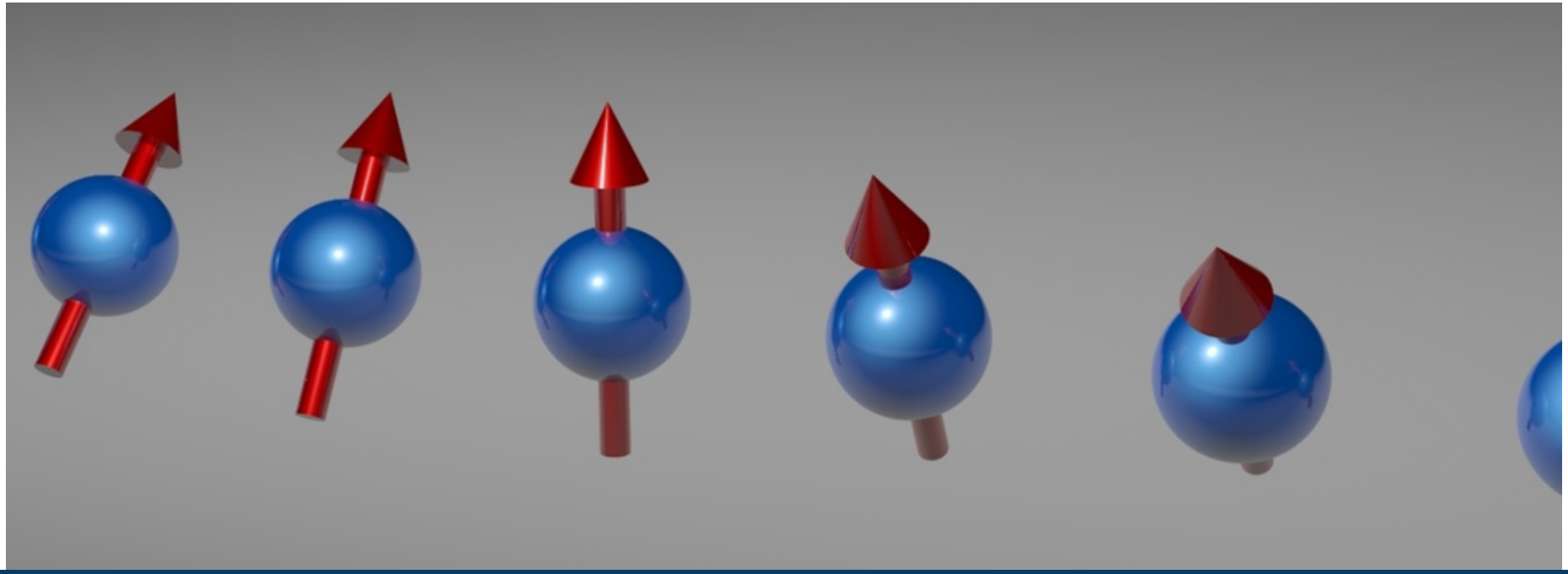


EFFICIENT STATE PREPARATION FOR THE SCHWINGER MODEL

09/06/2023 | Giovanni Pederiva, Alexei Bazavov, Brandon Henke, Leon Hostetler, Dean Lee, Huey-Wen Lin, Andrea Shindler |

Quantum Computing in a Nutshell

The Schwinger Model

We consider the Schwinger model, a $U(1)$ gauge theory in $1+1$ dimension, with the inclusion of a θ-term[1].
The Lagrangian can be written as

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g\theta}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}\psi \tag{1}$$

We use Gauss's Law and the boundary conditions to remove the gauge degrees of freedom.

[1] Following Chakraborty et al. "Classically emulated digital quantum simulation of the Schwinger model with a topological term via adiabatic state preparation". DOI: PhysRevD.105.094503).

Schwinger Hamiltonian

We need to apply the Legendre Transform to get the Hamiltonian for QC.

$$H = \int dx \left[-i\bar{\psi}\gamma^1(\partial_1 + igA_1)\psi + m\bar{\psi}e^{i\theta\gamma_5}\psi + \frac{1}{2}E^2 \right], \tag{2}$$

Then we employ staggered fermions for discretizing H

$$H = -i\sum_{n=1}^{N-1}\left(\frac{1}{2a} - (-1)^n\frac{m}{2}\,\theta\right)\left[\chi_n^\dagger e^{i\phi_n}\chi_{n+1} - \text{h.c.}\right]$$
$$+ m\,\cos\theta\sum_{n=1}^{N}(-1)^n\chi_n^\dagger\chi_n + \frac{g^2a}{2}\sum_{n=1}^{N-1}L_n^2 \tag{3}$$

with $A^1(x) \to -\phi_n/(ag)$ and $E(x) \to gL_n$, while the fermions $\chi_n = \psi_u(x)$ for n even and $\chi_n = \psi_d(x)$ for n odd.

Discretization of the Hamiltonian

The final Hamiltonian in terms of spin variables can be written, after a Jordan-Wigner transformation, as $H = H_{ZZ} + H_{\pm} + H_Z$ where:

$$H_{ZZ} = \frac{J}{2} \sum_{n=2}^{N-1} \sum_{1 \le k < l \le n} Z_k Z_l$$

$$H_{\pm} = \frac{1}{2} \sum_{n=1}^{N-1} \left(w - (-1)^n \frac{m}{2} \sin\theta \right) \left[X_n X_{n+1} + Y_n Y_{n+1} \right]$$

$$H_Z = m \cos\theta \sum_{n=1}^{N} (-1)^n Z_n - \frac{J}{2} \sum_{n=1}^{N-1} (n \bmod 2) \sum_{l=1}^{n} Z_l,$$

where we defined the constants $w = \frac{1}{2a}$ and $J = \frac{g^a}{2}$ and (X_n, Y_n, Z_n) are the Pauli matrices at site n. The gauge field is removed using Gauss's law and open boundary conditions.

Adiabatic State Preparation

Adiabatic State Preparation (ASP) is a well established method for state preparation.

1. Identify a simpler problem that can be easily solve
2. Find a transition between the simple Hamiltonian and the target one
3. Initialize the system in the GS of the simple Hamiltonian
4. Slowly change the Hamiltonian to the target one

$$|\Omega\rangle = \lim_{T\to\infty} \exp\left(-i\int_0^T dt H_A(t)\right)|\Omega_0\rangle \tag{4}$$

For example, we considered the initial Hamiltonian $H_0 = H_{ZZ} + H_Z|_{m\to m_0,\theta\to 0}$, which is very simple to analyze.

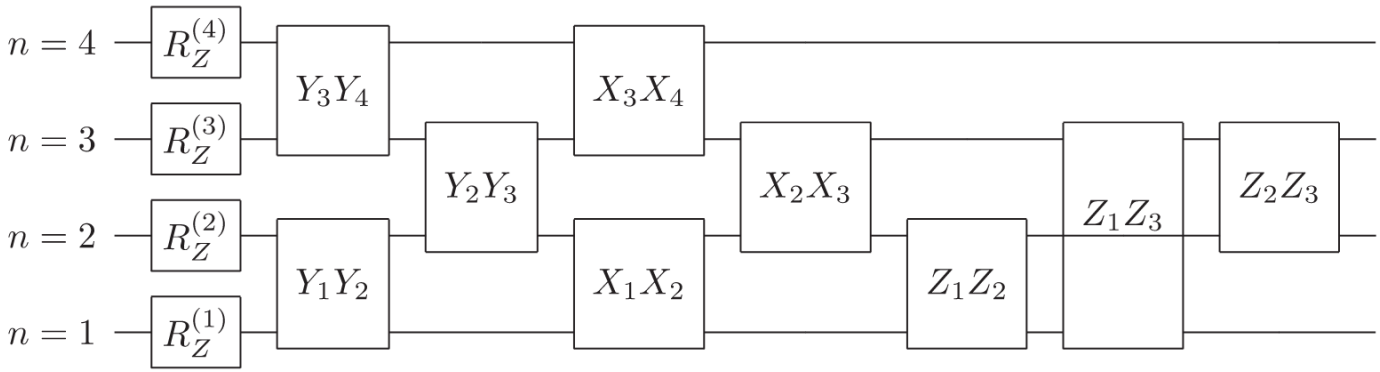
Adiabatic State Preparation

In the simplest case on defines this operator as $U(t) = e^{-iH_A(t)\delta t}$ where $H_A(t)$ is the adiabatic Hamiltonian, which interpolates between H and H_0 by making the constants w, θ and m time dependent. A linear interpolation would be, for a final time T:

$$w \to \frac{t}{T}w \quad \theta \to \frac{t}{T}\theta \quad m \to \left(1 - \frac{t}{T}\right) m_0 + \frac{t}{T}m$$

but other interpolations are possible and also more efficient, such as a $\sin^2$ or $\cos^2$.

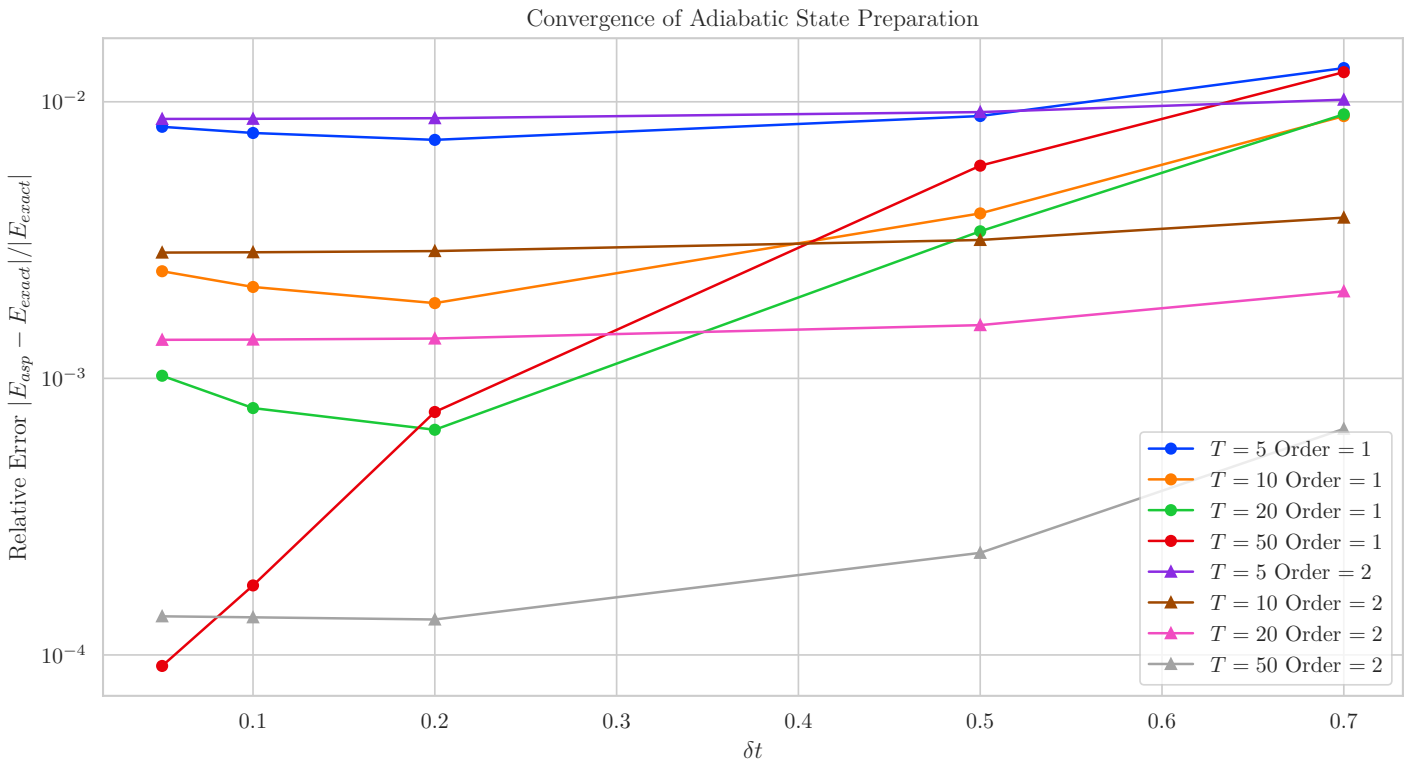
Circuit Realization of an ASP Step



Trotter Order	$N = 4$	$N = 8$
1	9	18
2	16	26

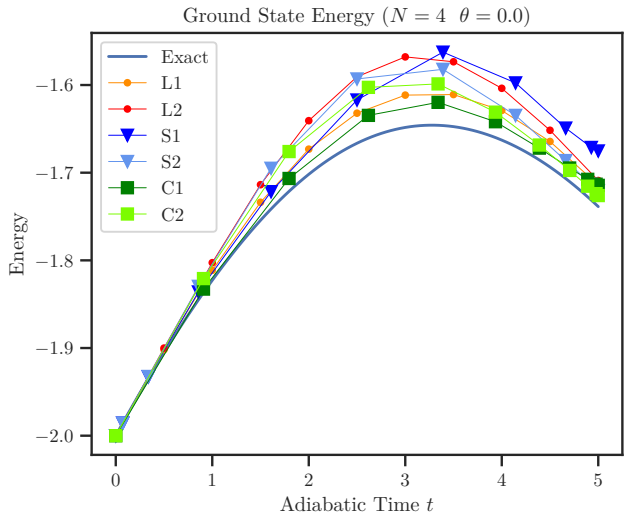
Average number of CNOT gates per time evolution step

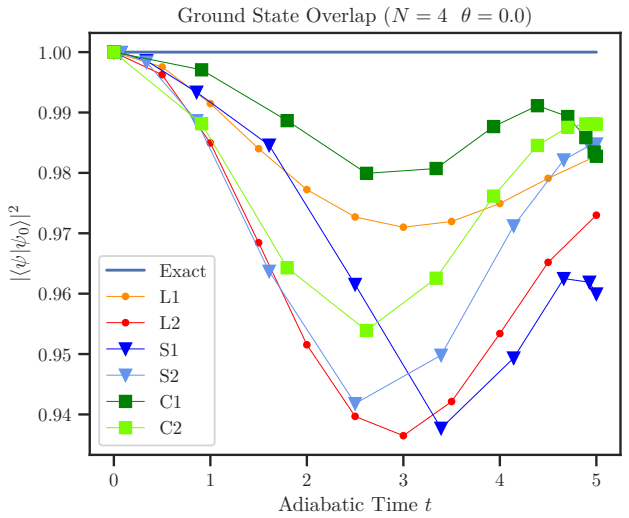
ASP Results

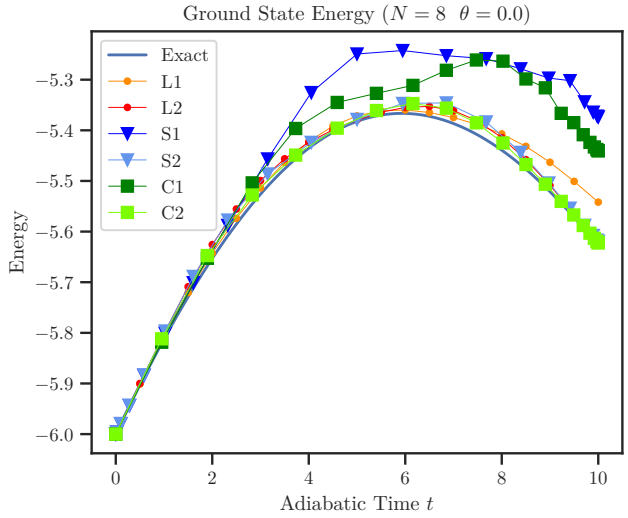


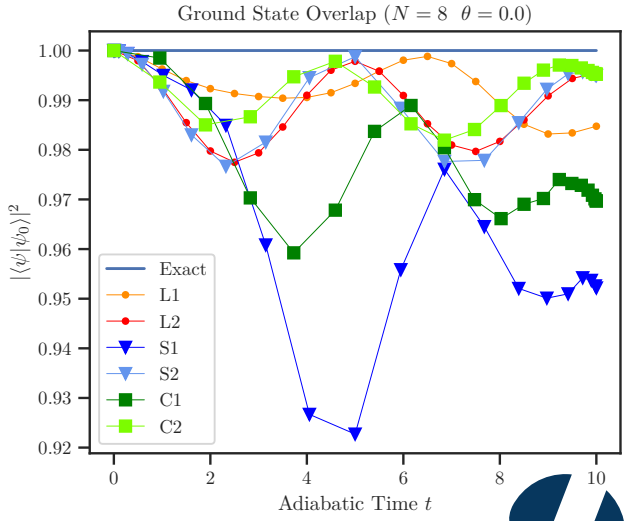
Errors given by δt for the Trotter-Suzuki truncation and by T for the adiabatic approximations $\rightarrow$ both need to be optimized

ASP Results









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Quantum Approximate Optimization Algorithm

The Quantum Approximate Optimization Algorithm (QAOA) is a quantum optimization algorithm that can be used also for state preparation. It relies, just as ASP, on the existence of a simple trivially solvable Hamiltonian to use as a starting point.
The ansatz for the state is given by:

$$|\psi_M(\vec{\gamma},\vec{\beta})\rangle = \left(\prod_{k=0}^{M-1} e^{-i\beta_{M-k}H_0}e^{-i\gamma_{M-k}H}\right)|\psi_0\rangle$$

The problem is reduced to finding the optimal values for $\overrightarrow{\gamma}^*$ and $\overrightarrow{\beta}^*$ such that $|\psi_N(\overrightarrow{\gamma}^*,\overrightarrow{\beta}^*)\rangle$ is a good approximation of the desired state. For this work we used simulated annealing, minimizing the energy of the system, as in a variational problem.

$$\langle\psi_M(\vec{\gamma},\vec{\beta})|\, H\, |\psi_M(\vec{\gamma},\vec{\beta})\rangle = E_0^V \geq E_0 \tag{5}$$

QAOA Results

The QAOA method has the advantage to permit to set the number of steps in the evolution to a very small number, provided one can find the optimal parameters for such evolution. For instance, for the same systems shown before we obtained comparable results with just two steps

Method	N	(θ, m)	M	CNOT/qubit	Rel. Err. E_0	GS Overlap
QAOA	4	$(0, 0)$	2	24	0.0029	0.9975
QAOA	4	$(0, 0)$	3	36	0.0041	0.9968
QAOA	4	$(\pi/4, 1)$	2	24	0.00045	0.9996
QAOA	4	$(\pi/4, 1)$	3	36	0.0031	0.9971
QAOA	8	$(0, 0)$	2	56	0.0089	0.9701
QAOA	8	$(0, 0)$	3	84	0.0068	0.9846
QAOA	8	$(\pi/4, 1)$	2	56	0.00047	0.9988
QAOA	8	$(\pi/4, 1)$	3	84	0.00040	0.9989

Blocked QAOA

One option to decrease the number of CNOT gates per qubit even further would be to use custom optimized 2-qubit gates. To deal with the non-local term we define a modified Hamiltonian H_B, which we will call "blocked," where only the diagonal and nearest-neighbor terms of the full Hamiltonian are kept:

$$H_B = H_\pm + H_Z + H'_{ZZ}$$

$$|\psi_M(\vec{\gamma},\vec{\beta})\rangle = e^{-i\beta_M H_0} e^{-i\gamma_M H} \left(\prod_{k=1}^{M-1} e^{-i\beta_{M-k} H_0} e^{-i\gamma_{M-k} H_B} \right) |\psi_0\rangle$$

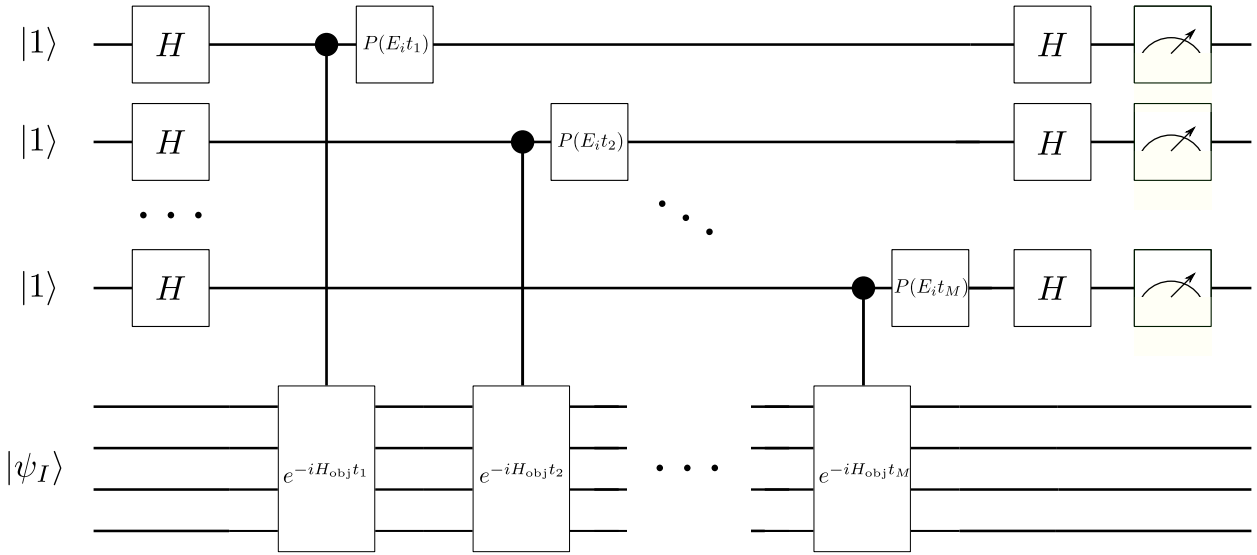
where the first $M-1$ unitary application contain the H_B while only one application of the full Hamiltonian is applied on the last step.
This can hopefully be used to scale the system to larger values of N.

Blocked QAOA Results

N	# CNOT/qubit	Rel. Err. E_0	GS Overlap
$4*$	16	0.0043	0.9960
6	22	0.0083	0.9273
8	27	0.0168	0.7516
10	32	0.0231	0.5138

QAOA Blocked with $M = 3$ steps. The results for the $N = 4$ are obtained after parameter optimization; the results for $N = 6, 8, 10$ have been computed using the same optimal parameters for $N = 4$.

The Rodeo Algorithm



A recently proposed algorithm that couples the system with a set of M ancilla qubits. For each a controlled time evolution is made with a random time t_n, followed by a phase gate.

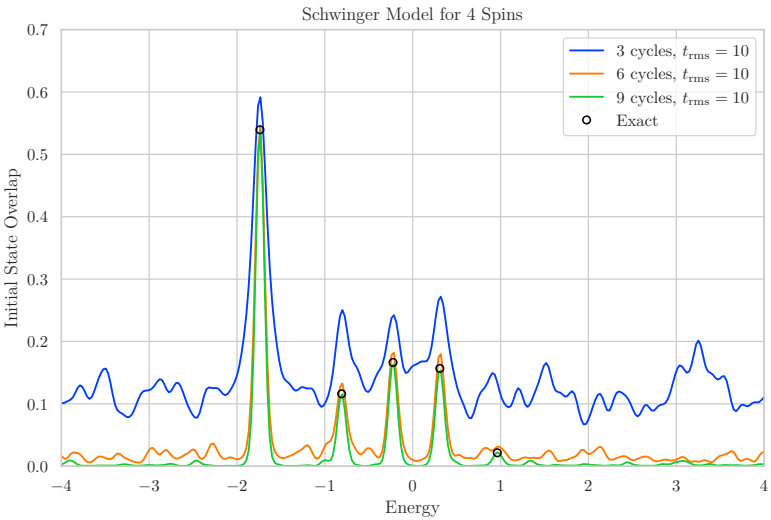
The Rodeo Algorithm

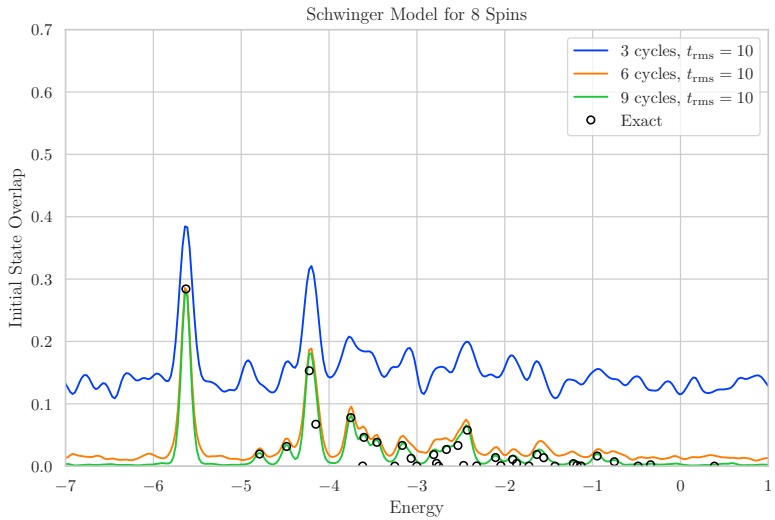
- The probability of measuring the ancilla qubit in the $|1\rangle$ state for a given eigenvalue ϵ_j is $\cos^2\left[(\epsilon_j - E_i)\frac{t}{2}\right]$
- For M qubits, each with a random time t_m then we have a cosine filter:

$$P_M = \prod_{m=1}^{M} \cos^2\left[(\epsilon_j - E_i)\frac{t_m}{2}\right] \tag{6}$$

- By “scanning” over different values of the energy, we can get the spectrum and the overlap factors for H_{obj} of an initial state $|\psi_I\rangle$.

Rodeo Algorithm Results





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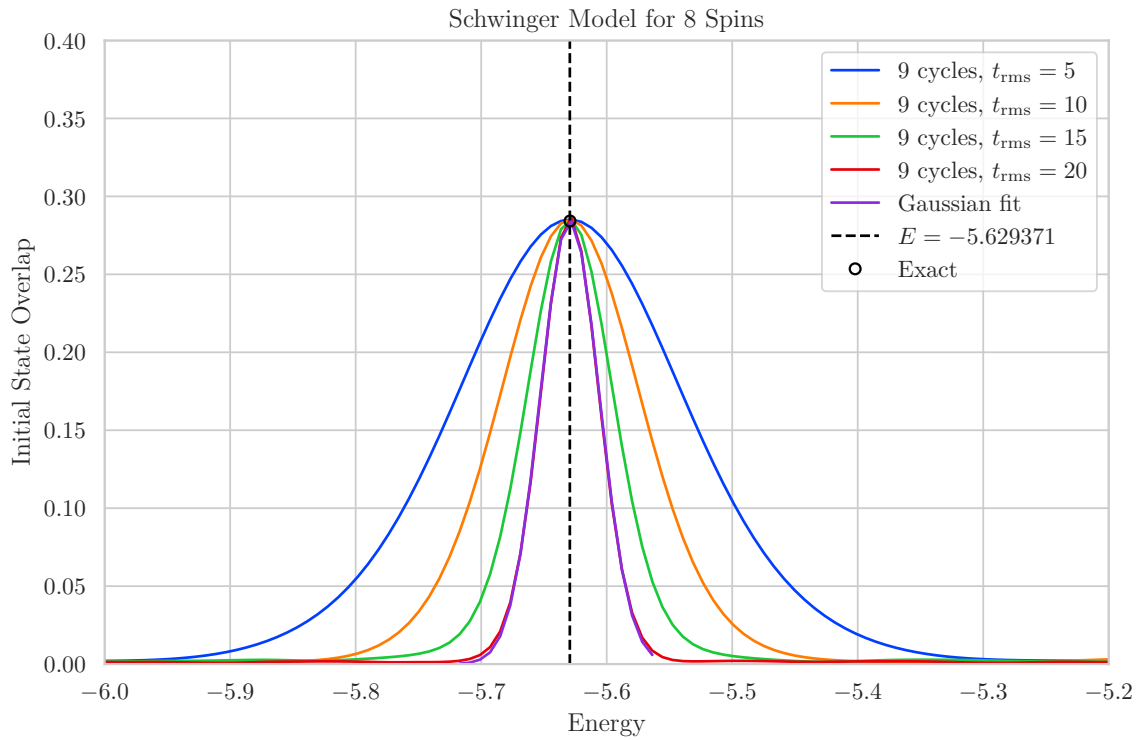
Rodeo Algorithm for State Preparation



The efficiency of the algorithm depends on the overlap with the initial state $|\psi_I\rangle$.

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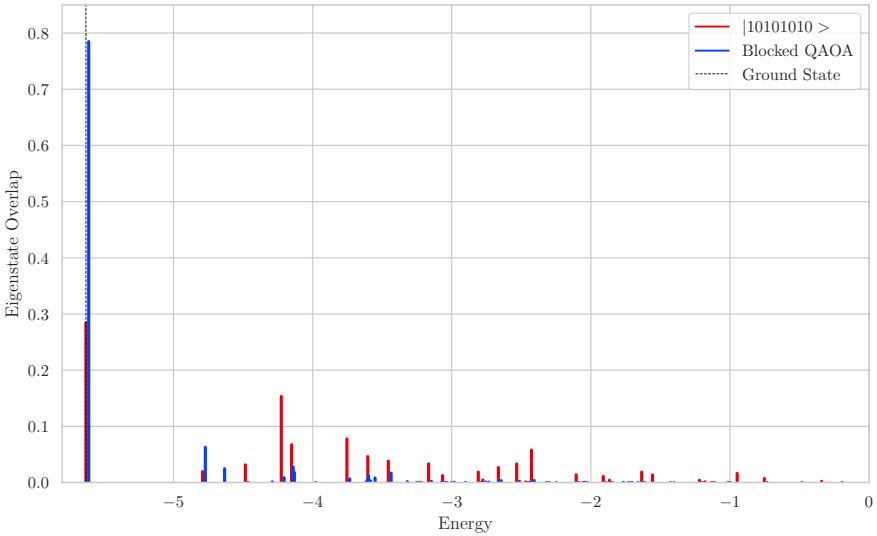
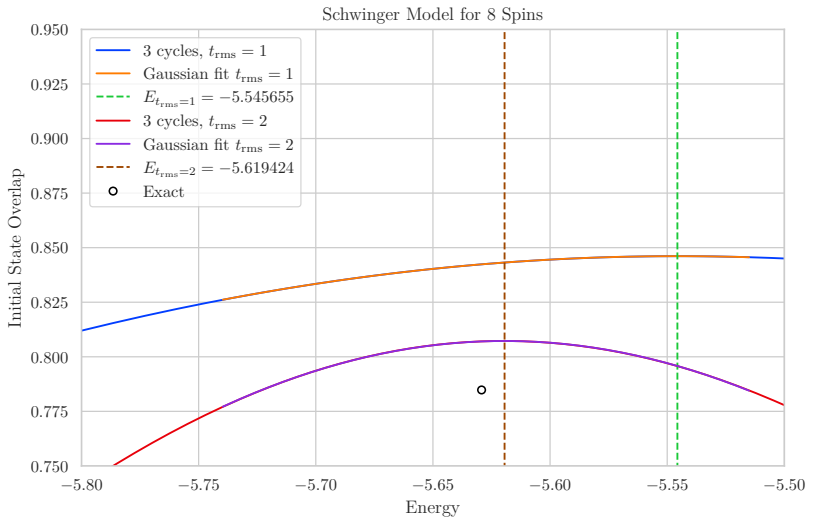
Combining QAOA and RA

To alleviate the issues with the Rodeo Algorithm we can use a “preconditioner”, i.e. a better initial state that has higher overlap with the GS. We use the blocked QAOA since it is very cheap:





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- The RA can be used for preparing any state, not just the ground state, but it results in very long algorithms.
- Can chain together different algorithms to obtain better results
- Scaling with larger systems? Efficient excited states? More complicated initial states?