChPT} = \frac{1}{\Delta_{\eta\pi}}\left(1 - 0.21 + O\left(p^4\right)\right)$$

$$h_2^{ChPT} = \frac{1}{\Delta_{\eta\pi}^2}\left(4.9 + O\left(p^4\right)\right)$$

$$h_3^{ChPT} = \frac{1}{\Delta_{\eta\pi}^2}\left(1.3 + O\left(p^4\right)\right)$$

$$\left[h_i \equiv \frac{H_i}{H_0} \right]$$

⇨

$$\chi^2_{theo} = \sum_{i=1}^{3}\left(\frac{h_i - h_i^{ChPT}}{\sigma_{h_i^{ChPT}}}\right)^2$$

$$\sigma_{h_i^{ChPT}} = 0.3\left|h_i^{NLO} - h_i^{LO}\right|$$

Isospin breaking corrections

- Dispersive calculations in the isospin limit ⇨ to fit to data one has to include isospin breaking corrections

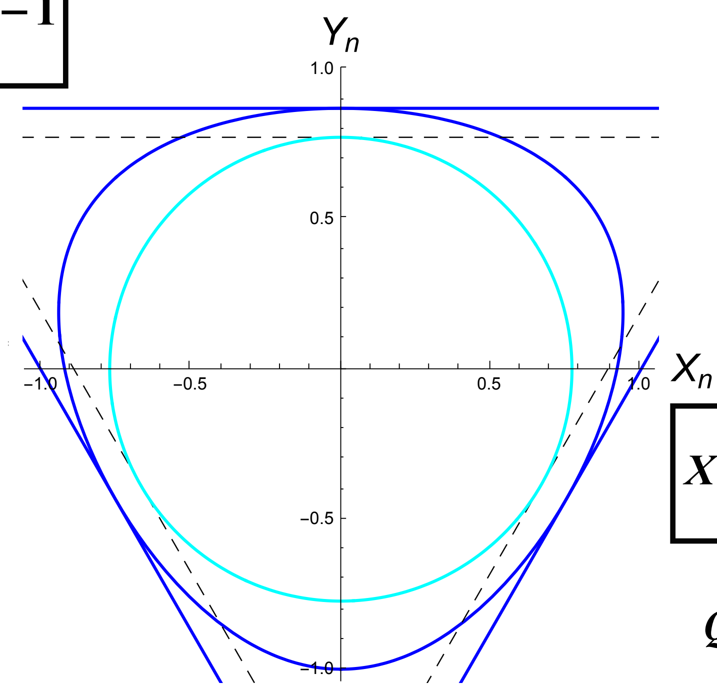
- $$M_{c/n}(s,t,u) = M_{disp}(s,t,u)\frac{M_{DKM}(s,t,u)}{\tilde{M}_{GL}(s,t,u)}$$

with M_{DKM} : amplitude at one loop with $\mathcal{O}(e^2 m)$ effects

Ditsche, Kubis, Meissner'09

$$Y_n = \frac{3T_3}{Q_n} - 1$$

Neutral channel



M_{GL}: amplitude at one loop in the isospin limit

Gasser & Leutwyler' 85

Kinematic map:
isospin symmetric boundaries
⇨ physical boundaries

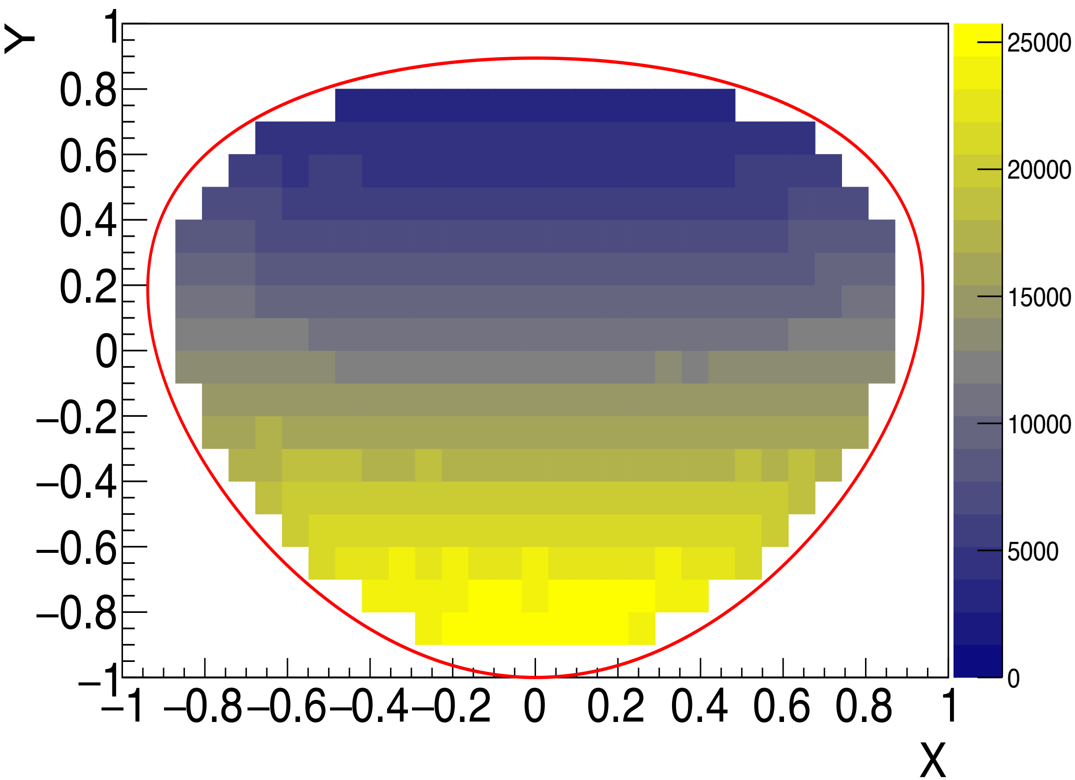
$$X_n = \sqrt{3}\frac{T_2 - T_1}{Q_n}$$

$$M_{GL} \rightarrow \tilde{M}_{GL}$$

$$Q_n \equiv M_\eta - 3M_{\pi^0}$$

2.4 $\eta \to 3\pi$ Dalitz plot

- In the charged channel: experimental data from *WASA, KLOE, BESIII*



KLOE'16

$$\left|A(s,t,u)\right|^2 = N\left(\begin{array}{l}1 + aY + bY^2 \\ +dX^2 + fY^3 + \ldots\end{array}\right)$$

$$X = \sqrt{3}\,\frac{T_+ - T_-}{Q_c} = \frac{\sqrt{3}}{2M_\eta Q_c}(u-t)$$

$$Y = \frac{3T_0}{Q_c} - 1 = \frac{3}{2M_\eta Q_c}\left(\left(M_\eta - M_{\pi^0}\right)^2 - s\right) - 1$$

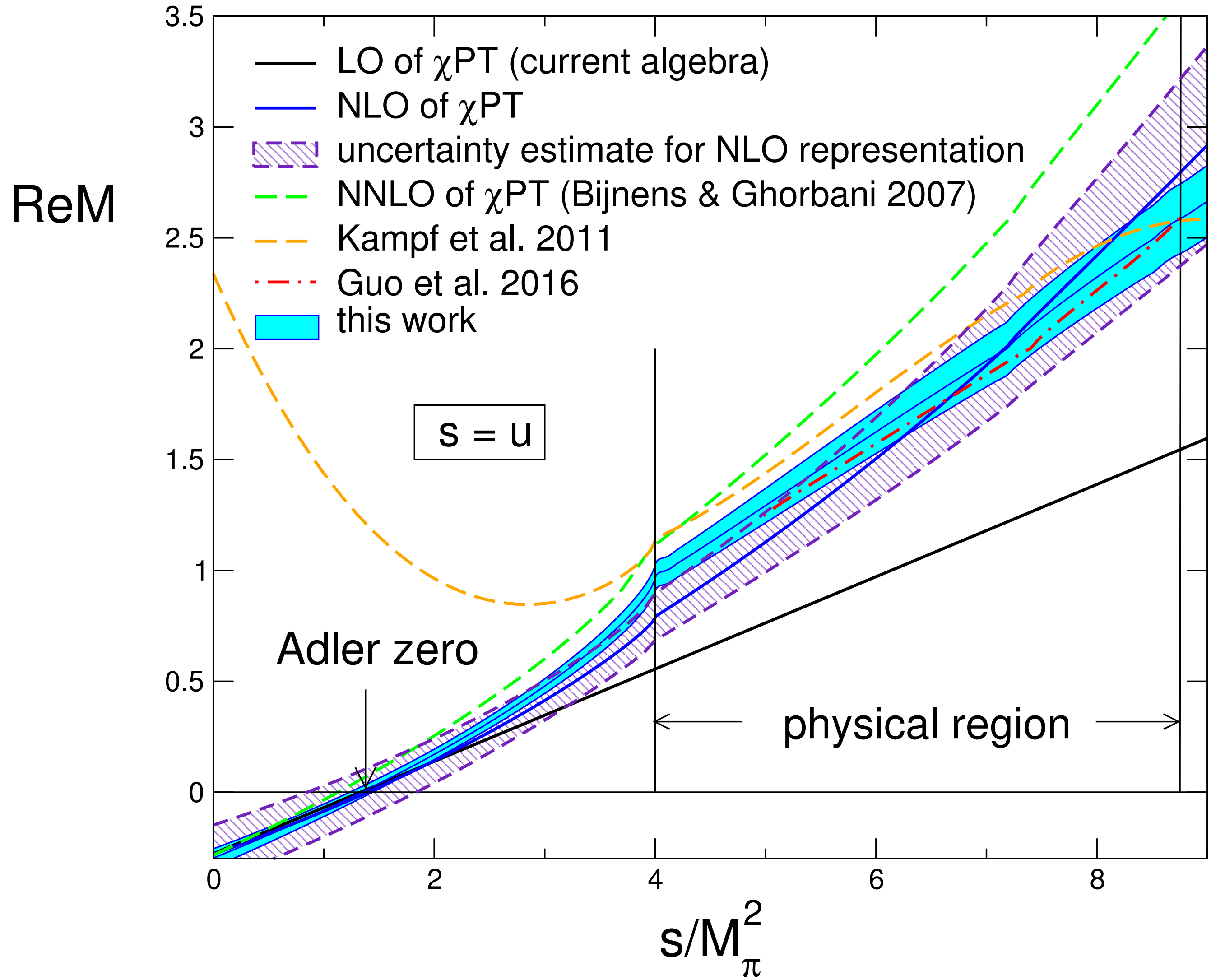
➡ See talk by *S. Giovannella*

- New data expected from *CLAS* and *GlueX* with very different systematics

➡ See talk by *S. Taylor*

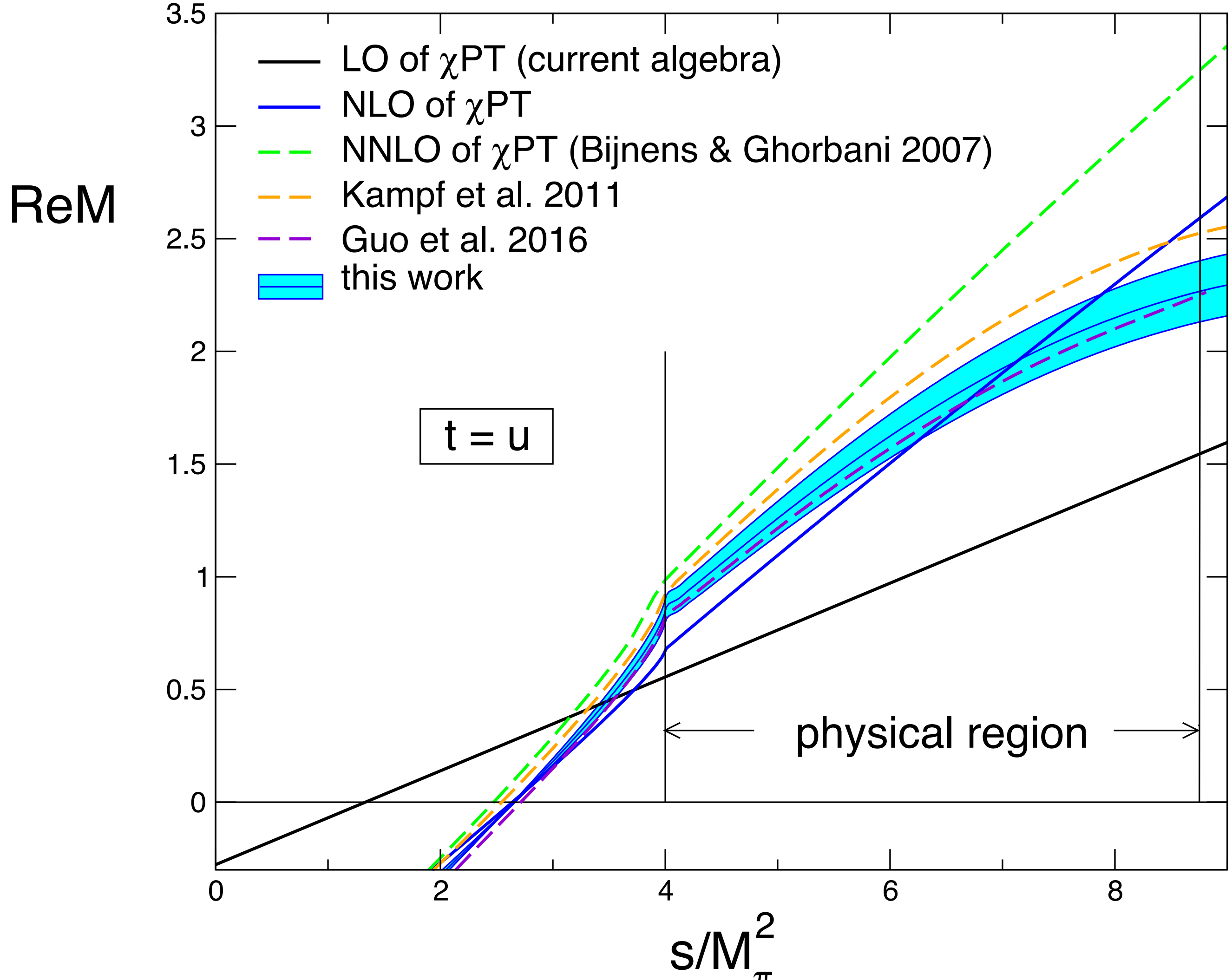
2.5 Results: Amplitude for $\eta \rightarrow \pi^+ \pi^- \pi^0$ decays

- The amplitude along the line s = u :

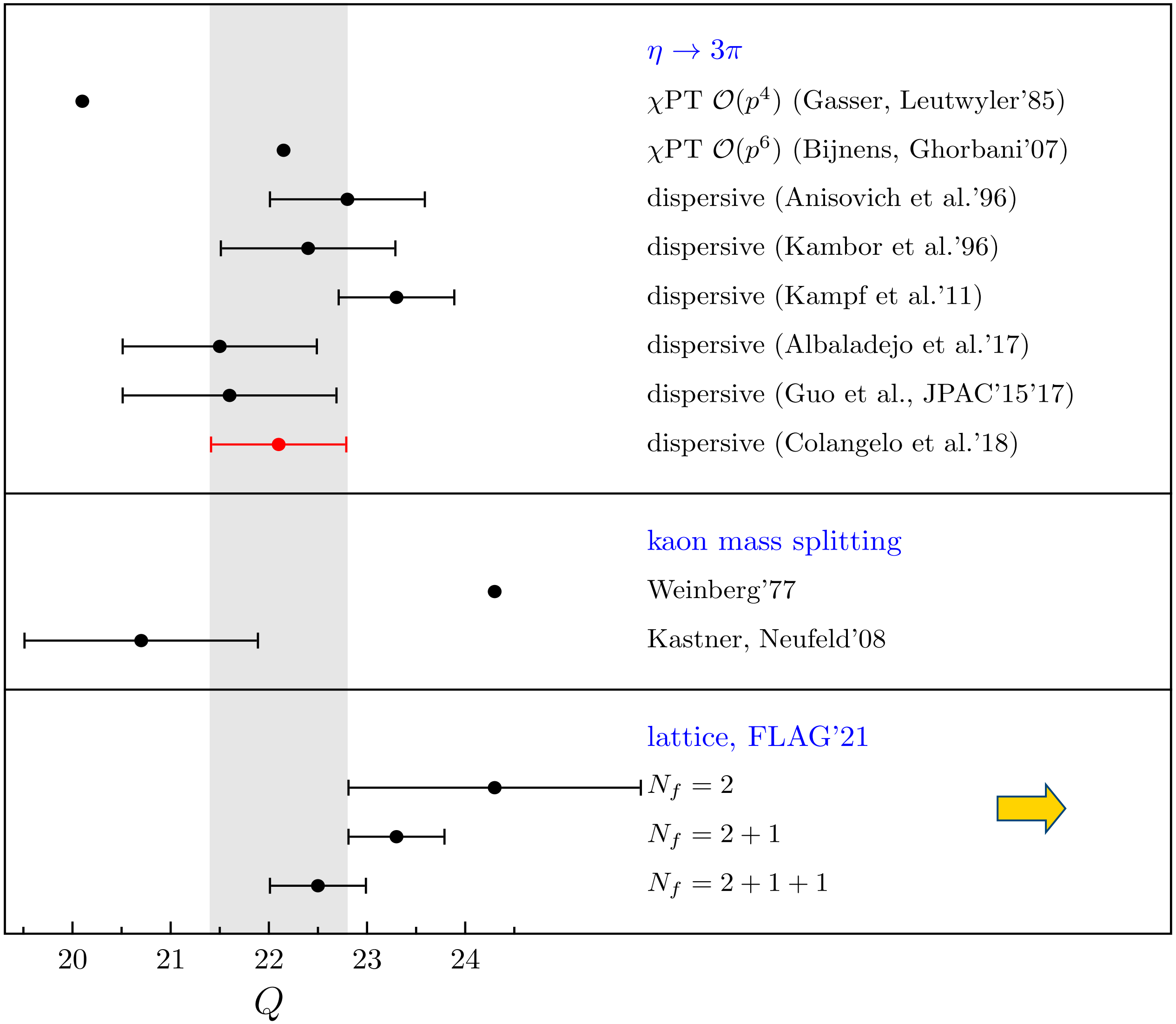


2.5 Results: Amplitude for $\eta \to \pi^+ \pi^- \pi^0$ decays

- The amplitude along the line t = u :



Quark mass ratio



$Q = 22.1 \pm 0.7$

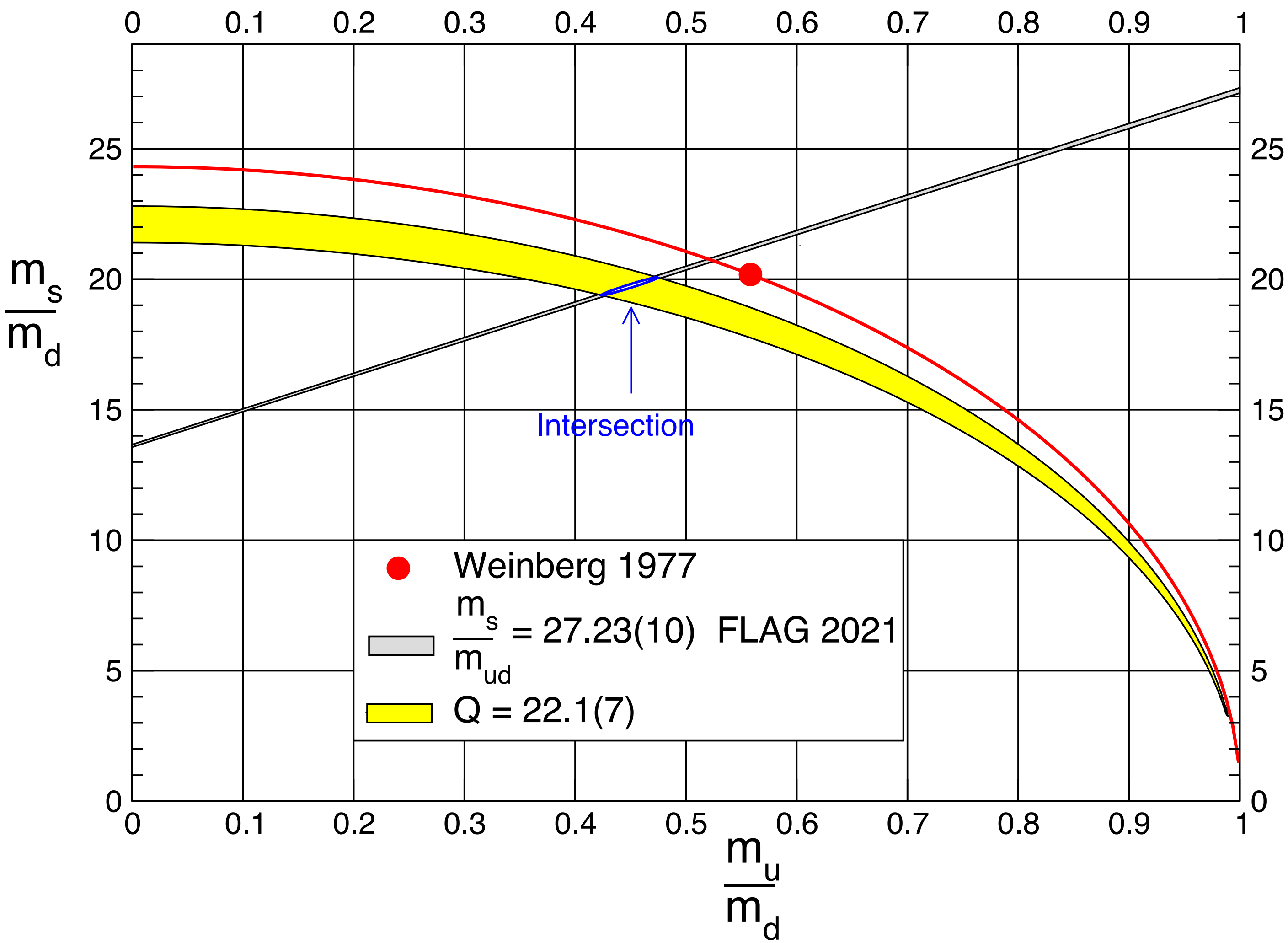
New lattice results

Shift of Q towards smaller values

Better *agreement* with $\eta \to 3\pi$ result

- Experimental systematics needs to be taken into account

Light quark masses



$Q = 22.1 \pm 0.7$

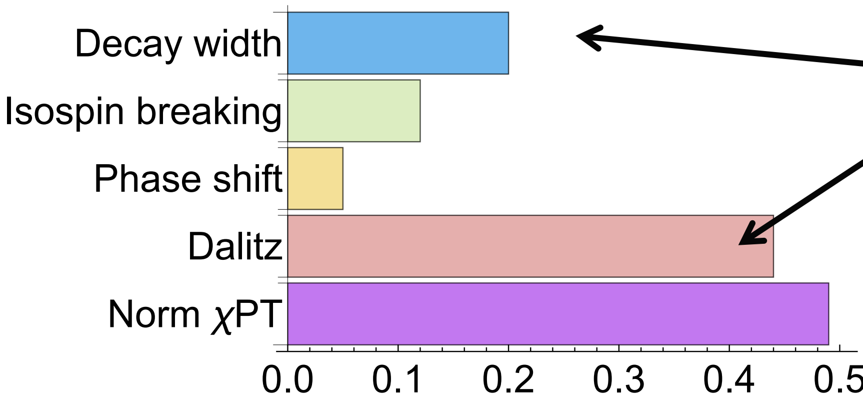
$\frac{m_u}{m_d} = 0.44 \pm 0.03$

- Smaller values for Q ⇨ smaller values for m_s/m_d and m_u/m_d than LO ChPT

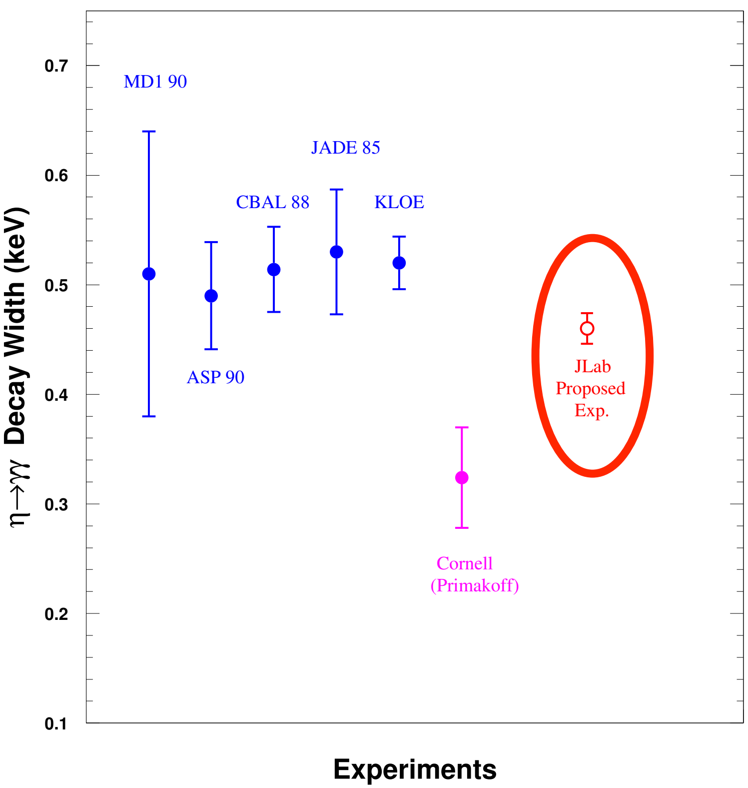
2.6 Prospects

Gan, Kubis, E. P., Tulin'22

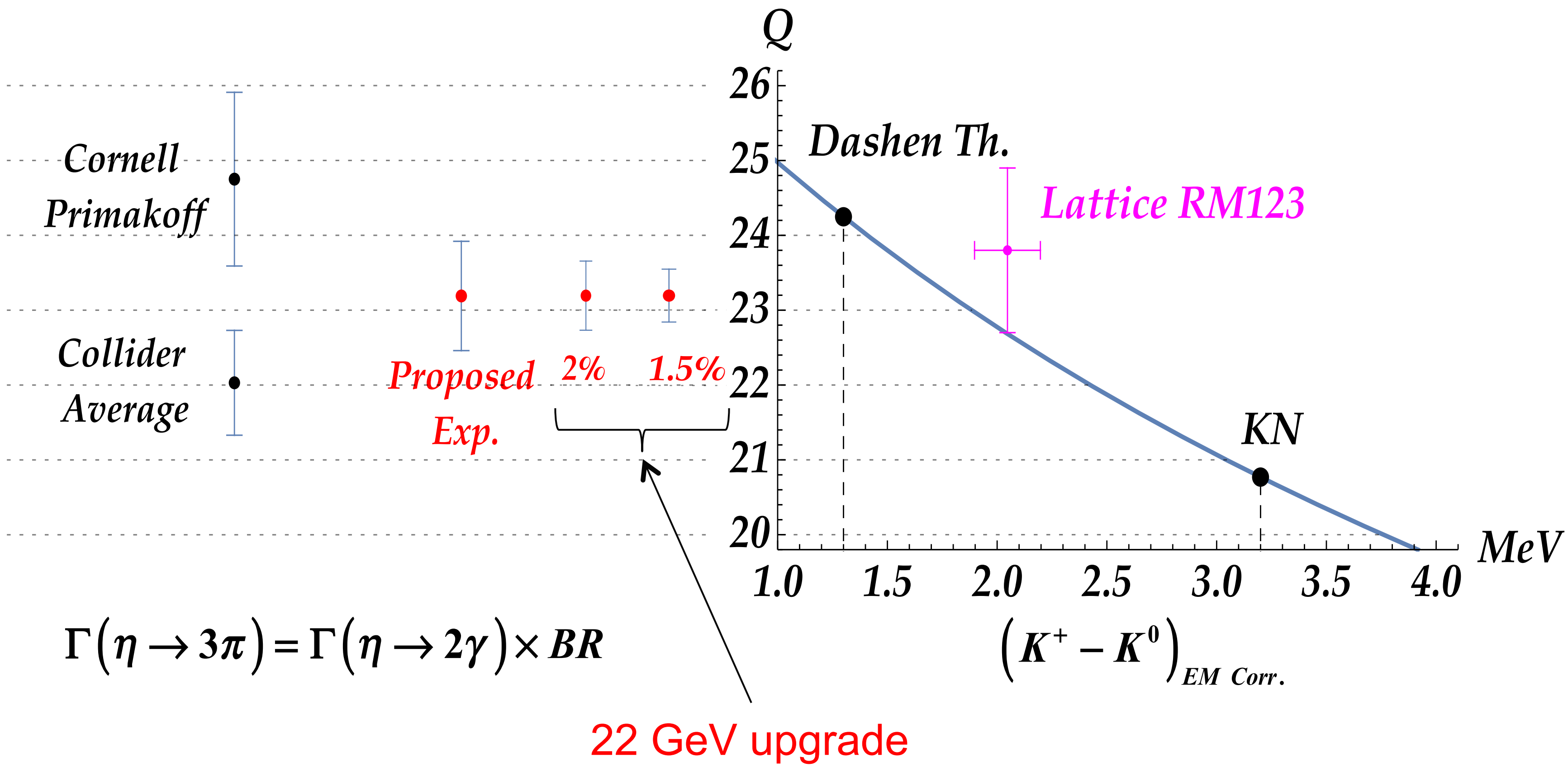
- Uncertainties in the quark mass ratio



Can be investigated and reduced at *future facilities*



2.7 Expected Impact of JLab 22 GeV program



$$\Gamma(\eta \to 3\pi) = \Gamma(\eta \to 2\gamma) \times BR$$

See talk by *L. Gan*
+ *JLab White Paper*
on ArXiv soon

2.8 Studying C & CP violation with $\eta \to 3\pi$ asymetries

Gardner & Shi'19
Akdag, Isken, Kubis'21
Akdag, Kubis, Wirzba'22

- $\eta(I^G = 0^+) \to 3\pi(I^G = 1^-)$ breaks G parity
  - In the SM: C conserved, isospin broken
  - Now in BSM: C broken, isospin either conserved or broken

$$\mathcal{M}(s,t,u) = \mathcal{M}_1^{C}(s,t,u) + \mathcal{M}_0^{\not C}(s,t,u) + \mathcal{M}_2^{\not C}(s,t,u)$$

- 2 additional amplitudes which are C violating:
  interference: $\pi^+ \leftrightarrow \pi^-$ asymmetries linear in BSM couplings

- Use KT approach to determine the hadronic amplitudes

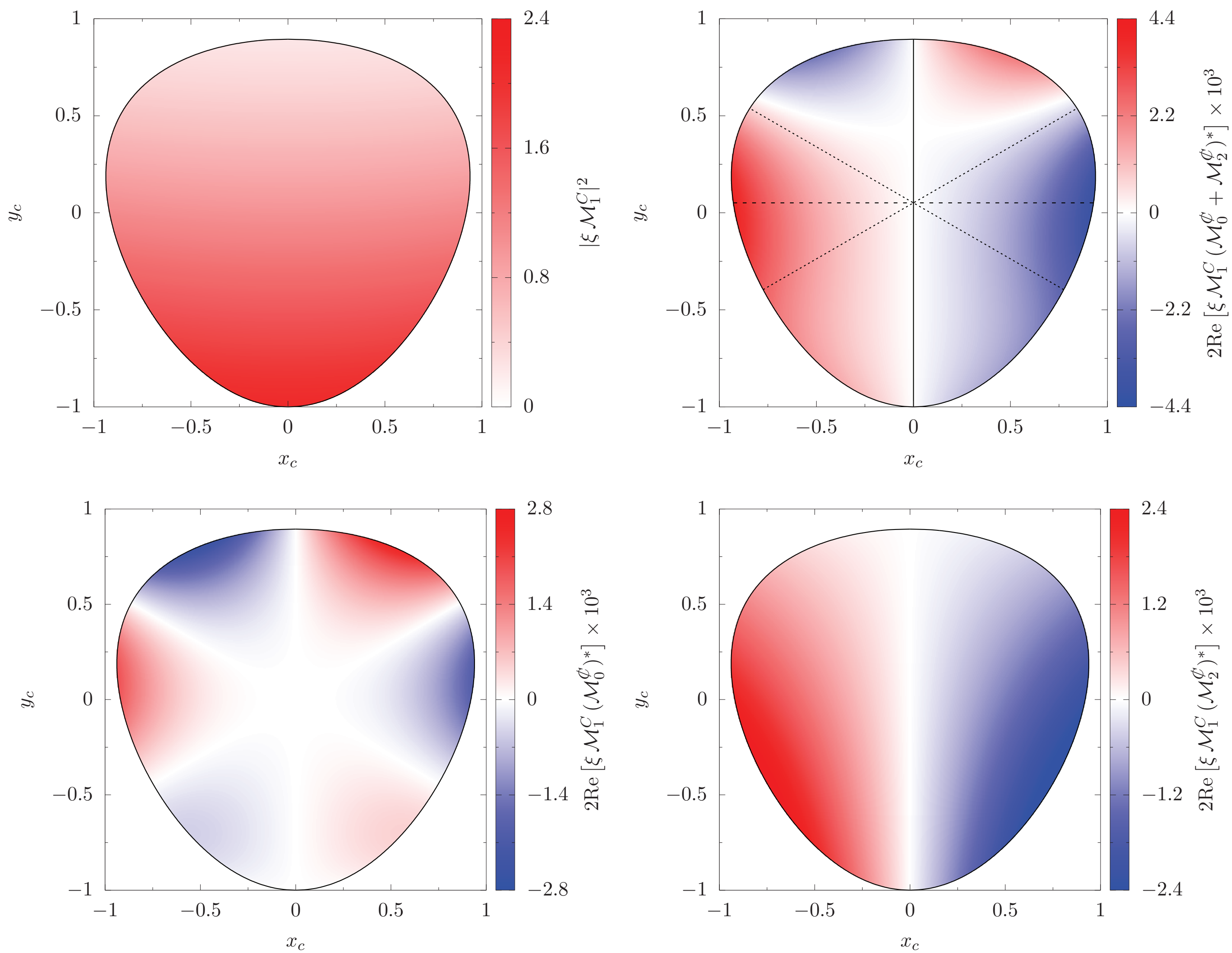
$$\left|\mathcal{M}_c\right|^2 \approx \left|\mathcal{M}_1^{C}\right|^2 + 2\mathrm{Re}\left[\mathcal{M}_1^{C}\left(\mathcal{M}_0^{\not C}\right)^*\right] + 2\mathrm{Re}\left[\mathcal{M}_1^{C}\left(\mathcal{M}_2^{\not C}\right)^*\right]$$

- $\mathcal{M}_0^{\not C}$ and $\mathcal{M}_2^{\not C}$ lead to different interference patterns

➡ See talk by *H. Akdag* and *S. Gardner*

2.8 Studying C & CP violation with $\eta \to 3\pi$ asymetries

Akdag, Isken, Kubis'21

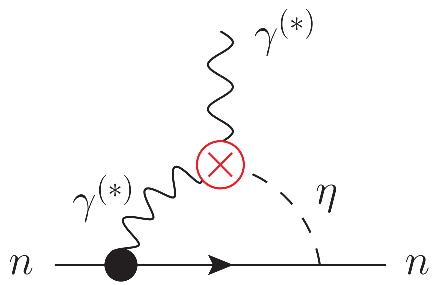


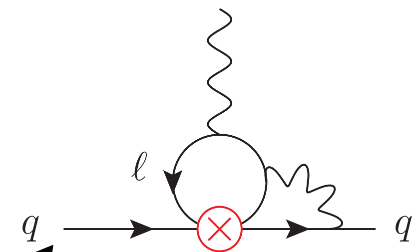
- Asymmetries constrained to the *permille* level ⇨ See talk by *H. Akdag*

3. Fundamental Symmetry tests: CP violation in $\eta \rightarrow \mu^+\mu^-$

Studying of P & CP violation with $\eta \to \mu^+\mu^-$

- A large number of P & CP-violating $\eta(')$ decays indirectly excluded from extremely stringent neutron EDM bounds





- The only exception: investigation of the *muon polarization asymmetries* in $\eta\to\mu^+\mu^-$: EDM constraints at 2 loop order

Sanchez-Puertas'19

$$\mathcal{L}_{\rm eff} = \frac{1}{2v^2}\,{\rm Im}\, c^{2222}_{\ell edq}\left[(\bar\mu\mu)(\bar s i\gamma^5 s) - (\bar\mu i\gamma^5\mu)(\bar s s)\right] + [u\text{-}, d\text{-quarks}]$$

Probe flavour-conserving CP-violation in the second generation

Constraint from EDM for strange quarks weakest: $|{\rm Im}\, c^{2222}_{\ell edq}| < 0.04$

⇨ possible with *REDTOP* statistics, see *Elam et al, Snowmass WP'22*

- Test of CPV in
  - *Escribano et al.'22*
    - $\eta(') \to \pi^0\mu^+\mu^-$
    - $\eta' \to \eta\mu^+\mu^-$
  - *Zillinger et al.'22*
    - $\eta(') \to \pi^+\pi^-\mu^+\mu^-$

⇨ See talks by *P. Sanchez-Puertas* and *H. Schäfer*

4. $\eta' \rightarrow \eta\pi\pi$ and chiral dynamics

In collaboration with
S. Gonzalez-Solis (LANL)
Eur. Phys. J. C78 (2018) no.9, 758

4.1 Why is it interesting to study $\eta' \to \eta\pi\pi$?

PDG'21
Gan, Kubis, E. P., Tulin'22

$$M_{\eta'} = 957.78(6)\ \text{MeV}$$

$\eta' \to 2\gamma$	$(2.20 \pm 0.08)\%$	chiral anomaly
$\eta' \to 3\gamma$	$< 1.0 \times 10^{-4}$	C, CP violation
$\eta' \to e^+e^-\gamma$	$< 9 \times 10^{-4}$	χPT, dark photon (BSM)
$\eta' \to 2\pi^0$	$< 4 \times 10^{-4}$	P, CP violation
$\eta' \to \pi^+\pi^-$	$< 1.8 \times 10^{-5}$	P, CP violation
$\eta' \to 3\pi^0$	$(2.14 \pm 0.20)\%$	$m_u - m_d$
$\eta' \to \pi^+\pi^-\pi^0$	$(3.8 \pm 0.4) \times 10^{-3}$	$m_u - m_d$, CP violation
$\eta' \to \eta\pi^+\pi^-$	$(42.6 \pm 0.7)\%$	RχPT, anomaly, $\eta - \eta'$ mixing
$\eta' \to \eta\pi^0\pi^0$	$(22.8 \pm 0.8)\%$	RχPT, anomaly, $\eta - \eta'$ mixing
$\eta' \to \pi^0 e^+e^-$	$< 1.4 \times 10^{-3}$	C violation
$\eta' \to \pi^+\pi^- e^+e^-$	$(2.4^{+1.3}_{-1.0}) \times 10^{-3}$	P, CP violation
$\eta' \to \pi^0\gamma\gamma$	$< 8 \times 10^{-4}$	χPT, leptophobic B boson (BSM)
$\eta' \to \eta e^+e^-$	$< 2.4 \times 10^{-3}$	C violation

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4.1 Why is it interesting to study $\eta' \to \eta\pi\pi$?

- Main decay channel of the η′:

PDG'21

$$\mathbf{BR(\eta' \to \eta\pi^0\pi^0) = 22.8(8)\%}$$ and $$\mathbf{BR(\eta' \to \eta\pi^+\pi^-) = 42.6(7)\%}$$

- Precise measurements became available: recent results on
  - neutral channel by *A2 collaboration* : 1.2 x 10^5 events
  - neutral and charged channel by *BESIII* collaboration: 351 016 events

- More to come from *GlueX* ⇒ See talk by *O. Cortes Becerra*

4.2 Method

- Main decay channel of the η′:

PDG'21

$$\mathbf{BR}(\eta' \to \eta\pi^0\pi^0) = \mathbf{22.8(8)\%}$$ and $$\mathbf{BR}(\eta' \to \eta\pi^+\pi^-) = \mathbf{42.6(7)\%}$$

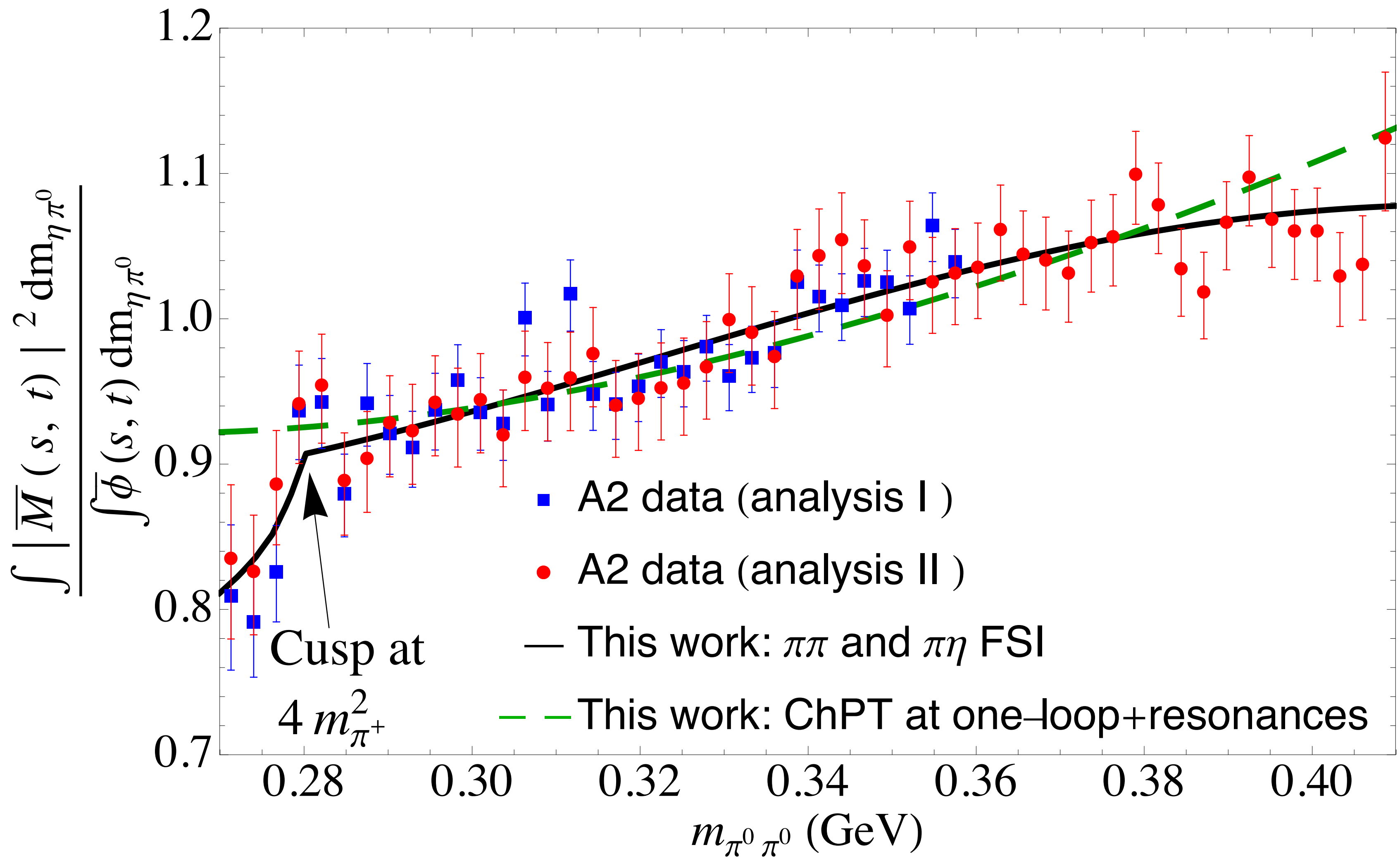
- Precise measurements became available: recent results on
  - neutral channel by *A2 collaboration* : 1.2 x 10^5 events
  - Neutral and charged channel by *BESIII* collaboration: 351 016 events

- Studying this decay allows
  - to test any of the extensions of ChPT e.g. resonance chiral theory, Large-N_C U(3) ChPT etc
  - to study the effects of the ππ and πη final-state interactions

- Method Used: U(3) ChPT with resonances at one-loop + Final-state interaction through N/D unitarization method with D waves + kaon loops

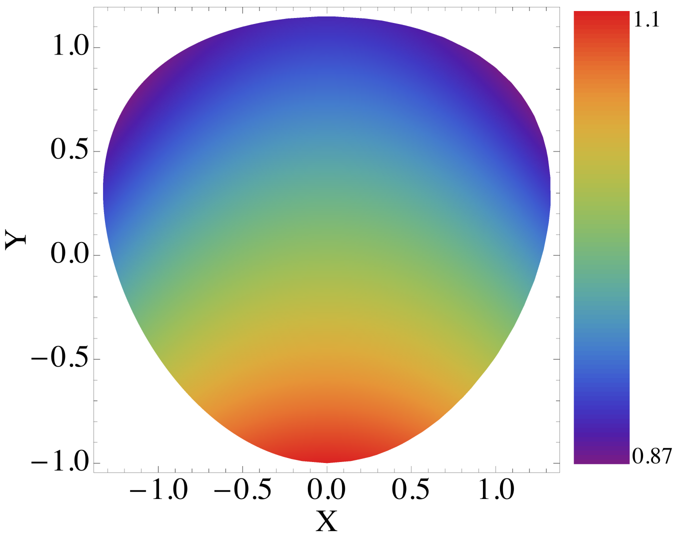
  N.B.: For KT framework see *Isken et al'17*

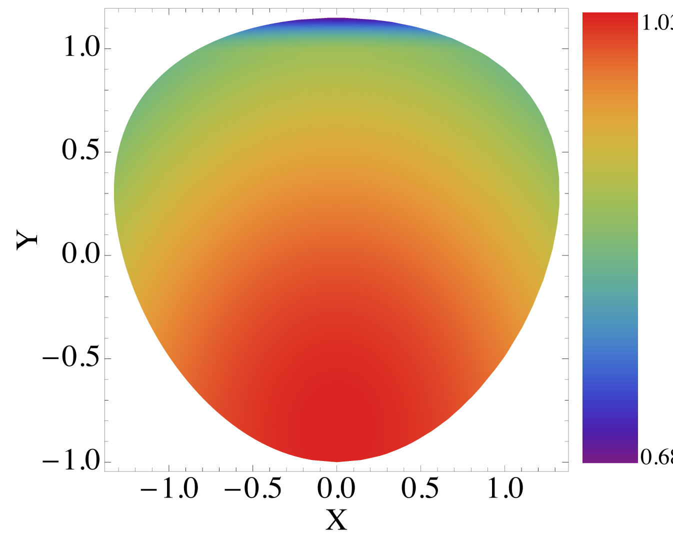
4.3 Results



1.2
1.1
1.0
0.9
0.8
0.7
0.28
0.30
0.32
0.34
0.36
0.38
0.40
$m_{\pi^0\pi^0}$ (GeV)
$\frac{\int |\overline{M}(s,t)|^2 dm_{\eta\pi^0}}{\int \overline{\phi}(s,t) dm_{\eta\pi^0}}$
Cusp at
$4\,m_{\pi^+}^2$
A2 data (analysis I)
A2 data (analysis II)
This work: $\pi\pi$ and $\pi\eta$ FSI
This work: ChPT at one–loop+resonances

4.3 Results



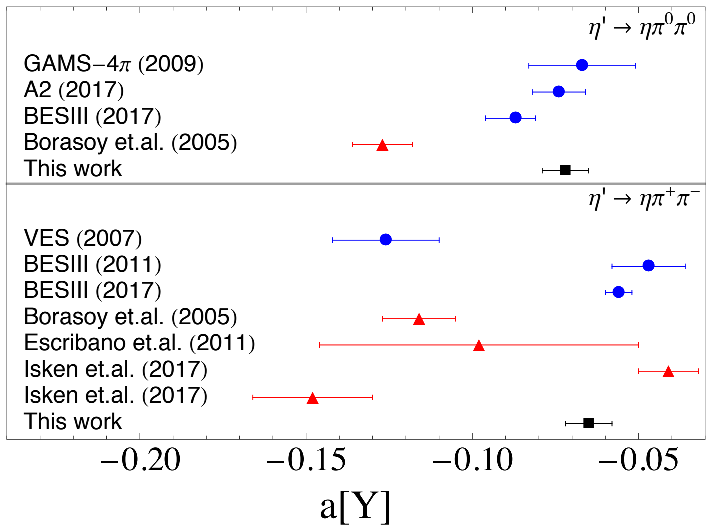


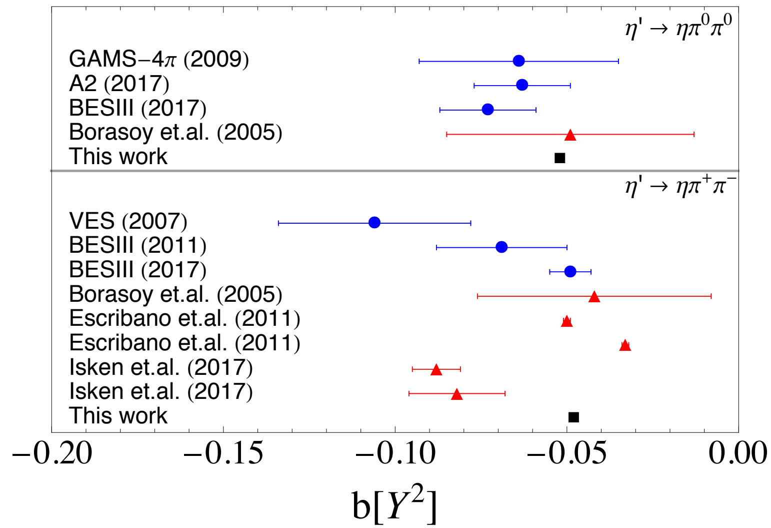
Dalitz slope parameters

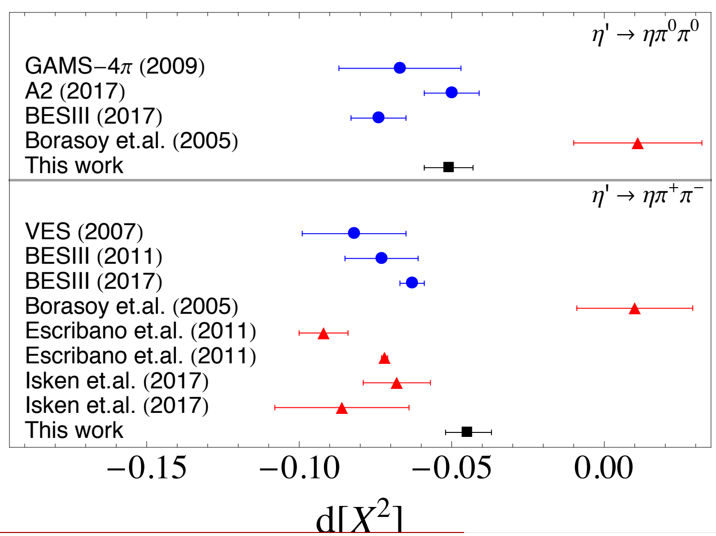
ChPT		Final-state interactions
$a[Y] = -0.095(6)$		$a[Y] = -0.073(7)(5)$
$b[Y^2] = 0.005(1)$	$\Rightarrow$	$b[Y^2] = -0.052(1)(2)$
$d[X^2] = -0.037(5)$		$d[X^2] = -0.052(8)(5)$

$$|A(s,t,u)|^2 = N\left(1 + aY + bY^2 + dX^2 + fY^3 + \ldots\right)$$

4.3 Results







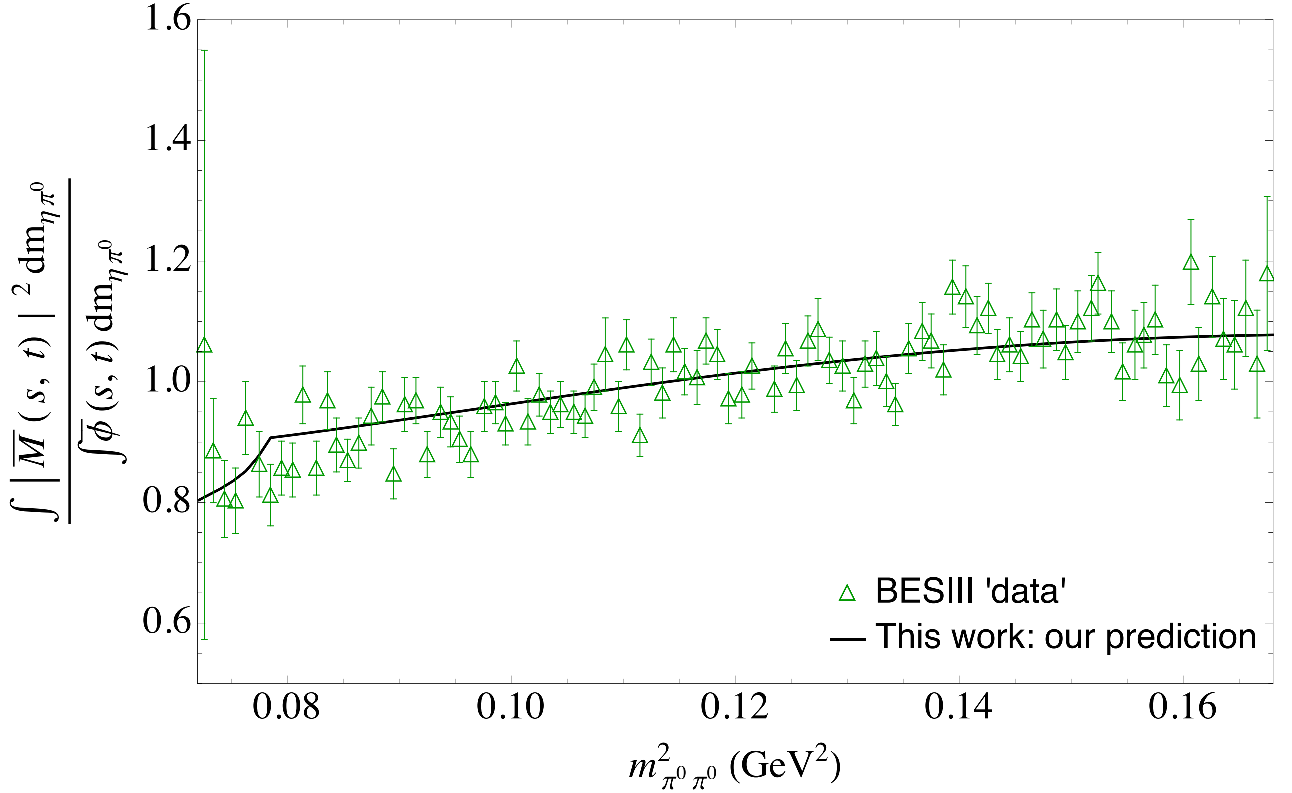
$$\left|A(s,t,u)\right|^2 = N\left(1 + aY + bY^2 + dX^2 + fY^3 + \ldots\right)$$

$$X = \sqrt{3}\,\frac{T_- - T_+}{Q_{\eta'}} = \frac{\sqrt{3}}{2M_{\eta'}Q_{\eta'}}\left(t-u\right)$$

$$Y = \frac{\left(M_\eta + 2M_\pi\right)}{M_\pi}\frac{T_\eta}{Q_{\eta'}} - 1 = \frac{\left(M_\eta + 2M_\pi\right)}{M_\pi}\frac{\left(\left(M_{\eta'} - M_\eta\right)^2 - s\right)}{2M_{\eta'}Q_{\eta'}} - 1$$

4.4 Prospects

- Comparison to BESIII data



- Simultaneous fit by experimental collaborations to the neutral and charged channels etc

5. Conclusion and Outlook

5.1 Conclusion

- η and η' allows to study the fundamental properties of QCD and test the SM
  - Extraction of fundamental parameters of the SM,
    ⇨ e.g. light quark masses
  - Study of chiral dynamics
  - Study of CP violation

- To studies η and η' with the best precision: Development of amplitude analysis techniques consistent with analyticity, unitarity, crossing symmetry dispersion relations allow to take into account *all rescattering effects* being as model independent as possible combined with ChPT ⇨ Provide parametrization for experimental studies

- In this talk, illustration with $\eta \to 3\pi$ and extraction of the light quark masses and $\eta' \to \eta\pi\pi$

- Examples of constraints on *CP violation* from:
  - $\eta \to 3\pi$ asymmetries : C & CP violation
  - $\eta \to \mu^+\mu^-$: P & CP violation ⇨ constraints on sμ operators
- Many more topics I did not have time to address e.g. inputs for g-2, light BSM, sorry! ⇨ See talks at the workshop!

5.2 Outlook

- New η and η' programs *JEF* and *REDTOP* — *Gan, Kubis, E. P., Tulin'22*
- In our opinion the most promising channels to study:

Decay channel	Standard Model	Discrete symmetries	Light BSM particles
$\eta \to \pi^+\pi^-\pi^0$	light quark masses	C/CP violation	scalar bosons (also η')
$\eta^{(\prime)} \to \gamma\gamma$	η–η' mixing, precision partial widths		
$\eta^{(\prime)} \to \ell^+\ell^-\gamma$	$(g-2)_\mu$		Z' bosons, dark photon
$\eta \to \pi^0\gamma\gamma$	higher-order χPT, scalar dynamics		$U(1)_B$ boson, scalar bosons
$\eta^{(\prime)} \to \mu^+\mu^-$	$(g-2)_\mu$, precision tests	CP violation	
$\eta \to \pi^0\ell^+\ell^-$		C violation	scalar bosons
$\eta^{(\prime)} \to \pi^+\pi^-\ell^+\ell^-$	$(g-2)_\mu$		ALPs, dark photon
$\eta^{(\prime)} \to \pi^0\pi^0\ell^+\ell^-$		C violation	ALPs

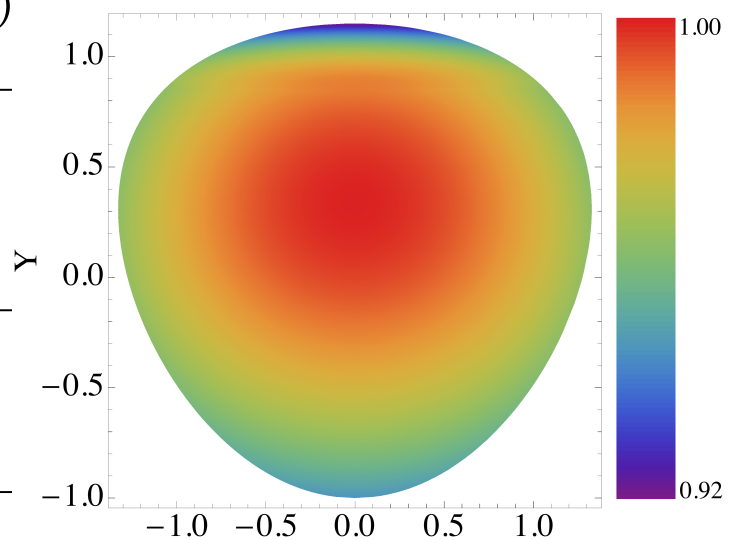
- Synergies between different physics:
  - Standard Model precision analyses
  - Discrete symmetry tests
  - Search for light BSM particles

6. Back-up

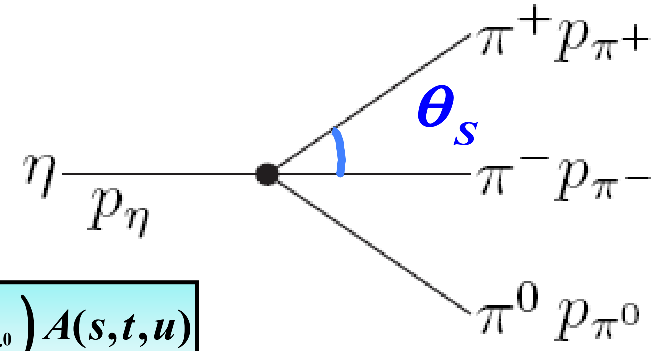
3.4 Role of the D-wave ππ FSI

Parameter	Analysis I Fit 1 (with D-wave)	Fit 1 (w/o D-wave)
M_S	$1017(68)(24)$	$996(66)(25)$
c_d	$30.4(4.8)(9)$	$23.3(3.5)(1.5)$
c_m	$= c_d$	$= c_d$
$\tilde{c}_d$	$17.6(2.8)(5)$	$13.5(2.0)(9)$
$\tilde{c}_m$	$= \tilde{c}_d$	$= \tilde{c}_d$
$a_{\pi\pi}$	$0.76(61)(6)$	$2.01(1.61)(71)$
$\chi^2_{\rm dof}$	1.12	1.24
$a[Y]$	$\mathbf{-0.074(7)(8)}$	$\mathbf{-0.091(9)(4)}$
$b[Y^2]$	$\mathbf{-0.049(1)(2)}$	$\mathbf{-0.013(1)(5)}$
$c[X]$	$\mathbf{0}$	$\mathbf{0}$
$d[X^2]$	$\mathbf{-0.047(8)(4)}$	$\mathbf{-0.031(6)(3)}$
$\kappa_{03}[Y^3]$	0.001	0.001
$\kappa_{21}[YX^2]$	-0.004	-0.001
$\kappa_{22}[Y^2X^2]$	0.001	0.0004

$|M(X,Y)_{\rm Full}|^2/|M(X,Y)_{\rm D-wave=0}|^2$



2.1 Definitions



- η decay: $\eta \to \pi^+ \pi^- \pi^0$

$$\left\langle \pi^+\pi^-\pi^0{}_{out} \middle| \eta \right\rangle = i(2\pi)^4 \delta^4\left(p_\eta - p_{\pi^+} - p_{\pi^-} - p_{\pi^0}\right) A(s,t,u)$$

- Mandelstam variables $s = \left(p_{\pi^+} + p_{\pi^-}\right)^2,\ t = \left(p_{\pi^-} + p_{\pi^0}\right)^2,\ u = \left(p_{\pi^0} + p_{\pi^+}\right)^2$

  ⇨ only two independent variables

$$s + t + u = M_\eta^2 + M_{\pi^0}^2 + 2M_{\pi^+}^2 \equiv 3s_0$$

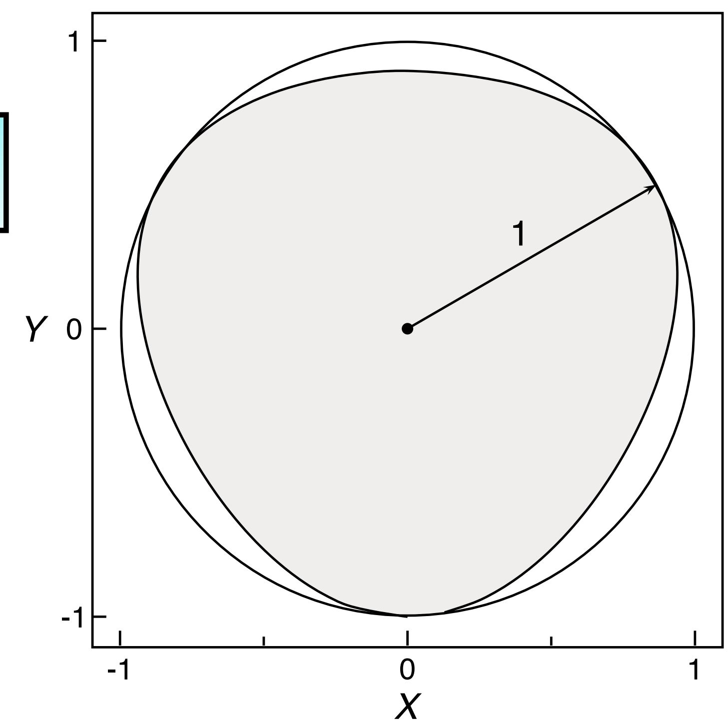
- 3 body decay ⇨ Dalitz plot

$$\left|A(s,t,u)\right|^2 = N\left(1 + aY + bY^2 + dX^2 + fY^3 + \ldots\right)$$

Expansion around X=Y=0

$$X = \sqrt{3}\,\frac{T_+ - T_-}{Q_c} = \frac{\sqrt{3}}{2M_\eta Q_c}(u - t)$$

$$Y = \frac{3T_0}{Q_c} - 1 = \frac{3}{2M_\eta Q_c}\left(\left(M_\eta - M_{\pi^0}\right)^2 - s\right) - 1$$



$$Q_c \equiv M_\eta - 2M_{\pi^+} - M_{\pi^0}$$

2.3 Computation of the amplitude

- What do we know?
- Compute the amplitude using ChPT :

$$\Gamma_{\eta\to 3\pi} = \left(66 + 94 + ... + ...\right)\text{eV} = \left(300 \pm 12\right)\text{eV}$$

LO NLO NNLO

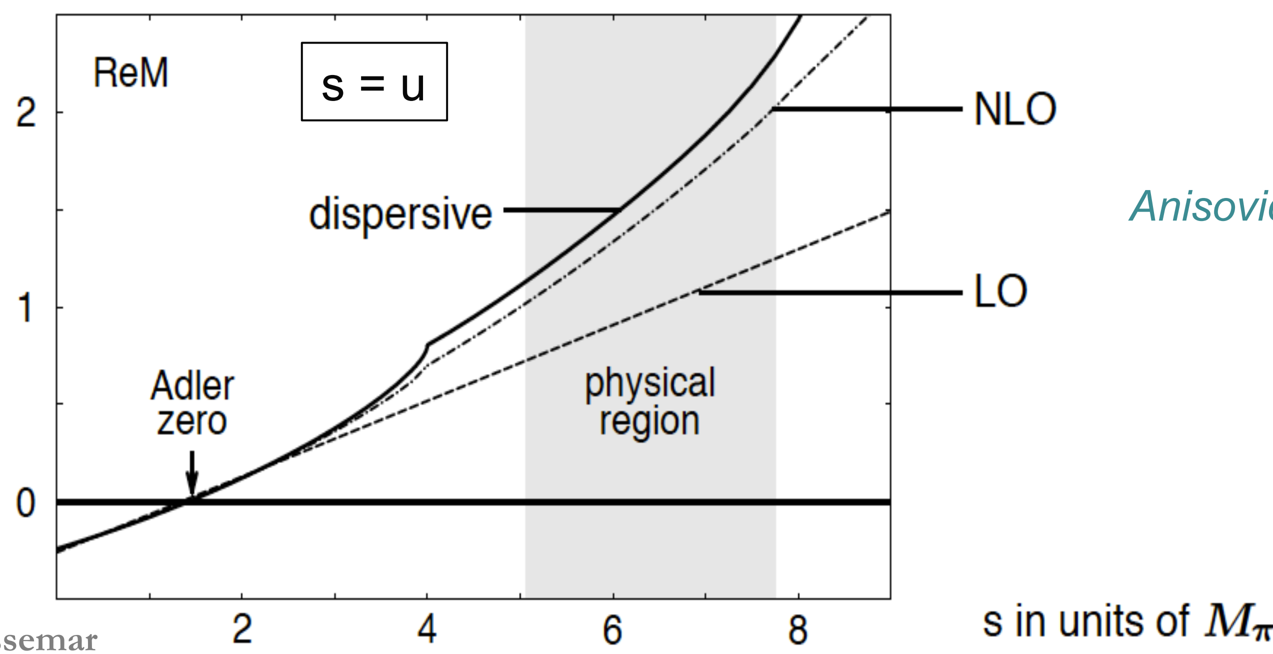
PDG'16

LO: *Osborn, Wallace'70*

NLO: *Gasser & Leutwyler'85*

NNLO: *Bijnens & Ghorbani'07*

The Chiral series has convergence problems



Anisovich & Leutwyler'96

2.3 Computation of the amplitude

- What do we know?
- The amplitude has an Adler zero: soft pion theorem *Adler'85*

  ⇒ Amplitude has a zero for :

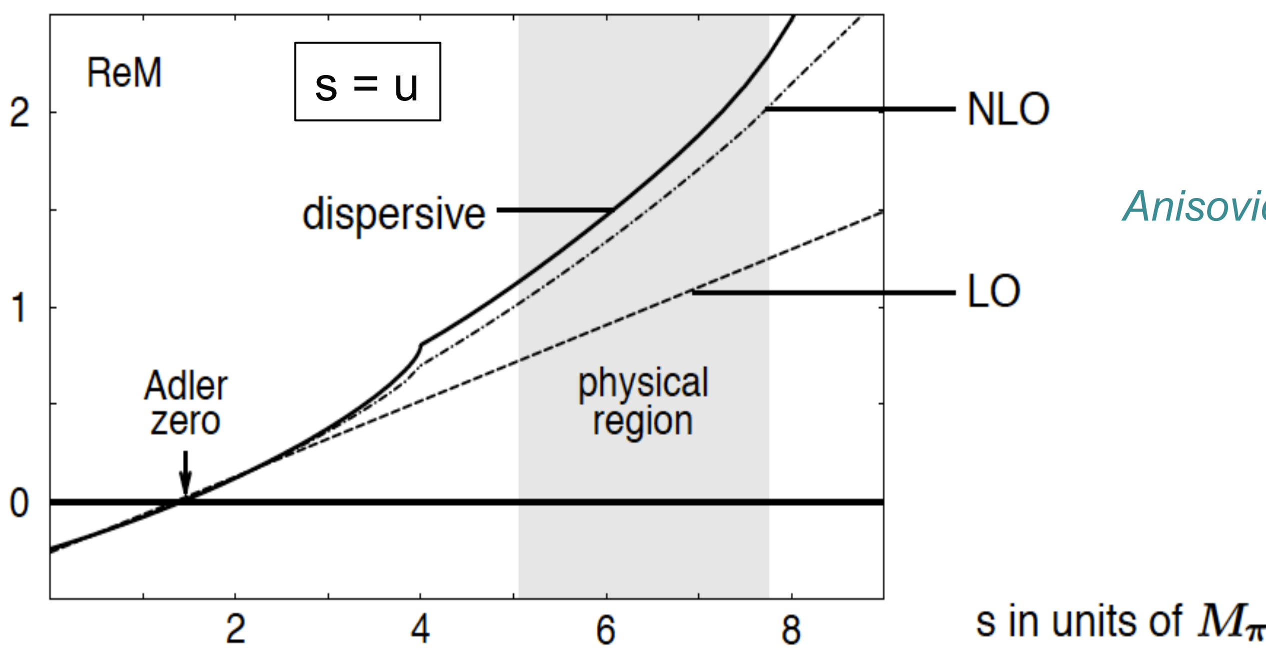
$$p_{\pi^+} \to 0 \;\Rightarrow\; s = u = 0,\; t = M_\eta^2$$

$$p_{\pi^-} \to 0 \;\Rightarrow\; s = t = 0,\; u = M_\eta^2$$

$$M_\pi \neq 0 \;\Rightarrow\; s = u = \frac{4}{3}M_\pi^2,\; t = M_\eta^2 + \frac{M_\pi^2}{3}$$

$$s = t = \frac{4}{3}M_\pi^2,\; u = M_\eta^2 + \frac{M_\pi^2}{3}$$

SU(2) corrections



Anisovich & Leutwyler'96

2.4 Neutral channel : $\eta \rightarrow \pi^0 \pi^0 \pi^0$

- What do we know?

- We can relate charged and neutral channels

$$\overline{A}(s,t,u) = A(s,t,u) + A(t,u,s) + A(u,s,t)$$

⇨ *Correct formalism should be able to reproduce both charged and neutral channels*

- Ratio of decay width precisely measured

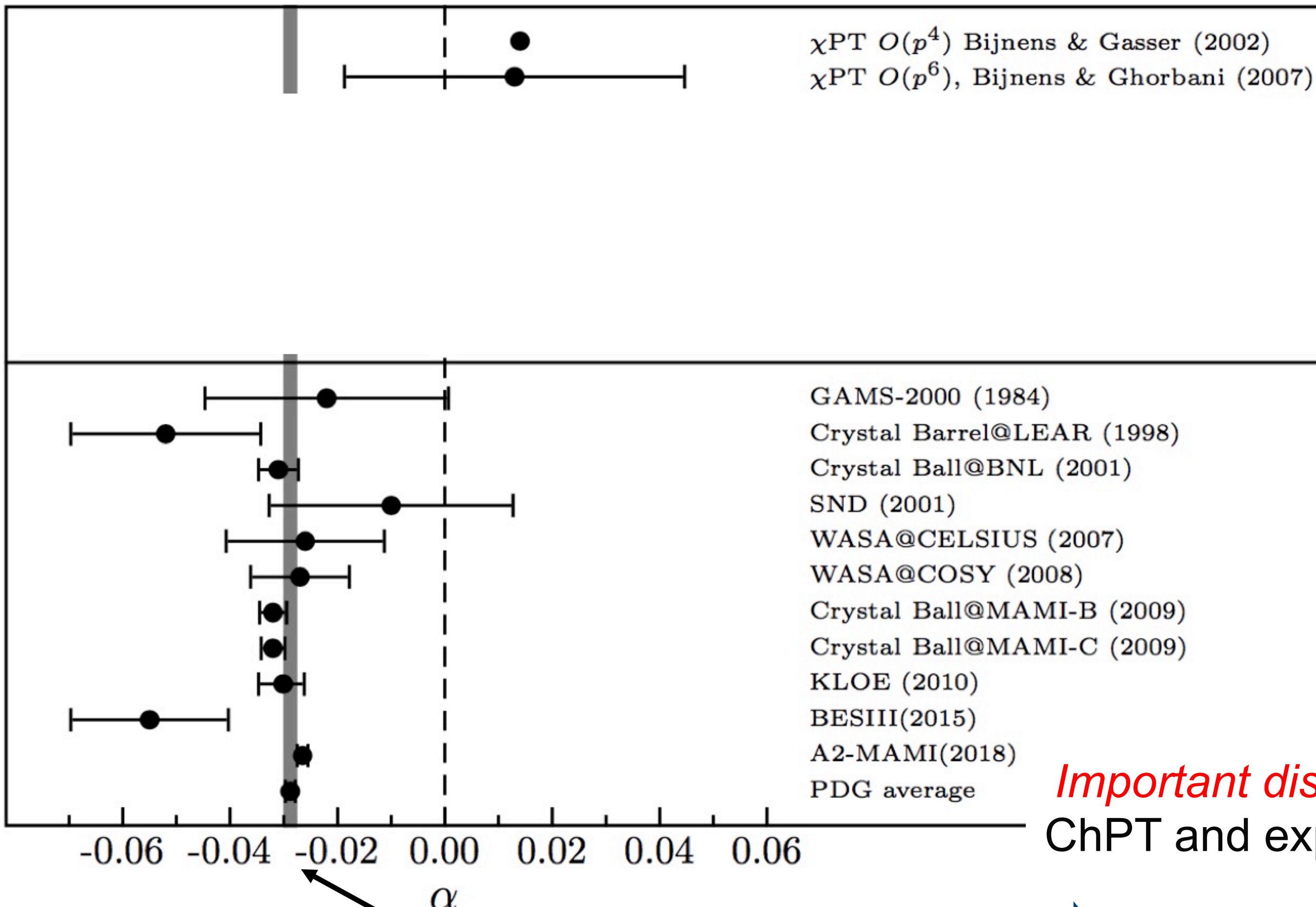
$$r = \frac{\Gamma\left(\eta \to \pi^0\pi^0\pi^0\right)}{\Gamma\left(\eta \to \pi^+\pi^-\pi^0\right)} = 1.426 \pm 0.026$$

PDG'19

2.4 Neutral Channel : $\eta \rightarrow \pi^0 \pi^0 \pi^0$

$$Q_n \equiv M_\eta - 3M_{\pi^0}$$

- Decay amplitude $\Gamma_{\eta\rightarrow 3\pi} \propto \left|\bar{A}\right|^2 \propto 1 + 2\alpha Z$ with $Z = \frac{2}{3}\sum_{i=1}^{3}\left(\frac{3T_i}{Q_n} - 1\right)^2$



$\alpha \sim 0.015$

$\alpha = -0.0288 \pm 0.0012$

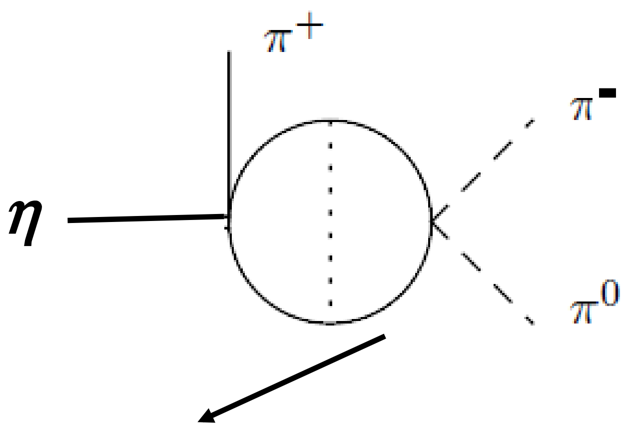
Important discrepancy between ChPT and experiment!

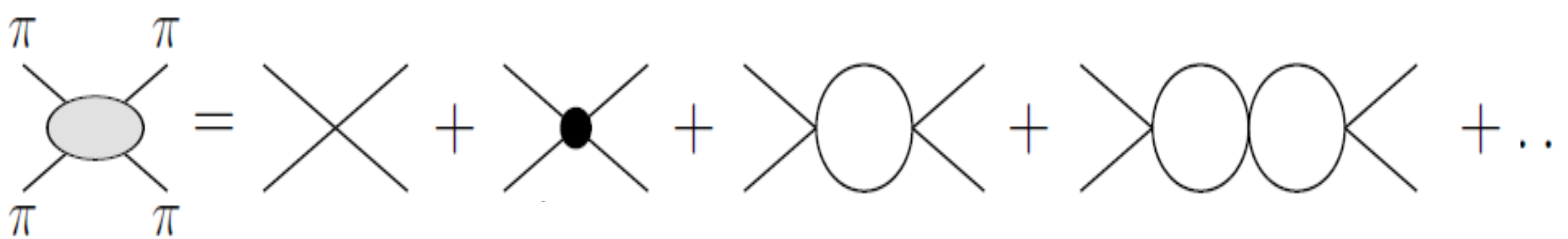
⇨ Help of a dispersive treatment?

2.5 Dispersive treatment

- The Chiral series has convergence problems

  ⇨ Large *ππ final state interactions* *Roiesnel & Truong'81*

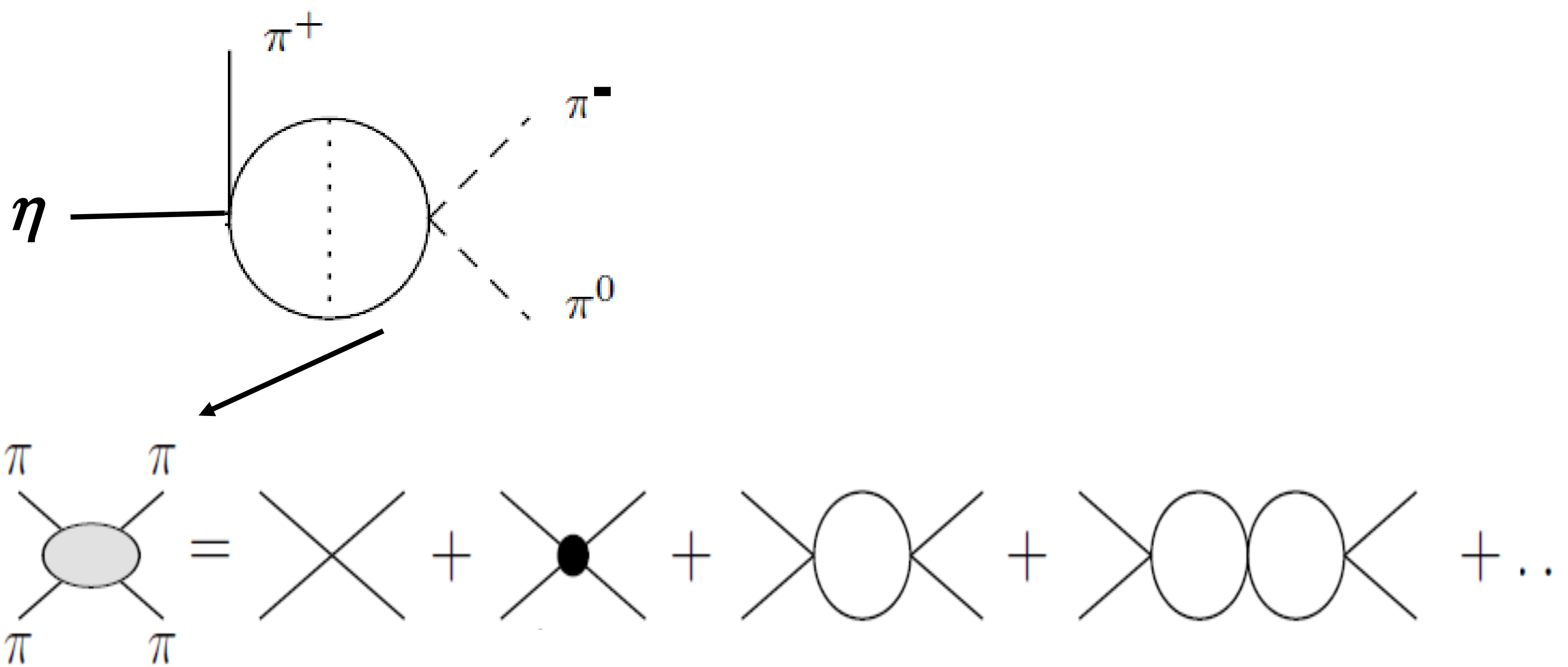




2.5 Dispersive treatment

- The Chiral series has convergence problems

  ⇨ Large *ππ final state interactions* *Roiesnel & Truong'81*



- *Dispersive treatment :*
  - analyticity, unitarity and crossing symmetry
  - Take into account all the rescattering effects

2.6 Why a new dispersive analysis?

- Several new ingredients:
  - New inputs available: extraction $\pi\pi$ phase shifts has improved

    Ananthanarayan et al'01, Colangelo et al'01
    Descotes-Genon et al'01
    Kaminsky et al'01, Garcia-Martin et al'09

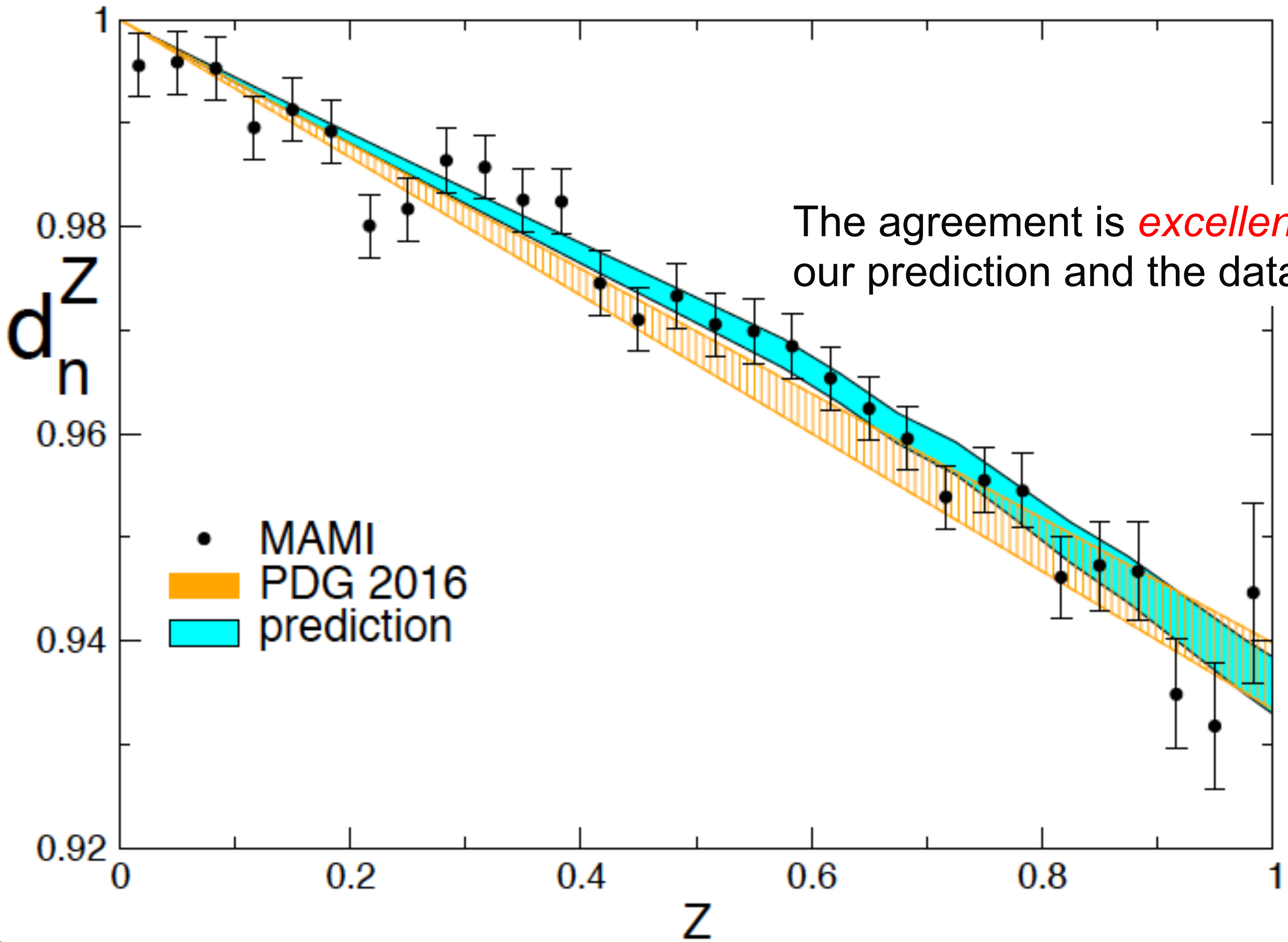
  - New experimental programs, precise Dalitz plot measurements

    TAPS/CBall-MAMI (Mainz), WASA-Celsius (Uppsala), WASA-Cosy (Juelich)
    CBall-Brookhaven, CLAS, GlueX (JLab), KLOE I-II (Frascati)
    BES III (Beijing)

  - Many improvements needed in view of very precise data: inclusion of
    - Electromagnetic effects ($\mathcal{O}(e^2m)$) *Ditsche, Kubis, Meissner'09*
    - Isospin breaking effects *Gullstrom, Kupsc, Rusetsky'09, Schneider, Kubis, Ditsche'11*
    - Inelasticities *Albaladejo & Moussallam'15*

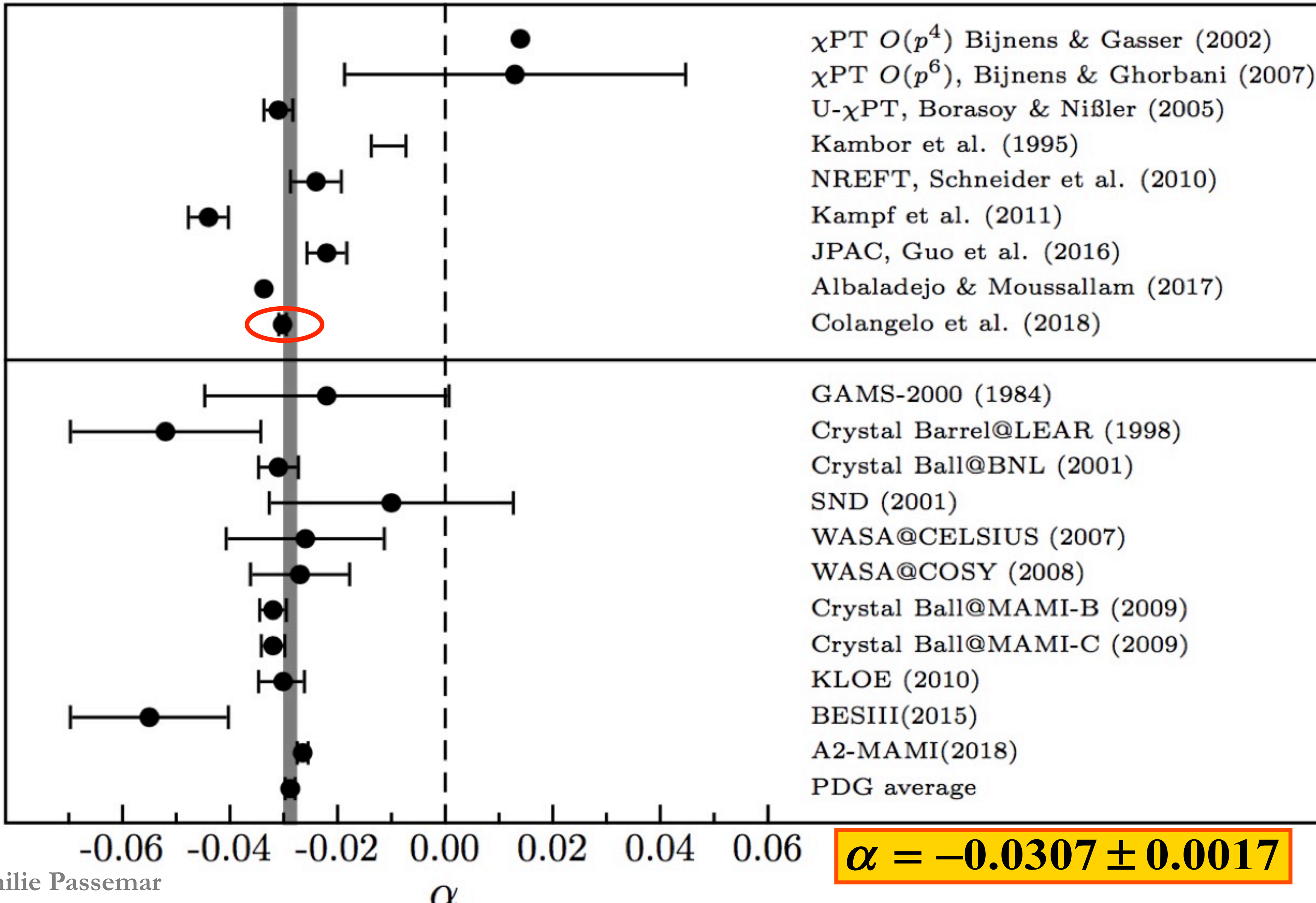
2.11 Z distribution for $\eta \rightarrow \pi^0 \pi^0 \pi^0$ decays

- The amplitude squared in the neutral channel is



The agreement is *excellent* between our prediction and the data!

2.12 Comparison of results for α



$$\alpha = -0.0307 \pm 0.0017$$

Experimental Facilities and Role of JLab 12

M. J. Amaryan et al.
CLAS Analysis Proposal, (2014)

π	$e^+e^-\gamma$			
η	$e^+e^-\gamma$	$\pi^+\pi^-\gamma$	$\pi^+\pi^-\pi^0$, $\pi^+\pi^-$	$\pi^+\pi^-e^+e^-$
η'	$e^+e^-\gamma$	$\pi^+\pi^-\gamma$	$\pi^+\pi^-\pi^0$, $\pi^+\pi^-$	$\pi^+\pi^-\eta$, $\pi^+\pi^-e^+e^-$
ρ		$\pi^+\pi^-\gamma$		
ω	$e^+e^-\pi^0$	$\pi^+\pi^-\gamma$	$\pi^+\pi^-\pi^0$	
φ			$\pi^+\pi^-\pi^0$	$\pi^+\pi^-\eta$

2.3 Computation of the amplitude

- What do we know?

- Compute the amplitude using ChPT : the effective theory that describe dynamics of the Goldstone bosons (kaons, pions, eta) at low energy

- Goldstone bosons interact weakly at low energy and $m_u, m_d \ll m_s < \Lambda_{QCD}$
Expansion organized in external momenta and quark masses

Weinberg's power counting rule

$$\mathcal{L}_{eff} = \sum_{d \geq 2} \mathcal{L}_d \,,\, \mathcal{L}_d = \mathcal{O}\left(p^d\right) \,,\, p \equiv \left\{q, m_q\right\}$$

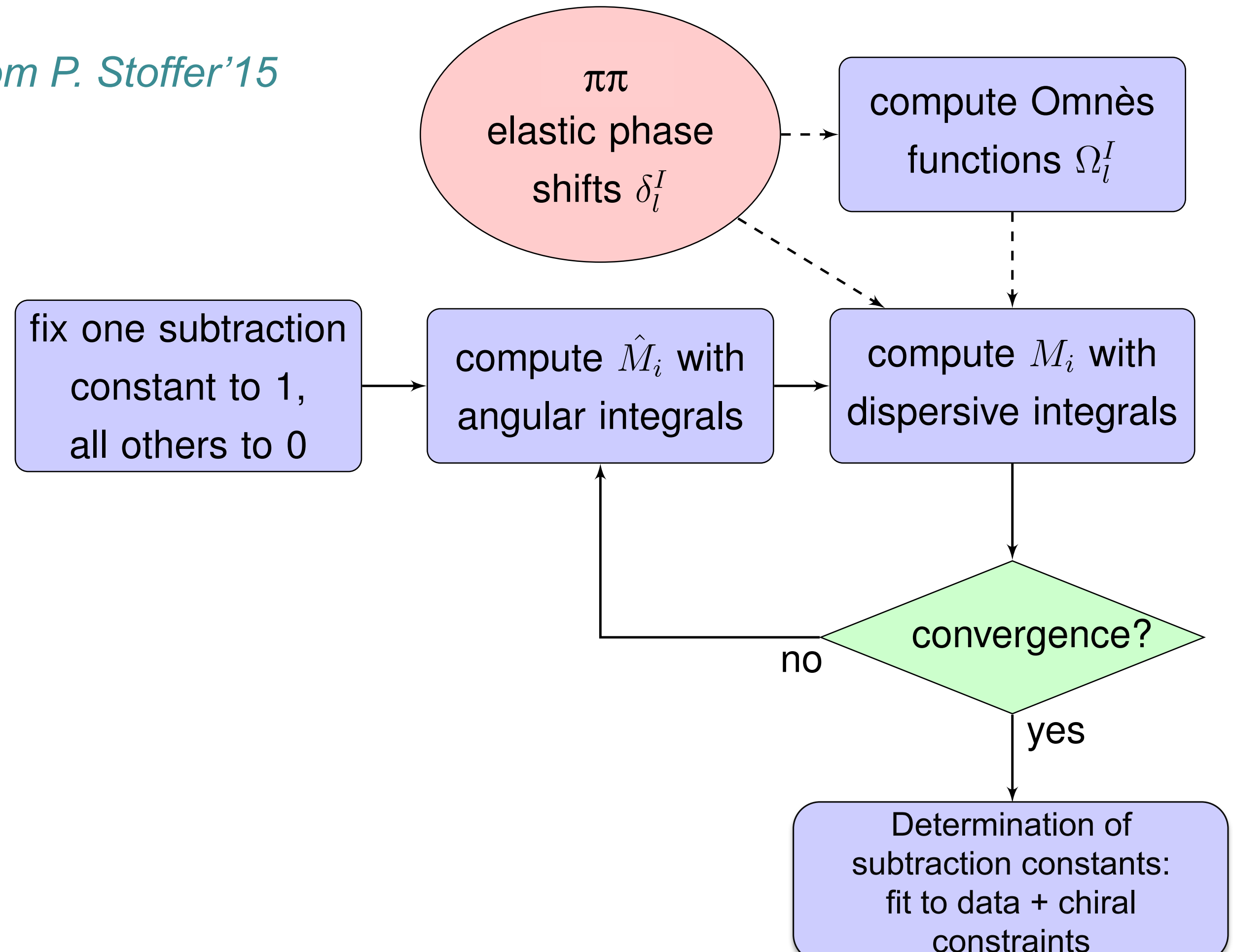
$$\mathrm{p} << \Lambda_H = 4\pi F_\pi \sim 1 \text{ GeV}$$

2.5 Iterative Procedure

- Solution *linear* in the *subtraction constants* *Anisovich & Leutwyler'96*

$$M(s,t,u)=\alpha_0 M_{\alpha_0}(s,t,u)+\beta_0 M_{\beta_0}(s,t,u)+\ldots$$ ⇨ makes the fit much easier

Adapted from P. Stoffer'15



2.6 Subtraction constants

- Extension of the numbers of parameters compared to *Anisovich & Leutwyler'96*

$$P_0(s) = \alpha_0 + \beta_0 s + \gamma_0 s^2 + \delta_0 s^3$$
$$P_1(s) = \alpha_1 + \beta_1 s + \gamma_1 s^2$$
$$P_2(s) = \alpha_2 + \beta_2 s + \gamma_2 s^2$$

- In the work of *Anisovich & Leutwyler'96* matching to one loop ChPT
  Use of the SU(2) x SU(2) chiral theorem
  ⇨ The amplitude has an *Adler zero* along the line s=u

- Now data on the Dalitz plot exist from KLOE, WASA, MAMI and BES III

  ⇨ Use the data to directly fit the subtraction constants

- However normalization to be fixed to ChPT!

2.7 Subtraction constants

- The subtraction constants are

$$P_0(s) = \alpha_0 + \beta_0 s + \gamma_0 s^2 + \delta_0 s^3$$
$$P_1(s) = \alpha_1 + \beta_1 s + \gamma_1 s^2$$
$$P_2(s) = \alpha_2 + \beta_2 s + \gamma_2 s^2 + \delta_0 s^3$$

Only *6 coefficients* are of physical relevance

- They are determined from combining ChPT with a fit to KLOE Dalitz plot

- Taylor expand the dispersive M_I
  Subtraction constants ⟺ Taylor coefficients

$$M_0(s) = A_0 + B_0 s + C_0 s^2 + D_0 s^3 + \ldots$$
$$M_1(s) = A_1 + B_1 s + C_1 s^2 + \ldots$$
$$M_2(s) = A_2 + B_2 s + C_2 s^2 + D_2 s^3 +$$

- Gauge freedom in the decomposition of M(s,t,u)

2.7 Subtraction constants

- Build some gauge independent combinations of Taylor coefficients

$$
\begin{aligned}
H_0 &= A_0 + \frac{4}{3}A_2 + s_0\left(B_0 + \frac{4}{3}B_2\right) \\
H_1 &= A_1 + \frac{1}{9}(3B_0 - 5B_2) - 3C_2 s_0 \\
H_2 &= C_0 + \frac{4}{3}C_2, \qquad H_3 = B_1 + C_2 \\
H_4 &= D_0 + \frac{4}{3}D_2, \qquad H_5 = C_1 - 3D_2
\end{aligned}
$$

⇨

$$H_0^{ChPT} = 1 + 0.176 + O\left(p^4\right)$$

$$h_1^{ChPT} = \frac{1}{\Delta_{\eta\pi}}\left(1 - 0.21 + O\left(p^4\right)\right)$$

$$h_2^{ChPT} = \frac{1}{\Delta_{\eta\pi}^2}\left(4.9 + O\left(p^4\right)\right)$$

$$h_3^{ChPT} = \frac{1}{\Delta_{\eta\pi}^2}\left(1.3 + O\left(p^4\right)\right)$$

$$\left[h_i \equiv \frac{H_i}{H_0}\right]$$

⇨

$$\chi^2_{theo} = \sum_{i=1}^{3}\left(\frac{h_i - h_i^{ChPT}}{\sigma_{h_i^{ChPT}}}\right)^2$$

$$\sigma_{h_i^{ChPT}} = 0.3\left|h_i^{NLO} - h_i^{LO}\right|$$

Isospin breaking corrections

- Dispersive calculations in the isospin limit ⇨ to fit to data one has to include isospin breaking corrections

- $$M_{c/n}(s,t,u) = M_{disp}(s,t,u)\frac{M_{DKM}(s,t,u)}{\tilde{M}_{GL}(s,t,u)}$$

with M_{DKM} : amplitude at one loop with $\mathcal{O}(e^2 m)$ effects

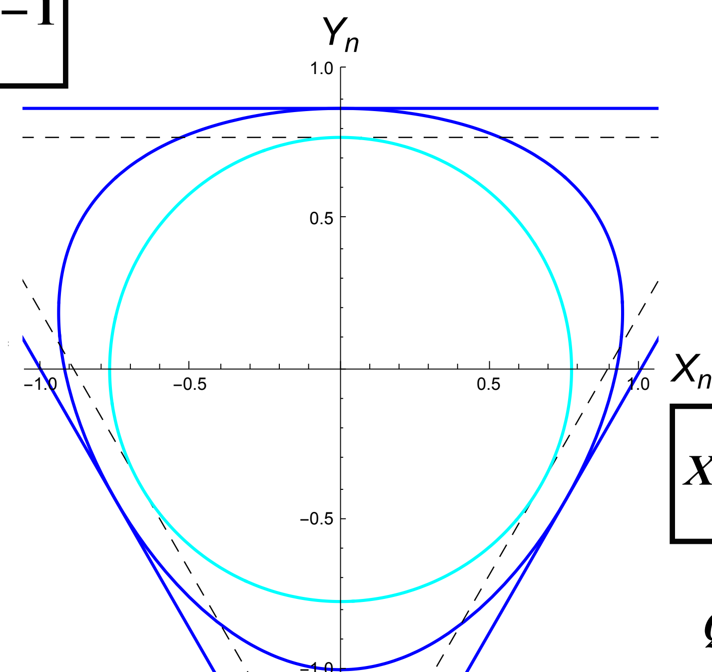
Ditsche, Kubis, Meissner'09

M_{GL}: amplitude at one loop in the isospin limit

Gasser & Leutwyler' 85

$$Y_n = \frac{3T_3}{Q_n} - 1$$

Neutral channel



$$X_n = \sqrt{3}\,\frac{T_2 - T_1}{Q_n}$$

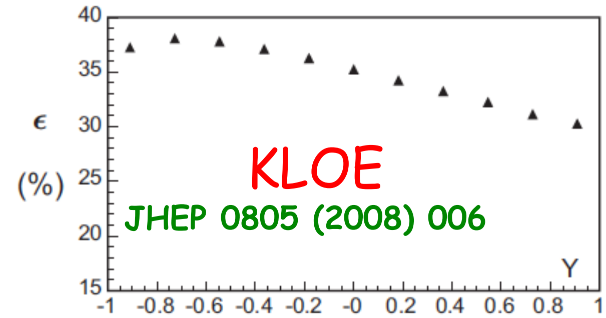
$$Q_n \equiv M_\eta - 3M_{\pi^0}$$

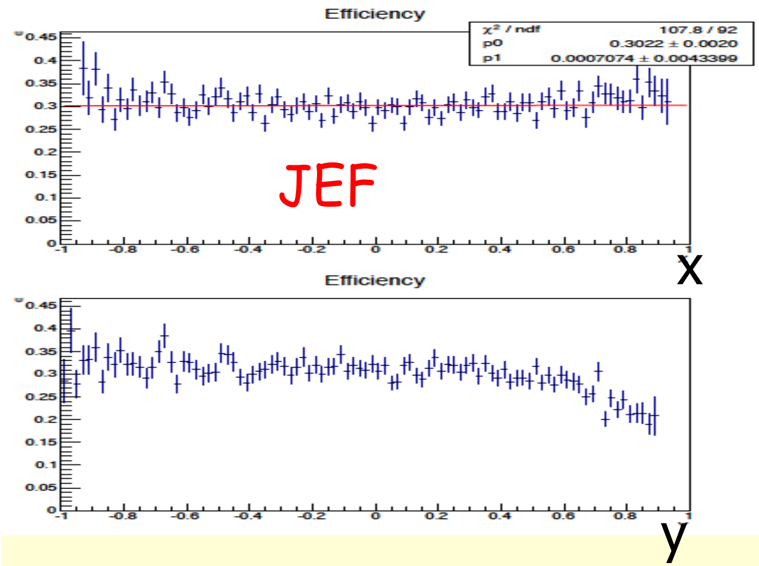
Kinematic map:
isospin symmetric boundaries
⇨ physical boundaries

$$M_{GL} \rightarrow \tilde{M}_{GL}$$

2.15 Prospects

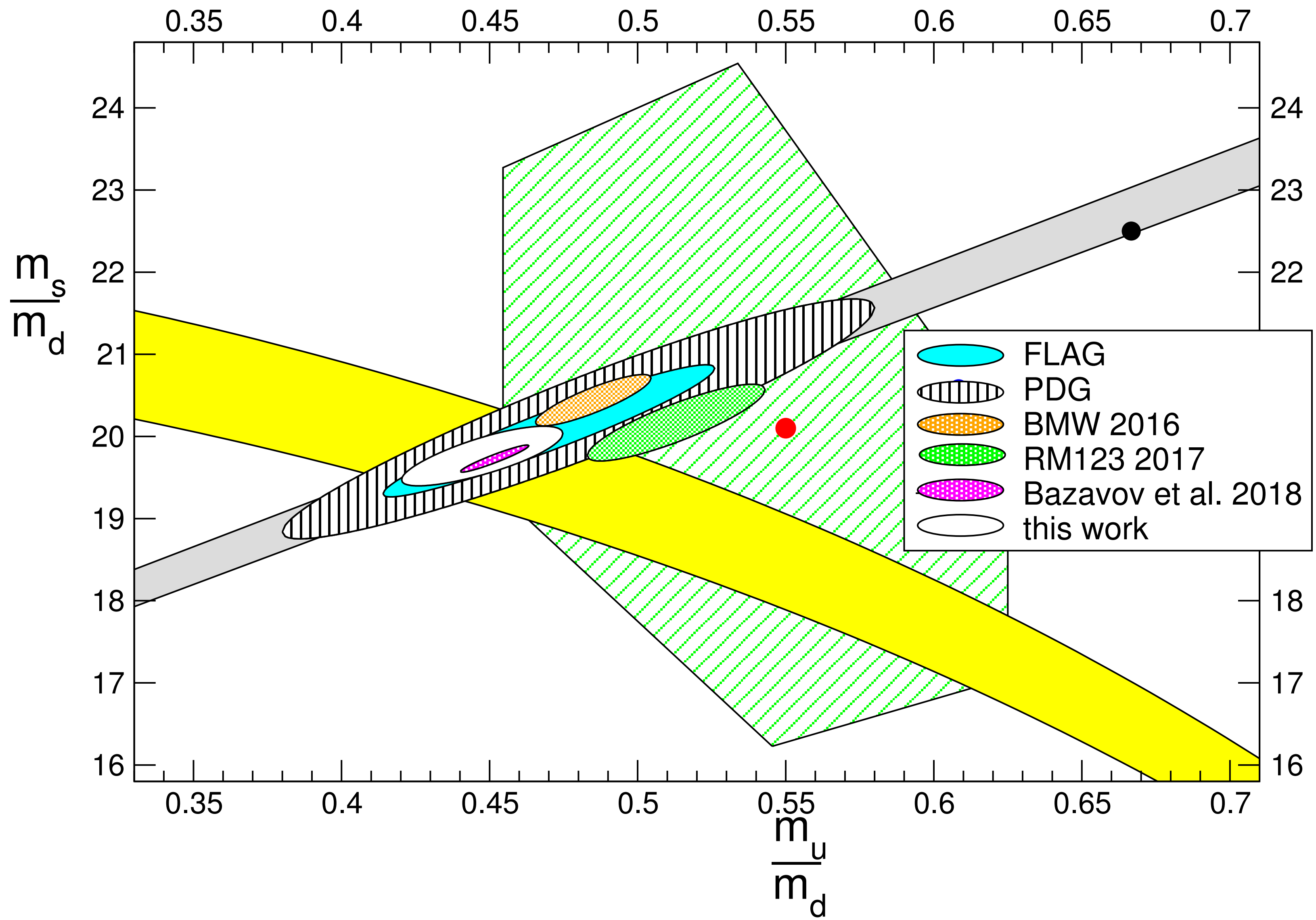
Exp.	$3\pi^0$ Events (10^6)	$\pi^+ \pi^- \pi^0$ Events (10^6)
Total world data (include prel. WASA and prel. KLOE)	6.5	6.0
GlueX+PrimEx-η +JEF	20	19.6





- Existing data from the low energy facilities are sensitive to the detection threshold effects
- JEF at high energy has uniform detection efficiency over Dalitz phase space
- JEF will offer large statistics and different systematics

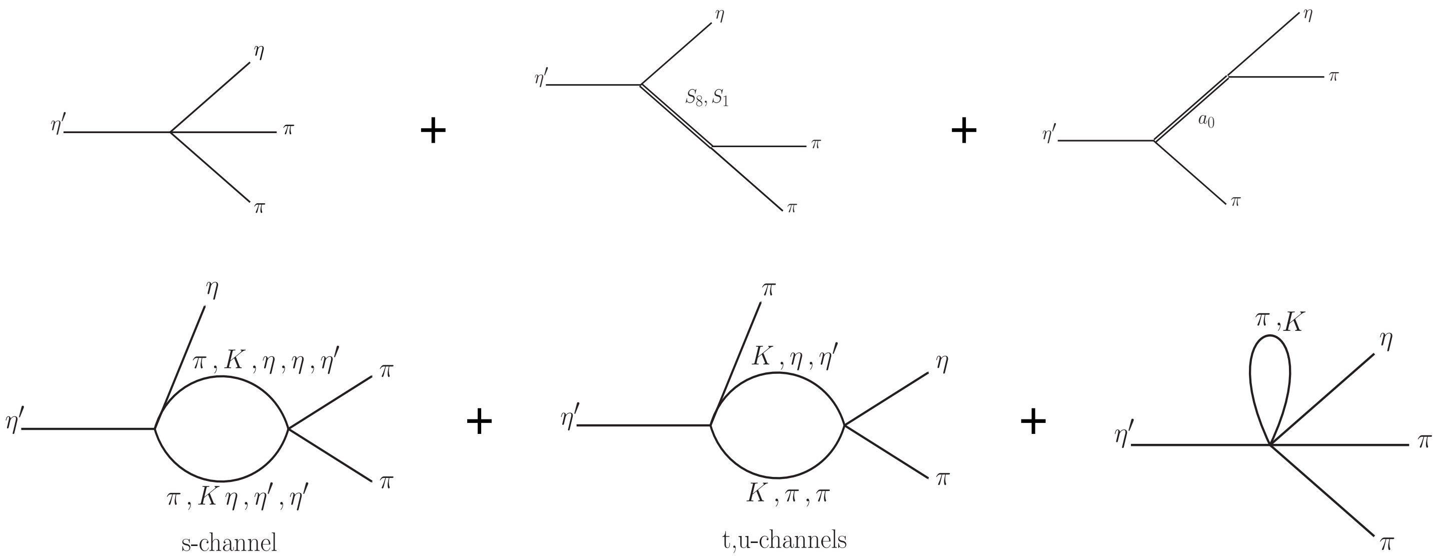
2.14 Comparison with Lattice

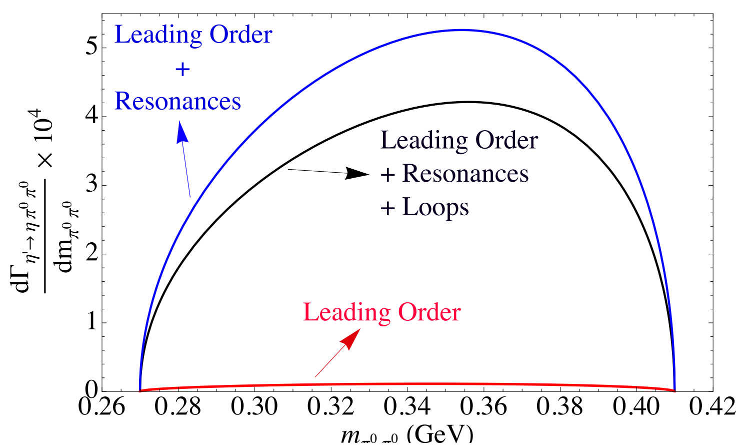


3.2 Theoretical Framework

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}$$

- U(3) ChPT with resonances at one-loop

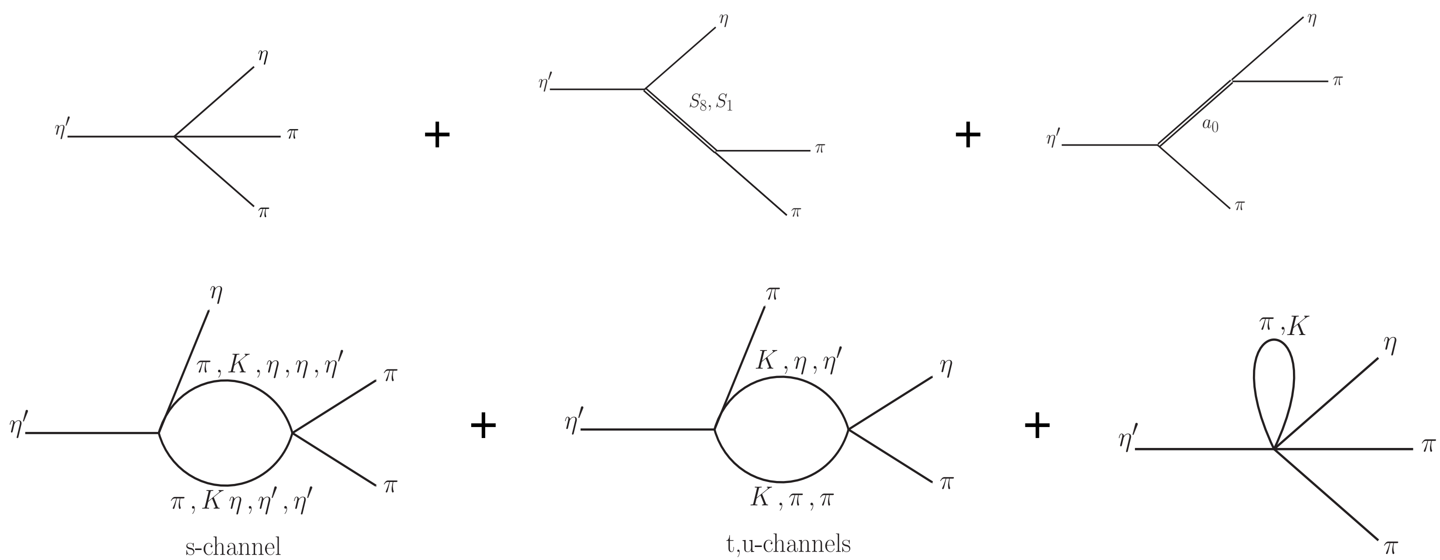


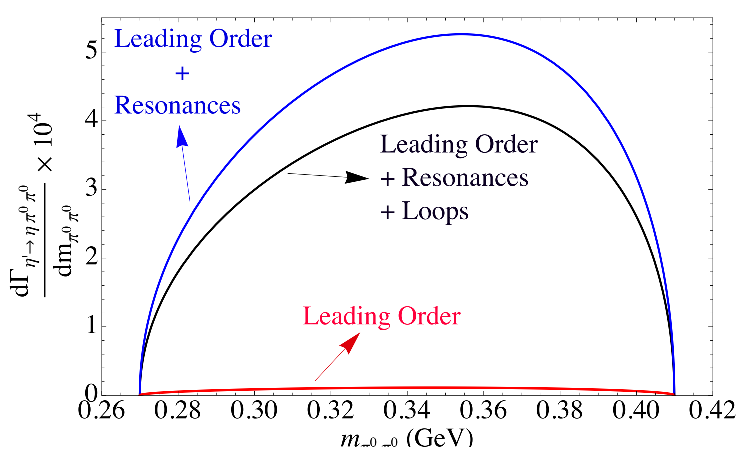


3.2 Theoretical Framework

$$\begin{pmatrix}\eta \\ \eta'\end{pmatrix} = \begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\eta_8 \\ \eta_1\end{pmatrix}$$

- U(3) ChPT with resonances at one-loop



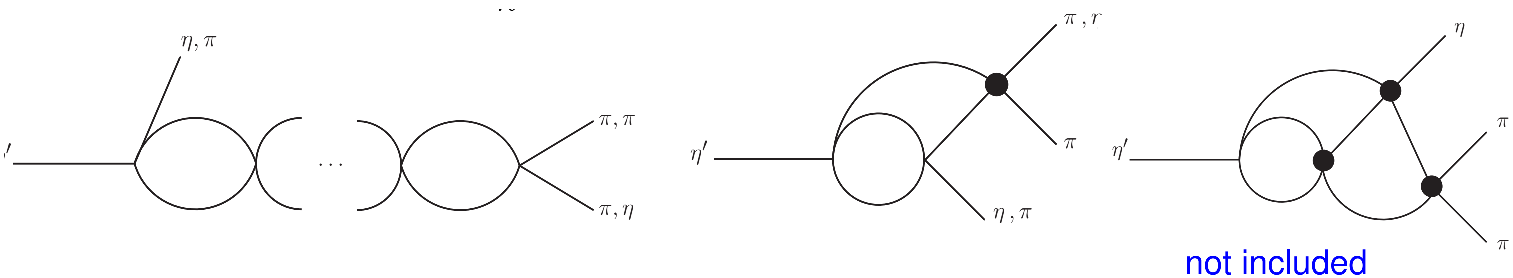


Final-state interaction through the N/D unitarization method

3.2 Theoretical Framework

- Unitarity relations

$$\mathrm{Im}\mathcal{M}_{\eta'\to\eta\pi\pi} = \frac{1}{2}\sum_n (2\pi)^4\,\delta^4\left(p_\eta + p_1 + p_2 - p_n\right)\mathcal{T}^*_{n\to\eta\pi\pi}\mathcal{M}_{\eta'\to n}$$



- A dispersive analysis also exists by *Isken et al.'17* but here we include D waves as well as kaon loops