The rare decay $\eta \to \pi^0\gamma\gamma$ in a dispersive approach

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Introduction:

- Interest in $\eta, \eta' \to \pi^0\gamma\gamma$ in relation to light boson searches (could couple to $\pi^0\gamma$), also gives lower bound to rare mode $\eta \to \pi^0 l^+ l^-$ (also induced by C-viol. operators)[Ng,Peters (1992)]

  SM dynamics not so trivial

- Recent developments:
  Width result by KLOE collab. : $\Gamma = (0.159 \pm 0.041)$ eV [P.Gauzzi,PoS (ICHEP 2022) 791]
  disagreement with previous measurements:

Crystal Ball @AGS (2008)	:	$\Gamma = (0.285 \pm 0.068)$ eV
A2 @ MAMI (2014)	:	$\Gamma = (0.330 \pm 0.030)$ eV

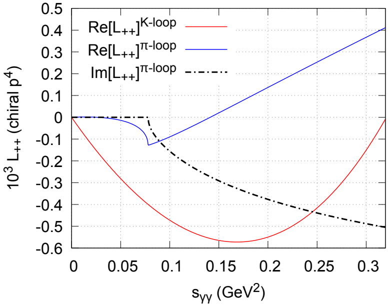
- KLOE in agreement with recent model prediction[R.Escribano et al., PR D102,034026 (2020)] (which also agrees with $\eta' \to \pi^0\gamma\gamma$, $\eta' \to \eta\gamma\gamma$).

- Dispersive approach:
  Eff.theory type but:
  - → Makes use of soft constraints
  - → Applies in $\gamma\gamma$ scattering region: (+ implements S-wave unitarity) use $\gamma\gamma \to \pi\eta$, $\gamma\gamma \to K\overline{K}$ exp. results

Chiral expansion:

- η decays can be described (in principle) by chiral expansion:[G.Ecker (1989),Ll.Amettler et al.,PL B276,185 (1992)]
  exp. in powers q^2/m_V^2, $m_\eta^2/m_V^2 \simeq 0.49$

  Order p^4: no tree contrib., one-loop graphs are finite (Kaon loop + pion loop)

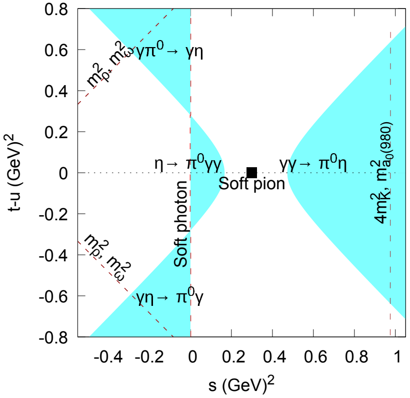


→ π-loop isospin violating (but not negligible) visible cusp ?

→ Decay width $\Gamma^{p^4}_{\eta\to\pi^0\gamma\gamma}$ too small by factor 100.

Important dynamical features:

- Mandelstam plane ($s \equiv s_{\gamma\gamma}$):



- Soft photon line $s = 0$ ($q_1 = 0$ or $q_2 = 0$)
  Soft pion point $p_1 = 0$: $s = m_\eta^2,\ t = u$

- Resonances: $\rho, \omega, \phi,\ a_0(980),\ a_2(1230)$

Soft constraints

- Soft photon limit: [H.Abarbanel,M.Goldberger, PR D5,2345 (1972)]

Structure of amplitudes:

$$L_{\lambda\lambda'} = \epsilon_\mu(q_1,\lambda)\epsilon_\nu(q_2,\lambda')W^{\mu\nu}(q_i,p_i)$$

with

$$W^{\mu\nu}(q_i,p_i) = \int d^4x d^4y e^{iq_1x+iq_2y}\langle\pi^0(p_1)|T[j^\mu(x)j^\nu(y)]|\eta(p_2)\rangle$$

Ward identities:

$$q_{1\mu}W^{\mu\nu} = q_{2\nu}W^{\mu\nu} = 0$$

$W^{\mu\nu}$ is then expressed with two independent tensors

$$W^{\mu\nu}(q_i,p_i) = A(s,t)\,T_1^{\mu\nu} + B(s,t)\,T_2^{\mu\nu}$$

$A(s,t)$, $B(s,t)$ analytic (no pole at $s=0$)

- Expression of $T_1^{\mu\nu}$:

$$T_1^{\mu\nu} = \frac{1}{2} s g^{\mu\nu} - q_1^{\nu} q_2^{\mu}$$

Vanishes in soft photon limit ($s = 2q_1.q_2 = 0$)

- Expression of $T_2^{\mu\nu}$:

$$T_2^{\mu\nu} = 2s\Delta^{\mu}\Delta^{\nu} + 4q_1.\Delta\, q_2.\Delta\, g^{\mu\nu} - 4q_2.\Delta\, q_1^{\nu}\Delta^{\mu} - 4q_1.\Delta\, q_2^{\mu}\Delta^{\nu}$$

with $\Delta = p_1 - p_2$: has a kinematical singularity at $s = 0$

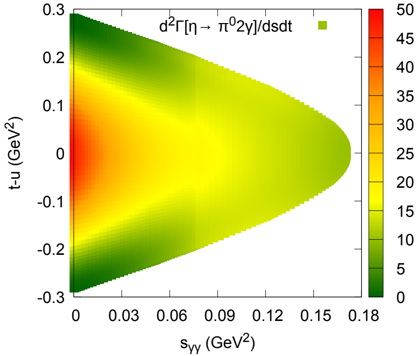
$$\Delta = \frac{1}{\sqrt{s}} \begin{pmatrix} m_\pi^2 - m_\eta^2 \\ \lambda_{\pi\eta}^{1/2}(s)\,\hat{v} \end{pmatrix}$$

$T_2^{\mu\nu}$ does not vanish in soft photon limit

→ At $s = 0$: $L_{++} = 0$ (order p^4, $J \geqslant 0$)

$L_{+-} \neq 0$ (order p^6 and $J \geqslant 2$)

- Consequences:
  - → Explains (partly) failure of p^4 ChPT
  - → Unusual shape of Dalitz plot:



- Soft pion limit: $p_1 = 0$

→ $s_A = m_\eta^2, t_A = u_A = 0,$ $T_2^{\mu\nu} = 2s_A T_1^{\mu\nu}$

→ Combination involved:

$$A(s_A, t_A) + 2s_A B(s_A, t_A) \propto L_{++}(s_A, t_A) + O(m_\pi^2)$$

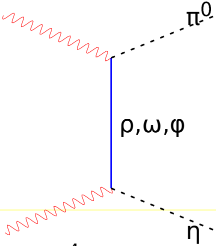
→ Using standard soft-pion technique: $L_{++}(s_A, t_A) =$

$$-\frac{i}{F_\pi}\langle\gamma(q_1)\gamma(q_2)|[Q_5^3, \frac{1}{2}(m_u - m_d)(\bar{u}u - \bar{d}d)]|\eta(p_2)\rangle$$

– Physical Isospin conserving part of $L_{++}(s_A, t_A) = 0$
$s_A = m_\eta^2 + O(m_\pi^2),\ t_A, u_A \sim O(m_\pi^2)$.

– Isospin violating part of $L_{++}(s_A, t_A)$ can be estimated using $O(p^4)$ ChPT.

Light vector meson exchanges (t,u channels)



- At small s: leading contributions to L_{+-} +substantial contribution to S-wave l_{0++}

- From resonance chiral Lagrangian:
  - → Adler zero in L_{++}

$$s_A = m_\eta^2 + m_\pi^2 (1 - \frac{2m_\eta^2}{3m_V^2}) + O(m_\pi^4)$$

  - → Left-hand cut in partial-waves, e.g.

$$\frac{1}{\pi}\mathrm{Im}\,[l^V_{0++}(s)] = 2C_{V\pi}C_{V\eta}\frac{sm_V^2}{\sqrt{\lambda_{\pi\eta}(s)}}\theta(s_V - s)$$

(zero width limit), with

$$s_V = -(m_V^2 - m_\eta^2)\left(1 - \frac{m_\pi^2}{m_V^2}\right) \simeq -0.3 GeV^2 (V = \rho, \omega)$$

- C_{VP} couplings from radiative widths:

	Γ (keV)	$C_{VP}(\mathrm{GeV}^{-1})$
$\rho^0 \to \pi^0\gamma$	69(9)	0.368(24)
$\rho^0 \to \eta\gamma$	44(3)	0.789(30)
$\omega \to \pi^0\gamma$	713(26)	1.160(20)
$\omega \to \eta\gamma$	3.8(4)	0.222(11)
$\phi \to \pi^0\gamma$	5.5(2)	0.067(1)
$\phi \to \eta\gamma$	55(1)	0.345(4)
$K^{*\pm} \to K^{\pm}\gamma$	50(5)	0.418(22)
$K^{*0} \to K^0\gamma$	116(11)	−0.636(30)

→ Relative signs assuming $SU(3)$ flavour symmetry

Tensor meson $a_2(1320)$ exchange (s-channel)

- $J = 2$ partial-wave

$$l^T_{2++}(s) = \frac{C^T_{\pi\eta}\, D^T_{\gamma\gamma}}{60 m_T^2}\, \frac{s\,\lambda_{\pi\eta}(s)}{m_T^2 - s}$$
$$l^T_{2+-}(s) = \frac{\sqrt{6}\, C^T_{\pi\eta}\, C^T_{\gamma\gamma}}{60}\, \frac{\lambda_{\pi\eta}(s)}{m_T^2 - s}$$

below $\pi\eta$ threshold (above: include width, Blatt-Weisskopf factors)

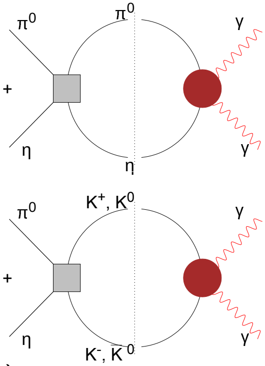
- Coupling constants:

$$C^T_{\pi\eta} = (10.8 \pm 0.5)\ \text{GeV}^{-1}$$

$C^T_{\gamma\gamma}$, $D^T_{\gamma\gamma}$ from fit to $\gamma\gamma$ data (also determines their signs)

S-wave (I_{0++}) construction:

- Unitarity in $a_0(980)$ region requires two channels: ($\pi\eta$, $K\overline{K}$)

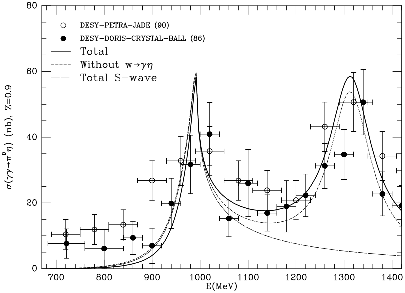


- Muskhelishvili-Omnès formalism ensures correct unitarity, analyticity + includes t,u-channels resonances.

- Previous work ($\gamma\gamma \to \pi\eta$, $\eta \to \pi^0\gamma\gamma$)
  [I.Danilkin et al., PR D96,114018 (2017)]
  [N.Achasov,G.Shestakov PR D81,094029 (2010)]

  → UChPT T-matrix model: also probe $\gamma\gamma \to K^+K^-$, $\gamma\gamma \to K^0\bar{K}^0$: [J.A.Oller,E.Oset, NP A629,739 (1998), E.Oset et al., Phys.Rev.D 67, 073013 (2003), Phys.Rev.D 77, 073001 (2008)]

- Illustration of Oller,Oset (1998) result:



→ Data from [Crystal Ball, PR D33,1847 (1986), JADE, ZP C47,343 (1990)]

→ Much more precise data nowadays available

→ Low energy $\pi\eta$ scattering can be probed, important for η decay region

Coupled-channel T-matrix description:

- Oller,Oset(1998) model:

$$\mathbb{T} = (\mathbb{1} - \mathbb{K}_{(2)}\Psi)^{-1}\mathbb{K}_{(2)}$$

– $\mathbb{K}_{(2)}$: $O(p^2)$ amplitudes

– $\Psi = diag(J_{\pi\eta}(s), J_{KK}(s))$ (1 cutoff parameter)

- Extended model[M.Albaladejo,B.M,EPJ C75,488 (2015)]

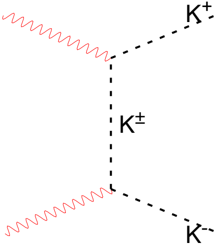
$$\mathbb{K} = \mathbb{K}_{(2)} + \mathbb{K}_{(4)} + \mathbb{K}_{(6)}$$

– 4 parameters in Ψ matrix+ 2 in $\mathbb{K}_{(6)}$ (pole m_8, residue λ)

– Reproduces p^4 exp. of $T^{\pi\eta\to\pi\eta}$, $T^{\pi\eta\to K\bar{K}}$ exactly , $T^{K\bar{K}\to K\bar{K}}$ approximately

–T-matrix has two resonance poles

Coupled-channel MO:



- $\gamma\gamma \to K^+K^-$ has a $K^\pm$ pole,

$$k^{1,Born}_{0++}(s) = -\frac{2\sqrt{2}m_K^2}{s\beta_K(s)} \log\frac{1+\beta_K(s)}{1-\beta_K(s)}$$

- We choose MO of following form

$$\begin{pmatrix} l_{0++}(s) \\ k_{0++}(s) \end{pmatrix} = \begin{pmatrix} 0 \\ k^{1,Born}_{0++}(s) \end{pmatrix} + s\,\Omega(s)\begin{pmatrix} b_l + L_1(s) + R_1(s) \\ b_K + L_2(s) + R_2(s) \end{pmatrix}$$

$k^{1,Born}_{0++}(s)$ dominates near $s = 0$.

Integrals over left cut $[-\infty, s_V]$:

$$L_i(s) = \frac{s - s_A}{\pi} \int_{-\infty}^{s_V} ds' \frac{D_{i1}(s')\mathrm{Im}\,[l^V_{0++}(s')] + D_{i2}(s')\mathrm{Im}\,[k^V_{0++}(s')]}{s'(s' - s_A)(s' - s)}$$

with $\mathbb{D} = \Omega^{-1}$.

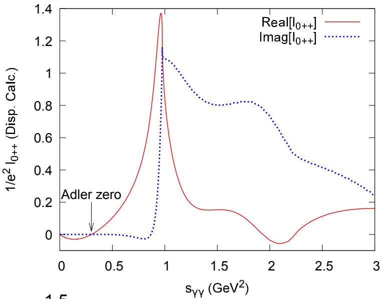
Integrals over right-cut $[4m_K^2, \infty]$

$$R_i(s) = -\frac{s - s_A}{\pi} \int_{4m_K^2}^{\infty} ds' \frac{\mathrm{Im}\,[D_{i2}(s')]\, k^{1,Born}_{0++}(s')}{s'(s' - s_A)(s' - s)}$$

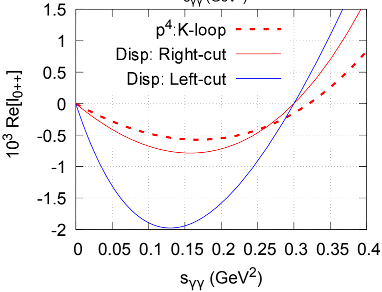
- Representation satisfies soft constraints, analyticity, two-channel unitarity

  Depends on two parameters: b_K, s_A.

- A look at the solution:



- Anatomy at low energy:



Part induced by R_1, R_2 close to chiral K-loop

Part induced by L_1, L_2 (vector mesons) larger

Fits to the data

- Parameters:
  s_A fixed: assumed in range $[m_\eta^2 - 3m_\pi^2, m_\eta^2 + 3m_\pi^2]$

  Fitted parameters: 6 [T-matrix]
  7 $[b_K,\ M_{a_2},\ \Gamma_{a_2},\ C^T_{2\gamma}, D^T_{2\gamma}]$

- Input experimental data: Differential cross-sections
  [Belle, PR D80,032001 (2009),PTEP 12,123C01 (2013)]
  $\pi\eta$: $N_{dat} = 448$ $E = [0.85, 1.39]$ GeV
  $K_S K_S$: $N_{dat} = 240$ $E = [1.105, 1.395]$ GeV

  [ARGUS: Z.P. C48,183 (1990)]
  K^+K^- : $N_{dat} = 7$ $E = [1.17, 1.47]$ GeV

  Total: $N_{data} = 697$

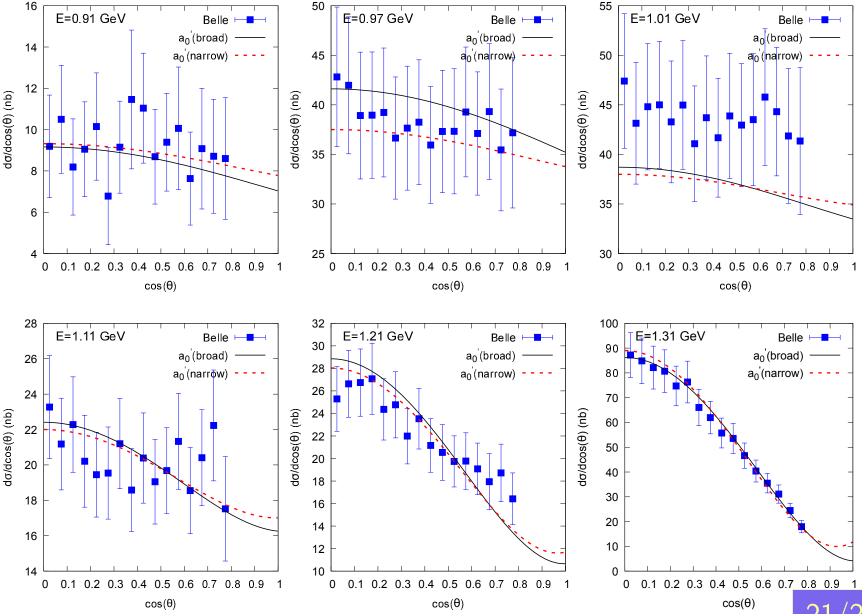
- Influence of Adler zero position:

$$\begin{aligned} s_A &= m_\eta^2 - 3m_\pi^2 & \chi^2 &= 455 \\ s_A &= m_\eta^2 & \chi^2 &= 437 \\ s_A &= m_\eta^2 + 3m_\pi^2 & \chi^2 &= 421 \end{aligned}$$

Larger value favoured

Influence of s_A not so significant

Some fit results: differential cross-sections



- Two χ^2 minimums: ambiguity in $a_0(1450)$ determination

Scalar resonance complex poles:

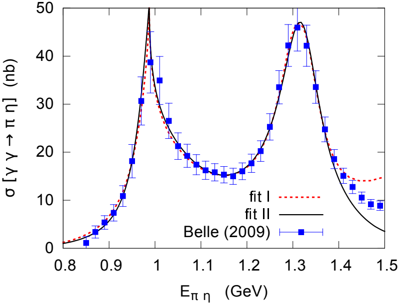
	$a_0(980)$	$a_0(1450)$
Narrow a_0'	1020.3-i 49.3	1314.4-i 24.5
Broad a_0'	1000.7-i 36.6	1420.9-i 174.4

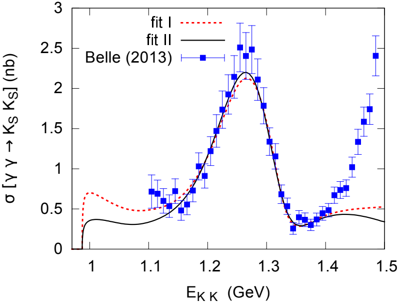
→ In first line $a_0(1450)$ mass/width very close to $a_2(1320)$

→ Old ambiguity (see PDG)

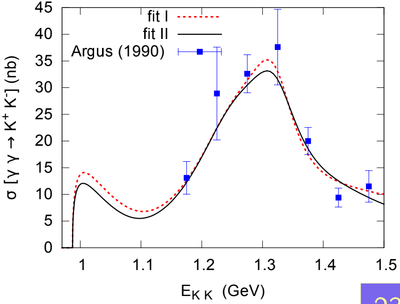
→ Broad $a_0(1450)$ likely more physical

Integrated cross-sections ($\pi\eta$, K^+K^-, K_SK_S)



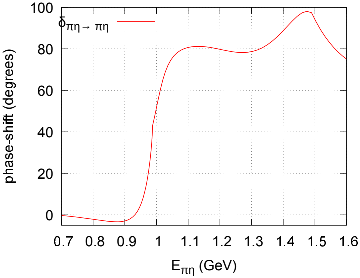


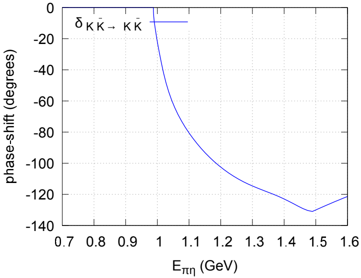
- → $a_0(980)$: cusp peak at $\sqrt{s} = 2m_{K^+}$

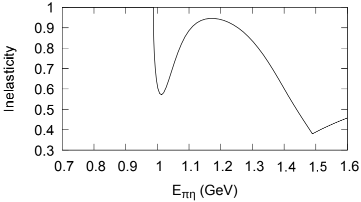


- → Agreement with exp. for $\pi^0\eta$ at low energy

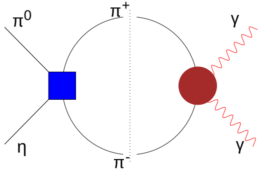
Fit results for $\pi\eta - K\bar{K}$ scattering







$\eta \to \pi^0\gamma\gamma$: isospin violating part



- $\pi^+\pi^-$ discontinuity:

$$\mathrm{disc}[l_{0++}]^{\pi\pi} = \frac{\sqrt{3}\epsilon_L}{32\pi}\sqrt{\frac{s-4m_\pi^2}{s}}\Big[(h^0_{0++}(s))^*(M_0(s)+\hat{M}_0(s)) + (h^2_{0++}(s))^*(M_2(s)+\hat{M}_2(s))\Big]$$

where: $\epsilon_L = \frac{(m^2_{K^0}-m^2_{K^+})_{QCD}}{3\sqrt{3}F_\pi^2}$

$M_I + \hat{M}_I$: $\boxed{\eta\pi \to (\pi\pi)_I}$ partial-wave (see [A.Anisovich,H.Leutwyler, PL B375,335 (1996)])

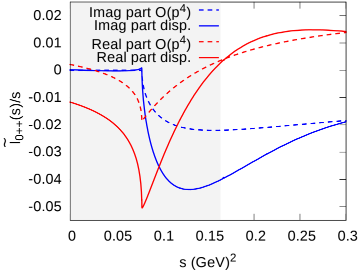
h^I_{0++} : $\boxed{\gamma\gamma \to (\pi\pi)_I}$ partial-waves (enhanced at low energy because of pion pole)

- Dispersive represention with one subtraction:

$$\tilde{l}_{0,++}(s) \equiv s\left(\tilde{\lambda} + \frac{s - s_A}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'(s' - s_A)(s' - s)} \, \mathrm{disc}[l_{0,++}(s')]^{\pi\pi}\right).$$

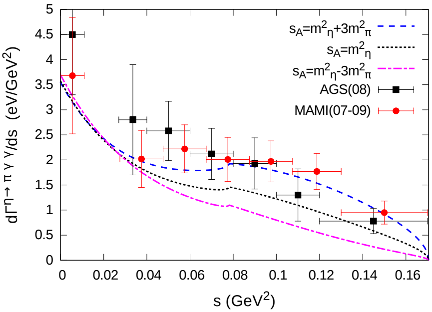
$\tilde{\lambda}$ estimated from one-loop chiral formula (at $s = s_A$)

Result:
(Inputs from [R. Garcia-Martin, BM, EPJ C70,155 (2010), M.Albaladejo, BM, EPJ C77,508 (2017)])



Energy distribution in $\eta \to \pi^0\gamma\gamma$ decay

- Comparison with Crystal Ball@AGS and A2@MAMI



[Junxu Lu, BM, EPJ C80, 436 (2020)]

Region $s > 0.6$ GeV^2 sensitive to s_A: larger value favoured

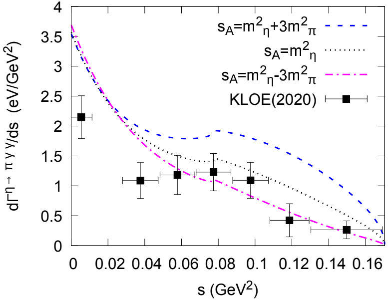
- Comparison with KLOE

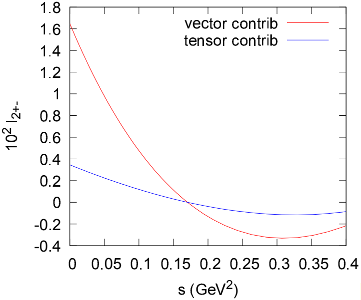
Larger disagreement near $s = 0$

D-wave contributions: vector exchange + tensor ($a_2(1320)$) exchange

Both positive: relative sign fixed from fit

Absence of $a_2(1320)$ in [R.Escribano et al. (2020)] explains difference





Conclusions

- Model for $\gamma\gamma \to \pi^0\eta, K_S K_S, K^+ K^-$ with analyticity/unitarity for the S-wave
- D-waves description more phenomenological
- Decay $\eta \to \pi^0\gamma\gamma$ predicted
  Sensitive to Adler zero position
  Sensitive to D-waves near $s = 0$
- Reasonable agreement with Crystal Ball@AGS and A2@MAMI but tension with new results by KLOE