

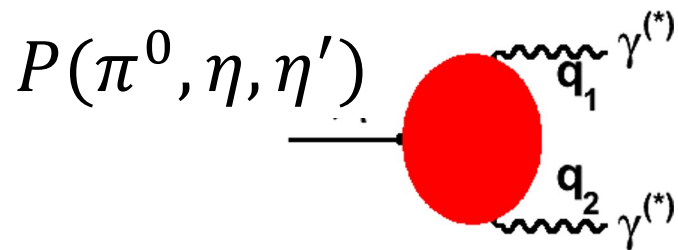


UPPSALA  
UNIVERSITET

# Transition form factors

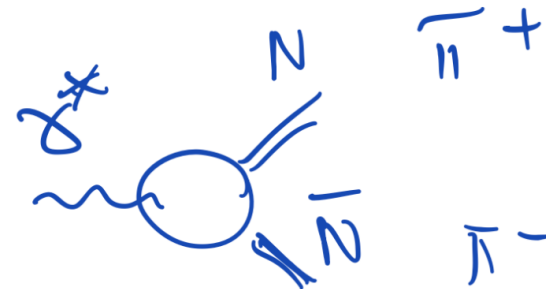
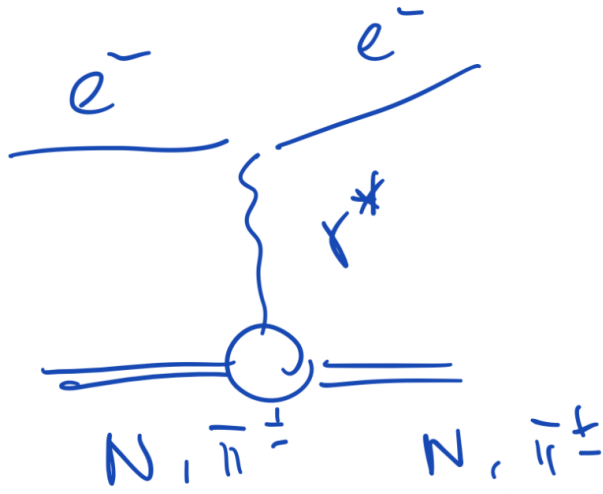
Andrzej Kupsc

Electromagnetic TFF  $\gamma^* M_1 M_2$



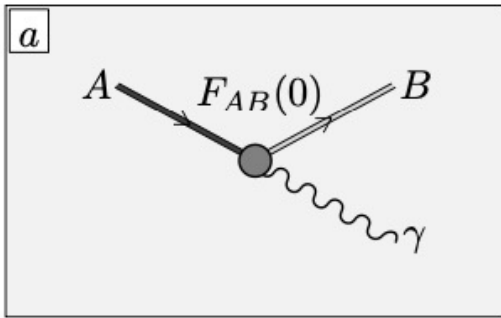
# Electromagnetic Form factors (FFs)

Elastic FFs

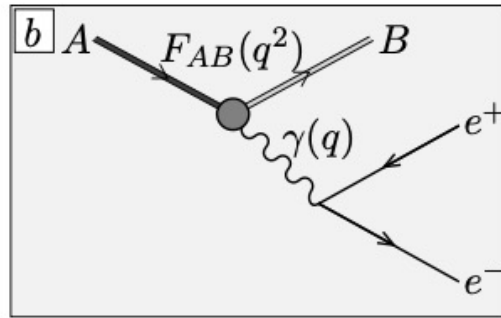


Transition FFs...

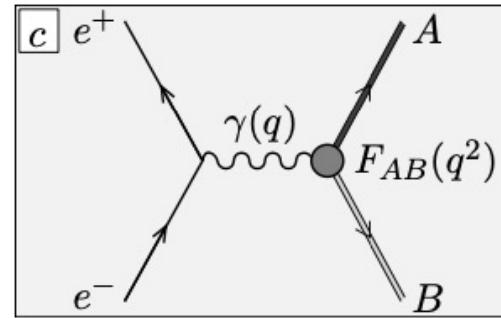
# Transition form factors (TFFs)



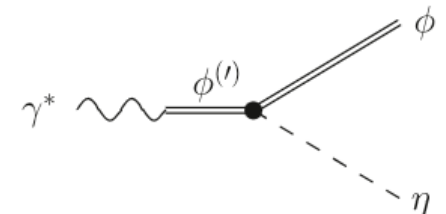
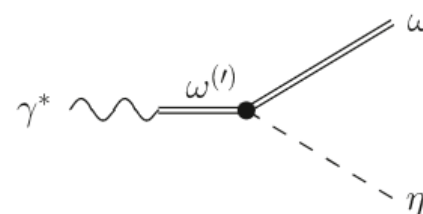
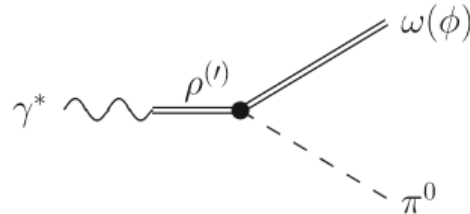
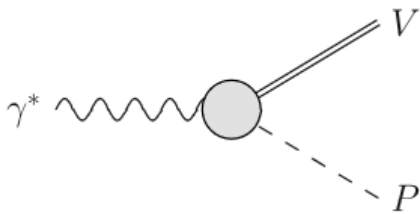
$$q^2 = 0$$



$$(2m_e)^2 < q^2 < (M_A - M_B)^2$$



$$q^2 > (M_A + M_B)^2$$



$$\Gamma(V \rightarrow P\gamma) = \frac{\alpha}{3} E_\gamma^3 |F_{VP}(0)|^2,$$

$$\Gamma(P \rightarrow V\gamma) = \alpha E_\gamma^3 |F_{VP}(0)|^2,$$

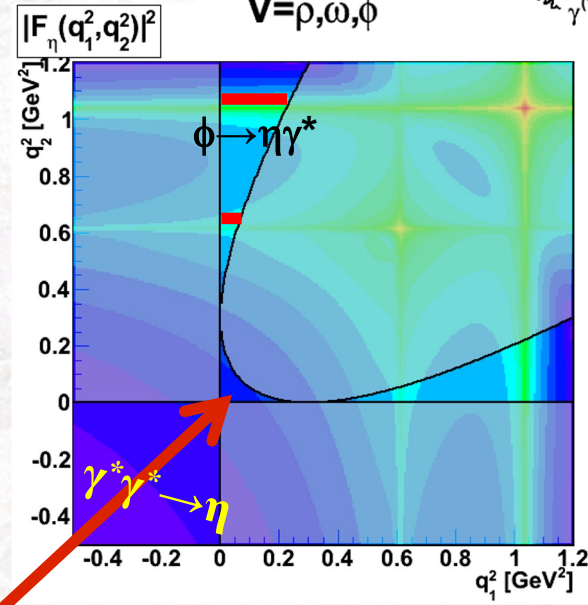
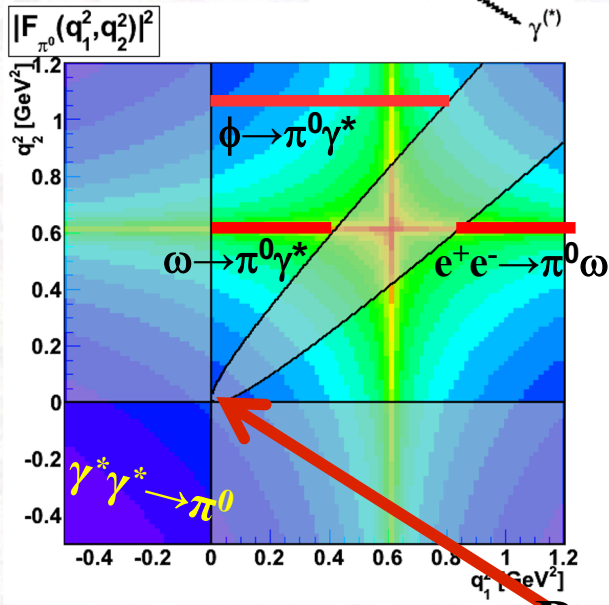
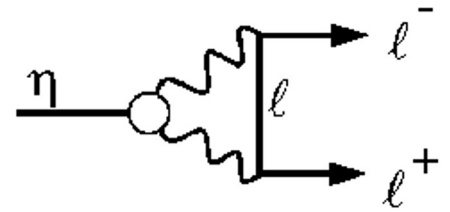
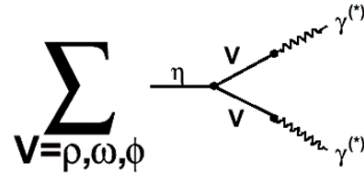
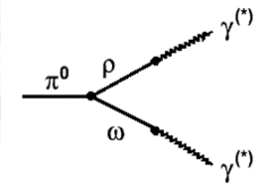
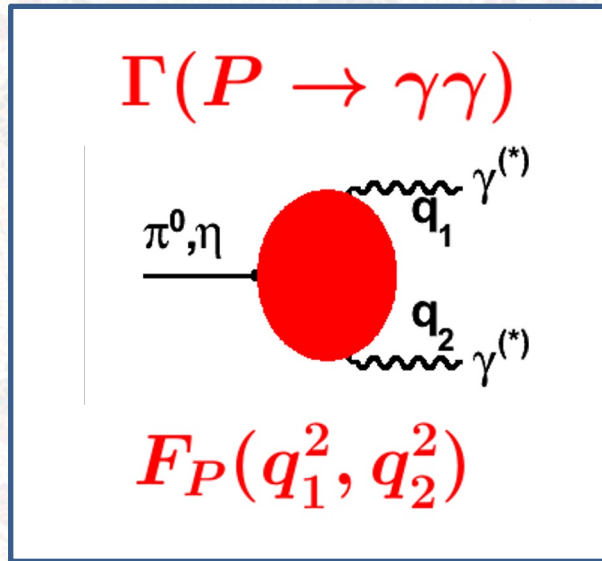
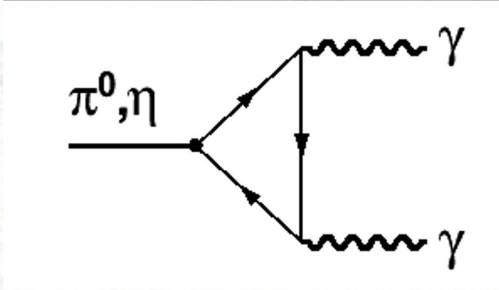
$$\sigma(e^+e^- \rightarrow VP) = \frac{4\pi\alpha^2}{3s^{3/2}} |F_{VP}(s)|^2 P_f(s)$$

Landsberg Phys. Rep., 128 (1985) 301

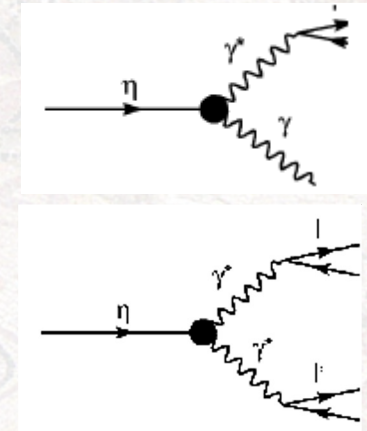
Pacetti Eur.Phys.J.A 38 (2008) 331

Fang, Kubis, AK Prog.Part.Nucl.Phys. 120 (2021) 103884

# $\pi^0, \eta, \eta'$ Transition Form Factors



$P \rightarrow \gamma\gamma$



# Radiative widths of $\eta, \pi^0$

$\eta: 5 \times 10^{-19} \text{ s}; \Gamma = 1.3 \text{ keV}$	$\eta \rightarrow \gamma\gamma$
$\pi^0: 8 \times 10^{-17} \text{ s}; c\tau = 25 \text{ nm}$	$\pi^0 \rightarrow \gamma\gamma$

Two exp. techniques:

$\gamma Z \rightarrow \eta, \pi^0$  Primakoff

PrimEx(II) PRL 106,162303(2011),  
Science 368,506 (2020)

$e^+e^-: \gamma\gamma \rightarrow \pi^0$

KLOE-2 Taggers

$\Gamma(\pi^0 \rightarrow \gamma\gamma) = 7.802 \text{ eV} \pm 0.052 \text{ stat.} \pm 0.105 \text{ syst.}$

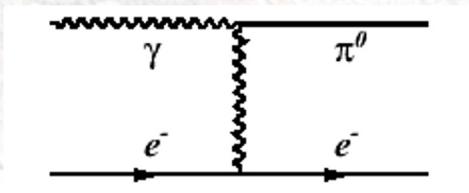
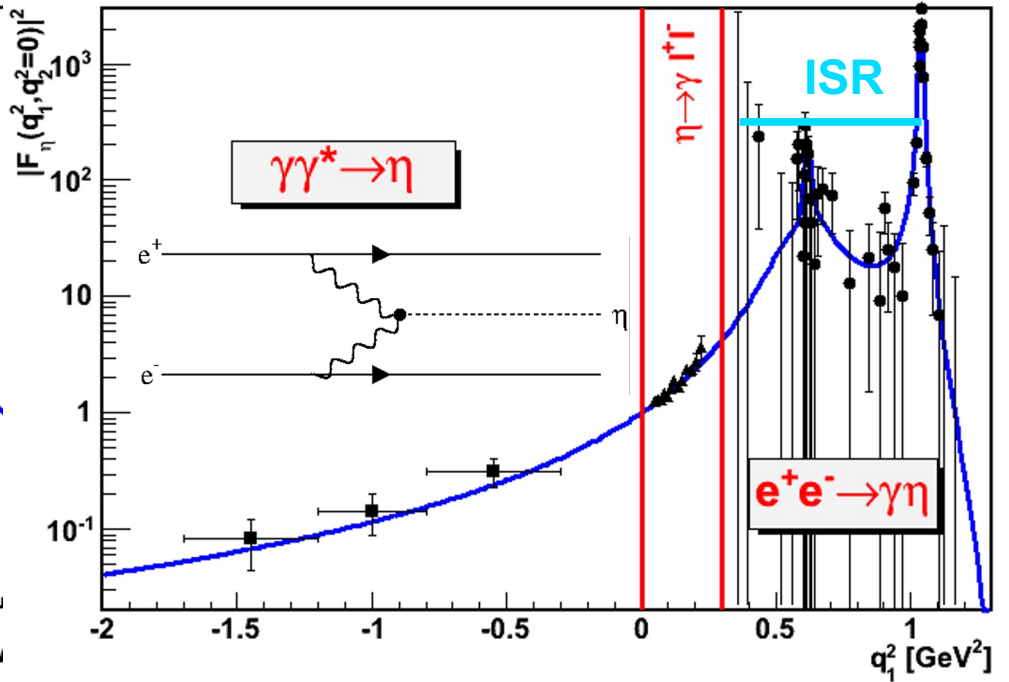
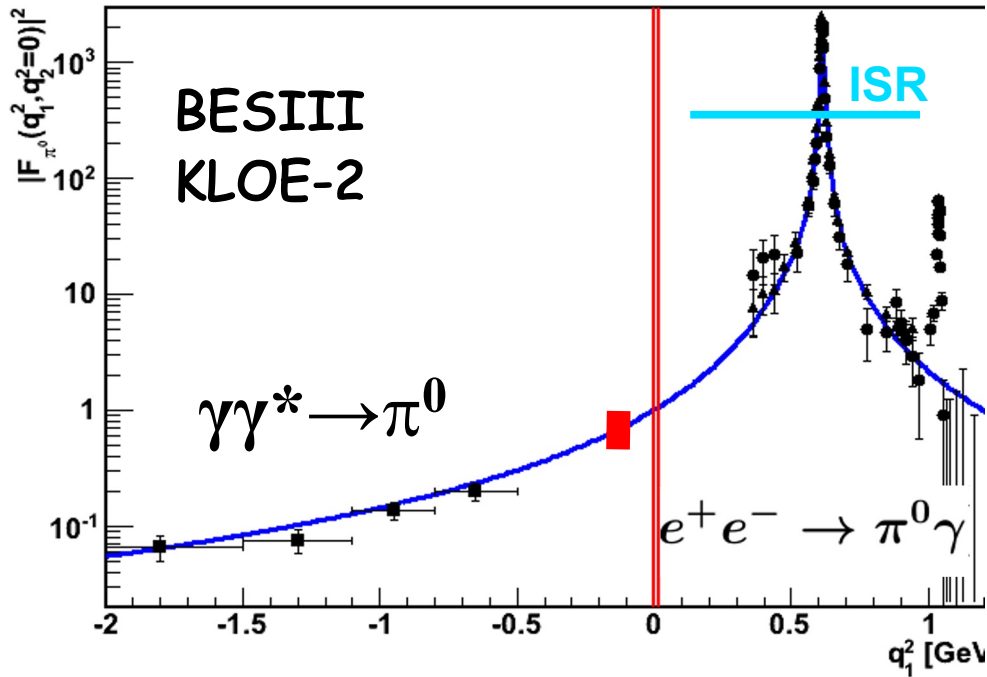
$$\Gamma(\pi^0 \rightarrow \gamma\gamma) = 7.11 \pm 0.44_{\text{stat}} \pm 0.21_{\text{syst}} \text{ eV}$$

$$\Gamma(\eta \rightarrow \gamma\gamma) = 338 \pm 94_{\text{stat}} \pm 35_{\text{syst}} \text{ eV}$$

$$\Gamma(\eta' \rightarrow \gamma\gamma) = 3.4 \pm 1.0_{\text{stat}} \pm 0.4_{\text{syst}} \text{ keV.}$$

→LQCD: 2305.04570

# $\eta, \pi^0$ single off shell TFF

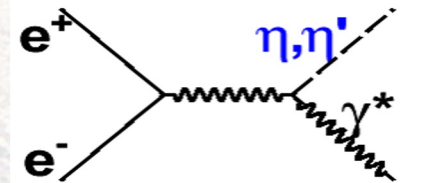


$$\frac{d\sigma}{dt}(e^- \gamma \rightarrow e^- P) = \frac{16 \pi \alpha}{3 s m_P^3} \Gamma_{\gamma\gamma} |F_P(t, 0)|^2 \frac{s - m_P^2 + t}{t}$$

$P \rightarrow \gamma^* \gamma$   
Dalitz decays:  
KLOE, WASA, CBall, BESIII  
CLAS, NA48

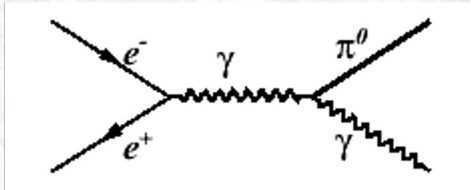
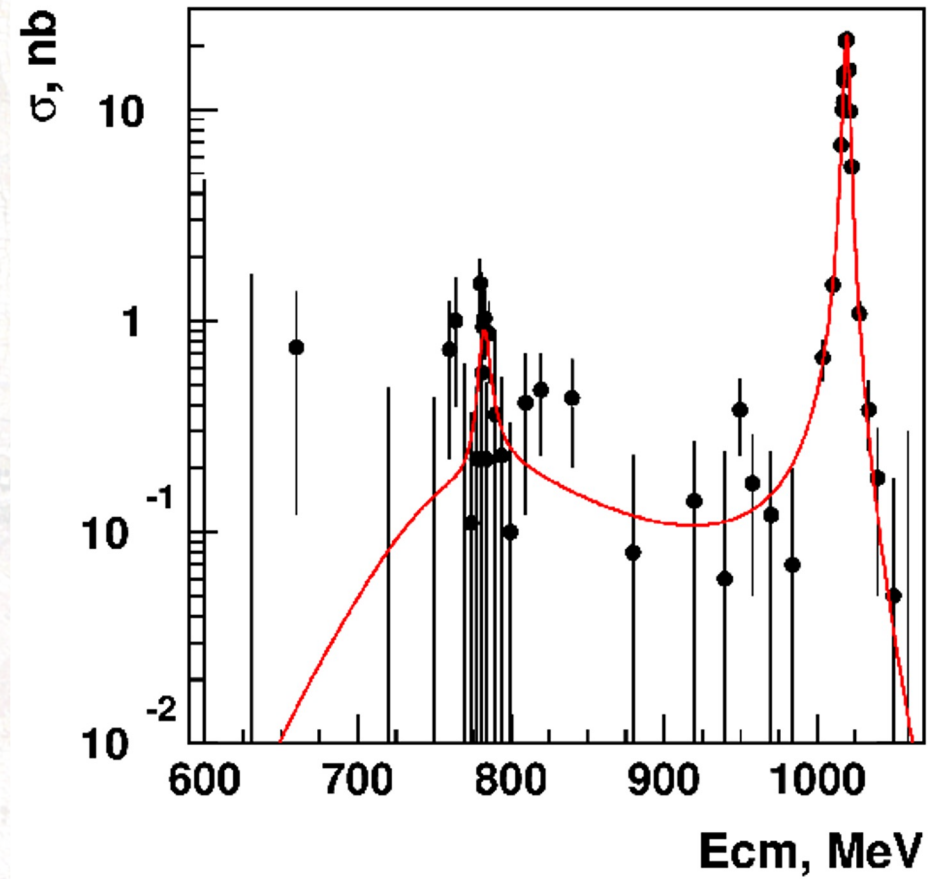
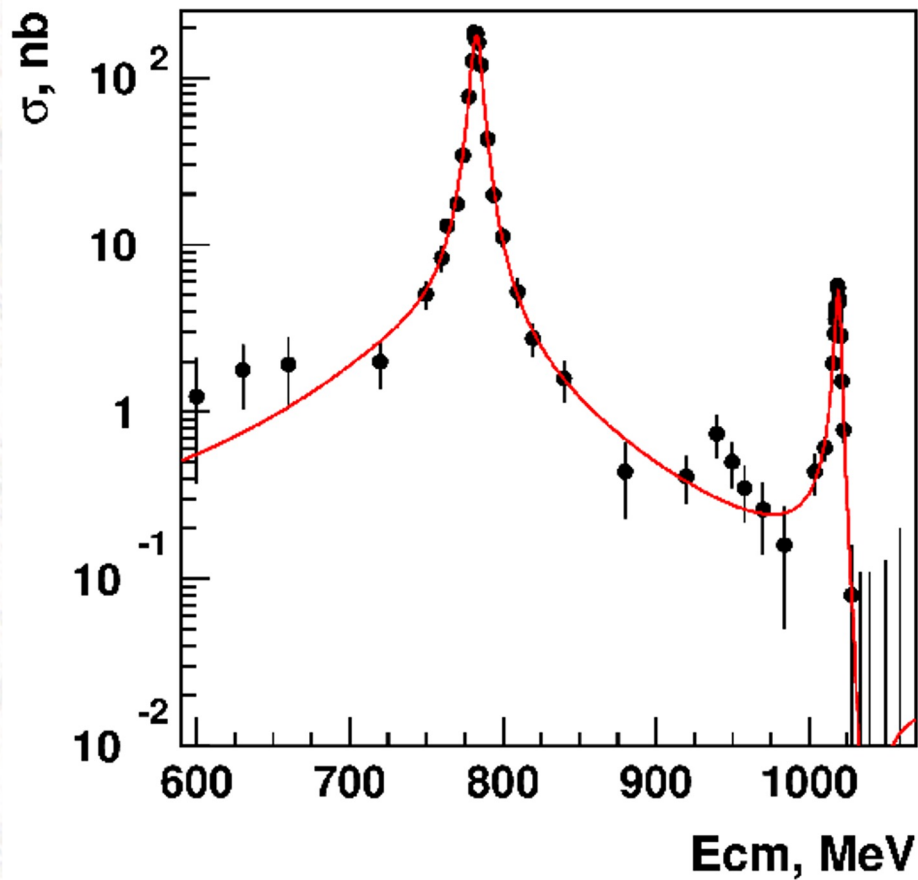
$\gamma^* \rightarrow P\gamma$   
VEPP 2000 0.3-2GeV  
KLOE-2 ISR, BESIII

$$\sigma(e^+e^- \rightarrow P\gamma) = \frac{8}{3} \pi \alpha \Gamma_{\gamma\gamma} |F_P(s, 0)|^2 \left( \frac{s - m_P^2}{s m_P} \right)^3$$



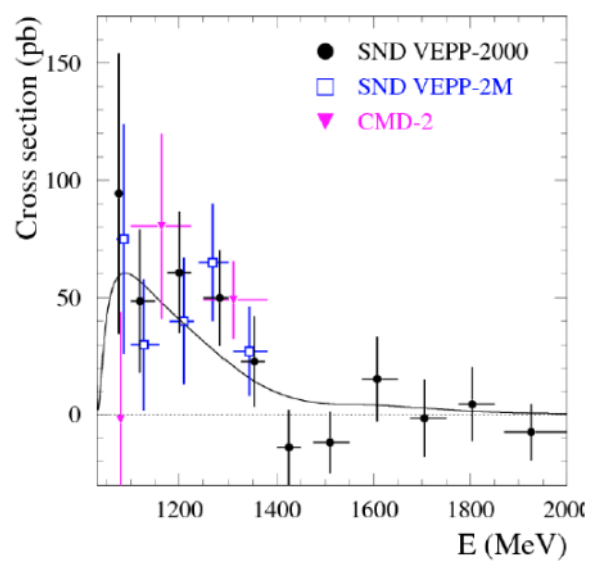
Data: CELLO, NA60, CB-MAMI, CMD-2, SND

# $\sigma(e^+e^- \rightarrow \pi^0\gamma, \eta\gamma)$



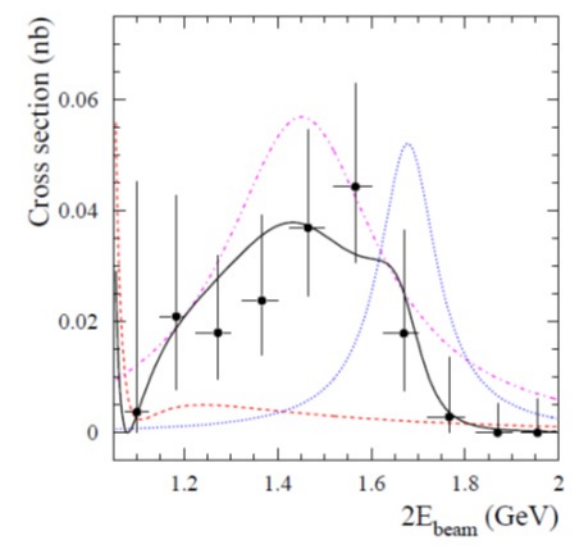
$$\sigma(e^+e^- \rightarrow P\gamma) = \frac{8}{3}\pi\alpha \Gamma_{\gamma\gamma} |F_P(s, 0)|^2 \left( \frac{s - m_P^2}{sm_P} \right)^3$$

$e^+e^- \rightarrow \pi^0\gamma$  at SND



The first search above 1.4 GeV, preliminary  
 No signal above the background

$e^+e^- \rightarrow \eta\gamma$  at SND



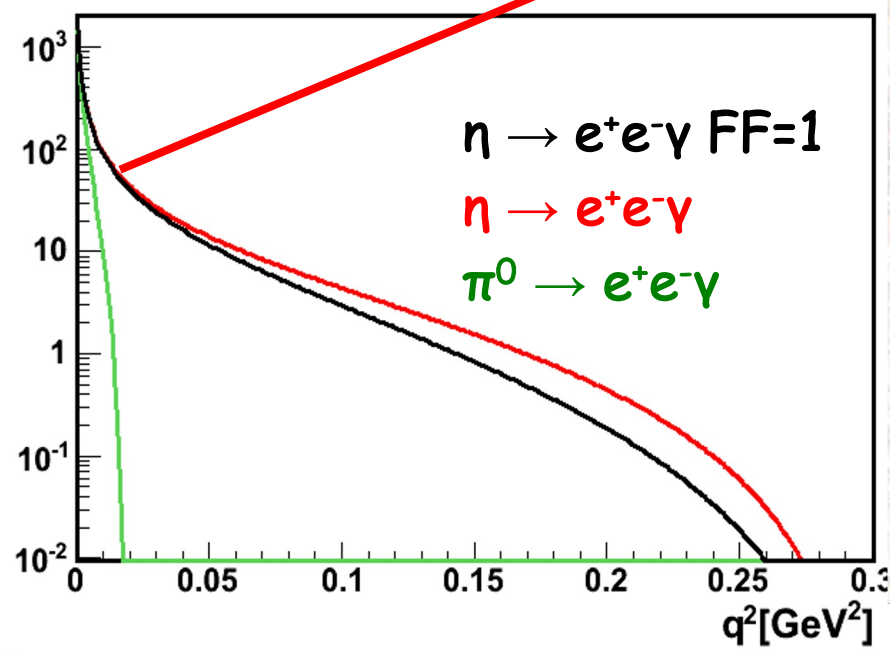
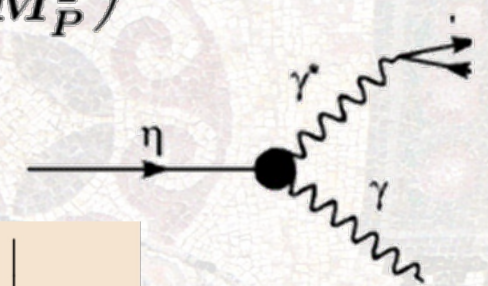
The first measurement above 1.4 GeV, Phys. Rev. D90 (2014) 032002  
 Dominated by the  $\rho(1450)$  and  $\phi(1680)$  mesons



# Dalitz decays

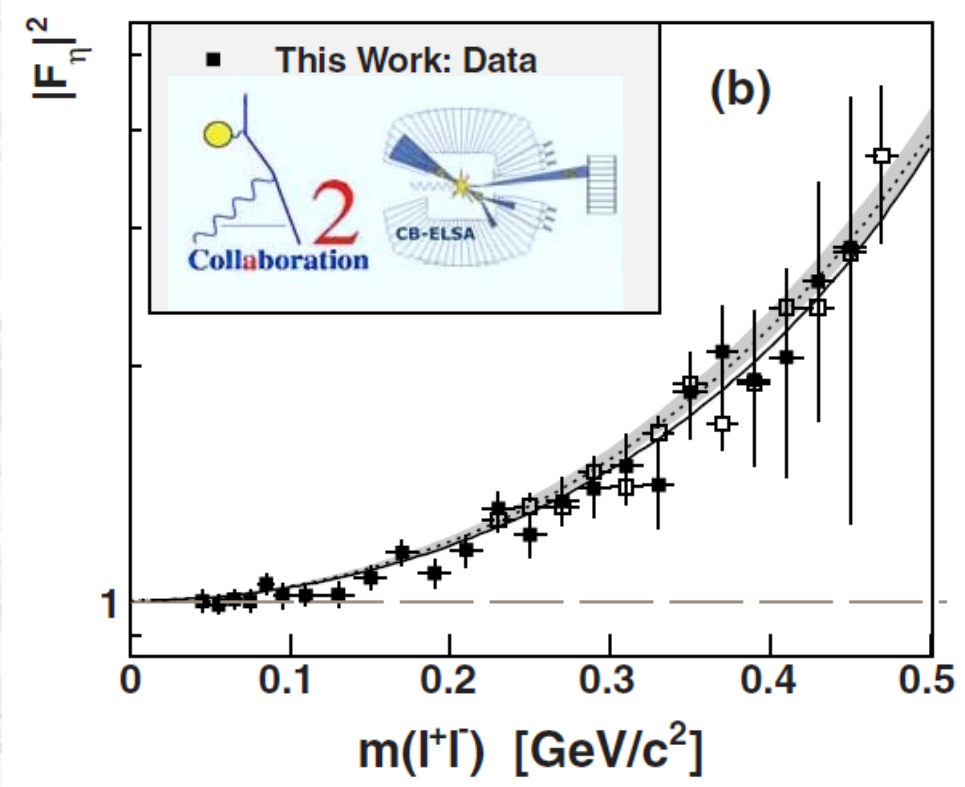
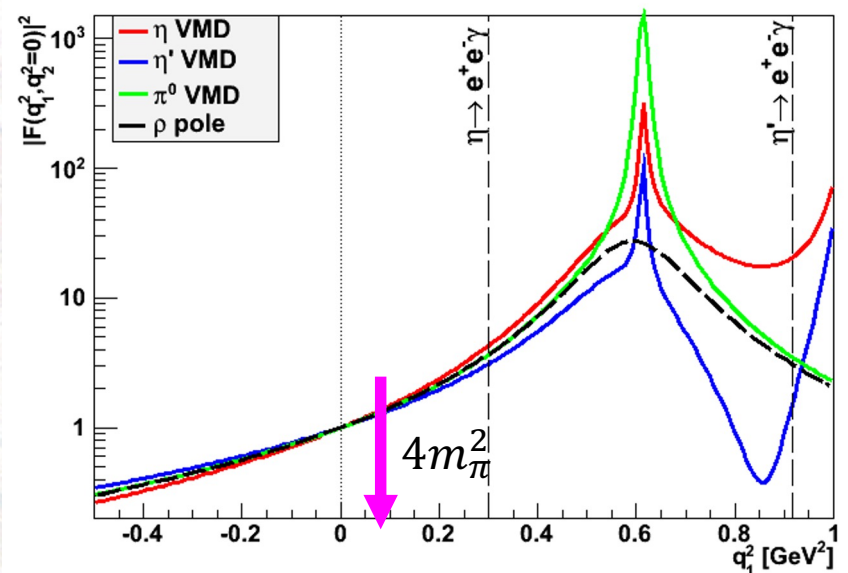
## Single Dalitz decays

$$\frac{d\Gamma(P \rightarrow \ell^+\ell^-\gamma)}{dq^2\Gamma_{\gamma\gamma}} = \frac{2\alpha}{3\pi} \frac{1}{q^2} \sqrt{1 - \frac{4m_\ell^2}{q^2}} \left(1 + \frac{2m_\ell^2}{q^2}\right) \left(1 - \frac{q^2}{M_P^2}\right)^3 |F_P(q^2, 0)|^2$$

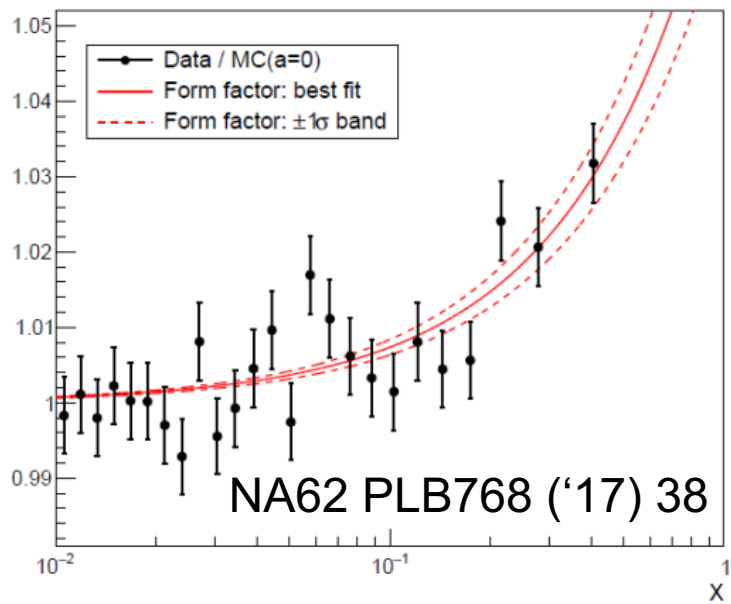


$$b_P = \left. \frac{d \ln |F_P(q^2)|}{dq^2} \right|_{q^2=0}$$

CB/TAPS: PRC89, 044608 (2014)



# $\pi^0 \rightarrow e^+e^-\gamma$



$$a_\pi = (3.68 \pm 0.57) \times 10^{-2}$$

A2 PRC 95 ('17) 025202

$$a_\pi = (3.0 \pm 1.0) \times 10^{-2}$$

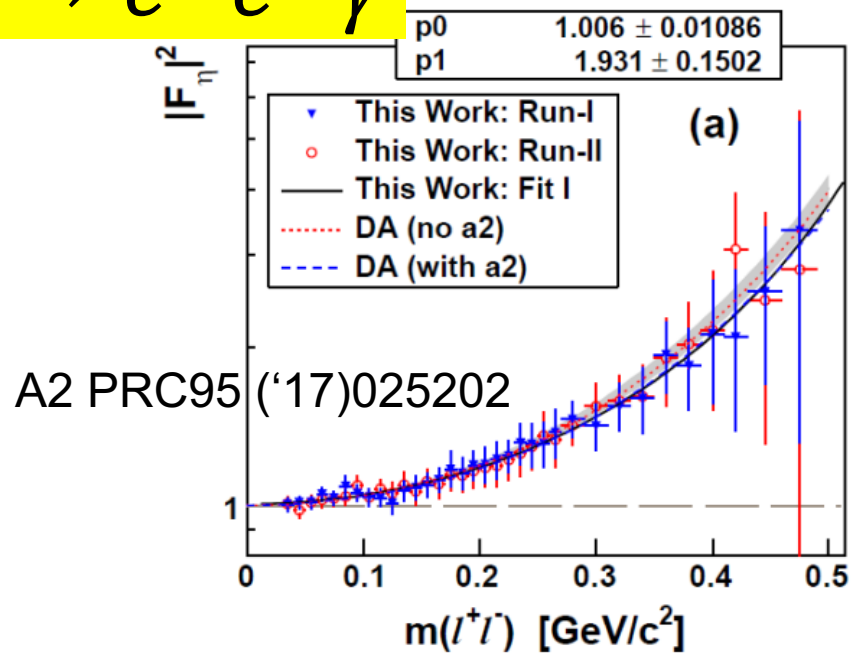
PDG<sub>AVG</sub>

$$a_\pi = (3.35 \pm 0.31) \times 10^{-2}$$

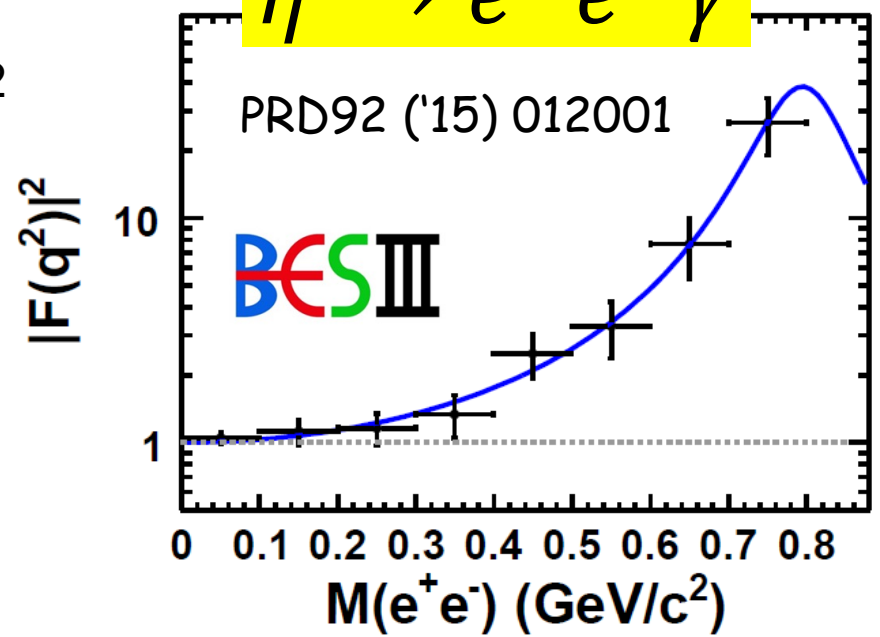
$$\text{DR: } a_\pi = (3.15 \pm 0.09) \times 10^{-2} \quad \text{JHEP 10 (2018) 141}$$

$$F_{\pi^0\gamma}(m_{ee}) = 1 + a_\pi \frac{m_{ee}^2}{m_{\pi^0}^2}$$

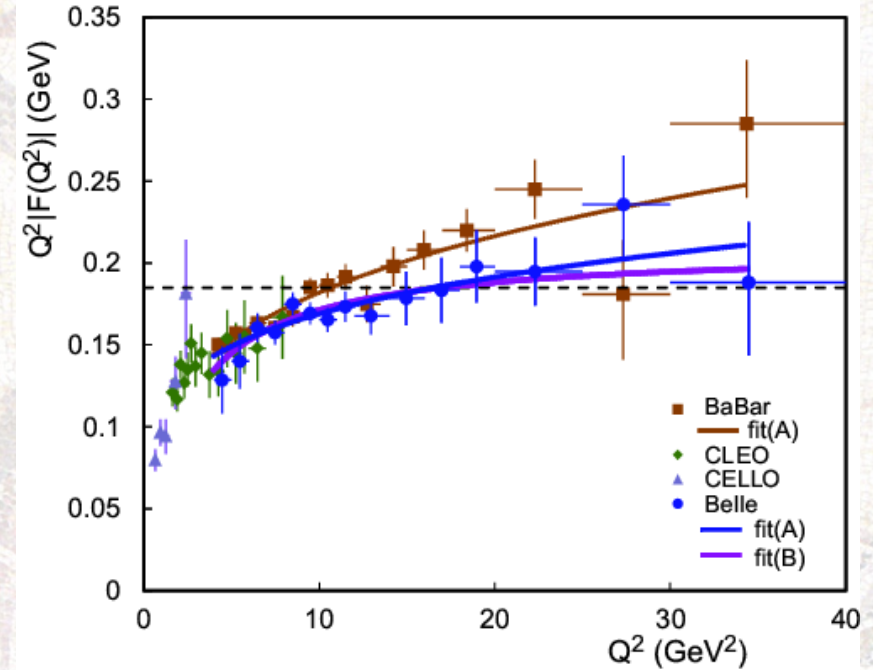
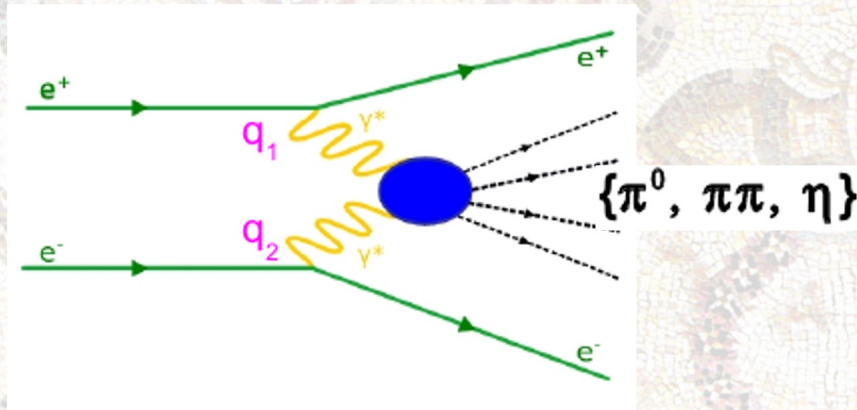
# $\eta \rightarrow e^+e^-\gamma$



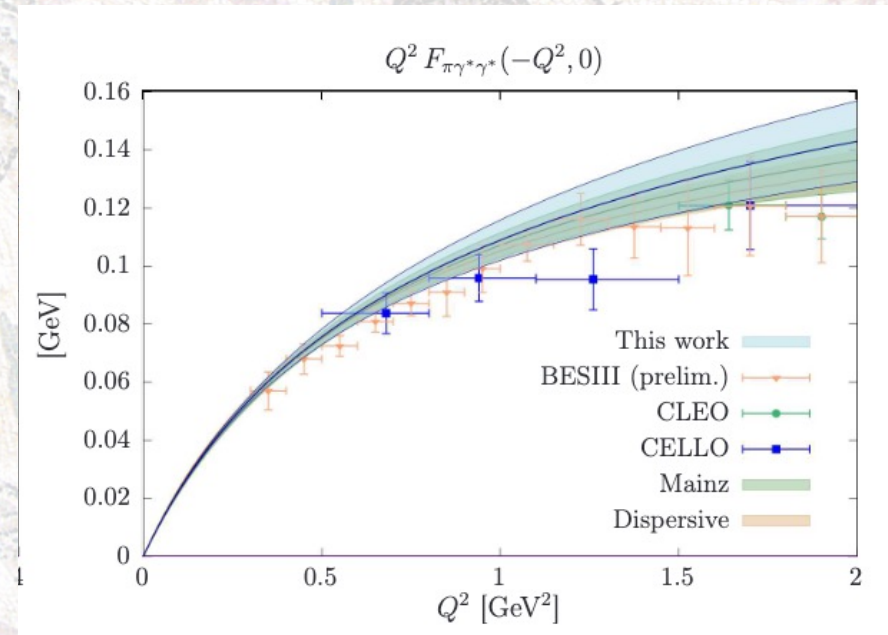
# $\eta' \rightarrow e^+e^-\gamma$



$$\gamma\gamma^* \rightarrow \pi^0$$



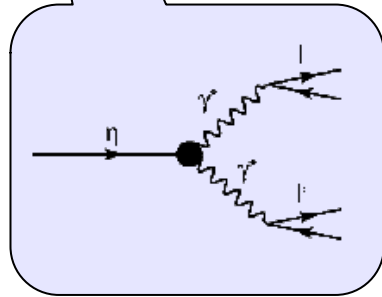
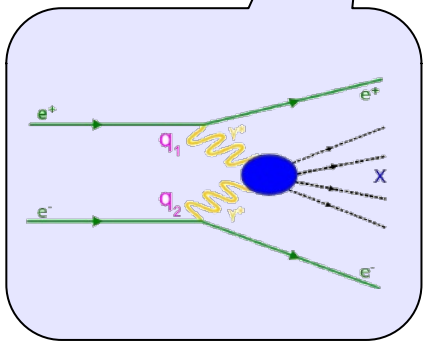
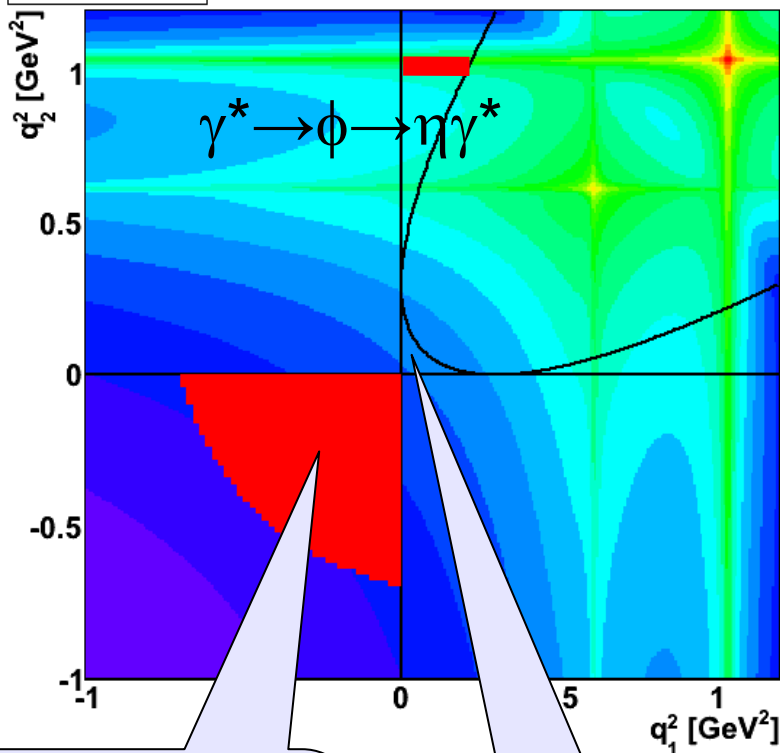
→ Antoine LQCD talk



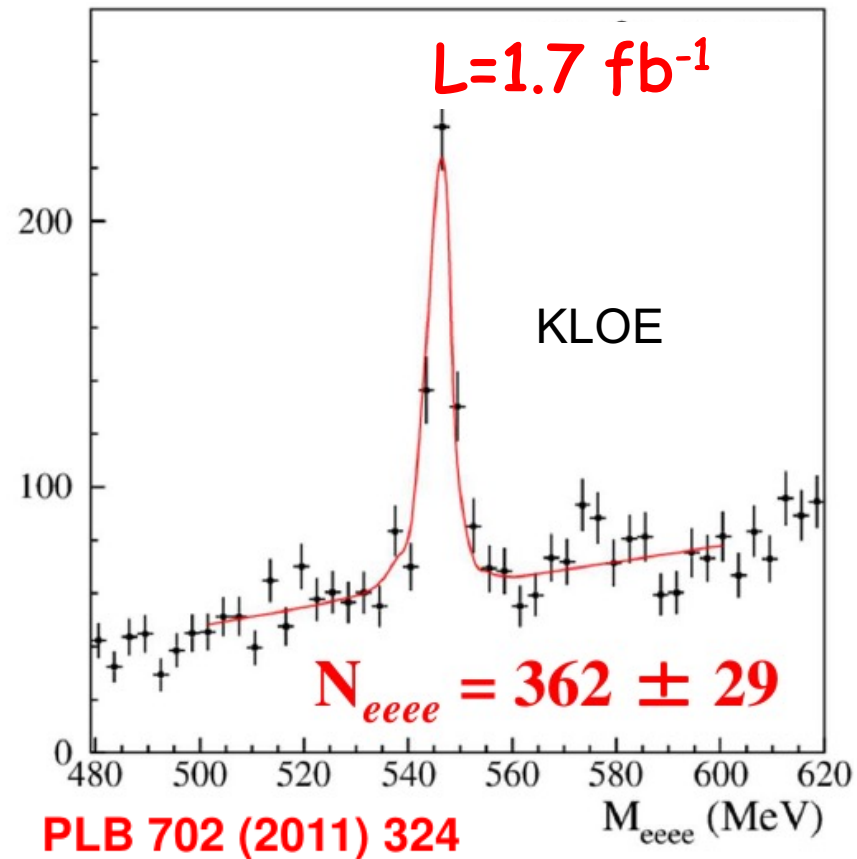
# Double off shell TFF

$\eta$

$$|F_\eta(q_1^2, q_2^2)|^2$$



$$\eta \rightarrow e^+ e^- e^+ e^-$$



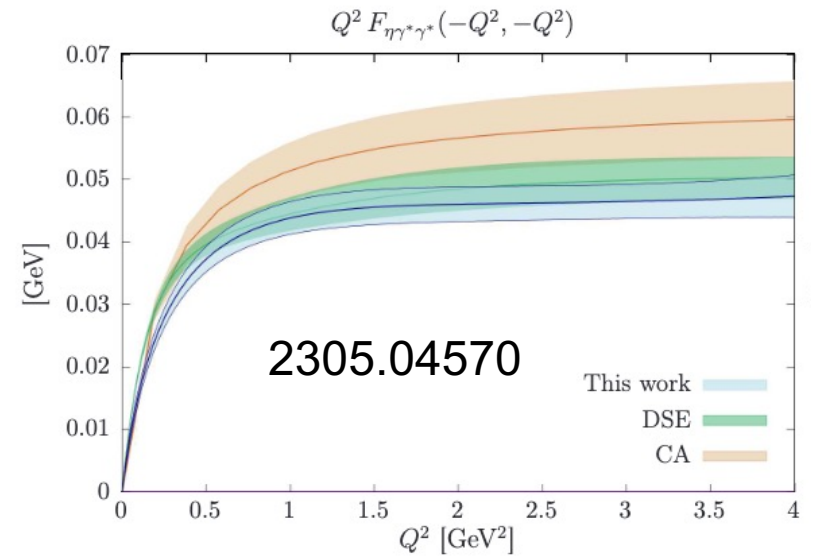
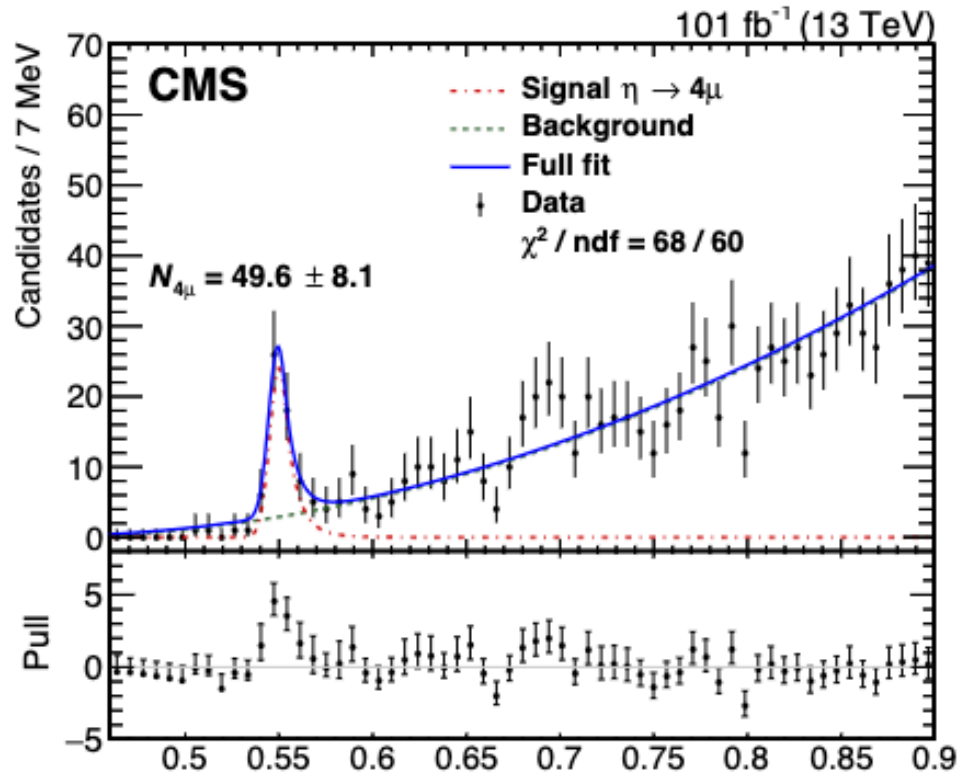
$$\text{BR}(\eta \rightarrow e^+ e^- e^+ e^- (\gamma)) = (2.4 \pm 0.2_{\text{stat}} \pm 0.1_{\text{syst}}) \times 10^{-5}$$

Shuangshi BESIII talk

BESIII, PRD 105 (2022) 112010

$$\mathcal{B}(\eta' \rightarrow e^+ e^- e^+ e^-) = (4.5 \pm 1.0(\text{stat.}) \pm 0.5(\text{sys.})) \times 10^{-6}$$

$$\eta \rightarrow \mu^+ \mu^- \mu^+ \mu^-$$



$$\frac{\mathcal{B}_{4\mu}}{\mathcal{B}_{2\mu}} = (0.86 \pm 0.14 (\text{stat}) \pm 0.12 (\text{syst})) \times 10^{-3}$$

CMS arXiv:2305.04904

$$\Gamma(\eta \rightarrow \mu^+ \mu^- \mu^+ \mu^-) / \Gamma_{\gamma\gamma} (10^{-9}) \quad \text{FF=1} \quad 6.511 \text{ mVMD} \quad 9.7(1)$$

$$\Gamma(\eta \rightarrow \mu^+ \mu^-) / \Gamma_{\gamma\gamma} (10^{-6}) \quad \text{mVMD} \quad 13(1) \quad \text{Exp.} \quad 15(2)$$

T.Petri arxiv:1010.2378

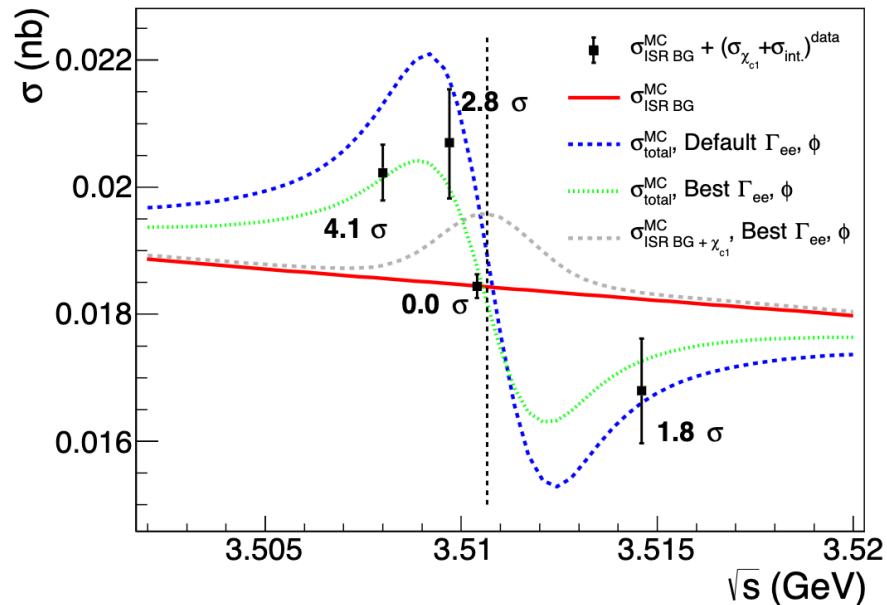
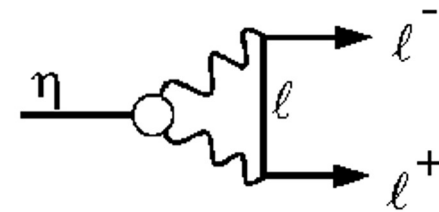
$$e^+e^- \rightarrow \eta, \eta', f_1 \dots (C - \text{even})$$

CMD-3 *Phys.Lett.B* 740 (2015) 273  
 SND, *Phys.Rev.D* 91 (2015) 092010

SND *Phys.Rev.D* 98 (2018) 5, 052007

$$B(\eta' \rightarrow e^+e^-) < 5.6 \times 10^{-9} \text{ 90\% CL}$$

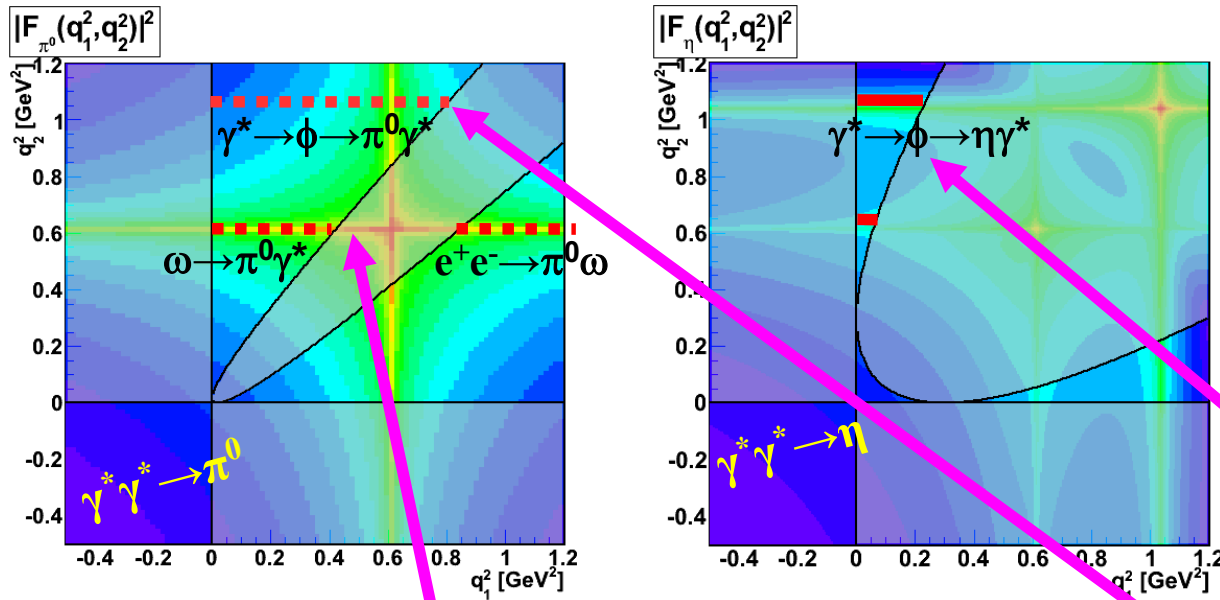
$$B(\eta \rightarrow e^+e^-) < 7.7 \times 10^{-7} \text{ 90\% CL}$$



BESIII *Phys.Rev.Lett.* 129 (2022) 12

*Interference  $e^+e^- \rightarrow J/\psi \gamma \rightarrow \gamma \mu^+ \mu^-$   
 and  $e^+e^- \rightarrow \chi_{c1}$*

# $V \rightarrow P\gamma^*$ and $e^+e^- \rightarrow PV$ processes



**KLOE**

result  $b_{\eta}(m_{\phi}^2)$   
 $\phi \rightarrow \eta \gamma^*$  BR  $10^{-4}$

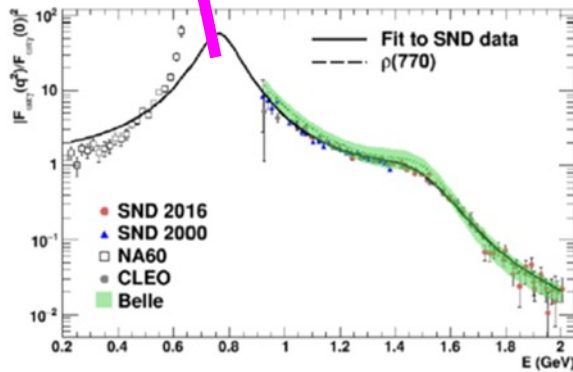
Phys.Lett. B742 (2015) 1-6

$$b_{\phi\eta} = (1.17 \pm 0.10_{-0.11}^{+0.07}) \text{GeV}^{-2}$$

$b_{\pi^0}(m_{\phi}^2)$   $\phi \rightarrow \pi^0 \gamma^*$  BR  $10^{-5}$

Phys.Lett. B757 (2016) 362-367

$$b_{\phi\pi^0} = (2.02 \pm 0.11) \text{GeV}^{-2}$$

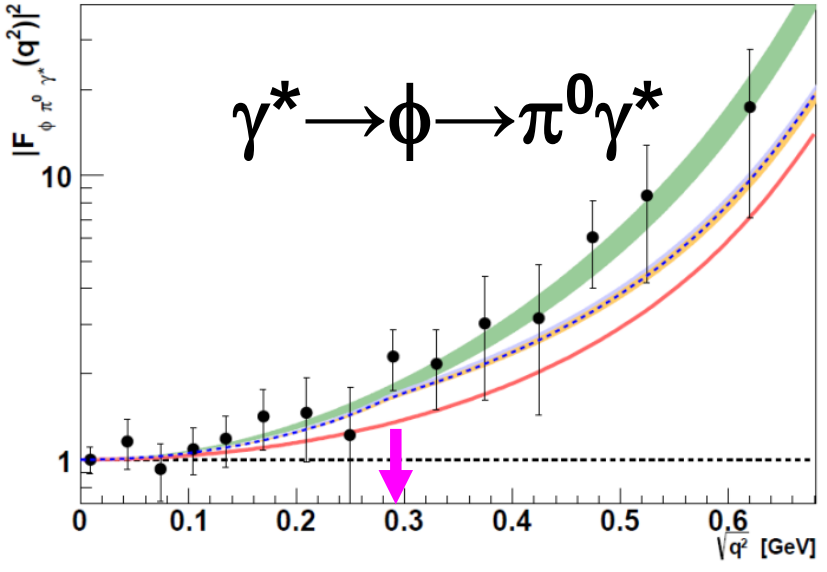
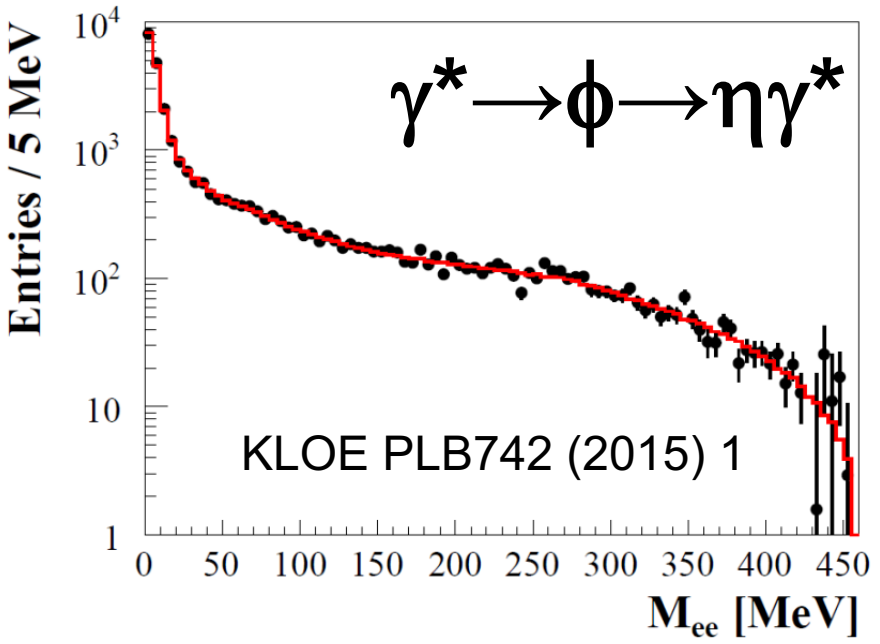


In addition to SND and NA60, other data on  $\mathcal{F}(\gamma\omega\pi)$  exist:

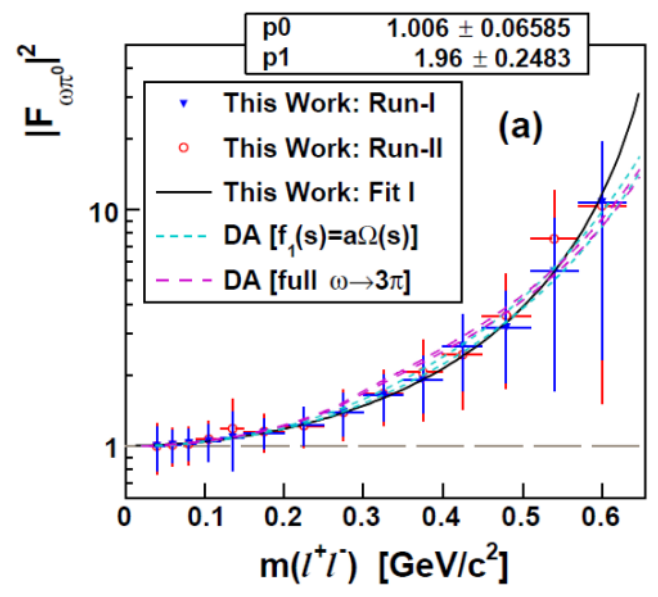
CLEO ( $\tau^- \rightarrow \omega\pi^- \nu_{\tau}$ ), K.W. Edwards et al., Phys. Rev. D61 (2000) 072003

$$F_{\omega\pi}(s) = \frac{g_{\rho\omega\pi}}{g_{\rho}} \left( \text{BW}_{\rho}^{\text{GS}}(s) + c_1 \text{BW}_{\rho'}(s) + c_2 \text{BW}_{\rho''}(s) + \dots \right)$$

from Simon Eidelman



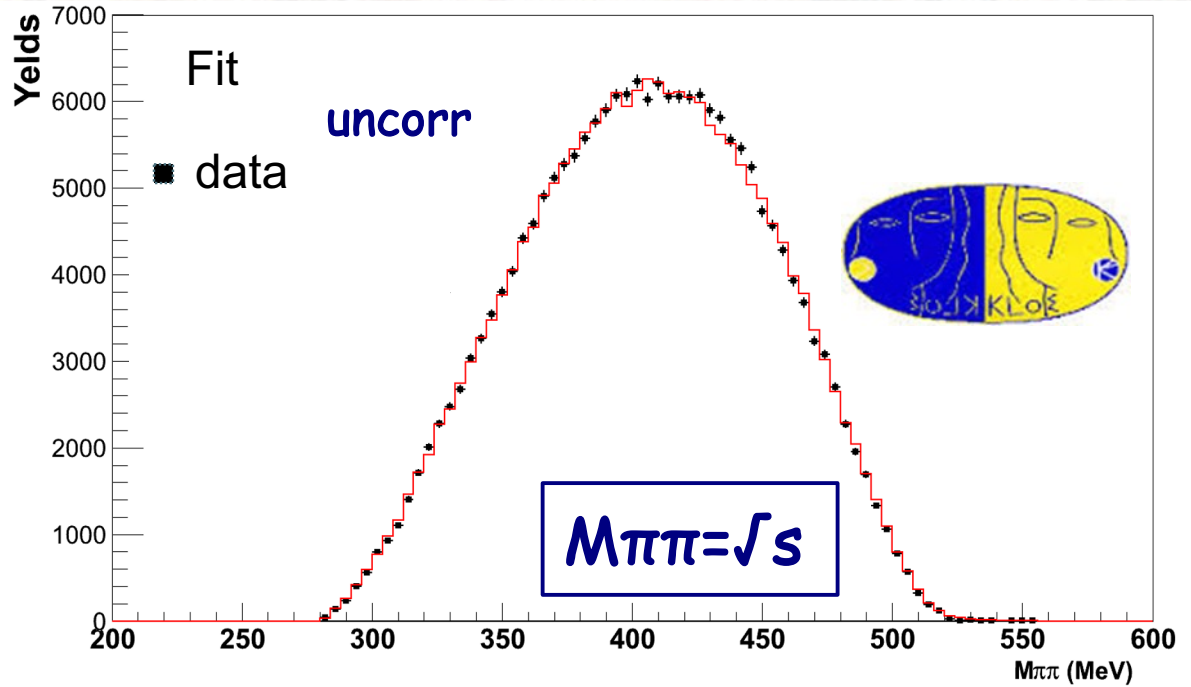
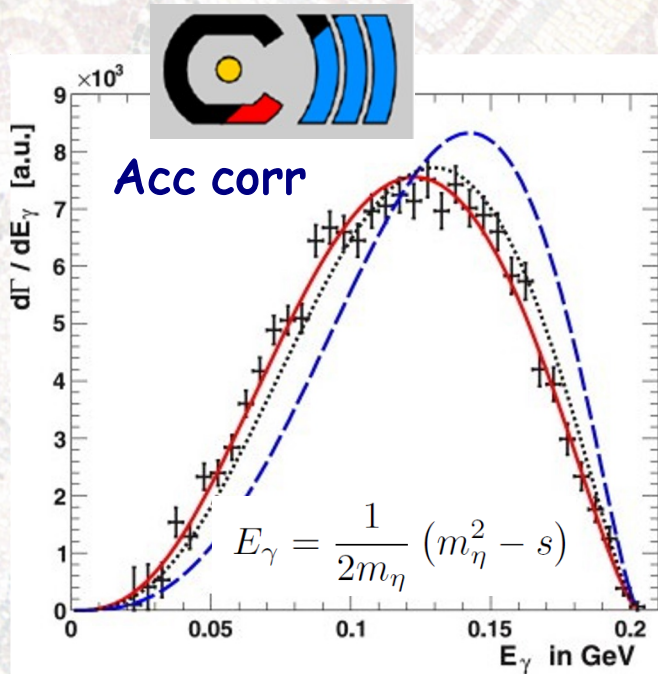
$\gamma^* \rightarrow \pi^0 \omega$



A2 Phys.Rev. C95 (2017) 025202



# TFF from radiative processes (ex $\eta \rightarrow \pi^+\pi^-\gamma$ )

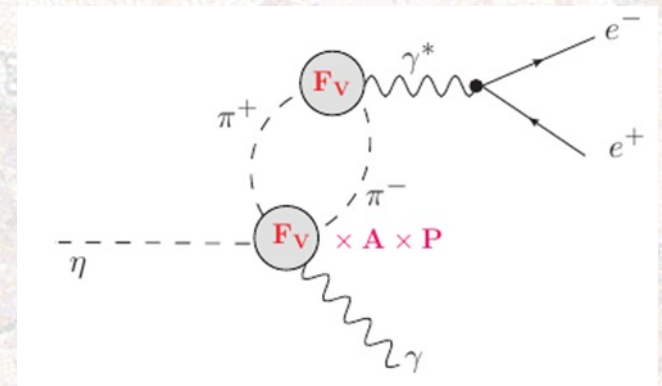


## Model independent parametrization:

$$\frac{d\Gamma}{ds} = |A(1 + \alpha s + \dots)F_V(s)|^2 K_P(s)$$

PLB707 (2012) 184

$e^+e^- \rightarrow \pi^+\pi^-$



$\alpha = 1.89 \pm 0.25_{\text{stat}} \pm 0.59_{\text{syst}} \text{ GeV}^{-2}$

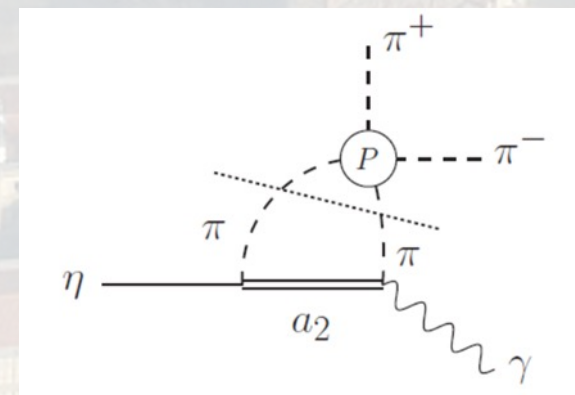
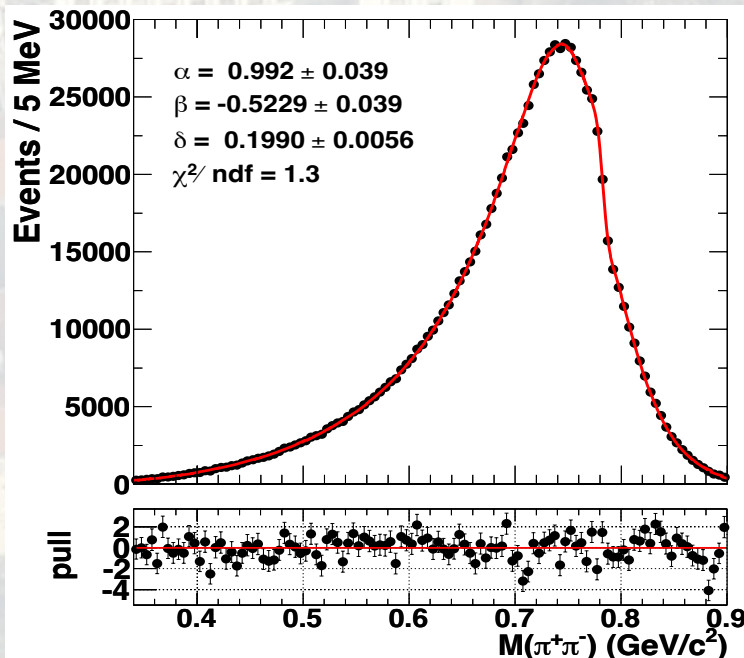
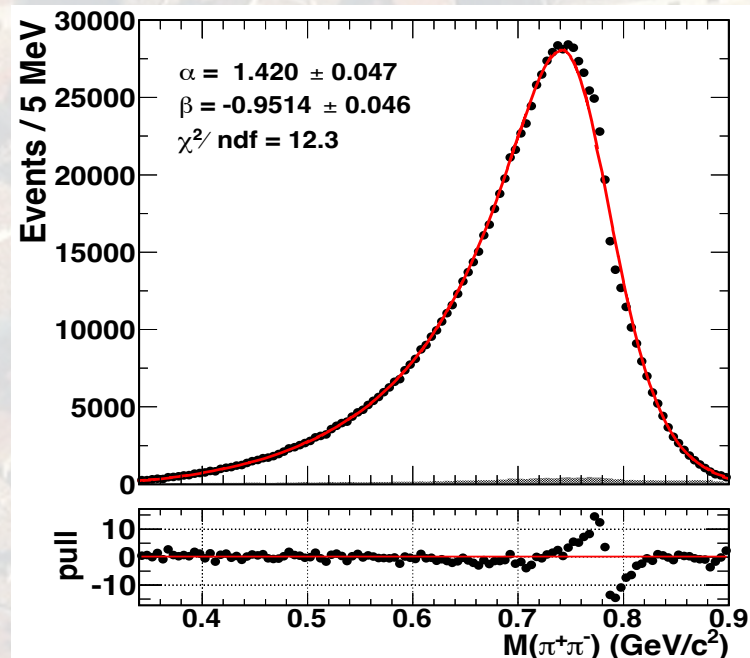
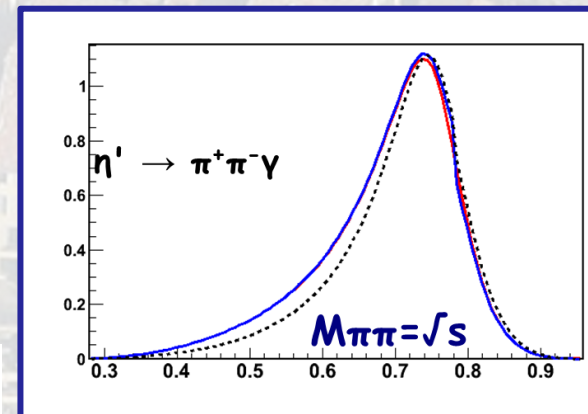
$\alpha = 1.31 \pm 0.08_{\text{stat}} \pm 0.40_{\text{syst}} \text{ GeV}^{-2}$

# Analysis based on $0.9 \times 10^6 \eta' \rightarrow \pi^+\pi^-\gamma$

$$\frac{d\Gamma}{ds_{\pi\pi}} = |AP(s_{\pi\pi})F_V(s_{\pi\pi})|^2 \Gamma_0(s_{\pi\pi})$$

$$P(s_{\pi\pi}) = 1 + \alpha s_{\pi\pi} + \beta s_{\pi\pi}^2$$

$$P(s_{\pi\pi}) = 1 + \alpha s_{\pi\pi} + \beta s_{\pi\pi}^2 + \delta \text{BW}_\omega$$

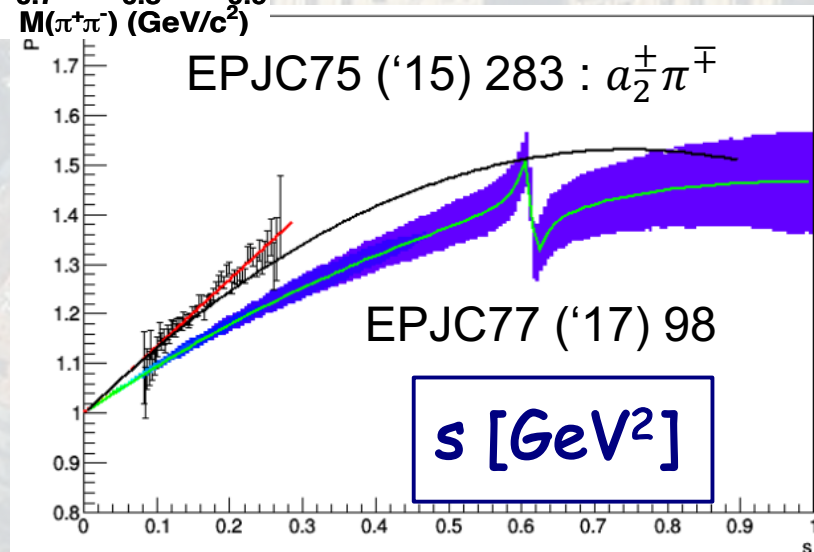


$\omega$  contribution necessary

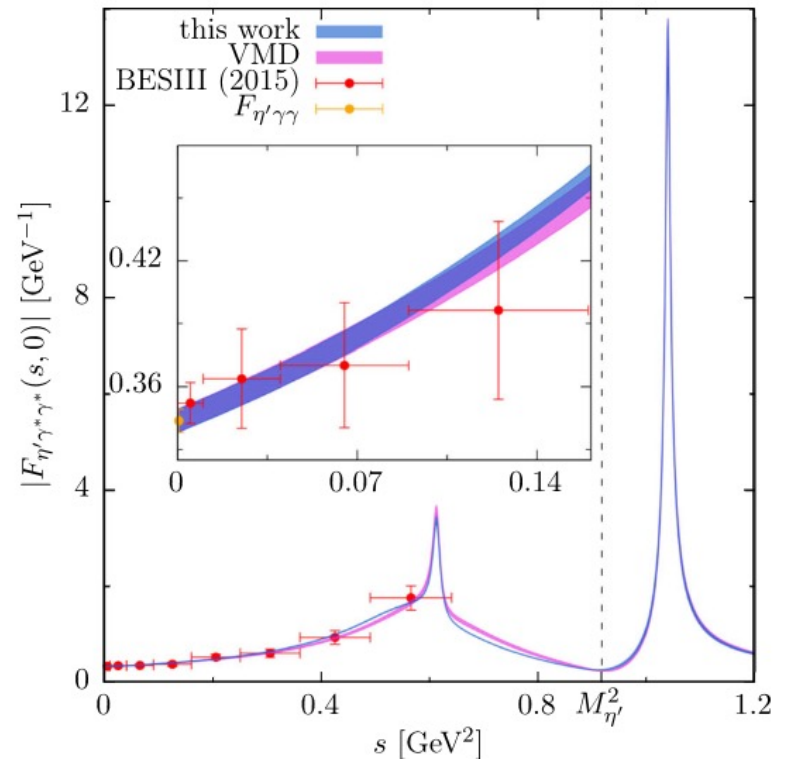
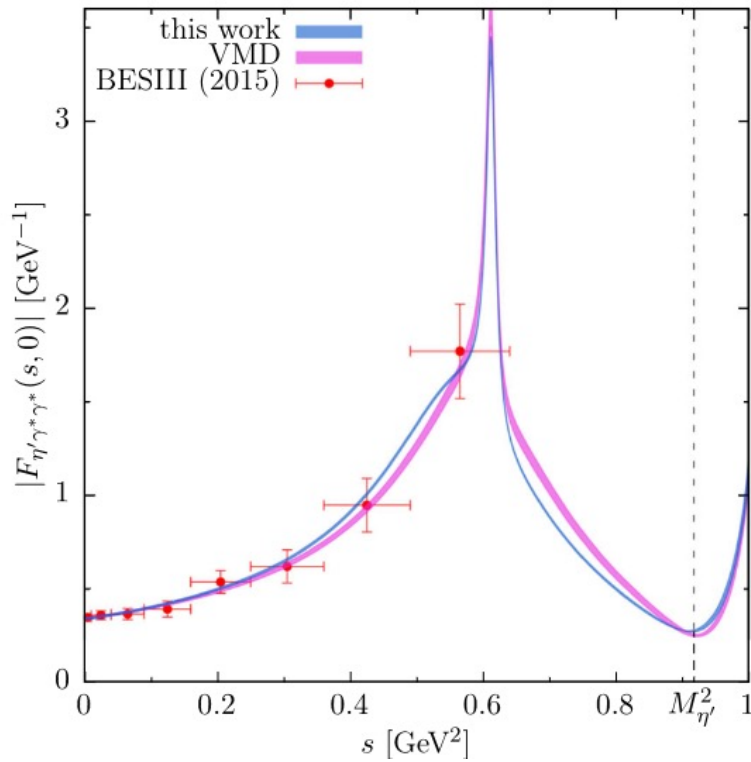
Linear polynomial is insufficient...

Phys.Rev.Lett. 120 (2018) 242003

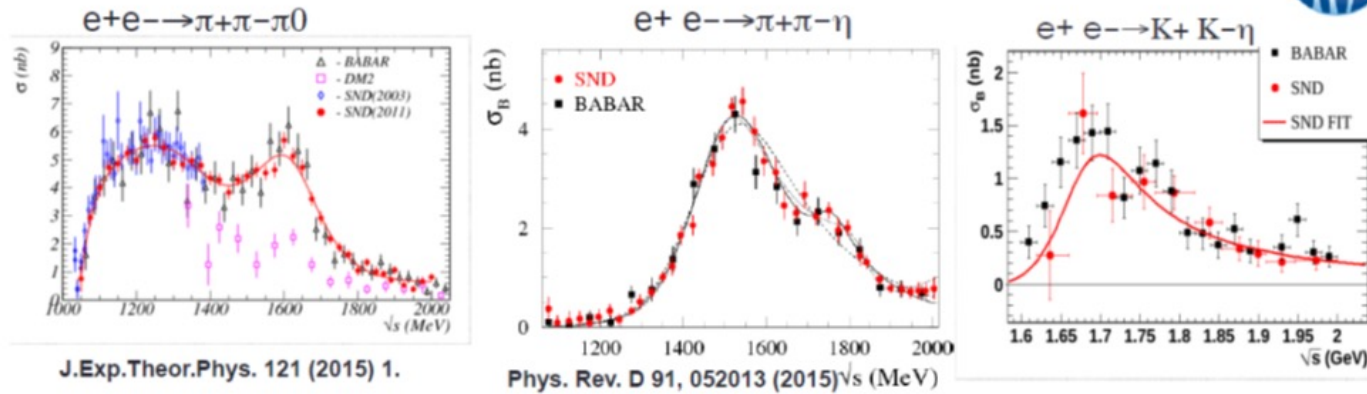
**BES III**



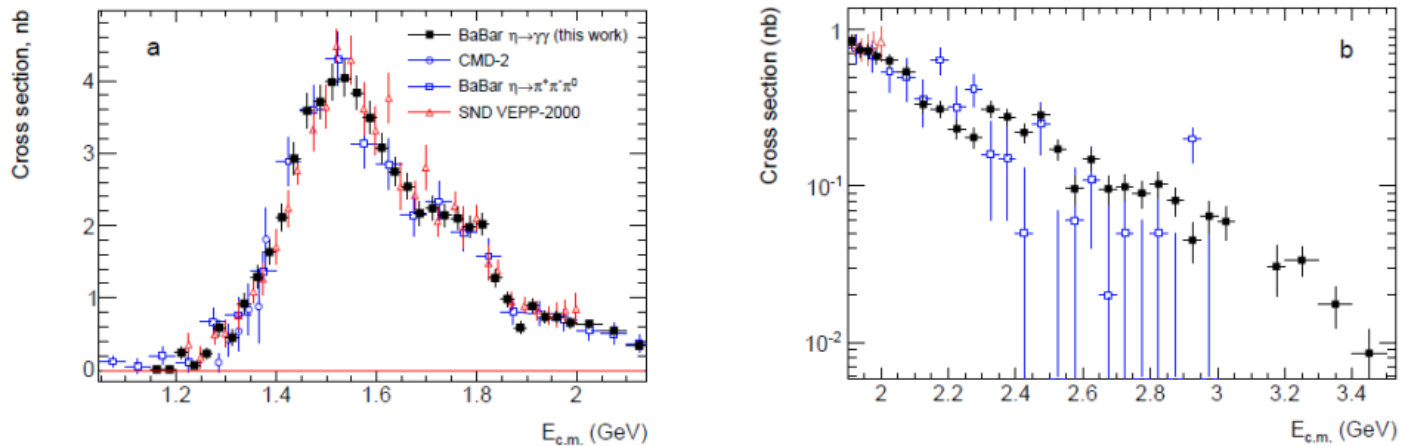
$$\eta' \rightarrow e^+ e^- \gamma$$



# Some SND results overview



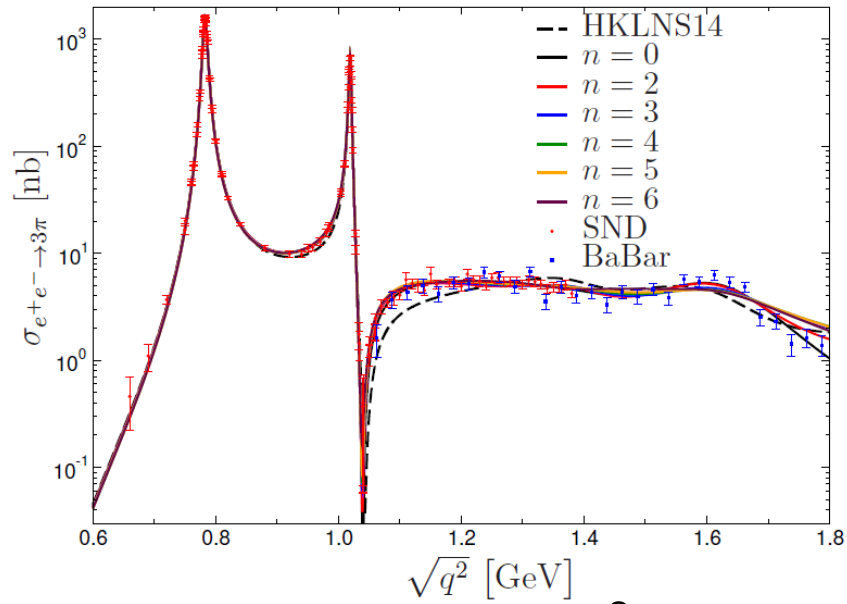
$e^+e^- \rightarrow \eta\pi^+\pi^-$  at BaBar



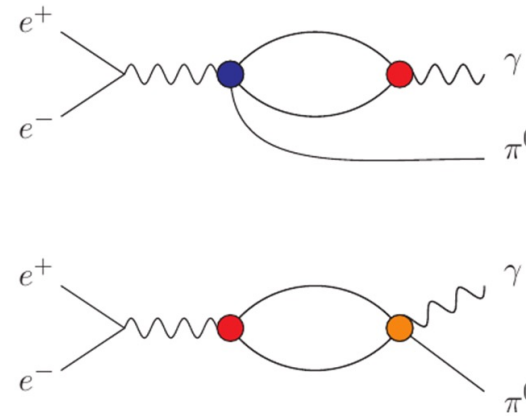
More precise result  $\Rightarrow$  first observation of the  $\rho(1700)$  in  $\eta\pi^+\pi^-$

J.P. Lees et al., Phys. Rev. D97 (2018) 052007

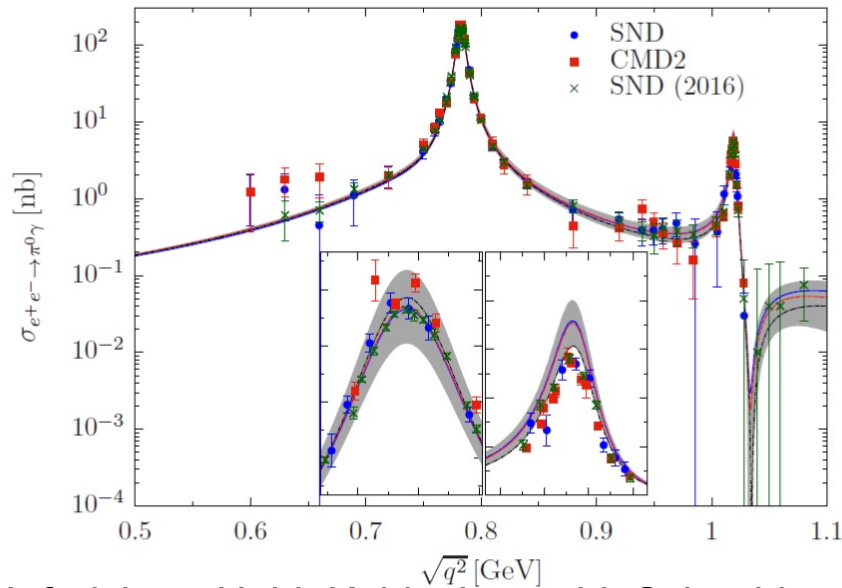
# DR for $\pi^0$ TFF



$$e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$$

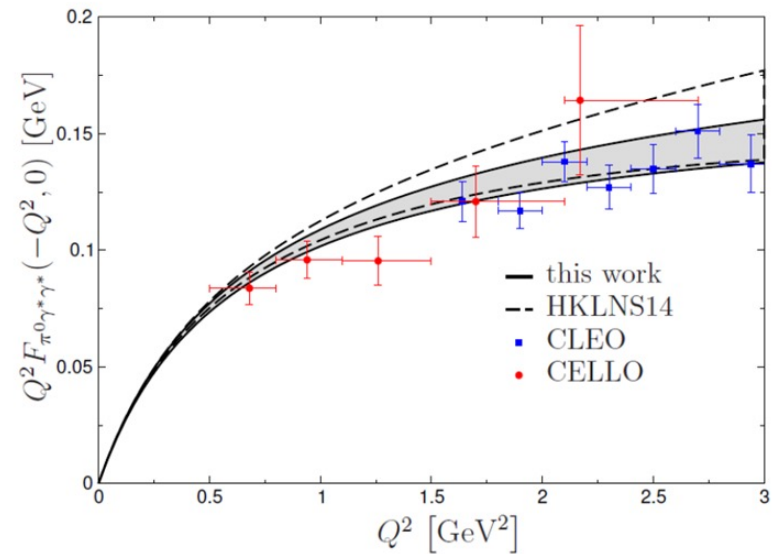


$\pi\pi$  phase shifts +  $e+e^- \rightarrow 3\pi$  data  
 Eur.Phys.J. C74 (2014) 3180



Hoferichter, Hoid, Kubis, Leupold, Schneider

*JHEP* 10 (2018) 141



## Electromagnetic TFF

Dispersive/ChPT calculations:

**Exp input:**  $\Gamma(P \rightarrow \gamma\gamma)$ , hadronic and single radiative processes

Exp double (virtual) radiative processes less precise  
and exp is mainly used for crosscheck

**except** (maybe) CMS with  $> 10^{11} \eta$

Lattice: **Precision results**

Relation to weak TFF...

