C and CP violation in light-meson decays

Hakan Akdag

15th June 2023

HISKP (Theory), University of Bonn

Asymmetry between matter and antimatter

- Origin of matter from baryogenesis presumes C and CP violation [*Sakharov 1967*]
- In SM: violation from weak interaction is not sufficient to create observed asymmetry

Search for new sources of CP violation:

- In EDM analyses: T-odd and P-odd quark operators at dimension 6
- Mostly neglected since 1960s: T-odd P-even (ToPe) operators
- Focus on the η meson: eigenstate of C with strongly reduced background from SM

How to investigate ToPe forces?

1. Effective field theories (EFTs)
2. Dispersion theory

1. C and CP violation in effective theories

   [*HA, Bastian Kubis, and Andreas Wirzba 2022, arXiv:2212.07794[hep-ph]*]

2. Patterns of C and CP violation in hadronic η and η' three-body decays

   [*HA, Tobias Isken, and Bastian Kubis 2021, JHEP 02 (2022) 137 , arXiv:2111.02417[hep-ph]*]

3. Correlations of C and CP violation in $\eta \to \pi^0\ell^+\ell^-$ and $\eta' \to \eta\ell^+\ell^-$

   [*HA, Bastian Kubis, and Andreas Wirzba 2023 (in preparation)*]

1 | C and CP violation in effective theories

[*HA, Bastian Kubis, and Andreas Wirzba 2022, arXiv:2212.07794[hep-ph]*]

New physics

$p \geq \Lambda$

$\mathcal{L}_{\mathrm{NP}} = ?$

Appelquist–Carazzone: high-energy phenomena decouple from low-energy physics

New physics $\longrightarrow$ SMEFT

$p \geq \Lambda$ $\qquad$ $\Lambda > p \geq \Lambda_{\mathrm{EW}}$

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \frac{1}{\Lambda^3}\mathcal{L}_7 + \frac{1}{\Lambda^4}\mathcal{L}_8 + \ldots, \quad \text{with} \quad \mathcal{L}_D \equiv \sum_i C_i^D \hat{Q}_i^D$$

[*Grzadkowski et al. 2010, Lehman et al. 2014, Murphy 2020, ...*]

Appelquist–Carazzone: high-energy phenomena decouple from low-energy physics

New physics $\longrightarrow$ SMEFT

$p \geq \Lambda$ $\qquad$ $\Lambda > p \geq \Lambda_{\text{EW}}$

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \frac{1}{\Lambda^3}\mathcal{L}_7 + \frac{1}{\Lambda^4}\mathcal{L}_8 + \dots\,, \qquad \text{with} \qquad \mathcal{L}_D \equiv \sum_i C_i^D \hat{Q}_i^D$$

[*Grzadkowski et al. 2010, Lehman et al. 2014, Murphy 2020, ...*]

Until now: "ToPe operators start at *dimension 7*"

$$\bar{\psi}\overleftrightarrow{D}_\mu\gamma_5\psi\bar{\chi}\gamma^\mu\gamma_5\chi\,, \qquad \bar{\psi}\sigma_{\mu\nu}\lambda^a\psi F^{\mu\lambda}G^{a\nu}_{\lambda}\,, \qquad \bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\lambda}Z^{\nu}_{\lambda}$$

[*Khriplovic 1991, Engel et al. 1996, Ramsey-Musolf 1999, ...*]

$\psi\,, \chi$: quarks, $\qquad F\,, G\,, Z$: gauge fields

Appelquist–Carazzone: high-energy phenomena decouple from low-energy physics

New physics $\longrightarrow$ SMEFT

$p \geq \Lambda$ $\qquad$ $\Lambda > p \geq \Lambda_{\text{EW}}$

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \frac{1}{\Lambda^3}\mathcal{L}_7 + \frac{1}{\Lambda^4}\mathcal{L}_8 + \ldots, \quad \text{with} \quad \mathcal{L}_D \equiv \sum_i C_i^D \hat{Q}_i^D$$

Until now: "ToPe operators start at *dimension 7*"

$$\underbrace{\bar{\psi}\overleftrightarrow{D}_\mu\gamma_5\psi\bar{\chi}\gamma^\mu\gamma_5\chi}_{\text{chirality-violating!}}, \qquad \underbrace{\bar{\psi}\sigma_{\mu\nu}\lambda^a\psi F^{\mu\lambda}G^{a\nu}_\lambda}_{\text{chirality-violating!}}, \qquad \underbrace{\bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\lambda}Z^\nu_\lambda}_{\text{chirality-violating!}}$$

$\Longrightarrow$ couple to a Higgs, are dimension 8 in SMEFT $\Longrightarrow$ list of operators not complete!

[*Gardner, Shi 2020*]

Appelquist–Carazzone: high-energy phenomena decouple from low-energy physics

New physics $\longrightarrow$ SMEFT $\longrightarrow$ LEFT

$p \geq \Lambda$ | $\Lambda > p \geq \Lambda_{\text{EW}}$ | $\Lambda_{\text{EW}} > p \geq \Lambda_\chi$

$$\mathcal{L}_{\text{LEFT}} = v\tilde{\mathcal{L}}_3 + \mathcal{L}_{\nu\,\text{kin}} + \mathcal{L}_{\text{QCD+QED}} + \frac{1}{v}\tilde{\mathcal{L}}_5 + \frac{1}{v^2}\tilde{\mathcal{L}}_6 + \frac{1}{v^3}\tilde{\mathcal{L}}_7 + \frac{1}{v^4}\tilde{\mathcal{L}}_8 + \dots$$

[*Jenkins et al. 2018, Liao et al. 2021, Murphy 2021, ...*]

- Relevant degrees of freedom in LEFT: check more than $45 \cdot 10^3$ operators up to dimension 8
- Proper dependence on Higgs vev v and Λ:

chirality violating $\dfrac{v}{\Lambda^4}$, chirality conserving $\dfrac{1}{\Lambda^4}$

Appelquist–Carazzone: high-energy phenomena decouple from low-energy physics

New physics ⟶ SMEFT ⟶ LEFT ⟶ χPT

$p \geq \Lambda$ | $\Lambda > p \geq \Lambda_{\text{EW}}$ | $\Lambda_{\text{EW}} > p \geq \Lambda_\chi$ | $\Lambda_\chi \geq p$

How to match LEFT onto χPT?

- Rely on external source method [*Gasser, Leutwyler 1984*]
- Treat coefficients of LEFT as spurions $\lambda^{(\dagger)}$, $\lambda_{L,R}$ (analogous to mass term χ)
- Ensure invariance $SU(3)_L \otimes SU(3)_R$, hermiticity, and correct discrete symmetries

$$\frac{v}{\Lambda^4} c_{\psi\chi}^{(a)} \bar{\psi} \overleftrightarrow{D}_\mu \gamma_5 \psi \bar{\chi} \gamma^\mu \gamma_5 \chi \longrightarrow \mathcal{L}_\chi = \frac{v}{\Lambda^4} c_{\psi\chi}^{(a)} i g_1^{(a)} \langle \lambda D_\mu U^\dagger + \lambda^\dagger D_\mu U \rangle \langle \lambda_L D^\mu U^\dagger U + \lambda_R D^\mu U U^\dagger \rangle + \ldots$$

- List of ToPe operators in LEFT and matching to χPT in [*Akdag et al. 2022*]

Applications

Decay	Mesonic operator	Lowest order	Current measurement	Theoretical estimate
$\eta^{(\prime)} \to \pi^0\pi^+\pi^-$	$i\,\eta^{(\prime)}\partial^\mu\pi^0(\pi^+\partial_\mu\pi^- - \pi^-\partial_\mu\pi^+)$	$p^2\,(\delta^0)$	$g_2 = -9.3(4.5)\cdot 10^3/\text{TeV}^2$	$\lvert g_2\rvert \sim 3\cdot 10^{-4}\,\text{TeV}^2/\Lambda^4$
$\eta' \to \eta\pi^+\pi^-$	$i\,\eta'\partial^\mu\eta(\pi^+\partial_\mu\pi^- - \pi^-\partial_\mu\pi^+)$	$p^2\,(\delta^1)$	$g_1 = 7(10)\cdot 10^5/\text{TeV}^2$	$\lvert g_1\rvert \sim 3\cdot 10^{-4}\,\text{TeV}^2/\Lambda^4$
$\eta \to \pi^0 e^+e^-$	$\eta\partial_\mu\pi^0\,\bar{e}\gamma^\mu e$	$p^2\,(\delta^1)$	$\text{BR} < 7.5\cdot 10^{-6}$	$\text{BR} \sim 7\cdot 10^{-27}\,\text{TeV}^8/\Lambda^8$
$\eta' \to \eta e^+e^-$	$\eta'\partial_\mu\eta\,\bar{e}\gamma^\mu e$	$p^2\,(\delta^1)$	$\text{BR} < 2.4\cdot 10^{-3}$	$\text{BR} \sim 9\cdot 10^{-29}\,\text{TeV}^8/\Lambda^8$
$\eta \to \pi^+\pi^-\gamma$	$\epsilon_{\alpha\beta\mu\nu}\,\eta(\partial^\nu\pi^+\partial^\rho\partial^\mu\pi^- + \partial^\nu\pi^-\partial^\rho\partial^\mu\pi^+)\partial_\rho F^{\alpha\beta}$	$p^6\,(\delta^2)$	$A_{LR} = 0.009(4)$	$\lvert A_{LR}\rvert \sim 5\cdot 10^{-16}\,\text{TeV}^4/\Lambda^4$
$\eta \to \pi^0\pi^0\gamma$	$\epsilon_{\alpha\beta\mu\nu}\,\eta(\partial^\nu\pi^0\partial^\rho\partial^\mu\pi^0 + \partial^\nu\pi^0\partial^\rho\partial^\mu\pi^0)\partial_\rho F^{\alpha\beta}$	$p^6\,(\delta^3)$	$\text{BR} < 5\cdot 10^{-4}$	$\text{BR} \sim 1\cdot 10^{-29}\,\text{TeV}^8/\Lambda^8$
$\eta' \to \eta\pi^0\gamma$	$\epsilon_{\alpha\beta\mu\nu}\,\eta'\partial^\mu\eta\partial^\nu\pi^0 F^{\alpha\beta}$	$p^4\,(\delta^3)$	–	$\text{BR} \sim 2\cdot 10^{-28}\,\text{TeV}^8/\Lambda^8$
$\eta' \to \eta\pi^0\pi^0\gamma$	$\eta'\partial_\mu\eta\pi^0\partial_\nu\pi^0 F^{\mu\nu}$	$p^4\,(\delta^2)$	–	$\text{BR} \sim 2\cdot 10^{-32}\,\text{TeV}^8/\Lambda^8$
$\eta \to 3\pi^0\gamma$	$\partial_\mu\eta\partial_\nu\pi^0\partial_\alpha\pi^0\pi^0\partial^\alpha F^{\mu\nu}$	$p^6\,(\delta^3)$	$\text{BR} < 6\cdot 10^{-5}$	$\text{BR} \sim 1\cdot 10^{-35}\,\text{TeV}^8/\Lambda^8$
$\eta \to 3\gamma$	$\epsilon^{\mu\nu\rho\sigma}\partial_\alpha\eta(\partial^\gamma F^{\alpha\beta})(\partial_\gamma\partial_\beta F_{\rho\sigma})F_{\mu\nu}$	$p^{10}\,(\delta^4)$	$\text{BR} < 4\cdot 10^{-5}$	$\text{BR} \sim 1\cdot 10^{-36}\,\text{TeV}^8/\Lambda^8$

[*... and many more in Akdag et al. 2022*]

2 | Patterns of C and CP violation in hadronic η and η' three-body decays

[*HA, Tobias Isken, and Bastian Kubis 2021, arXiv:2111.02417[hep-ph]*]

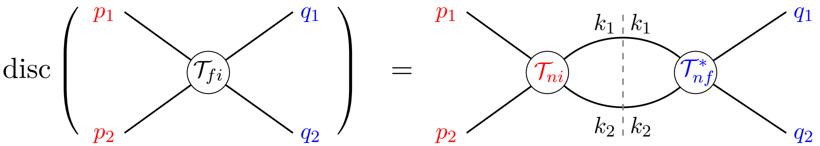
Dispersive Framework

Evaluate three-particle decay in dispersive (*Khuri-Treiman*) framework:

Model independent and non-perturbative re-summation of final-state interactions, based on

1. Unitarity ($\sim$ probability conservation) gives rise to *optical theorem*:

$$\mathcal{T}_{fi} - \mathcal{T}_{if}^* = i \sum_n \int \mathrm{d}\Pi_n (2\pi)^4 \delta^4 \Big(\sum_{i,n} p_i - k_n \Big) \mathcal{T}_{ni} \mathcal{T}_{nf}{}^*$$



2. Analyticity ($\sim$ causality)

   Dispersion relations: reconstruct whole amplitude with knowledge about discontinuity

Idea: derive $2 \to 2$ scattering amplitude and analytically continue to realm of $1 \to 3$ decay

- $\eta \to \pi^0\pi^+\pi^-$ *breaks* G-parity; in the Standard Model consider transition with $\Delta I = 1$

$$\mathcal{M}(s,t,u) = \xi \mathcal{M}_1^C(s,t,u)$$ [*cf. talks by I. Danilkin and E. Passemar*]

[*Colangelo et al. 2018; Albaladejo et al. 2017; Guo et al. 2017; ...*]

- $\eta \to \pi^0\pi^+\pi^-$ *breaks* G-parity; in the Standard Model consider transition with $\Delta I = 1$
- For C-violating parts consider $C = -(-1)^{\Delta I}$, i.e., need even *total* isospin [*Gardner, Shi 2020*]

$$\mathcal{M}(s,t,u) = \xi\mathcal{M}_1^{C}(s,t,u) + \mathcal{M}_0^{\not{C}}(s,t,u) + \mathcal{M}_2^{\not{C}}(s,t,u)$$

$\eta \to \pi^0\pi^+\pi^-$: Amplitude Decomposition

- $\eta \to \pi^0\pi^+\pi^-$ *breaks* G-parity; in the Standard Model consider transition with $\Delta I = 1$
- For C-violating parts consider $C = -(-1)^{\Delta I}$, i.e., need even *total* isospin [*Gardner, Shi 2020*]

$$\mathcal{M}(s,t,u) = \xi \mathcal{M}_1^{C}(s,t,u) + \mathcal{M}_0^{\not{C}}(s,t,u) + \mathcal{M}_2^{\not{C}}(s,t,u)$$

- Bose symmetry: odd (even) $\pi\pi$-isospin must have odd (even) partial wave
- Reconstruction theorem: expand for fixed *two-body* isospin and partial wave

$$\mathcal{M}_1^{C}(s,t,u) = \mathcal{F}_0(s) + (s-u)\mathcal{F}_1(t) + (s-t)\mathcal{F}_1(u) + \mathcal{F}_2(t) + \mathcal{F}_2(u) - \frac{2}{3}\mathcal{F}_2(s)$$

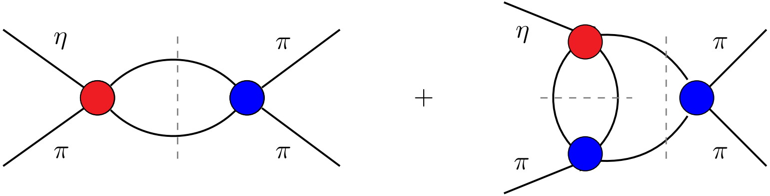
$$\mathcal{M}_0^{\not{C}}(s,t,u) = (t-u)\mathcal{G}_1(s) + (u-s)\mathcal{G}_1(t) + (s-t)\mathcal{G}_1(u)$$

$$\mathcal{M}_2^{\not{C}}(s,t,u) = 2(u-t)\mathcal{H}_1(s) + (u-s)\mathcal{H}_1(t) + (s-t)\mathcal{H}_1(u) - \mathcal{H}_2(t) + \mathcal{H}_2(u)$$

- C-even terms are symmetric and C-odd ones antisymmetric under $t \leftrightarrow u$
- Note: $\mathcal{F}_I$, $\mathcal{G}_I$ and $\mathcal{H}_I$ are completely independent

- Single-variable amplitudes $\mathcal{A} \in \{\mathcal{F}, \mathcal{G}, \mathcal{H}\}$ obey discontinuity relation

$$\text{disc}\,\mathcal{A}_I(s) = 2i\,\theta(s - 4M_\pi^2)\,\big[\mathcal{A}_I(s) + \hat{\mathcal{A}}_I(s)\big]\,\sin\delta_I(s)\,e^{-i\delta_I(s)}$$



- Full solution

$$\mathcal{A}_I(s) = \Omega_I(s)\left(P_{n-1}(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\mathrm{d}x}{x^n}\,\frac{\sin\delta_I(x)\,\hat{\mathcal{A}}_I(x)}{|\Omega_I(x)|\,(x-s)}\right)$$

- Subtraction polynomial P_{n-1} fixed by asymptotics imposed on $\mathcal{A}_I$ and δ_I and by data

Regression to Dalitz plot [*KLOE-2 2016*]

The SM amplitude $\mathcal{M}_1^C$:

- Minimal subtraction scheme 3 dof: $\chi^2_{\text{red}} \approx 1.054$
- Observables agree with data [*Colangelo et al. 2018, PDG 2020*]

$\Longrightarrow$ subtraction scheme justified, apply also to $\mathcal{M}_{0,2}^{\not{C}}$

Regression to Dalitz plot [*KLOE-2 2016*]

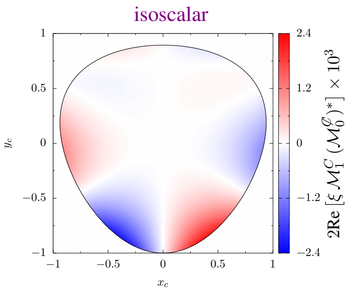
The SM amplitude $\mathcal{M}_1^C$:

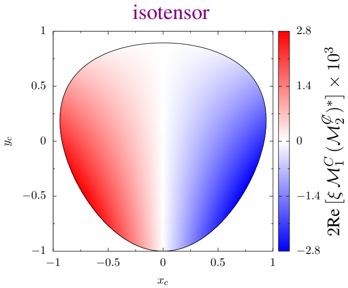
- Minimal subtraction scheme 3 dof: $\chi^2_{\text{red}} \approx 1.054$
- Observables agree with data [*Colangelo et al. 2018, PDG 2020*]

$\Longrightarrow$ subtraction scheme justified, apply also to $\mathcal{M}_{0,2}^{\not C}$

The BSM amplitude $\mathcal{M} = \xi\mathcal{M}_1^C + \mathcal{M}_0^{\not C} + \mathcal{M}_2^{\not C}$:

- Fix $\mathcal{M}_{0,2}^{\not C}$ by one *real*-valued dof each (phase is fixed!)
- Full amplitude 5 dof: $\chi^2_{\text{red}} \approx 1.048$
- Upper limit on ToPe effects in per mille level
- All C- and CP-violating signals vanish within 1-2σ





- Match dispersive amplitude to ToPeχPT

$$\mathcal{M}_0^{\not{C}} \sim i\, g_0 (s-t)(t-u)(u-s) + \mathcal{O}(p^8)$$

$$\mathcal{M}_2^{\not{C}} \sim i\, g_2 (t-u) + \mathcal{O}(p^4)$$

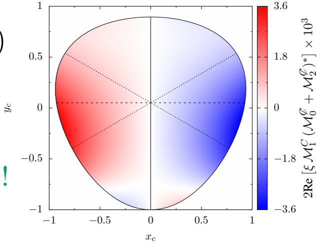
  Phase fixed by T-violation and hermiticity
  $\Longrightarrow g_0, g_2 \in \mathbb{R}$, no asymmetries without strong rescattering phases!

- Obtain couplings by a Taylor expansion of $\mathcal{M}_0^{\not{C}}$, $\mathcal{M}_2^{\not{C}}$:

$$g_0 = -2.8(4.5)\,\text{GeV}^{-6}\,, \qquad g_2 = -9.3(4.6)\cdot 10^{-3}\,\text{GeV}^{-2}$$

- $\mathcal{M}_0^{\not{C}}$ kinematically suppressed: data less sensitive to $g_0 \approx g_2 \cdot 10^3\,\text{GeV}^{-4}$

$\eta \to \pi^0\pi^+\pi^-$: *BSM Couplings*



- Match dispersive amplitude to ToPeχPT

$$\mathcal{M}_0^{\not{\mathcal{C}}} \sim i\, g_0 (s-t)(t-u)(u-s) + \mathcal{O}(p^8)$$

$$\mathcal{M}_2^{\not{\mathcal{C}}} \sim i\, g_2 (t-u) + \mathcal{O}(p^4)$$

  Phase fixed by T-violation and hermiticity
  $\Longrightarrow g_0, g_2 \in \mathbb{R}$, no asymmetries without strong rescattering phases!

- Obtain couplings by a Taylor expansion of $\mathcal{M}_0^{\not{\mathcal{C}}}$, $\mathcal{M}_2^{\not{\mathcal{C}}}$:

$$g_0 = -2.8(4.5)\,\text{GeV}^{-6}\,, \qquad g_2 = -9.3(4.6)\cdot 10^{-3}\,\text{GeV}^{-2}$$

- $\mathcal{M}_0^{\not{\mathcal{C}}}$ kinematically suppressed: data less sensitive to $g_0 \approx g_2 \cdot 10^3\,\text{GeV}^{-4}$

- Dalitz-plot asymmetries in 10^{-4} (g_0 in GeV^{-6}, g_2 in $10^3\,\text{GeV}^{-2}$):

$$A_{\text{left-right}} = -0.300\, g_0 + 0.936\, g_2 = -7.9(4.5)$$

$$A_{\text{quadrant}} = 0.443\, g_0 - 0.336\, g_2 = 1.9(2.5)$$

$$A_{\text{sextant}} = -0.850\, g_0 + 0.043\, g_2 = 2.0(3.8)$$

- $\eta' \to \eta\pi^+\pi^-$ *conserves* G-parity; in the Standard Model consider transition with $\Delta I = 0$

$$\mathcal{M}(s,t,u) = \mathcal{M}_0^C(s,t,u)$$

[*Isken et al. 2017, Isken et al. 2023 (in preparation)*]

- $\eta' \to \eta\pi^+\pi^-$ *conserves* G-parity; in the Standard Model consider transition with $\Delta I = 0$
- C-violation driven by isospin $\Delta I = 1$

$$\mathcal{M}(s,t,u) = \mathcal{M}_0^{C}(s,t,u) + \mathcal{M}_1^{\not{C}}(s,t,u)$$

- $\eta' \to \eta\pi^+\pi^-$ *conserves* G-parity; in the Standard Model consider transition with $\Delta I = 0$
- C-violating driven by isospin $\Delta I = 1$

$$\mathcal{M}(s,t,u) = \mathcal{M}_0^{C}(s,t,u) + \mathcal{M}_1^{\not{C}}(s,t,u)$$

- Sensitive to a different class of BSM operators! (different normalizations in ToPeχPT)

$\eta' \to \eta\pi^+\pi^-$: *Amplitude Decomposition*

- $\eta' \to \eta\pi^+\pi^-$ *conserves* G-parity; in the Standard Model consider transition with $\Delta I = 0$
- C-violating driven by isospin $\Delta I = 1$

$$\mathcal{M}(s,t,u) = \mathcal{M}_0^{C}(s,t,u) + \mathcal{M}_1^{\not C}(s,t,u)$$

- Sensitive to a different class of BSM operators
- Reconstruction theorem: two different intermediate states

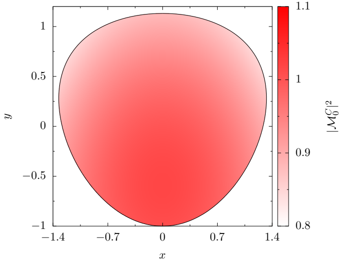
$$\mathcal{M}_0^{C}(s,t,u) = \mathcal{F}_{\pi\pi}(s) + \mathcal{F}_{\eta\pi}(t) + \mathcal{F}_{\eta\pi}(u)$$
$$\mathcal{M}_1^{\not C}(s,t,u) = (t-u)\,\mathcal{G}_{\pi\pi}(s) + \mathcal{G}_{\eta\pi}(t) - \mathcal{G}_{\eta\pi}(u)$$

- $\mathcal{F}_{\pi\pi}$, $\mathcal{F}_{\eta\pi}$, and $\mathcal{G}_{\eta\pi}$ in S-waves, $\mathcal{G}_{\pi\pi}$ in P-wave
- Solution analogous to $\eta \to 3\pi$

Regression to Dalitz plot [*BESIII, 2018*]

The SM amplitude $\mathcal{M}_0^C$:

- Minimal subtraction scheme fails to describe data accurately
- Need at least 4 dof: $\chi^2_{\text{red}} \approx 0.994$



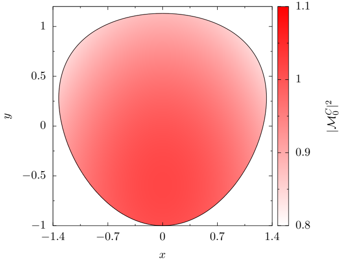
Regression to Dalitz plot [*BESIII, 2018*]

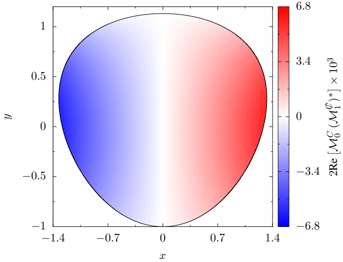
The SM amplitude $\mathcal{M}_0^C$:

- Minimal subtraction scheme fails to describe data accurately
- Need at least 4 dof: $\chi^2_{\text{red}} \approx 0.994$

Apply same subtraction scheme for BSM amplitude:

- Fix $\mathcal{M}_1^{\not{C}}$ by two *real* coefficients
- Full amplitude 6 dof: $\chi^2_{\text{red}} \approx 0.994$
- Upper limit on TOPE effects in per mille level
- All C- and CP-violating signals vanish within $< 1.5\sigma$





- Effective BSM operator

$$X_1^{\not{C}} \sim i\, g_1\, (t-u)\, (1 + s\, \delta g_1) + \mathcal{O}(p^6)$$

- Obtain coupling and s-dependent correction by a Taylor expansion of $\mathcal{M}_1^{\not{C}}$:

$$g_1 = 0.7(1.0)\,\text{GeV}^{-2}$$

$$\delta g_1 = -5.5(7.3)\,\text{GeV}^{-2}$$

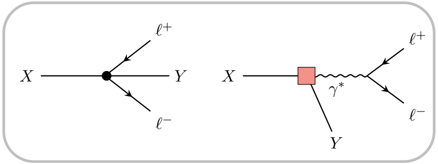
- Dalitz-plot asymmetry in 10^{-3} (g_1 and δg_1 in GeV^{-2}):

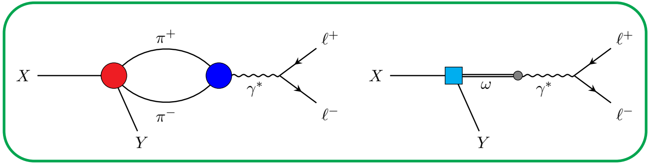
$$A_{\text{left}-\text{right}} = -6.6\, g_1\, (1 + 0.10\, \delta g_1) = -2.3(1.7)$$

3 | Correlations of C and CP violation in $\eta \to \pi^0 \ell^+ \ell^-$ and $\eta' \to \eta \ell^+ \ell^-$

[*HA, Bastian Kubis, and Andreas Wirzba 2023 (in preparation)*]

- Current effort as two-photon process in SM [*R. Escribano et al. 2022; H. Schäfer et al. 2023 (in preparation)*]
- ToPe forces driven by one-photon exchange

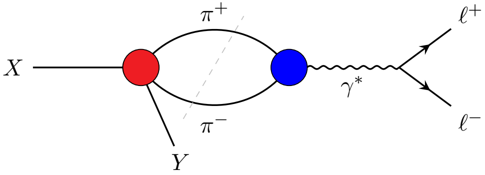




- *Predict* isovector and isoscalar form factor of hadronic intermediate states

$$F_{XY}(s) = F_{XY}^{(1)}(s) + F_{XY}^{(0)}(s)$$

through correlations with C and CP violation in $\eta \to 3\pi$ and $\eta' \to \eta\pi\pi$

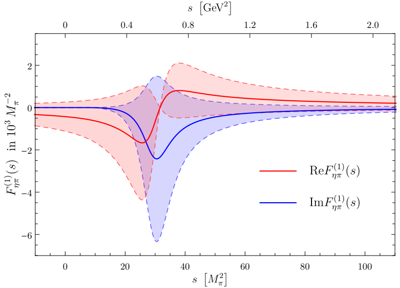


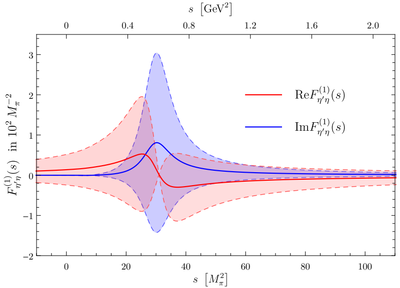
[*cf.* $\omega \to \pi^0\gamma^*$ *in talk by M. Albaladejo*]

$$F_{XY}^{(1)}(s) = \frac{i}{48\pi^2}\int_{4M_\pi^2}^{\infty} \mathrm{d}x\, \sigma_\pi^3(x) F_\pi^{V*}(x)\, \frac{f_{XY}(x)}{x-s-i\epsilon}$$

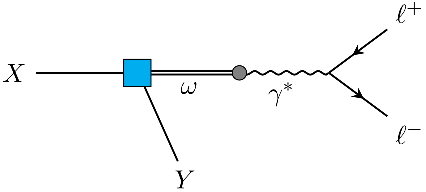
- P-wave $f_{XY}(s)$ known from C- and CP-odd $\eta \to 3\pi$ and $\eta' \to \eta\pi\pi$
- Pion vector form factor

$$F_\pi^V(s) = \Omega_1(s) = \exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} \frac{\delta_1(x)}{x(x-s)}\,\mathrm{d}x\right)$$





Symmetry-driven vector-meson dominance (VMD) model



$$F^{(0)}_{\eta\pi}(s) = \frac{g_{\omega Y X}}{2 g_{\omega\gamma}} \frac{M_\omega^2}{M_\omega^2 - s}$$

Extract resonance coupling by analytic continuation of

$\eta' \to \eta\pi^+\pi^-$ $\xleftrightarrow{\text{analytic cont.}}$ $g_{\rho\eta\eta'}$ $\xleftrightarrow{\text{NDA, isospin}}$ $g_{\omega\pi\eta}$ $\xleftrightarrow{\text{VMD}}$ $\eta \to (\omega\pi^0 \to)\ \pi^0\gamma^*$

$\eta \to \pi^0\pi^+\pi^-$ $\xleftrightarrow{\text{analytic cont.}}$ $g_{\rho\pi\eta}$ $\xleftrightarrow{\text{NDA, isospin}}$ $g_{\omega\eta\eta'}, g_{\phi\eta\eta'}$ $\xleftrightarrow{\text{VMD}}$ $\eta' \to (\eta\omega, \eta\phi \to)\ \eta\gamma^*$

- Decrease systematic experimental errors by $R_{X\ell} \equiv \frac{\Gamma(X \to Y \ell^+ \ell^-)}{\Gamma(X \to Y \pi^+ \pi^-)}$
- $\eta \to \pi^0 \ell^+ \ell^-$

$$R^{(1)}_{\eta e} < 8.8 \cdot 10^{-5} \qquad R_{\eta e} < 12.6 \cdot 10^{-5} \qquad R^{\text{exp}}_{\eta e} < 3.3 \cdot 10^{-5}$$ [WASA-at-COSY 2018]

$$R^{(1)}_{\eta \mu} < 3.1 \cdot 10^{-5} \qquad R_{\eta \mu} < 4.5 \cdot 10^{-5} \qquad R^{\text{exp}}_{\eta \mu} < 2.6 \cdot 10^{-5}$$ [Dzhelyadin et al. 1980]

- $\eta' \to \eta \ell^+ \ell^-$

$$R^{(1)}_{\eta' e} < 1.1 \cdot 10^{-5} \qquad R_{\eta' e} < 2.2 \cdot 10^{-5} \qquad R^{\text{exp}}_{\eta' e} < 5.6 \cdot 10^{-5}$$ [CLEO 1999]

$$R^{(1)}_{\eta' \mu} < 4.5 \cdot 10^{-6} \qquad R_{\eta' \mu} < 9.2 \cdot 10^{-6} \qquad R^{\text{exp}}_{\eta' \mu} < 3.5 \cdot 10^{-5}$$ [Dzhelyadin et al. 1980]

- Experiments for $X \to Y \ell^+ \ell^-$ and $X \to Y \pi^+ \pi^-$ share similiar sensitivity to ToPe effects
- Isovector and isoscalar contribution are of the same order of magnitude

- Decrease systematic experimental errors by $R_{X\ell} \equiv \frac{\Gamma(X \to Y \ell^+ \ell^-)}{\Gamma(X \to Y \pi^+ \pi^-)}$
- $\eta \to \pi^0 \ell^+ \ell^-$

$$R_{\eta e}^{(1)} < 8.8 \cdot 10^{-5} \qquad R_{\eta e} < 12.6 \cdot 10^{-5} \qquad R_{\eta e}^{\text{exp}} < 3.3 \cdot 10^{-5}$$ [*WASA-at-COSY 2018*]

$$R_{\eta \mu}^{(1)} < 3.1 \cdot 10^{-5} \qquad R_{\eta \mu} < 4.5 \cdot 10^{-5} \qquad R_{\eta \mu}^{\text{exp}} < 2.6 \cdot 10^{-5}$$ [*Dzhelyadin et al. 1980*]

- Refine regression to $\eta \to \pi^0 \pi^+ \pi^-$ with isovector part of $\eta \to \pi^0 \ell^+ \ell^-$ as constraint:

$$g_0 = -2.8(4.5)\,\text{GeV}^{-6} \quad \longrightarrow \quad -1.2(4.5)\,\text{GeV}^{-6}$$

$$A_{\text{left-right}} = -7.9(4.5) \quad \longrightarrow \quad -8.3(4.5)$$

$$A_{\text{quadrant}} = 1.9(2.5) \quad \longrightarrow \quad 2.6(2.5)$$

$$A_{\text{sextant}} = 2.0(3.8) \quad \longrightarrow \quad 0.6(3.8)$$

Model-independent description of C and CP violation in η decays

1. ToPeχPT
   - Set of C- and CP-odd quark-level operators (start at dimension 8 in SMEFT)
   - Can access every ToPe η-decay into SM particles, hierarchies, correlations, and BSM scale Λ
2. Dispersive approach
   - Non-perturbative, based on fundamental principles of analyticity, unitarity, and crossing
   - Extracted effective BSM couplings in $\eta \to \pi^0\pi^+\pi^-$ and $\eta' \to \eta\pi^+\pi^-$ (match to ToPeχPT)
   - Correlated $\eta \to \pi^0\pi^+\pi^-(\eta' \to \eta\pi^+\pi^-)$ with $\eta \to \pi^0\ell^+\ell^-(\eta' \to \eta\ell^+\ell^-)$

Summary & Outlook

Model-independent description of C and CP violation in η decays

1. ToPeχPT
   - Set of C- and CP-odd quark-level operators (start at dimension 8 in SMEFT)
   - Can access every ToPe η-decay into SM particles, hierarchies, correlations, and BSM scale Λ
2. Dispersive approach
   - Non-perturbative, based on fundamental principles of analyticity, unitarity, and crossing
   - Extracted effective BSM couplings in $\eta \to \pi^0\pi^+\pi^-$ and $\eta' \to \eta\pi^+\pi^-$ (match to ToPeχPT)
   - Correlated $\eta \to \pi^0\pi^+\pi^-(\eta' \to \eta\pi^+\pi^-)$ with $\eta \to \pi^0\ell^+\ell^-(\eta' \to \eta\ell^+\ell^-)$

Future theoretical interest:

- Formalism of ToPeχPT can be extended to arbitrary mesonic/baryonic processes
- Interpret future ToPe signals appropriately

Summary & Outlook

Model-independent description of C and CP violation in η decays

1. ToPeχPT
   - Set of C- and CP-odd quark-level operators (start at dimension 8 in SMEFT)
   - Can access every ToPe η-decay into SM particles, hierarchies, correlations, and BSM scale Λ
2. Dispersive approach
   - Non-perturbative, based on fundamental principles of analyticity, unitarity, and crossing
   - Extracted effective BSM couplings in $\eta \to \pi^0\pi^+\pi^-$ and $\eta' \to \eta\pi^+\pi^-$ (match to ToPeχPT)
   - Correlated $\eta \to \pi^0\pi^+\pi^-(\eta' \to \eta\pi^+\pi^-)$ with $\eta \to \pi^0\ell^+\ell^-(\eta' \to \eta\ell^+\ell^-)$

Future theoretical interest:

- Formalism of ToPeχPT can be extended to arbitrary mesonic/baryonic processes
- Interpret future ToPe signals appropriately

From experimental point of view:

- *JLab Eta Factory* (JEF)
- *Rare Eta Decays with a TPC for Optical Photons* (REDTOP)

Thank you very much for your attention!

Backup

$$\mathcal{O}^{(a)}_{\psi\chi} \equiv \frac{v}{\Lambda^4} c^{(a)}_{\psi\chi}\, \bar\psi \overleftrightarrow{D}_\mu \gamma_5 \psi \bar\chi \gamma^\mu \gamma_5 \chi$$

$$\mathcal{O}^{(b)}_{\psi} \equiv \frac{v}{\Lambda^4} c^{(b)}_{\psi}\, \bar\psi T^a \sigma^{\mu\nu} \psi F_{\mu\rho} G^{a\rho}_{\nu}$$

$$\mathcal{O}^{(n)}_{\psi\chi} \equiv \frac{1}{\Lambda^4} c^{(n)}_{\psi\chi}\, \bar\psi \gamma^\mu \psi \bar\chi \gamma^\nu \chi F_{\mu\nu}$$

$$\mathcal{O}^{(o)}_{\psi\chi} \equiv \frac{1}{\Lambda^4} c^{(o)}_{\psi\chi}\, \bar\psi \gamma^\mu \gamma_5 \psi \bar\chi \gamma^\nu \gamma_5 \chi F_{\mu\nu}$$

$$\mathcal{O}^{(p)}_{\psi\chi} \equiv \frac{1}{\Lambda^4} c^{(p)}_{\psi\chi}\, \bar\psi \gamma^\mu T^a \psi \bar\chi \gamma^\nu T^a \chi F_{\mu\nu}$$

$$\mathcal{O}^{(q)}_{\psi\chi} \equiv \frac{1}{\Lambda^4} c^{(q)}_{\psi\chi}\, \bar\psi \gamma^\mu \gamma_5 T^a \psi \bar\chi \gamma^\nu \gamma_5 T^a \chi F_{\mu\nu}$$

$$\mathcal{O}^{(r)}_{\psi\chi} \equiv \frac{1}{\Lambda^4} c^{(r)}_{\psi\chi}\, i[\bar\psi \chi \bar\chi \sigma^{\mu\nu} \psi + \bar\psi \gamma_5 \chi \bar\chi \sigma^{\mu\nu} \gamma_5 \psi - (\psi \leftrightarrow \chi)] F_{\mu\nu}$$

$$\mathcal{O}^{(s)}_{\psi} \equiv \frac{1}{\Lambda^4} c^{(s)}_{\psi}\, \bar\psi \gamma^\mu i \overleftrightarrow{D}^\nu T^a \gamma_5 \psi F_{\mu\rho} \tilde{G}^{a\,\rho}_{\nu}$$

$$\mathcal{O}^{(t)}_{\psi} \equiv \frac{1}{\Lambda^4} c^{(t)}_{\psi}\, \bar\psi \gamma^\mu i \overleftrightarrow{D}^\nu T^a \gamma_5 \psi F_{\nu\rho} \tilde{G}^{a\,\rho}_{\mu}$$

$$\mathcal{O}^{(e)}_{\psi} \equiv \frac{1}{\Lambda^4} c^{(e)}_{\psi}\, f_{abc} \bar\psi \gamma^\mu i \overleftrightarrow{D}^\nu T^a \psi G^b_{\mu\rho} G^{c\,\rho}_{\nu}$$

$$\mathcal{O}^{(f)}_{\psi\chi} \equiv \frac{1}{\Lambda^4} c^{(f)}_{\psi\chi}\, \bar\psi \gamma^\mu \psi \bar\chi \gamma^\nu T^a \chi G^a_{\mu\nu}$$

$$\mathcal{O}^{(g)}_{\psi\chi} \equiv \frac{1}{\Lambda^4} c^{(g)}_{\psi\chi}\, \bar\psi \gamma^\mu \gamma_5 \psi \bar\chi \gamma^\nu \gamma_5 T^a \chi G^a_{\mu\nu}$$

$$\mathcal{O}^{(h)}_{\psi\chi} \equiv \frac{1}{\Lambda^4} c^{(h)}_{\psi\chi}\, f_{abc} \bar\psi \gamma^\mu \gamma_5 T^a \psi \bar\chi \gamma^\nu T^b \chi \tilde{G}^c_{\mu\nu}$$

$$\mathcal{O}^{(i)}_{\psi\chi} \equiv \frac{1}{\Lambda^4} c^{(i)}_{\psi\chi}\, d_{abc} \bar\psi \gamma^\mu T^a \psi \bar\chi \gamma^\nu T^b \chi G^c_{\mu\nu}$$

$$\mathcal{O}^{(j)}_{\psi\chi} \equiv \frac{1}{\Lambda^4} c^{(j)}_{\psi\chi}\, d_{abc} \bar\psi \gamma^\mu \gamma_5 T^a \psi \bar\chi \gamma^\nu \gamma_5 T^b \chi G^c_{\mu\nu}$$

$$\mathcal{O}^{(k)}_{\psi\chi} \equiv \frac{1}{\Lambda^4} c^{(k)}_{\psi\chi}\, i[\bar\psi T^a \chi \bar\chi \sigma^{\mu\nu} \psi + \bar\psi \gamma_5 T^a \chi \bar\chi \sigma^{\mu\nu} \gamma_5 \psi - (\psi \leftrightarrow \chi)] G^a_{\mu\nu}$$

$$\mathcal{O}^{(l)}_{\psi\chi} \equiv \frac{1}{\Lambda^4} c^{(l)}_{\psi\chi}\, i[\bar\psi \chi \bar\chi \sigma^{\mu\nu} T^a \psi + \bar\psi \gamma_5 \chi \bar\chi \sigma^{\mu\nu} \gamma_5 T^a \psi - (\psi \leftrightarrow \chi)] G^a_{\mu\nu}$$

$$\mathcal{O}^{(m)}_{\psi\chi} \equiv \frac{1}{\Lambda^4} c^{(m)}_{\psi\chi}\, [\bar\psi \sigma^{\lambda\mu} T^a \chi \bar\chi \sigma_{\mu\nu} \psi + \bar\psi \sigma^{\lambda\mu} \gamma_5 T^a \chi \bar\chi \sigma_{\mu\nu} \gamma_5 \psi + (\psi \leftrightarrow \chi)] G^{a\,\nu}_{\lambda}$$

$$\mathcal{O}^{(c)}_{\ell\psi} \equiv \frac{v}{\Lambda^4} c^{(c)}_{\ell\psi}\, \bar\ell \overleftrightarrow{D}_\mu \gamma_5 \ell \bar\psi \gamma^\mu \gamma_5 \psi$$

$$\mathcal{O}^{(d)}_{\ell\psi} \equiv \frac{v}{\Lambda^4} c^{(d)}_{\ell\psi}\, \bar\ell \gamma^\mu \gamma_5 \ell \bar\psi \overleftrightarrow{D}_\mu \gamma_5 \psi$$

$$\mathcal{O}^{(w)}_{\ell\psi} \equiv \frac{1}{\Lambda^4} c^{(w)}_{\ell\psi}\, \bar\ell \gamma^\mu \ell \bar\psi \gamma^\nu \psi F_{\mu\nu}$$

$$\mathcal{O}^{(x)}_{\ell\psi} \equiv \frac{1}{\Lambda^4} c^{(x)}_{\ell\psi}\, \bar\ell \gamma^\mu \gamma_5 \ell \bar\psi \gamma^\nu \gamma_5 \psi F_{\mu\nu}$$

$$\mathcal{O}^{(u)}_{\ell\psi} \equiv \frac{1}{\Lambda^4} c^{(u)}_{\ell\psi}\, \bar\ell \gamma^\mu \ell \bar\psi \gamma^\nu T^a \psi G^a_{\mu\nu}$$

$$\mathcal{O}^{(v)}_{\ell\psi} \equiv \frac{1}{\Lambda^4} c^{(v)}_{\ell\psi}\, \bar\ell \gamma^\mu \gamma_5 \ell \bar\psi \gamma^\nu \gamma_5 T^a \psi G^a_{\mu\nu}$$

In terms of chiral irreducible representations:

$$\mathcal{O}_{\psi\chi}^{(a)} = \frac{v}{\Lambda^4}c_{\psi\chi}^{(a)}\big[(\bar{q}_L\overleftrightarrow{D}_\mu\lambda^\dagger q_R)(\bar{q}_R\gamma^\mu\lambda_R q_R) - (\bar{q}_R\overleftrightarrow{D}_\mu\lambda q_L)(\bar{q}_R\gamma^\mu\lambda_R q_R) - (\bar{q}_R\overleftrightarrow{D}_\mu\lambda q_L)(\bar{q}_L\gamma^\mu\lambda_L q_L) + (\bar{q}_L\overleftrightarrow{D}_\mu\lambda^\dagger q_R)(\bar{q}_L\gamma^\mu\lambda_L q_L)\big]$$

Full ToPeχPT operator at leading order:

$$\begin{aligned} X_{\psi\chi}^{(a)} = \frac{v}{\Lambda^4}c_{\psi\chi}^{(a)}\big[\, & ig_1^{(a)}\langle\lambda D_\mu U^\dagger + \lambda^\dagger D_\mu U\rangle\langle\lambda_L D^\mu U^\dagger U + \lambda_R D^\mu U U^\dagger\rangle \\ & + ig_2^{(a)}\langle(\lambda D^2 U^\dagger U\lambda_L U^\dagger + \lambda^\dagger U D^2 U^\dagger\lambda_R U) - (\lambda^\dagger U\lambda_L D^2 U^\dagger U + \lambda U^\dagger\lambda_R D^2 U U^\dagger)\rangle \\ & + ig_3^{(a)}\langle(\lambda D_\mu U^\dagger D^\mu U\lambda_L U^\dagger + \lambda^\dagger D^\mu U D_\mu U^\dagger\lambda_R U) - (\lambda^\dagger U\lambda_L D_\mu U^\dagger D^\mu U + \lambda U^\dagger\lambda_R D^\mu U D_\mu U^\dagger)\rangle \\ & + ig_4^{(a)}\langle(\lambda D_\mu U^\dagger U\lambda_L D^\mu U^\dagger + \lambda^\dagger D^\mu U U^\dagger\lambda_R D_\mu U) - (\lambda^\dagger D_\mu U\lambda_L U^\dagger D^\mu U + \lambda D^\mu U^\dagger\lambda_R U D_\mu U^\dagger)\rangle \\ & + \mathcal{O}(p^4)\big] \end{aligned}$$