Connection with $g-2$

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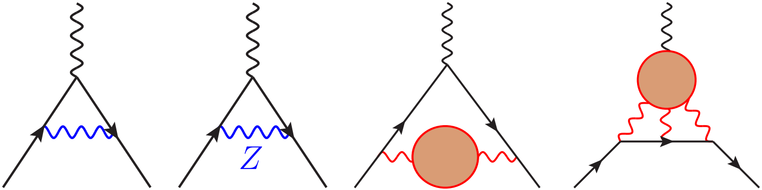
Precision Tests of Fundamental Physics with Light Mesons

ECT* Trento

Anomalous magnetic moments of charged leptons



- **SM prediction for $(g-2)_\ell$**

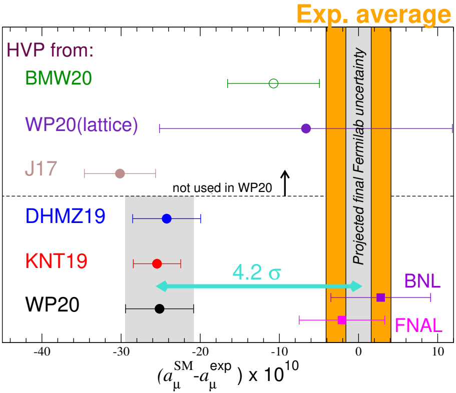
$$a_\ell^{\text{SM}} = a_\ell^{\text{QED}} + a_\ell^{\text{EW}} + a_\ell^{\text{had}} \qquad a_\ell^{\text{had}} = a_\ell^{\text{HVP}} + a_\ell^{\text{HLbL}}$$

- For the muon: by far main uncertainty from the hadronic contributions
- What does this have to do with meson decays?
    - Pseudoscalar poles in HLbL

      $\hookrightarrow$ **transition form factors** talks by Antoine Gérardin, Bai-Long Hoid, Simon Holz, Andrzej Kupść
    - Some connection to HVP

      $\hookrightarrow$ $e^+e^- \to 3\pi, \pi^0\gamma$ and π^0 TFF, $e^+e^- \to \eta\pi\pi, \eta\gamma$ for η TFF
- This talk: mainly overview of HLbL, some interplay with HVP

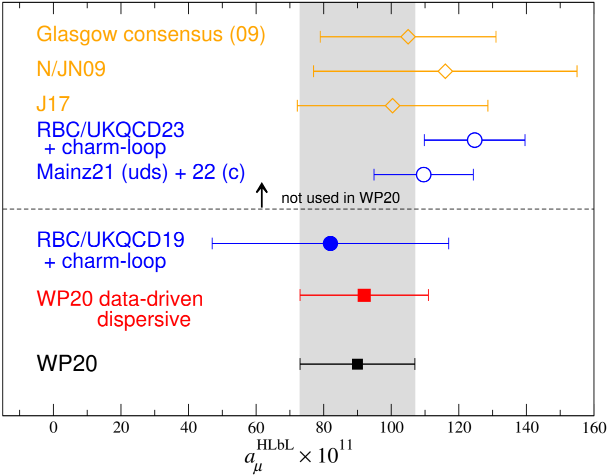
Status of $(g-2)_\mu$: hadronic vacuum polarization



- **Experiment** talk by Saskia Charity
  - BNL confirmed by Fermilab Run 1
  - Run 2+3 in late summer
- **Theory**
  - 4.2σ if HVP from $e^+e^- \to$ hadrons data
  - e^+e^- data in 2.1σ tension with BMWc
  - CMD-3 result for $e^+e^- \to \pi^+\pi^-$

$\hookrightarrow$ Many puzzles in HVP, won't address in this talk

HLbL scattering: status



- Lattice QCD Mainz 2021, 2022:

$$a_\mu^{\text{HLbL}}[uds] = 107(15) \times 10^{-11}$$

$$a_\mu^{\text{HLbL}}[c] = 2.8(5) \times 10^{-11}$$

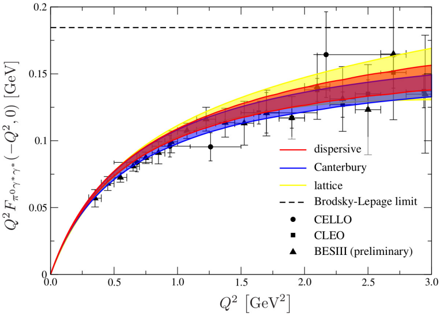
- News from RBC/UKQCD 2023:

$$a_\mu^{\text{HLbL}}[uds] = 122(15) \times 10^{-11}$$

- Good agreement between lattice QCD and phenomenology at $\simeq 20 \times 10^{-11}$
- Need another factor of 2 for final Fermilab precision

HLbL scattering: white paper details

Contribution	PdRV(09)	N/JN(09)	J(17)	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	} − 1(3)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)



- Pion pole from data MH et al. 2018, Masjuan, Sánchez-Puertas 2017 and lattice Gérardin et al. 2019

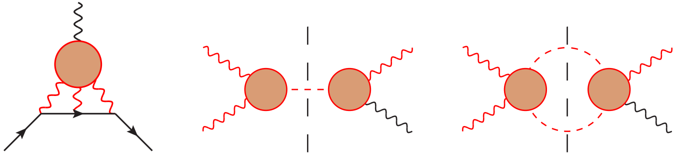
$$a_\mu^{\pi^0\text{-pole}}\Big|_{\text{dispersive}} = 63.0^{+2.7}_{-2.1} \times 10^{-11} \qquad a_\mu^{\pi^0\text{-pole}}\Big|_{\text{Canterbury}} = 63.6(2.7) \times 10^{-11}$$

$$a_\mu^{\pi^0\text{-pole}}\Big|_{\text{lattice+PrimEx}} = 62.3(2.3) \times 10^{-11} \qquad a_\mu^{\pi^0\text{-pole}}\Big|_{\text{lattice}} = 59.7(3.6) \times 10^{-11}$$

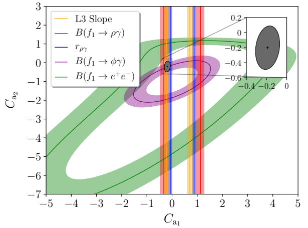
$\hookrightarrow$ agree within uncertainties well below Fermilab goal

- Singly-virtual results agree well with BESIII measurement

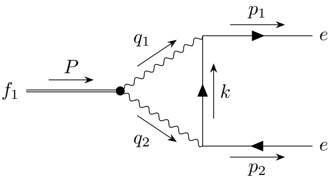
HLbL scattering: data-driven, dispersive evaluations

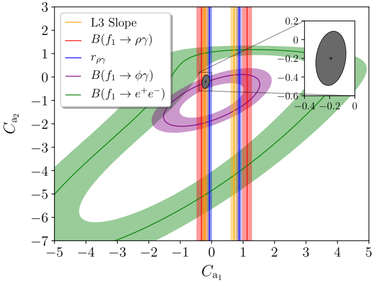


- Organized in terms of **hadronic intermediate states**, in close analogy to HVP Colangelo et al. 2014, ...
- Leading channels implemented with **data input for $\gamma^*\gamma^* \to$ hadrons**, e.g., $\pi^0 \to \gamma^*\gamma^*$
- Progress on dispersive evaluations of **η, η' poles**
  talk by Simon Holz
- Uncertainty dominated by subleading channels
  $\hookrightarrow$ **axial-vector mesons** $f_1(1285)$, $f_1(1420)$, $a_1(1260)$
- Transition form factors accessible in e^+e^- collisions
  $\hookrightarrow$ BESIII, Belle II (?)



Zanke, MH, Kubis 2021





- **Axial-vector TFFs**
  - $e^+e^- \to e^+e^- f_1$ (space-like)
  - $f_1 \to \rho\gamma$, $f_1 \to \phi\gamma$
  - $f_1 \to e^+e^-$
- Result/limit from SND on $f_1 \to e^+e^-$

$$\text{Br}[f_1 \to e^+e^-] = 5.1^{+3.7}_{-2.7} \times 10^{-9} \qquad \text{Br}[f_1 \to e^+e^-] < 9.4 \times 10^{-9} \text{ at 90\% CL}$$

A new source of information on axial-vector TFFs

- Three independent TFFs

  ↪ combined analysis in VMD model so far

- Most information available for f_1

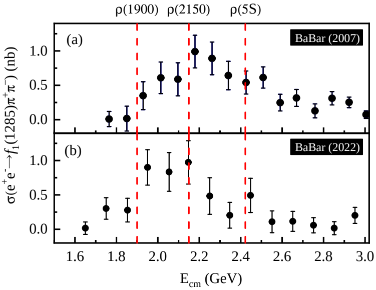
  ↪ f_1' and a_1 from $U(3)$ symmetry

- Also measured: $\boldsymbol{e^+e^- \to f_1\pi\pi}$
  - Constraints on excited ρ resonances
  - Sensitive to all TFFs

  ↪ should provide useful upper bound

- Global analysis of all of this in progress

  MH, Kubis, Zanke in preparation

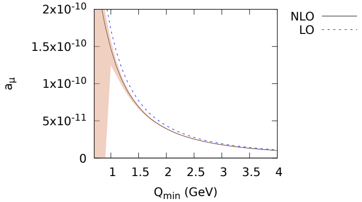


Liu, Zhou, Wang 2022

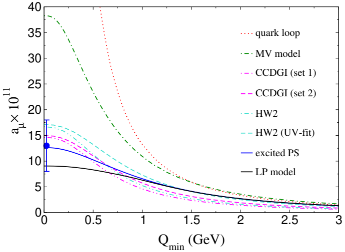
- **Higher-order short-distance constraints**
  - Two-loop α_s corrections
  - Higher-order OPE corrections
  - Higher-order terms in Melnikov–Vainshtein limit
- **Implementation of SDCs**
  - Large-N_c Regge models Colangelo . . .
  - Holographic QCD Leutgeb, Rebhan, Cappiello, . . .
  - Interpolants Lüdtke, Procura

$\hookrightarrow$ reasonable agreement on longitudinal component

- **Transverse component/axial-vectors**
  - SDCs MH, Stoffer 2020
  - Implementation of axial-vectors, new HLbL basis, new dispersive formalism
  - Determination of TFFs



Bijnens, Hermansson-Truedsson, Laub, Rodríguez-Sánchez 2021



Colangelo, Hagelstein, MH, Laub, Stoffer 2021

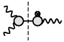
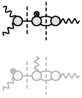
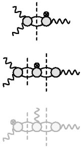
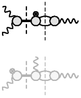
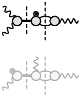
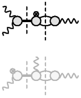
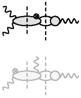
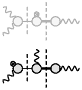
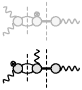
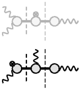
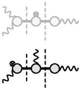
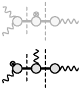
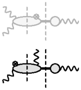
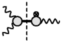
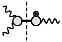
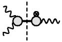
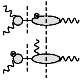
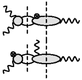
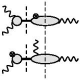
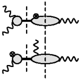
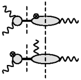
- Recall discussions with MV about the **definition of the pion pole**

$$\frac{F_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2)F_{\pi^0\gamma^*\gamma^*}(q_3^2,0)}{q_3^2-M_\pi^2} \quad \text{vs.} \quad \frac{F_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2)F_{\pi^0\gamma^*\gamma^*}(M_\pi^2,0)}{q_3^2-M_\pi^2}$$

- Comparison in Colangelo, Hagelstein, MH, Laub, Stoffer 2019:
  - First variant: dispersion relation in four-point kinematics
  - Second variant: dispersion relation in $g-2$ ("triangle") kinematics
- Triangle variant looks attractive because of SDCs, but very complicated in low-energy region due to missing $2\pi,\ldots$ cuts
- **Kinematic singularities**
  - Disappear in four-point kinematics only for the entire HLbL tensor due to sum rules
    $\hookrightarrow$ higher partial waves, axial-vectors, tensors
  - For axial-vectors: can find a basis manifestly free of kinematic singularities
    $\hookrightarrow$ ideal for axial-vectors, but need to check other contributions; not possible for tensors

$\hookrightarrow$ complementary information from triangle kinematics Lüdtke, Procura, Stoffer 2023

HLbL dispersion relation in triangle vs. four-point kinematics

	DR in four-point kinematics					
triangle-DR	π^0, η, η'	2π	S	A	T	$\dots$
π^0, η, η'		$\times$	$\times$	$\times$	$\times$	$\times$
2π	$\times$ 		$\times$ 	$\times$ 	$\times$ 	$\times$ 
V						
S	$\times$	$\times$		$\times$	$\times$	$\times$
A	$\times$	$\times$	$\times$		$\times$	$\times$
T	$\times$	$\times$	$\times$	$\times$		$\times$
$\dots$						$\dots$

Lüdtke, Procura, Stoffer 2023

ρ–ω mixing in $e^+e^- \to 3\pi$

- Cross check between lattice QCD and $e^+e^- \to$ hadrons

  $\hookrightarrow$ **isospin-breaking corrections**

- Signal claimed by BaBar 2021 based on sum of Breit–Wigner functions

  $\hookrightarrow a_\mu^{3\pi}[\rho\text{–}\omega] \simeq -0.6 \times 10^{-10}$

- Check with dispersive parameterization
  - 3π rescattering in $\gamma^* \to 3\pi$ via Khuri–Treiman equations
  - Information on ω, ϕ in normalization function $a(q^2)$
  - Same formalism used for π^0 TFF
  - How to ensure consistency between $e^+e^- \to 2\pi, 3\pi$ and not spoil analytic properties?

ρ–ω mixing in $e^+e^- \to 3\pi$

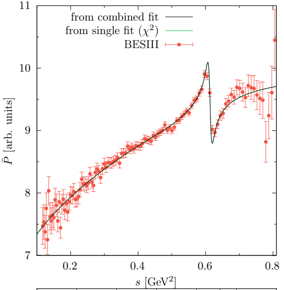
- A coupled-channel system for $\{2\pi, \ell^+\ell^-, 3\pi\}$
  Holz, Hanhart, MH, Kubis 2022
- Developed for consistent description of $\eta' \to \pi\pi\gamma, \ell^+\ell^-\gamma$
  $\hookrightarrow \eta'$ transition form factor and HLbL
- $\epsilon_{\rho\omega}$ now consistent

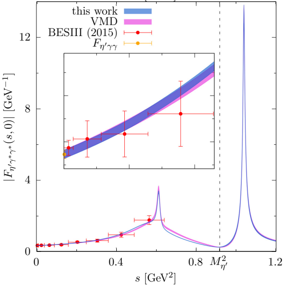
$$\text{Re}\,\epsilon_{\rho\omega}\big|_{e^+e^-\to 2\pi} = 1.97(3)\times 10^{-3}$$
$$\epsilon_{\rho\omega}\big|_{\eta'\to\pi\pi\gamma} = 2.00(7)\times 10^{-3}$$

- By-product: ρ–ω mixing in $e^+e^- \to 3\pi$ should enter as

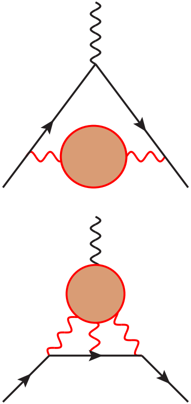
$$1+\epsilon_{\rho\omega} g_{\omega\gamma}^2 \frac{s}{48\pi^2}\int_{4M_\pi^2}^\infty ds' \frac{\left(1-\frac{4M_\pi^2}{s'}\right)^{3/2}|F_\pi^V(s')|^2}{s'(s'-s-i\epsilon)}$$

- Preliminary results:
  - BaBar fit improves significantly
  - $\epsilon_{\rho\omega}$ (largely) consistent with $e^+e^- \to 2\pi$
  - $a_\mu^{3\pi}[\rho$–$\omega]$ sizable (and negative)



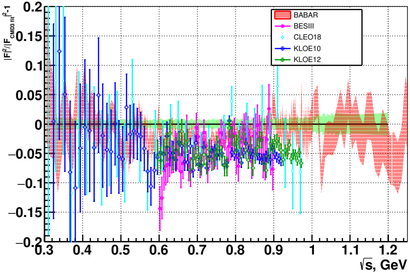


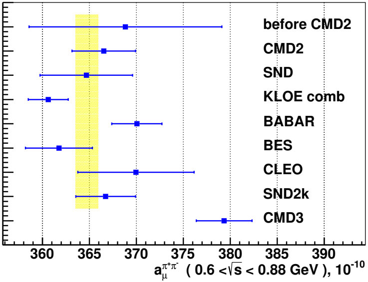
- **Muon $g-2$ and rare decays**:
  - For HLbL agreement between lattice and phenomenology
    $\hookrightarrow$ another factor 2 looks feasible
  - Improvements for η, η' TFFs to establish agreement at same level as for pion pole
  - TFFs also probed in di-lepton decays
  - Some lessons transfer to axial-vector decays
  - WP update in preparation, with CMD-3 timeline unclear, but still aimed for 2023
    $\hookrightarrow$ will include update for HLbL



Sixth plenary TI workshop

A new puzzle: $e^+e^- \to \pi^+\pi^-$ from CMD-3

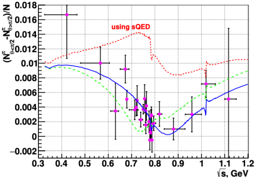




CMD-3, 2302.08834

generally shows larger pion form factor in the whole energy range under discussion. The most significant difference to other energy scan measurements, including previous CMD-2 measurement, is observed at the left side of ρ-meson ($\sqrt{s} = 0.6 - 0.75$ GeV), where it reach up to 5%, well beyond the combined systematic and statistical errors of the new and previous results. The source of this difference is unknown at the moment.

Discrepancy with Calculation of Radiative Corrections



Measured forward-backward asymmetry in e^+e $^+$ disagrees with standard sQED code

https://indico.cern.ch/event/1204084 CMD-3 Collaboration, arXiv:2302.08834

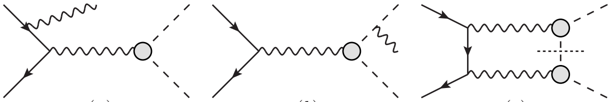
John Ellis, "The future of particle physics", ALPS 2023

The charge asymmetry in the $\pi^+\pi^-$ final state was extracted using forward-backward parts of measured cross sections, and the strong deviation was observed from the prediction based on the conventional sQED approach for radiative correction calculations. The improved GVMD model was proposed in the paper [49], which gives the remarkable agreement with the experimental data. The significant corrections beyond sQED was also confirmed by the calculation in a dispersive formalism in the paper [50]. It will be still interesting to understand the difference in C-odd radiative correction between obtained in the dispersive formalism and the GVMD model prediction, which is sensed by the experimental statistical precision. The obtained result shows the importance of the appropriate choice of the model for the calculation of the radiative corrections for the $\pi^+\pi^-$ channel. It is important to revise the possible effect of sQED limitations for other calculations including two photon exchange processes. The observed difference in charge asymmetries for $\pi^+\pi^-$ and e^+e^- events between the measured value and predicted are $\delta A^{\pi^+\pi^-} = -0.00029 \pm 0.00023$ and $\delta A^{e^+e^-} = -0.00060 \pm 0.00026$, averaged over $\sqrt{s} = 0.7 \div 0.82$ GeV energy range. This consistency better than 0.1% should additionally ensure our θ angle related systematic uncertainty estimation for the $|F_\pi|^2$ measurement.

- **Forward–backward asymmetry**:

$$A_{\text{FB}}(z) = \frac{\frac{d\sigma}{dz}(z) - \frac{d\sigma}{dz}(-z)}{\frac{d\sigma}{dz}(z) + \frac{d\sigma}{dz}(-z)} \qquad \frac{d\sigma}{dz}\bigg|_{C\text{-odd}} = \frac{d\sigma_0}{dz}\Big[\delta_{\text{soft}}(\lambda^2, \Delta) + \delta_{\text{virt}}(\lambda^2)\Big] + \frac{d\sigma}{dz}\bigg|_{\text{hard}}(\Delta)$$

Radiative corrections: forward–backward asymmetry



(a) (b) (c)

- δ_{soft} in point-like approximation for final-state photon in (b), but pion VFF always included otherwise

  $\hookrightarrow$ **FsQED**

- Previously, (c) evaluated in sQED, not FsQED

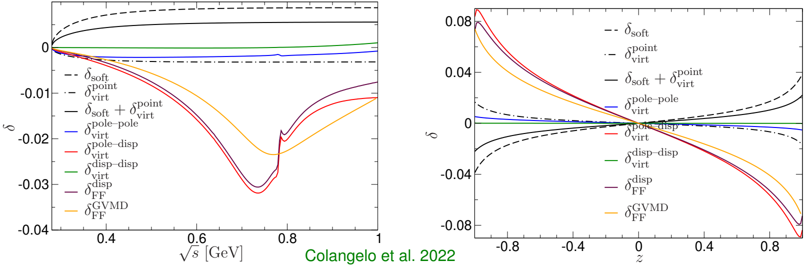
  $\hookrightarrow$ CMD-3 use generalized vector meson dominance instead Ignatov, Lee 2022

- Problem: unphysical imaginary parts below 2π threshold in loop integral

- Our approach: use **dispersive representation of pion VFF**

$$\frac{F_\pi^V(s)}{s} = \frac{1}{s} + \frac{1}{\pi}\int_{4M_\pi^2}^\infty ds' \frac{\text{Im}\, F_\pi^V(s')}{s'(s'-s)} \to \frac{1}{s-\lambda^2} - \frac{1}{\pi}\int_{4M_\pi^2}^\infty ds' \frac{\text{Im}\, F_\pi^V(s')}{s'} \frac{1}{s-s'}$$

$\hookrightarrow$ captures all the structure-dependent, infrared-enhanced effects



- Actually good agreement between dispersive formulation and GVMD!

  $\hookrightarrow$ why do the unphysical imaginary parts not matter more?

- FsQED describes the data well, actually confirms common lore

- Are there relevant effects being missed in the *C*-even contributions?

CMD-3 with dispersive constraints

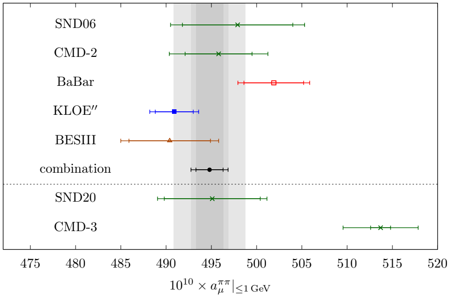
The pion form factor from dispersion relations

$$F_\pi^V(s) = \underbrace{\Omega_1^1(s)}_{\text{elastic } \pi\pi \text{ scattering}} \times \underbrace{G_\omega(s)}_{\text{isospin-breaking } 3\pi \text{ cut}} \times \underbrace{G_{\text{in}}(s)}_{\text{inelastic effects: } 4\pi, \ldots}$$

- $e^+e^- \to \pi^+\pi^-$ cross section subject to strong constraints from **analyticity**, **unitarity**, **crossing symmetry**, leading to dispersive representation with few parameters Colangelo, MH, Stoffer, 2018, 2021, 2022, work in progress
  - **Elastic $\pi\pi$ scattering**: two values of phase shifts
  - **ρ–ω mixing**: ω pole parameters and residue
  - **Inelastic states**: conformal polynomial

$\hookrightarrow$ cross check on data, functional form for all $s \leq 1\,\text{GeV}^2$

CMD-3 with dispersive constraints

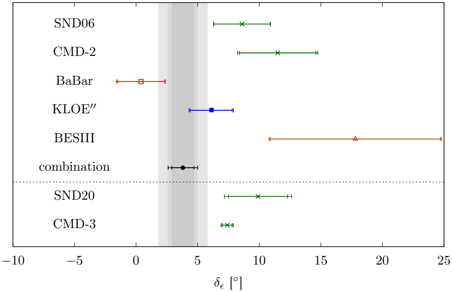


	$a_\mu^{\pi\pi}\big\vert_{\leq 1\text{ GeV}}$	$a_\mu^{\pi\pi}\big\vert_{[0.60,0.88]\text{ GeV}}$	$a_\mu^{\pi\pi}\big\vert_{\text{win}}$
SND06	1.7σ	1.8σ	1.7σ
CMD-2	2.0σ	2.3σ	2.1σ
BaBar	2.9σ	3.3σ	3.1σ
KLOE″	4.8σ	5.6σ	5.4σ
BESIII	2.8σ	3.0σ	3.1σ
SND20	2.1σ	2.2σ	2.2σ
comb	3.7σ [5.0σ]	4.2σ [6.1σ]	3.8σ [5.7σ]

- Tensions in $a_\mu^{\pi\pi}\big|_{\leq 1\text{ GeV}}$ compared to CMD-3:
  - Inner/outer error: experiment/total (also shown: combination + BaBar/KLOE error)
  - Theory error dominated by order in conformal polynomial N
- No red flags for CMD-3 so far, but:
  - Large systematic error from N, correlated/anticorrelated for BaBar/other experiments
  - $\pi\pi$ phase shifts remain reasonable, main change in conformal polynomial

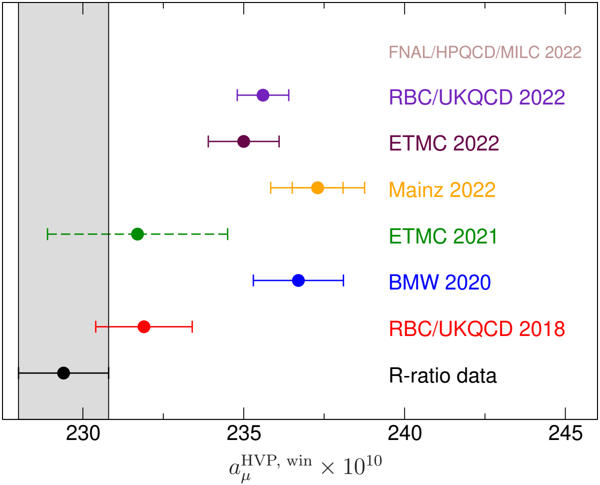
$\hookrightarrow$ further constraints from inelastic channels, $e^+e^- \to 4\pi, \pi\omega, \ldots$?

Phase of the ρ–ω mixing parameter



- Can also study consistency of hadronic parameters
  $\hookrightarrow$ **phase of the ρ–ω mixing parameter δ_ϵ**
  - δ_ϵ observable, since defined as a phase of a residue
  - δ_ϵ vanishes in isospin limit, but can be non-vanishing due to $\rho \to \pi^0\gamma, \eta\gamma, \pi\pi\gamma, \ldots \to \omega$
  - Combined-fit $\delta_\epsilon = 3.8(2.0)[1.2]^\circ$ agrees well with narrow-width expectation $\delta_\epsilon = 3.5(1.0)^\circ$, but **considerable spread among experiments**
  - Mass of the ω systematically too low compared to $e^+e^- \to 3\pi$

On to the next puzzle: e^+e^- vs. lattice QCD in the intermediate window



RBC/UKQCD 2022 supersedes RBC/UKQCD 2018

ETMC 2022 supersedes ETMC 2021

FNAL/HPQCD/MILC 2022 agrees for *ud* connected contribution, same for Aubin et al. 2022, χQCD 2022

R-ratio result from Colangelo et al. 2022

Role of isospin breaking: phenomenological estimates

	SD window		int window		LD window		full HVP	
	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$
$\pi^0\gamma$	0.16(0)	–	1.52(2)	–	2.70(4)	–	4.38(6)	–
$\eta\gamma$	0.05(0)	–	0.34(1)	–	0.31(1)	–	0.70(2)	–
ρ–ω mixing	–	0.05(0)	–	0.83(6)	–	2.79(11)	–	3.68(17)
FSR (2π)	0.11(0)	–	1.17(1)	–	3.14(3)	–	4.42(4)	–
M_{π^0} vs. $M_{\pi^\pm}$ (2π)	0.04(1)	–	−0.09(7)	–	−7.62(14)	–	−7.67(22)	–
FSR (K^+K^-)	0.07(0)	–	0.39(2)	–	0.29(2)	–	0.75(4)	–
kaon mass (K^+K^-)	−0.29(1)	0.44(2)	−1.71(9)	2.63(14)	−1.24(6)	1.91(10)	−3.24(17)	4.98(26)
kaon mass ($\bar{K}^0K^0$)	0.00(0)	−0.41(2)	−0.01(0)	−2.44(12)	−0.01(0)	−1.78(9)	−0.02(0)	−4.62(23)
total	0.14(1)	0.08(3)	1.61(12)	1.02(20)	−2.44(16)	2.92(17)	−0.68(29)	4.04(39)
BMWc 2020	–	–	−0.09(6)	0.52(4)	–	–	−1.5(6)	1.9(1.2)
RBC/UKQCD 2018	–	–	0.0(2)	0.1(3)	–	–	−1.0(6.6)	10.6(8.0)
JLM 2021	–	–	–	–	–	–	–	3.32(89)

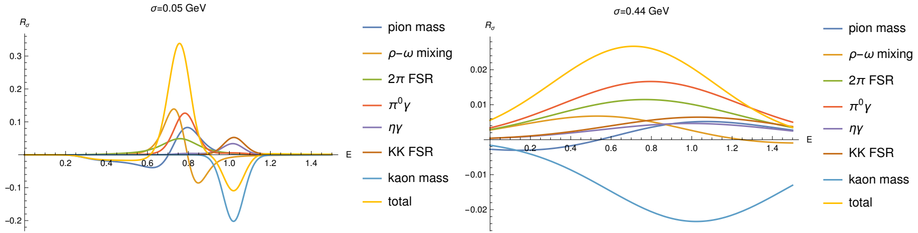
- Reasonable agreement with BMWc 2020, RBC/UKQCD 2018, and James, Lewis, Maltman 2021

  ↪ if anything, the result would become even larger with pheno estimates

- Adding 3π (FSR and ρ–ω mixing) will remove tension in $\mathcal{O}(\delta)$

- **Cancellation of individually sizable corrections!**

Role of isospin breaking: energy dependence



- Alternative to windows: **Gaussian smearing** ETMC 2022

$$R_\sigma(s) = \int_0^\infty ds' \, G_\sigma(\sqrt{s'} - \sqrt{s}) R(s') \qquad G_\sigma(\omega) = \frac{e^{-\omega^2/(2\sigma^2)}}{\sqrt{2\pi\sigma^2}}$$

- Cancellation for a_μ seems to involve a delicate balance with kernel $K(s)$
- Question: Is Gaussian smearing (expected to be) advantageous compared to linear combinations of windows? The inverse Laplace problem should persist . . .