Radiative corrections and form factors (not only) in Dalitz decays of lightest mesons

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Overview

Hadronic parameters from experiment

- QED corrections left out entirely
- some of the relevant terms neglected
- approximative results (leading logs, soft-photon approximation)

Artificial discrepancies between theory and experiment
$\hookrightarrow$ measured observables or related hadronic parameters may include unsubtracted QED part

QED radiative corrections in the low-energy QCD sector
$\hookrightarrow$ π^0, $\eta^{(\prime)}$, Σ^0, K^+, ...
$\hookrightarrow$ direct application to experiment

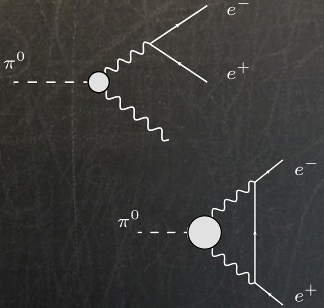
Radiative corrections for π^0 decays

Rare decay of π^0

Introduction

Decay modes of the neutral pion:

Process	Branching ratio
$\pi^0 \to \gamma\gamma$	$(98.823 \pm 0.034)\,\%$
$\pi^0 \to e^+e^-\gamma$	$(1.174 \pm 0.035)\,\%$
$\pi^0 \to e^+e^+e^-e^-$	$(3.34 \pm 0.16) \times 10^{-5}$
$\pi^0 \to e^+e^-$	$(6.46 \pm 0.33) \times 10^{-8}$



Rare decay $\pi^0 \to e^+e^-$

- interesting way to study low-energy (long-distance) dynamics in the SM
- systematic theoretical treatment dates back to *Drell*, NC (1959)
- suppressed compared to the decay $\pi^0 \to \gamma\gamma$ by a factor of $2(\alpha m_e/M_\pi)^2$

  $\hookrightarrow$ one-loop structure + helicity suppression
  $\hookrightarrow$ may be sensitive to possible effects of new physics

Rare decay of π^0

KTeV measurement

KTeV-E799-II experiment at Fermilab (*Abouzaid et al.*, PRD 75 (2007))

↪ precise measurements of branching ratio $\pi^0 \to e^+e^-$ (794 candidates)

$$\frac{\Gamma(\pi^0 \to e^+e^-(\gamma),\ x > 0.95)}{\Gamma(\pi^0 \to e^+e^-\gamma,\ x > 0.232)} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}$$

↪ final result for lowest order (no final state radiation)

$$B^{\text{no-rad}}_{\text{KTeV}}(\pi^0 \to e^+e^-) = (7.48 \pm 0.29 \pm 0.25) \times 10^{-8}$$

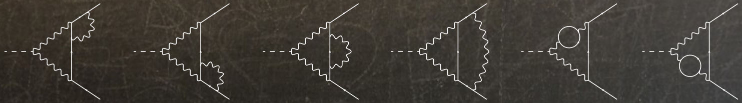
Comparison with SM prediction (*Dorokhov and Ivanov*, PRD 75 (2007))

$$B^{\text{no-rad}}_{\text{SM}}(\pi^0 \to e^+e^-) = (6.23 \pm 0.09) \times 10^{-8}$$

↪ interpreted as $3.3\,\sigma$ discrepancy between theory and experiment

Radiative corrections for $\pi^0 \to e^+e^-$

Final results



Vaško and Novotný, JHEP 1110 (2011)

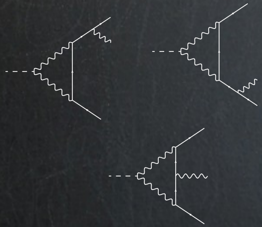
Size of the radiative corrections (newly calculated)

$$\delta^{\text{NLO}}(0.95) \equiv \delta^{\text{virt.}} + \delta^{\text{BS}}(0.95) = (-5.5 \pm 0.2)\,\%$$

- can be thought as model-independent
- differs significantly from previous approximate calculations

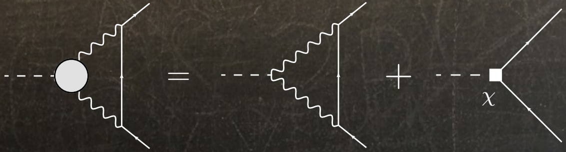
  Bergström, Z.Ph.C 20 (1983): $\delta(0.95) = -13.8\,\%$
  Dorokhov et al., EPJC 55 (2008): $\delta(0.95) = -13.3\,\%$
- original KTeV vs. SM discrepancy reduced to the 2σ level
- contact interaction coupling finite part set to

$$\chi^{(\text{r})}_{\text{LMD}}(M_\rho) = 2.2 \pm 0.9$$



TH, Kampf and Novotný,
EPJC 74 (2014)

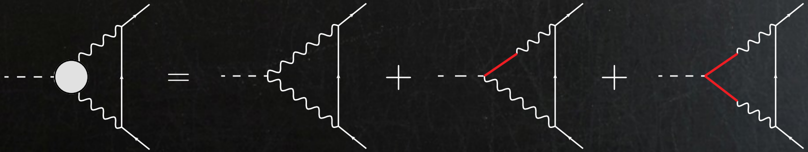
Resonances



Chiral Perturbation Theory (χPT)

$\Updownarrow$

Resonance Chiral Theory (RχT)



THS model for PVV correlator

1) Ansatz for pseudoscalar-vector-vector (PVV) correlator

- Two-hadron saturation (THS) - 2 meson multiplets per channel

$$\Pi^{\mathrm{THS}}(r^2;p^2,q^2) \sim \frac{1}{r^2(r^2-M_P^2)}\,\frac{P(r^2;p^2,q^2)}{(p^2-M_{V_1}^2)(p^2-M_{V_2}^2)(q^2-M_{V_1}^2)(q^2-M_{V_2}^2)}$$

- numerator: general polynomial symmetrical in p^2 and q^2

$\hookrightarrow$ simple requirement: correlator must drop at large momenta
$\hookrightarrow$ 22 free parameters

$$P(r^2;p^2,q^2) = c_0 p^2 q^2 + c_1[(p^2)^3 q^2 + (q^2)^3 p^2] + c_2 (r^2)^2 p^2 q^2 + \ldots$$

2) High- and low-energy limits to constrain the parameters c_i

- operator-product expansion (OPE)
- Brodsky–Lepage (BL) scaling limit
- chiral anomaly

TH and S. Leupold, EPJC 75 (2015)

THS and $\mathcal{F}_{\pi^0\gamma^*\gamma^*}$ form factor

Form factor is in general related to PVV correlator as

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(p^2,q^2) \sim \lim_{r^2\to 0} r^2\Pi(r^2;p^2,q^2)$$

↪ in our case complicated, but with only one free parameter

$$\mathcal{F}^{\mathsf{THS}}_{\pi^0\gamma^*\gamma^*}(p^2,q^2) = -\frac{N_c}{12\pi^2 F}\left[\frac{M_{V_1}^4 M_{V_2}^4}{(p^2-M_{V_1}^2)(p^2-M_{V_2}^2)(q^2-M_{V_1}^2)(q^2-M_{V_2}^2)}\right]$$
$$\times\left\{1+\frac{\kappa}{2N_c}\frac{p^2q^2}{(4\pi F)^4}-\frac{4\pi^2F^2(p^2+q^2)}{N_c M_{V_1}^2 M_{V_2}^2}\left[6+\frac{p^2q^2}{M_{V_1}^2 M_{V_2}^2}\right]\right\}$$

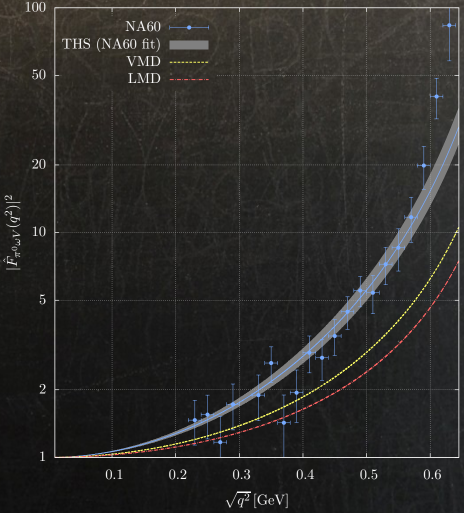
κ determined from fit to ω-π transition form factor measurements
↪ *NA60*, PLB 677 (2009)

$$\kappa = 21 \pm 3$$

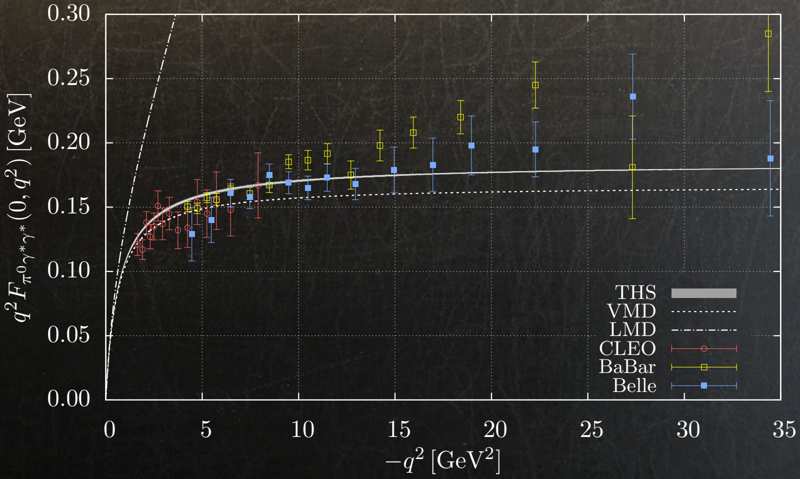
$M_{V_1} \sim \rho, \omega$ vector-meson mass
$M_{V_2} \sim$ between physical masses of first and second vector-meson excitations

$$M_{V_2} \in [1400, 1740]\ \text{MeV}$$

Fit of κ to NA60 data



Form-factor data



Results

Theoretical prediction within THS model

$$B^{\mathsf{THS}}(\pi^0 \to e^+e^-(\gamma),\, x_{\mathrm{D}} > 0.95) = (5.76 \pm 0.7 \pm 2.0) \times 10^{-8}$$

$$\Updownarrow$$

$$B^{\text{no-rad}}_{\mathsf{THS}}(\pi^0 \to e^+e^-) = 6.11(7)(20) \times 10^{-8}$$

- experimental value: $B^{\mathsf{KTeV}} = 6.44(33) \times 10^{-8}$ ⇔ $B^{\text{no-rad}}_{\mathsf{KTeV}} = 6.82(35) \times 10^{-8}$
  ↪ disagreement at the level of only 1.8 σ
- matching on LO χPT gives $\chi^{(\mathrm{r})}_{\mathsf{THS}}(M_\rho) = 2.2 \pm 0.7$
- if KTeV result confirmed (e.g. by NA62) → two scenarios are conceivable:

  a) some aspects of the THS approach not well-suited for $\pi^0 \to e^+e^-$
  b) beyond-Standard Model physics influences the rare pion decay significantly
- under the present circumstances the current discrepancy is inconclusive

Extract $\chi^{(r)}(M_\rho)$ from various models and measurements

VMD	LMD	THS	CLEO-based	KTeV (old)	KTeV (upd.)
2.87	2.29(±0.9)	2.2 ± 0.7	2.6 ± 0.3	6.0 ± 1.0	4.5 ± 1.0

$\pi^0 \to e^+e^-$ branching ratio

Standard Model prediction

$$\frac{B(\pi^0 \to e^+e^-)}{B(\pi^0 \to \gamma\gamma)}$$

$$= 2\beta\left(\frac{\alpha m}{\pi M}\right)^2 \left\{\left[-\frac{5}{2} + \chi^{(\mathrm{r})}(\mu) + \frac{3}{2}\log\frac{m^2}{\mu^2} + \frac{1}{2\beta}\left(\mathrm{Li}_2\, z - \mathrm{Li}_2\,\frac{1}{z}\right)\right]^2 + \left[\frac{\pi}{2\beta}\log(-z)\right]^2\right\}$$

$$z = -\frac{1-\beta}{1+\beta}, \quad \beta = \sqrt{1-\frac{4m^2}{M^2}}$$

$$B(\pi^0 \to e^+e^-) \approx (6.21 + 0.15\widetilde{\chi}) \times 10^{-8}, \qquad \widetilde{\chi} \equiv 2\left[\chi^{(\mathrm{r})}(770\,\mathrm{MeV}) - \tfrac{5}{2}\right] \in (-1, 1)$$

$$B_{\text{KTeV}}^{\text{no-rad}} = 6.82(35) \times 10^{-8}$$

Theoretical predictions and models suggest $\chi^{(\mathrm{r})}(770\,\mathrm{MeV}) \sim$ 2–3

Knecht et al., PRL 83 (1999)
Dorokhov and Ivanov, PRD 75 (2007)
TH and Leupold, EPJC 75 (2015)
Hoferichter et al., PRL 128 (2022)

Dalitz decay of the neutral pion

Introduction

Quantity really measured by KTeV

$$\left.\frac{\Gamma(\pi^0\to e^+e^-(\gamma)\,,\; x>0.95)}{\Gamma(\pi^0\to e^+e^-\gamma(\gamma)\,,\; x>0.2319)}\right|_{\text{KTeV}} = (1.685\pm 0.064\pm 0.027)\times 10^{-4}$$

↪ Dalitz decay comes into play

- second most important decay channel of the neutral pion
  ↪ branching ratio $(1.174\pm 0.035)\,\%$
- first studied by *Richard H. Dalitz*, PPSA 64 (1951)
- experimental data of this process provide the information about singly-virtual pion transition form factor $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,q^2)$
  ↪ in particular about its slope parameter a_π

$$\frac{\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,M^2x)}{\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,0)} \simeq 1 + a_\pi x$$

$$x = \frac{(p_{e^+}+p_{e^-})^2}{M_{\pi^0}^2}\,, \qquad y = -\frac{2}{M_{\pi^0}^2}\frac{p_\gamma\cdot(p_{e^-}-p_{e^+})}{1-x}$$

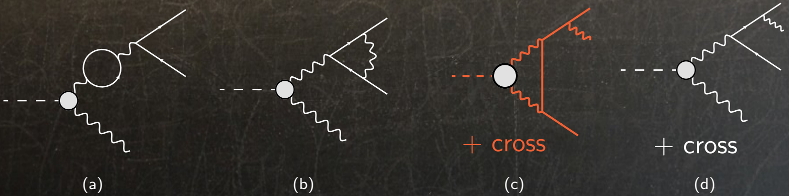
Radiative corrections for $\pi^0 \to e^+e^-\gamma$

Introduction

- radiative corrections to the total decay rate of the Dalitz decay
  $\hookrightarrow$ first addressed (numerically) by *Joseph*, NC 16 (1960)
- pioneering study of corrections to the differential decay rate
  $\hookrightarrow$ *Lautrup and Smith*, PRD 3 (1971)
  $\hookrightarrow$ soft-photon approximation
- extended by *Mikaelian and Smith*, PRD 5 (1972)
  $\hookrightarrow$ hard-photon corrections
  $\hookrightarrow$ whole range of bremsstrahlung photon energy
  $\hookrightarrow$ table of values

Radiative corrections for $\pi^0 \to e^+e^-\gamma$

Novel approach



- new calculations motivated by needs of NA48/NA62 experiments at CERN
  $\hookrightarrow$ measure the slope of $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,q^2)$: *Lazzeroni et al.*, PLB 768 (2017)

$$a_\pi^{\text{NA62}} = 3.68(57)\,\%$$

- unlike before no approximation was used + 1γIR correction
- C++ code returns the correction for any given x and y
  $\hookrightarrow$ propagated into MC generator of NA62 experiment
- *TH, Kampf and Novotný*, PRD 92 (2015)

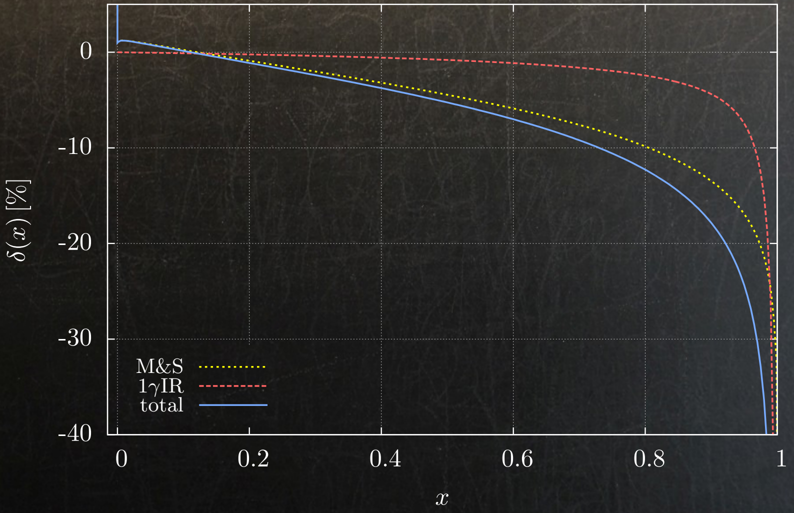
Radiative corrections for $\pi^0 \to e^+e^-\gamma$

The overall NLO correction $\delta(x,y)$ given in percent (Dalitz-plot corrections)

y x	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.99
0.01	2.761	2.714	2.599	2.449	2.273	2.061	1.786	1.402	0.803	-0.357	-5.657
0.02	2.756	2.720	2.622	2.480	2.300	2.073	1.774	1.355	0.703	-0.546	-5.859
0.03	2.669	2.639	2.552	2.419	2.242	2.012	1.704	1.267	0.586	-0.716	-6.125
0.04	2.558	2.531	2.452	2.327	2.155	1.925	1.611	1.164	0.464	-0.874	-6.372
0.05	2.437	2.412	2.340	2.221	2.053	1.824	1.509	1.054	0.341	-1.025	-6.601
0.06	2.311	2.288	2.221	2.108	1.944	1.717	1.400	0.940	0.216	-1.172	-6.815
0.07	2.184	2.163	2.099	1.990	1.830	1.605	1.288	0.824	0.092	-1.315	-7.017
0.08	2.056	2.036	1.975	1.870	1.714	1.491	1.173	0.707	-0.033	-1.455	-7.211
0.09	1.928	1.909	1.851	1.749	1.596	1.374	1.057	0.588	-0.157	-1.593	-7.397
0.10	1.801	1.783	1.726	1.628	1.477	1.257	0.940	0.469	-0.281	-1.729	-7.578
0.15	1.170	1.154	1.105	1.016	0.874	0.661	0.345	-0.131	-0.900	-2.394	-8.424
0.20	0.546	0.532	0.486	0.402	0.266	0.057	-0.258	-0.738	-1.520	-3.048	-9.219
0.25	-0.079	-0.092	-0.135	-0.217	-0.350	-0.556	-0.871	-1.355	-2.148	-3.704	-9.995
0.30	-0.713	-0.726	-0.768	-0.847	-0.978	-1.184	-1.499	-1.988	-2.790	-4.372	-10.77
0.35	-1.366	-1.378	-1.419	-1.497	-1.627	-1.833	-2.149	-2.641	-3.454	-5.058	-11.56
0.40	-2.044	-2.056	-2.097	-2.174	-2.304	-2.509	-2.827	-3.324	-4.146	-5.773	-12.37
0.45	-2.759	-2.771	-2.811	-2.887	-3.017	-3.222	-3.543	-4.044	-4.875	-6.525	-13.22
0.50	-3.521	-3.533	-3.572	-3.648	-3.777	-3.983	-4.306	-4.811	-5.653	-7.324	-14.12
0.55	-4.344	-4.356	-4.395	-4.470	-4.599	-4.806	-5.130	-5.640	-6.492	-8.186	-15.08
0.60	-5.249	-5.261	-5.299	-5.373	-5.501	-5.708	-6.034	-6.549	-7.410	-9.128	-16.12
0.65	-6.262	-6.273	-6.310	-6.383	-6.510	-6.717	-7.044	-7.563	-8.435	-10.18	-17.28
0.70	-7.425	-7.435	-7.470	-7.541	-7.666	-7.871	-8.198	-8.721	-9.603	-11.37	-18.60
0.75	-8.802	-8.811	-8.844	-8.910	-9.031	-9.232	-9.558	-10.08	-10.98	-12.77	-20.14
0.80	-10.51	-10.52	-10.54	-10.60	-10.72	-10.91	-11.23	-11.76	-12.66	-14.49	-22.02
0.85	-12.78	-12.78	-12.80	-12.85	-12.95	-13.13	-13.44	-13.96	-14.86	-16.72	-24.47
0.90	-16.21	-16.21	-16.21	-16.23	-16.29	-16.43	-16.71	-17.21	-18.11	-20.00	-28.00
0.95	-23.17	-23.14	-23.08	-23.01	-22.96	-22.98	-23.14	-23.53	-24.36	-26.26	-34.45
0.99	-54.29	-54.07	-53.44	-52.50	-51.35	-50.15	-49.03	-48.16	-47.76	-48.47	-55.83

Radiative corrections for $\pi^0 \to e^+e^-\gamma$

Results



Radiative corrections for $\pi^0 \to e^+e^-\gamma$

Determination of ratio R

Precise and reliable determination of $R \equiv \frac{\Gamma(\pi^0 \to e^+e^-\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)}$

↪ for small slope and up to NLO radiative corrections

$$R \simeq \frac{\alpha}{\pi} \iint (1 + a_\pi x)^2 (1 + \delta(x,y)) \frac{(1-x)^3}{4x} \left[1 + y^2 + \frac{4m_e^2}{M_\pi^2 x} \right] \mathrm{d}x\mathrm{d}y$$

Conservative estimate for uncertainty (a_π, NNLO): $R = 1.1978(5)(3)\,\%$

↪ chosen $a_\pi^{\text{univ}} = 3.55(70)\,\%$, covers

source	VMD	LMD	THS	dispers.	Padé aps.	NA62	A2	PDG
a_π [%]	3.00	2.45	2.92(4)	3.15(9)	3.21(19)	3.68(57)	3.0(1.0)	3.35(31)
b_π [10^{-3}]	0.90	0.74	0.87(2)	1.14(4)	1.04(22)	×	×	×

Constraint: $1 \simeq \mathcal{B}(\pi^0 \to \gamma\gamma) + \mathcal{B}(\pi^0 \to e^+e^-\gamma(\gamma)) + \mathcal{B}(\pi^0 \to e^+e^-e^+e^-)$

$$\mathcal{B}(\pi^0 \to \gamma\gamma) = 98.8131(6)\,\%\,, \quad \mathcal{B}(\pi^0 \to e^+e^-\gamma(\gamma)) = 1.1836(6)\,\%$$

TH, Goudzovski and Kampf, PRL 122 (2018)

↪ could be used in future exp. analysis of $K^+ \to \pi^+e^+e^-$

PDG

$$R = 1.188(35)\,\%\,, \quad \mathcal{B}(\pi^0 \to \gamma\gamma) = 98.823(34)\,\%\,, \quad \mathcal{B}(\pi^0 \to e^+e^-\gamma) = 1.174(35)\,\%$$

‘Recent’ KTeV measurement

Abouzaid et al., PRD 100 (2019)
↪ based on 1999 data and *E. Abouzaid*, Ph.D. thesis (2007)

$$\frac{\Gamma(\pi^0 \to e^+e^-\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)} = (1.1559 \pm 0.0046 \pm 0.0106)\,\%$$

TH, Goudzovski and Kampf, PRL 122 (2018)
Conservative estimate for uncertainty (a_π, NNLO): $R = 1.1978(5)(3)\,\%$
↪ chosen $a_\pi^{\text{univ}} = 3.55(70)\,\%$

$\Longrightarrow$ $3.6\,\sigma$ discrepancy between theory and experiment

PDG average: $R = 1.188(35)\,\%$
↪ most recent (archived ALEPH data) *Beddall and Beddall*, EPJC 54 (2008)

Need for new measurements
↪ R, improvement on a_π (maybe b_π) welcome

Radiative corrections for $\eta^{(\prime)}$ Dalitz decays

Radiative corrections for $\eta^{(\prime)} \to \ell^+\ell^-\gamma$ decays

What should be taken into account?

naive rad. corrections for $\eta \to e^+e^-\gamma$: *Mikaelian and Smith*, PRD 5 2890 (1972)

- numerical values correspond to simple change $M_{\pi^0} \to M_\eta$
  ↪ π^0 case: *Mikaelian and Smith*, PRD 5 1763 (1972)

$\eta^{(\prime)}$ case compared to π^0

- larger rest mass

  ↪ M_η above muon-pair threshold: $M_\eta > 2m_\mu$
  ↪ $M_{\eta'}$ above lowest-lying resonances: $M_{\eta'} > M_\rho, M_\omega$
    ↪ sensitive to the widths of resonances
    ↪ ω narrow, ρ broad resonance in $\pi\pi$ scattering

- strange-flavor content

  ↪ quark-flavor basis
  Feldmann et al., PLB 449 (1999), *Escribano et al.*, JHEP 06 (2005)

$$j^\ell \equiv \frac{i}{2}\left[\bar{u}\gamma_5 u + \bar{d}\gamma_5 d\right], \quad j^{\mathrm{s}} \equiv \frac{i}{\sqrt{2}}\left[\bar{s}\gamma_5 s\right]$$

- η-η' mixing: $\langle 0|j^A|\eta^B\rangle = B_0 F_\pi f_A \delta^{AB}$, $\langle \eta^A|\eta^B\rangle = \delta^{AB}$, $A, B \in \{\ell, \mathrm{s}\}$

$$|\eta\rangle = \cos\phi\,|\eta^\ell\rangle - \sin\phi\,|\eta^{\mathrm{s}}\rangle$$
$$|\eta'\rangle = \sin\phi\,|\eta^\ell\rangle + \cos\phi\,|\eta^{\mathrm{s}}\rangle$$

Radiative corrections for $\eta^{(\prime)} \to \ell^+\ell^-\gamma$ decays

Novel approach

Full set of NLO QED radiative corrections:
TH, Kampf, Leupold and Novotný, PRD 97 (2018)

- compared to previous approach:
  $\hookrightarrow$ muon loops + hadronic VP
  $\hookrightarrow$ 1γIR at one-loop level
  $\hookrightarrow$ form-factor effects (also in BS)
  $\hookrightarrow$ higher orders in the final-state-lepton mass not neglected
- general framework: three additional processes
  $\hookrightarrow$ also muon decay modes

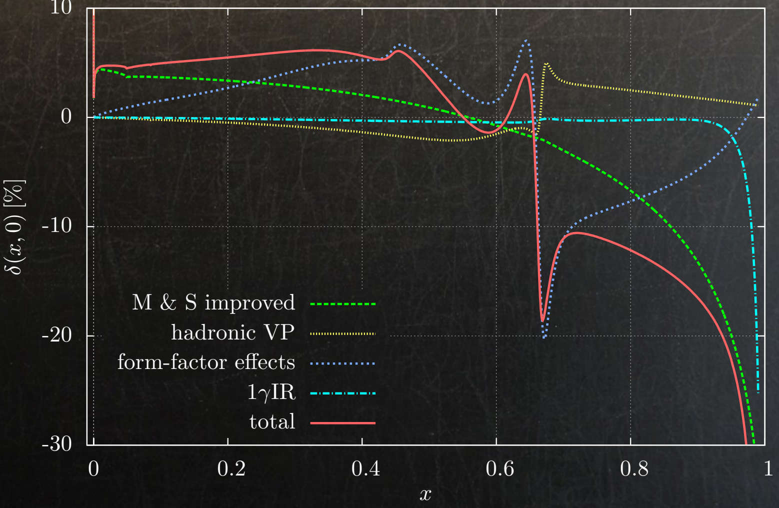
η case: most of the ingredients in *TH, Kampf and Novotný*, PRD 92 (2015)

η' case: real challenge

$\hookrightarrow$ resulting framework also applicable to the π^0 case (numerically compatible)
  $\hookrightarrow$ overkill (correction to the correction of order 1 %)

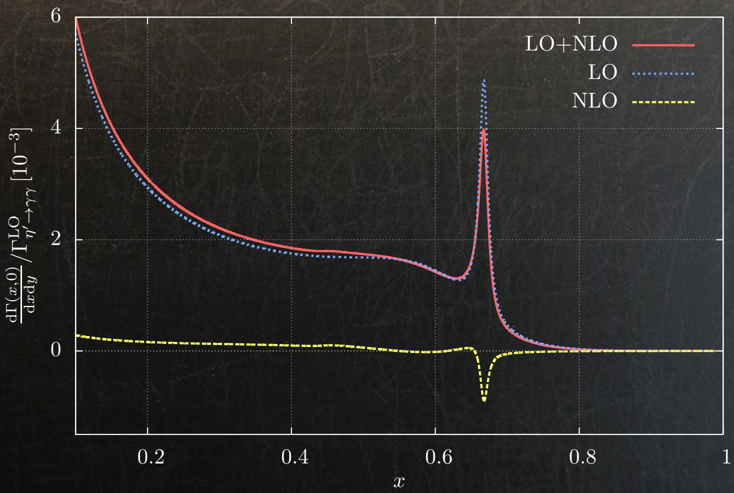
Radiative corrections for $\eta' \to e^+e^-\gamma$ decays

The overall NLO correction $\delta(x,0)$ in comparison to its constituents



Radiative corrections for $\eta' \to e^+e^-\gamma$ decays

The two-fold differential decay width $\mathrm{d}\Gamma(x,0)$ at NLO



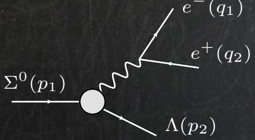
Radiative corrections for Σ^0 Dalitz decay

Dalitz decay of Σ^0

Introduction

Dalitz decay $\Sigma^0 \to \Lambda e^+ e^-$

- e^+e^- invariant mass only up to $M_\Sigma - M_\Lambda \simeq 77\,\text{MeV}$
- provides electric and magnetic transition form factors of $\Sigma^0 \to \Lambda$ transition
- extracting transition radii challenging
    - high-precision measurement required
    - competing with QED radiative corrections



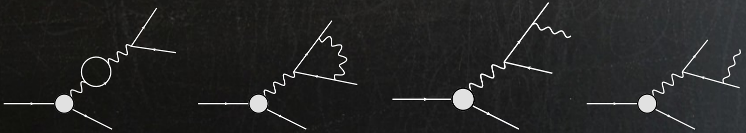
Predictions of electric and magnetic radii

- *Kubis and Meissner*, EPJC 18 (2001)
- *Granados, Leupold and Perotti*, EPJA 53 (2017)

Radiative corrections for $\Sigma^0 \to \Lambda e^+ e^-$

Introduction

Radiative corrections to the differential decay width in soft-photon approximation
$\hookrightarrow$ *Sidhu and Smith*, PRD 4 3344 (1971)

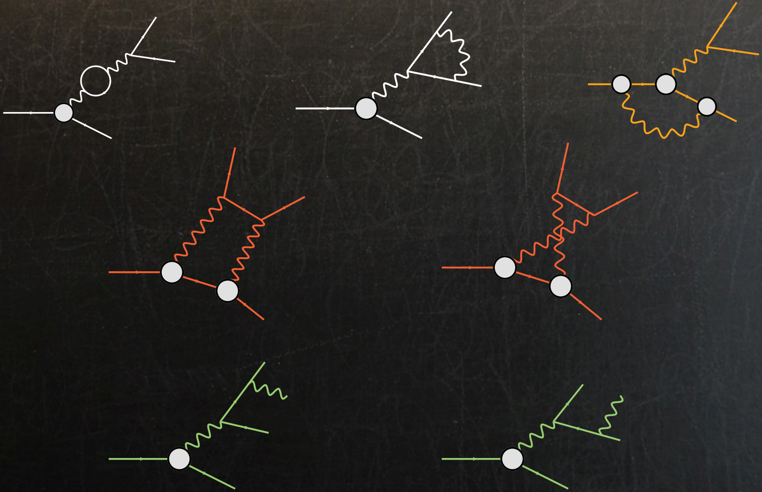


Radiative corrections for $\Sigma^0 \to \Lambda e^+ e^-$

Diagrams

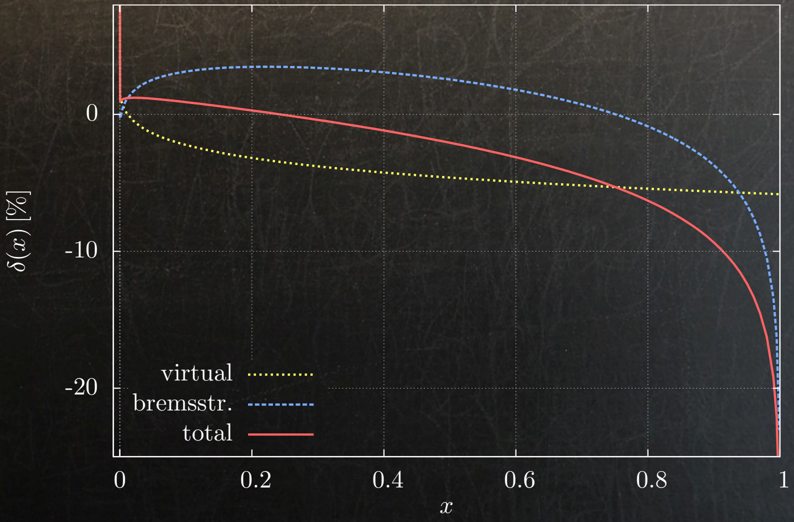
TH and Leupold, EPJC 80 (2020)

$\hookrightarrow$ inclusive radiative corrections beyond the soft-photon approximation



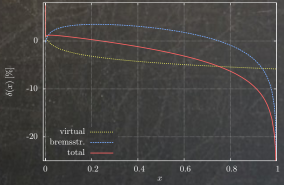
Radiative corrections for $\Sigma^0 \to \Lambda e^+ e^-$

Results



Radiative corrections for $\Sigma^0 \to \Lambda e^+ e^-$

Correction to the form-factor slope



Estimate of size of correction to the magnetic form-factor slope a

- take half of the slope of the curve in the low-x region
- farther from the threshold ($\nu^2 \ll x_0 \ll 1$):

$$\Delta a \equiv a_{(+\mathsf{QED})} - a \simeq \frac{1}{2}\frac{\mathrm{d}\delta(x)}{\mathrm{d}x}\bigg|_{x=x_0}$$

$a_{(+\mathsf{QED})}$: measured value implicitly containing the QED radiative correction

$$\frac{1}{2}\frac{\mathrm{d}\delta(x)}{\mathrm{d}x}\bigg|_{x=x_0} \approx -3.5\,\%$$

- bigger than the estimate on the slope $a \equiv \frac{1}{6}\langle r_{\mathsf{M}}^2\rangle\Delta_M^2$ itself ($a \approx 1.8(3)\,\%$)

Using no radiative corrections in the experimental analysis

- one expects “measured” radius $\langle r_{\mathsf{M}}^2\rangle_{(+\mathsf{QED})}$ to be negative

$$\langle r_{\mathsf{M}}^2\rangle_{(+\mathsf{QED})} = \langle r_{\mathsf{M}}^2\rangle + \frac{6}{\Delta_M^2}\Delta a\,, \quad \text{with } \frac{6}{\Delta_M^2}\Delta a \approx -35\,\mathsf{GeV}^{-2}$$

$(\chi\mathsf{PT}:\ \langle r_{\mathsf{M}}^2\rangle = 18.5(2.6)\,\mathsf{GeV}^{-2})$; in general for hadronic radii: $\langle r^2\rangle \le (1\,\mathsf{fm})^2 \approx 25\,\mathsf{GeV}^{-2}$

Radiative corrections for $\Sigma^0 \to \Lambda e^+ e^-$

Integrated decay width

Integrate over the Dalitz plot (values from *Kubis and Meissner*, EPJC 18 (2001))

$$R \equiv \frac{\Gamma(\Sigma^0 \to \Lambda e^+ e^-)}{\Gamma(\Sigma^0 \to \Lambda \gamma)} = 5.541(2) \times 10^{-3}$$

↪ neglect electric form factor → expansion in magnetic form-factor slope $a \equiv \frac{1}{6}\langle r_{\mathrm{M}}^2 \rangle \Delta_M^2$

$$R = R_0 + aR_1 + \mathcal{O}(a^2)$$

+ higher order corrections as additional uncertainty

$$R = [5.530(3) + 0.626(2)a] \times 10^{-3}$$

↪ consistent with the NLO result in *Sidhu and Smith*, PRD 4 3344 (1971)

$$R_{\mathrm{S\&S}} = (5.532 + 0.627a) \times 10^{-3}\,[\approx 5.544 \times 10^{-3}]$$

From $\mathcal{B}(\Sigma^0 \to \Lambda\gamma) + \mathcal{B}(\Sigma^0 \to \Lambda e^+ e^-) + \mathcal{B}(\Sigma^0 \to \Lambda\gamma\gamma) \simeq 1$

$$\mathcal{B}(\Sigma^0 \to \Lambda\gamma) = [99.4501(3) - 0.0619(2)a]\,\%$$

$$\mathcal{B}(\Sigma^0 \to \Lambda e^+ e^-) = [0.5499(3) + 0.0619(2)a]\,\%$$

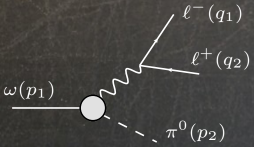
$a = 0.02(2)$: $\mathcal{B}(\Sigma^0 \to \Lambda\gamma) = 99.449(2)\,\%$, $\mathcal{B}(\Sigma^0 \to \Lambda e^+ e^-) = 0.551(2)\,\%$

Radiative corrections for $\omega \to \pi^0 \ell^+ \ell^-$ decays

Radiative corrections for $\omega \to \pi^0 \ell^+ \ell^-$ decays

Dalitz decay $\omega \to \pi^0 \ell^+ \ell^-$

- $\pi\omega V$ correlator measured in the past
  $\hookrightarrow$ NA60: Arnaldi et al., PLB 677 (2009)
  $\hookrightarrow$ A2: Adlarson et al., PRC 95 (2017)
- radiative corrections not included (not available)



Inclusive NLO QED radiative corrections beyond the soft-photon approximation

$$\left|\mathcal{F}_{(+\mathrm{QED})}(q^2)\right|^2 = \left|\mathcal{F}(q^2)\right|^2[1+\delta(x)]$$

Bachelor project: *Jacob Lindahl* (2021)

Radiative corrections for $K^+ \to \pi^+ \ell^+ \ell^-$ decays

Radiative decays $K^+ \to \pi^+\ell^+\ell^-$

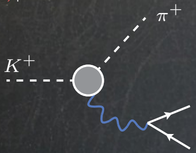
Form-factor parametrization

LO appears at $\mathcal{O}(p^4)$ + unitarity loop correction from $\pi\pi$ rescattering ↓

$\hookrightarrow$ universally used parametrization for the fit: $V_+(x) = a_+ + b_+x + V_+^{\pi\pi}(x)$

$\hookrightarrow$ *Ecker et al.*, NPB 291 (1987), *D'Ambrosio et al.*, JHEP 08 (1998)

$$\frac{\mathrm{d}\Gamma_+}{\mathrm{d}x} = \frac{G_\mathsf{F}^2\alpha^2M_K^5}{3(4\pi)^5}\lambda^{3/2}(x)\sqrt{1-\frac{4r_\ell^2}{x}}\left(1+\frac{2r_\ell^2}{x}\right)|V_+(x)|^2$$



LFU

$\hookrightarrow$ a_+s and b_+s should be the same for both (e and μ) channels

$\hookrightarrow$ discrepancy due to NP via SD effects

Moreover, the ratio deviates significantly from the VMD ansatz

$$\text{VMD:}\quad \frac{b_+}{a_+} = \frac{M_K^2}{M_\rho^2} \approx 0.4\,, \qquad \text{exp.:}\quad \frac{b_+}{a_+} \approx 1.25$$

Measurement of quadratic term c_+x^2 may further test the VMD hypothesis

ℓ	a_+	b_+	exp.
e	$-0.587(10)$	$-0.655(44)$	E865
e	$-0.578(16)$	$-0.779(66)$	NA48/2
μ	$-0.575(39)$	$-0.813(145)$	NA48/2
μ	$-0.575(13)$	$-0.722(43)$	NA62(2022)

← JHEP 11 (2022) 011

improve precision $\longrightarrow$ radiative corrections, studied earlier: *Kubis and Schmidt*, EPJC 70 (2010)

$K^+ \to \pi^+ 4e$ decay

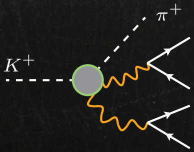
$K^+ \to \pi^+ e^+ e^- e^+ e^-$

Standard Model prediction: Topologies

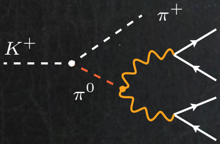
One-photon-exchange topology

(1)

Two-photon-exchange topology



(2a)

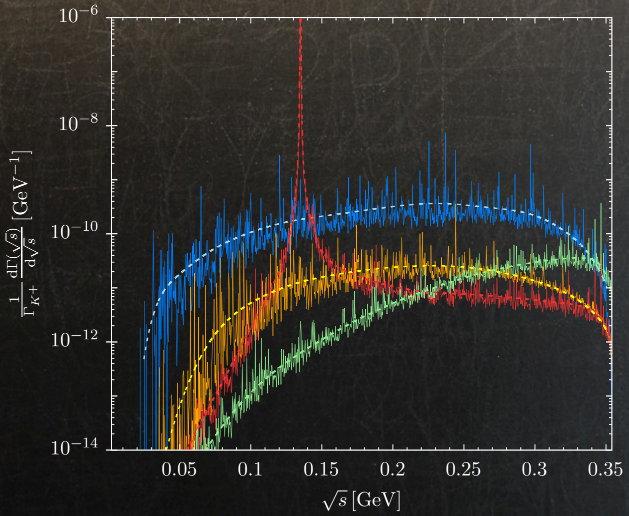


(2b)

TH, PRD 106 (2022)

$K^+ \to \pi^+ e^+ e^- e^+ e^-$

Contributions to the branching ratio



[large MC samples generated by A. Shaikhiev, E. Goudzovski]

$K^+ \to \pi^+ e^+ e^- e^+ e^-$

Contributions to the branching ratio

Branching ratio calculated using Monte Carlo event generator technique:

$$B = \frac{1}{\Gamma_0} \frac{1}{4} \frac{1}{2M_K} \Phi_5 \frac{1}{N} \sum_{N \text{ events}} \overline{|\mathcal{M}|^2}$$

	$B(\sqrt{s} < 120\,\text{MeV})$	$B(\sqrt{s} > 150\,\text{MeV})$	B
(1)	5.60×10^{-12}	5.44×10^{-11}	6.70×10^{-11}
(2a)	3.11×10^{-13}	3.85×10^{-12}	4.60×10^{-12}
(2b)	1.40×10^{-13}	1.97×10^{-12}	$7.0(3) \times 10^{-6}$
κ	7.08×10^{-15}	3.69×10^{-12}	3.72×10^{-12}
$\sum$	$6.1(4) \times 10^{-12}$	$6.0(6) \times 10^{-11}$	$7.2(7) \times 10^{-11}$

$B(K^+ \to \pi^+ 4e) \simeq B(K^+ \to \pi^+ \pi^0) B(\pi^0 \to 4e)$
$\hookrightarrow B(K^+ \to \pi^+ \pi^0) = 20.67(8)\,\%$ and $B(\pi^0 \to 4e) = 3.38(16) \times 10^{-5}$

TH, PRD 106 (2022)

Summary

NLO QED radiative corrections for discussed decays are now available

Meson sector

- $\pi^0 \to e^+e^-$
  Vaško and Novotný, JHEP 1110 (2011)
  TH, Kampf and Novotný, EPJC 74 (2014)
  $\hookrightarrow$ THS model: *TH and S. Leupold*, EPJC 75 (2015)
  $\hookrightarrow$ measure $B(\pi^0 \to e^+e^-)$, extract $\chi^{(r)}(M_\rho)$
- $\pi^0 \to e^+e^-\gamma$
  TH, Kampf and Novotný, PRD 92 (2015)
  $\hookrightarrow$ precise determination of R: *TH, Goudzovski and Kampf*, PRL 122 (2018)
  $\hookrightarrow$ could be used in future exp. analysis of $K^+ \to \pi^+e^+e^-$
- $\eta^{(\prime)} \to \ell^+\ell^-\gamma$
  TH, Kampf, Leupold and Novotný, PRD 97 (2018)
- $K^+ \to \pi^+\ell^+\ell^-$
  TH, in preparation, used in recent NA62 analysis $\longrightarrow$ JHEP 11 (2022) 011
  $\hookrightarrow$ SM estimate of $B(K^+ \to \pi^+4e)$: *TH*, PRD 106 (2022) 7

Baryon sector

- $\Sigma^0 \to \Lambda e^+e^-$
  TH and Leupold, EPJC 80 (2020)

Summary

NLO QED radiative corrections for discussed decays are now available

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Baryon sector

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Thank you for listening!