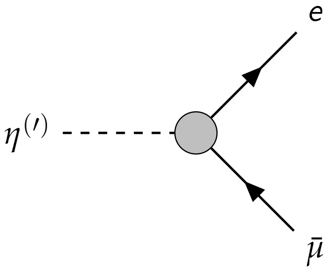
Lepton-flavor violation in eta decays

Frederic Noël

Universität Bern, Institute for Theoretical Physics



ECT*: Precision tests of fundamental physics with light mesons

15.06.2023

partially based on
[Hoferichter, Menéndez, Noël; Phys. Rev. Lett. 130 (2023)]

Outline

Overview of LFV (in $\eta^{(\prime)}$ decays):

1. Motivation
2. Experiments and Current limits
3. Strategy

Constraining $P \to \bar{\mu}e$ from $\mu \to e$ conversion:

4. Formalism
5. Master Formulae
6. Results

Part I

Overview of LFV (in $\eta^{(\prime)}$ decays)

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ν_e	ν_μ	ν_τ

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    → only indirectly via ν_i

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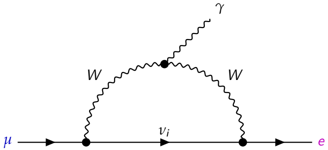

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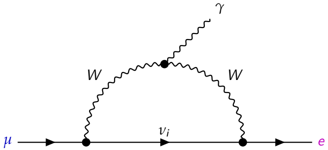

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Very clean BSM signal (no competing SM)

LFV Experiments and current limits

LFV process	current limit	(planed) experiments
$\pi^0 \to \bar{\mu} e$	$< 3.6 \cdot 10^{-10}$ [KTeV]	JEF, REDTOP
$\eta \to \bar{\mu} e$	$< 6 \cdot 10^{-6}$ [SPEC]	
$\eta' \to \bar{\mu} e$	$< 4.7 \cdot 10^{-4}$ [CLEO II]	
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$\mu \to e\gamma$ $\mu \to 3e$ $\tau \to \ell\gamma, 3\ell, \ell P, \ldots$	$< 4.2 \cdot 10^{-13}$ [MEG] $< 1.0 \cdot 10^{-12}$ [SINDRUM] $\lesssim 10^{-8}$ [Belle, LHCb, ...]	MEG II Mu3e Belle 2, ...

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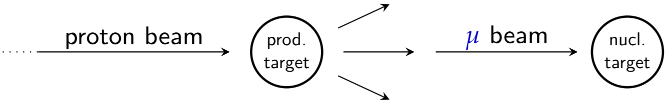
$\tau \to \ell\gamma, 3\ell, \ell P, \ldots$	$\lesssim 10^{-8}$ [Belle, LHCb, ...]	Belle 2, ...
Au $\mu^- \to$ Au e^-	$< 7 \cdot 10^{-13}$ [SINDRUM II]	
Ti $\mu^- \to$ Ti e^-	$< 6.1 \cdot 10^{-13}$ [SINDRUM II]	
Al $\mu^- \to$ Al e^-	$\lesssim 10^{-17}$ (projected)	Mu2e, COMET

$\to$ stringent bounds on LFV

What is $\mu \to e$ conversion?

- Experimental Setup:



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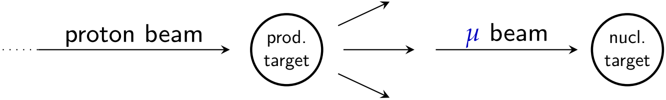

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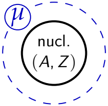

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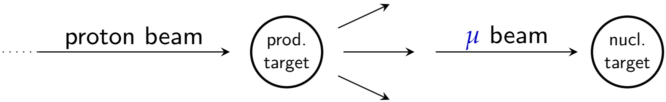

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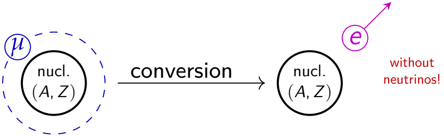

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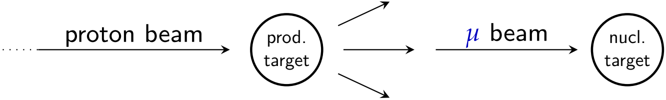

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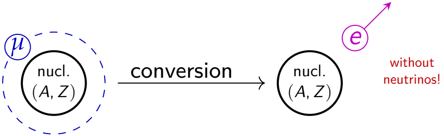

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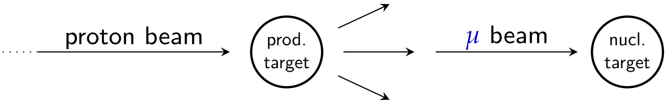

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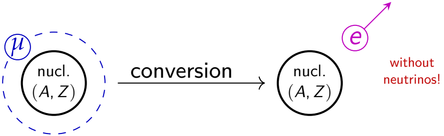

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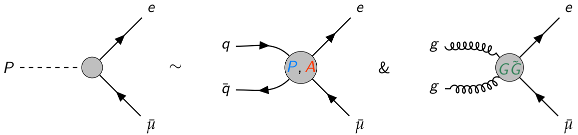
- Distinction between:
  - Spin-independent (SI) $\to$ coherent enhancement ($\sim \#N$)
  - Spin-dependent (SD) $\to$ non-coherent (only $J_{\text{nucl.}} \neq 0$)

Strategy

- Decays of light pseudoscalars $P = \pi^0, \eta, \eta'$:



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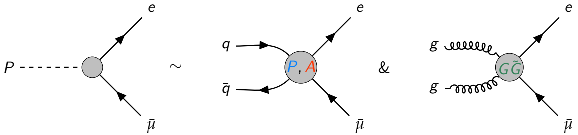

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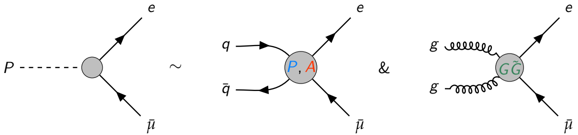

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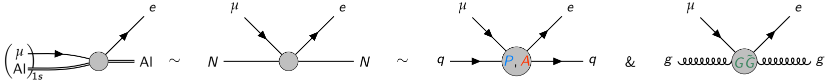

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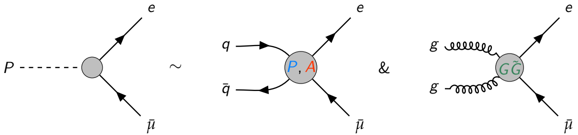

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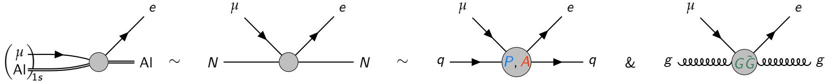

Goal: Use limits on $\mu \to e$ conversion to derive limits on $P \to \bar{\mu} e$

- also suggested in [Gan et al., 2022]

Part II

Constraining $P \to \bar{\mu} e$ from $\mu \to e$ conversion

Standard Model EFT

- Model-independent effective field theory description of BSM physics with higher dimensional operators obeying all SM symmetries:

$$\mathcal{L}^{\text{SM EFT}} = \mathcal{L}^{\text{SM}} + \frac{1}{\Lambda}\mathcal{L}^{(5)} + \frac{1}{\Lambda^2}\mathcal{L}^{(6)} + \dots$$

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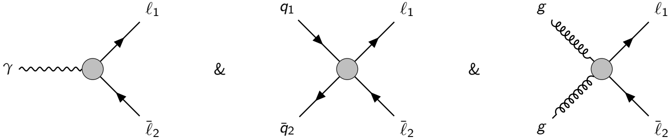

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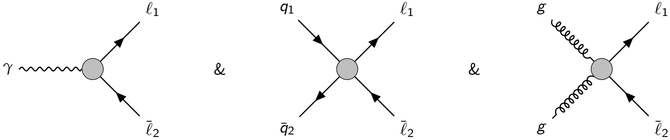

Can be used to describe all LFV processes in a model-independent way

Procedure

Observation

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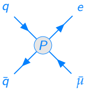
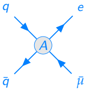
- relevant part of effective Lagrangian:

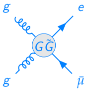
$$\mathcal{L}_{\text{eff}} \supset \frac{1}{\Lambda^2} \sum_{\substack{Y=L,R \\ q=u,d,s}} \left[C_Y^{P,q} (\overline{e_Y} \mu)(\bar{q}\gamma_5 q) + C_Y^{A,q} (\overline{e_Y} \gamma^\mu \mu)(\bar{q}\gamma_\mu \gamma_5 q) \right] + \frac{i\alpha_s}{\Lambda^3} \sum_{Y=L,R} C_Y^{G\tilde{G}} (\overline{e_Y} \mu) G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} + \text{h.c.}$$



pseudoscalar



axialvector



gluonic

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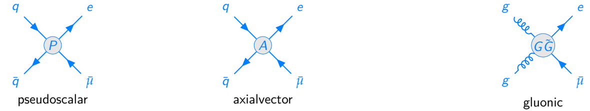


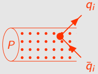
Fundamental Idea

Translate the strong limits of $\mu \to e$ conversion onto $P \to \bar{\mu} e$

$\to$ Need: Masterformula for both processes in terms of these operators

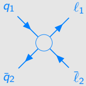
Master Formula: $P \to \mu e$

Decay Rate =



hadronic matrix elements

$\otimes$



(short distance) EFT operator

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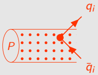

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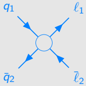

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$$\mathrm{Br}_{P\to\mu^\mp e^\pm} = \frac{(M_P^2 - m_\mu^2)^2}{16\pi\Gamma_P M_P^3} \sum_{Y=L,R} |C_Y^P|^2$$

$$C_Y^P = \sum_q \frac{b_q}{\Lambda^2}\left(\pm C_Y^{A,q} f_P^q m_\mu - C_Y^{P,q} \frac{h_P^q}{2m_q}\right) + \frac{4\pi}{\Lambda^3} C_Y^{G\tilde{G}} a_P$$

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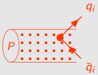

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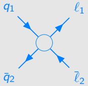

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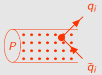
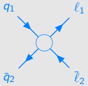
Decay Rate =  $\otimes$ 

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- hadronic matrix elements from lattice-QCD and phenomenology
- Ward identity:

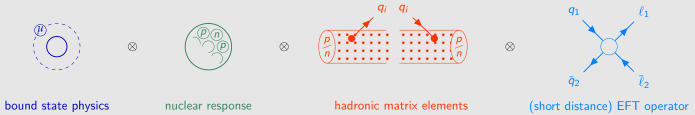
  $b_q f_P^q M_P^2 = b_q h_P^q - a_P$

	π	η		η'	
		Pheno	Lattice	Pheno	Lattice
$\frac{b_u f_P^u}{F_\pi}$	1	0.80	0.77	0.66	0.56
$\frac{b_d f_P^d}{F_\pi}$	-1	0.80	0.77	0.66	0.56
$\frac{b_s f_P^s}{F_\pi}$	0	-1.26	-1.17	1.45	1.50
a_P [GeV^3]	0	–	-0.017	–	-0.038
a_P^{FKS} [GeV^3]	0	-0.022	-0.021	-0.056	-0.048
h_P^q	Ward identity				

Phenomenology: [Escribano et al., 2016]
Lattice-QCD: [Bali et al., 2021]

Master Formula: SD $\mu \to e$ conversion

Conversion Rate =



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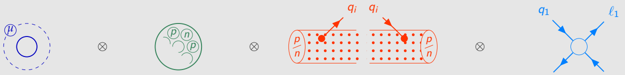
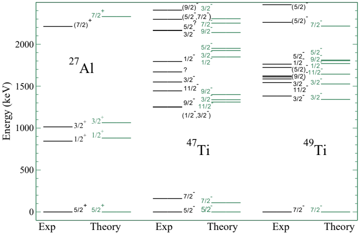


bound state physics ⊗ nuclear response ⊗ hadronic matrix elements ⊗ (short distance) EFT operator

$$\mathrm{Br}^{\mathrm{SD}}_{\mu\to e} = \frac{4 m_\mu^5 \alpha^3 Z^3}{\pi \Gamma_{\mathrm{cap}} (2J+1)} \left(\frac{Z_{\mathrm{eff}}}{Z}\right)^4 \times \sum_{\substack{Y=L,R \\ \tau=\mathcal{L},\mathcal{T}}} \left[C_Y^{\tau,00} S_{00}^{\tau} + C_Y^{\tau,11} S_{11}^{\tau} + C_Y^{\tau,01} S_{01}^{\tau} \right]$$

- numerical solution of Dirac equation:

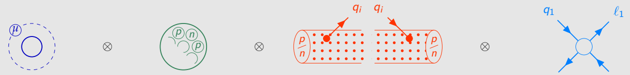
  $Z_{\mathrm{eff}}^{\mathrm{Al}} = 11.64, \qquad Z_{\mathrm{eff}}^{\mathrm{Ti}} = 17.65$ [Kitano et al., 2002]



shell-model

Master Formula: SD $\mu \to e$ conversion

Conversion Rate = bound state physics ⊗ nuclear response ⊗ hadronic matrix elements ⊗ (short distance) EFT operator



$$\mathrm{Br}^{\mathrm{SD}}_{\mu\to e} = \frac{4 m_\mu^5 \alpha^3 Z^3}{\pi \Gamma_{\mathrm{cap}}(2J+1)} \left(\frac{Z_{\mathrm{eff}}}{Z}\right)^4 \times \sum_{\substack{Y=L,R\\ \tau=\mathcal{L},\mathcal{T}}} \left[C_Y^{\tau,00} S_{00}^\tau + C_Y^{\tau,11} S_{11}^\tau + C_Y^{\tau,01} S_{01}^\tau \right]$$

$$C_Y^{\mathcal{T},ij} = \left[\bar{C}_Y^{A,i}(1+\delta')^i \pm 2\bar{C}_Y^{T,i}\right] \times (i \leftrightarrow j); \quad C_Y^{\mathcal{L},ij} = \left[\bar{C}_Y^{A,i}(1+\delta'')^i - \frac{m_\mu}{2m_N}\bar{C}_Y^{P,i} \pm 2\bar{C}_Y^{T,i}\right] \times (i \leftrightarrow j)$$

- numerical solution of Dirac equation:
  $Z_{\mathrm{eff}}^{\mathrm{Al}} = 11.64, \quad Z_{\mathrm{eff}}^{\mathrm{Ti}} = 17.65$ [Kitano et al., 2002]
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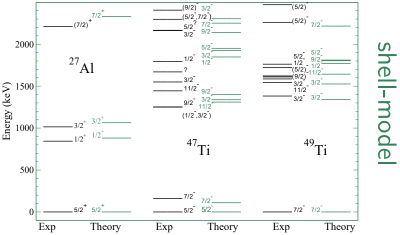
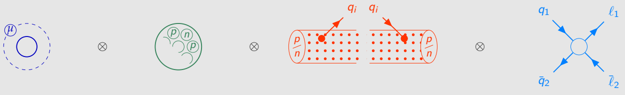


Master Formula: SD $\mu \to e$ conversion

Conversion Rate $=$ (bound state physics) $\otimes$ (nuclear response) $\otimes$ (hadronic matrix elements) $\otimes$ ((short distance) EFT operator)



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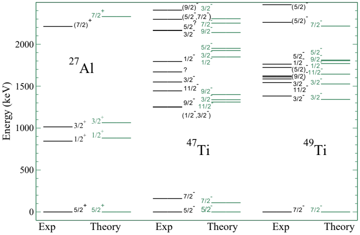
$g_A^{u,p}$	$g_A^{d,p}$	$g_A^{s,N}$	$\tilde{a}_N$ [GeV]	$g_5^{q,N}$
0.842(12)	-0.427(13)	-0.085(18)	-0.39(12) $[N_C \to \infty]$	Ward identity

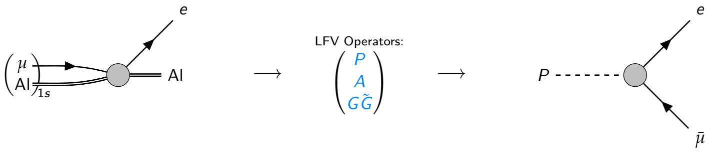
[HERMES, 2007]



shell-model

Deduced Limits (individual)

- Use limits on $\mu \to e$ conversion to derive limits on $P \to \bar{\mu} e$



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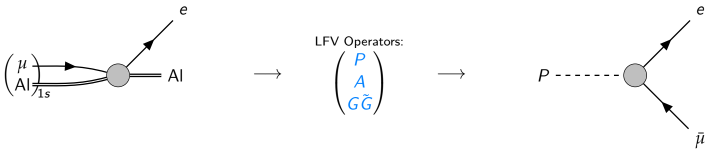

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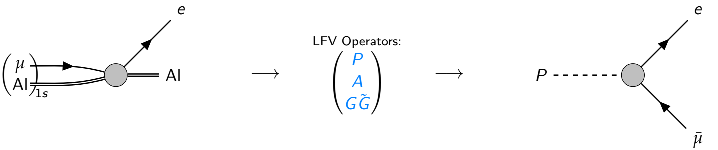

- In general the operators do not appear in the same linear combinations
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$\mu \to e$ (exp.)	$P \to \bar{\mu} e$ (derived)	current limit
	$\mathrm{BR}_{\pi^0} \lesssim 4 \times 10^{-17}$	$< 3.6 \times 10^{-10}$
$\mathrm{BR}_{\mathrm{Ti}} < 6.1 \times 10^{-13}$	$\mathrm{BR}_{\eta} \lesssim 5 \times 10^{-13}$	$< 6.0 \times 10^{-6}$
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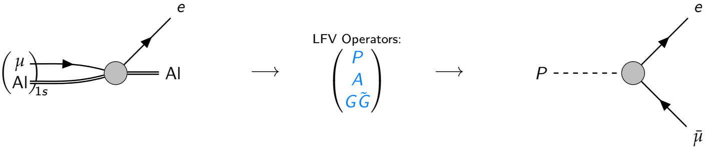

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Derived limits are several orders of magnitude better!

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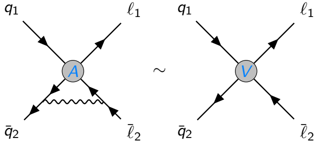
- easily spoilt by RG corrections
- contributing to SI $\mu \to e$ conversion



Outlook

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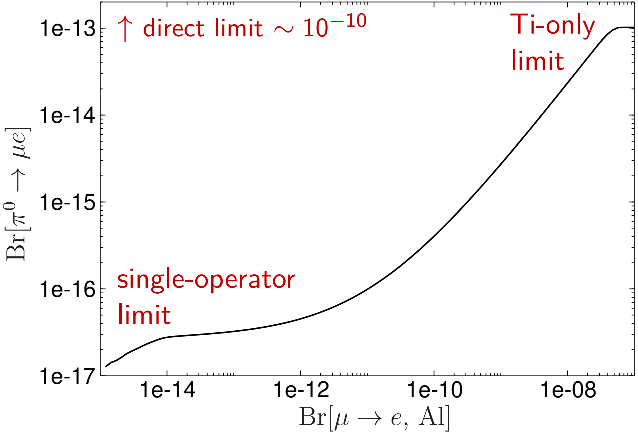
- Combining the limits from Ti and Al we find:



Conclusion

Summary:

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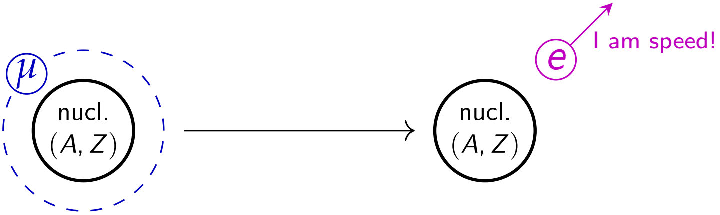
- Future results from Mu2e and COMET can further improve these limits
- General treatment of $\mu \to e$ conversion: beyond SI or SD, combining nuclear **and** bound state physics
- nucleus calculations from ab-initio methods

Thank you for your attention!



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Backup-Slides

Description of $\mu \to e$ conversion

- No interaction with elementary particles, but with a whole nucleus

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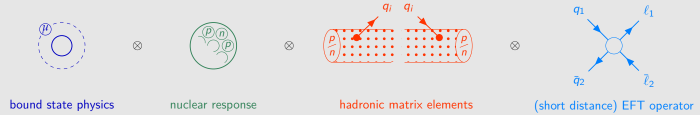
Effective description by separation of the appearing scales

Conversion Rate =



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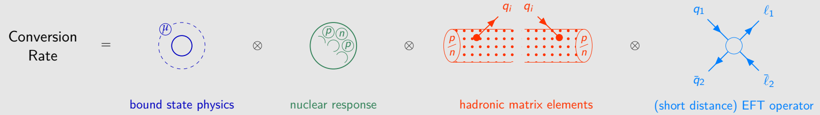

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$$\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2} \sum_\Gamma \; C_q^\Gamma \; \left(L^\Gamma \cdot Q^{\Gamma,q}\right)$$

$$L^\Gamma \in \{\bar{e}_Y \mu, \bar{e}_Y \gamma_\mu \mu, \bar{e}_Y \sigma_{\mu\nu} \mu\}, \quad (\Gamma = S, P, V, A, T, D, GG, G\tilde{G})$$

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- hadronic matrix elements:

  $$\langle N|\, Q^{\Gamma,q}\,|N\rangle \to\sim F^{\Gamma,i}_{q,N}\; \bar{u}_N \mathcal{O}_i u_N \xrightarrow{\text{non.rel.}} \sim \bar{u}_N^{\text{NR}} \mathcal{O}_i^{\text{NR}} u_N^{\text{NR}}$$

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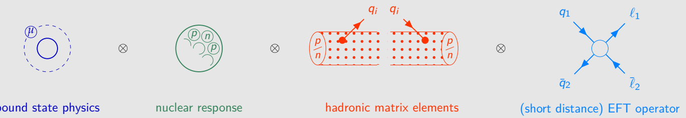

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  $\langle M | \, \mathcal{O}_i^{\text{NR}} \, | M \rangle \to\sim \mathcal{F}^{\mathcal{S}_N}$ $\quad \mathcal{S} \in \left\{M, \Sigma^{(\prime\prime)}, \Phi^{(\prime\prime)}, \Delta^{(\prime\prime)}, \Omega^{(\prime\prime)}, \Gamma^{(\prime\prime)}, \Pi^{(\prime\prime)}, \Theta^{(\prime\prime)}\right\}$

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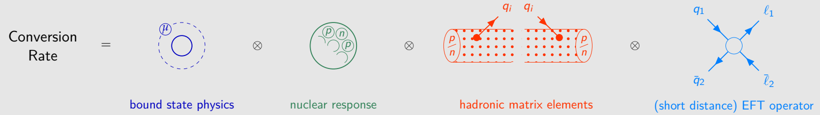

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- nuclear multipoles (shell-model): $\quad \mathcal{O}_i^{\text{NR}} \in \left\{\mathbb{1}, \vec{\sigma}, \vec{\nabla}, \text{...and all combinations}\right\}$

  $\langle M| \mathcal{O}_i^{\text{NR}} |M\rangle \to\sim \mathcal{F}^{\mathcal{S}_N}$ $\quad \mathcal{S} \in \left\{M, \Sigma^{(\prime\prime)}, \Phi^{(\prime\prime)}, \Delta^{(\prime\prime)}, \Omega^{(\prime\prime)}, \Gamma^{(\prime\prime)}, \Pi^{(\prime\prime)}, \Theta^{(\prime\prime)}\right\}$
- bound state physics (numerical):

  $\langle \tilde{e}| L^\Gamma |\mu(1s)\rangle \to\sim \overline{\Psi_e}\mathcal{O}_\Gamma\Psi_\mu$ with $\Psi_e, \Psi_\mu \xleftarrow{\text{Dirac-eq.}} V(r) \leftarrow \rho_{\text{ch}}(r)$

Outlook: General Masterformula

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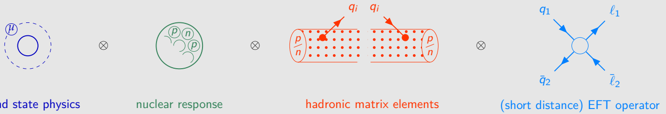
Conversion Rate =



$\mathcal{M} \sim$

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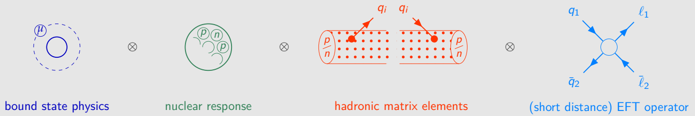
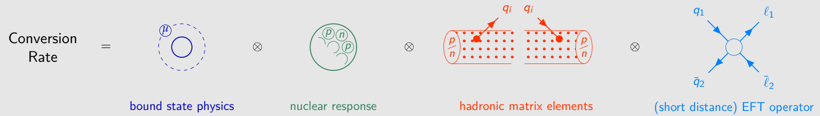


- effective Lagrangian with all possible quark and gluon operators:

$$\Gamma \in S, P, V, A, T, D, GG, G\tilde{G}$$

$$\mathcal{M} \sim \sum_{\Gamma, q,} C_q^{\Gamma}$$

Outlook: General Masterformula



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$$\mathcal{M} \sim \sum_{\Gamma,q,i,N,} C^{\Gamma}_{q} \cdot F^{\Gamma,i}_{q,N}(\vec{q})$$

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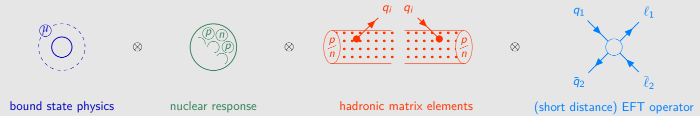

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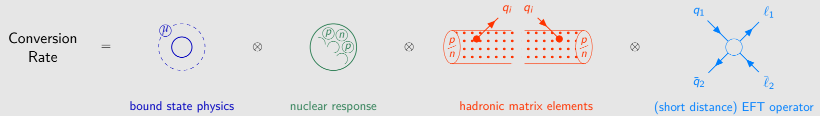

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$$\mathcal{M} \sim \int \frac{d^3q}{(2\pi)^3} \sum_{\Gamma,q,i,N,\mathcal{S}} C^{\Gamma}_q \cdot F^{\Gamma,i}_{q,N}(\vec{q}) \cdot \mathcal{F}^{\mathcal{S}_N}(\vec{q}) \cdot \widetilde{\overline{\Psi}_e \mathcal{O}_\Gamma \Psi_\mu}\,(\vec{q})$$

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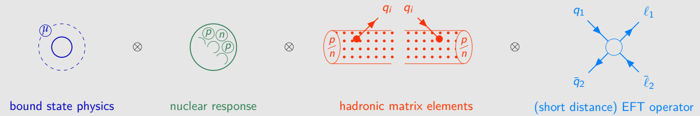

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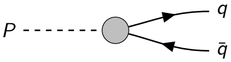

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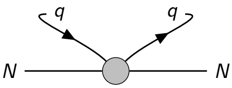

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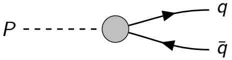

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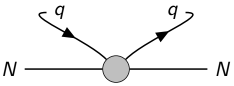

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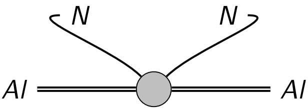

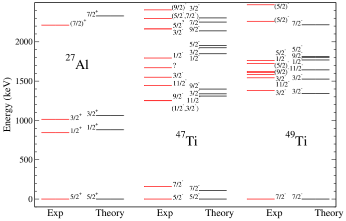

- ... in terms of multipoles, calculated in the shell-model

Formulas I

$$\langle 0|\bar{q}\gamma^\mu\gamma_5 q|P(k)\rangle = i b_q f_P^q k^\mu, \tag{1}$$

$$\langle 0|m_q \bar{q} i\gamma_5 q|P(k)\rangle = \frac{b_q h_P^q}{2}, \tag{2}$$

$$\langle 0|\frac{\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu}|P(k)\rangle = a_P, \tag{3}$$

$$\langle N|\bar{q}\gamma^\mu\gamma_5 q|N\rangle = g_A^{q,N}\langle N|\bar{N}\gamma^\mu\gamma_5 N|N\rangle, \tag{4}$$

$$m_q\langle N|\bar{q} i\gamma_5 q|N\rangle = m_N g_5^{q,N}\langle N|\bar{N} i\gamma_5 N|N\rangle, \tag{5}$$

$$\langle N|\bar{q}\sigma^{\mu\nu} q|N\rangle = f_{1,T}^{q,N}\langle N|\bar{N}\sigma^{\mu\nu} N|N\rangle, \tag{6}$$

$$\langle N|\frac{\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu}|N\rangle = \tilde{a}_N\langle N|\bar{N} i\gamma_5 N|N\rangle, \tag{7}$$

Formulas II

$$\mathrm{Br}_{\mathrm{SI}}[\mu \to e] = \frac{4 m_\mu^5}{\Gamma_{\mathrm{cap}}} \sum_{Y=L,R} \left| \sum_{\substack{N=p,n \\ \mathcal{O}=S,V}} \bar{C}_Y^{\mathcal{O},N} \mathcal{O}^{(N)} \right|^2, \tag{8}$$

$$\bar{C}_Y^{S,N} = \frac{1}{\Lambda^2} \sum_q C_Y^{S,q} \frac{m_N}{m_q} f_q^N + \frac{4\pi}{\Lambda^3} C_Y^{GG} a_N, \tag{9}$$

$$\bar{C}_Y^{V,N} = \frac{1}{\Lambda^2} \sum_q C_Y^{V,q} f_{V_q}^N, \tag{10}$$

$$S^{(N)} = V^{(N)} = \frac{(\alpha Z)^{3/2}}{4\pi} \left(\frac{Z_{\mathrm{eff}}}{Z} \right)^2 \mathcal{F}_N^M (m_\mu^2), \tag{11}$$

Formulas III

$$\bar{C}^0 = \frac{\bar{C}^p + \bar{C}^n}{2}, \qquad \bar{C}^1 = \frac{\bar{C}^p - \bar{C}^n}{2}, \tag{12}$$

$$g_A^{q,N} = g_5^{q,N} - \frac{\tilde{a}_N}{2m_N}, \tag{13}$$

$$\tilde{a}_N = -2m_N g_A^{u,0} = -0.39(12)\,\text{GeV}, \tag{14}$$

Formulas IV

$$C_Y^{A,u} = C_Y^{A,d}, \qquad C_Y^{A,s} = -\frac{2C_Y^{A,u} g_A^{u,0}}{g_A^{s,N}}, \tag{15}$$

$$\frac{C_Y^{P,u}}{m_u} = \frac{C_Y^{P,d}}{m_d}, \qquad \frac{C_Y^{P,s}}{m_s} = \frac{4\pi}{\Lambda} C_Y^{G\tilde{G}} \frac{2g_A^{u,0}}{g_A^{u,0} - g_A^{s,N}}. \tag{16}$$

Formulas V

$$S_{00}^{\mathcal{T}} = \sum_L \left[\mathcal{F}_{+}^{\Sigma_L'}(q^2)\right]^2, \qquad S_{00}^{\mathcal{L}} = \sum_L \left[\mathcal{F}_{+}^{\Sigma_L''}(q^2)\right]^2, \tag{17}$$

$$S_{11}^{\mathcal{T}} = \sum_L \left[\mathcal{F}_{-}^{\Sigma_L'}(q^2)\right]^2, \qquad S_{11}^{\mathcal{L}} = \sum_L \left[\mathcal{F}_{-}^{\Sigma_L''}(q^2)\right]^2, \tag{18}$$

$$S_{01}^{\mathcal{T}} = \sum_L 2\mathcal{F}_{+}^{\Sigma_L'}(q^2)\,\mathcal{F}_{-}^{\Sigma_L'}(q^2), \tag{19}$$

$$S_{01}^{\mathcal{L}} = \sum_L 2\mathcal{F}_{+}^{\Sigma_L''}(q^2)\,\mathcal{F}_{-}^{\Sigma_L''}(q^2), \tag{20}$$

Table

	π^0	η	η'
$C_Y^{A,3}$	1.3×10^{-17}	–	–
$C_Y^{A,8}$	–	1.5×10^{-17}	4.0×10^{-20}
$C_Y^{A,0}$	–	2.9×10^{-19}	2.1×10^{-19}
$C_Y^{P,3}$	4.1×10^{-17}	–	–
$C_Y^{P,8}$	–	1.6×10^{-12}	2.1×10^{-14}
$C_Y^{P,0}$	–	4.1×10^{-12}	5.4×10^{-13}
$C_Y^{G\tilde{G}}$	–	5.8×10^{-15}	4.7×10^{-16}