The decays $\eta^{(\prime)} \to \pi^0 \ell^+ \ell^-$ and $\eta' \to \eta \ell^+ \ell^-$ in the standard model

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June 13, 2023

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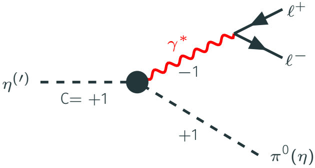
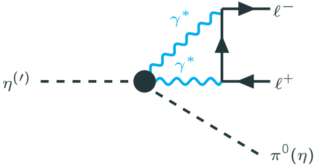
Motivation: C and CP violation

- Matter-antimatter asymmetry in the universe needs C and CP violation Sakharov 1967
- CP violation in the weak (CKM phase, C and CP violation) and strong (θ term, P and CP violation) interaction not enough to explain observed asymmetry – search for **additional sources of C and CP violation**
- η is C and P eigenstate: ideal test case
- Semileptonic decay channels C and CP violating for γ^* intermediate state → see talk of H. Akdag
- C preserved for $\gamma^*\gamma^*$ intermediate state



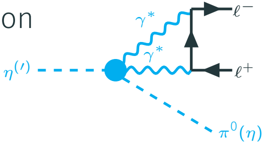
tree-level



one-loop

Semileptonic $\eta^{(\prime)}$ decays in the standard model

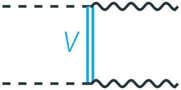
- Calculation of $\eta^{(\prime)} \to \pi^0(\eta)\ell^+\ell^-$ based on $\eta^{(\prime)} \to \pi^0(\eta)\gamma^*\gamma^*$ (similar to $P \to \ell^+\ell^-$)
- Transition form factor needs to regularize loop integral



- Several theory calculations Llewellyn Smith 1967; Smith 1968; Cheng 1967 , … , most recently Escribano and Royo 2020
- Models: effective operators, dispersive reconstruction, vector-meson dominance (VMD), unitarity bounds
- Today: better understanding of $\eta^{(\prime)} \to \pi^0(\eta)\gamma^*\gamma^*$, more experimental input, computational resources
- Improved calculation HS, Zanke, Korte, Kubis, in preparation

Modeling the hadronic part

- In ChPT, $\eta \to \pi^0\gamma\gamma$ is suppressed Ametller et al. 1992, Jetter 1996
  - No $\mathcal{O}(p^2)$ and $\mathcal{O}(p^4)$ tree level diagrams as $\eta^{(\prime)}, \pi^0$ neutral
  - Loop contributions suppressed (π loops violate isospin symmetry, K loops by $1/M_K^2$ and combinatorical factors)
- Dominant contribution: counter terms at $\mathcal{O}(p^6)$, estimated by resonance saturation $\to$ **vector-meson dominance**
- Scalar rescattering contributions of moderate size Oset et al. 2003, 2008; Lu and Moussallam 2020
- Similar for η' decays Escribano, Gonzàlez-Solís, et al. 2020
- Consider lowest-lying neutral VM as exchange particles: $\rho, \omega, \phi \to$ left-hand cut in the amplitude



Matrix element for $\eta(p) \to \pi^0(p_\pi)\ell^+(p_+)\ell^-(p_-)$

$$\mathcal{M} = i\frac{\alpha^2}{\pi^2}\sum_{V\in\{\rho,\omega,\phi\}}\int \mathrm{d}^4k\; g^{\beta\tilde\beta}\epsilon_{\mu\nu\alpha\beta}\epsilon_{\tilde\mu\tilde\nu\tilde\alpha\tilde\beta}P^\alpha k^\mu(P^{\tilde\alpha}k^{\tilde\mu} - P^{\tilde\alpha}l^{\tilde\mu} + k^{\tilde\alpha}l^{\tilde\mu})$$

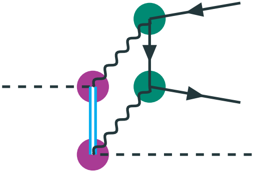
$$P_V((P-k)^2)\mathcal{F}_{V\eta}(k^2)\mathcal{F}_{V\pi^0}((l-k)^2)P_\gamma(k^2)P_\gamma((l-k)^2)$$

$$\bar u_s\left[\gamma^{\tilde\nu}\frac{\not k - \not p_+ + m_\ell}{(k-p_+)^2 - m_\ell^2}\gamma^\nu + \gamma^\nu\frac{\not p_- - \not k + m_\ell}{(p_- - k)^2 - m_\ell^2}\gamma^{\tilde\nu}\right]v_r$$

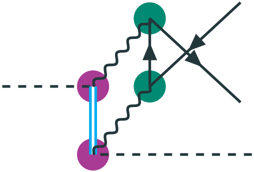
$l = p_+ + p_-$

P_V vector-meson propagator

P_γ photon propagator



t-channel

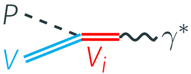


u-channel

Form factor $\mathcal{F}_{VP}$: definition

$$\langle P(p)|j_\mu(0)|V(p_V)\rangle = e\,\epsilon_{\mu\nu\alpha\beta}\epsilon^\nu(p_V)p^\alpha q^\beta \mathcal{F}_{VP}(q^2) \qquad q{=}p_V{-}p$$

- Previous VMD calculations: constant instead of energy-dependent form factors – real photons
- Energy dependence – virtual photons; otherwise, loop integral generally not convergent
- Form factor $\mathcal{F}_{VP}$ modeled via VMD



- Possible combinations of V and V_i determined by isospin and U(3) symmetry

	$V\pi^0\gamma$			$V\eta^{(\prime)}\gamma$		
V	ρ	ω	ϕ	ρ	ω	ϕ
V_i	$\omega^{(\prime)}$	$\rho^{(\prime)}$	$\rho^{(\prime)}$	$\rho^{(\prime)}$	$\omega^{(\prime)}$	$\phi^{(\prime)}$

Form factor $\mathcal{F}_{VP}$: parametrization

- Coupling for real photons fixed phenomenologically by $V \to P\gamma / P \to V\gamma$ decays, signs by U(3) flavor symmetry

$$\mathcal{F}_{VP}(q^2 = 0) = C_{VP\gamma} \equiv \mathcal{F}_{VP}^{\mathrm{PL}} \qquad \text{point-like (PL)}$$

- Simplest parametrization with $\mathcal{O}(q^{-2})$ asymptotic behavior: **monopole model (MP)**

$$\mathcal{F}_{VP}^{\mathrm{MP}}(q^2) = C_{VP\gamma} M_{V_i}^2 P_{V_i}(q^2)$$

- Expected high energy behavior $\mathcal{O}(q^{-4})$ Cherniyak et al. 1984 $\to$ **dipole model (DP)**, include next-higher multiplet of VM, $\rho' = \rho^0(1450), \omega' = \omega(1420), \phi' = \phi(1680)$

$$\mathcal{F}_{VP}^{\mathrm{DP}}(q^2) = C_{VP\gamma}\left[(1-\epsilon_{V_i}) M_{V_i}^2 P_{V_i}(q^2) + \epsilon_{V_i} M_{V_i'}^2 P_{V_i'}(q^2)\right],$$

ϵ_{V_i} tuned such that $\mathcal{O}(q^{-2})$ terms cancel

Spectral representation of vector meson propagators

- ω and ϕ narrow resonances $\to$ Breit-Wigner (BW) propagators good description

$$P_V^{\mathrm{BW}}(q^2) = \frac{1}{q^2 - M_V^2 + \mathrm{i} M_V \Gamma_V}$$

- Not true for $\rho^{(\prime)}, \omega', \phi' \to$ dispersively improved propagator

$$P_V^{\mathrm{disp}}(q^2) = -\frac{1}{\pi} \int_{s_{\mathrm{thr}}}^{\infty} \mathrm{d}x \, \frac{\mathrm{Im}\,[P_V^{\mathrm{BW}}(x)]}{q^2 - x + \mathrm{i}\epsilon}$$

$$\text{with} \quad \mathrm{Im}\,[P_V^{\mathrm{BW}}(x)] = \frac{-\sqrt{x}\,\Gamma_V(x)}{(x - M_V^2)^2 + x \Gamma_V(x)^2}$$

- Energy-dependent widths avoid unphysical imaginary parts

- Separation of spinor part in matrix element

$$\mathcal{M} = 16\pi^2\alpha^2\left[\mathcal{M}^{uv}_{\mathrm{QED}}\mathcal{M}^{uv}_{\mathrm{H}} + \mathcal{M}^{u0v}_{\mathrm{QED}}\mathcal{M}^{u0v}_{\mathrm{H}}\right]$$

$$\mathcal{M}^{uv}_{\mathrm{QED}} = \bar{u}_s v_r, \quad \mathcal{M}^{u0v}_{\mathrm{QED}} = \bar{u}_s \not{p}_0 v_r$$

- Separation of hadronic part according to coupling constants → uncertainty estimation: ΔC_V dominant

$$\mathcal{M}^{u(0)v}_{\mathrm{H}} = \sum_V C_V \mathcal{M}^{u(0)v}_V, \quad C_V = C_{V\eta^{(\prime)}\gamma} C_{V[\pi^0/\eta]\gamma}$$

- Passarino-Veltman decomposition Passarino, Veltman; 't Hooft, Veltman 1979 of $\mathcal{M}^{u(0)v}_V$ using package *FeynCalc* Mertig 1990, Shtabovenko 2016, 2020
- Numerical evaluation with *Collier* Denner and Dittmaier 2003, 2006, 2011 [*C++* interface for native *Fortran* library]

Observables: branching ratio BR and $\widehat{\mathrm{BR}}$

- $\mathrm{BR}(\eta^{(\prime)} \to [\pi^0/\eta]\ell^+\ell^-) = \frac{\Gamma}{\Gamma_{\eta^{(\prime)}}}$
- $\widehat{\mathrm{BR}}(\eta^{(\prime)} \to [\pi^0/\eta]\ell^+\ell^-) = \frac{\mathrm{BR}(\eta^{(\prime)} \to [\pi^0/\eta]\ell^+\ell^-)}{\mathrm{BR}(\eta^{(\prime)} \to [\pi^0/\eta]\gamma\gamma)}$

  $\to$ ΔC_V cancel partially
- Calculation of $\mathrm{BR}(\eta^{(\prime)} \to [\pi^0/\eta]\gamma\gamma)$ in the same framework

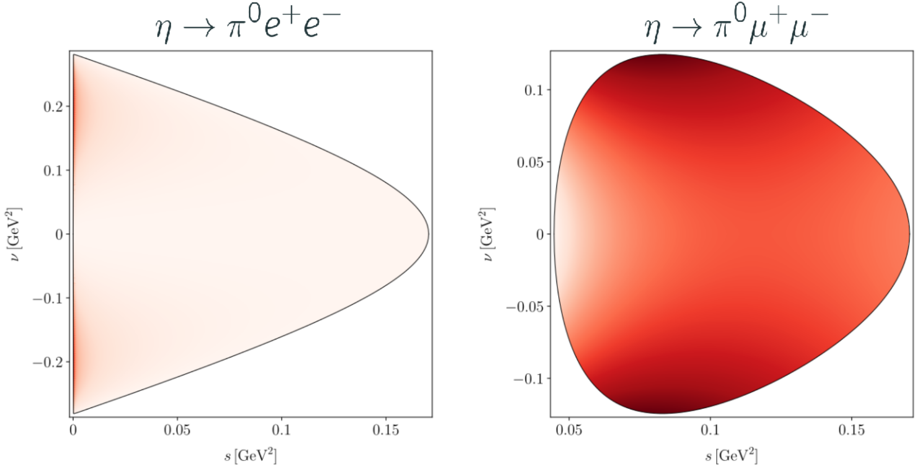
$$\mathrm{d}\Gamma = \frac{1}{(2\pi)^3}\frac{1}{64M^3_{\eta^{(\prime)}}}|\overline{\mathcal{M}}|^2 \mathrm{d}s\,\mathrm{d}\nu \quad \text{and} \quad \mathrm{d}\Gamma_\gamma = \frac{1}{(2\pi)^3}\frac{1}{32M^3_{\eta^{(\prime)}}}|\overline{\mathcal{M}}_\gamma|^2 \mathrm{d}s_\gamma\,\mathrm{d}t_\gamma$$

$$\Gamma_{(\gamma)} = C_\rho^2 \Gamma^{(\gamma)}_{\rho,\rho} + C_\omega^2 \Gamma^{(\gamma)}_{\omega,\omega} + C_\phi^2 \Gamma^{(\gamma)}_{\phi,\phi} + C_\rho C_\omega \Gamma^{(\gamma)}_{\rho,\omega} + C_\rho C_\phi \Gamma^{(\gamma)}_{\rho,\phi} + C_\omega C_\phi \Gamma^{(\gamma)}_{\omega,\phi}$$

$$s = m^2_{\ell\ell}, \nu = t - u$$

- Numerical integration (phase space and spectral parameters) with *Cuhre* and *Vegas* algorithms from *Cuba* library Hahn 2005 $\to$ numerical uncertainties negligible

Results: Dalitz plot



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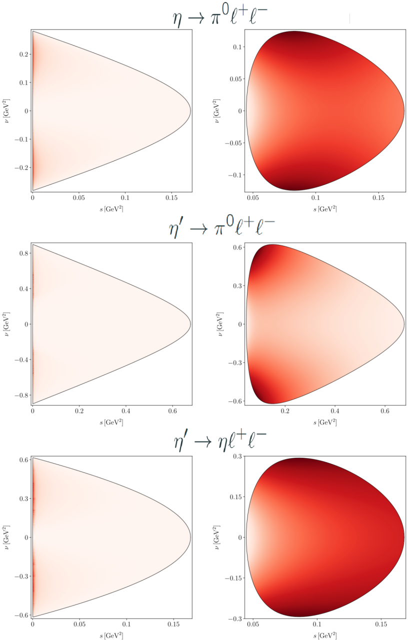
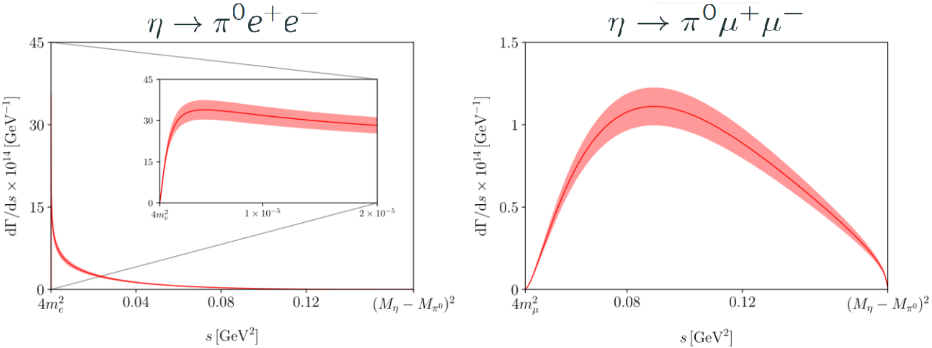


- Electrons in the final state: largest contribution for $s = m_{\ell\ell}^2$ very close to lower threshold
  $\rightarrow$ important for numerical integration and experimental measurement
- Muons: distribution more centered and spread out

Results: singly-differential decay width



- log(s) singularity from $\gamma^*\gamma^*$ intermediate state present in e^+e^- channel
- Further away from $\mu^+\mu^-$-channel phase space

Results: branching ratio

		PL / 10^{-9}	MP / 10^{-9}	DP / 10^{-9}	ER / 10^{-9}
$\eta \to \pi^0 e^+ e^-$	CW	2.10(23)	1.35(15)	1.33(15)	2.0(1)(1)(1)
	VW	2.06(22)	1.40(15)	1.36(15)	
$\eta \to \pi^0 \mu^+ \mu^-$	CW	1.37(15)	0.70(8)	0.66(7)	1.1(1)(1)(1)
	VW	1.32(14)	0.71(8)	0.67(7)	
$\eta' \to \pi^0 e^+ e^-$	CW	3.82(33)	3.08(27)	3.14(27)	4.5(3)(4)(4)
	VW	3.81(33)	3.30(28)	3.30(28)	
$\eta' \to \pi^0 \mu^+ \mu^-$	CW	2.57(23)	1.69(15)	1.68(15)	1.7(1)(2)(2)
	VW	2.53(23)	1.81(16)	1.81(16)	
$\eta' \to \eta e^+ e^-$	CW	0.53(4)	0.48(4)	0.49(4)	0.43(3)(2)(18)
	VW	0.51(4)	0.50(4)	0.50(4)	
$\eta' \to \eta \mu^+ \mu^-$	CW	0.287(26)	0.213(18)	0.207(18)	0.15(1)(1)(5)
	VW	0.280(25)	0.225(20)	0.240(21)	

ER: Escribano and Royo 2020

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ER: Escribano and Royo 2020

Comparison to experiment

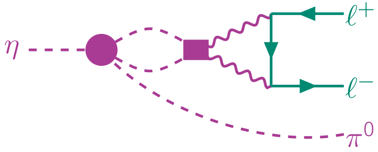
	Branching ratio		
	DP VW	exp.limit	
$\eta \to \pi^0 e^+ e^-$	$1.36(15) \times 10^{-9}$	$< 7.5 \times 10^{-6}$	Adlarson et al. 2018
$\eta \to \pi^0 \mu^+ \mu^-$	$6.7(7) \times 10^{-10}$	$< 5 \times 10^{-6}$	Dzhelyadin et al. 1981
$\eta' \to \pi^0 e^+ e^-$	$3.30(28) \times 10^{-9}$	$< 1.4 \times 10^{-3}$	Briere et al. 2000
$\eta' \to \pi^0 \mu^+ \mu^-$	$1.81(16) \times 10^{-9}$	$< 6 \times 10^{-5}$	Dzhelyadin et al. 1981
$\eta' \to \eta e^+ e^-$	$5.0(4) \times 10^{-10}$	$< 2.4 \times 10^{-3}$	Briere et al. 2000
$\eta' \to \eta \mu^+ \mu^-$	$2.40(21) \times 10^{-10}$	$< 1.5 \times 10^{-5}$	Dzhelyadin et al. 1981

REDTOP might provide comparable results Gatto 2019

Ratio of branching ratios $\widehat{\mathrm{BR}} = \mathrm{BR}_{\ell^+\ell^-}/\mathrm{BR}_{\gamma\gamma}$

	$\mathrm{BR}_{\gamma\gamma}$		$\widehat{\mathrm{BR}}$
$\eta \to \pi^0\gamma\gamma$	$1.18(13) \times 10^{-4}$	$\eta \to \pi^0 e^+e^-$	$1.1531(4) \times 10^{-5}$
		$\eta \to \pi^0 \mu^+\mu^-$	$5.647(5) \times 10^{-6}$
$\eta' \to \pi^0\gamma\gamma$	$2.81(18) \times 10^{-3}$	$\eta' \to \pi^0 e^+e^-$	$1.18(6) \times 10^{-6}$
		$\eta' \to \pi^0 \mu^+\mu^-$	$6.5(4) \times 10^{-7}$
$\eta' \to \eta\gamma\gamma$	$1.10(8) \times 10^{-4}$	$\eta' \to \eta e^+e^-$	$4.56(7) \times 10^{-6}$
		$\eta' \to \eta \mu^+\mu^-$	$2.18(4) \times 10^{-6}$

- Tension in $\mathrm{BR}(\eta \to \pi^0\gamma\gamma)$, experimentally and theoretically
  – our result is compatible with latest KLOE-2 result
  $\mathrm{BR}(\eta \to \pi^0\gamma\gamma) = (1.21 \pm 0.13 \pm 0.28) \times 10^{-4}$ Gauzzi 2022
- $\mathrm{BR}(\eta' \to [\pi^0/\eta]\gamma\gamma)$ compatible with PDG results Zyla et al. 2020



- Helicity suppressed: coupling of scalar resonance to leptons needs spin flip $\sim m_\ell$
- Start from $\eta \to \pi^0\gamma\gamma$ (cf. B. Moussallam's talk, Lu and Moussallam 2020)

$$\langle\gamma(q_1,\lambda)\gamma(q_2,\lambda')|\, S\,|\eta(P)\pi^0(p_0)\rangle \sim e^{i(\lambda-\lambda')\varphi} L_{\lambda\lambda'}(s,t)$$

- Helicity amplitudes L_{++} and L_{+-}, neglect D and higher waves $\to \mathcal{M}^{\text{scalar}} \sim L_{++} \sim \ell^0_{++}(s)$
- Dispersion relation for S-wave helicity amplitude $\ell^0_{++}(s)$ with coupled-channel formalism ($\pi^0\eta$ and $K\bar{K}$)
- Isolate S-wave amplitude by subtracting VMD contribution

Scalar rescattering corrections: results

$$|\overline{\mathcal{M}}|^2 = |\overline{\mathcal{M}^{\text{VMD}}}|^2 + |\overline{\mathcal{M}^{\text{resc}}}|^2 + 2\text{Re}\left(\mathcal{M}^{\text{resc}}\mathcal{M}^{\text{VMD}*}\right)$$

	Branching ratio		
	VMD	rescattering	mixed
$\eta \to \pi^0 e^+ e^-$	$1.36(15) \times 10^{-9}$	$2.51(9) \times 10^{-13}$	$4.63(31) \times 10^{-13}$
$\eta \to \pi^0 \mu^+ \mu^-$	$6.7(7) \times 10^{-10}$	$2.76(11) \times 10^{-11}$	$-2.63(15) \times 10^{-11}$

- BRs, as expected, of $\mathcal{O}(10^{-2}) - \mathcal{O}(10^{-4})$ of the VMD results
- Expect results of similar order for η' channels
- Negligible compared to experimental uncertainties from the coupling constants

Discussion

- Results in the same range as previous calculations
- Higher confidence with results as we took into account
  - photon virtualities $\rightarrow$ energy-dependent form factors
  - correct high-energy behavior of form factors
  - dispersively improved vector meson propagators
  - compared different parameterizations
  - compared different integration algorithms
  - estimated S-channel correction to be indeed small
- Prospect of improved experimental precision
  $\rightarrow$ comparison to experimental results in the future?

Spares

Energy-dependent widths

- ρ meson

$$\Gamma_\rho(q^2) = \theta(q^2 - 4M_{\pi^\pm}^2)\frac{\gamma_{\rho\to\pi^+\pi^-}(q^2)}{\gamma_{\rho\to\pi^+\pi^-}(M_\rho^2)} f(q^2)\Gamma_\rho$$

$$\text{with } \gamma_{\rho\to\pi^+\pi^-}(q^2) = \frac{(q^2 - 4M_{\pi^\pm}^2)^{3/2}}{q^2}$$

$$\text{barrier factors } f(q^2) = \frac{\sqrt{q^2}}{M_\rho}\frac{M_\rho^2 - 4M_{\pi^\pm}^2 + 4p_R^2}{q^2 - 4M_{\pi^\pm}^2 + 4p_R^2}$$

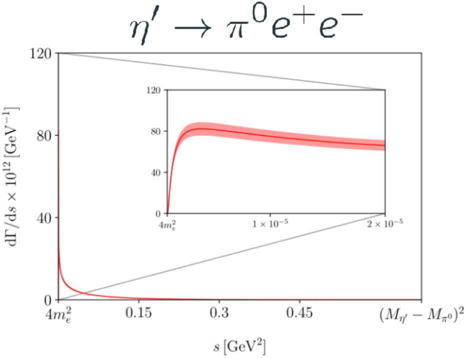
Adolph et al. 2017; Hippel and Quigg 1972

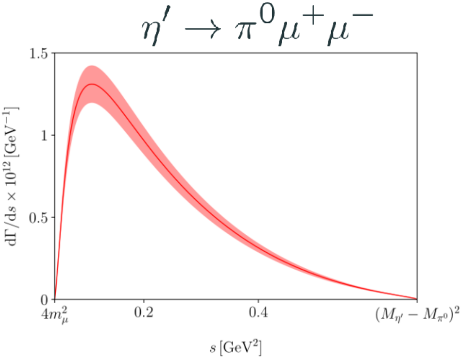
- ρ', ω', ϕ' mesons

$$\Gamma_{V'}(q^2) = \theta(q^2 - (M_V + M_P)^2)\frac{\gamma_{V'\to VP}(q^2)}{\gamma_{V'\to VP}(M_{V'}^2)}\Gamma_{V'}$$

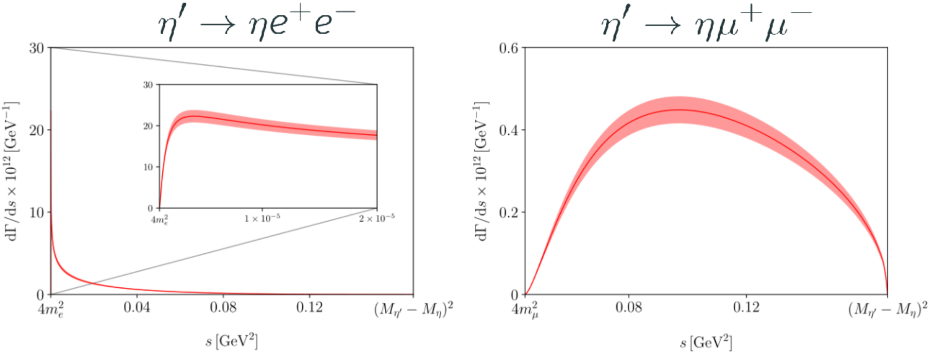
$$\text{with } \gamma_{V'\to VP}(q^2) = \frac{\lambda(q^2, M_V^2, M_P^2)^{3/2}}{(q^2)^{3/2}}$$

Results: singly-differential decay width



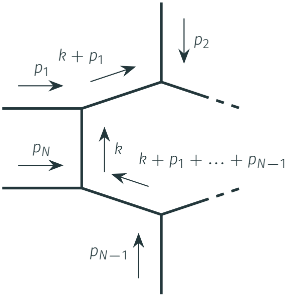


Results: singly-differential decay width



Passarino-Veltman reduction: general loop integral

$$T^N_{\mu_1\ldots\mu_M}(p_1,\ldots,p_N,m_0,\ldots,m_{N-1}) =$$



$$= \frac{(2\pi)^{4-D}}{i\pi^2}\int d^Dk$$

$$\frac{k_{\mu_1}\ldots k_{\mu_M}}{(k^2-m_0^2)[(k+p_1)^2-m_1^2]\ldots[(k+p_1+\ldots+p_N)^2-m_{N-1}^2]}$$

Passarino-Veltman reduction: scalar integrals

- Define
  - $A_0(m_0)$
  - $B_0(p, m_0, m_1)$
  - $C_0(p_1, p_2, m_0, m_1, m_2)$
  - $D_0(p_1, p_2, p_3, m_0, m_1, m_2, m_3)$

  as integrals without loop momentum in the numerator
- Can reduce any integral of shape T to those four scalar integrals
- Calculate coefficients
- General solutions to A_0, B_0, C_0, D_0 are known 't Hooft and Veltman 1979
- Numerical evaluation can be tedious; use e.g. *Collier* Denner and Dittmaier 2003, 2006, 2011

Classes of symmetry violations

CPT conserved in the Standard Model, but not individually

Different classes of violations:

- C, P, T violated (e.g. weak interaction $\rightarrow$ direct/indirect CP violation in e.g. kaon decays)
- T=CP even, C and P odd (e.g. w.i. without CKM phase)
- T=CP odd, P odd: TOPO (e.g. QCD θ term $\rightarrow$ neutron EDM)
- T=CP odd, P even ($\rightarrow$ C odd): TOPE