Quantum simulation \and entanglement/ of lattice gauge theories

ECT star

June 9th, 2023

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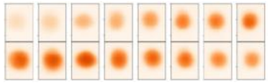
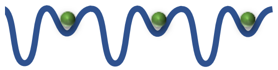
Quantum simulation of lattice gauge theories is seeing a breathtaking advance

	# qubits	platform type
Martinez, Muschik, Schindler, Nigg, Erhard, Heyl, Hauke, Dalmonte, Monz, Zoller, Blatt, *Nature* 2016	4	
Bernien, et al., *Nature* 2017	51	Rydberg atoms
Klco, Dumitrescu, McCaskey, Morris, Pooser, Sanz, Solano, Lougovski, Savage, *PRA* 2018	4	SC qbits
Kokail, Maier, van Bijnen, Brydges, Joshi, Jurcevic, Muschik, Silvi, Blatt, Roos, Zoller, *Nature* 2019	20	
Mil, Zache, Hegde, Xia, Bhatt, Oberthaler, Hauke, Berges, Jendrzejewski, *Science* 2020	2	
Yang, Sun, Ott, Wang, Zache, Halimeh, Yuan, Hauke, Pan, *Nature* 2020	70	

. . . many more!

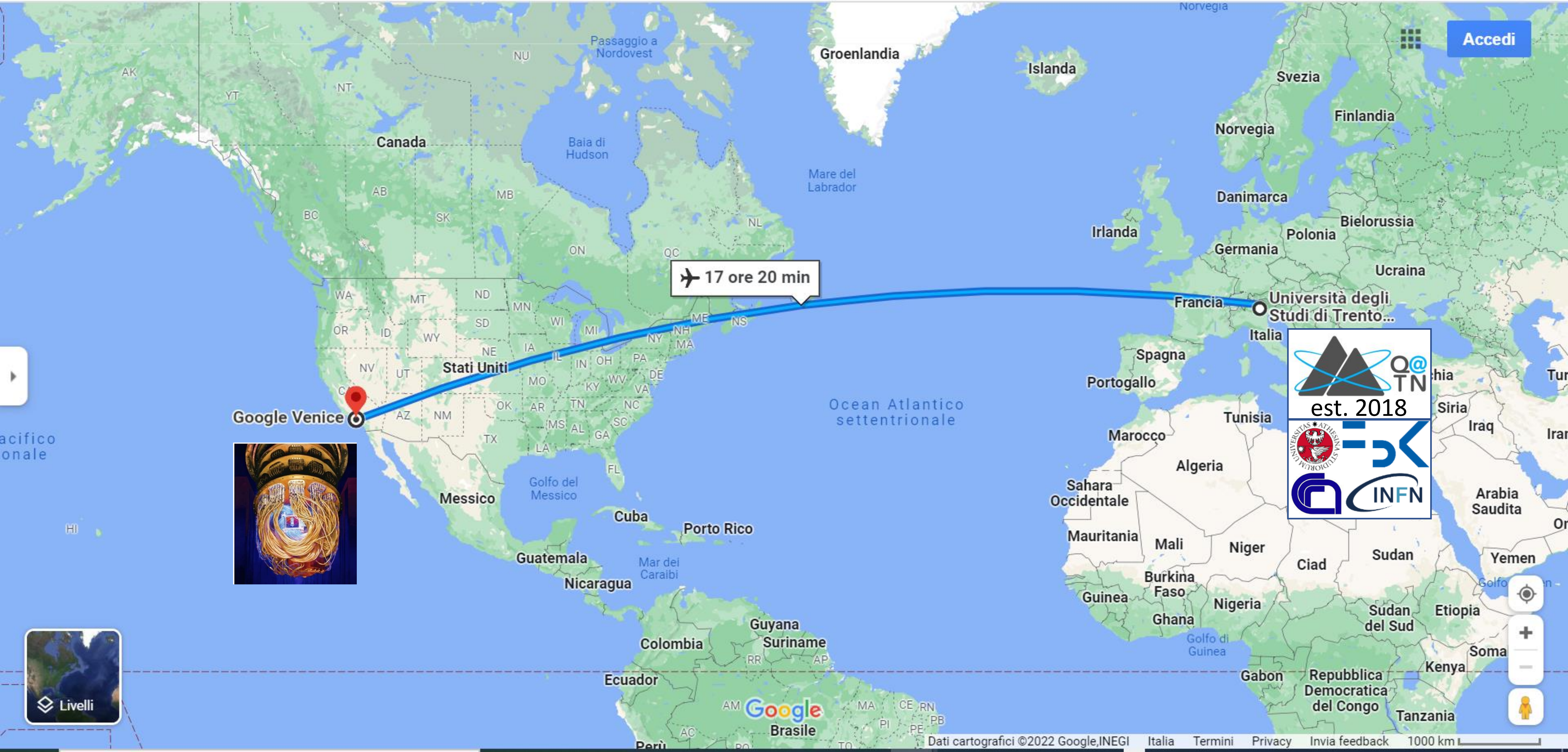
Some current key challenges

- What type of physics questions can we address?
- How to ensure error resilience (Trotter, gauge symmetry)?
- How to measure complex observables?
- More complex theories.

This talk

Google Quantum AI Early Access Program

Mildenberger, Mruczkiewicz, Halimeh, Jiang, Hauke, arXiv:2203.08905



Target model: $\mathbb{Z}_2$ lattice gauge theory

$$H = J\sum_i (\hat{\sigma}_i^+ \hat{\tau}_{i,i+1}^z \hat{\sigma}_{i+1}^- + \mathrm{h.c.}) + m\sum_i (-1)^i \hat{\sigma}_i^z - f\sum_i \tau_{i,i+1}^x \qquad G_i^{\mathbb{Z}_2} = -\tau_{i-1,i}^x \sigma_i^z \tau_{i,i+1}^x$$

matter-gauge field coupling

rest mass

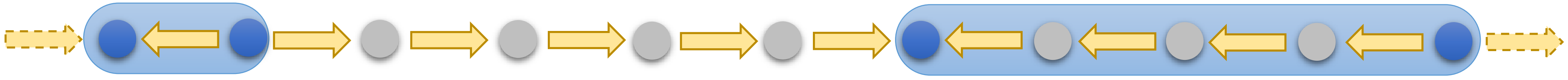
background field (in U(1): topological θ-angle)

Background field leads to confinement

Kebrič, Barbiero, Reinmoser, Schollwöck, Grusdt, *PRL* 2021; Borla, Verresen, Grusdt, Moroz, *PRL* 2020

$$H = J\sum_i(\hat{\sigma}_i^+ \hat{\tau}^z_{i,i+1}\hat{\sigma}^-_{i+1} + \mathrm{h.c.}) + m\sum_i(-1)^i\hat{\sigma}_i^z - f\sum_i \tau^x_{i,i+1} \qquad G_i^{\mathbb{Z}_2} = -\tau^x_{i-1,i}\sigma^z_i\tau^x_{i,i+1}$$

at $f = 0$ E-field energy independent of L

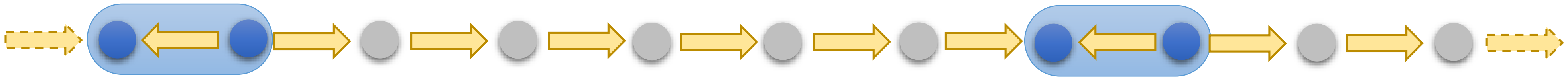


E-field "string"

"anti-string"

"string"

at $f > 0$ E-field energy $\propto L$



confined mesons

confinement in 1+1D spin models, e.g.,
Kormos, Collura, Takács, Calabrese, *Nat. Phys.* 2017
Vovrosh, Knolle, *Scientific Reports* 2021
Lencsés, Mussardo, Takács, *Phys. Lett. B* 2022
Knaute, Hauke, *Phys. Rev. A* 2022, . . .

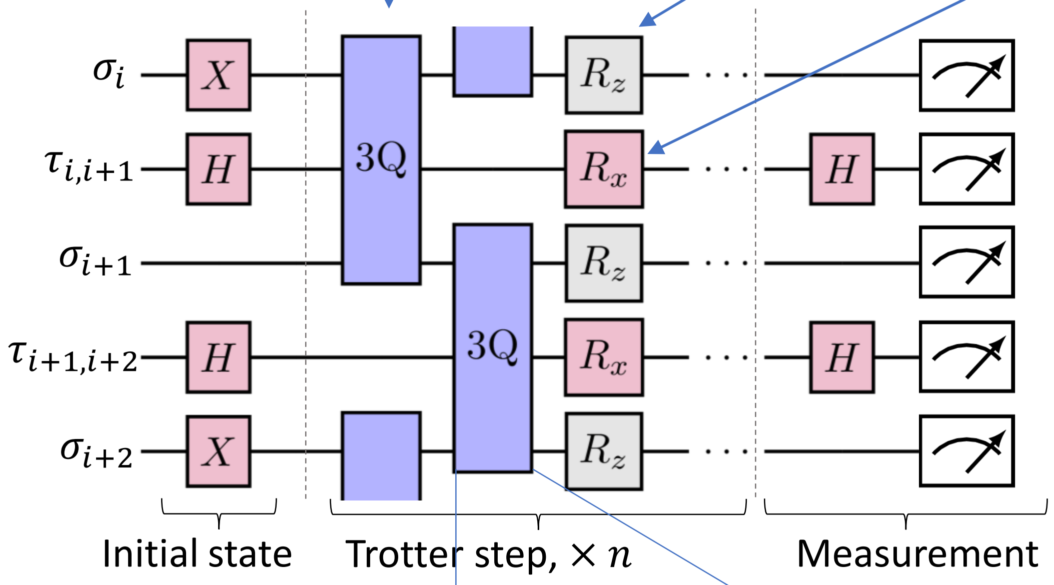
Different from confinement in higher dimensional gauge theories, see, e.g.,
Lumia et al., arXiv:2112.11787
Huffman, Garcia Vera, Banerjee, arXiv:2109.15065
Mueller, Zache, Ott, arXiv:2107.11416, . . .

Our implementation scheme

Mildenberger, Mruczkiewicz, Halimeh, Jiang, Hauke, arXiv:2203.08905

$$H = J\sum_i(\hat{\sigma}_i^+ \, \hat{\tau}_{i,i+1}^z \hat{\sigma}_{i,i+1}^- + \mathrm{h.c.}) + m\sum_i(-1)^i \hat{\sigma}_i^z - f\sum_i \tau_{i,i+1}^x$$

$$G_i^{\mathbb{Z}_2} = -\tau_{i-1,i}^x \sigma_i^z \tau_{i,i+1}^x$$



Challenge: implement this with native gate

$$\sqrt{iSWAP}^\dagger = e^{-\frac{i\pi}{4}(\sigma_1^+\sigma_2^- + h.c.)}$$

Perturbative scheme: Wang, Ge, Xiang, Song, Huang, Song, Guo, Su, Xu, Zheng, Fan, 2111.05048

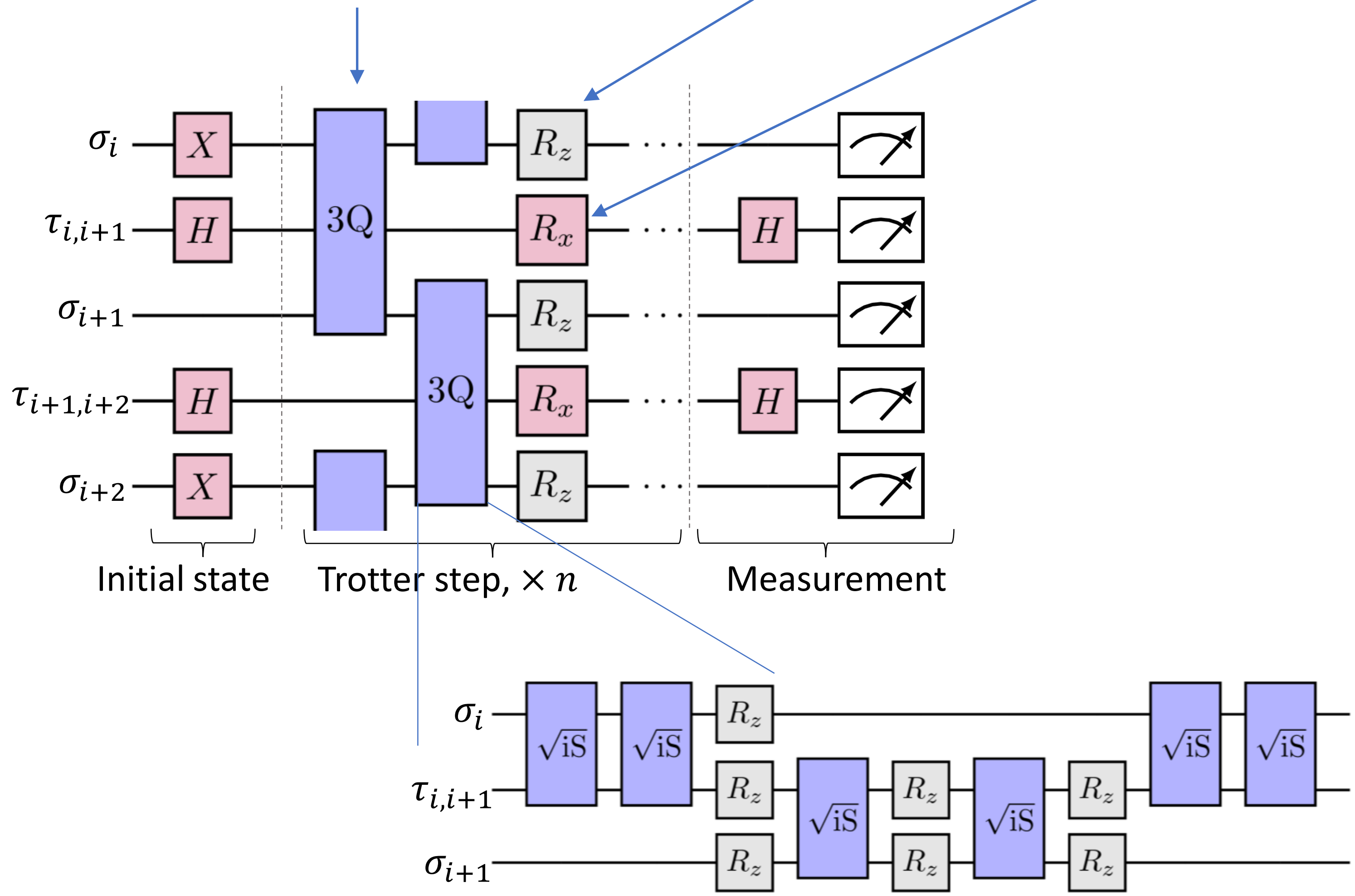
Floquet approach single site: Schweizer et al., *Nat. Phys.* 2019 (see also Görg et al., *Nat. Phys.* 2019)

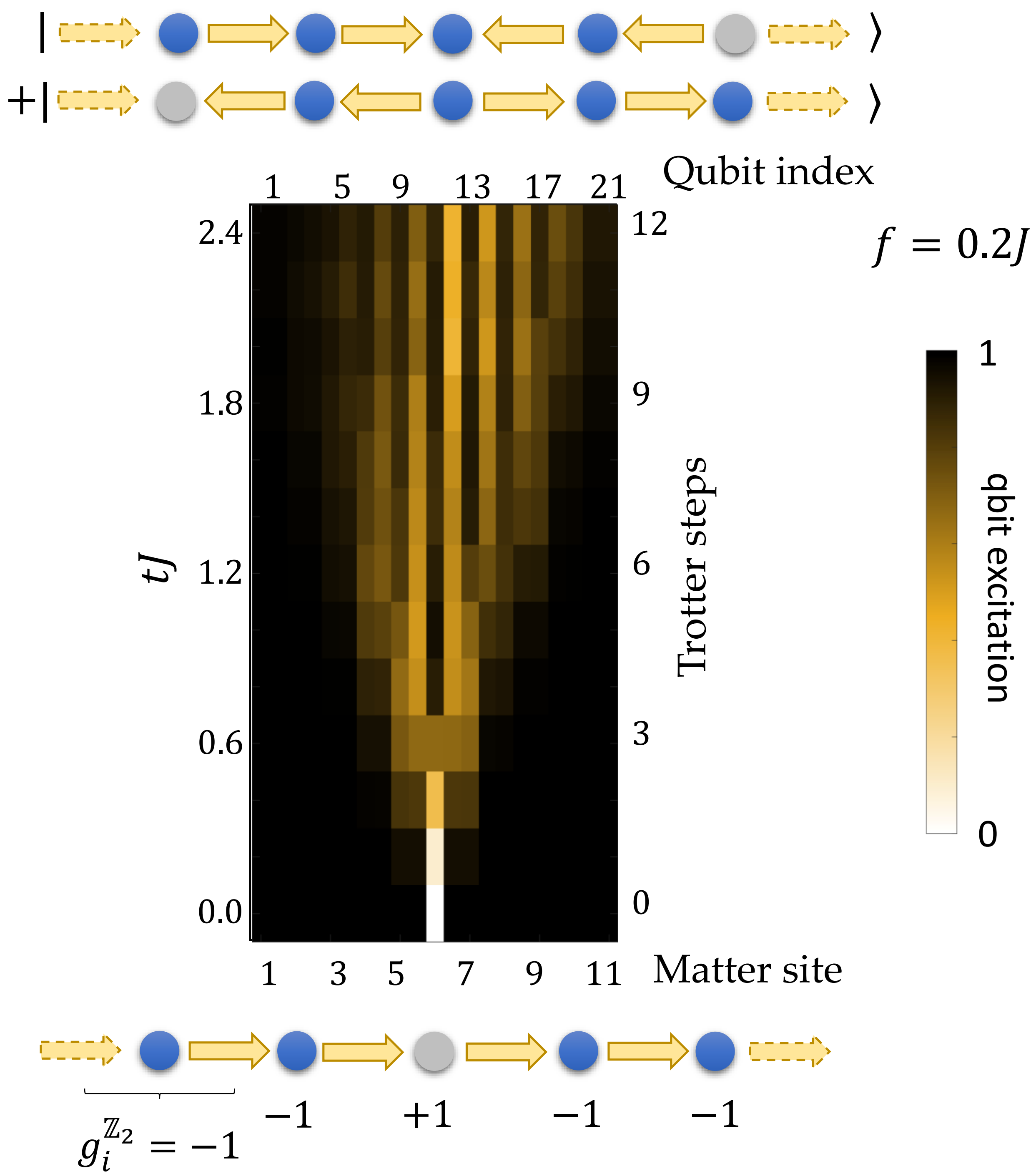
Our implementation scheme

Mildenberger, Mruczkiewicz, Halimeh, Jiang, Hauke, arXiv:2203.08905

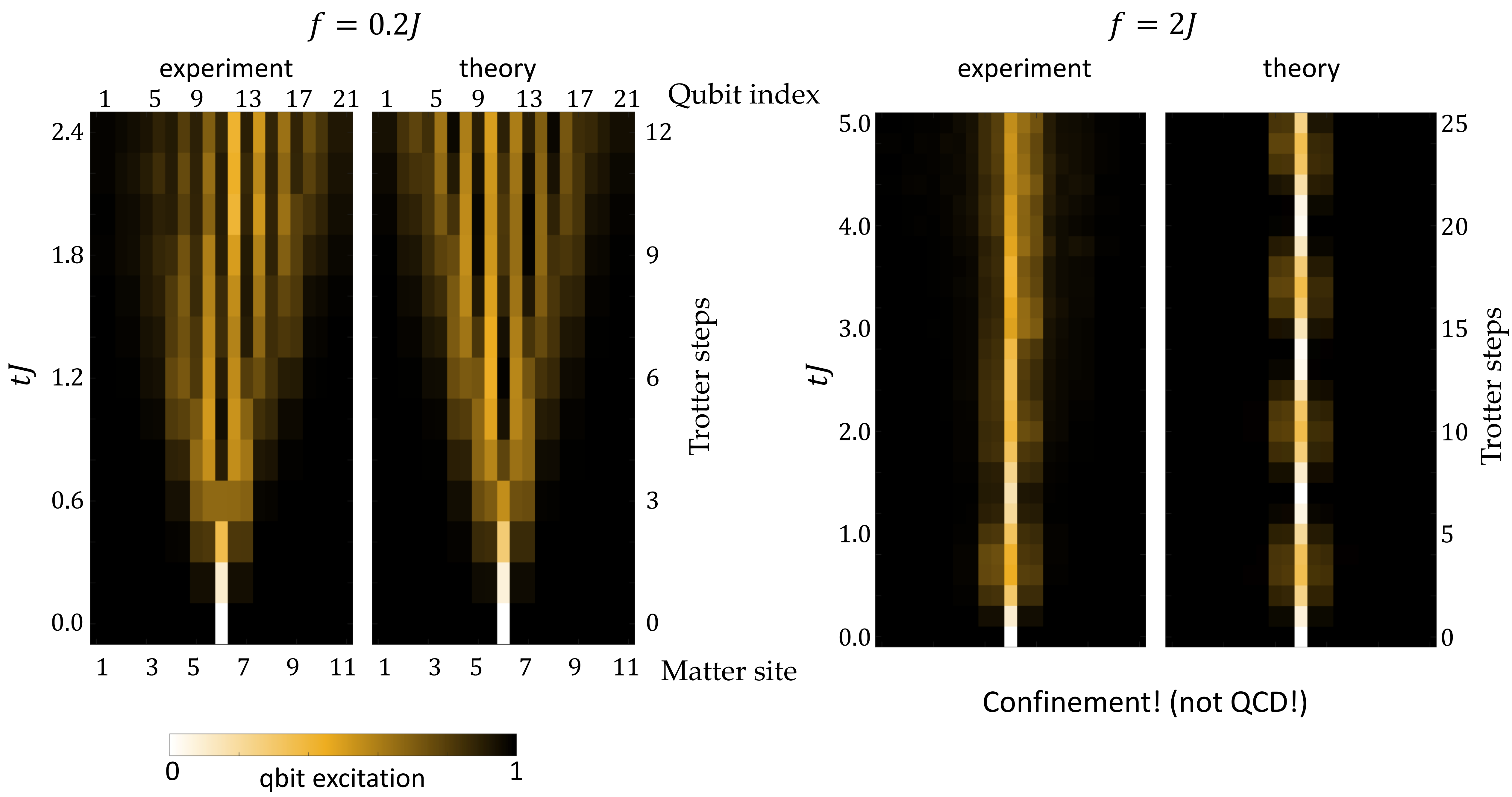
$$H = J\sum_i (\hat{\sigma}_i^+ \hat{\tau}_{i,i+1}^z \hat{\sigma}_{i,i+1}^- + \mathrm{h.c.}) + m\sum_i (-1)^i \hat{\sigma}_i^z - f\sum_i \tau_{i,i+1}^x$$

$$G_i^{\mathbb{Z}_2} = -\tau_{i-1,i}^x \sigma_i^z \tau_{i,i+1}^x$$





Qubit index
1 5 9 13 17 21
2.4
1.8
1.2
0.6
0.0
tJ
12
9
6
3
0
Trotter steps
1 3 5 7 9 11
Matter site
$f = 0.2J$
1
0
qbit excitation
$g_i^{\mathbb{Z}_2} = -1$
−1
+1
−1
−1



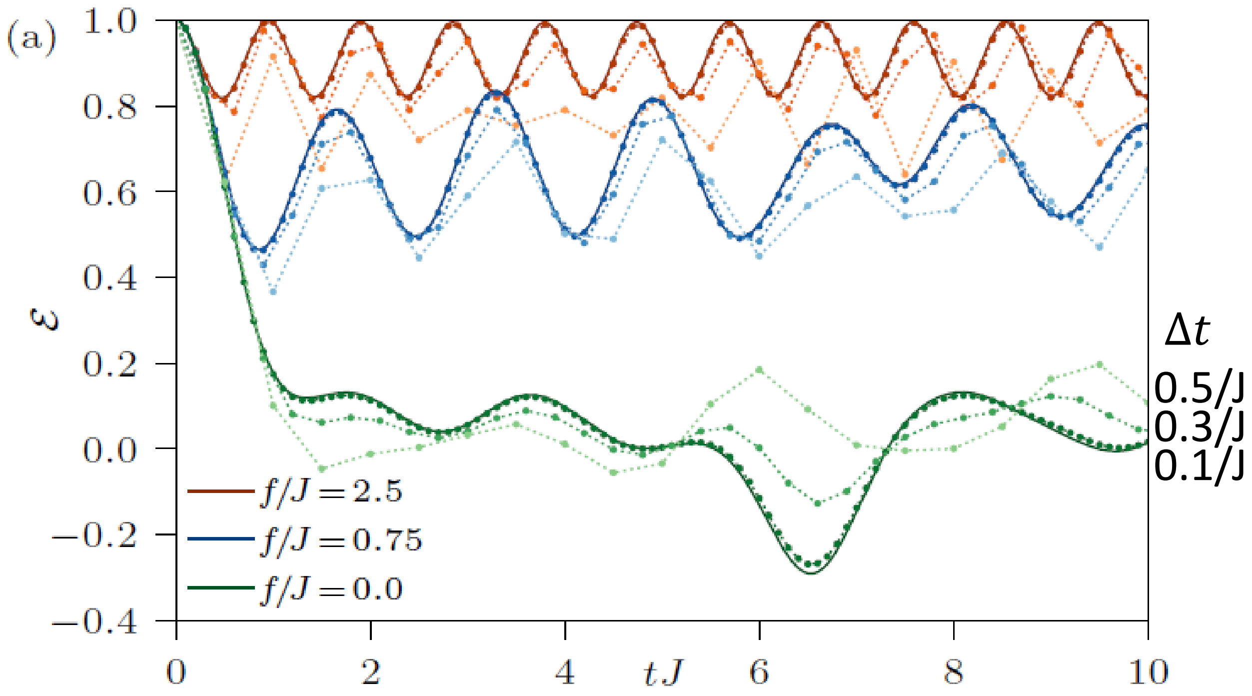
$f = 0.2J$
experiment
theory
Qubit index
1 5 9 13 17 21
tJ
2.4 1.8 1.2 0.6 0.0
Trotter steps
12 9 6 3 0
Matter site
1 3 5 7 9 11
0
qbit excitation
1
$f = 2J$
experiment
theory
5.0 4.0 3.0 2.0 1.0 0.0
25 20 15 10 5 0
Confinement! (not QCD!)

Excellent playground to test error mitigation strategies

Trotter errors

Heyl, Hauke, Zoller, *Science Adv.* 2019
Sieberer, Olsacher, Elben, Heyl, Hauke, Haake, Zoller, npj QInf. 2019
Chinni, Munoz-Arias, Poggi, Deutsch, PRX Quantum 2022
Kargi, Dehollain, Henriques, Sieberer, Olsacher, Hauke, Heyl, Zoller, Langford, Quantum 2023

Comparison of Trotterized time evolution to ideal one



Reality is much more benign than that!

Lloyd, Science 1996

$$||U(t) - U^{(n)}(t)|| = \frac{t^2}{2n} \sum_{l>m=1}^{M} ||[H_l, H_m]|| + \mathcal{O}\left(\frac{t^3}{n^2}\right)$$

Polynomial divergence in t and N (# of qubits)

Many works + refinements:

Aharonov and Ta-Shma, in Proc. 35th STOC

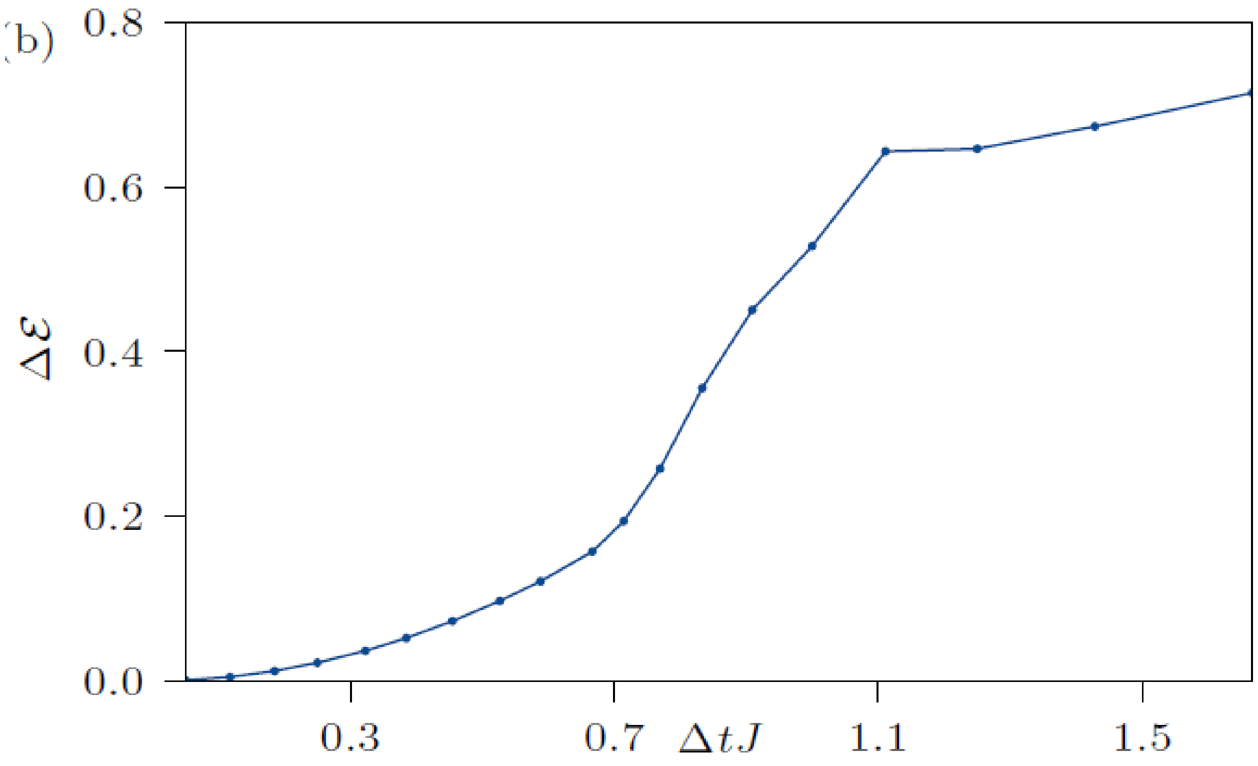
Berry, Ahokas, Cleve, and Sanders, Commun. Math. Phys. 2007

Brown, Munro, and Kendon, Entropy 2010

Childs and Kothari, Lecture Notes in Computer Science 2011

See talk Jacob Watkins

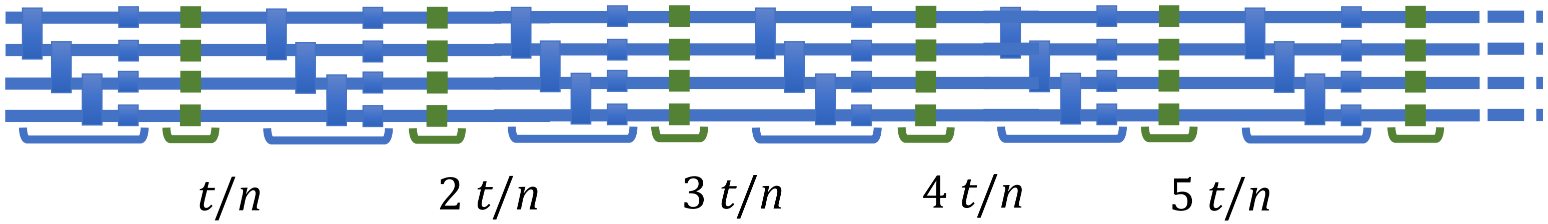
Deviation stays bounded even at large time



Why?

Interpret Trotter sequence as periodic driving

$$U^{(n)}(t) = \left[U_1\left(\frac{t}{n}\right)U_2\left(\frac{t}{n}\right)\cdots U_M\left(\frac{t}{n}\right)\right]^n$$



Period: $\tau = t/n$ ← small expansion parameter

Frequency: $\omega = \frac{2\pi}{\tau}$

See high-frequency expansion, e.g.,
Goldman and Dalibard, PRX 2014, Eckardt 2016, Holthaus 2016

For small period $t/n = \tau$, obtain effective Hamiltonian from perturbative Magnus expansion

$$U^{(n)}(t) = \left[U_1\left(\frac{t}{n}\right)U_2\left(\frac{t}{n}\right)\cdots U_M\left(\frac{t}{n}\right)\right]^n = \mathcal{T}\mathrm{e}^{-i\int_{-\infty}^{t}\mathrm{d}t' H(t')} = e^{-it\mathcal{H}(\tau)}$$

For small $t/n = \tau$ (Magnus)

$$U^{(n)}(t) = \mathrm{e}^{-i\int_{-\infty}^{t} dt_1 H(t_1) - \frac{1}{2}\int_{-\infty}^{t} dt_1 \int_{-\infty}^{t_1} dt_2 [H(t_1),H(t_2)] + \ldots}$$

$$\mathcal{H}(\tau) = H + i\frac{\tau}{2}\sum_{l>m}[H_l, H_m] + \mathcal{O}(\tau^2),$$

Zeroth order: time average = desired Hamiltonian

perturbative correction

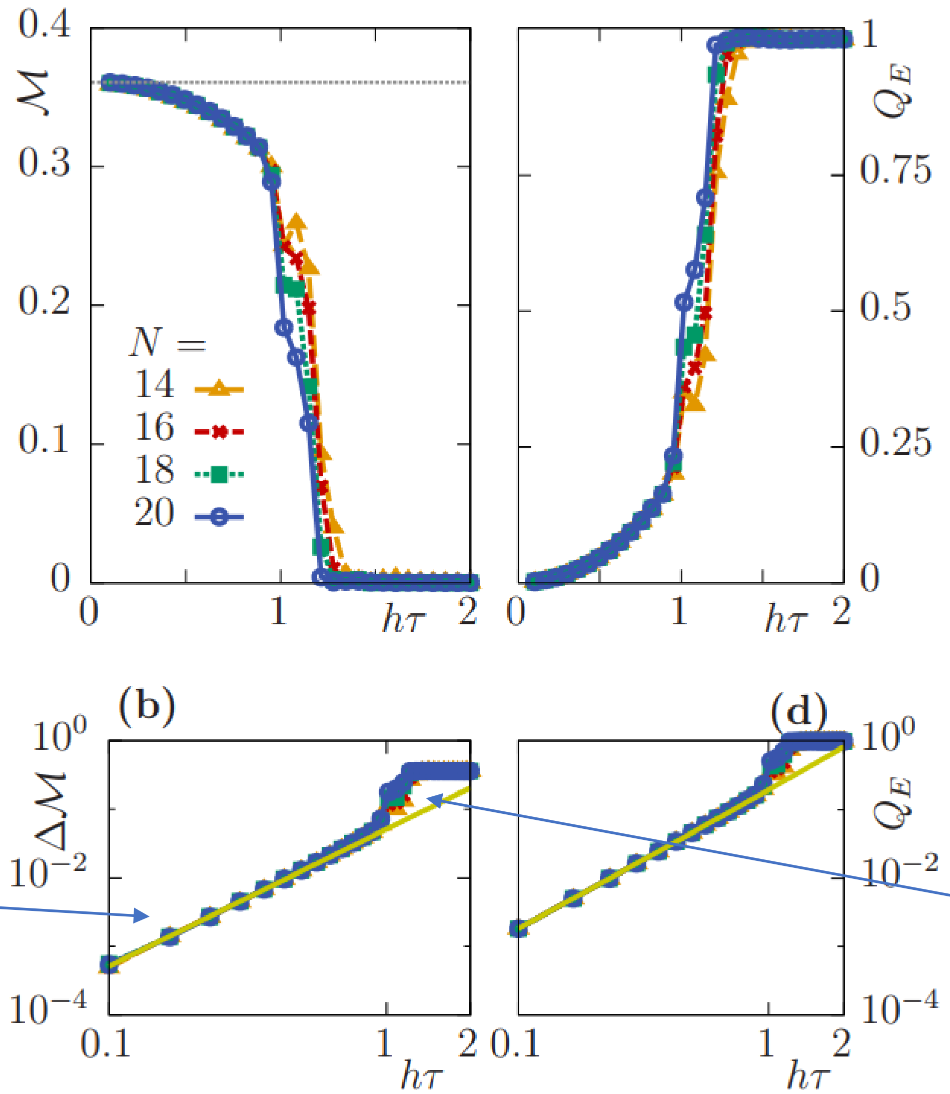
Main difference to Lloyd $||U(t) - U^{(n)}(t)|| = \frac{t^2}{2n}\sum_{l>m=1}^{M}||[H_l, H_m]|| + \mathcal{O}\left(\frac{t^3}{n^2}\right)$

expansion “downstairs” or “upstairs” (resummation of infinite series)

Perturbative expansion gives correct result even at infinite times

local observable

heat $Q_E(n\tau) \equiv \dfrac{E_\tau(n\tau) - E_0}{E_{T=\infty} - E_0}$



$$\Delta M(t) = \langle M(t)\rangle_\tau - \langle M(t)\rangle_{\tau=0}$$

Perturbative expansion in τ

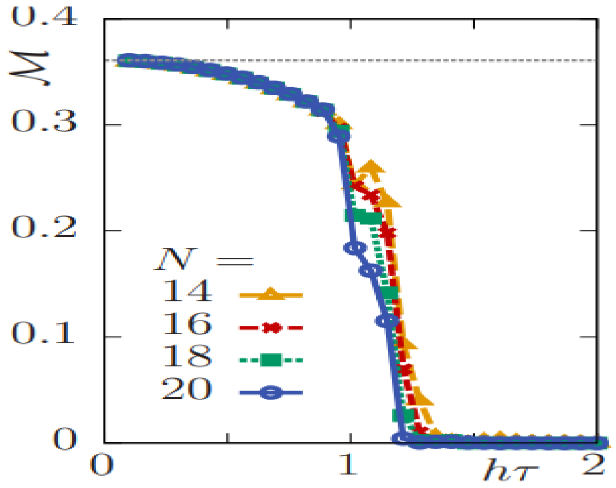
Breakdown of expansion

Caveat

Convergence of perturbative expansion guaranteed only for $t \left\| \tau \sum_{l,m} [H_l, H_m] \right\| \ll 1$

Good news

- Seems robust for testable system sizes
- Experiment can test with few Trotter step sizes whether it has nice scaling
- Reach large t with few Trotter steps



Many more insights and tests

Heyl, Hauke, Zoller, *Science Adv.* 2019

Sieberer, Olsacher, Elben, Heyl, Hauke, Haake, Zoller, npj QInf. 2019

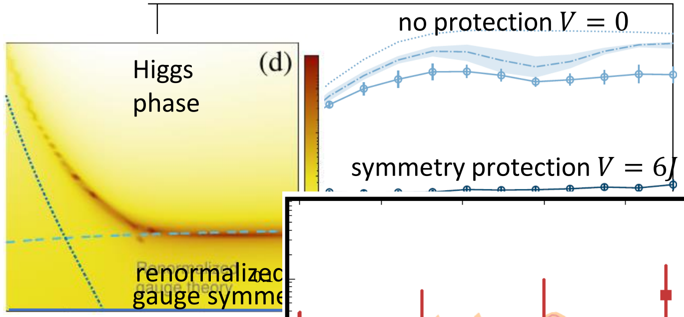
Chinni, Munoz-Arias, Poggi, Deutsch, PRX Quantum 2022

Kargi, Dehollain, Henriques, Sieberer, Olsacher, Hauke, Heyl, Zoller, Langford, Quantum 2023

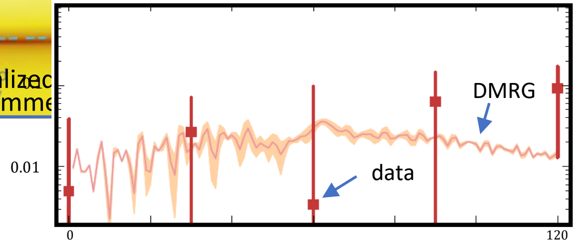
Mildenberger, Mruczkiewicz, Halimeh, Jiang, Hauke, arXiv:2203.08905

Excellent playground to test error mitigation strategies

Gauge-symmetry protection



Mildenberger et al., arXiv:2203.08905



Van Damme, Halimeh, Hauke, arXiv:2010.07338

Yang, Sun, Ott, Wang, Zache, Halimeh, Yuan, Hauke, Pan, *Nature* 2020

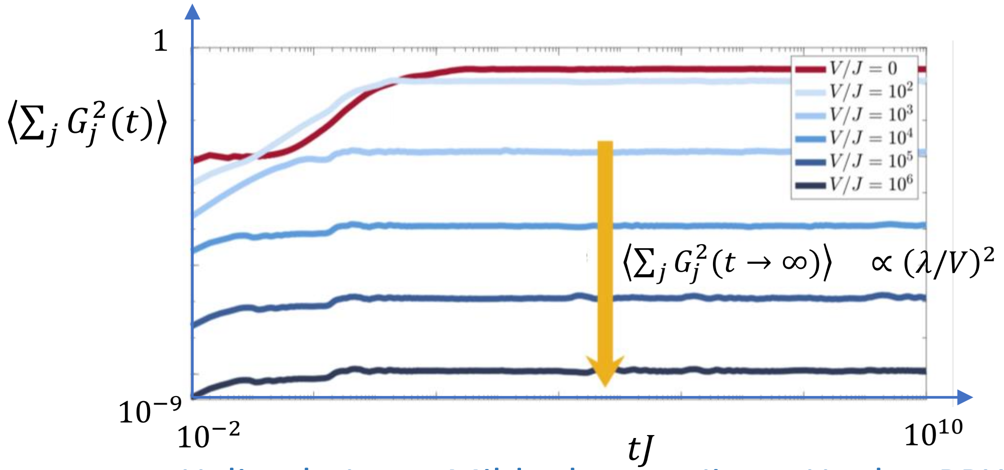
Realistic models always have violations of gauge symmetry

How to control these? suitable energy penalty

$$H_{\text{real}} = H_0 + \lambda H_1 + V H_G$$

$$H_G = \sum_j c_j G_j$$

$$[H_0, G_j] = 0, [H_1, G_j] \neq 0$$



Halimeh, Lang, Mildenberger, Jiang, Hauke, *PRX Quantum* 2021

Here

$$H_0 = J \sum_i (\psi_i^\dagger \tau_{i,i+1}^+ \psi_{i+1} + \text{h.c.}) + m \sum_i (-1)^i \psi_i^\dagger \psi_i$$

$$H_1 = \lambda \sum_i \tau_{i,i+1}^x$$

$$G_i^{U(1)} = (\sigma_i^z + \tau_{i-1,i}^z - \tau_{i,i+1}^z + (-1)^i)/2$$

See also

Stannigel, Hauke, Marcos, Hafezi, Diehl, Dalmonte, Zoller, *PRL* 2014
Yang, Sun, Ott, Wang, Zache, Halimeh, Yuan, Hauke, Pan, *Nature* 2020
Lamm, Lawrence, Yamauchi, *Phys. Rev. D* 2019
Tran, Su, Carney, Taylor, *PRX Quantum* 2021
Kasper, Zache, Jendrzejewski, Lewenstein, Zohar, arXiv:2012.08620, …

Follow ups

Halimeh, Homeier, Zhao, Bohrdt, Grusdt, Hauke, Knolle, PRX Quantum 2022
Halimeh, Lang, Hauke, New J. Phys. 2022
Halimeh, Homeier, Schweizer,. Aidelsburger, Hauke, Grusdt, PRR 2022
Van Damme, Mildenberger, Grusdt, Hauke, Halimeh, Comm. Phys. 2021

Explanations/bounds

- Degenerate perturbation theory $P_0 H_{\text{eff}} P_0 = P_0 H_0 P_0 + \mathcal{O}\left(\frac{\lambda^2}{V}\right)$
  Emergence of renormalized Gauss' law
  Van Damme, Lang, Hauke, Halimeh, arXiv:2104.07040

  Convergence not ensured at large L

- Emergent symmetry Chubb, Flammia, *J. Math. Phys.* 2017

  Given small breaking of symmetry, $\left\|\left[\mathrm{e}^{\mathrm{i}\alpha H_G}, H\right]\right\| < \lambda$,
  + energy gap between symmetry sectors $\sim V$
  $\Longrightarrow$ $\exists$ exact symmetry with $\left\|\left[\mathrm{e}^{\mathrm{i}\alpha \tilde{H}_G}, H\right]\right\| = 0$, $\left\|\mathrm{e}^{\mathrm{i}\alpha \tilde{H}_G} - \mathrm{e}^{\mathrm{i}\alpha H_G}\right\| < \left(\frac{\lambda}{V}\right)^2$

  Energy gap closes with L

- Quantum Zeno effect Facchi, Pascazio, *PRL* 2002; Tran, Su, Carney, Taylor, *PRX Quantum* 2021
  Halimeh, Lang, Hauke, *New J. Phys.* 2022

  $$\left\|U(t) O U(t)^\dagger - e^{-iH_0 t} O e^{iH_0 t}\right\| \le t L^2 \big(V_0(\lambda)\big)^2 / V$$

  Bound diverges with L, t

- Constrained dynamics Gong, Yoshioka, Shibata, Hamazaki, *PRL, PRA* 2020; Halimeh, Lang, Hauke, *NJP* 2022

  $$\left\|U(t) O U(t)^\dagger - e^{-iH_0 t} O e^{iH_0 t}\right\| \le t^2 \big(V_0(\lambda)\big)^3 / V$$

  Bound diverges with t

- Results from periodically driven systems Abanin, De Roeck, Ho, Huveneers, *Comm. in Math. Phys.* 2017
  Halimeh, Lang, Mildenberger, Jiang, Hauke, *PRX Quantum* 2021

  Issues at large L

We can use such protection also to tune the gauge symmetry

Add $U(1)$ gauge protection to $\mathbb{Z}_2$ LGT

$$H_{\mathbb{Z}_2} = J\sum_i(\hat{\sigma}_i^+ \hat{\tau}^z_{i,i+1}\hat{\sigma}^-_{i+1} + \text{h.c.}) + m\sum_i(-1)^i\hat{\sigma}_i^z - f\sum_i \tau^x_{i,i+1}$$

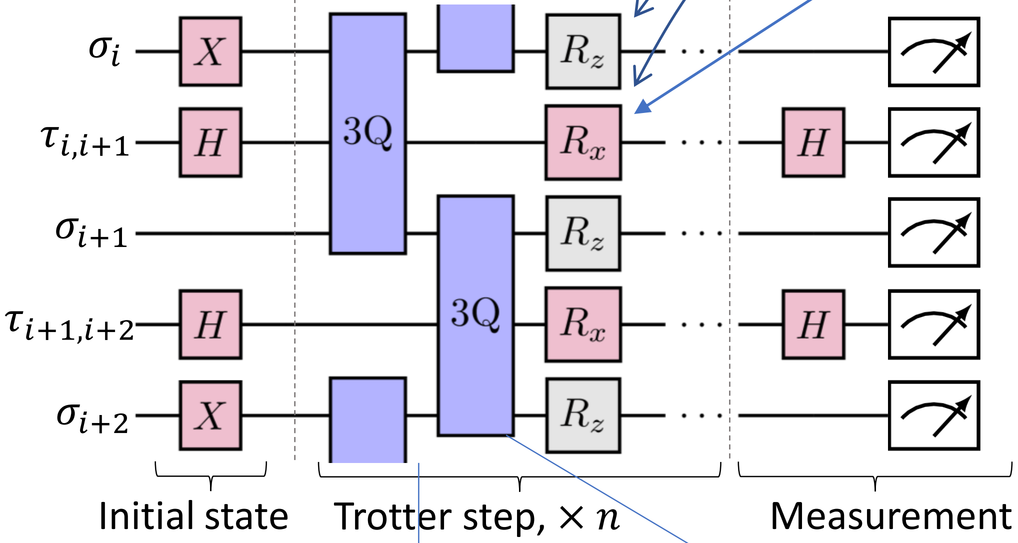
$$[H_{\mathbb{Z}_2}, G_i^{\mathbb{Z}_2}] = 0 \qquad G_i^{\mathbb{Z}_2} = -\tau^x_{i-1,i}\sigma^z_i\tau^x_{i,i+1}$$

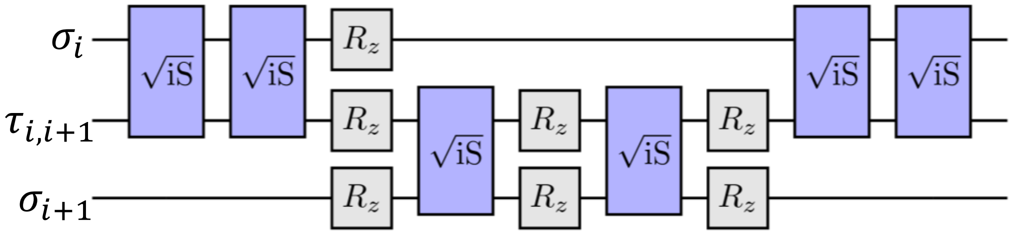
$$H = H_{\mathbb{Z}_2} + VH_G$$

$$[H_{\mathbb{Z}_2}, G_i^{U(1)}] \neq 0$$

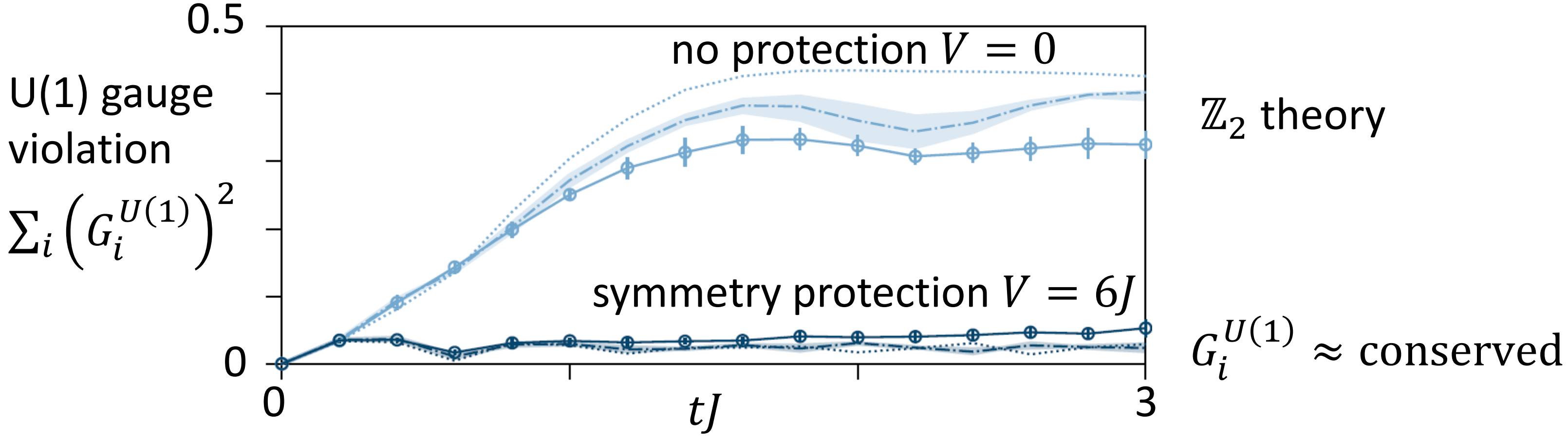
$$G_i^{U(1)} = (\sigma_i^z + \tau^x_{i-1,i} - \tau^x_{i,i+1} + (-1)^i)/2$$

$$H_G = \sum_i c_i\, G_i^{U(1)}$$



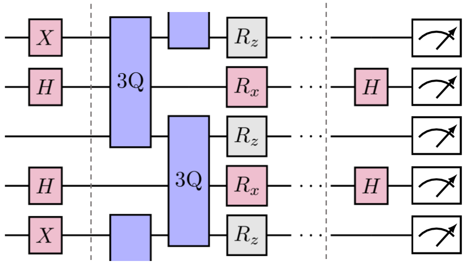


Gauge protection tunes symmetry

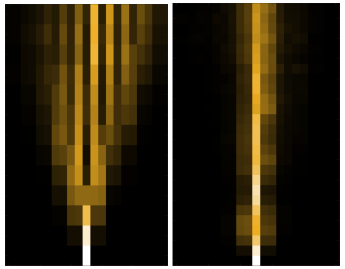


Our goals

1. Design gauge-theory implementations

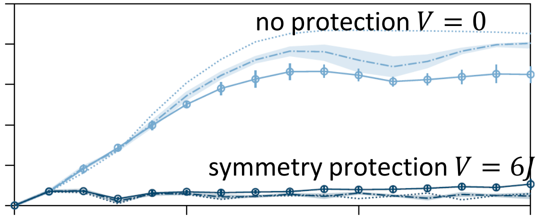


2. Do some interesting physics

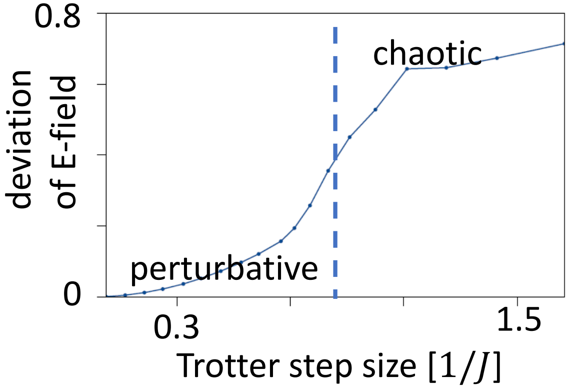


3. Test error mitigation strategies

gauge-symmetry protection



Trotter errors



Mildenberger, Mruczkiewicz, Halimeh, Jiang, Hauke, arXiv:2203.08905

Entanglement witnessing in LGTs

See session on Thursday (Caroline Robin, Morten Hjorth-Jensen, Niklas Mueller, Axel Pérez-Obiol,…)

Reminder: Usual definition of entanglement

Given $\mathcal{H} = \mathcal{H}^A \otimes \mathcal{H}^B$

$|\psi\rangle$ entangled $\Leftrightarrow |\psi\rangle \neq |\varphi_A\rangle \otimes |\zeta_B\rangle$ (= product)

ρ entangled $\Leftrightarrow \rho \neq \sum_\lambda \rho_\lambda^A \otimes \rho_\lambda^B$ (= separable)

Entanglement measures

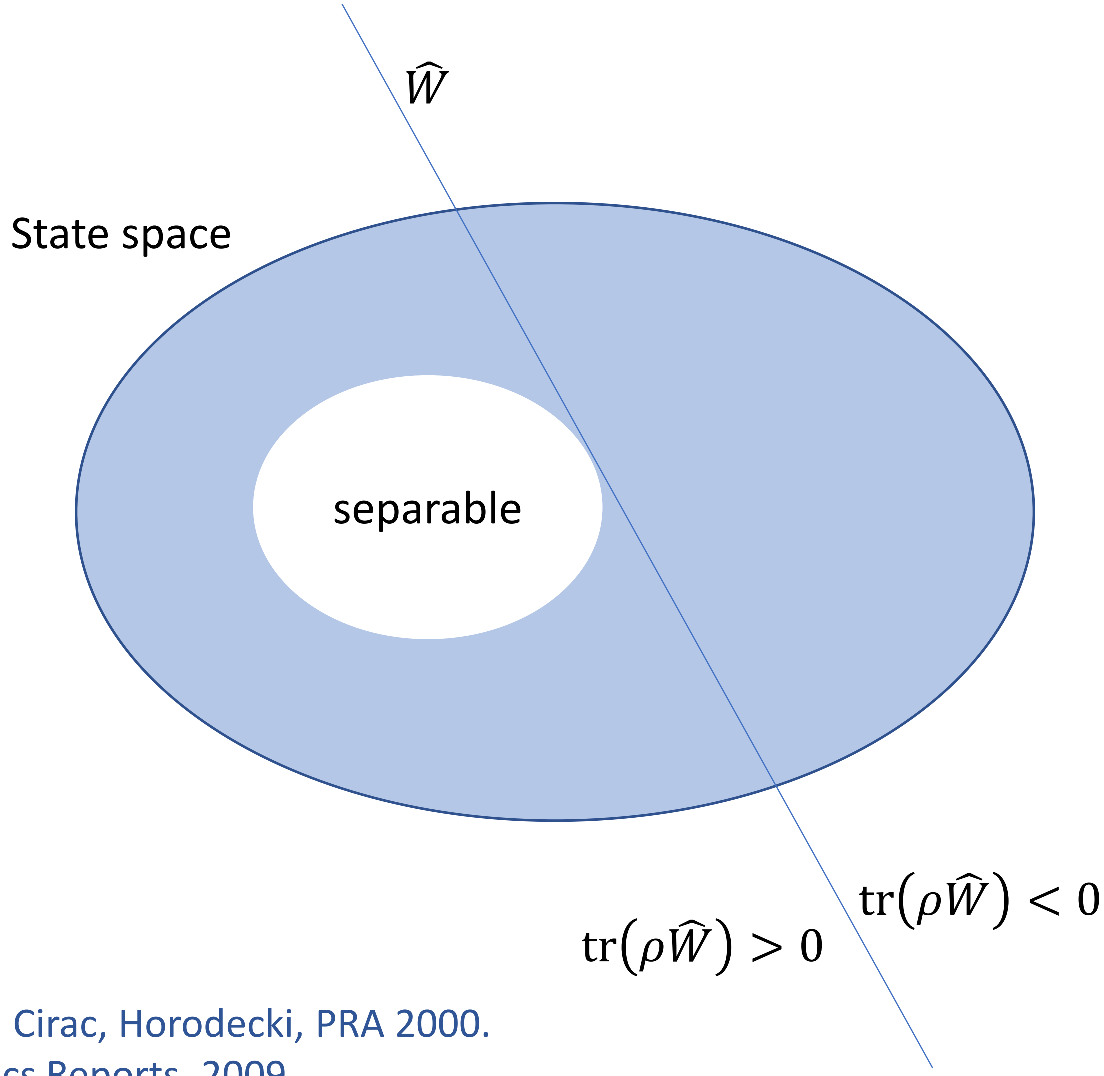
- Von Neumann entropy ($S_{\mathrm{vN}} = -\mathrm{tr}(\rho_A \log \rho_A)$)
- (logarithmic) negativity
- Entanglement cost
- Geometric measure of entanglement
- . . .

Non-linear functions of ρ !

Measurement methods

- Full state tomography (exponential scaling)
- Copies (e.g., theory Zoller group, experiments Greiner group)
- Random unitaries (see talk Niklas Mueller)

Simpler approach: entanglement witnessing



$\widehat{W}$ observable (simple measurement) with

$$\mathrm{tr}(\rho\widehat{W}) < 0 \quad \Longrightarrow \text{ entangled}$$

But

$$\mathrm{tr}(\rho\widehat{W}) > 0 \quad \not\Rightarrow \text{separable}$$

no ordering relation

Lewenstein, Kraus, Cirac, Horodecki, PRA 2000.
Gühne, Toth, Physics Reports, 2009.
Chruscinski, Sarbicki, Journal of Physics A 2014.

How to construct a witness?

Given $\mathcal{H} = \mathcal{H}^A \otimes \mathcal{H}^B$

Define $\hat{C} = \sum_\lambda \hat{C}_\lambda^A \otimes \hat{C}_\lambda^B$

Compute $c_1 \leq \mathrm{tr}\left(\rho_{\mathrm{sep}}\, \hat{C}\right) \leq c_2$

$\Longrightarrow$ $\widehat{W}_1 = \hat{C} - c_1$
$\widehat{W}_2 = c_2 - \hat{C}$

$\mathrm{tr}\left(\rho \widehat{W}_{1,2}\right) < 0 \Longrightarrow$ entangled

Need also to show

$\exists \rho_{\mathrm{ent}}$ s. t. $\mathrm{tr}\left(\rho_{\mathrm{ent}}\, \widehat{W}_{1,2}\right) < 0$

example

$\mathcal{H} = \mathbb{C}^2 \otimes \mathbb{C}^2$

$$\hat{C} = \sum_{\alpha = x,y,z} \sigma_\alpha^A \otimes \sigma_\alpha^B$$

$\mathrm{tr}\left(\rho_{\mathrm{sep}}\, \hat{C}\right) \leq 2$ (prove using Robertson–Schrödinger uncertainty relation)

$\Longrightarrow$ $\widehat{W}_2 = 2 - \hat{C}$

$|\Uparrow\Uparrow\rangle + |\Downarrow\Downarrow\rangle$ $\mathrm{tr}\left(\rho \widehat{W}_2\right) = -1 \Longrightarrow$ entangled

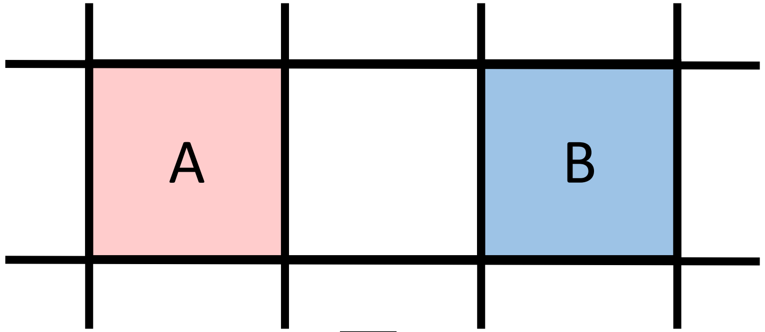
Hauke, Bonnes, Heyl, Lechner, Front. Phys. 2015
Hauke, Sewell, Mitchell, Lewenstein, PRA 2013
Cramer, Plenio, Wunderlich, PRL 2011

What about gauge theories?

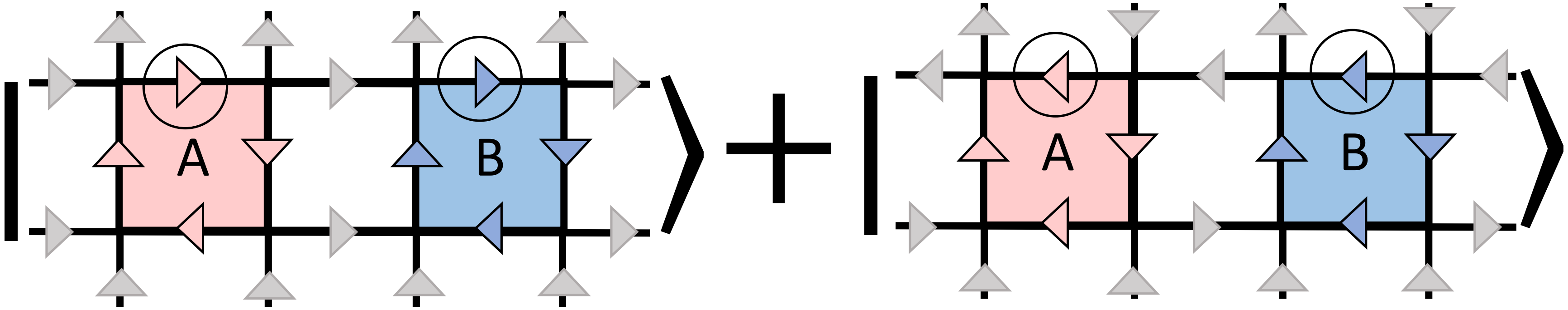
U(1) pure gauge, quantum link model (S=1/2 on links)

$$H = \sum_{\text{plaquettes } p} (U_p + U_p^\dagger) + \sum_{\text{links } l} E_l$$

$$U_p = \sigma_{l_1}^+ \sigma_{l_2}^+ \sigma_{l_3}^- \sigma_{l_4}^- \qquad E_l = \sigma_l^z$$



$$G_i^{U(1)} = \sum_{\ell \in \text{vertex } i} \sigma_\ell^z = \text{const}$$

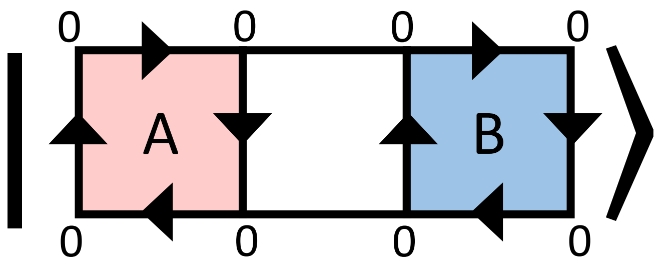


Is A entangled with B?

$|\Uparrow\Uparrow\rangle + |\Downarrow\Downarrow\rangle$

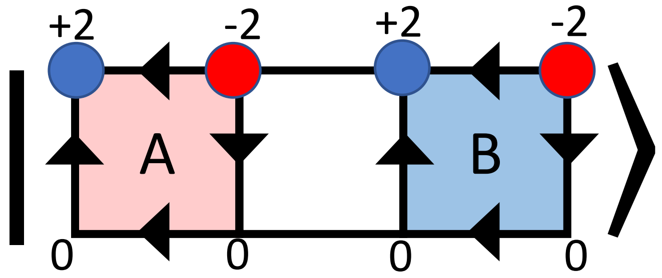
Need to keep into account different superselection sectors (background charges)!

$G_i^{U(1)} = \sum_{\ell \in \text{vertex } i} \sigma_\ell^z$ =const   Defines superselection sectors $\mathcal{Z}_A, \mathcal{Z}_B$



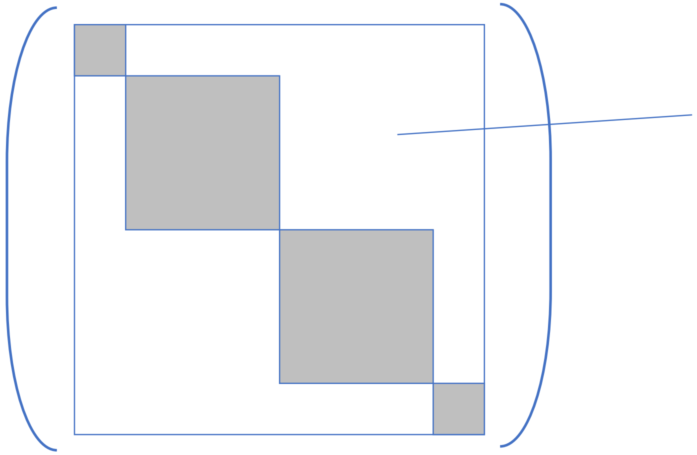
$\mathcal{Z}_A = \{0,0,0,0\}$ $\mathcal{Z}_B = \{0,0,0,0\}$

there is no gauge-invariant operation coupling these two states!



$\mathcal{Z}_A = \{0,0,-2,+2\}$ $\mathcal{Z}_B = \{0,0,-2,+2\}$

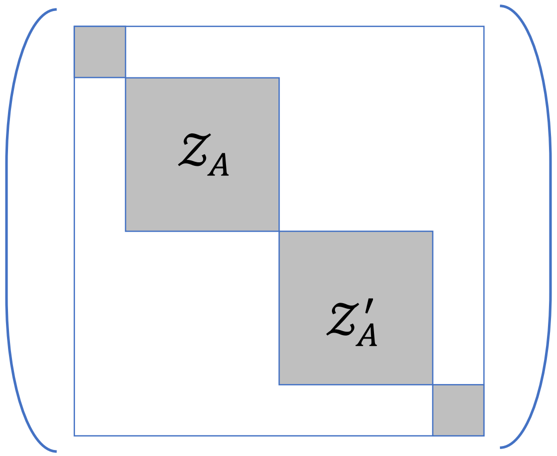
$$\rho_A \to \sum_{\mathcal{Z}_A} \Pi_{\mathcal{Z}_A} \rho_A \Pi_{\mathcal{Z}_A} =$$



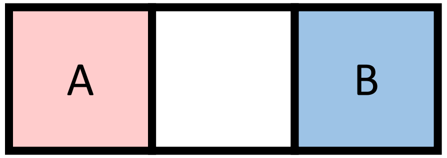
No coherences between superselection sectors

Definition of separable states

$\rho =$



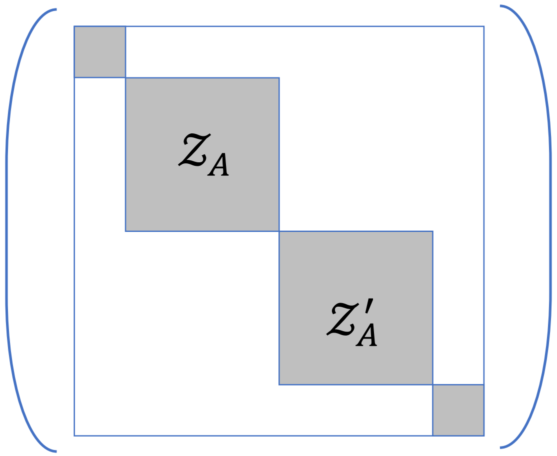
$$S = -\sum_{\mathcal{Z}_A} p(\mathcal{Z}_A) \log p(\mathcal{Z}_A) + \sum_{\mathcal{Z}_A} p(\mathcal{Z}_A)\, S_{\mathcal{Z}_A}$$



$$\rho_{\mathrm{sep}}|_{\mathcal{A}\cup\mathcal{B}} = \bigoplus_{(Z_{\mathcal{A}},Z_{\mathcal{B}})} p(Z_{\mathcal{A}}, Z_{\mathcal{B}})\rho_{\mathcal{A}}(Z_{\mathcal{A}}) \otimes \rho_{\mathcal{B}}(Z_{\mathcal{B}})$$

Buividovich and Polikarpov, Physics Letters B, 2008
Donnelly, Physical Review D, 2012
Casini, Huerta, Rosabal, Physical Review D, 2014
Ghosh, Soni, Trivedi, Journal of High Energy Physics, 2015
Aoki, Iritani, Nozaki, Numasawa, Shiba, Tasaki, Journal of High Energy Physics, 2015.
Van Acoleyen, Bultinck, Haegeman, Marien, Scholz, Verstraete,Physical Review Letters, 2016.
Radicevic, 2022.

Definition of separable states

$\rho =$ 

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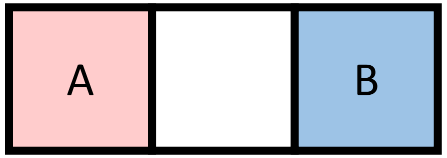

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Note: does not need to be postulated, can be **derived from measurable properties**

state is product $\Leftrightarrow \langle C_A \otimes C_B \rangle = \langle C_A \rangle \langle C_B \rangle \quad \forall\, C_A, C_B$ such that $C_A \in \mathcal{A}_g$ and $C_B \in \mathcal{B}_g$ (i.e., $[C_{A,B}\,, G_i] = 0$)

Algebras in A and B compatible with Gauss' law

Advantage: Never need to write unphysical state that is then projected $\sum_{\mathcal{Z}_A} \Pi_{\mathcal{Z}_A} \rho_A \Pi_{\mathcal{Z}_A}$

Panizza, Costa de Almeida, Hauke, JHEP, 10.1007/JHEP09(2022)196 (2022)
(adapting Banuls, Cirac, Wolf, PRA 2007 from fermions to LGT)

How to construct a witness in a LGT?

Panizza, Costa de Almeida, Hauke, JHEP, 10.1007/JHEP09(2022)196 (2022)

Given subsystems A, B and $C_\lambda^A \in \mathcal{A}_g$, $C_\lambda^B \in \mathcal{B}_g$

Define $$\hat{C} = \sum_\lambda \hat{C}_\lambda^A \otimes \hat{C}_\lambda^B$$

Compute $$c_\lambda^A(\mathcal{Z}_A) = \langle \psi_{\mathcal{Z}_A} | \hat{C}_\lambda^A | \psi_{\mathcal{Z}_A} \rangle \quad \forall \, |\psi\rangle \text{ with } \mathcal{Z}_A$$

$$c_\lambda^B(\mathcal{Z}_B) = \langle \psi_{\mathcal{Z}_B} | \hat{C}_\lambda^B | \psi_{\mathcal{Z}_B} \rangle \quad \forall \, |\psi\rangle \text{ with } \mathcal{Z}_B$$

Easy to compute: do it for each $\mathcal{Z}_A$ individually!

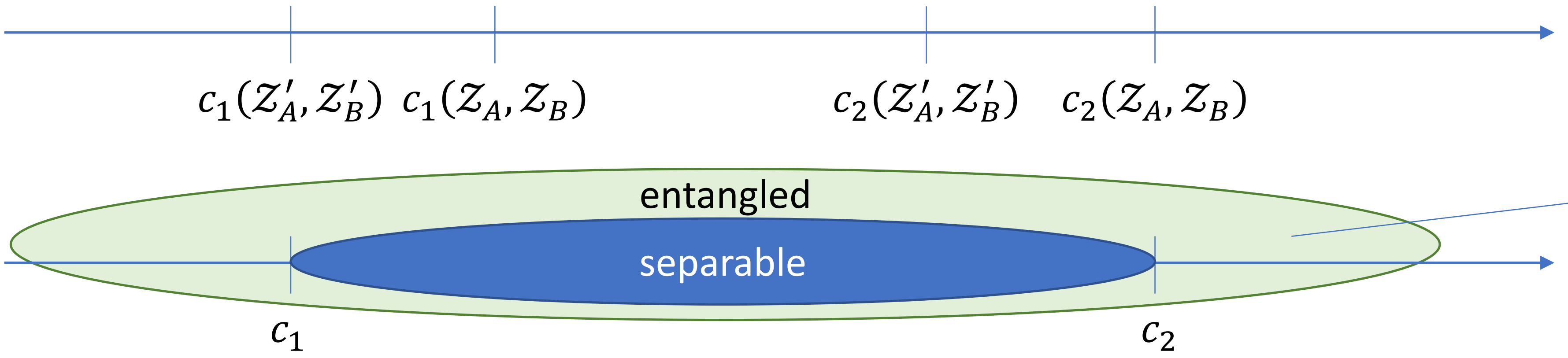
Find $$c_1(\mathcal{Z}_A, \mathcal{Z}_B) = \min_\psi \sum_\lambda c_\lambda^A(\mathcal{Z}_A) \times c_\lambda^B(\mathcal{Z}_B)$$

such that $\mathcal{Z}_A, \mathcal{Z}_B$ compatible

Find $$c_1 = \min_{(\mathcal{Z}_A, \mathcal{Z}_B)} c_{1,2}(\mathcal{Z}_A, \mathcal{Z}_B)$$

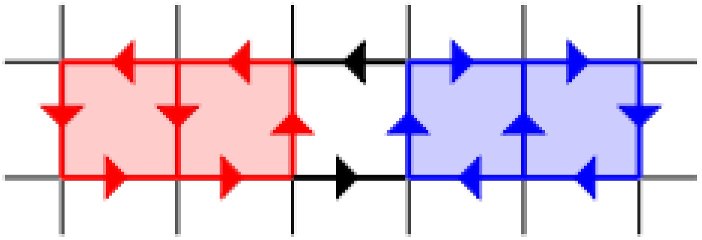
$$\Longrightarrow \quad \widehat{W}_1 = \hat{C} - c_1$$

Analogously with min $\widehat{W}_2 = c_2 - \hat{C}$

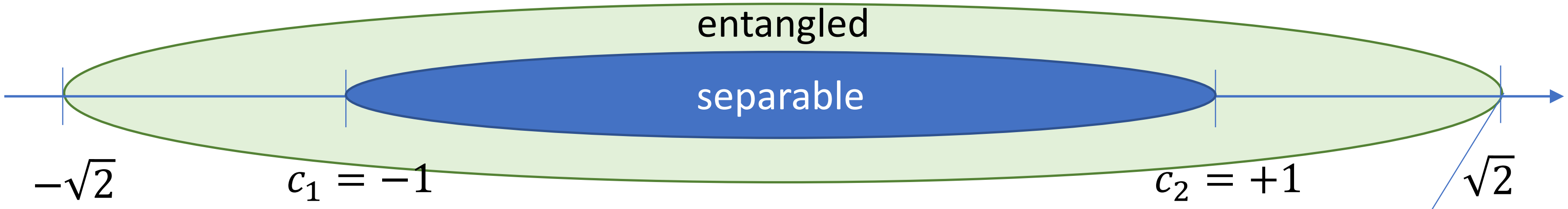


Need also to show that exists

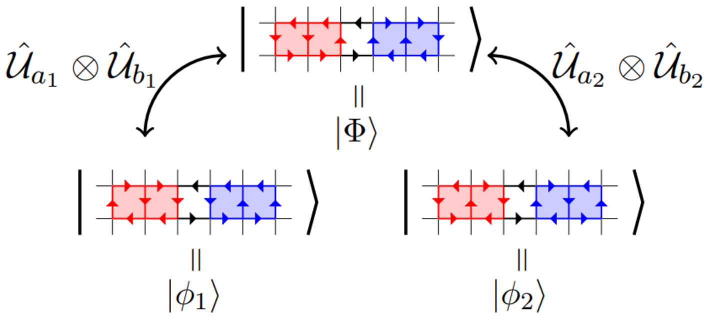
Example



$$\hat{C} = \hat{\mathcal{U}}_{a_1} \otimes \hat{\mathcal{U}}_{b_1} + \hat{\mathcal{U}}_{a_2} \otimes \hat{\mathcal{U}}_{b_2} \qquad \hat{\mathcal{U}}_{x_i} = \hat{U}_{x_i} e^{i\phi_{x_i}} + \hat{U}^{\dagger}_{x_i} e^{-i\phi_{x_i}}$$



detects $\quad |\xi\rangle = \frac{1}{2}|\phi_1\rangle + \frac{1}{\sqrt{2}}|\Phi\rangle + \frac{1}{2}|\phi_2\rangle$

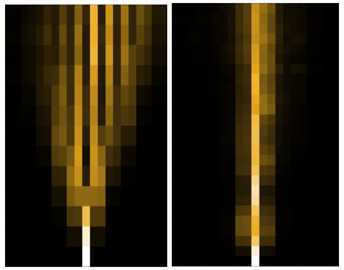


Further work

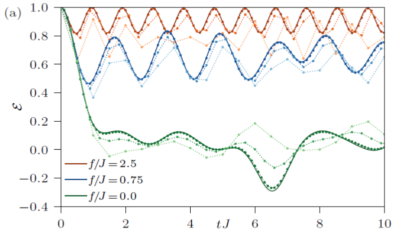
- Ways to maximize efficiently over superselection sectors for large subsystems?
- Which witnesses tell us something useful?
- Generalization to non-Abelian (for gauge theory with fermionic matter, see Panizza, Costa de Almeida, Hauke, JHEP 2022).
- Test in experiment

Conclusions

Take away messages



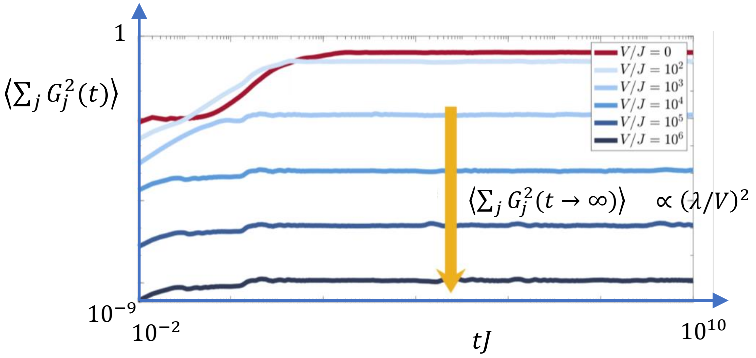
- Quantum simulation of gauge theory is reaching system sizes to do some interesting physics



- Much leeway for improving bounds on Trotter errors in practice
- Energy penalties can controllably suppress gauge-symmetry violations

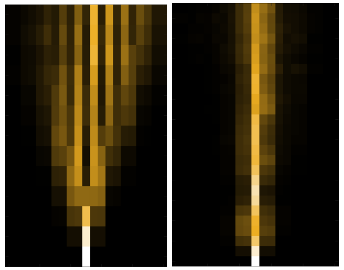
→ Deep questions about emergence of gauge invariance

Foerster, Nielsen, Ninomiya, *Physics Letters* 1980 („*Light from Chaos*“)
Fradkin, Shenker, *PRD* 1979
Poppitz, Shang, *Int. J. Mod. Phys. A* 2008
Komargodski, Sharon, Thorngren, Zhou, *arXiv* 2017
Göschl, Gattringer, Sulejmanpasic *arxiv* 2018
Unmuth-Yockey, Zhang, Bazavov, Meurice, S.-W. Tsai, *PRD* 2018
Wetterich, Nuclear Physics B 2017

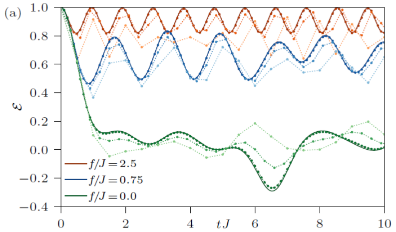


Take away messages

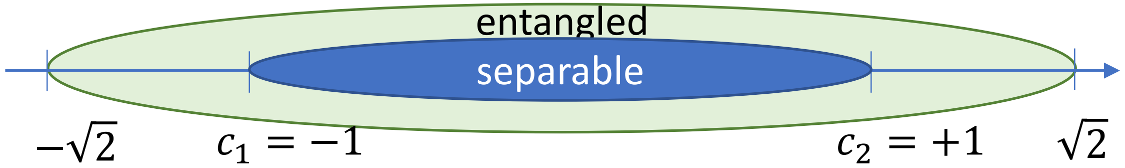
- Quantum simulation of gauge theory is reaching system sizes to do some interesting physics



- Much leeway for improving bounds on Trotter errors in practice



- Energy penalties can controllably suppress gauge-symmetry violations
- Witnesses may be an efficient way of accessing entanglement in LGTs



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Markus Heyl (Augsburg)
Peter Zoller + group (Innsbruck)
. . .

Thank you!

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erc ERC Starting Grant *StrEnQTh* (GA 804305)

Funded by the European Union under Horizon Europe Programme – Grant Agreement 101080086 — NeQST

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