

# A theoretical analysis of the semileptonic decays

$$\eta^{(\prime)} \rightarrow \pi^0 \ell^+ \ell^- \text{ and } \eta' \rightarrow \eta \ell^+ \ell^-$$

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Presentation based on:

- Escribano, Royo,  
*Eur. Phys. J. C* (2020) 80: 1190, [arXiv:2007.12467](https://arxiv.org/abs/2007.12467)
- Escribano, Royo, Sanchez-Puertas,  
*JHEP* 05 (2022) 147, [arXiv:2202.04886](https://arxiv.org/abs/2202.04886)



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de Barcelona**



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d'Altes Energies**

# Outline

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1. Motivation
2. SM Calculations & Results
3. Potential  $CP$  Violation
4. Summary

# 1. Motivation

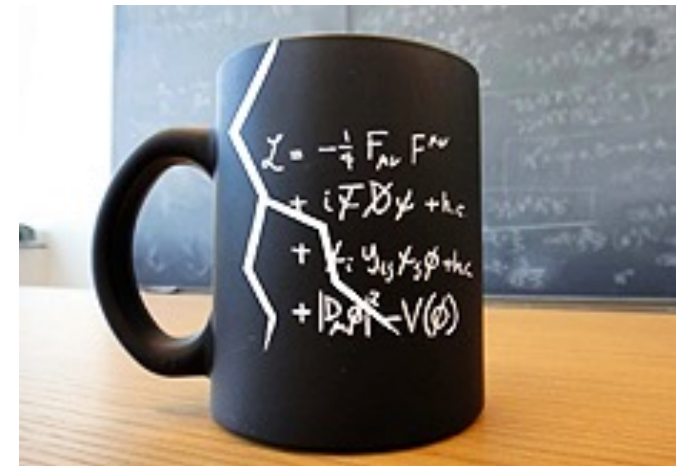
- Standard Model of particle physics is *too* successful!



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I	II	III	
2.4 MeV/c <sup>2</sup> $\frac{2}{3}$ <b>u</b> u-kvark	1.27 GeV/c <sup>2</sup> $\frac{2}{3}$ <b>c</b> c-kvark	171.2 GeV/c <sup>2</sup> $\frac{2}{3}$ <b>t</b> t-kvark	0 0 1 <b>γ</b> footon
4.8 MeV/c <sup>2</sup> $-\frac{1}{3}$ <b>d</b> d-kvark	104 MeV/c <sup>2</sup> $-\frac{1}{3}$ <b>s</b> s-kvark	4.2 GeV/c <sup>2</sup> $-\frac{1}{3}$ <b>b</b> b-kvark	0 0 1 <b>g</b> gluon
<2.2 eV/c <sup>2</sup> 0 $\frac{1}{2}$ <b>ν<sub>e</sub></b> elektron-neutriino	<0.17 MeV/c <sup>2</sup> 0 $\frac{1}{2}$ <b>ν<sub>μ</sub></b> müu-neutriino	<15.5 MeV/c <sup>2</sup> 0 $\frac{1}{2}$ <b>ν<sub>τ</sub></b> tau-neutriino	91.2 GeV/c <sup>2</sup> 0 1 <b>Z<sup>0</sup></b> Z-boson
0.511 MeV/c <sup>2</sup> -1 $\frac{1}{2}$ <b>e</b> elektron	105.7 MeV/c <sup>2</sup> -1 $\frac{1}{2}$ <b>μ</b> müuon	1.777 GeV/c <sup>2</sup> -1 $\frac{1}{2}$ <b>τ</b> taun	80.4 GeV/c <sup>2</sup> $\pm 1$ 1 <b>W<sup>±</sup></b> W-boson

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# 1. Motivation

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- A number of low-energy precision measurements are sensitive to BSM Physics
  - SM prediction for the measured quantity is precisely known
  - SM background is small
- Experiments currently underway
  - Muon  $g - 2$
  - nEDMs
  - Neutron decays
  - Etc.

# 1. Motivation

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- The  $\eta$  and  $\eta'$  mesons are special:
  - The  $\eta$  is a pseudo-Goldstone boson
  - The  $\eta'$  is largely influenced by the  $U(1)_A$  anomaly
  - The  $\eta$  and  $\eta'$  are eigenstates of the  $C$ ,  $P$ ,  $CP$  and  $G$  operators:  $I^G J^{PC} = 0^+ 0^{-+}$
  - All their additive quantum numbers are zero: flavour conserving decays
  - All their strong and EM decays are forbidden at lowest order
  - Decays are mostly free from SM background



**Perfect laboratory to stress-test the SM in search of physics BSM**

# 2. Calculations

*Eur. Phys. J. C* (2020) 80: 1190, [arXiv:2007.12467](https://arxiv.org/abs/2007.12467)

- The semileptonic decays  $\eta^{(\prime)} \rightarrow \pi^0 \ell^+ \ell^-$  and  $\eta' \rightarrow \eta \ell^+ \ell^-$  ( $\ell = e$  or  $\mu$ ) can be used as fine probes to assess physics BSM.
  - SM contributes through the  $C$ -conserving exchange of two photons that is highly suppressed (no contribution at tree-level, only corrections at one-loop and higher orders)
- Latest theoretical estimations for  $\eta \rightarrow \pi^0 \ell^+ \ell^-$  date back to the 90s
- No theoretical studies for  $\eta' \rightarrow \pi^0 \ell^+ \ell^-$  or  $\eta' \rightarrow \eta \ell^+ \ell^-$  to the best of our knowledge

# 2. Calculations

*Eur. Phys. J. C* (2020) 80: 1190, [arXiv:2007.12467](https://arxiv.org/abs/2007.12467)

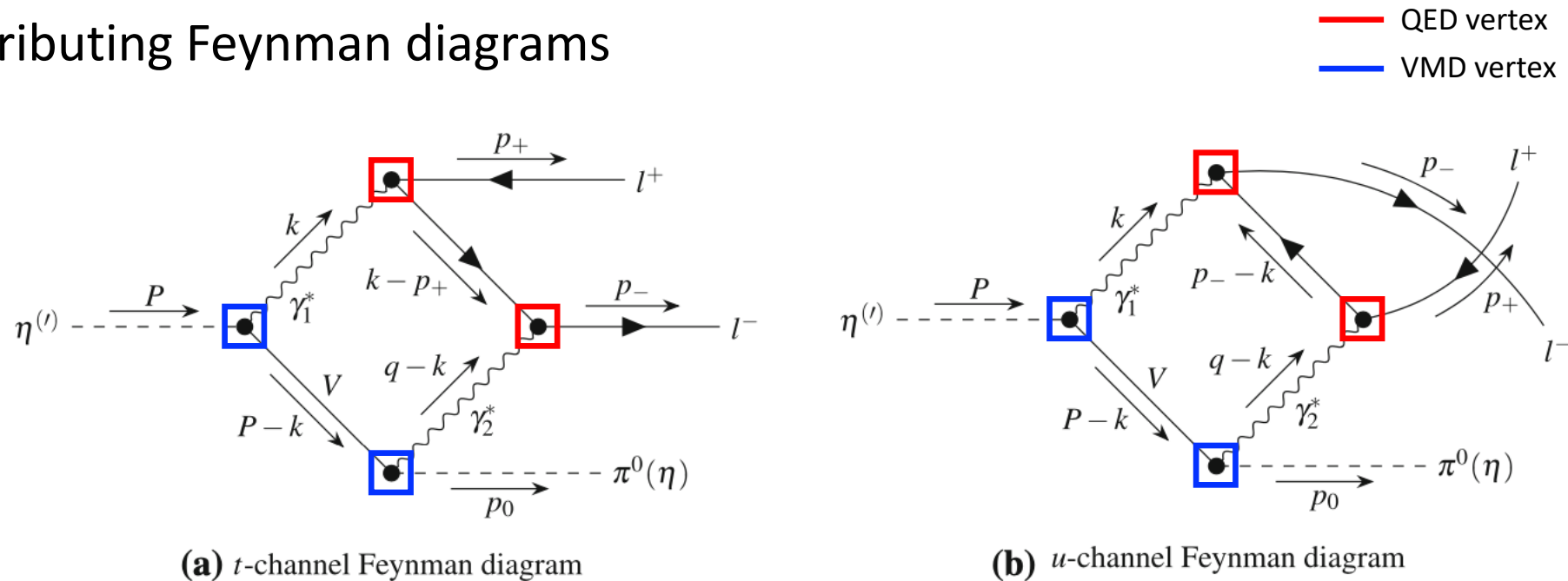
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- Calculations performed within the Vector Meson Dominance (VMD) framework
  - Decay processes dominated by the exchange of vector resonances
- VMD coupling constants parametrised using an existing phenomenological model
  - Numerical values obtained from an optimisation fit to  $V \rightarrow P\gamma$  and  $P \rightarrow V\gamma$  radiative decays ( $V = \rho^0, \omega, \phi$  and  $P = \pi^0, \eta, \eta'$ )
  - See *Phys. Lett. B* 807 (2020) 135534, [arXiv:2003.08379](https://arxiv.org/abs/2003.08379) for details

# 2. Calculations

*Eur. Phys. J. C* (2020) 80: 1190, [arXiv:2007.12467](https://arxiv.org/abs/2007.12467)

- Contributing Feynman diagrams



**Fig. 1** Feynman diagrams contributing to the *C*-conserving semileptonic decays  $\eta^{(\prime)} \rightarrow \pi^0 l^+ l^-$  and  $\eta' \rightarrow \eta l^+ l^-$  ( $l = e$  or  $\mu$ ). Note that  $q = p_+ + p_-$  and  $V = \rho^0, \omega, \phi$



# 2. Calculations

*Eur. Phys. J. C* (2020) 80: 1190, [arXiv:2007.12467](https://arxiv.org/abs/2007.12467)

- $VP\gamma$  interaction amplitude consistent with Lorentz,  $C$ ,  $P$  and EM gauge invariance can be written as

$$\mathcal{M}(V \rightarrow P\gamma) = g_{VP\gamma} \epsilon_{\mu\nu\alpha\beta} \epsilon_{(V)}^\mu p_V^\nu \epsilon_{(\gamma)}^{*\alpha} q^\beta \hat{F}_{VP\gamma}(q^2),$$

where  $g_{VP\gamma}$  is the coupling constant for the  $VP\gamma$  transition involving on-shell photons,  $\epsilon_{\mu\nu\alpha\beta}$  is the totally antisymmetric Levi-Civita tensor,  $\epsilon_{(V)}$  and  $p_V$  are the polarisation and 4-momentum vectors of the initial  $V$ ,  $\epsilon_{(\gamma)}^*$  and  $q$  are the corresponding ones for the final  $\gamma$ , and  $\hat{F}_{VP\gamma}(q^2) \equiv F_{VP\gamma}(q^2)/F_{VP\gamma}(0)$  is a normalised form factor to account for off-shell photons mediating the transition

# 2. Calculations

*Eur. Phys. J. C* (2020) 80: 1190, [arXiv:2007.12467](https://arxiv.org/abs/2007.12467)

- Invariant decay amplitude

$$\mathcal{M} = ie^2 \sum_{V=\rho^0, \omega, \phi} g_{V\eta^{(\prime)}\gamma} g_{V\pi^0(\eta)\gamma} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + i\epsilon} \frac{1}{(k-q)^2 + i\epsilon} \epsilon^{\mu\nu\alpha\beta} \left[ \frac{k^\mu (P-k)^\alpha (k-q)^\rho (P-k)^\delta}{(P-k)^2 - m_V^2 + i\epsilon} \right] \epsilon_{\rho\sigma\delta}{}^\beta$$

$$\bar{u}(p_-) \left[ \gamma^\sigma \frac{\not{k} - \not{p}_+ + m_l}{(k-p_+)^2 - m_l^2 + i\epsilon} \gamma^\nu + \gamma^\nu \frac{\not{p}_- - \not{k} + m_l}{(k-p_-)^2 - m_l^2 + i\epsilon} \gamma^\sigma \right] v(p_+),$$

where  $q = p_+ + p_-$  is the sum of lepton-antilepton pair 4-momenta,  $e$  is the electron charge, and  $g_{V\eta^{(\prime)}\gamma}$  and  $g_{V\pi^0(\eta)\gamma}$  are the corresponding VMD coupling constants

$$\mathcal{M} = \sum_{V=\rho^0, \omega, \phi} \mathcal{M}_{1V} + \mathcal{M}_{2V} = \overbrace{\Omega [\bar{u}(p_-) \not{P} v(p_+)]}^{\text{Vector current}} + \overbrace{m_l \Sigma [\bar{u}(p_-) v(p_+)]}^{\text{Scalar current}}$$

# 2. Calculations

*Eur. Phys. J. C* (2020) 80: 1190, [arXiv:2007.12467](https://arxiv.org/abs/2007.12467)

with

$$\begin{aligned}\Omega &= \sum_{V=\rho^0,\omega,\phi} \alpha_V + \sigma_V, & \alpha_V &= e^2 \frac{g_{V\eta^{(\prime)}} g_{V\pi^0} \gamma}{16\pi^2} \int dx dy dz \left[ \frac{2A_1}{\Delta_{1V} - i\epsilon} - \frac{B_1}{(\Delta_{1V} - i\epsilon)^2} \right] \\ \Sigma &= \sum_{V=\rho^0,\omega,\phi} \beta_V + \tau_V, & \beta_V &= e^2 \frac{g_{V\eta^{(\prime)}} g_{V\pi^0} \gamma}{16\pi^2} \int dx dy dz \left[ \frac{2C_1}{\Delta_{1V} - i\epsilon} - \frac{D_1}{(\Delta_{1V} - i\epsilon)^2} \right] \\ & & \sigma_V &= e^2 \frac{g_{V\eta^{(\prime)}} g_{V\pi^0} \gamma}{16\pi^2} \int dx dy dz \left[ \frac{2A_2}{\Delta_{2V} - i\epsilon} - \frac{B_2}{(\Delta_{2V} - i\epsilon)^2} \right] \\ & & \tau_V &= e^2 \frac{g_{V\eta^{(\prime)}} g_{V\pi^0} \gamma}{16\pi^2} \int dx dy dz \left[ \frac{2C_2}{\Delta_{2V} - i\epsilon} - \frac{D_2}{(\Delta_{2V} - i\epsilon)^2} \right]\end{aligned}$$

# 2. Calculations

*Phys. Lett. B* 807 (2020) 135534, [arXiv:2003.08379](https://arxiv.org/abs/2003.08379)

- VMD couplings parametrisation

$$g_{\rho^0\pi^0\gamma} = \frac{1}{3}g ,$$

$$g_{\rho^0\eta\gamma} = g z_{\text{NS}} \cos \phi_P ,$$

$$g_{\rho^0\eta'\gamma} = g z_{\text{NS}} \sin \phi_P ,$$

$$g_{\omega\pi^0\gamma} = g \cos \phi_V ,$$

$$g_{\phi\pi^0\gamma} = g \sin \phi_V ,$$

$$g_{\omega\eta\gamma} = \frac{1}{3}g \left( z_{\text{NS}} \cos \phi_P \cos \phi_V - 2 \frac{\bar{m}}{m_s} z_{\text{S}} \sin \phi_P \sin \phi_V \right) ,$$

$$g_{\omega\eta'\gamma} = \frac{1}{3}g \left( z_{\text{NS}} \sin \phi_P \cos \phi_V + 2 \frac{\bar{m}}{m_s} z_{\text{S}} \cos \phi_P \sin \phi_V \right) ,$$

$$g_{\phi\eta\gamma} = \frac{1}{3}g \left( z_{\text{NS}} \cos \phi_P \sin \phi_V + 2 \frac{\bar{m}}{m_s} z_{\text{S}} \sin \phi_P \cos \phi_V \right) ,$$

$$g_{\phi\eta'\gamma} = \frac{1}{3}g \left( z_{\text{NS}} \sin \phi_P \sin \phi_V - 2 \frac{\bar{m}}{m_s} z_{\text{S}} \cos \phi_P \cos \phi_V \right) ,$$

where  $g$  is a generic electromagnetic constant,  $\phi_P$  is the pseudoscalar  $\eta$ - $\eta'$  mixing angle in the quark-flavour basis,  $\phi_V$  is the vector  $\omega$ - $\phi$  mixing angle in the same basis,  $\bar{m}/m_s$  is the quotient of constituent quark masses, and  $z_{\text{NS}}$  and  $z_{\text{S}}$  are the *non-strange* and *strange* multiplicative factors accounting for the relative meson wavefunction overlaps

# 2. Calculations

*Phys. Lett. B* 807 (2020) 135534, [arXiv:2003.08379](https://arxiv.org/abs/2003.08379)

- Numerical values from optimisation fit to  $VP\gamma$  radiative decays

$$g = 0.70 \pm 0.01 \text{ GeV}^{-1}, \quad z_S \bar{m} / m_s = 0.65 \pm 0.01 ,$$
$$\phi_P = (41.4 \pm 0.5)^\circ, \quad \phi_V = (3.3 \pm 0.1)^\circ ,$$
$$z_{NS} = 0.83 \pm 0.02 .$$

# 2. Results

*Eur. Phys. J. C* (2020) 80: 1190, [arXiv:2007.12467](https://arxiv.org/abs/2007.12467)

- Decay widths and branching ratios

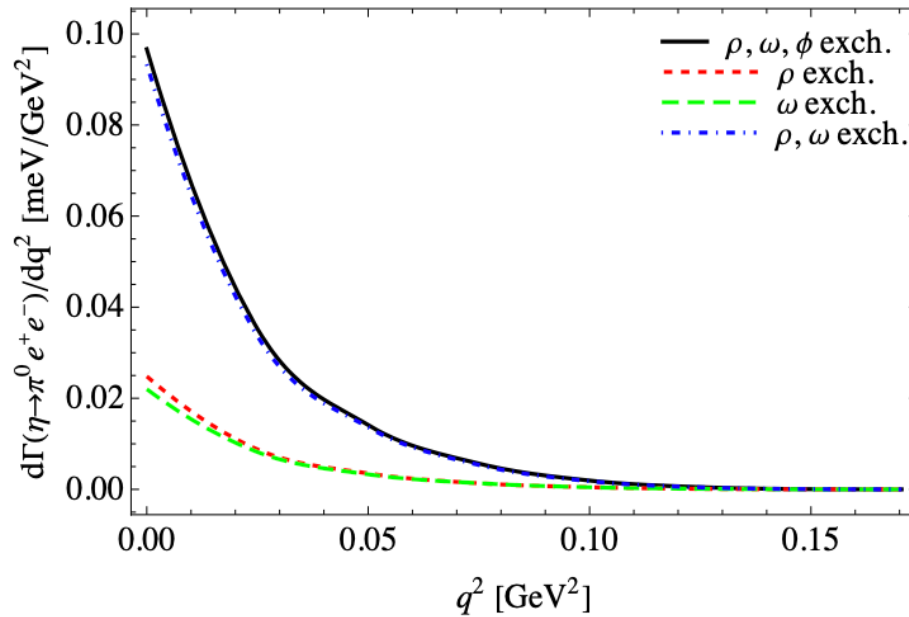
Decay	$\Gamma_{\text{th}}$	$\text{BR}_{\text{th}}$	$\text{BR}_{\text{exp}}$	
$\eta \rightarrow \pi^0 e^+ e^-$	$2.7(1)(1)(2) \times 10^{-6} \text{ eV}$	$2.0(1)(1)(1) \times 10^{-9}$	$< 7.5 \times 10^{-6} \text{ (CL=90\%)}$	WASA-at-COSY
$\eta \rightarrow \pi^0 \mu^+ \mu^-$	$1.4(1)(1)(1) \times 10^{-6} \text{ eV}$	$1.1(1)(1)(1) \times 10^{-9}$	$< 5 \times 10^{-6} \text{ (CL=90\%)}$	
$\eta' \rightarrow \pi^0 e^+ e^-$	$8.7(5)(6)(6) \times 10^{-4} \text{ eV}$	$4.5(3)(4)(4) \times 10^{-9}$	$< 1.4 \times 10^{-3} \text{ (CL=90\%)}$	PDG
$\eta' \rightarrow \pi^0 \mu^+ \mu^-$	$3.3(2)(4)(3) \times 10^{-4} \text{ eV}$	$1.7(1)(2)(2) \times 10^{-9}$	$< 6.0 \times 10^{-5} \text{ (CL=90\%)}$	
$\eta' \rightarrow \eta e^+ e^-$	$8.3(0.5)(0.1)(3.5) \times 10^{-5} \text{ eV}$	$4.3(0.3)(0.2)(1.8) \times 10^{-10}$	$< 2.4 \times 10^{-3} \text{ (CL=90\%)}$	
$\eta' \rightarrow \eta \mu^+ \mu^-$	$3.0(0.2)(0.1)(1.1) \times 10^{-5} \text{ eV}$	$1.5(1)(1)(5) \times 10^{-10}$	$< 1.5 \times 10^{-5} \text{ (CL=90\%)}$	

Table 1: Decay widths and branching ratios for the six  $C$ -conserving decays  $\eta^{(\prime)} \rightarrow \pi^0 l^+ l^-$  and  $\eta' \rightarrow \eta l^+ l^-$  ( $l = e$  or  $\mu$ ). First error is experimental, second is down to numerical integration and third is due to model dependency.

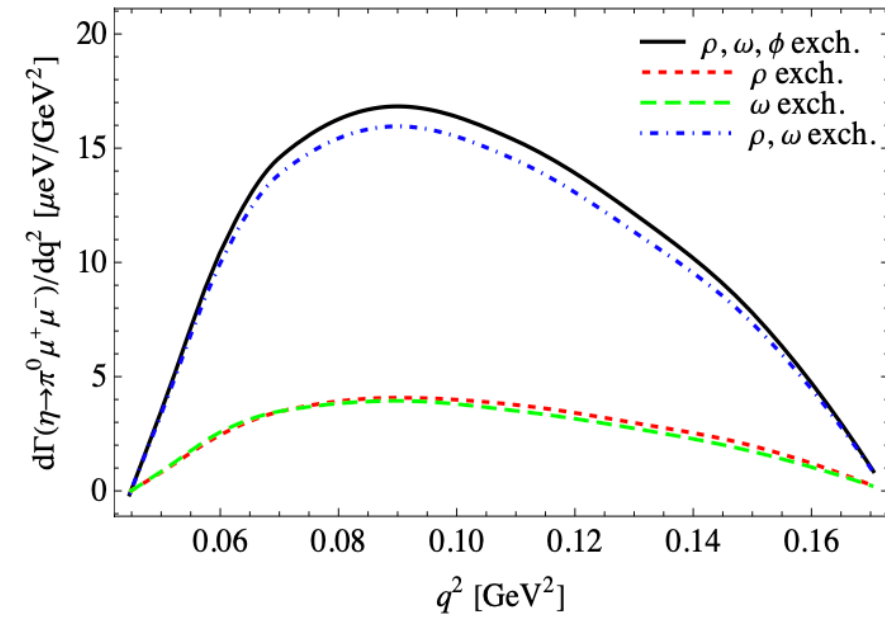
# 2. Results

*Eur. Phys. J. C* (2020) 80: 1190, [arXiv:2007.12467](https://arxiv.org/abs/2007.12467)

- $\eta \rightarrow \pi^0 \ell^+ \ell^-$  dilepton spectra



(a)  $\eta \rightarrow \pi^0 e^+ e^-$

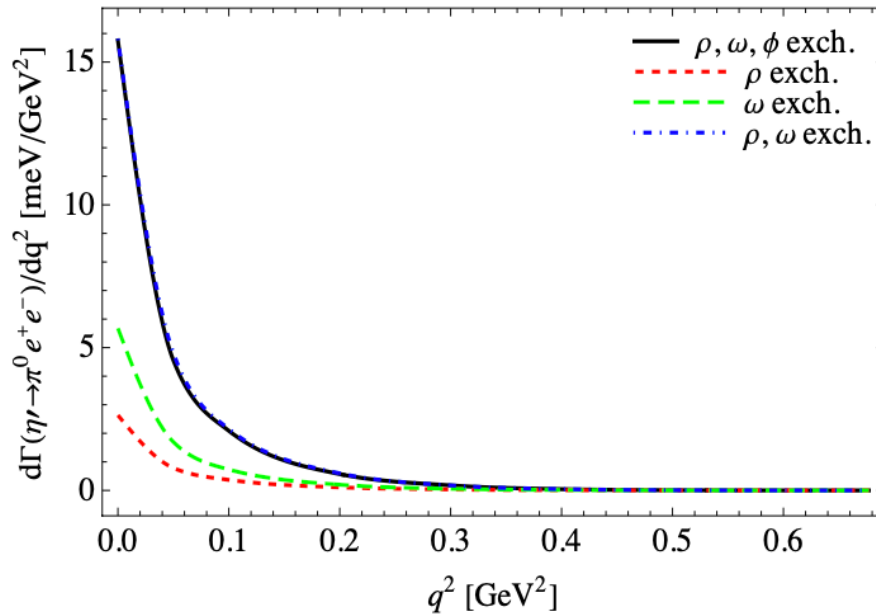


(b)  $\eta \rightarrow \pi^0 \mu^+ \mu^-$

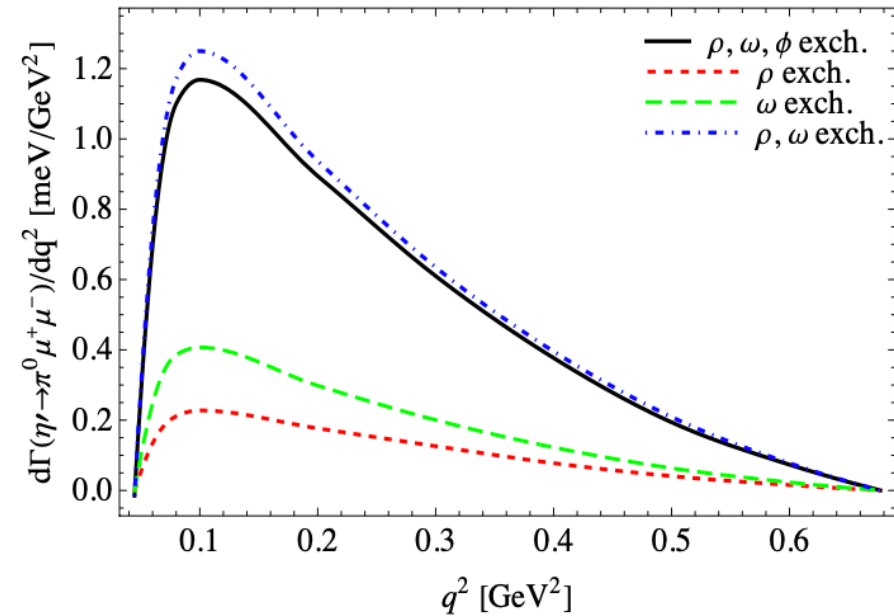
# 2. Results

*Eur. Phys. J. C* (2020) 80: 1190, [arXiv:2007.12467](https://arxiv.org/abs/2007.12467)

- $\eta' \rightarrow \pi^0 \ell^+ \ell^-$  dilepton spectra



(c)  $\eta' \rightarrow \pi^0 e^+ e^-$



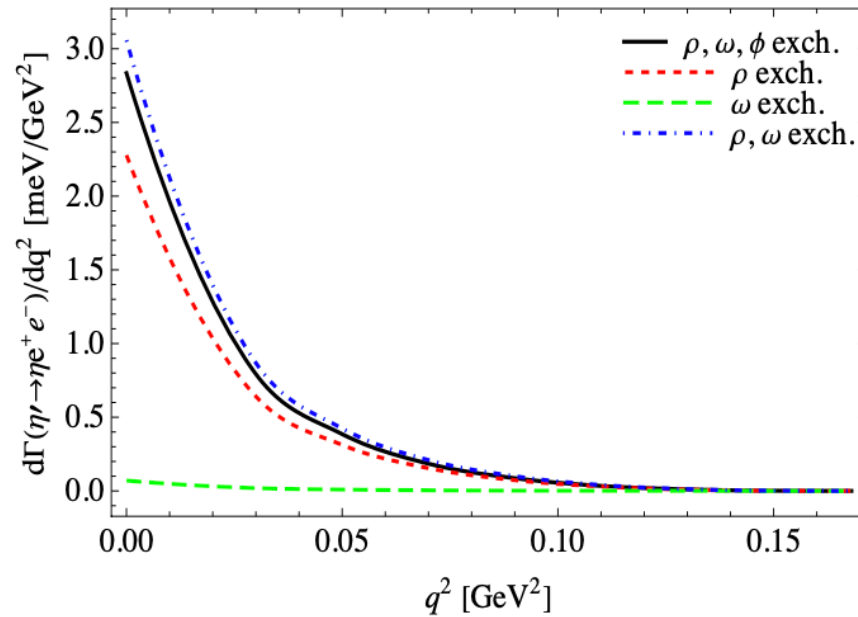
(d)  $\eta' \rightarrow \pi^0 \mu^+ \mu^-$



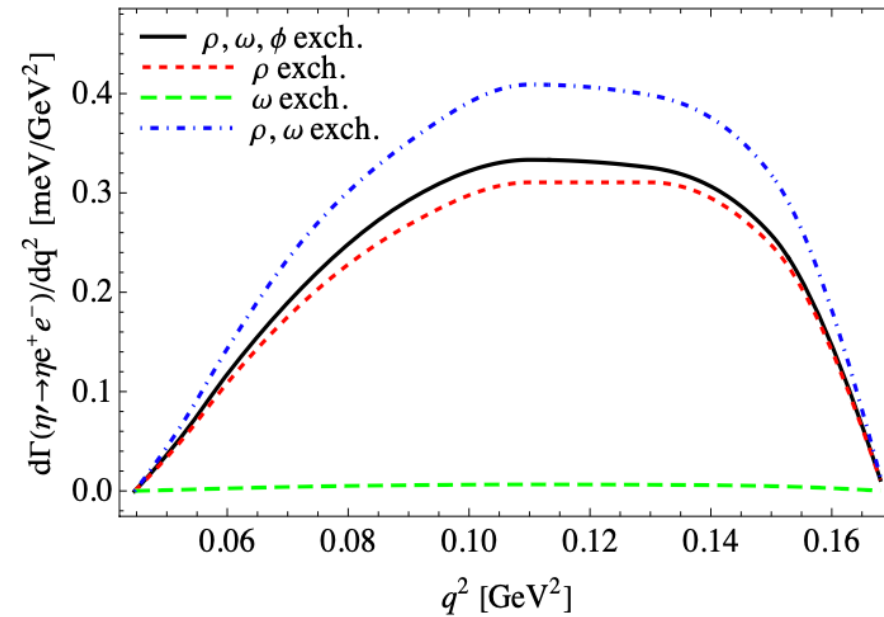
# 2. Results

*Eur. Phys. J. C* (2020) 80: 1190, [arXiv:2007.12467](https://arxiv.org/abs/2007.12467)

- $\eta' \rightarrow \eta \ell^+ \ell^-$  dilepton spectra



(e)  $\eta' \rightarrow \eta e^+ e^-$



(f)  $\eta' \rightarrow \eta \mu^+ \mu^-$

# 2. Results

*Eur. Phys. J. C* (2020) 80: 1190, [arXiv:2007.12467](https://arxiv.org/abs/2007.12467)

- REDTOP is a new Fermilab project that belongs to the high intensity class of experiments
  - It aims at detecting small variations from the SM by looking at a large number of events produced with very intense beams
- 1.8 GeV continuous proton beam impinging on a target made with 10 foils of beryllium to produce about  $2.5 \times 10^{13} \frac{\eta}{\text{year}}$  and  $2.5 \times 10^{11} \frac{\eta'}{\text{year}}$
- REDTOP may be able measure these BRs with significantly improved accuracy!
- More information about REDTOP can be found in <https://redtop.fnal.gov> and [arXiv:2203.07651](https://arxiv.org/abs/2203.07651)

# 3. $CP$ violation

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- The results from the previous section are used to fix the SM background



# 3. $CP$ violation

JHEP 05 (2022) 147, [arXiv:2202.04886](https://arxiv.org/abs/2202.04886)

- The SMEFT is a consistent EFT generalization of the SM constructed out of a series of  $SU_c(3) \times SU_L(2) \times U_Y(1)$  invariant higher dimensional operators, built out of SM fields\*

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \underbrace{\sum_i \frac{c_i}{\Lambda} \mathcal{O}_i^{d=5} + \sum_i \frac{c_i}{\Lambda^2} \mathcal{O}_i^{d=6}}_{\text{BSM effects}} + \mathcal{O}(\Lambda^{-3})$$

- Relevant operators for our study

$$\mathcal{O}_{ledq}^{prst} = (\bar{\ell}_p^i e_r)(\bar{d}_s q_t^i), \quad \mathcal{O}_{lequ}^{(1)prst} = (\bar{\ell}_p^i e_r)(\bar{q}_s^j u_t)\epsilon_{ij}$$

\* Phys. Rept. 793 (2019) 1-98, [arXiv:1706.08945](https://arxiv.org/abs/1706.08945)

# 3. $CP$ violation

JHEP 05 (2022) 147, [arXiv:2202.04886](https://arxiv.org/abs/2202.04886)

- The most general form factor decomposition for the three  $\eta^{(\prime)} \rightarrow \pi^0 \mu^+ \mu^-$  and  $\eta' \rightarrow \eta \mu^+ \mu^-$  processes

$$\langle \mu^+ \mu^- | iT | \eta^{(\prime)} \pi^0(\eta) \rangle = i\mathcal{M}(2\pi)^4 \delta(p_{\mu^+} + p_{\mu^-} - p_{\eta^{(\prime)}} - p_{\pi(\eta)})$$

is

$$\mathcal{M} = m_\ell(\bar{u}v)F_1 + (\bar{u}i\gamma^5 v)F_2 + (\bar{u}\not{k}v)F_3 + i(\bar{u}\not{k}\gamma^5 v)F_4,$$

where

$$k = p_{\eta^{(\prime)}} - p_{\pi(\eta)}$$

# 3. $CP$ violation

JHEP 05 (2022) 147, [arXiv:2202.04886](https://arxiv.org/abs/2202.04886)

- Noting that
  - within the SM only  $F_{1,3}$  terms contribute if one neglects electroweak effects,
  - contributions to  $F_4$  within the SM arise via electroweak loops which are negligible,
  - within the SMEFT contributions to  $F_4$  can appear at higher orders and, thus, are irrelevant for this study,

one arrives at

$$F_1 = \Sigma \quad (\text{cf. slide 11})$$

$$F_2 = \left[ \text{Im} c_{ledq}^{2211} \langle 0 | \bar{d}d | \eta^{(\prime)} \pi^0(\eta) \rangle + \text{Im} c_{ledq}^{2222} \langle 0 | \bar{s}s | \eta^{(\prime)} \pi^0(\eta) \rangle - \text{Im} c_{lequ}^{(1)2211} \langle 0 | \bar{u}u | \eta^{(\prime)} \pi^0(\eta) \rangle \right] / v^2$$

$$F_3 = \frac{1}{2} \Omega \quad (\text{cf. slide 11})$$

$$F_4 = 0$$

# 3. $CP$ violation

JHEP 05 (2022) 147, [arXiv:2202.04886](https://arxiv.org/abs/2202.04886)

- At NLO in  $LN_c \chi$ PT, the matrix elements in  $F_2$  can be expressed as

$$\langle 0 | \bar{u}u / \bar{d}d | \eta \pi^0 \rangle = \pm B_0 \left[ \left( 1 - \frac{m_\eta^2 - m_\pi^2}{M_S^2} \right) (c\phi_{23} \pm \epsilon_{13}s\phi_{23}) - \left( c\phi_{23} - \frac{s\phi_{23}}{\sqrt{2}} \right) \frac{\tilde{\Lambda}}{3} \right] \left( \frac{M_S^2}{M_S^2 - s} \right)$$

$$\langle 0 | \bar{s}s | \eta \pi^0 \rangle = -2B_0\epsilon_{13} \left[ \left( 1 - \frac{m_\eta^2 + 3m_\pi^2 - 4m_K^2}{M_S^2} \right) s\phi_{23} + \frac{\tilde{\Lambda}}{3} \left( \frac{c\phi_{23}}{\sqrt{2}} - s\phi_{23} - \frac{\epsilon_{12}s\phi_{23}}{\sqrt{2}\epsilon_{13}} \right) \right] \left( \frac{M_S^2}{M_S^2 - s} \right)$$

The corresponding expressions for  $\eta \rightarrow \eta'$  can be found by substituting  $\left\{ \begin{array}{l} c\phi_{23} \rightarrow s\phi_{23} \\ s\phi_{23} \rightarrow -c\phi_{23} \\ m_\eta \rightarrow m_{\eta'} \end{array} \right.$

# 3. $CP$ violation

JHEP 05 (2022) 147, [arXiv:2202.04886](https://arxiv.org/abs/2202.04886)

- At NLO in  $LN_c \chi$ PT, the matrix elements in  $F_2$  can be expressed as

$$\langle 0 | \bar{u}u / \bar{d}d | \eta' \eta \rangle = B_0 \left[ \left( 1 - \frac{m_{\eta'}^2 + m_{\eta}^2 - 2m_{\pi}^2}{M_S^2} \right) \left( \frac{s2\phi_{23}}{2} \mp \epsilon_{13}c2\phi_{23} \right) - \left( \frac{c2\phi_{23}}{\sqrt{2}} + s2\phi_{23} \right) \frac{\tilde{\Lambda}}{3} \right] \left( \frac{M_S^2}{M_S^2 - s} \right)$$

$$\langle 0 | \bar{s}s | \eta' \eta \rangle = -B_0 \left[ \left( 1 - \frac{m_{\eta'}^2 + m_{\eta}^2 + 2m_{\pi}^2 - 4m_K^2}{M_S^2} \right) s2\phi_{23} + \left( \sqrt{2}c2\phi_{23} - s2\phi_{23} \right) \frac{\tilde{\Lambda}}{3} \right] \left( \frac{M_S^2}{M_S^2 - s} \right)$$

where we have introduced the scale invariant parameter  $\tilde{\Lambda} = \Lambda_1 - 2\Lambda_2$ ,  $\phi_{23}$  is the  $\eta$ - $\eta'$  mixing angle in the quark-flavour basis,  $\epsilon_{12}$  and  $\epsilon_{13}$  are first order approximations to the corresponding  $\phi_{12}$  and  $\phi_{13}$  isospin-breaking mixing angles in the  $\pi^0$ - $\eta$  and  $\pi^0$ - $\eta'$  sectors, respectively, and  $M_S$  is the mass of a generic scalar resonance.



# 3. $CP$ violation

JHEP 05 (2022) 147, [arXiv:2202.04886](https://arxiv.org/abs/2202.04886)

- Differential decays widths for  $\eta^{(\prime)} \rightarrow \pi^0(\eta)\mu^+\mu^- \rightarrow \pi^0(\eta)e^+v_e\bar{\nu}_\mu e^-v_e\nu_\mu$

$$d\Gamma = \sum_{\lambda\bar{\lambda}} \underbrace{\frac{dsdc\theta}{64(2\pi)^3} \frac{\lambda_K^{1/2}\beta_\mu}{m_{\eta^{(\prime)}}^3} |\mathcal{M}(\lambda\mathbf{n}, \bar{\lambda}\bar{\mathbf{n}})|^2}_{\eta^{(\prime)} \rightarrow \pi^0(\eta)\mu^+\mu^-} \underbrace{\left[ \frac{d\Omega}{4\pi} dx n(x) (1 - \lambda b(x)\boldsymbol{\beta}\cdot\mathbf{n}) \right]}_{\mu^+ \rightarrow e^+v_e\bar{\nu}_\mu} \underbrace{\left[ \frac{d\bar{\Omega}}{4\pi} d\bar{x} n(\bar{x}) (1 + \bar{\lambda} b(\bar{x})\bar{\boldsymbol{\beta}}\cdot\bar{\mathbf{n}}) \right]}_{\mu^- \rightarrow e^-v_e\nu_\mu}$$

$$d\Gamma = \frac{dsdc\theta}{64(2\pi)^3} \frac{\lambda_K^{1/2}\beta_\mu}{m_{\eta^{(\prime)}}^3} \left[ \frac{d\Omega}{4\pi} dx n(x) \right] \left[ \frac{d\bar{\Omega}}{4\pi} d\bar{x} n(\bar{x}) \right] \left[ \tilde{c}_1|F_1|^2 + \tilde{c}_3|F_3|^2 + \tilde{c}_{13}^R \text{Re } F_1F_3^* + \tilde{c}_{13}^I \text{Im } F_1F_3^* \right. \\ \left. + \tilde{c}_2|F_2|^2 + \tilde{c}_{12}^R \text{Re } F_1F_2^* + \tilde{c}_{12}^I \text{Im } F_1F_2^* + \tilde{c}_{23}^R \text{Re } F_2F_3^* + \tilde{c}_{23}^I \text{Im } F_2F_3^* \right]$$

# 3. $CP$ violation

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- Longitudinal and transverse asymmetries:

$$A_L = \frac{N(c\theta_{e^+} > 0) - N(c\theta_{e^+} < 0)}{N(c\theta_{e^+} > 0) + N(c\theta_{e^+} < 0)} = -\frac{2 \int dsdc\theta \lambda_K^{1/2} \beta_\mu m_\mu \left[ \beta_\mu s \operatorname{Im} F_1 F_2^* + 2\lambda_K^{1/2} c\theta \operatorname{Im} F_3 F_2^* \right]}{64(2\pi)^3 m_{\eta^{(\prime)}}^3 \int d\Gamma}$$

$$A_T = \frac{N[s(\bar{\phi} - \phi) > 0] - N[s(\bar{\phi} - \phi) < 0]}{N[s(\bar{\phi} - \phi) > 0] + N[s(\bar{\phi} - \phi) < 0]} = \frac{\pi \int dsdc\theta \lambda_K^{1/2} \beta_\mu m_\mu \left[ \beta_\mu s \operatorname{Re} F_1 F_2^* + 2\lambda_K^{1/2} c\theta \operatorname{Re} F_3 F_2^* \right]}{64(2\pi)^3 m_{\eta^{(\prime)}}^3 \int d\Gamma}$$

where the polar angles  $\theta_{e^\pm}$  refer to those of the  $e^\pm$  in the  $\mu^\pm$  rest frames,  $\phi(\bar{\phi})$  correspond to the azimuthal  $e^\pm$  angles in the  $\mu^\pm$  rest frames, and  $N$  refers to the number of  $\eta^{(\prime)}$  decays.

# 3. $CP$ violation

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- Results @NLO in  $LN_c$   $\chi$ PT for  $\eta^{(\prime)} \rightarrow \pi^0(\eta)\mu^+\mu^-$

$$\begin{aligned}A_L^{\eta \rightarrow \pi^0 \mu^+ \mu^-} &= -0.19(6) \operatorname{Im} c_{lequ}^{(1)2211} - 0.19(6) \operatorname{Im} c_{ledq}^{2211} - 0.020(9) \operatorname{Im} c_{ledq}^{2222}, \\A_T^{\eta \rightarrow \pi^0 \mu^+ \mu^-} &= 0.07(2) \operatorname{Im} c_{lequ}^{(1)2211} + 0.07(2) \operatorname{Im} c_{ledq}^{2211} + 7(3) \times 10^{-3} \operatorname{Im} c_{ledq}^{2222}, \\A_L^{\eta' \rightarrow \pi^0 \mu^+ \mu^-} &= -0.04(8) \operatorname{Im} c_{lequ}^{(1)2211} - 0.04(8) \operatorname{Im} c_{ledq}^{2211} + 10(3) \times 10^{-3} \operatorname{Im} c_{ledq}^{2222}, \\A_T^{\eta' \rightarrow \pi^0 \mu^+ \mu^-} &= 3(6) \times 10^{-3} \operatorname{Im} c_{lequ}^{(1)2211} + 3(6) \times 10^{-3} \operatorname{Im} c_{ledq}^{2211} - 7(2) \times 10^{-4} \operatorname{Im} c_{ledq}^{2222}, \\A_L^{\eta' \rightarrow \eta \mu^+ \mu^-} &= -5(39) \times 10^{-3} \operatorname{Im} c_{lequ}^{(1)2211} + 5(46) \times 10^{-3} \operatorname{Im} c_{ledq}^{2211} - 0.08(1) \operatorname{Im} c_{ledq}^{2222}, \\A_T^{\eta' \rightarrow \eta \mu^+ \mu^-} &= 7(50) \times 10^{-5} \operatorname{Im} c_{lequ}^{(1)2211} - 6(65) \times 10^{-5} \operatorname{Im} c_{ledq}^{2211} \\&\quad + 1(19) \times 10^{-3} \operatorname{Im} c_{ledq}^{2222},\end{aligned}$$

where the error quoted accounts for both the numerical integration and the model-dependence uncertainties

# 3. $CP$ violation

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- REDTOP is currently studying the implementation of muon polarimetry, cf. [arXiv:2203.07651](https://arxiv.org/abs/2203.07651)
- A total production of  $2.5 \times 10^{13} \frac{\eta}{\text{year}}$  and  $2.5 \times 10^{11} \frac{\eta'}{\text{year}}$  is expected, with assumed reconstruction efficiencies of approximately 20%
- The expected SM asymmetry noise for the  $\eta^{(\prime)} \rightarrow \pi^0 \mu^+ \mu^-$  and  $\eta' \rightarrow \eta \mu^+ \mu^-$  is

$$\sigma_{\eta \rightarrow \pi^0 \mu^+ \mu^-} = 1.29 \times 10^{-2}$$

$$\sigma_{\eta' \rightarrow \pi^0 \mu^+ \mu^-} = 0.105$$

$$\sigma_{\eta' \rightarrow \eta \mu^+ \mu^-} = 0.354$$

# 3. $CP$ violation

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- Sensitivity to new physics at REDTOP

Process	Asymmetry	$\text{Im } c_{lequ}^{(1)2211}$	$\text{Im } c_{ledq}^{2211}$	$\text{Im } c_{ledq}^{2222}$
$\eta \rightarrow \pi^0 \mu^+ \mu^-$	$A_L$	0.0695	0.0720	0.686
	$A_T$	0.194	0.203	1.93
$\eta' \rightarrow \pi^0 \mu^+ \mu^-$	$A_L$	2.36	2.56	10.96
	$A_T$	33.1	35.8	154
$\eta' \rightarrow \eta \mu^+ \mu^-$	$A_L$	67.5	78.5	4.46
	$A_T$	5264	5549	328
$\eta \rightarrow \mu^+ \mu^-$	$A_L$	0.007	0.007	0.005
nEDM	—	$\leq 0.001$	$\leq 0.002$	$\leq 0.02$

**Table 1.** Summary of REDTOP sensitivities to (the imaginary parts of) the Wilson coefficients associated to the SMEFT  $CP$ -violating operators in eq. (2.2) for the processes studied in this work, as well as the  $\eta \rightarrow \mu^+ \mu^-$  decay analysed in ref. [4]. In addition, the upper bounds from nEDM experiments are given in the last row for comparison purposes.

# 4. Summary

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- The study of the  $\eta$  and  $\eta'$  phenomenology may provide a very effective window to find physics BSM
- Theoretical predictions within the SM framework have been presented for the six  $\eta^{(\prime)} \rightarrow \pi^0 \ell^+ \ell^-$  and  $\eta' \rightarrow \eta \ell^+ \ell^-$  semileptonic processes
- Theoretical estimations for the longitudinal and transverse asymmetries of the three  $\eta^{(\prime)} \rightarrow \pi^0 \mu^+ \mu^-$  and  $\eta' \rightarrow \eta \mu^+ \mu^-$  processes have been presented. The predicted statistics at REDTOP would fall short to detect  $CP$ -violating effects in these decays