

Random measurements and entanglement for nuclear and particle physics exploration


Niklas Mueller
University of Washington

based on work with Jacob Bringewatt, Jon Kunjummen, arXiv:2303.15519
and Torsten Zache, Robert Ott, Phys. Rev. Lett. 129 (2022) 011601

Shout-out and related developments

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- Quantum simulation of nuclear physics problems getting off the ground

 Quantum simulating gauge theories:

Savage, Zohar, Stryker, Meurice, Singh,
Yao, Hauke, Gonzales Cuadra, Ott

Quantum Simulation of Lattice Gauge Theories in more than 1+1 D	Erez Zohar
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next: 2+1d?	Tuesday
Aula Renzo Leonardi, ECT*	10:10 - 10:50

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10:10 - 10:50

- Entanglement is probably a good frontier

Does entanglement in equilibrium

Robin, Hjorth-Jensen, Yao, Perez-Obiol

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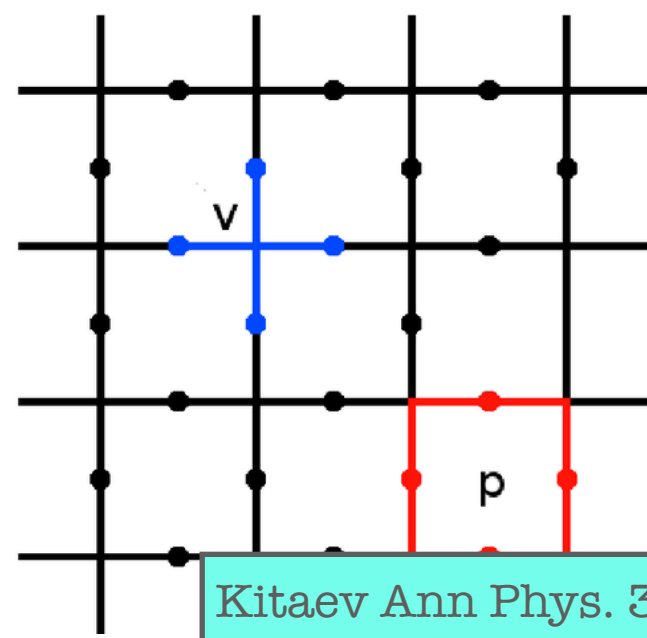
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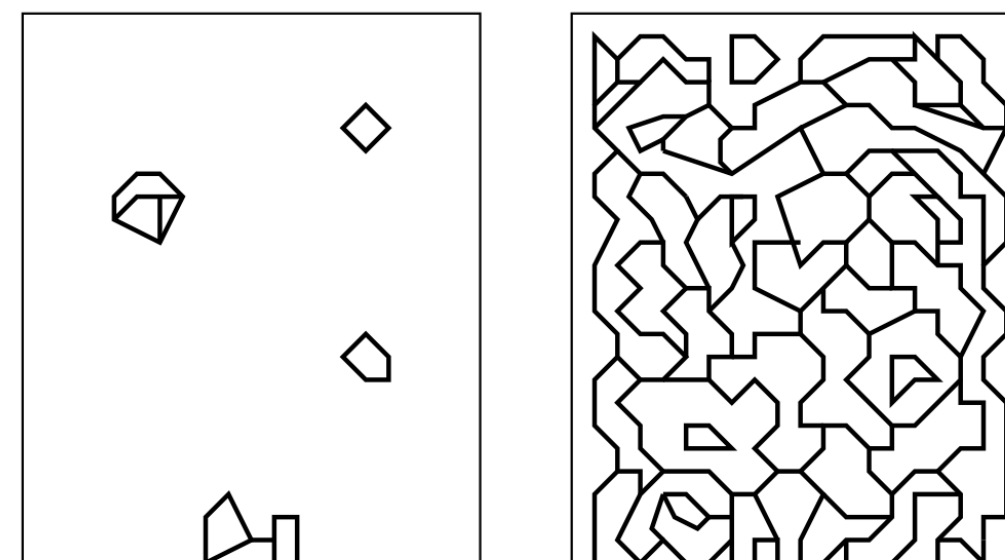
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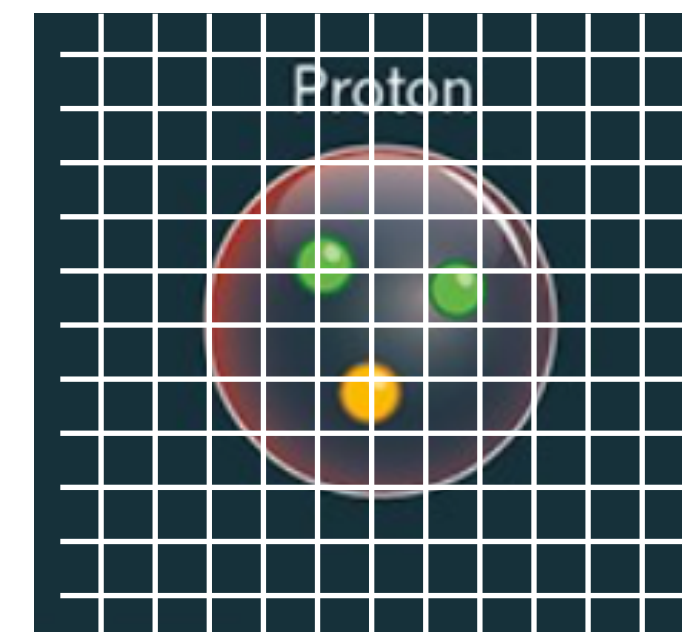
- Coincidentally, that's what others think, too



quantum information science and technology



condensed matter



high energy and nuclear physics

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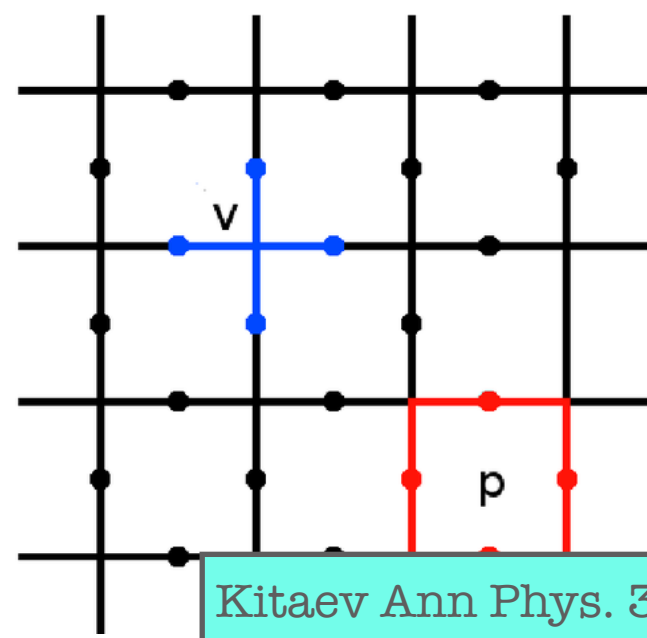
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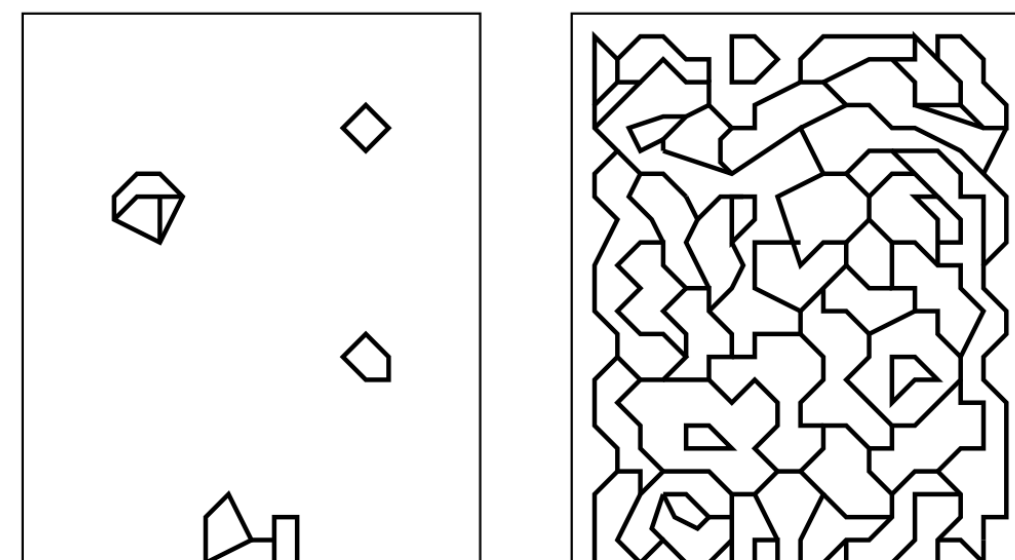
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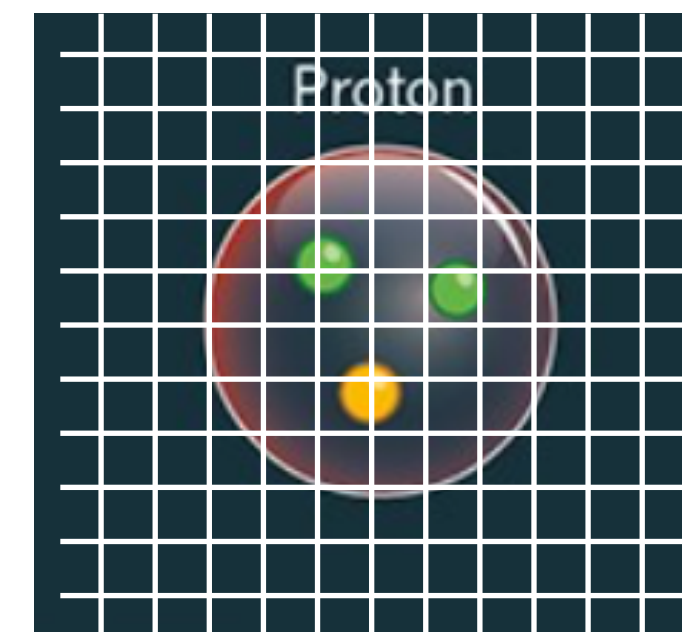


Kitaev Ann Phys. 303 (2003), 2



Levin, Wen PRB 71, 045110 (2005)

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Quantum Error Correction with Gauge Symmetries

Abhishek Rajput

Wednesday

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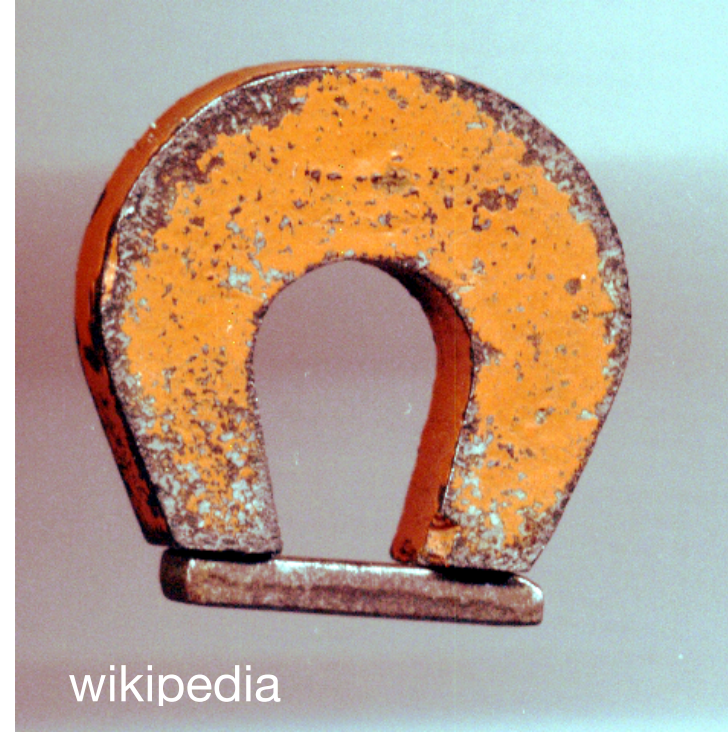
Entanglement

quantum phases

non-equilibrium problems
e.g. scattering, thermalization

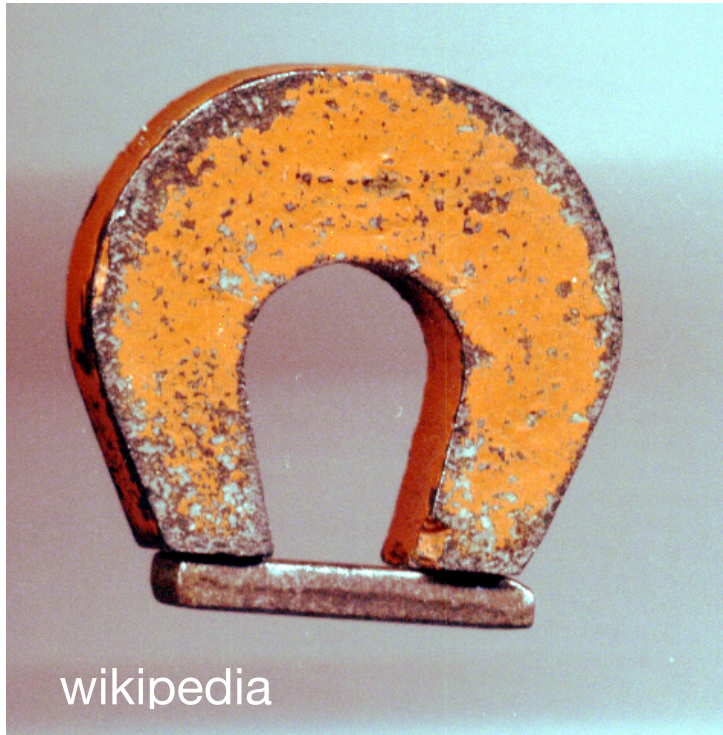
Entanglement: Quantum Phases

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Local Order Parameter: Landau theory of phase transitions

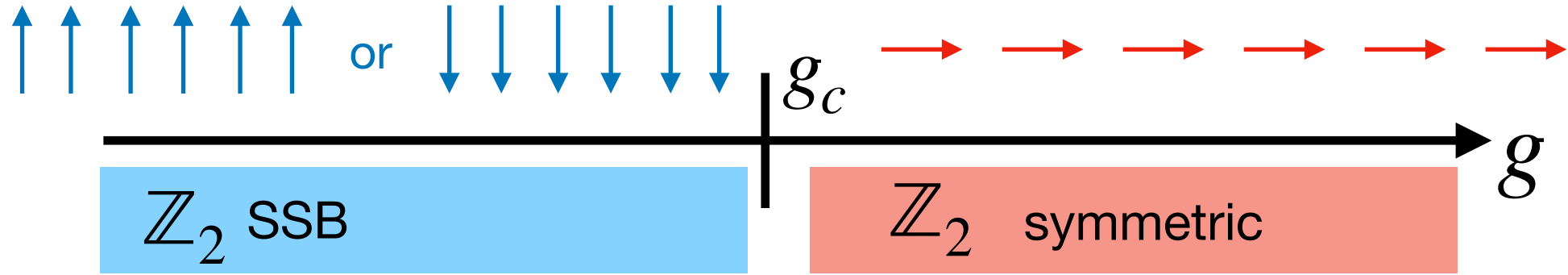
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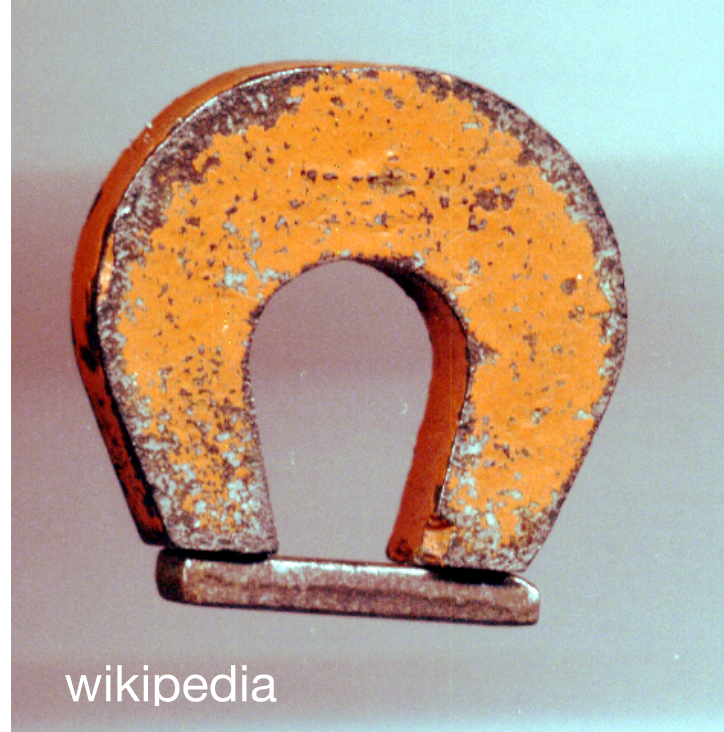
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Transverse Ising Model:

$$H = - \sum_i \sigma_i^z \sigma_{i+1}^z - g \sum_i \sigma_i^x$$



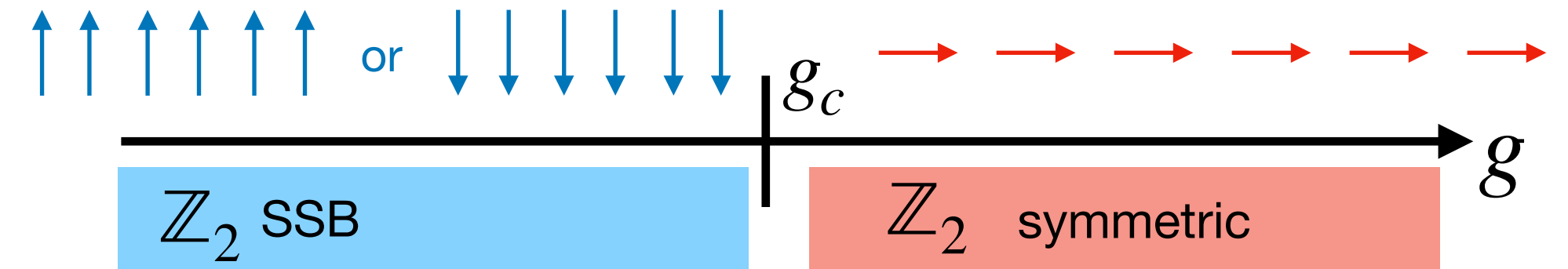
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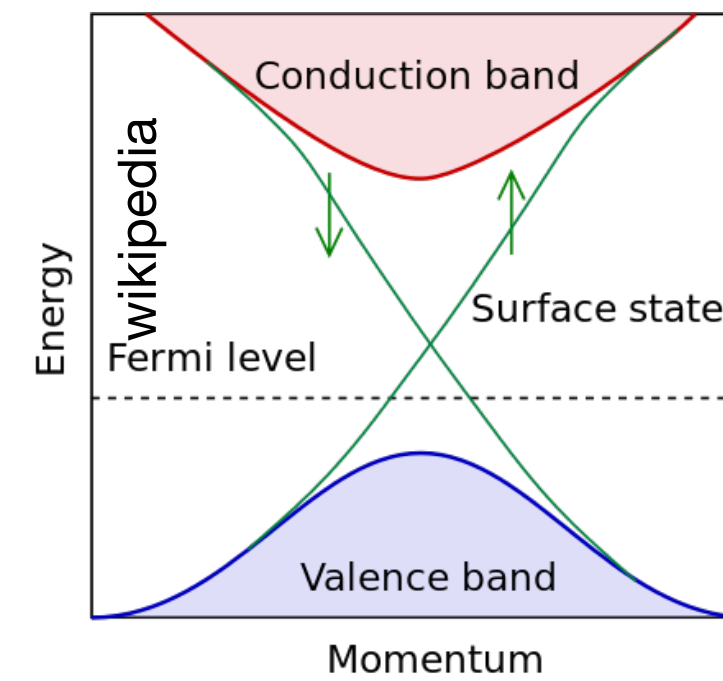
- **Symmetry protected**

Topological Insulators

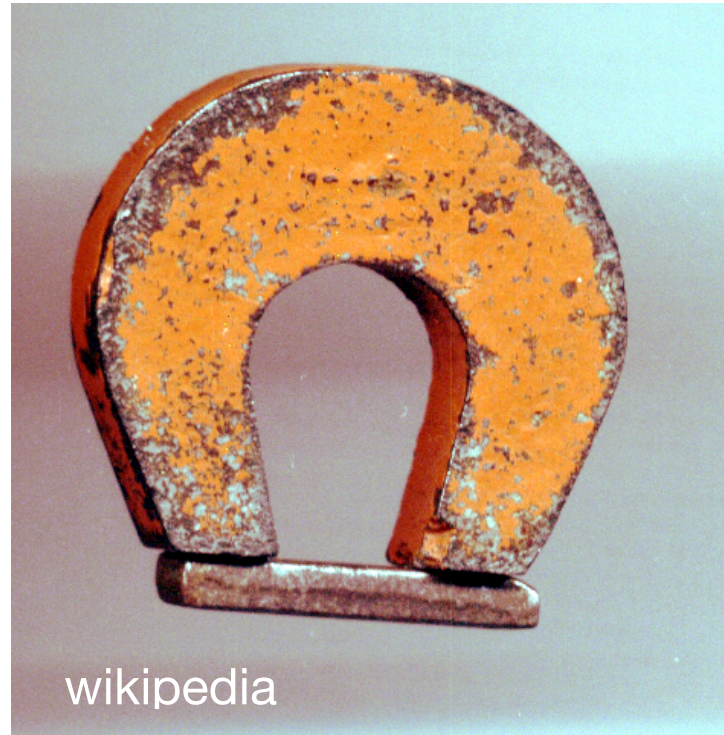
Kane, Mele, PRL 95 (2005) 226801
 Bernevig, Zhang, PRL 96 (2006) 106802

Haldane Phase

Haldane, PRL 50 (1983) 1153



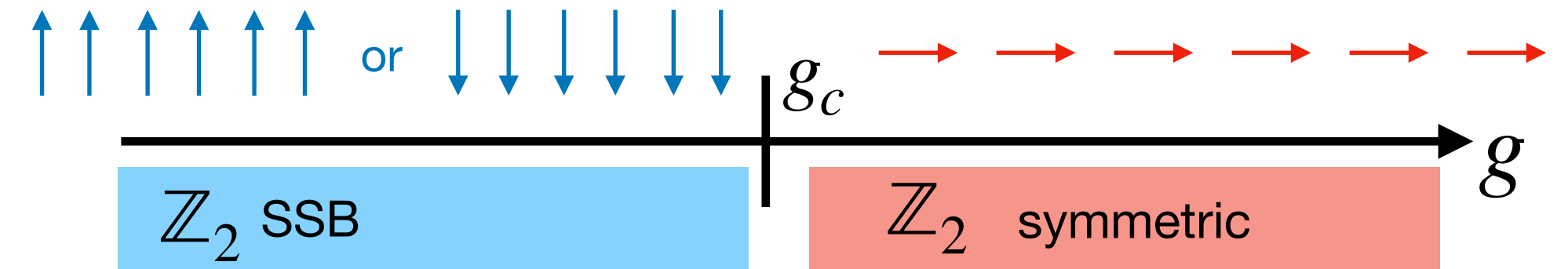
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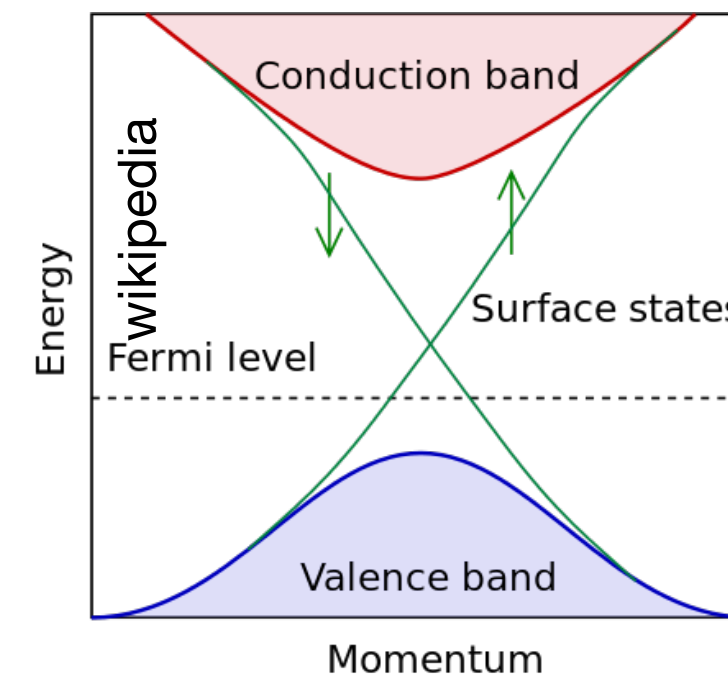
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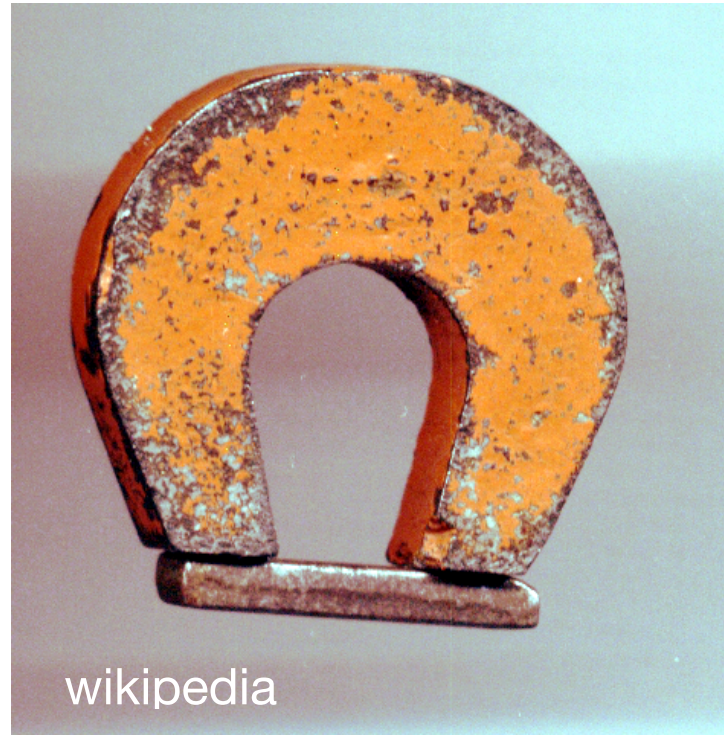
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- **Fracton Phases**

Chamon, PRL 94, 040402 (2005)
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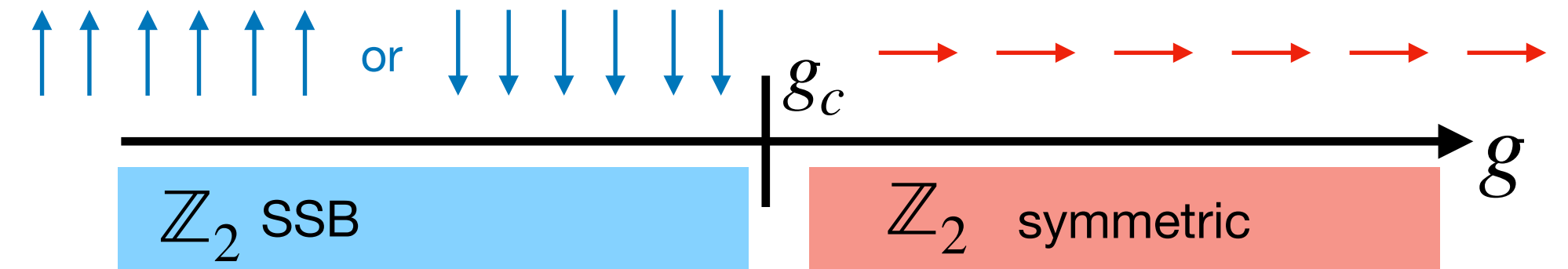
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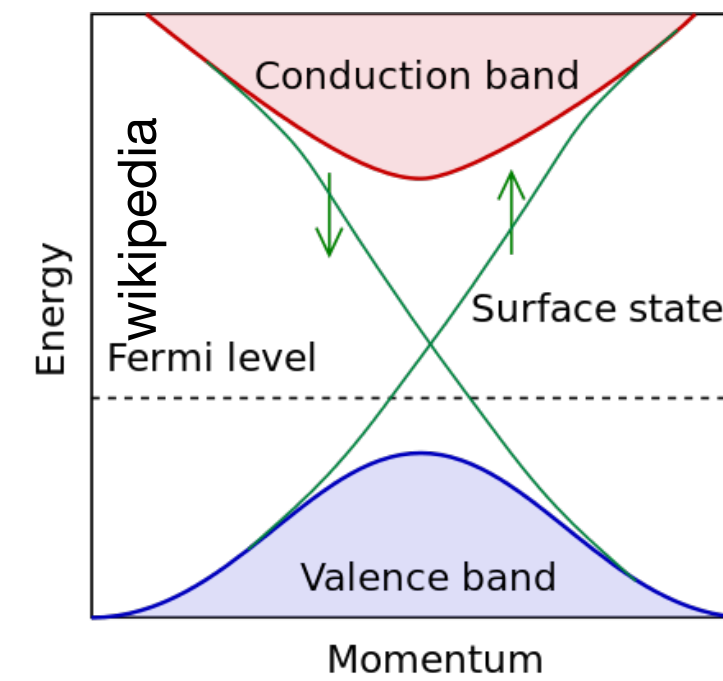
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- **Topological Order**

Spin Liquids

Anderson Mat. Res. Bull. 8 (1973) 153

Levin, Wen PRB 71 (2005)

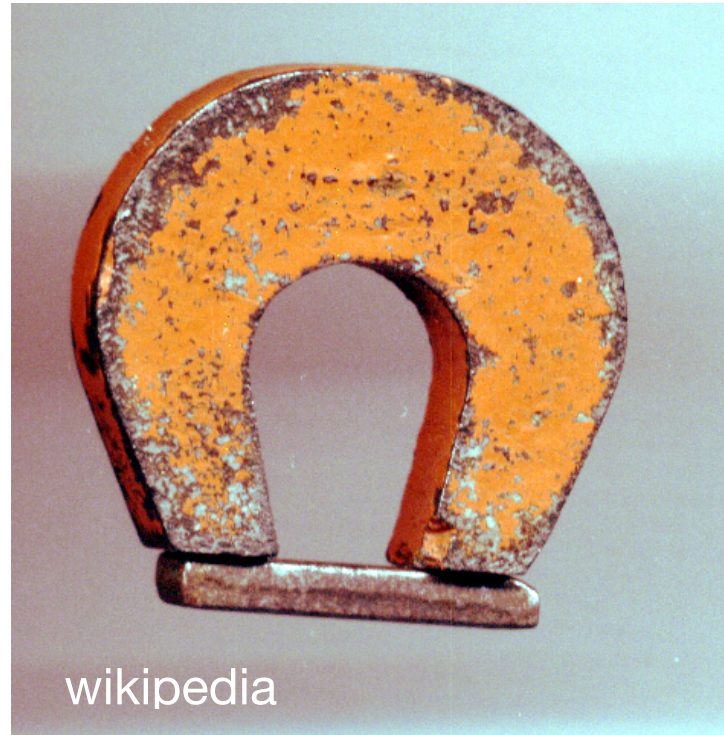
Fractional Quantum Hall states

Tsui, Stromer, Gossard PRL 48 (1982), 1559
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Quantum Error Correction /
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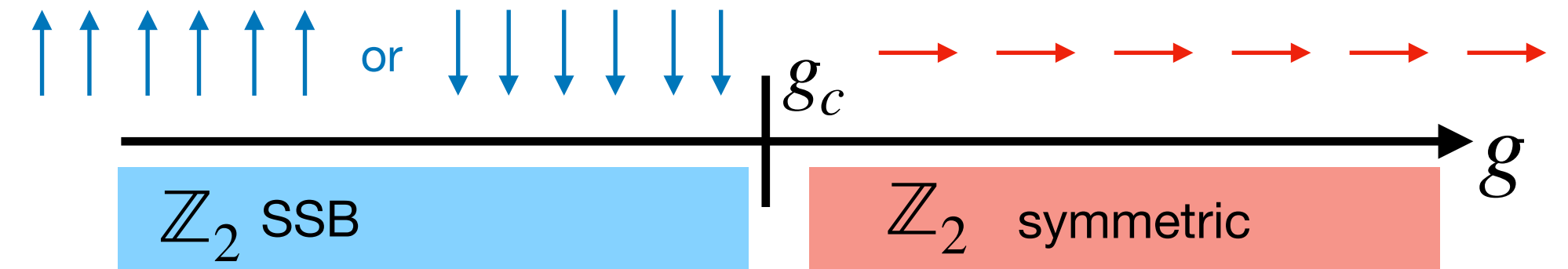
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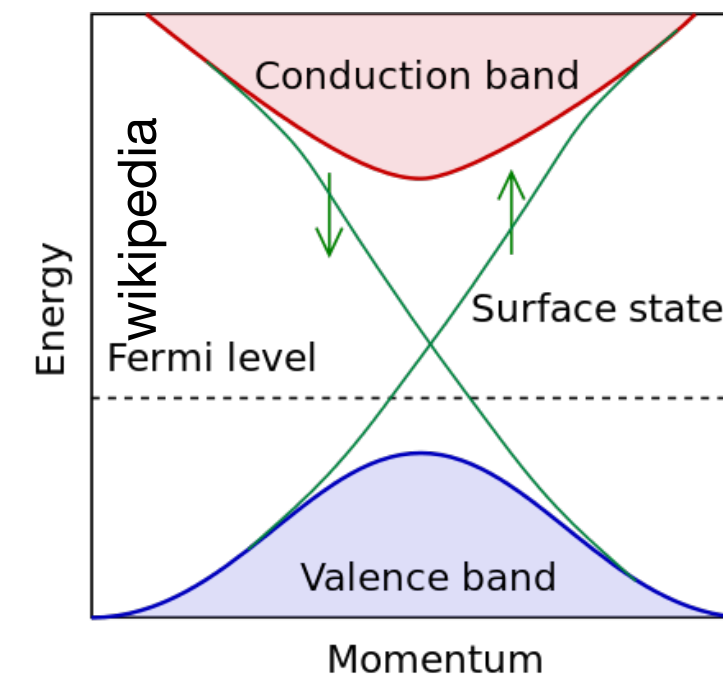
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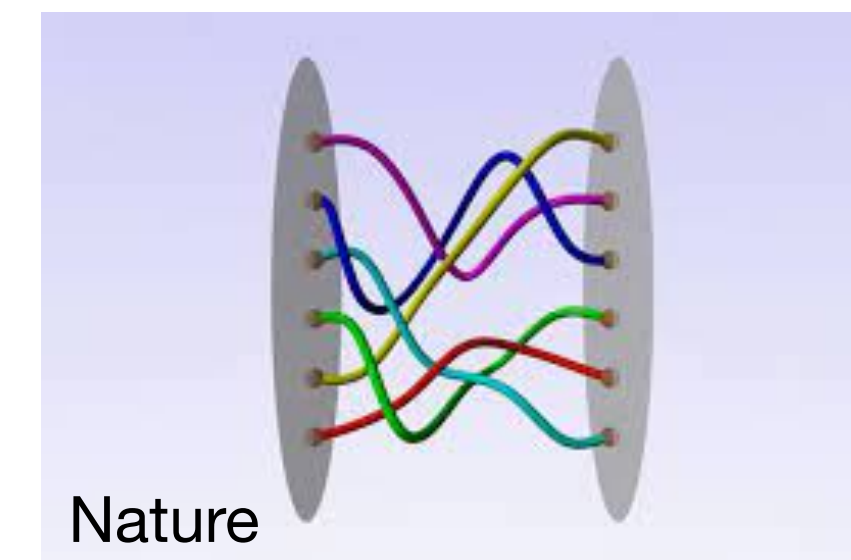
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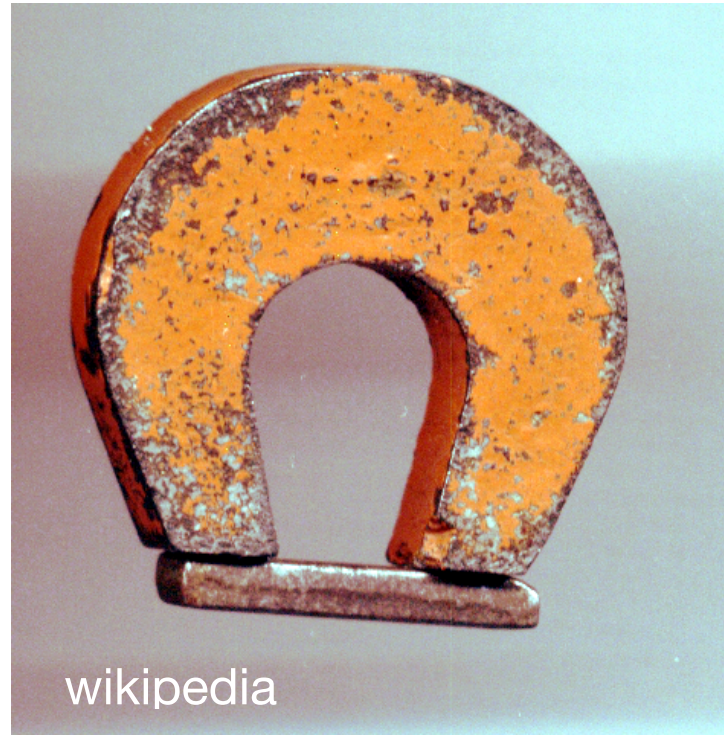
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Nature

anyonic / fractional
 excitations

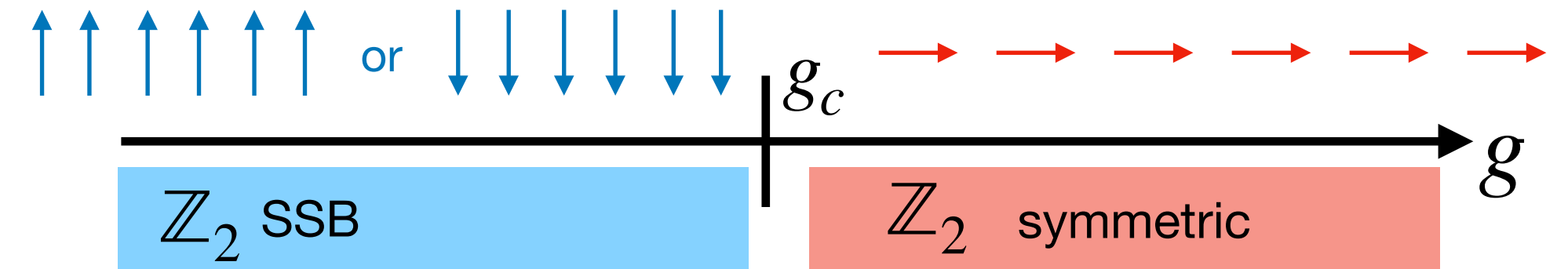
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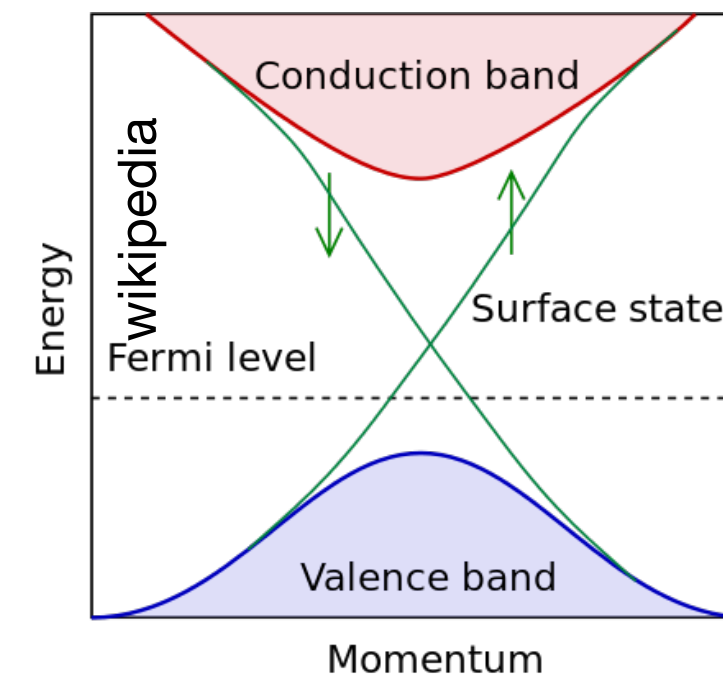
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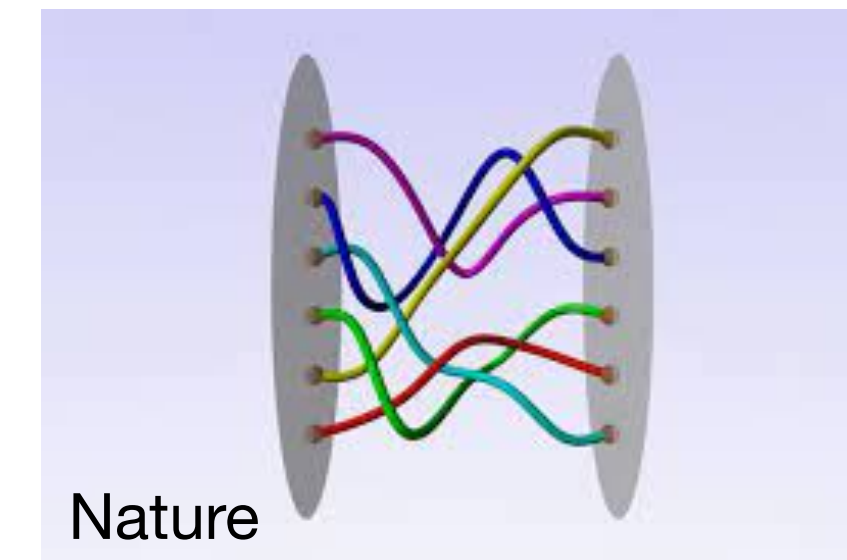
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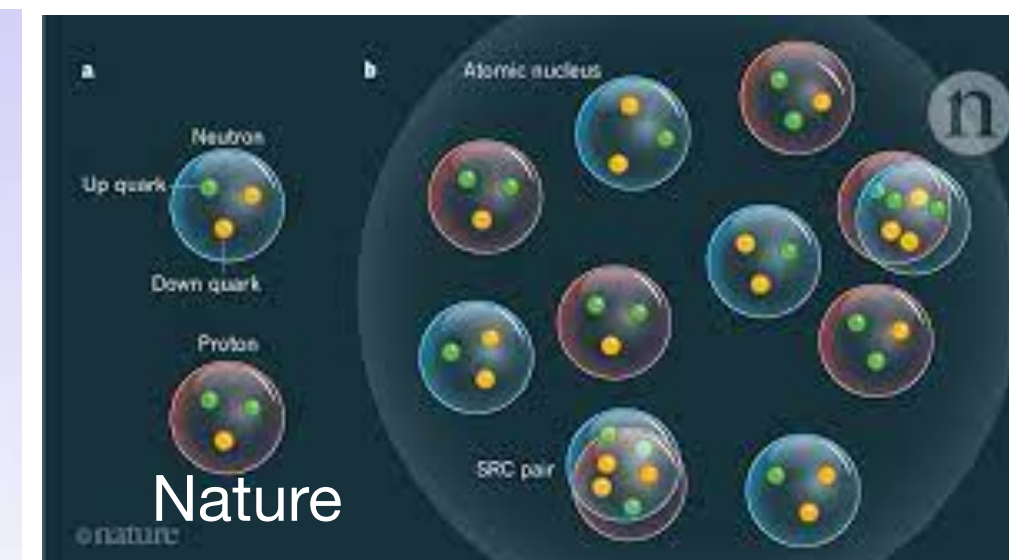
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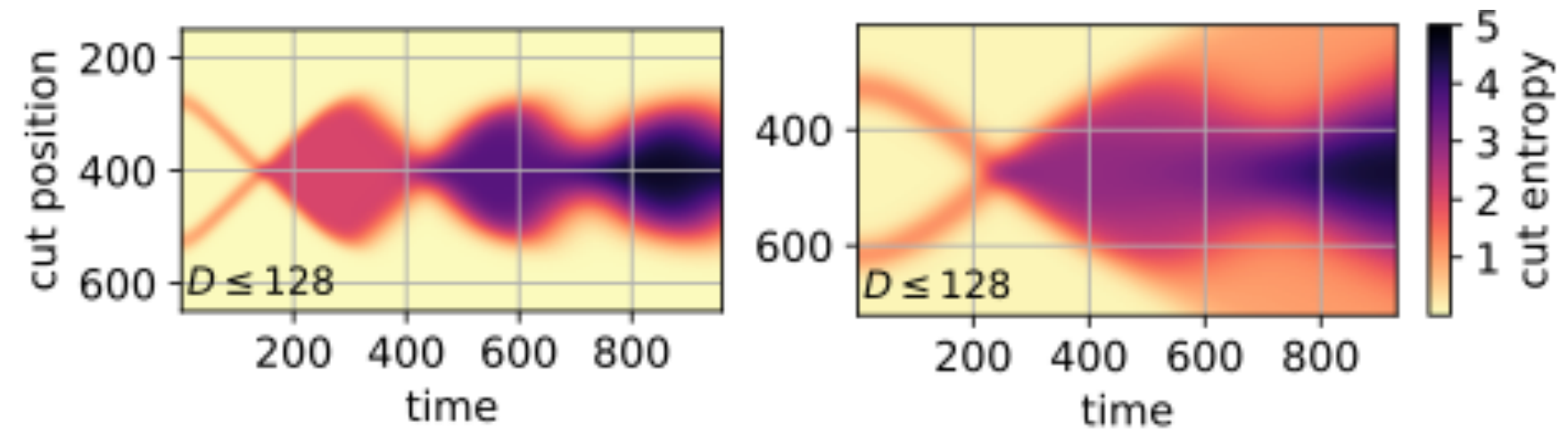


emergent gauge fields

Entanglement: Non-equilibrium problems

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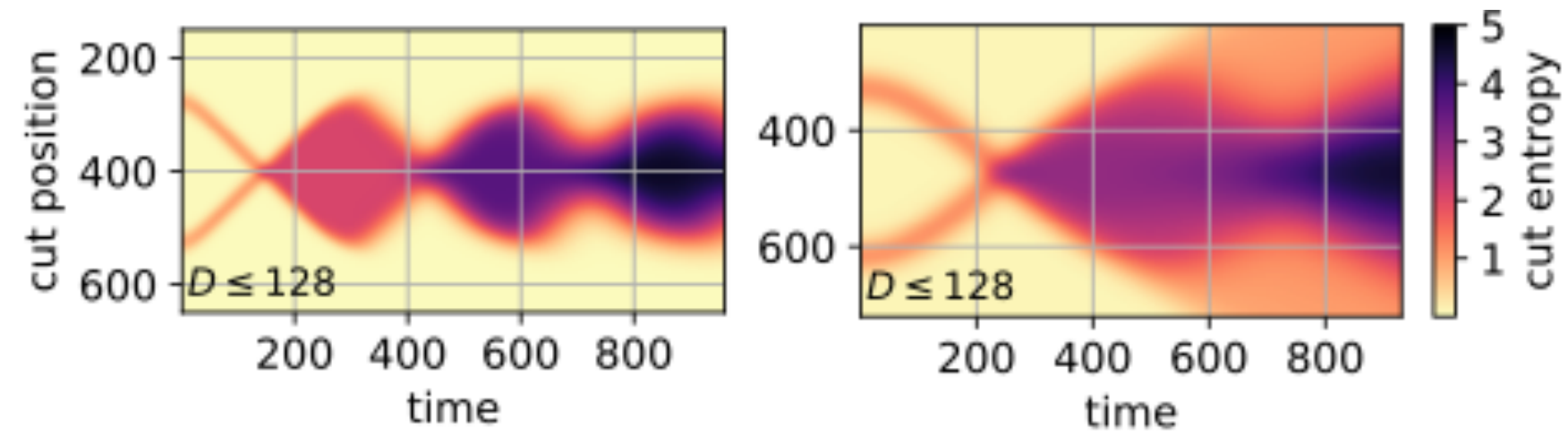
- **Scattering**



Milsted et al, PRX Quantum 3, 020316 (2022)

Entanglement: Non-equilibrium problems

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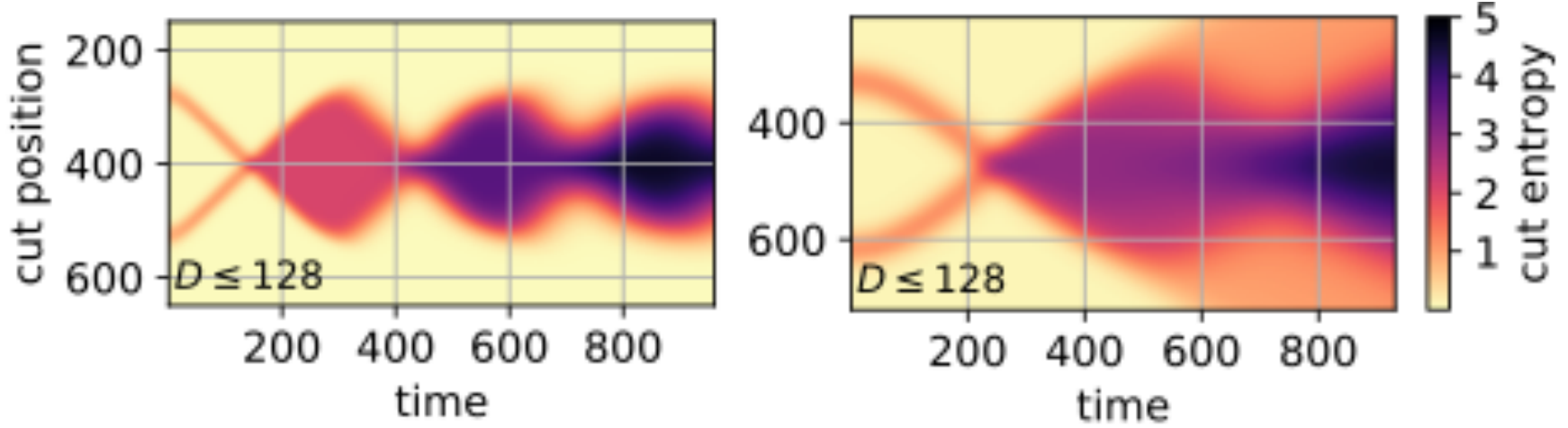
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Kharzeev, Levin PRD 95, 114008 (2017)
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Beane, Ehlers MPL A 35, 2050048 (2020)
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in nuclear and high energy physics

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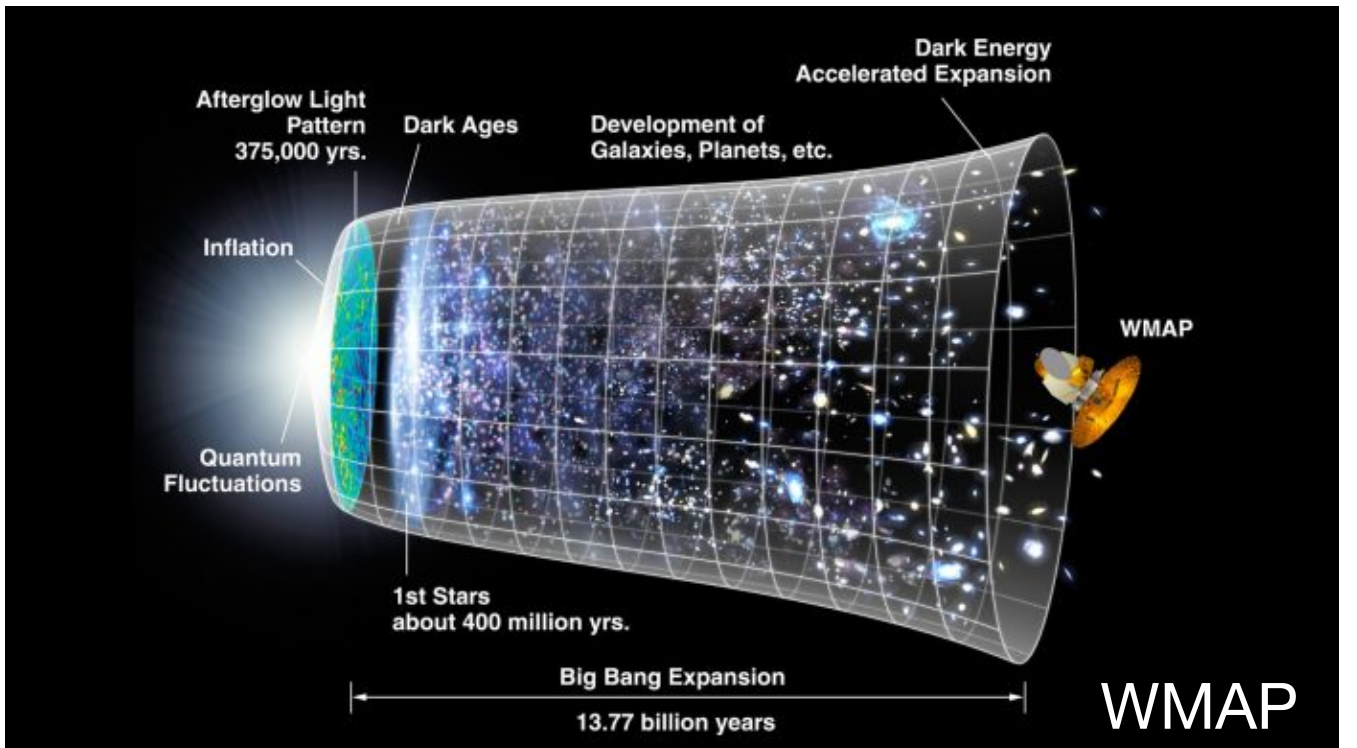
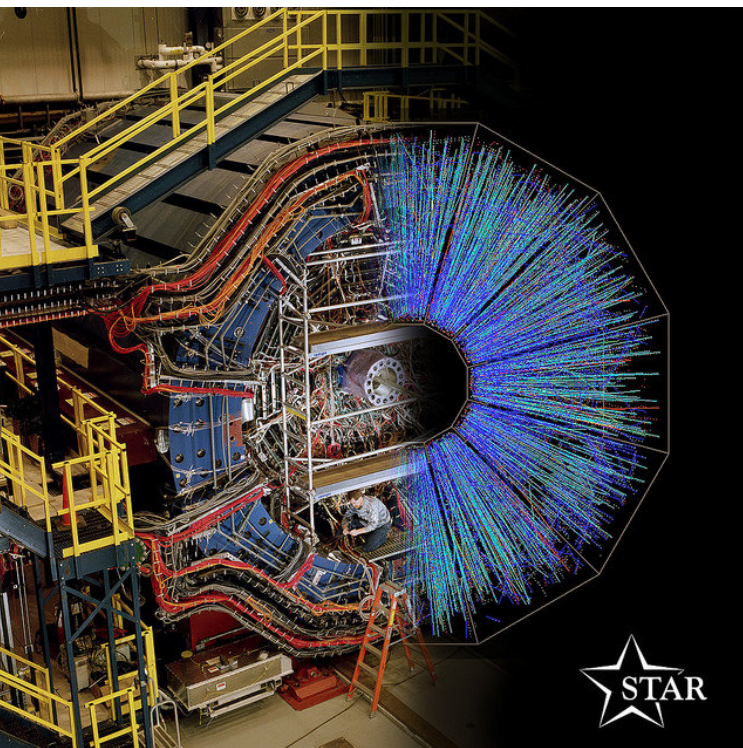
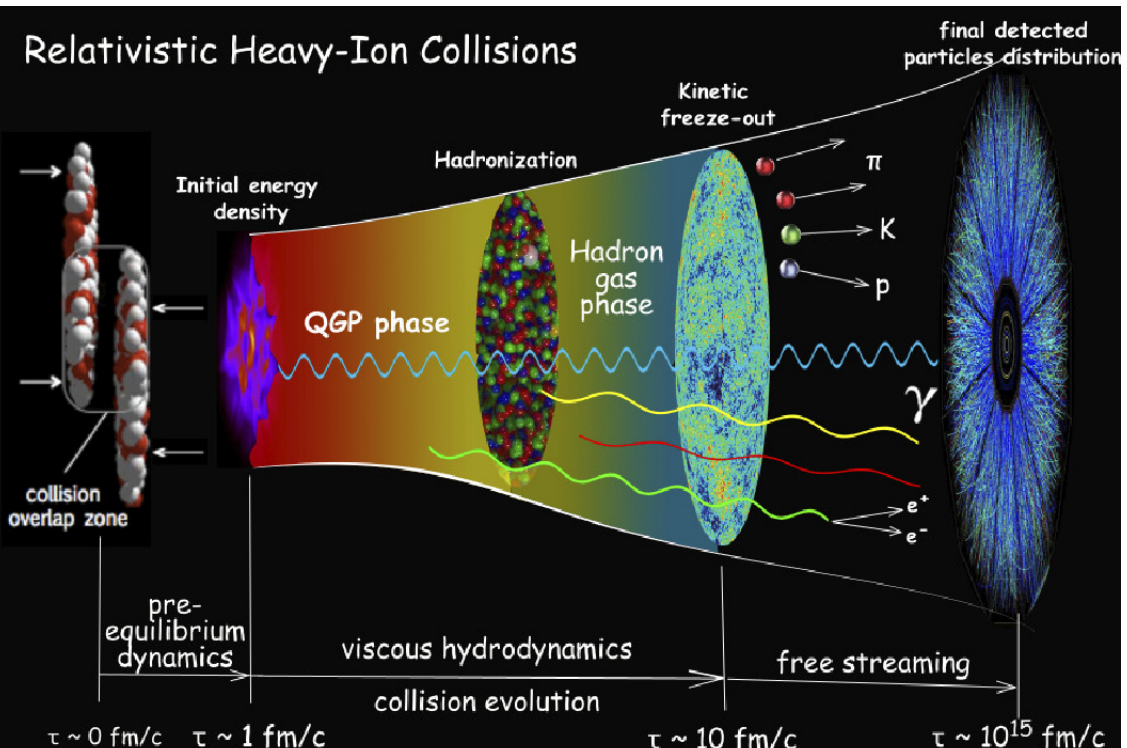


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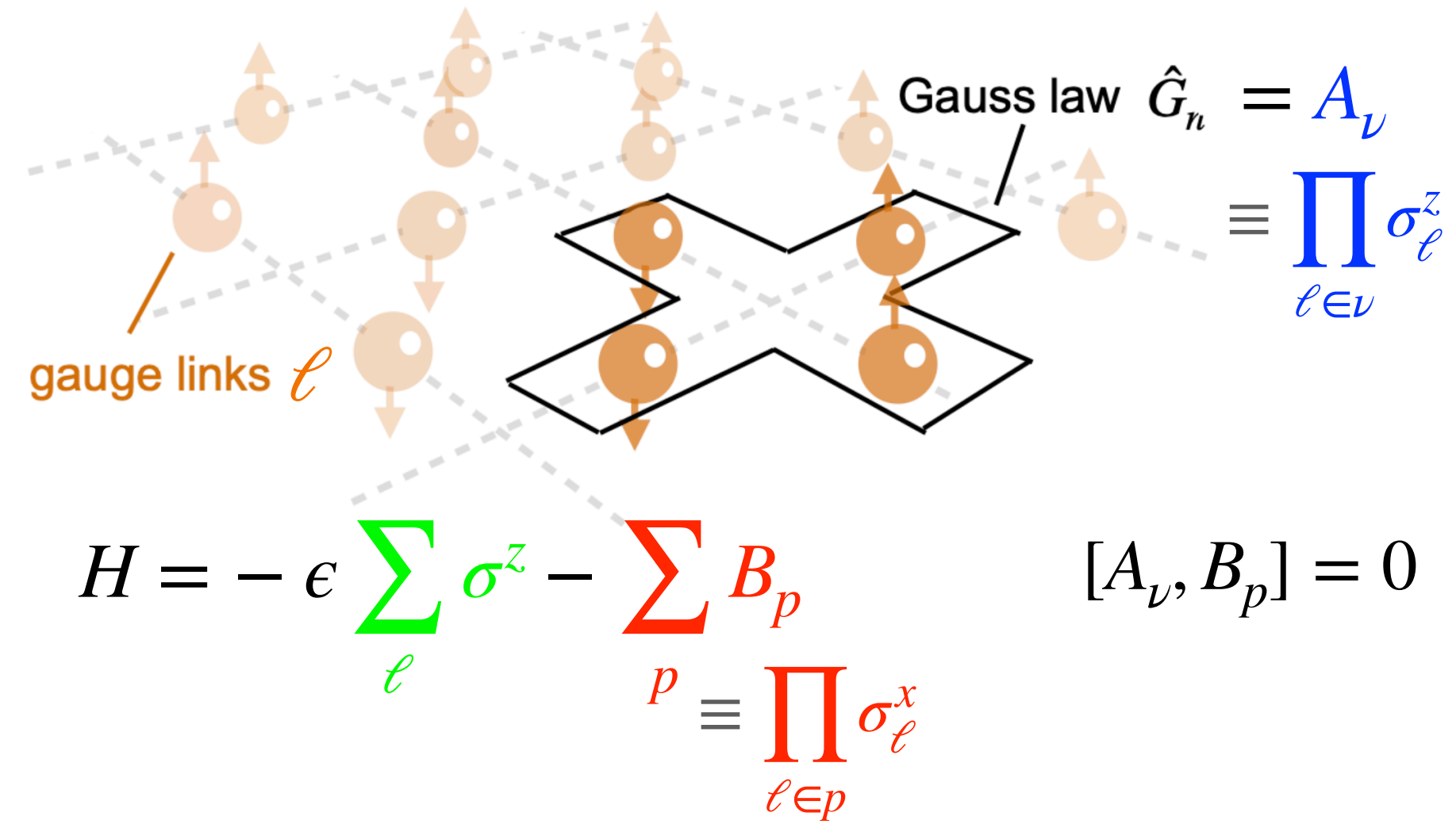
- **Thermalization**



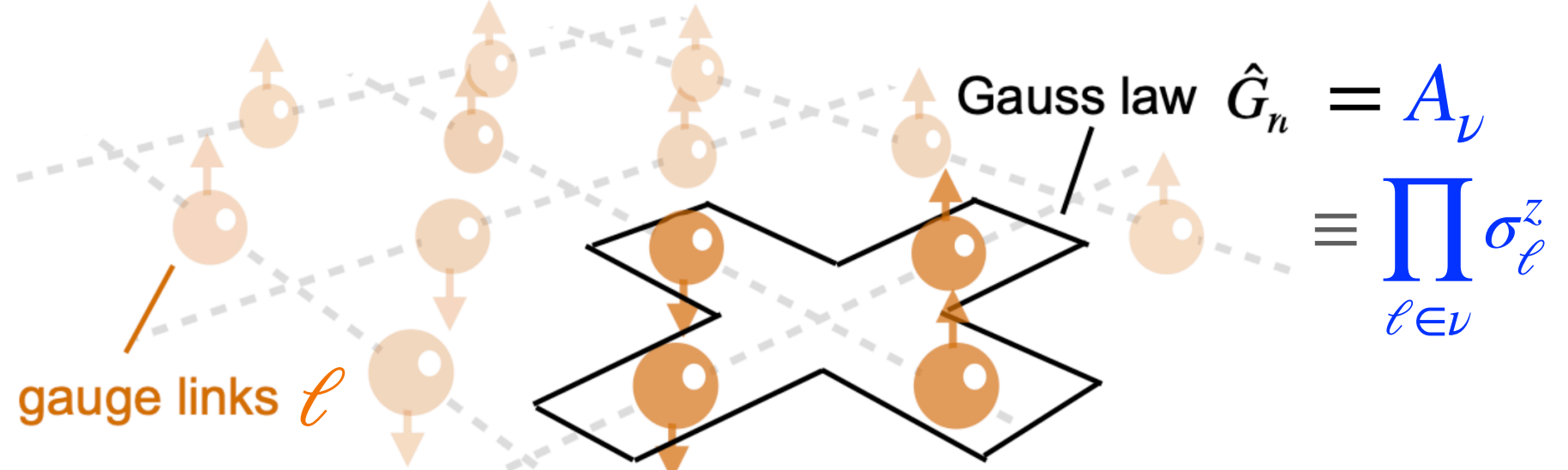
Arnold, Moore, Yaffe; JHEP 11 (2000) & 05 (2003)
 Baier, Mueller, Schiff, Son; PLB 502, 51 (2001)
 Berges, Heller, Mazeliauskas, Venugopalan, Rev. Mod. Phys. 93 (2021), 035003
 Keegan, Kurkela, Romatschke, van der Schee, JHEP 2016(4), 1

Entanglement: From topological order to Lattice Gauge theories

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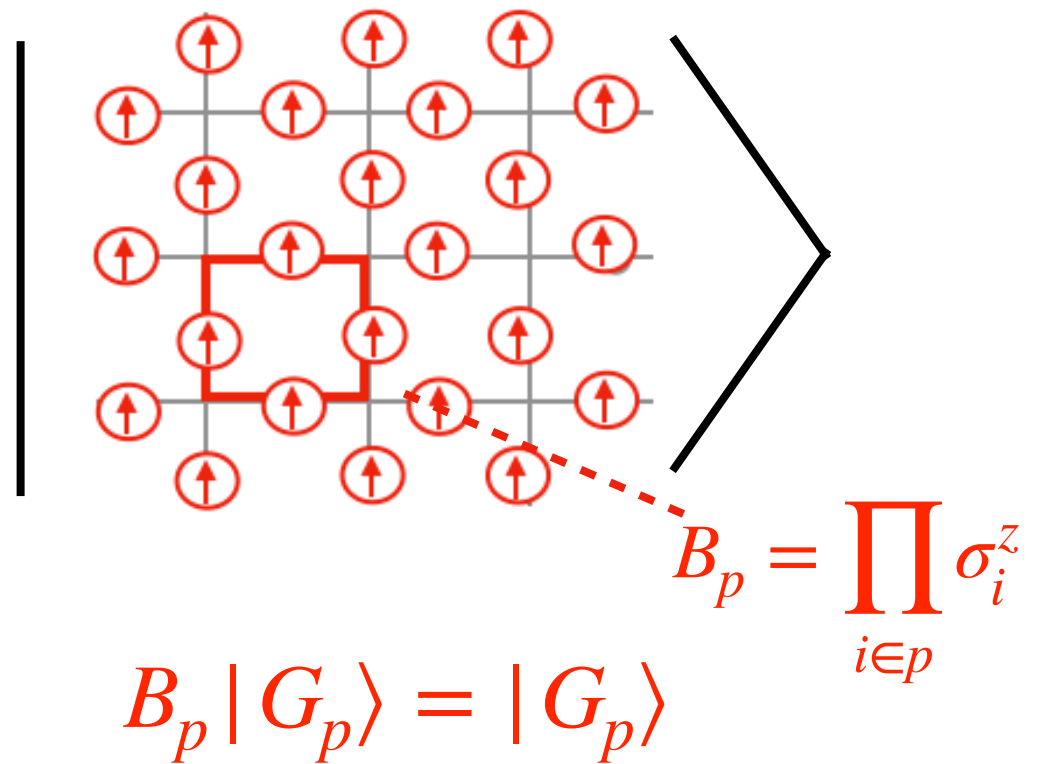


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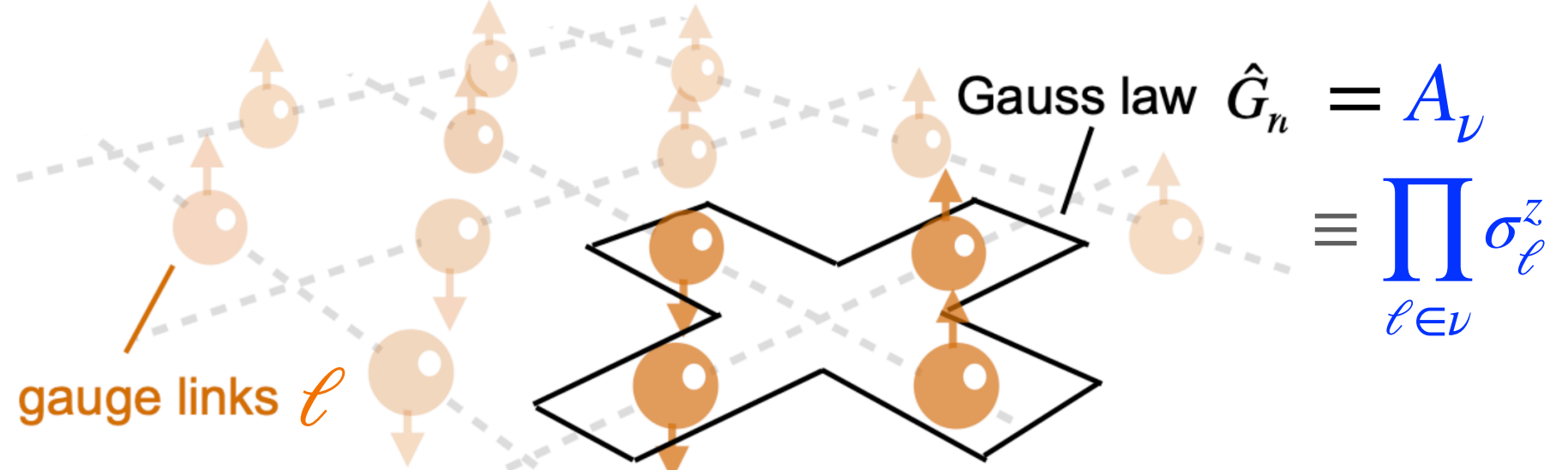


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$p \equiv \prod_{\ell \in p} \sigma_\ell^x$

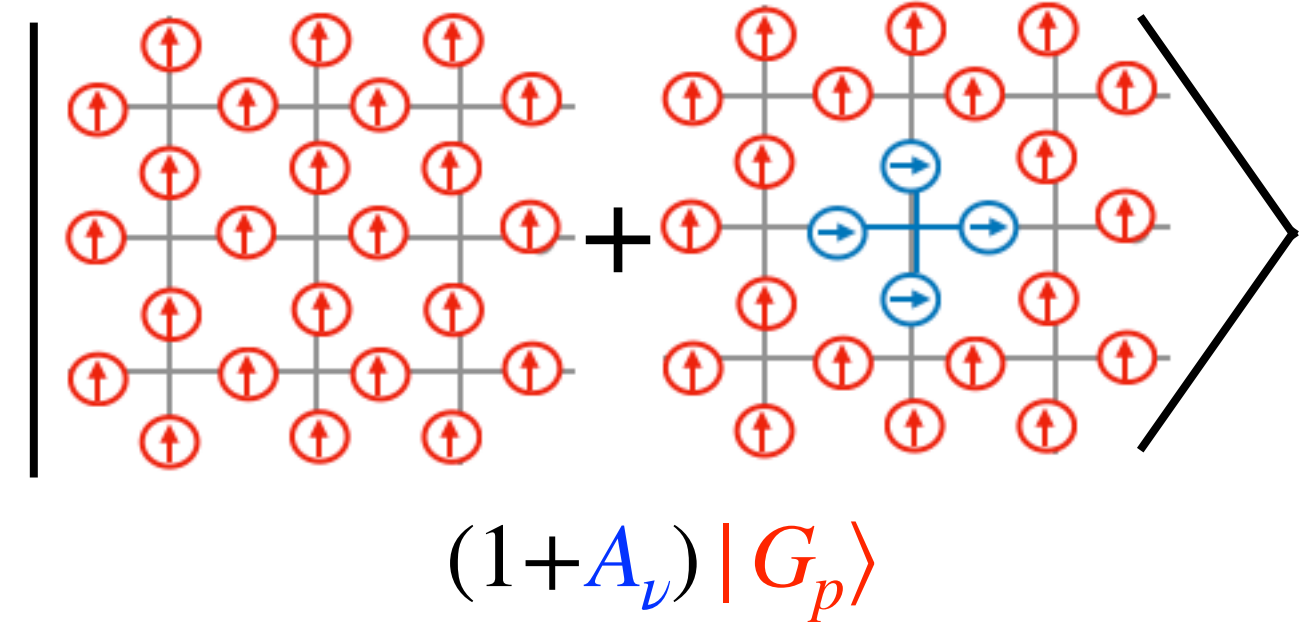
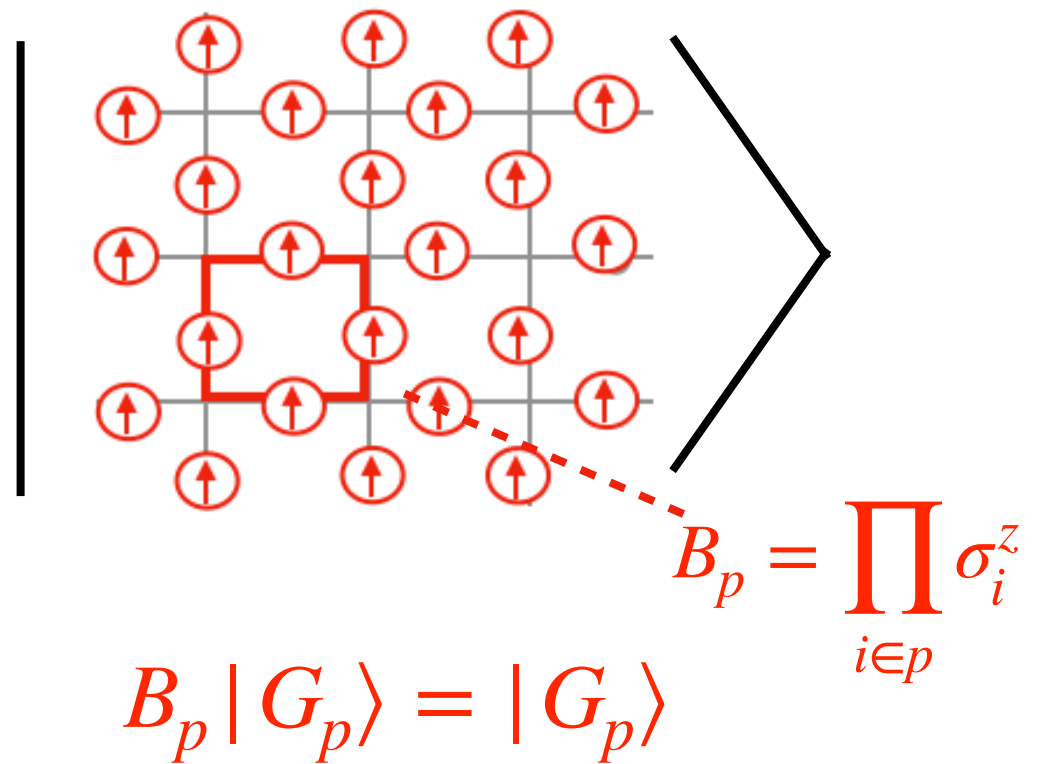


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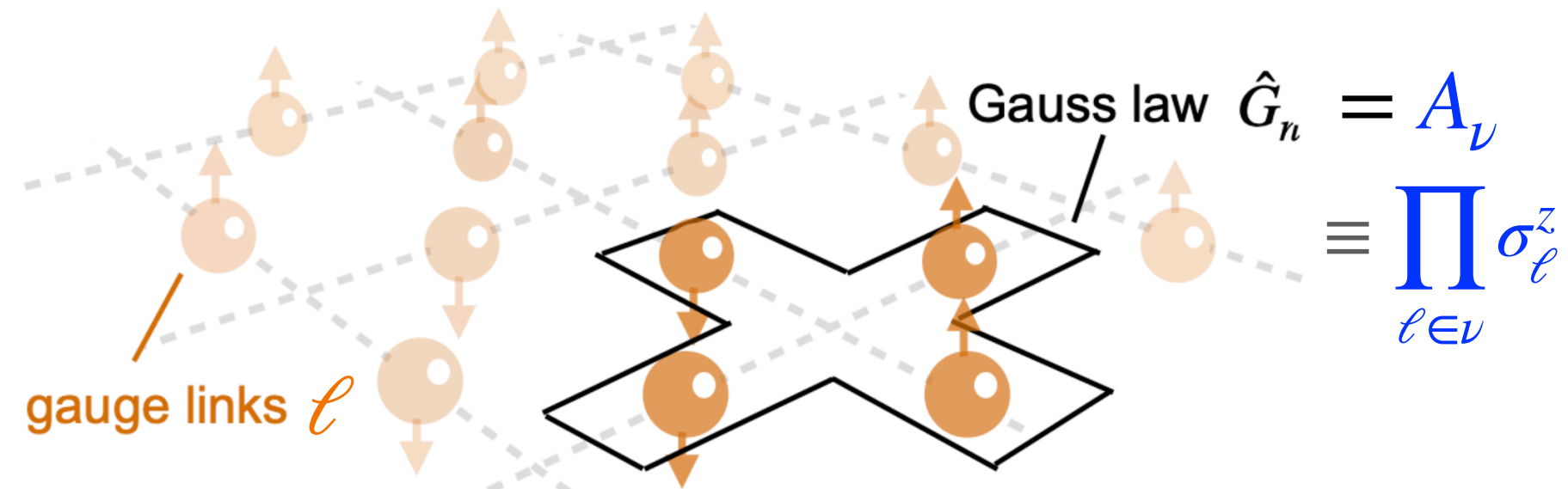


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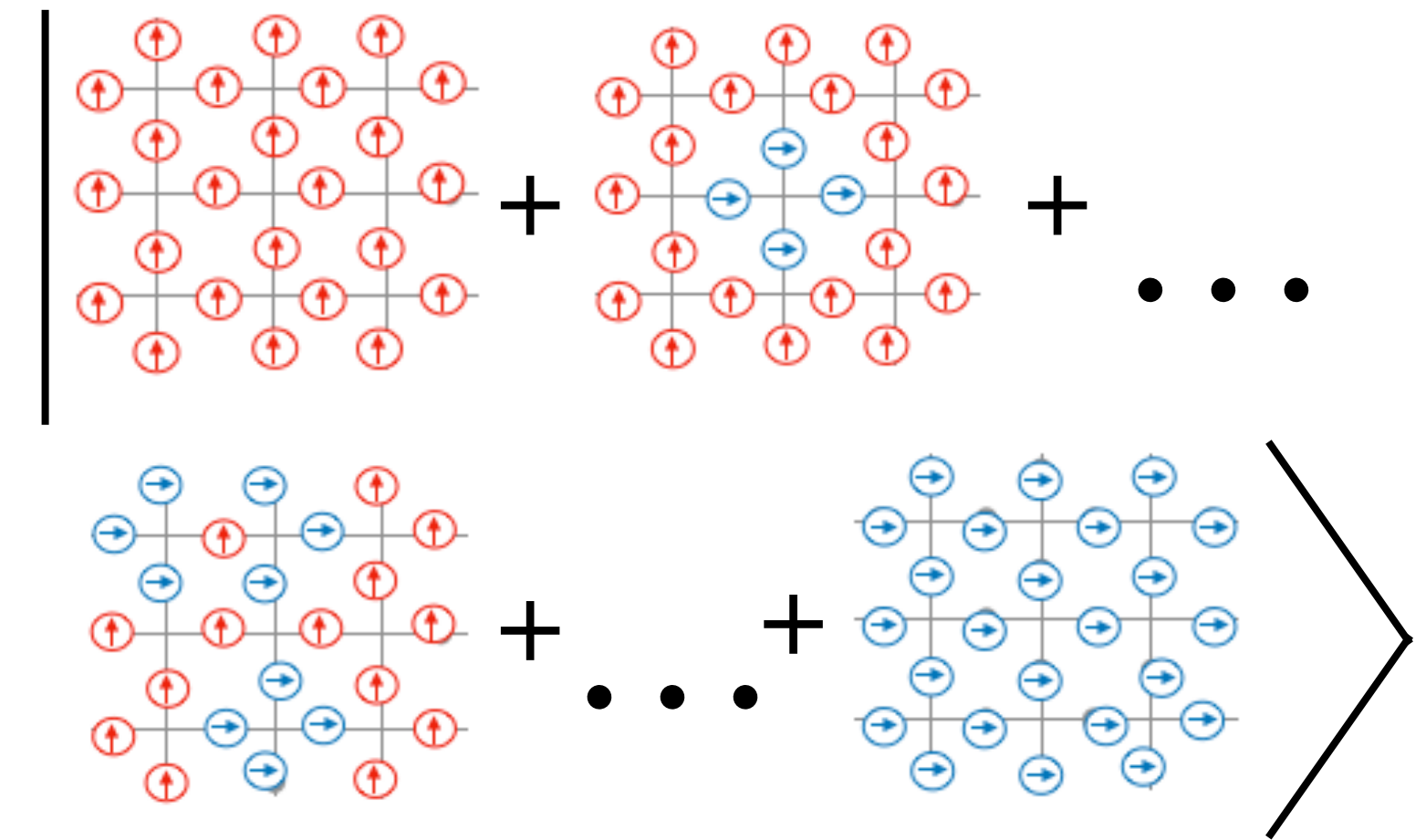
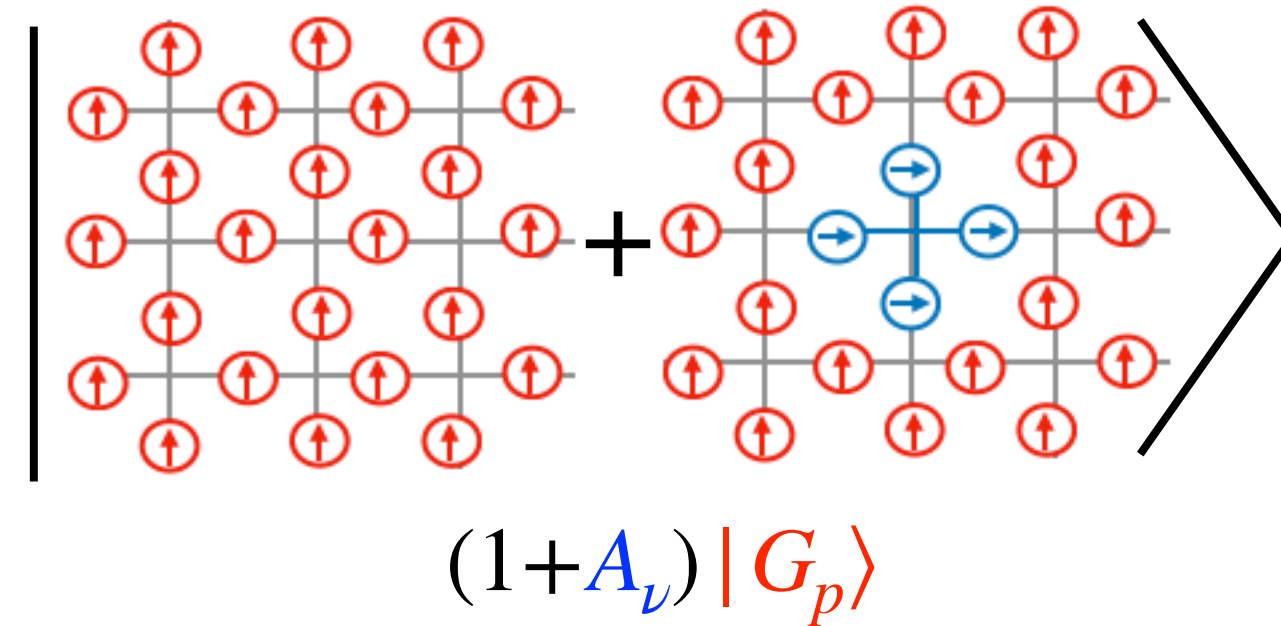
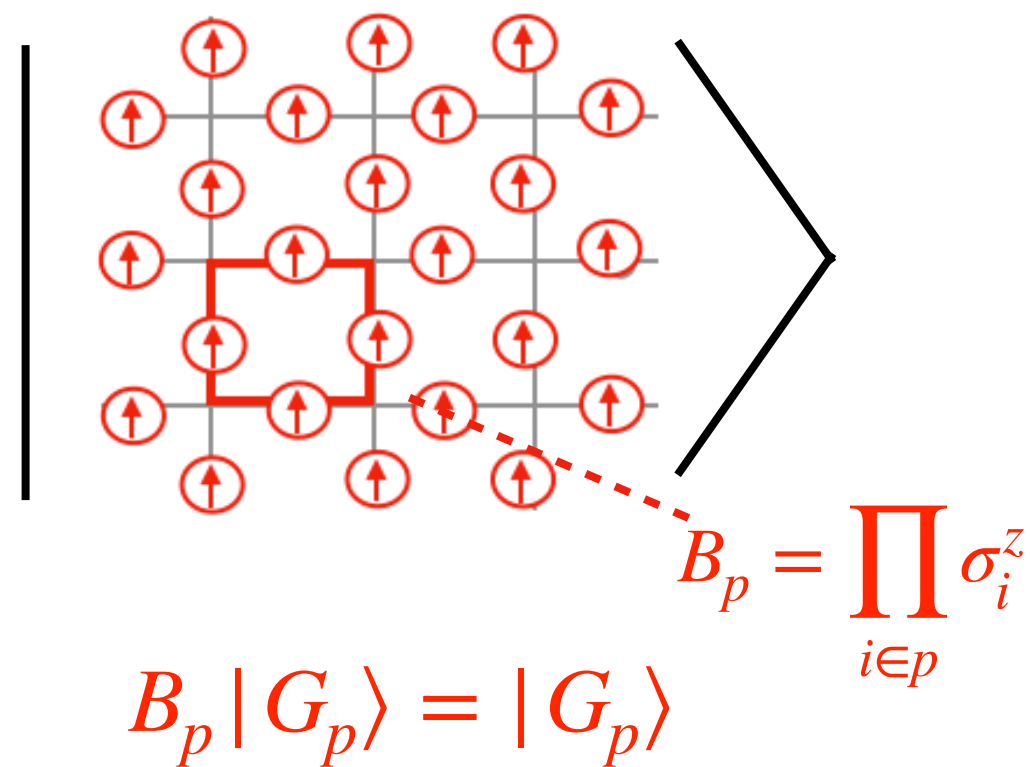


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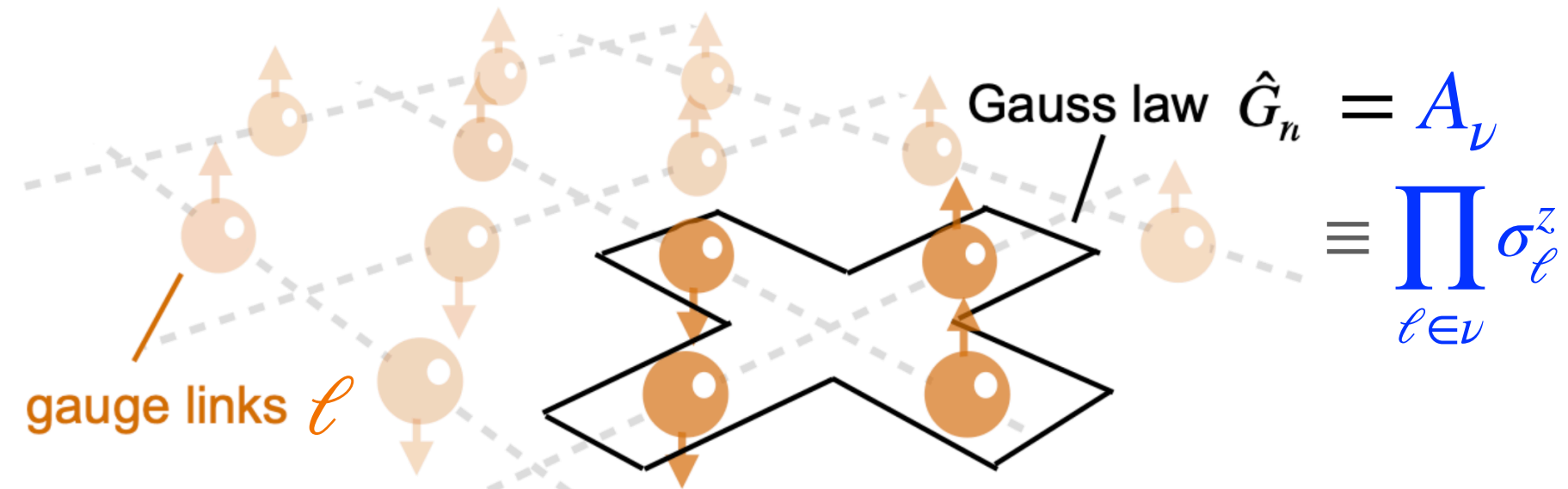


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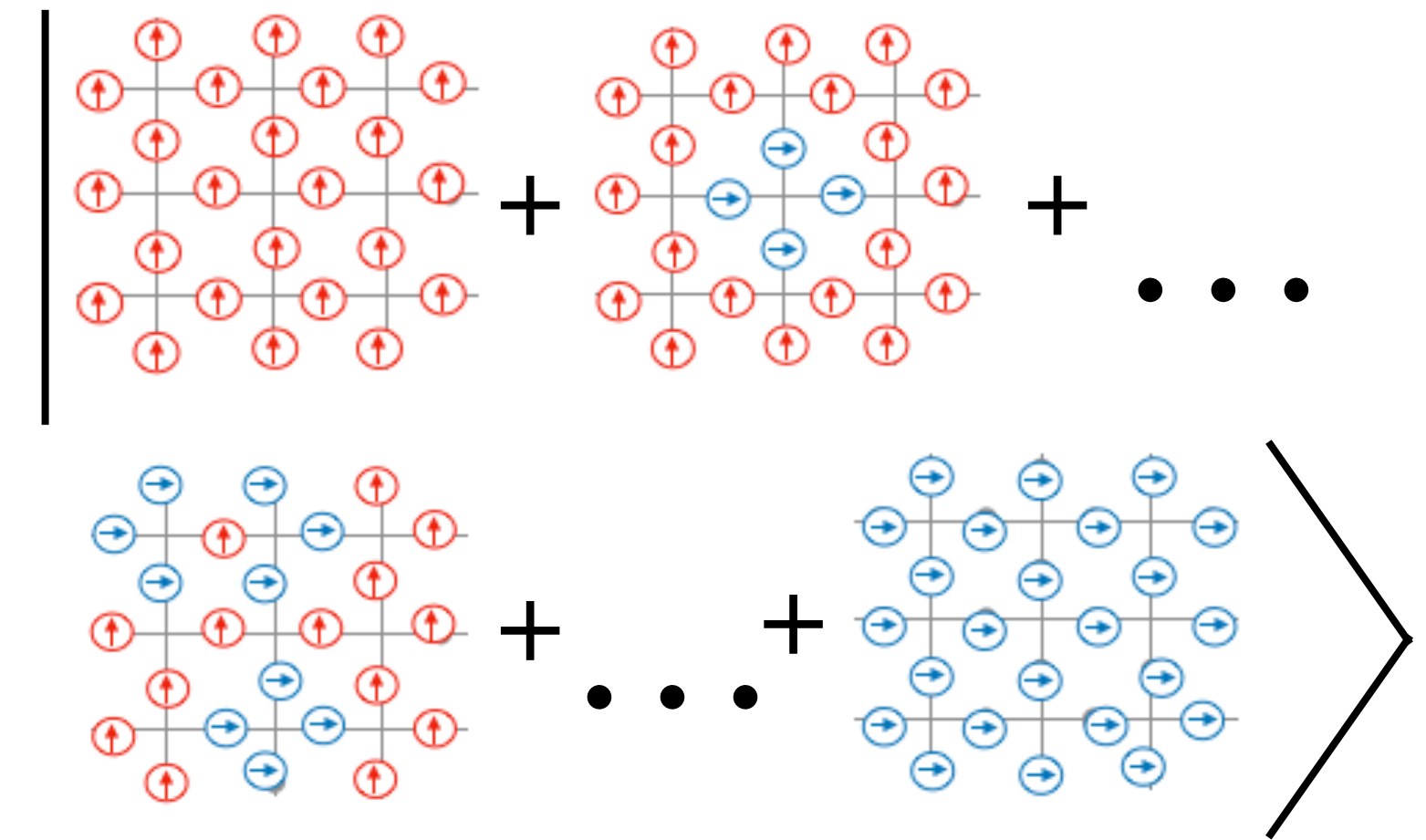
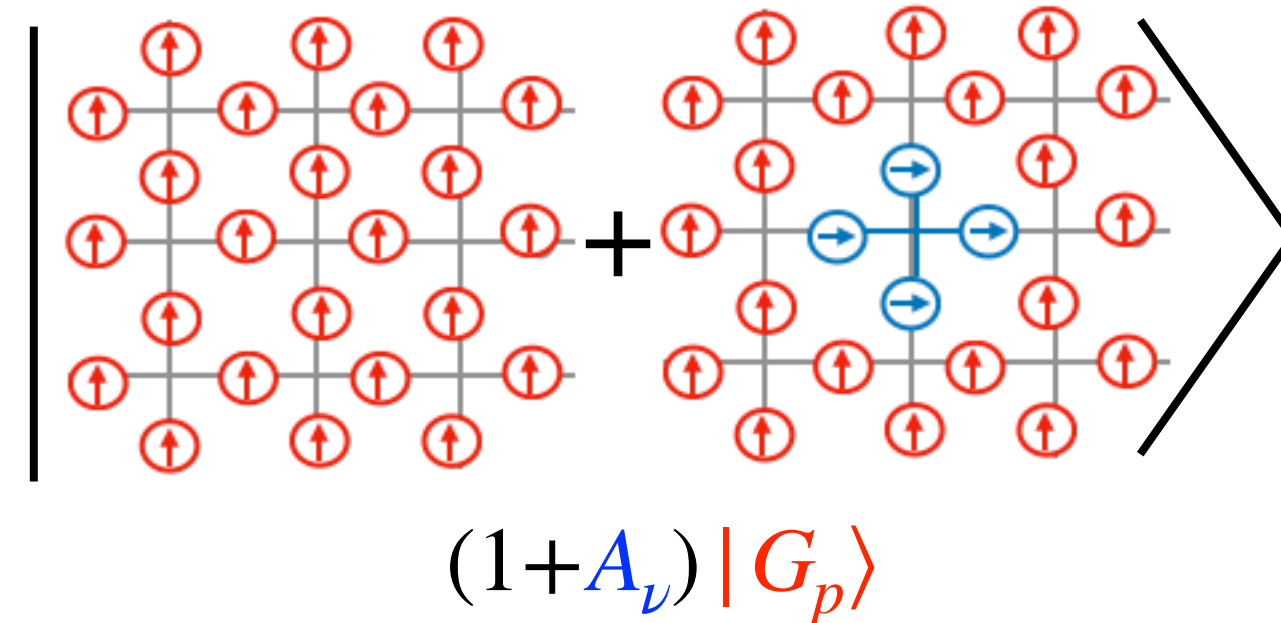
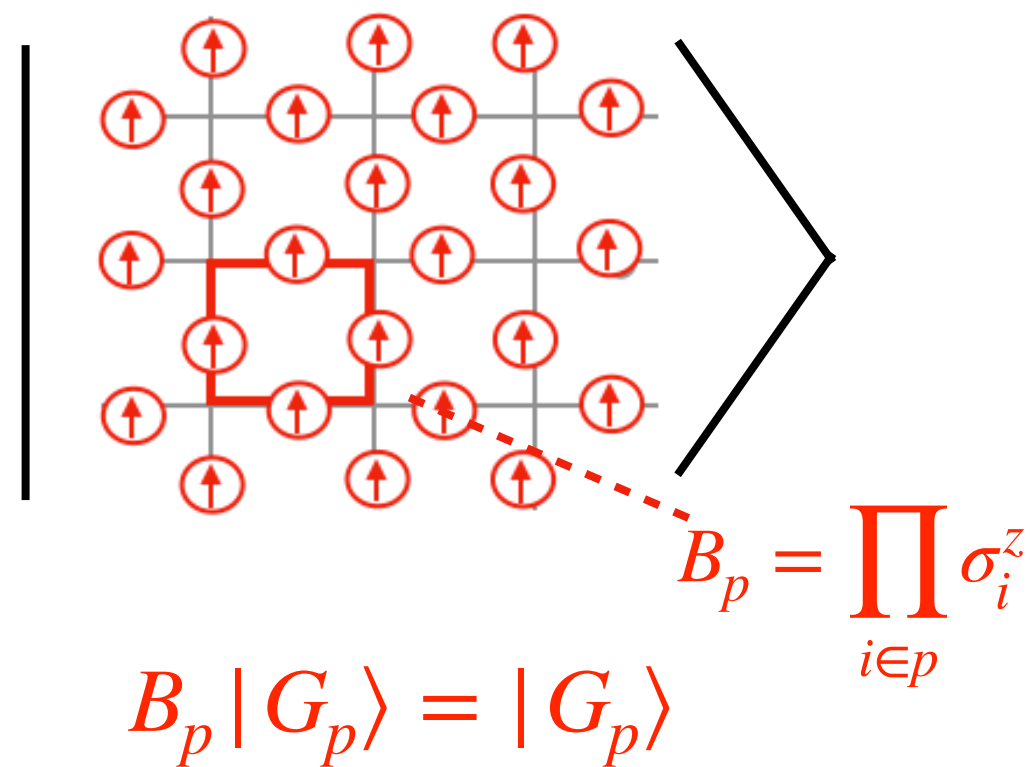


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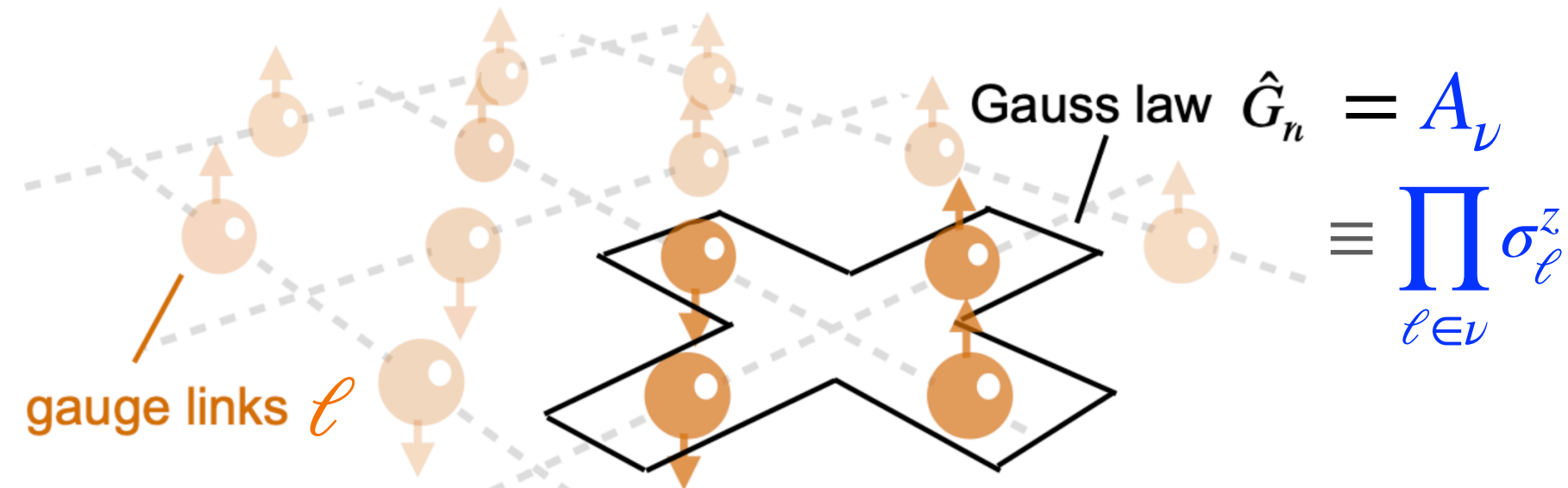
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$$|G\rangle \propto \prod_p (1+B_p) |G_p\rangle$$

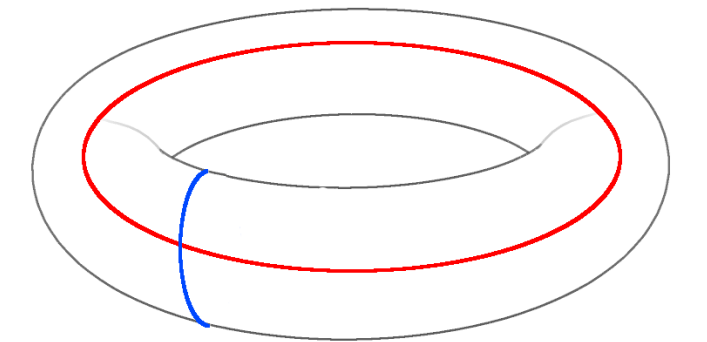
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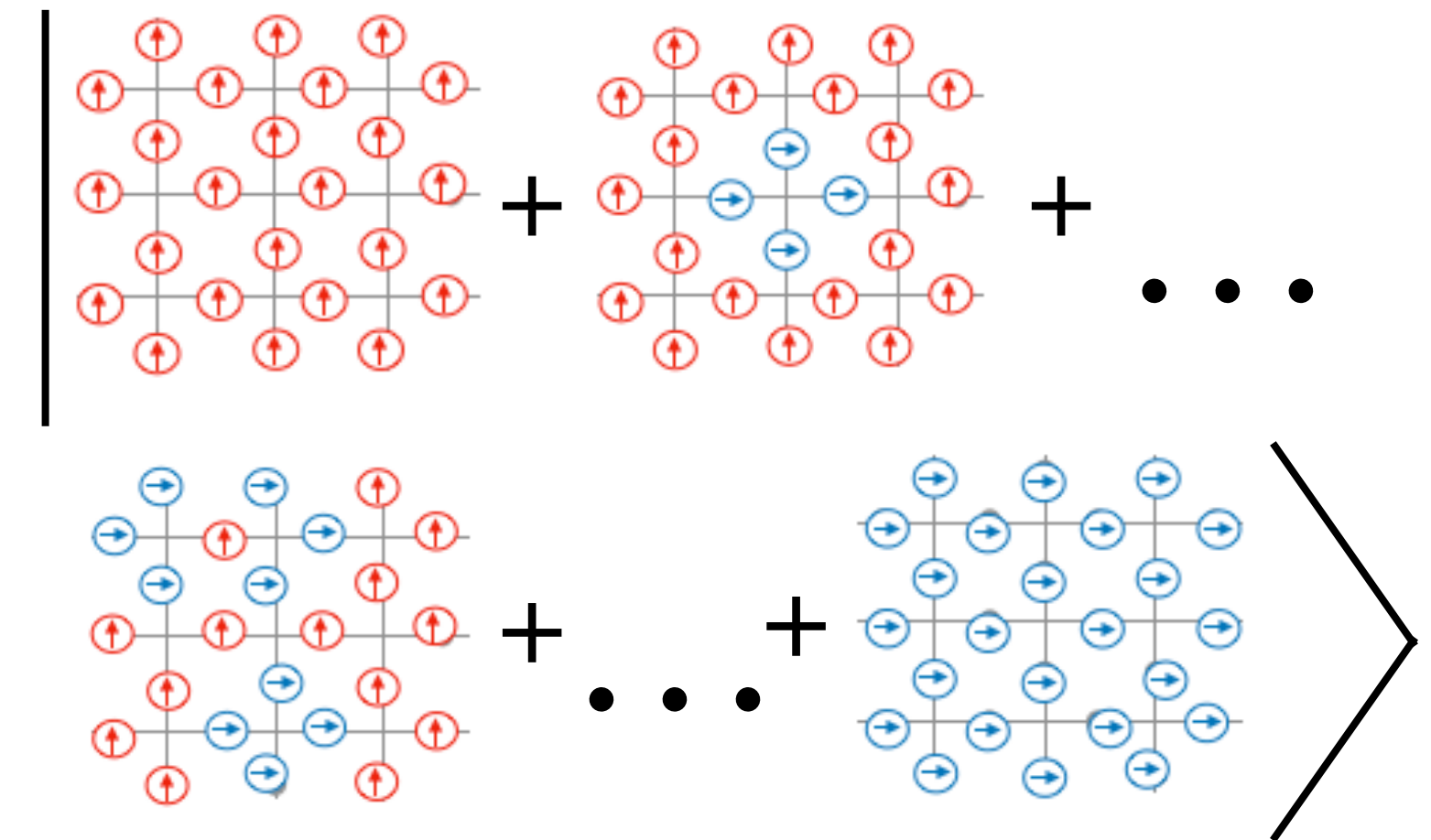
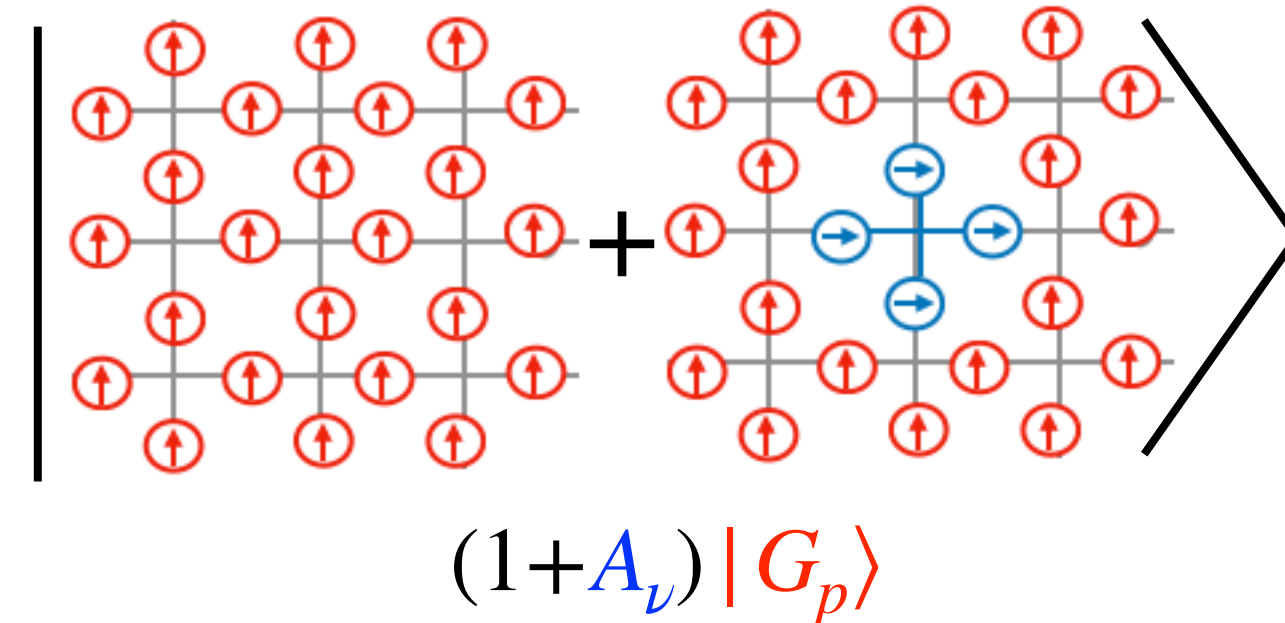
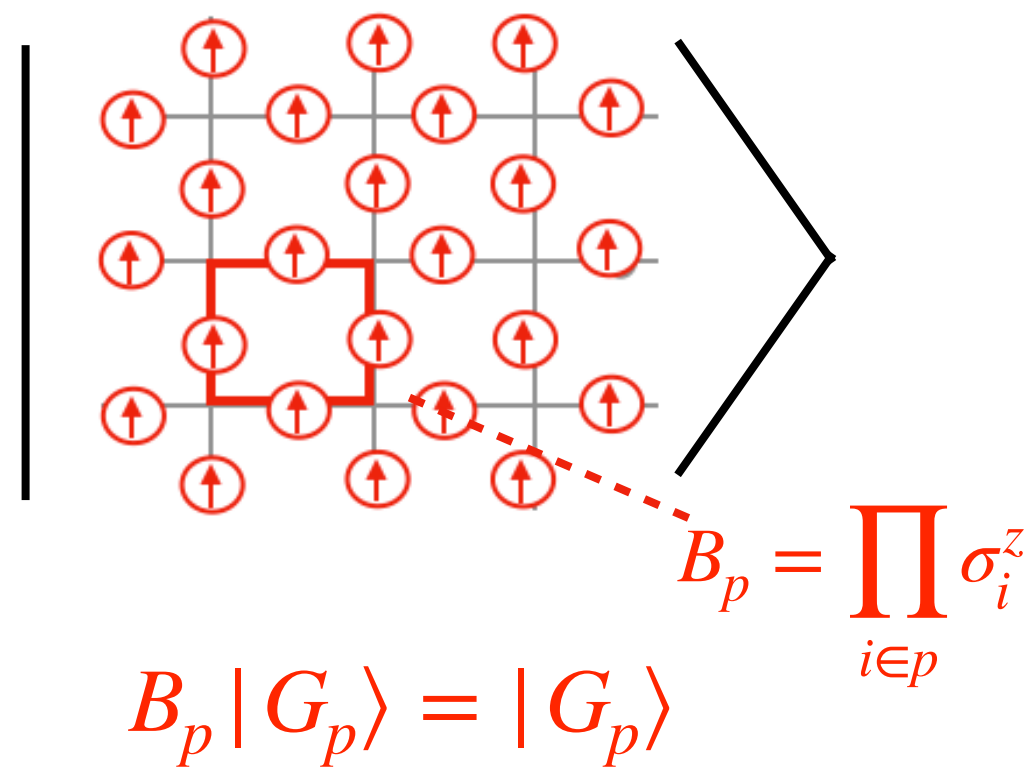
Classified by

- ground state degeneracy (torus)
- anyonic excitations and



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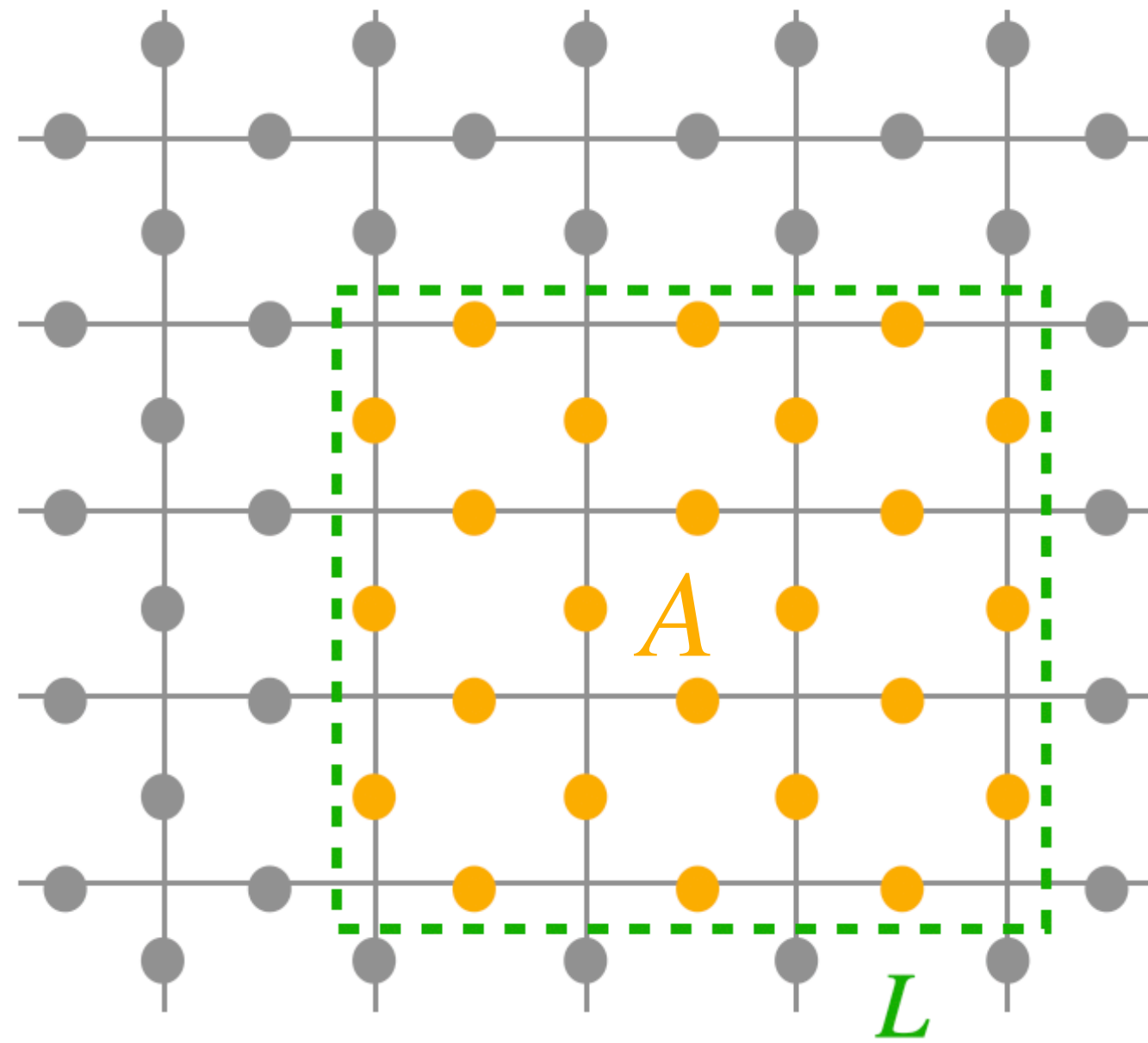
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... by Entanglement

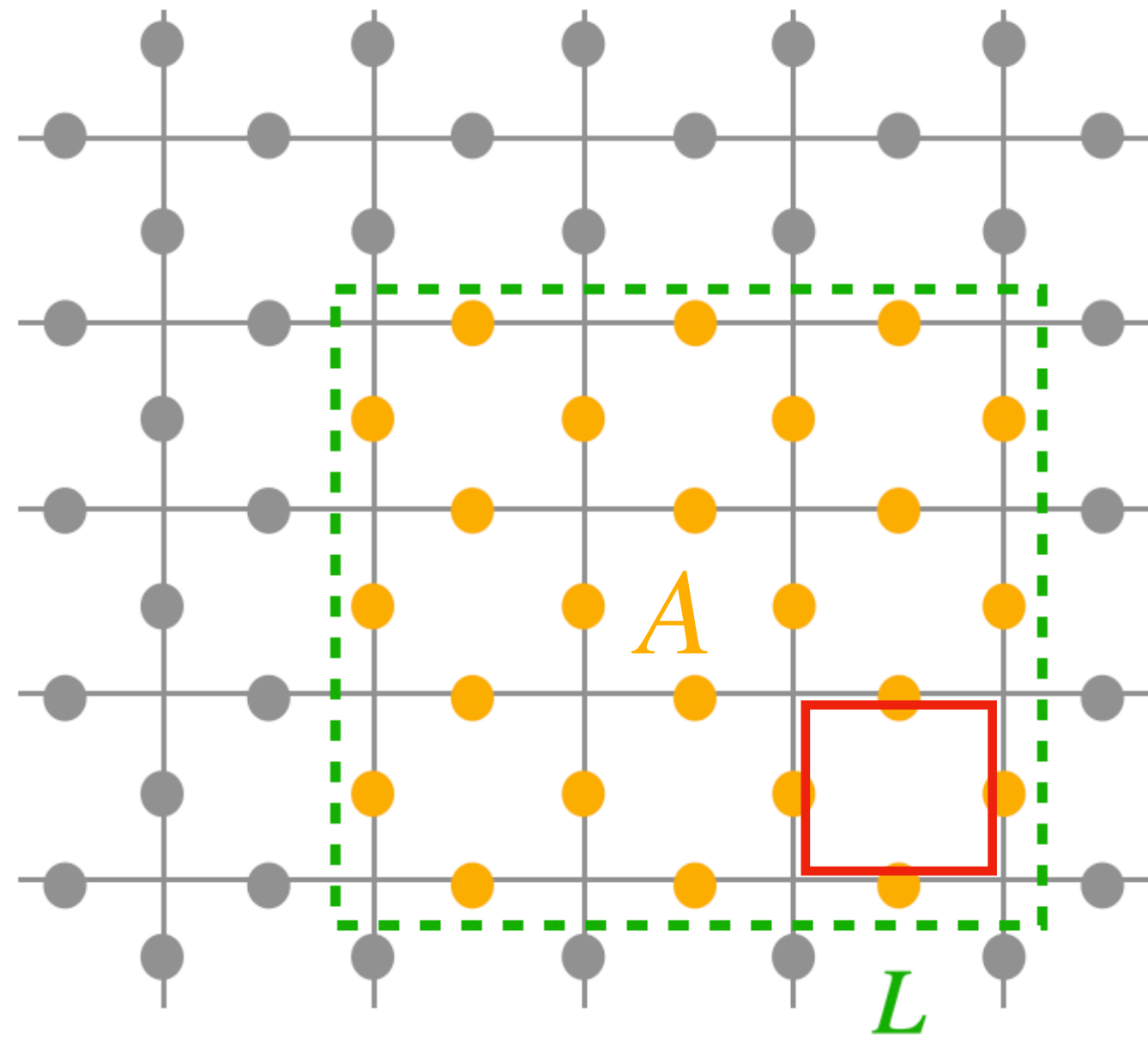
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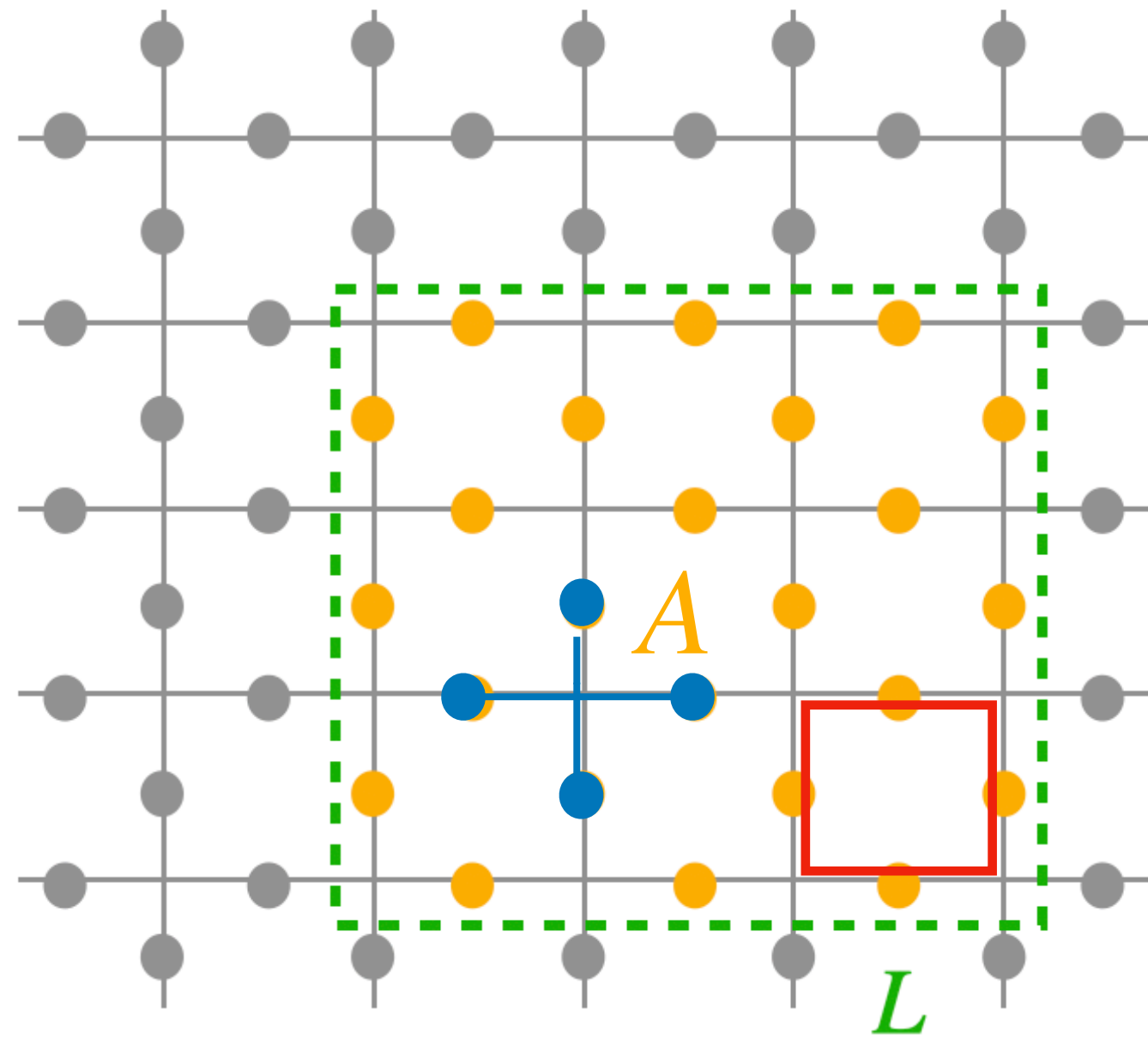
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$$\propto \prod_{p \in \mathcal{A}} (1 + B_p)$$

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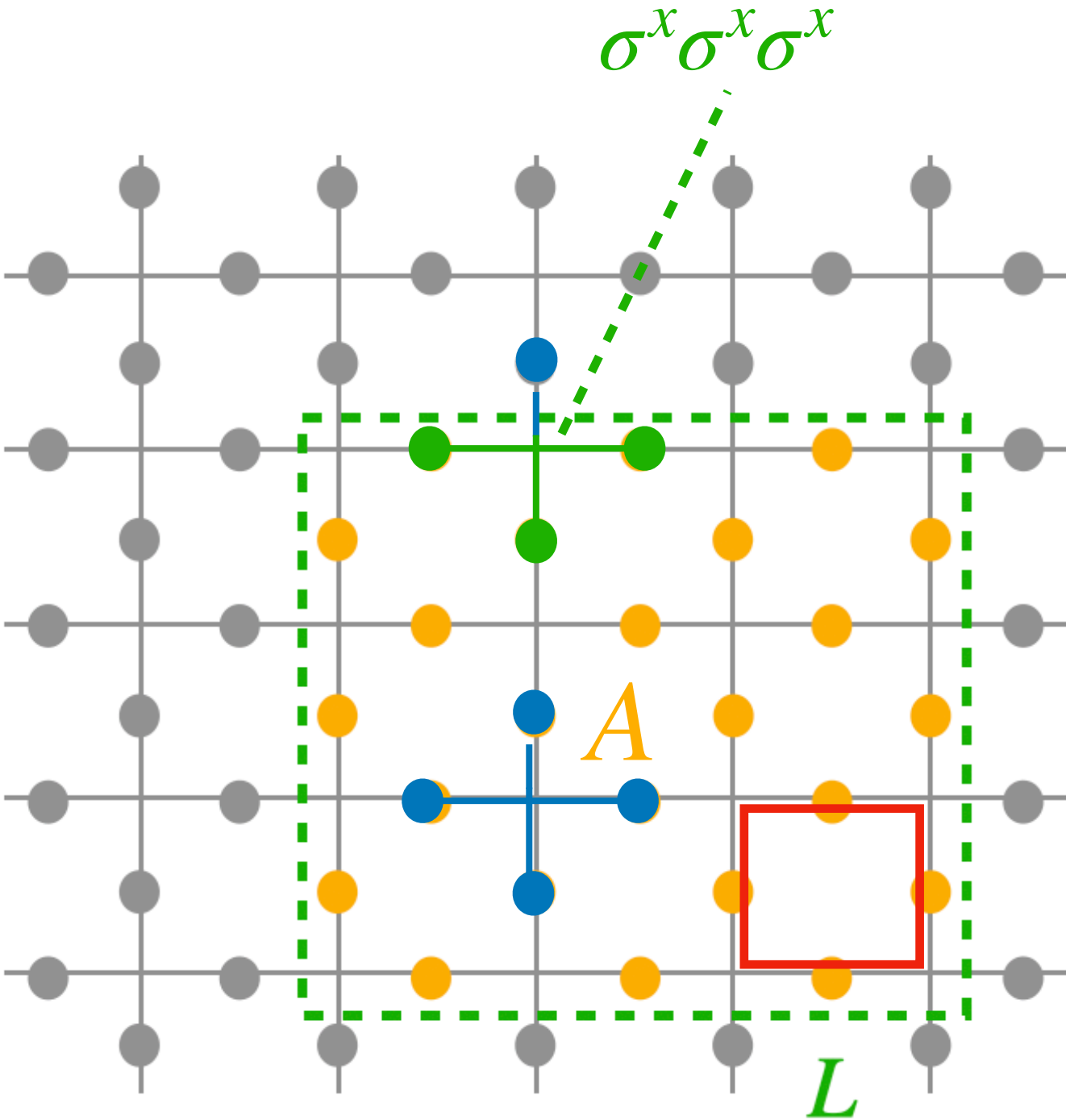
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$$\propto \prod_{p \in \mathcal{A}} (1 + B_p) \prod_{\nu \in \mathcal{A}} (1 + A_\nu)$$

Entanglement: From topological order to Lattice Gauge theories

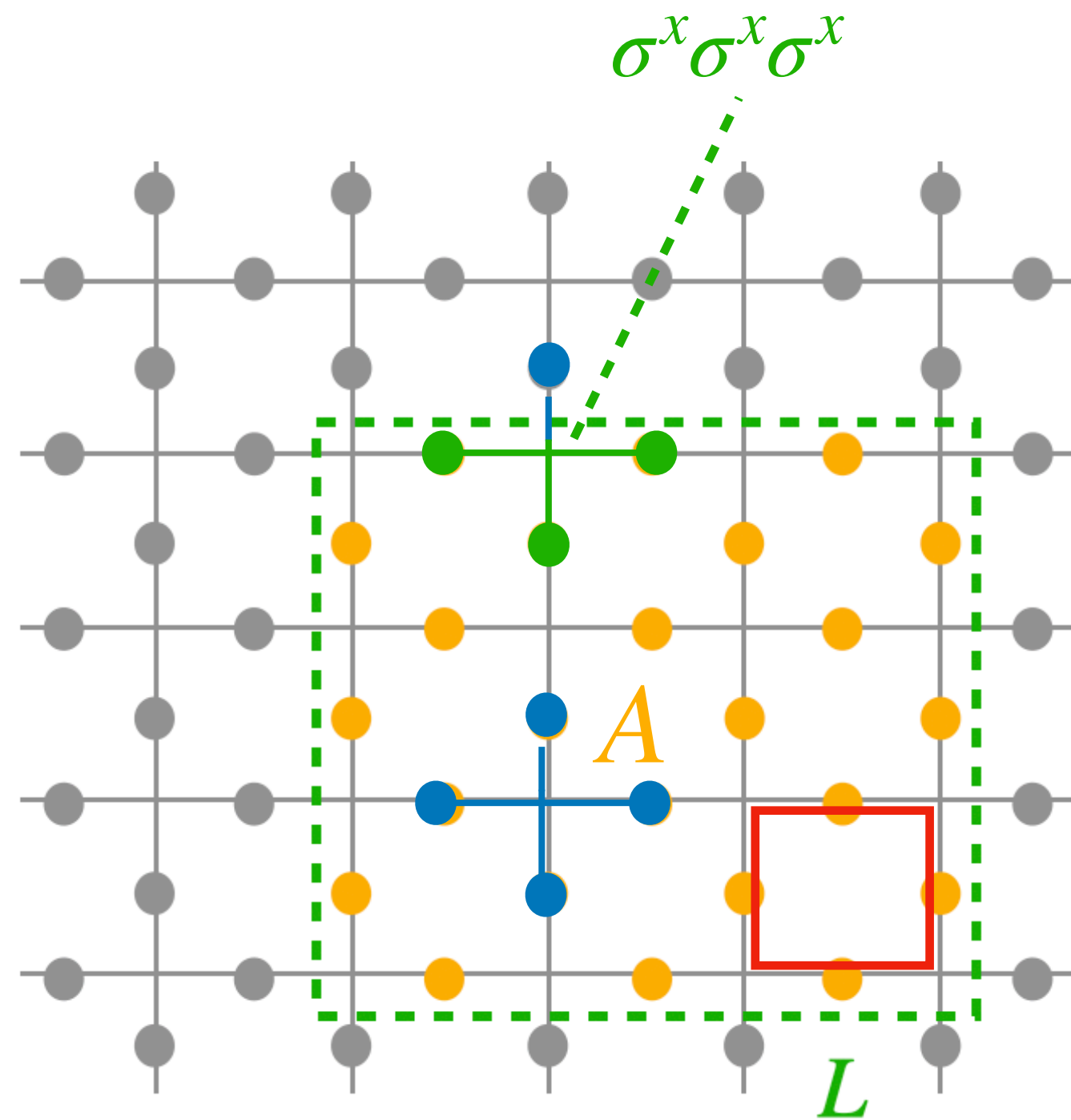
... by Entanglement



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Entanglement: From topological order to Lattice Gauge theories

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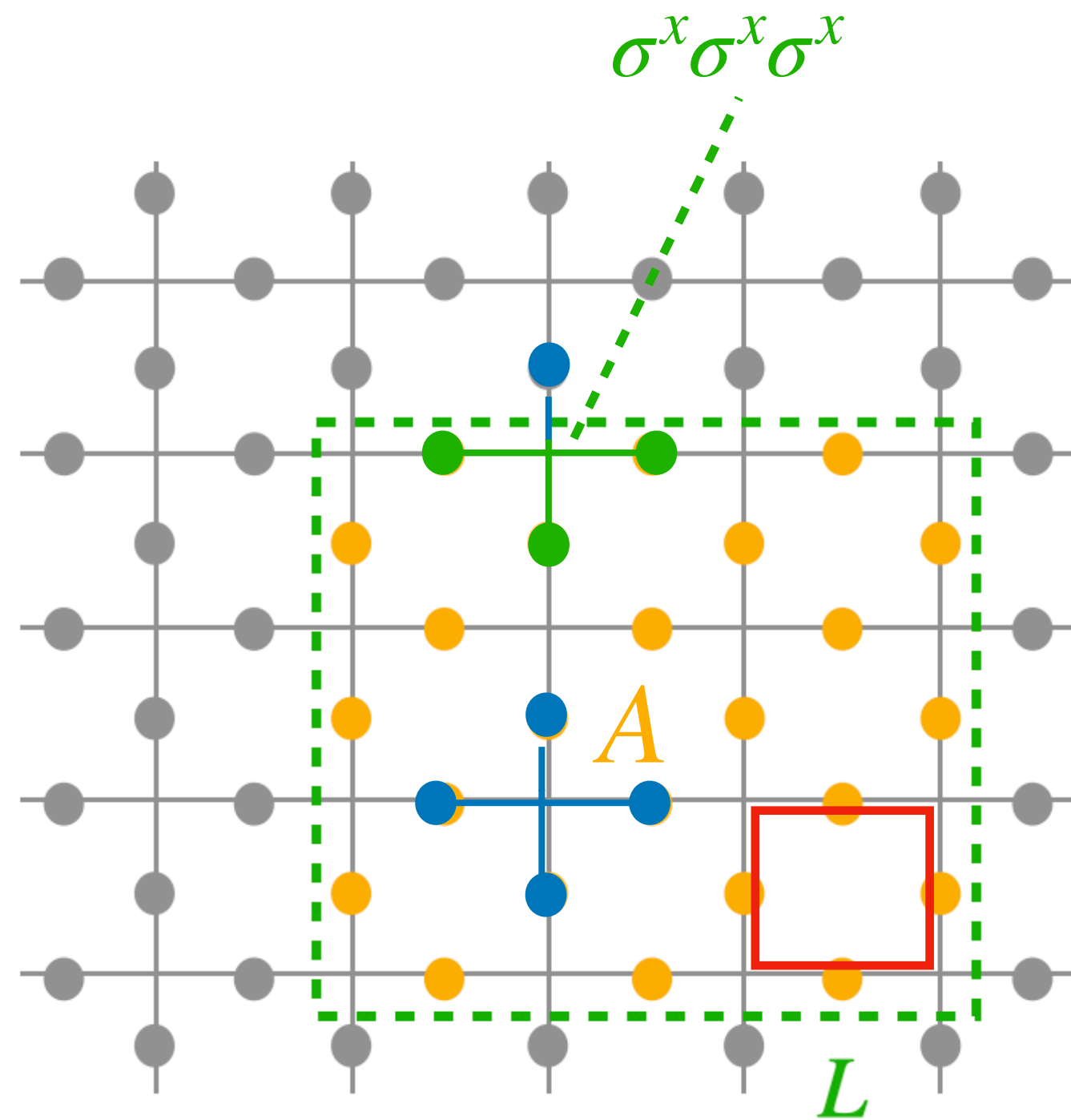


$$[\rho_A, \sigma^x \sigma^x \sigma^x] = 0$$

$$\propto \prod_{p \in \mathcal{A}} (1 + B_p) \prod_{\nu \in \mathcal{A}} (1 + A_\nu)$$

Entanglement: From topological order to Lattice Gauge theories

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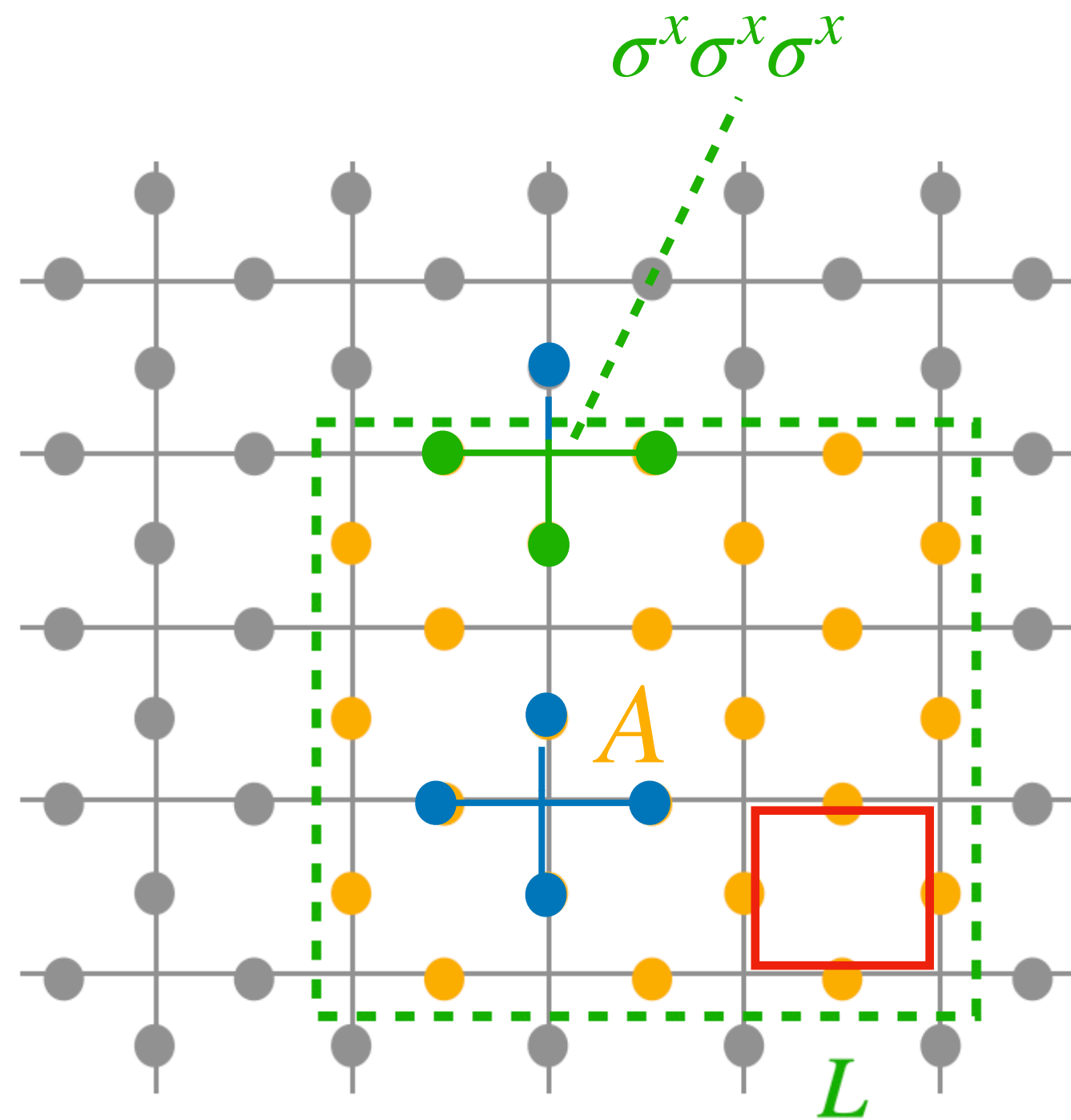
$$[\rho_A, \sigma^x \sigma^x \sigma^x] = 0$$

$$\rho_A = \begin{pmatrix} \square & \dots & \\ \vdots & \square & \\ & \dots & \ddots \end{pmatrix}$$

$$\propto \prod_{p \in \mathcal{A}} (1 + B_p) \prod_{\nu \in \mathcal{A}} (1 + A_\nu)$$

Entanglement: From topological order to Lattice Gauge theories

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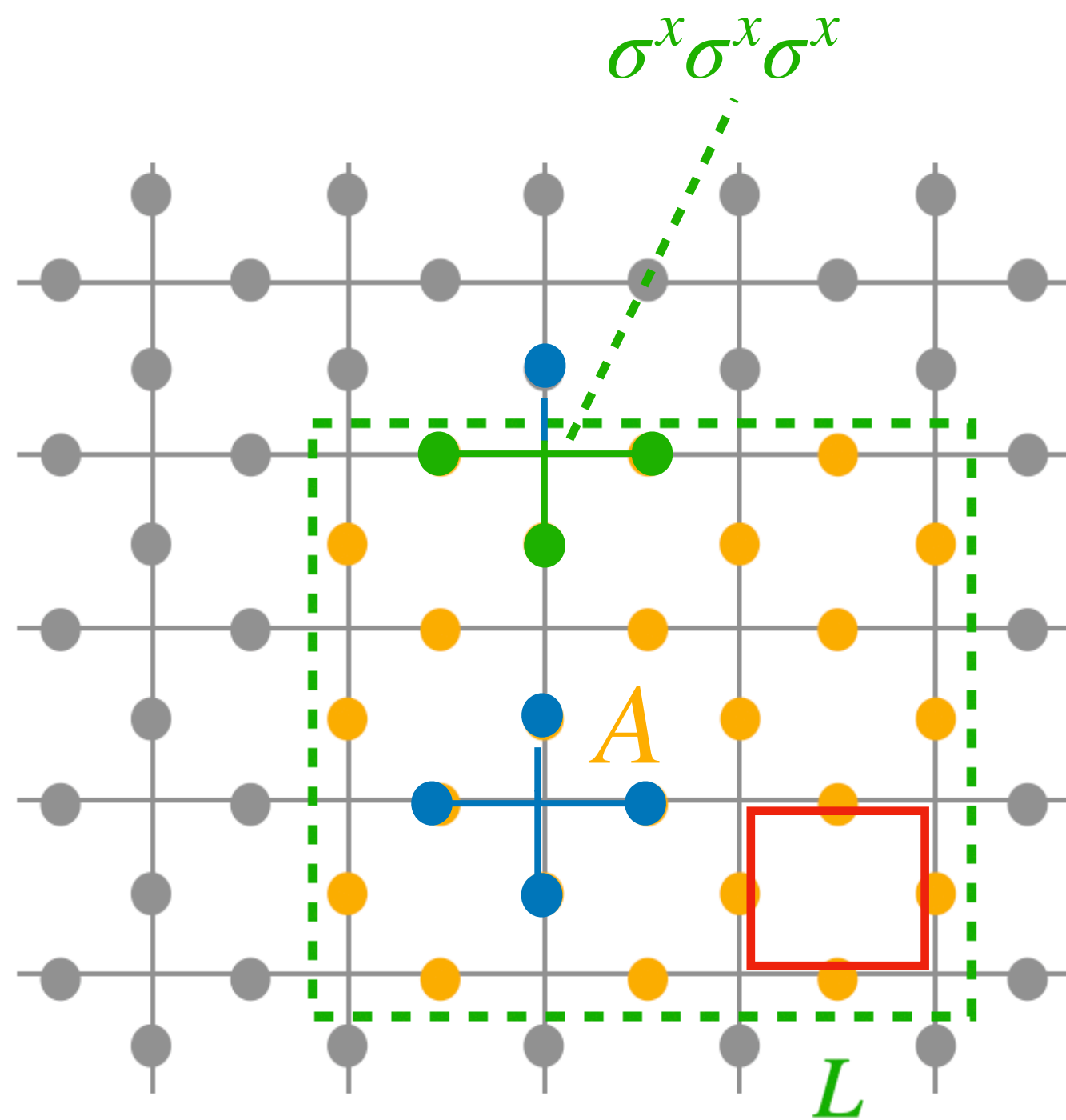
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Entanglement: From topological order to Lattice Gauge theories

... by Entanglement



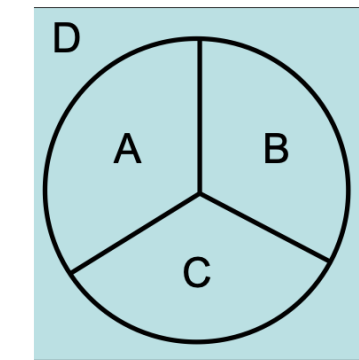
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Topological Entanglement Entropy

$$\prod_{\nu \in \partial A} A_\nu^\top = 1 \quad \# = L - 1$$



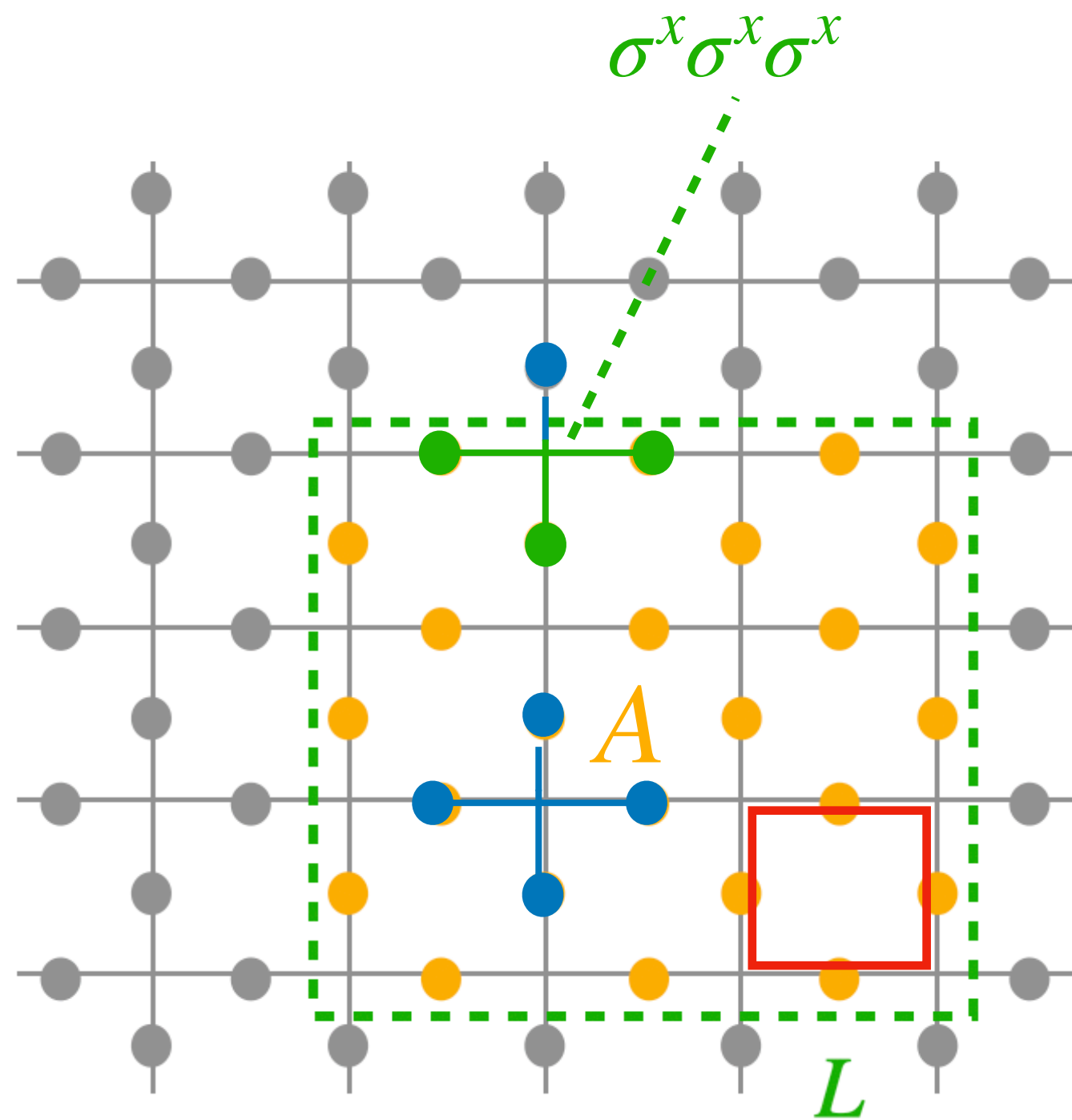
$$S = L \log(2) - \gamma$$

$$\gamma = \log(2)$$

Kitaev, Preskill PRL 96 (2006) 110404
Levin, Wen PRL 96, 110405 (2006)

Entanglement: From topological order to Lattice Gauge theories

... by Entanglement



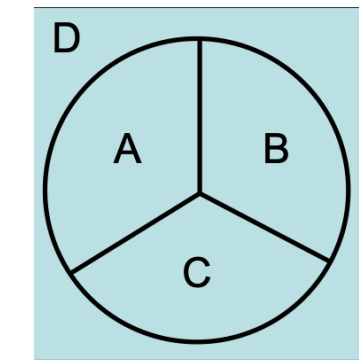
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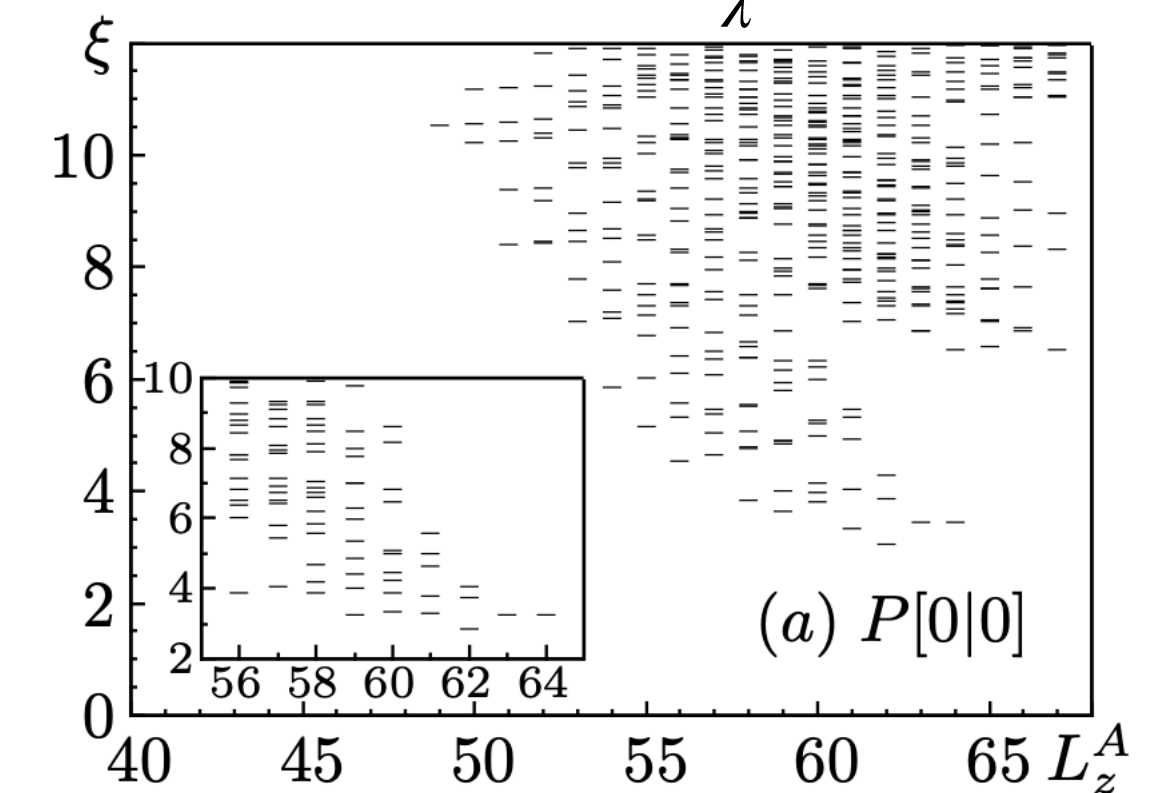
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Kitaev, Preskill PRL 96 (2006) 110404
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Entanglement Spectrum

$$\rho_A = \exp\{-H_A\} = \sum_{\lambda} e^{-\xi_\lambda} |\lambda\rangle \langle \lambda|$$



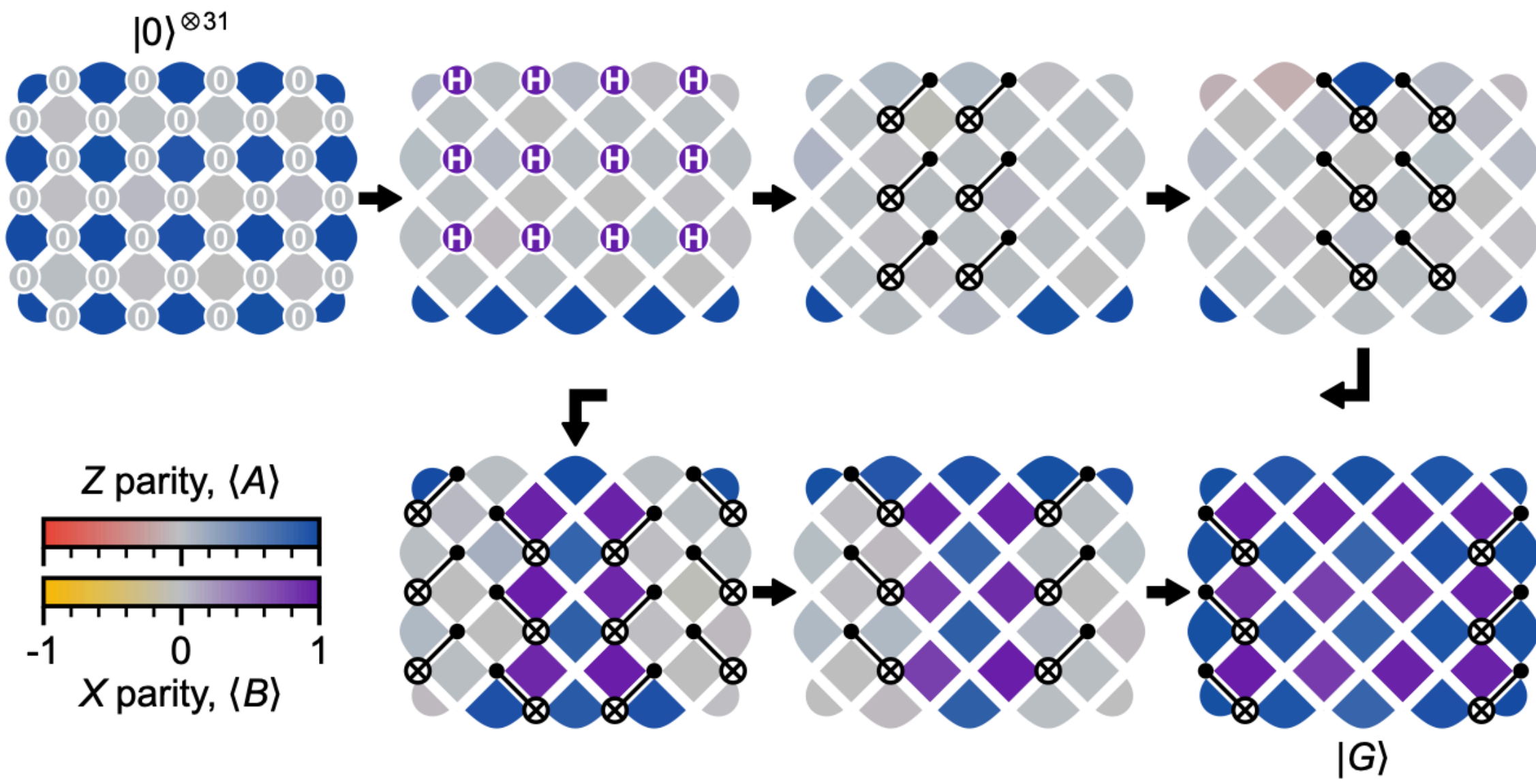
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Topological Order

Experimental realization

Topological Order

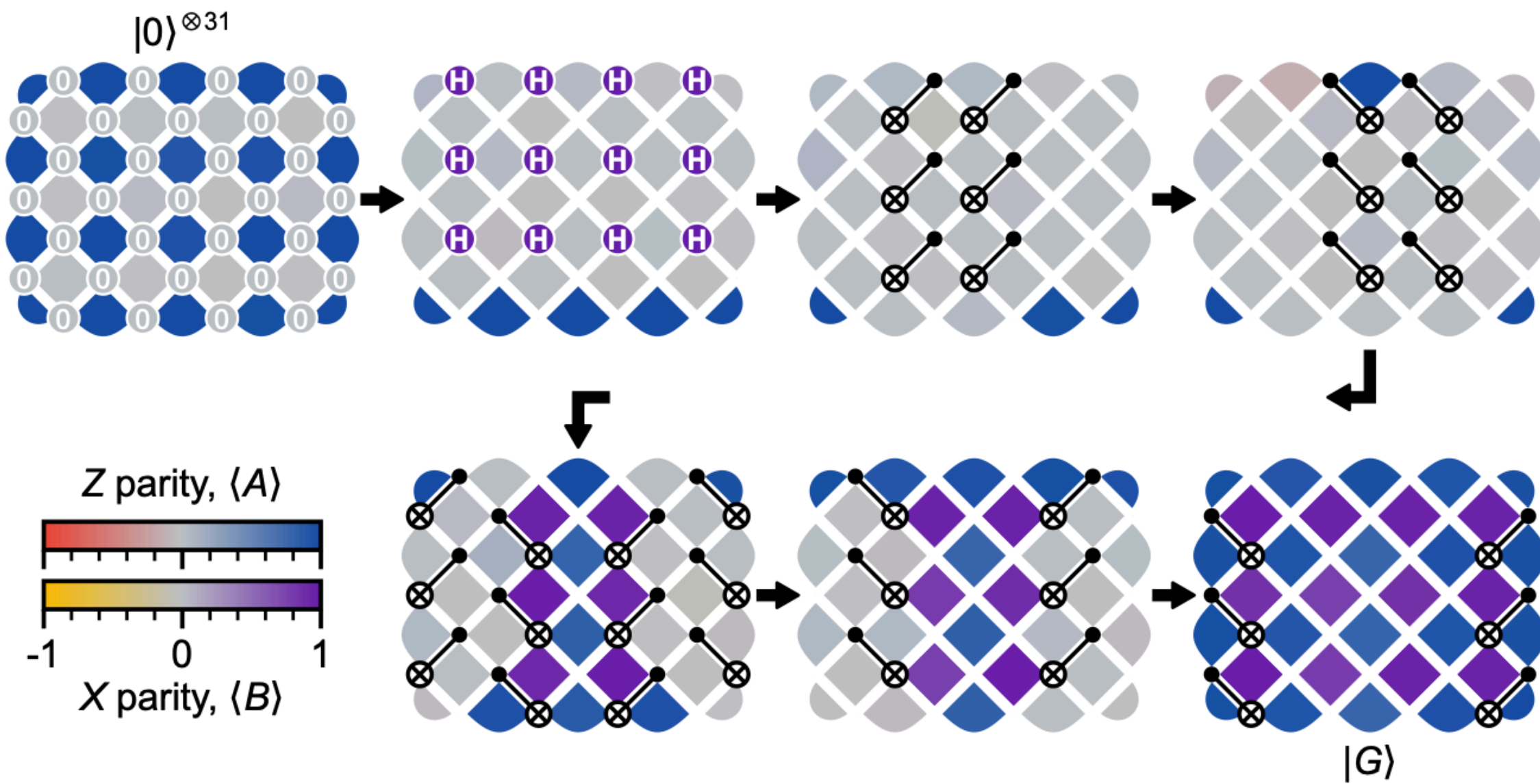
Experimental realization



Satzinger et al., Pollmann, Roushan Science 374, 123701241

Topological Order

Experimental realization



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- see also: **Spin liquid states with Rydberg arrays**

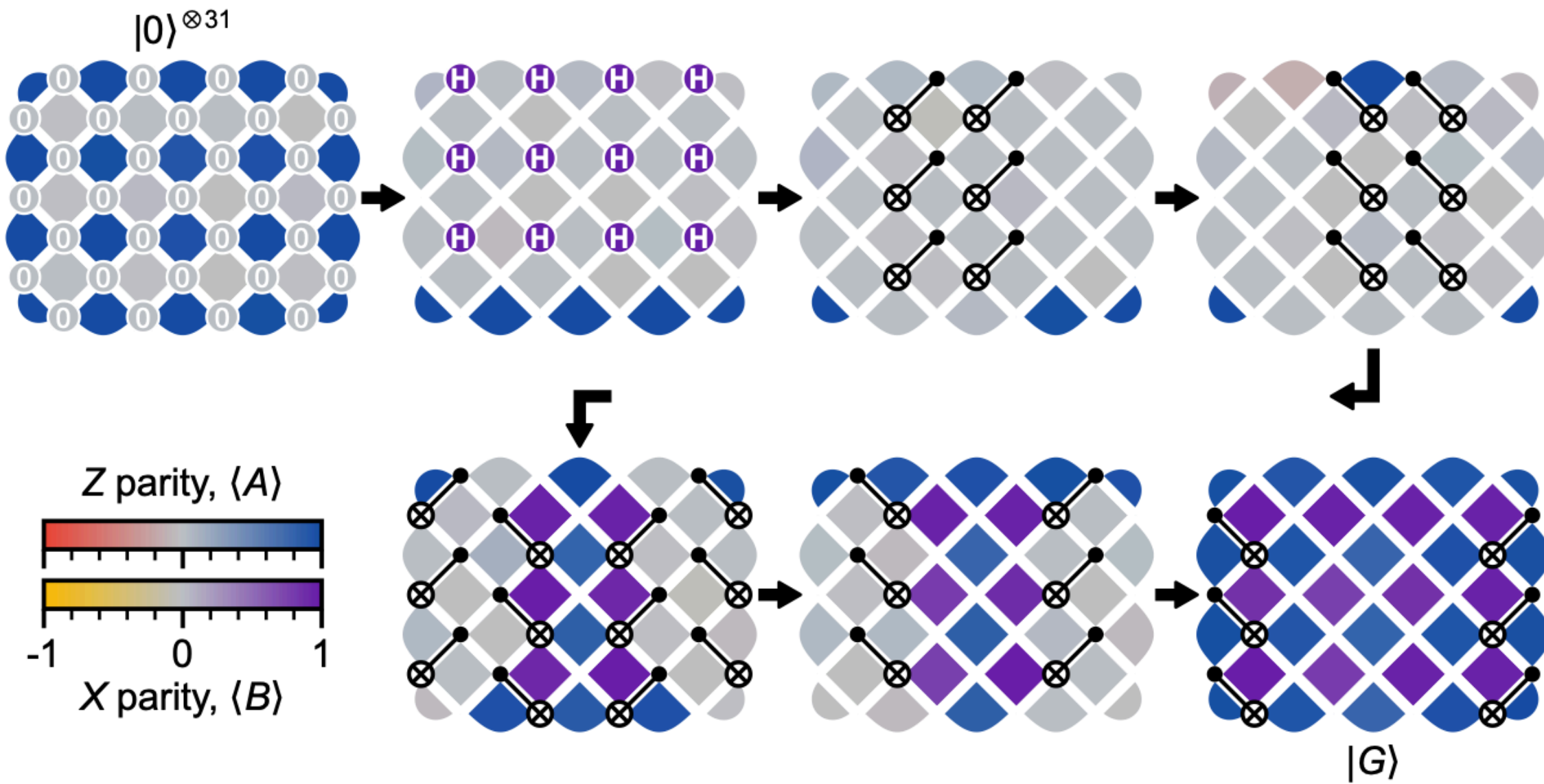
Semeghini et al Science 374, 1242

chiral spin liquids with cold atoms

Sun, Goldman, Aidelsburger, Bukov, PRX Quantum 4, 020329

Topological Order

Experimental realization



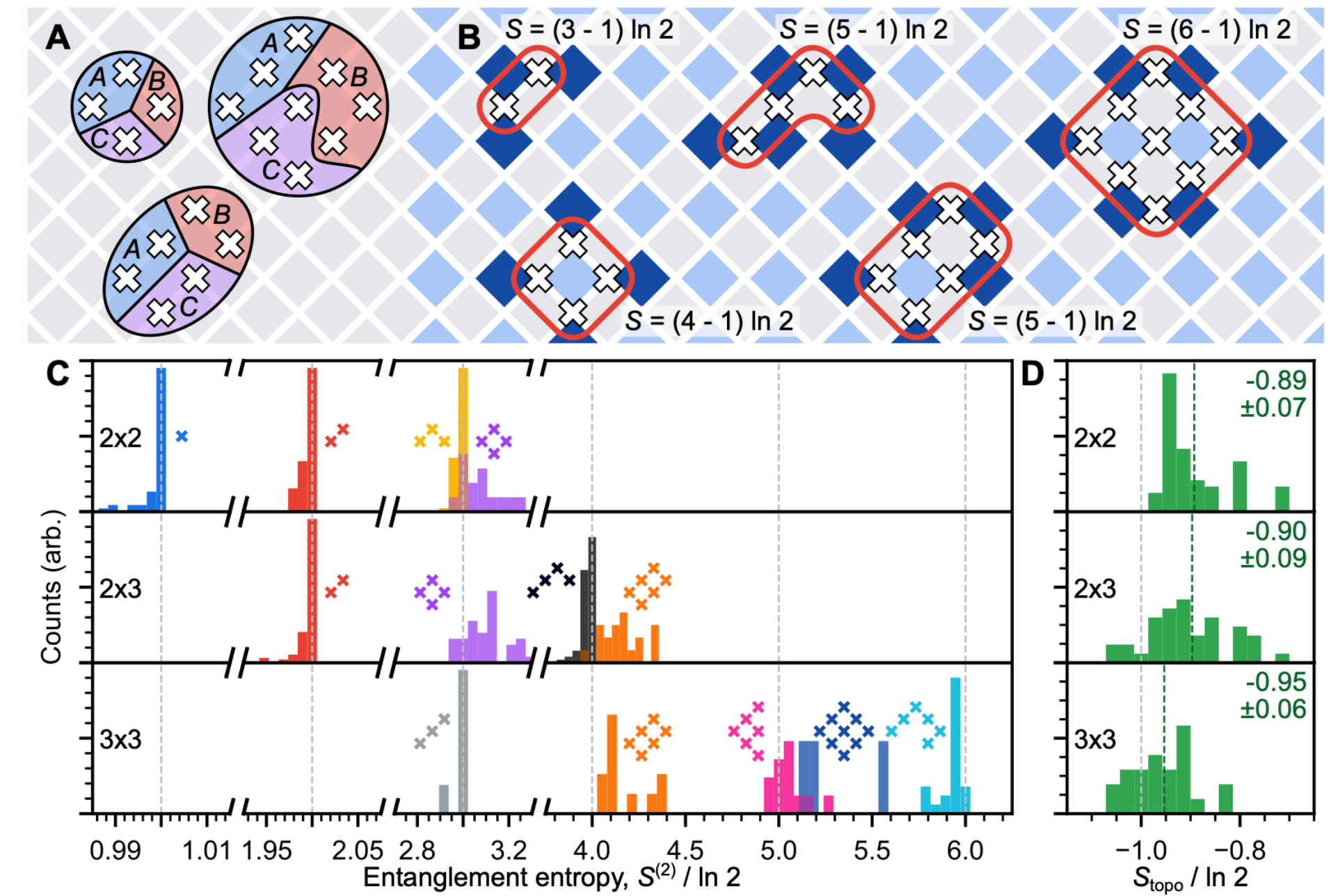
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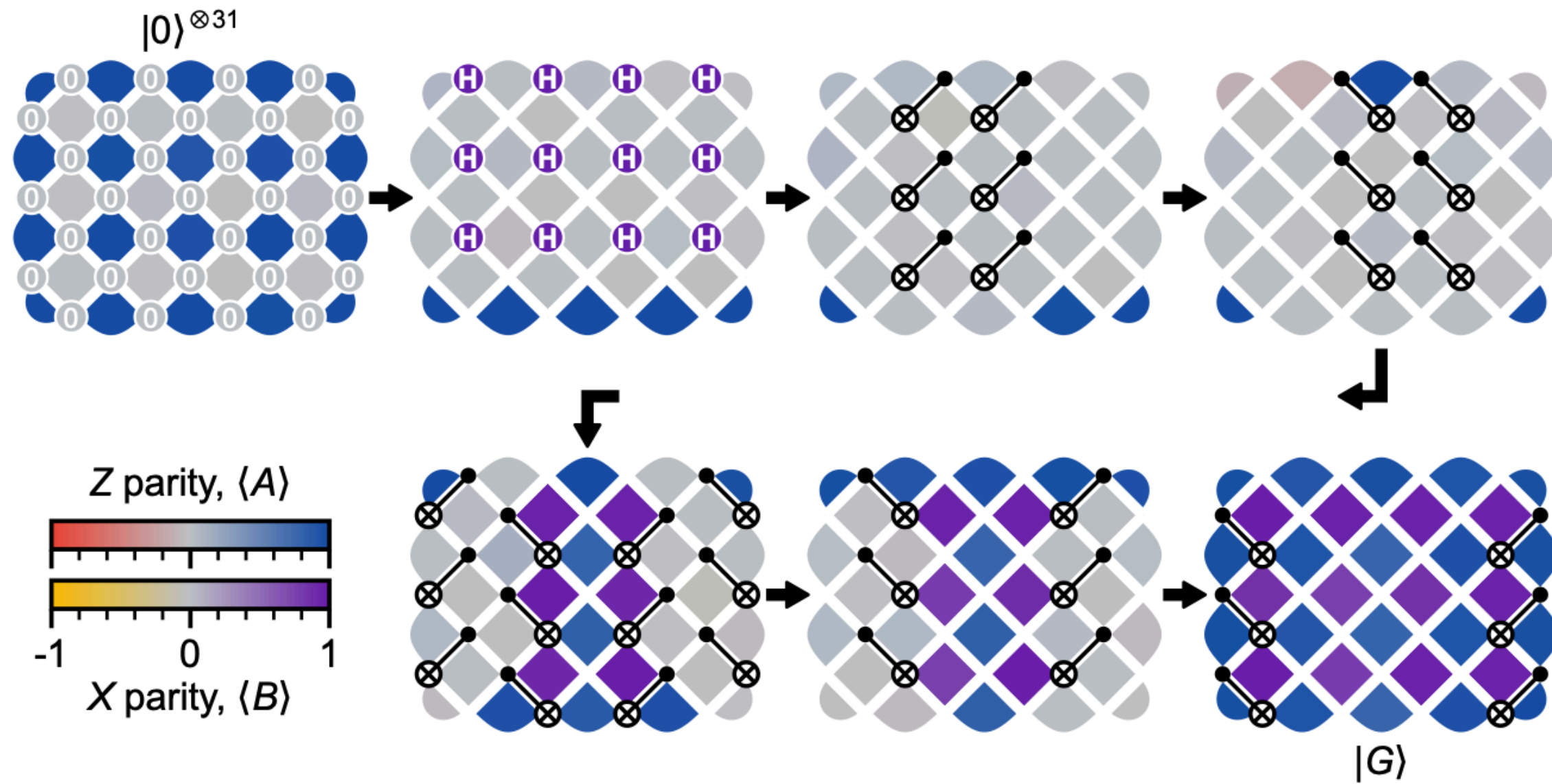
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Topological Order

Experimental realization



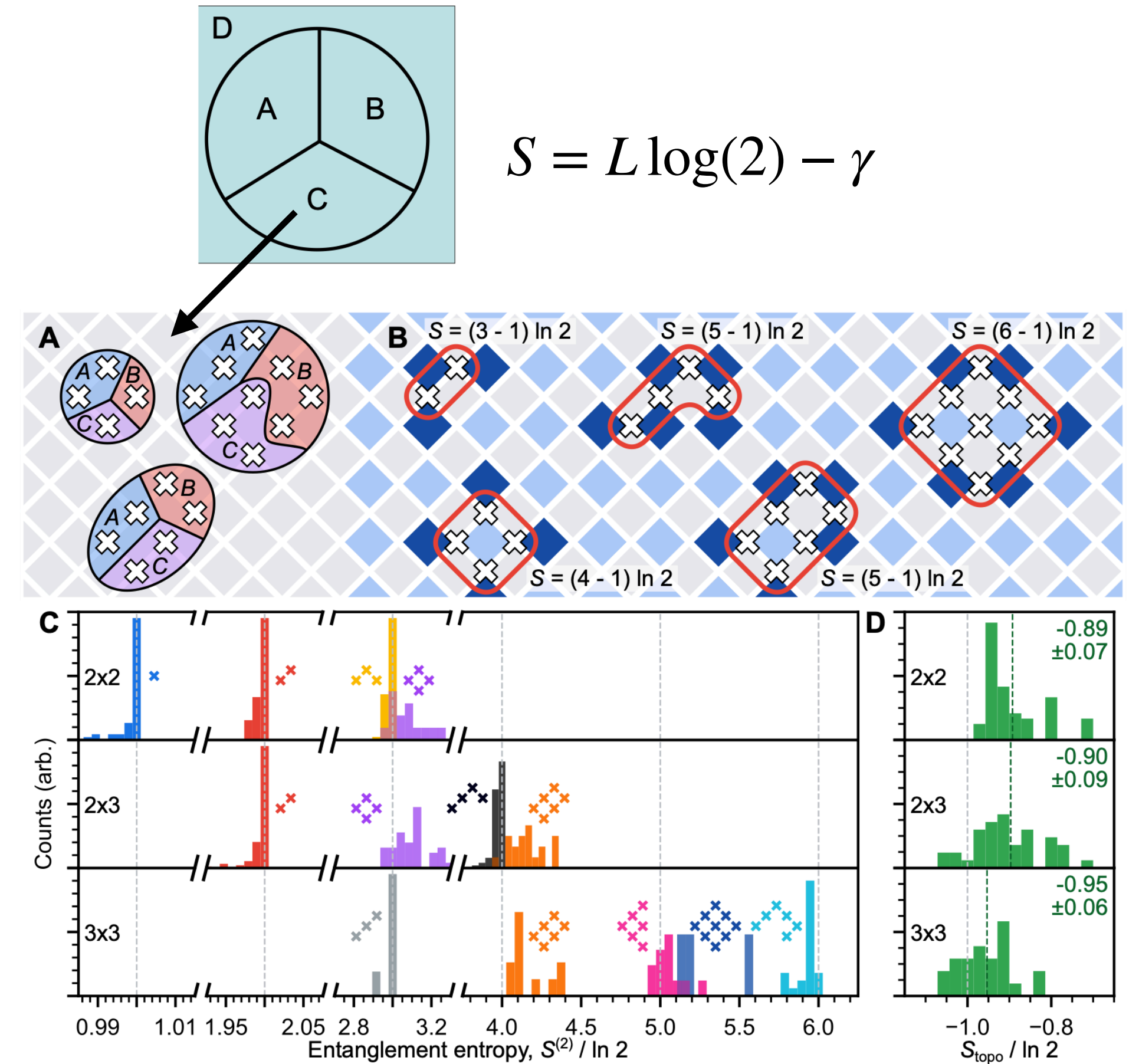
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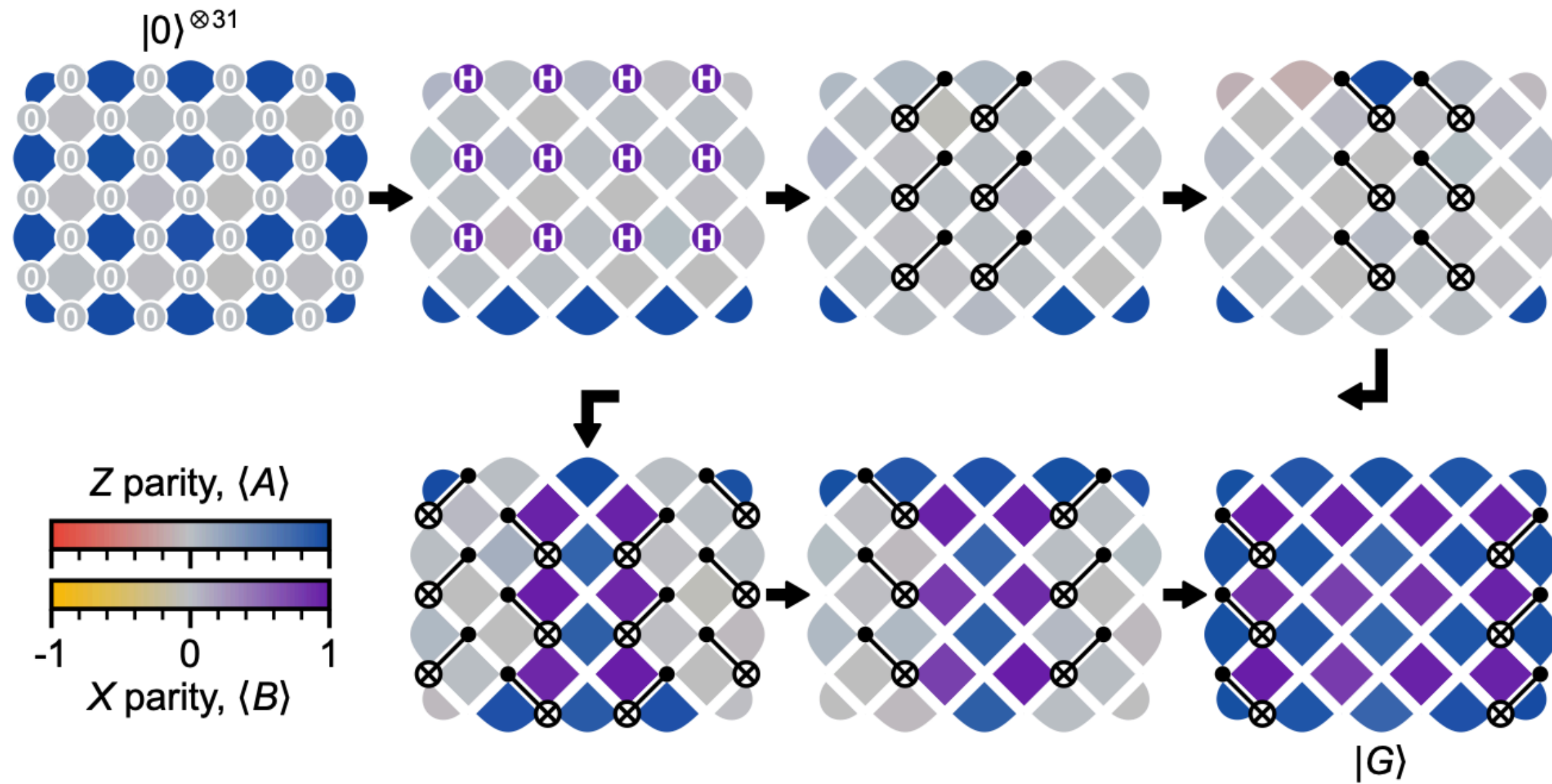
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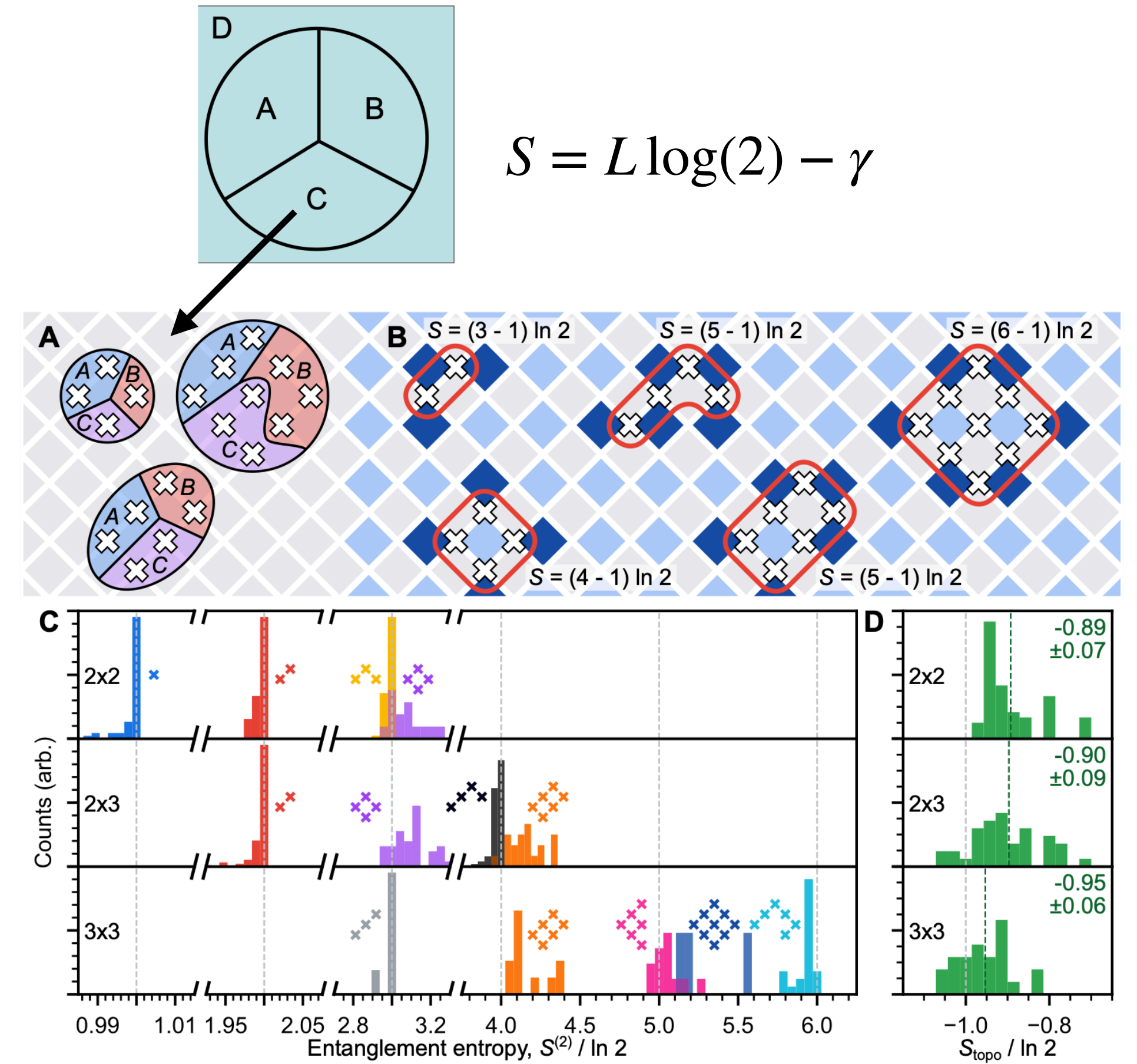
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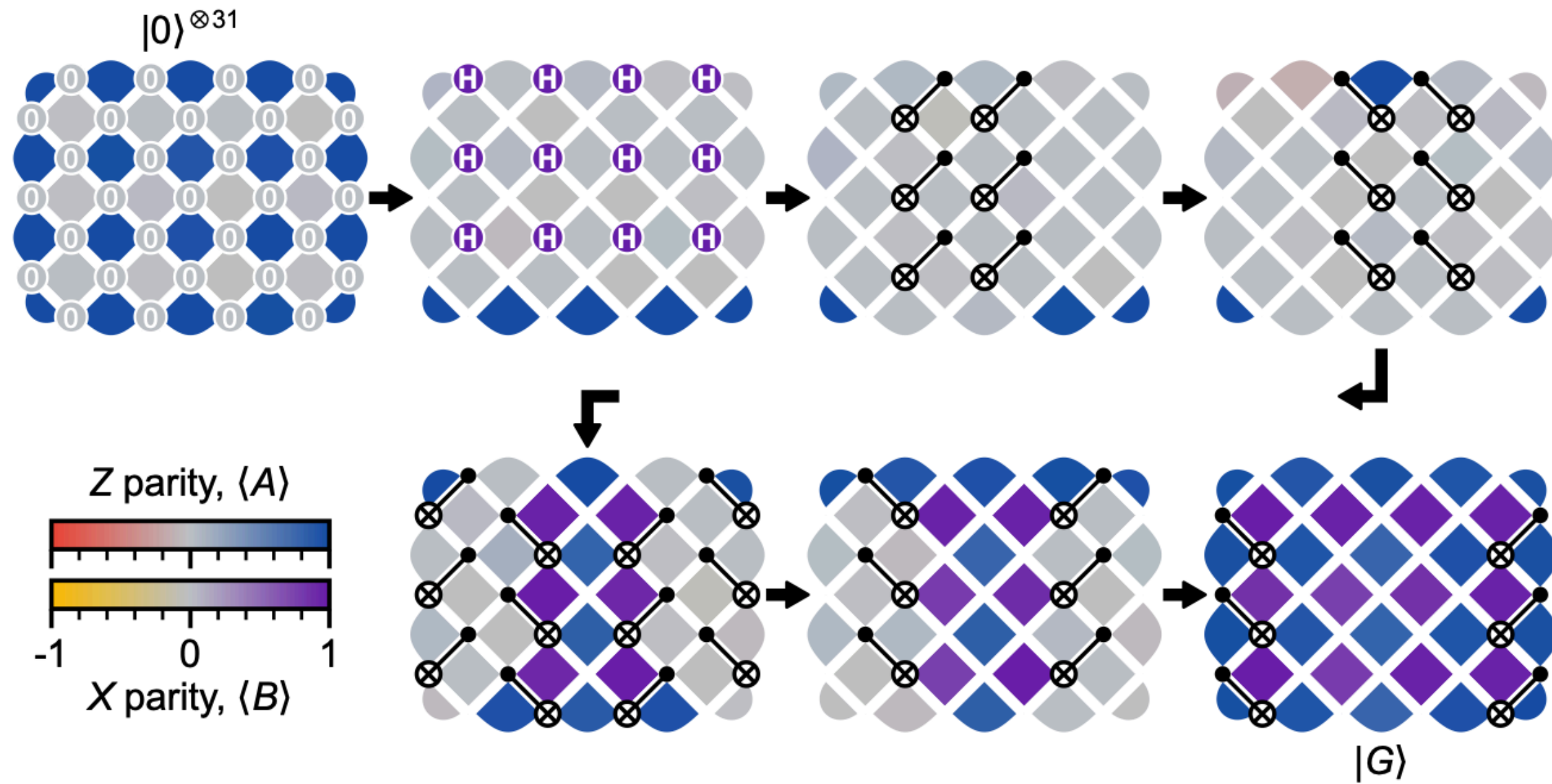


random measurement protocols

Brydges et al, Science 364, 260 (2019)

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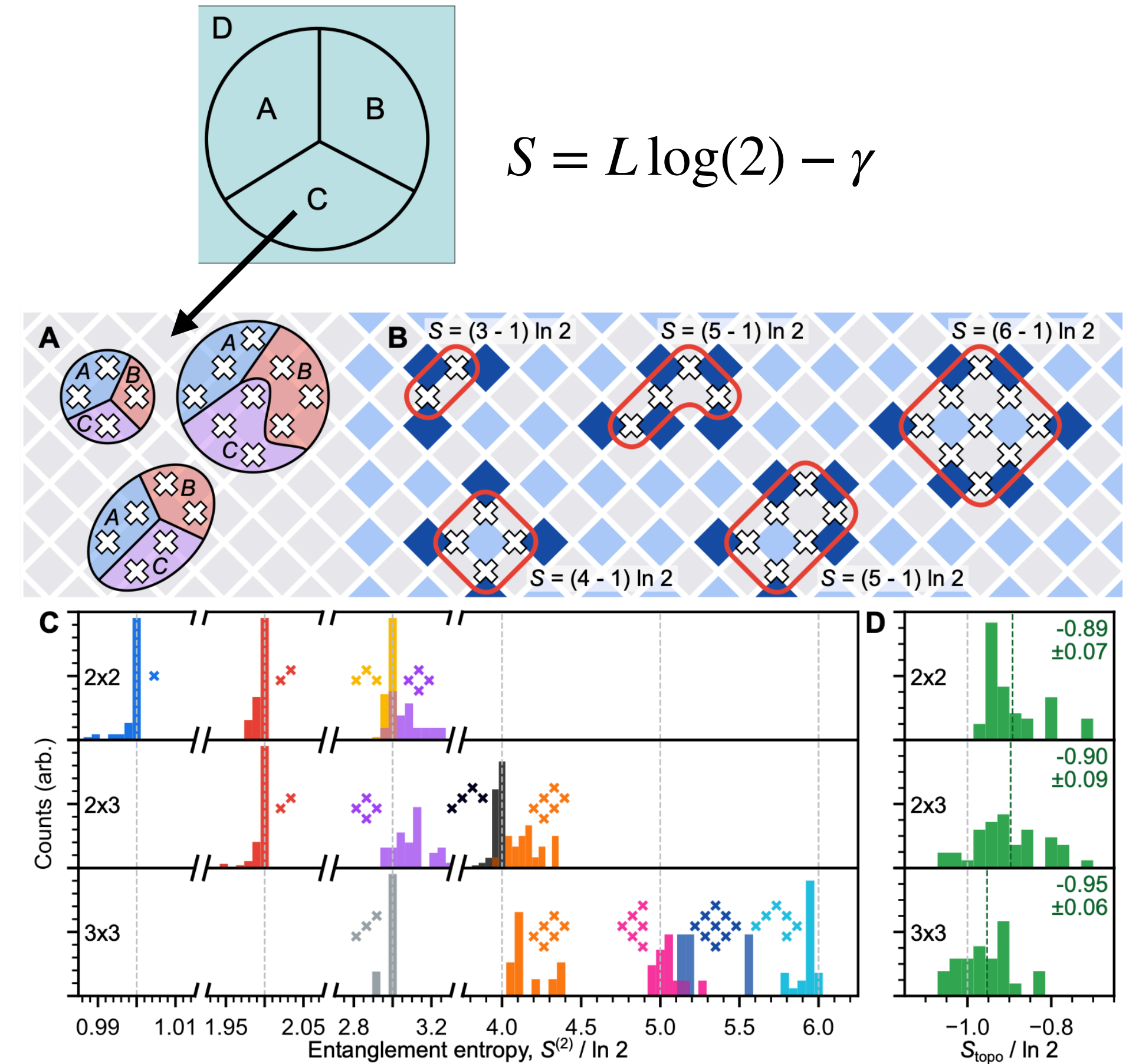
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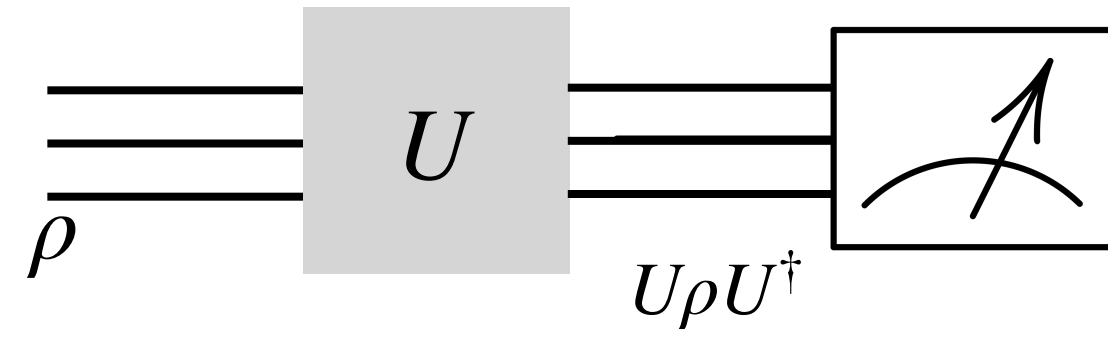


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Brydges et al, Science 364, 260 (2019)

Random Measurement: Entanglement

Random Measurement: Entanglement



Random Measurement: Entanglement

good intro: Richard Küng
<https://youtube.com/watch?v=FXdJoJ0qcZY>

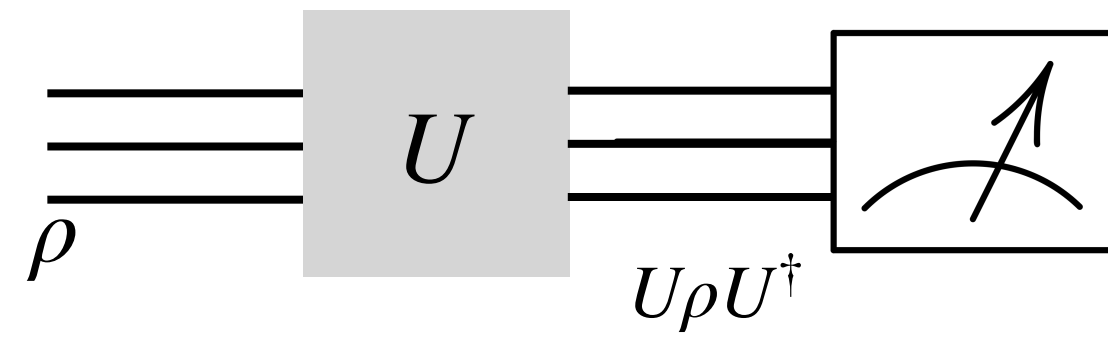


Tutorial
The Randomized
Measurement Toolbox

Richard Hueng
Johannes Kepler University Linz

March 5, 2022

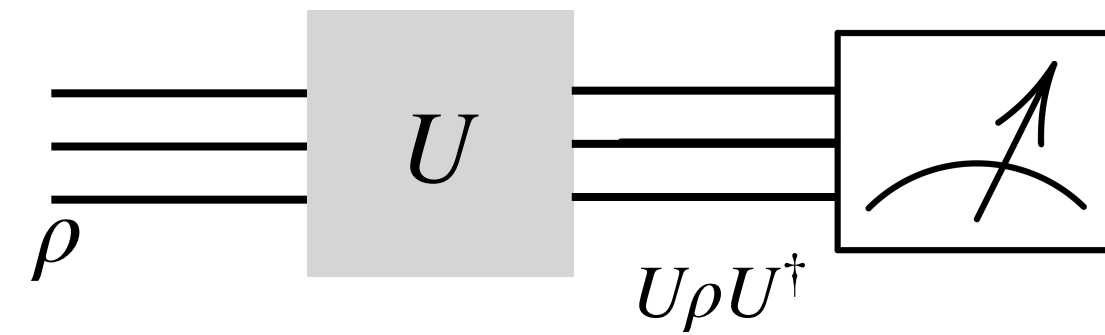
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Random Measurement: Entanglement

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VQE type algorithms

device verification

predict many observables from few measurements



Tutorial
The Randomized
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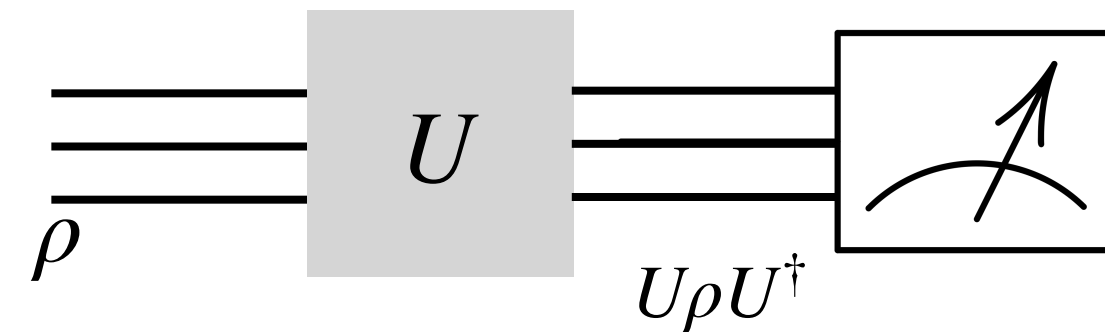
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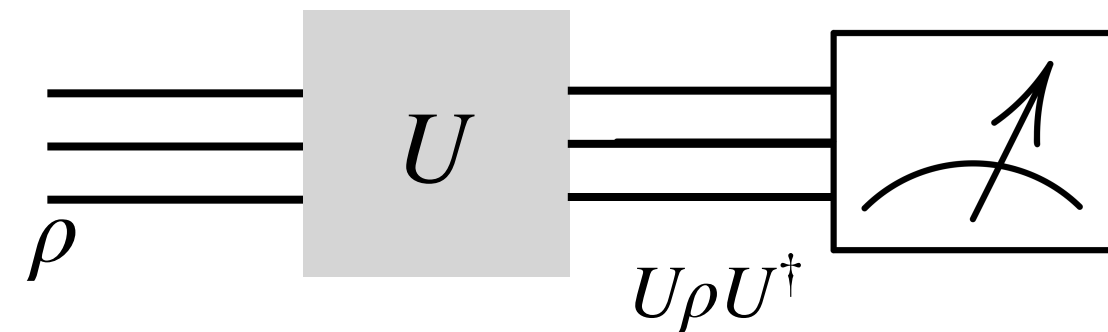


VQE type algorithms
device verification
predict many observables from few measurements
Entanglement Tomography: “k-designs give k-entropies”



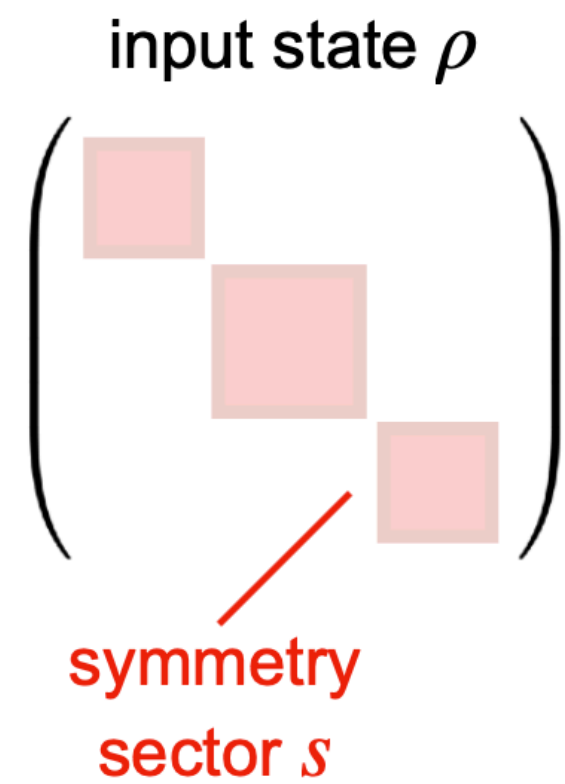
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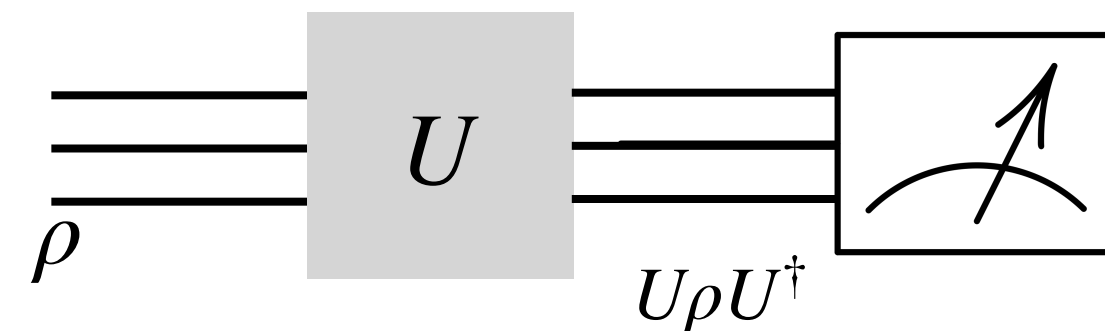
Symmetry ignorant versus conscious



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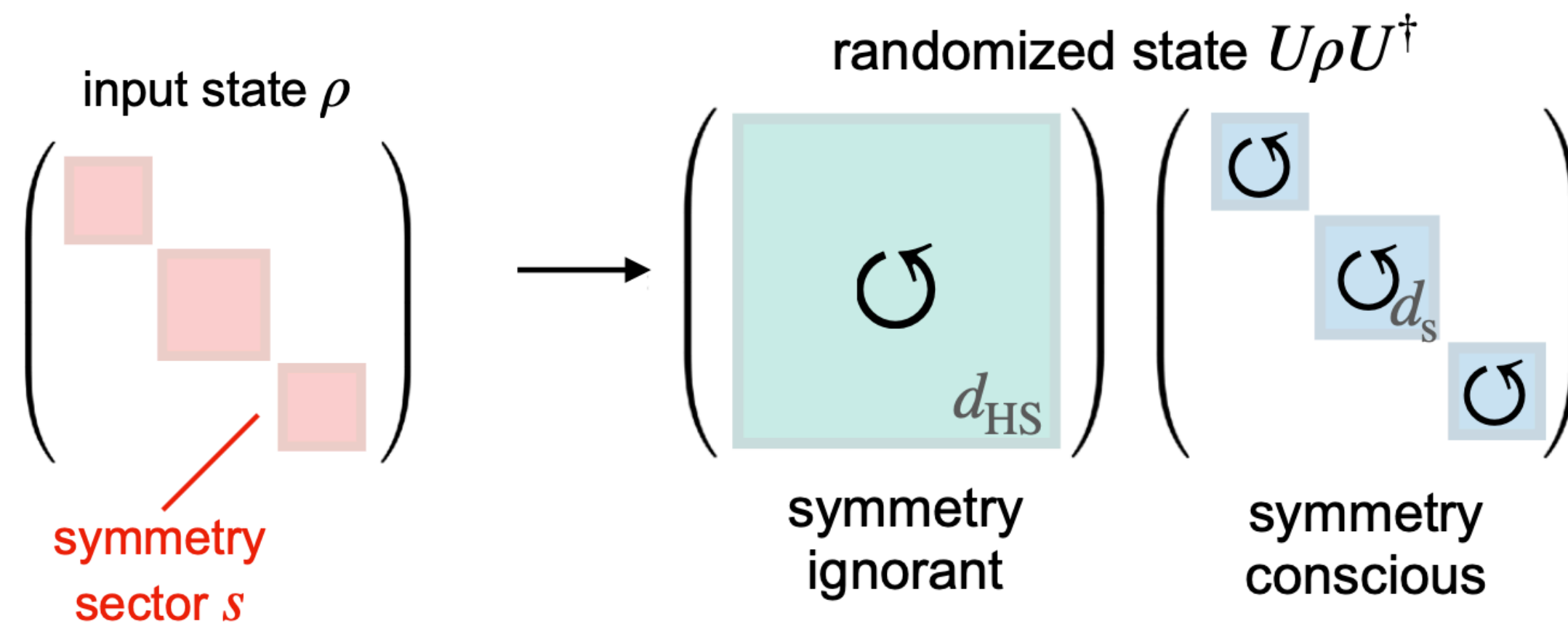
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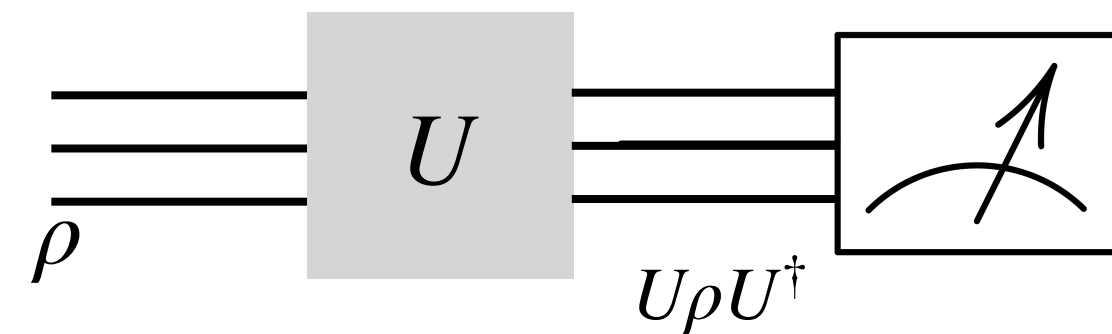
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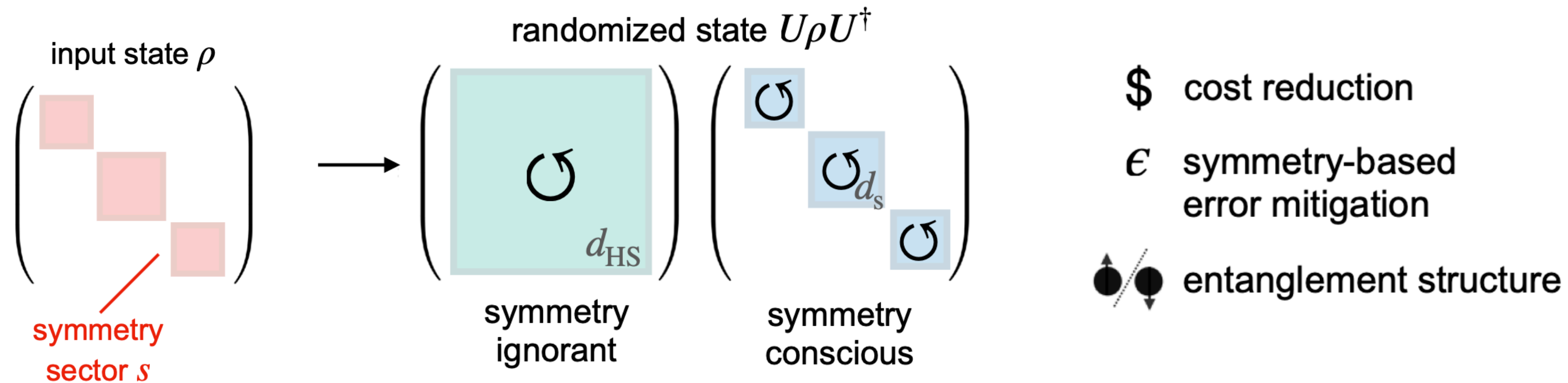
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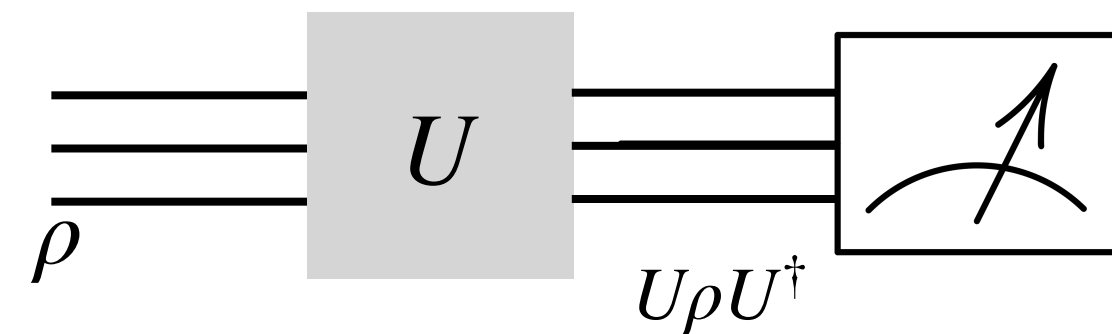
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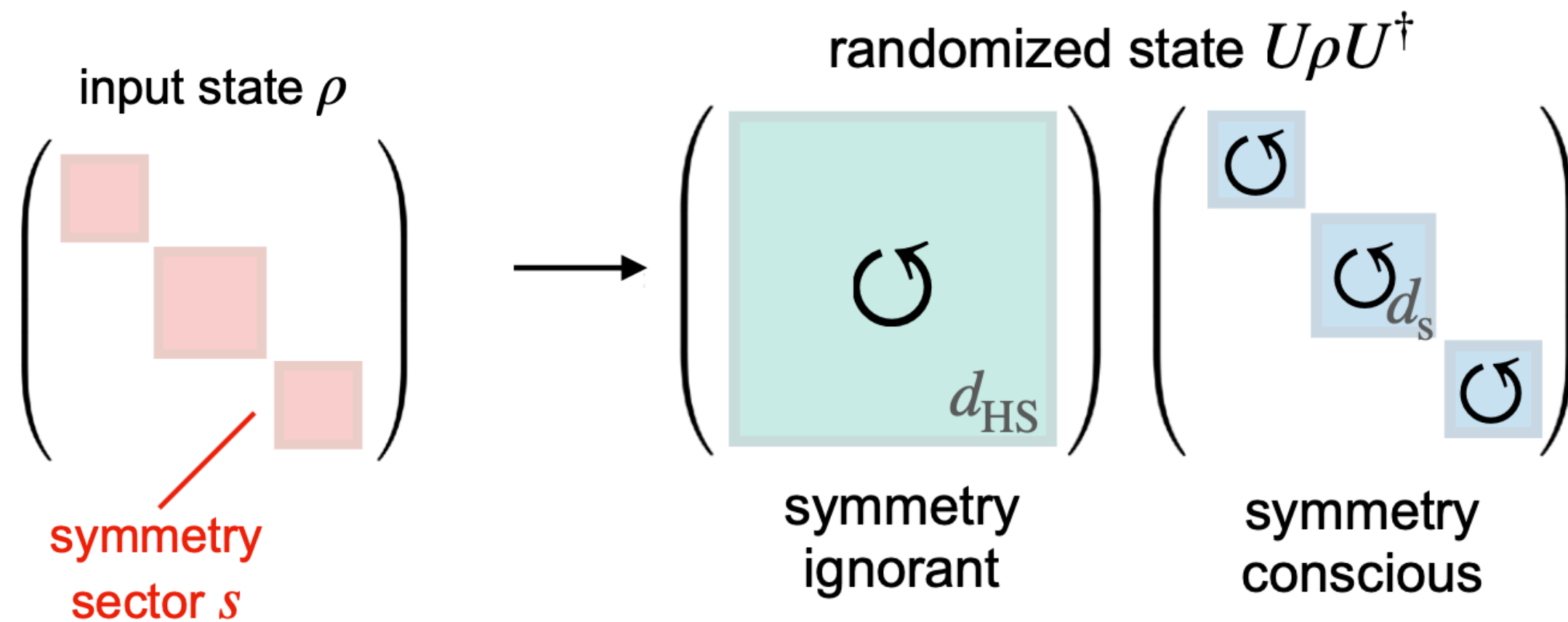
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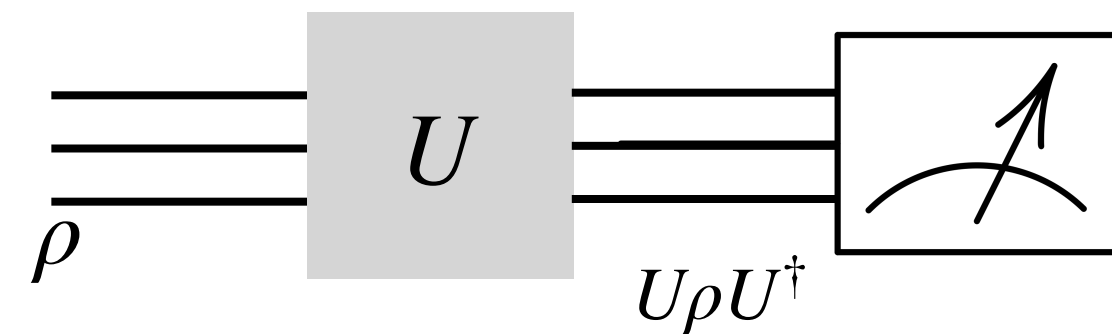


- \$ cost reduction
- ϵ symmetry-based error mitigation
- entanglement structure

d_s vs. d_{HS} (Z2: $2^V / 2^{2V} = 2^{-V}$)

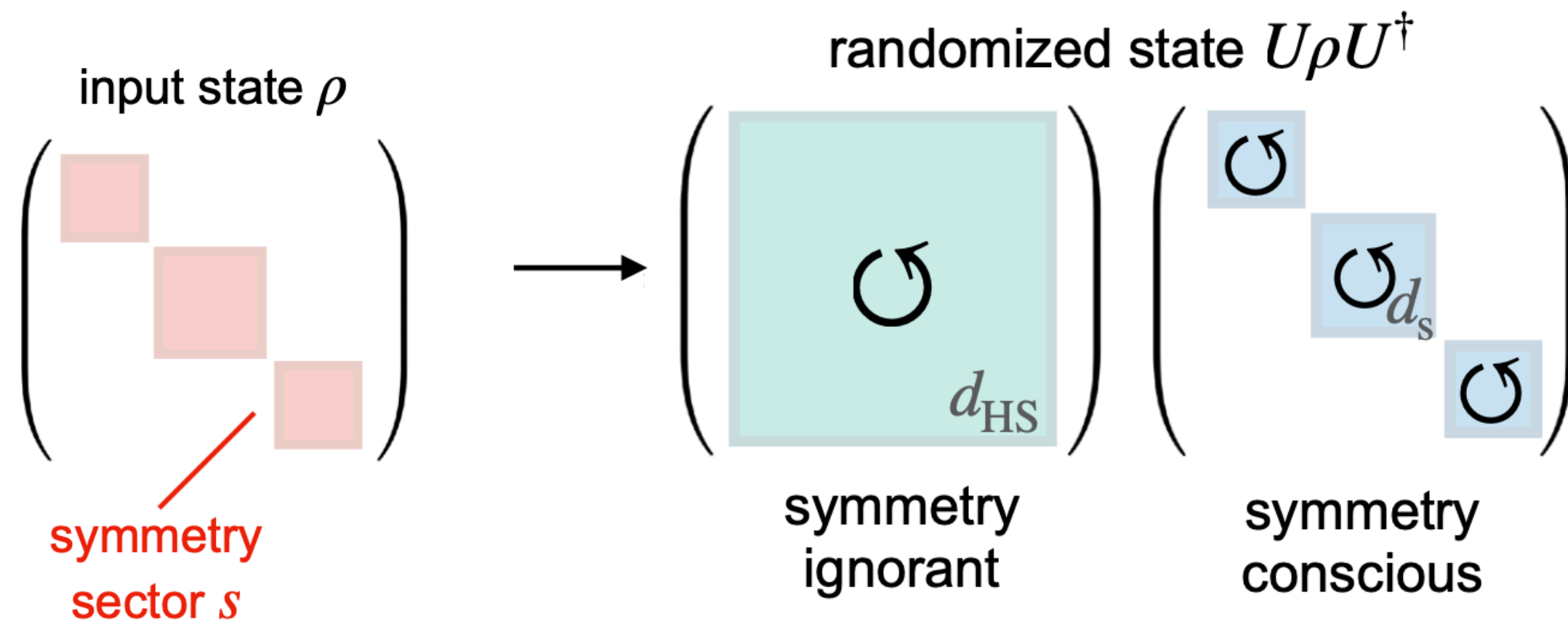
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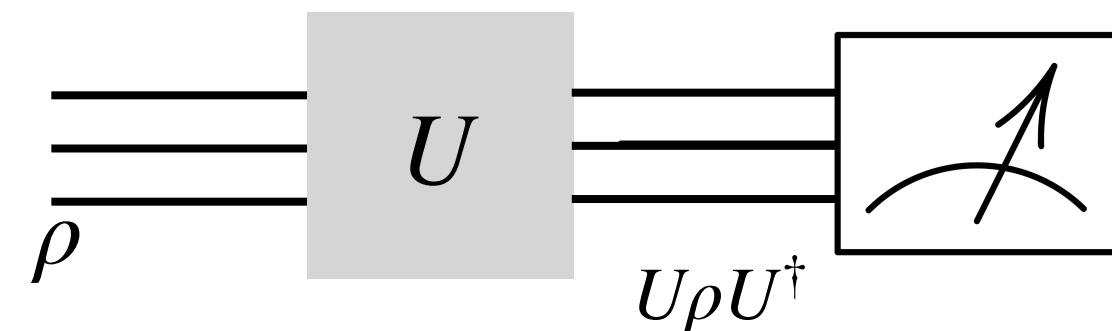
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Nuguyen et al. Davoudi, Linke
PRX Quantum 3, 023024 (2022)

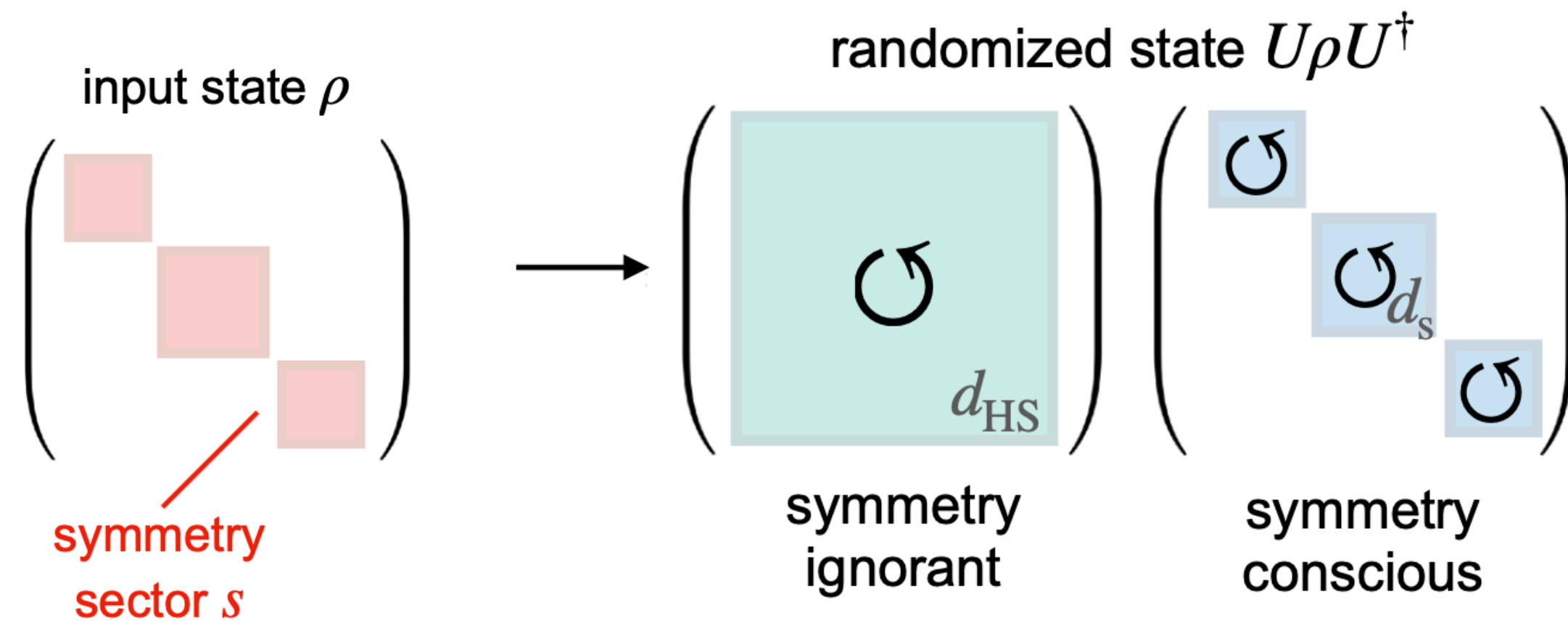
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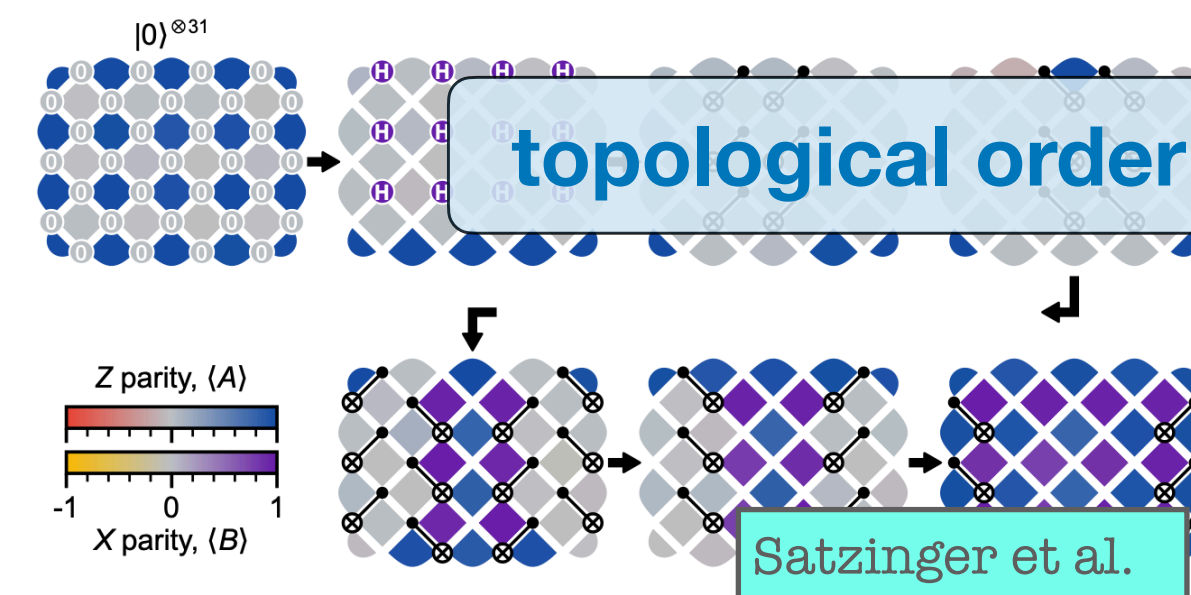


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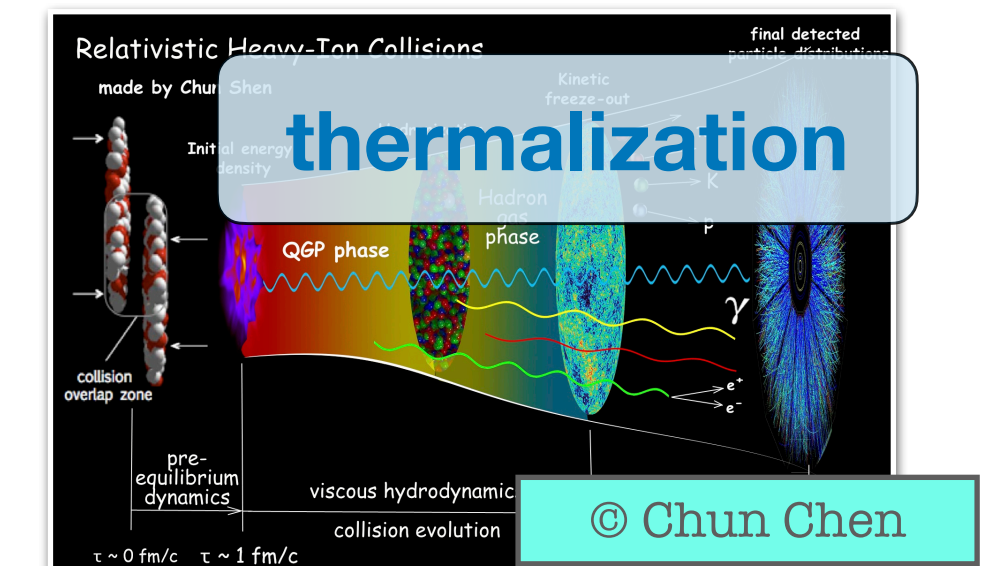
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Nuguyen et al. Davoudi, Linke
PRX Quantum 3, 023024 (2022)

this talk!



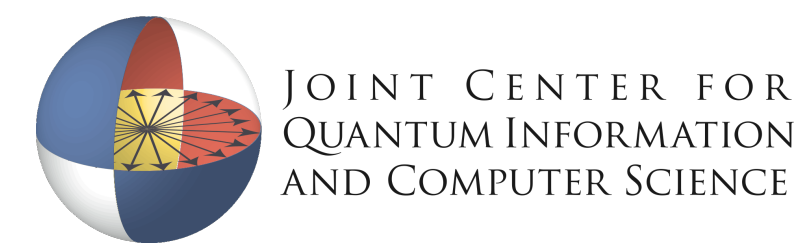
Satzinger et al.



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Random Measurement

Symmetry ignorant versus conscious



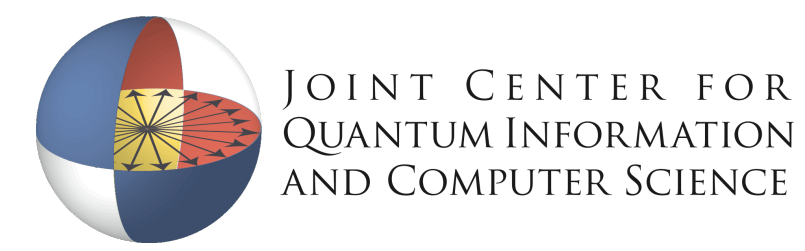
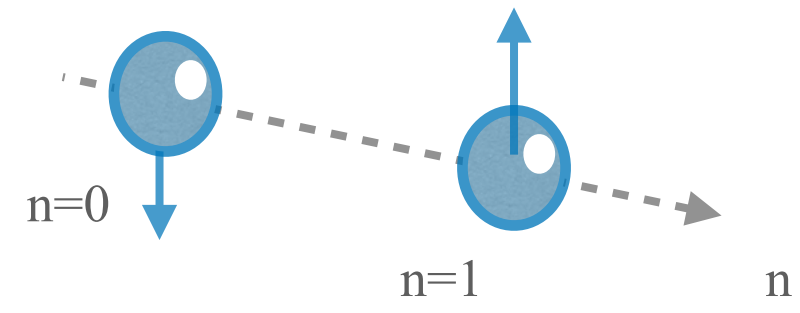
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Random Measurement

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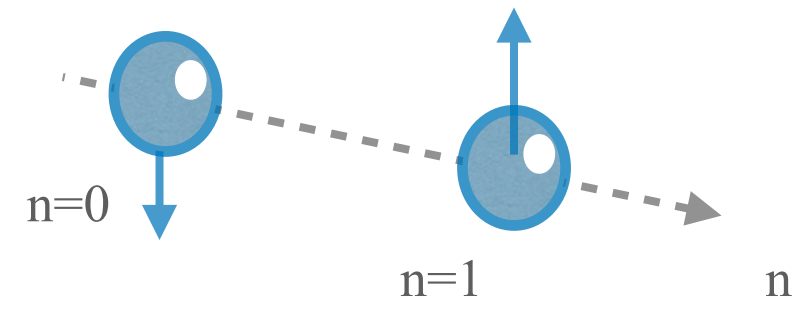
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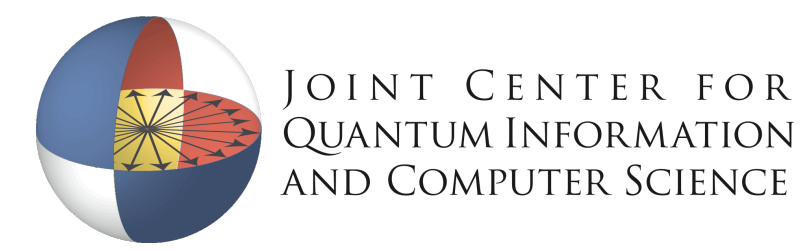
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Random Measurement

Symmetry ignorant versus conscious



$$\rho = \begin{pmatrix} \rho_2 & & \\ & [\rho_1] & \\ & & \rho_0 \end{pmatrix}$$



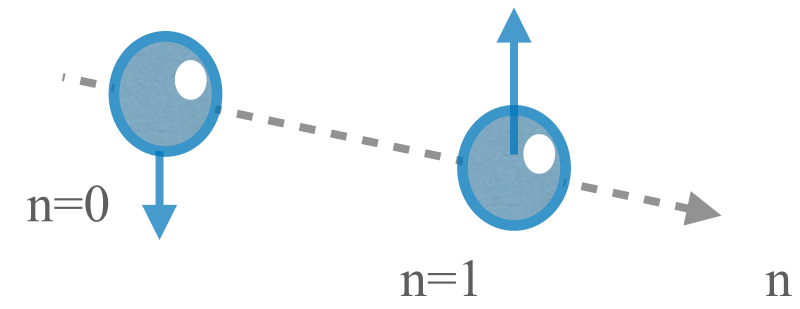
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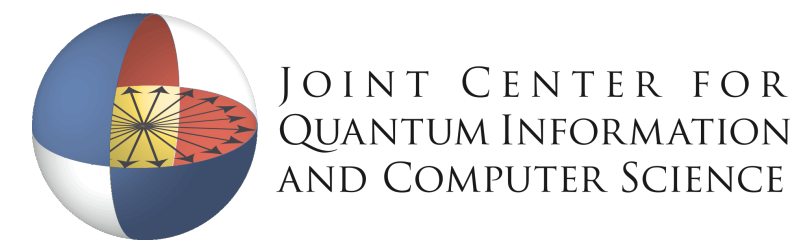
Random Measurement

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$$\rho = \begin{pmatrix} \rho_2 & & \\ & [\rho_1] & \\ & & \rho_0 \end{pmatrix}$$

← 2 particle
← 1 particle
← 0 particle



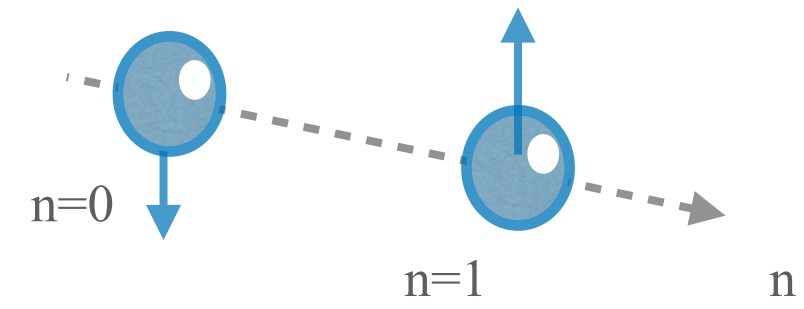
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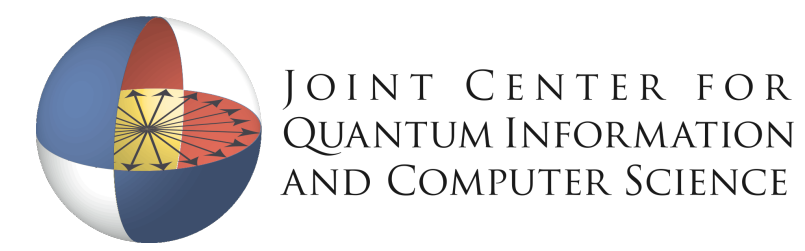
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$$u = \begin{pmatrix} e^{i\theta} & & \\ & [u_1] & \\ & & e^{i\phi} \end{pmatrix}$$



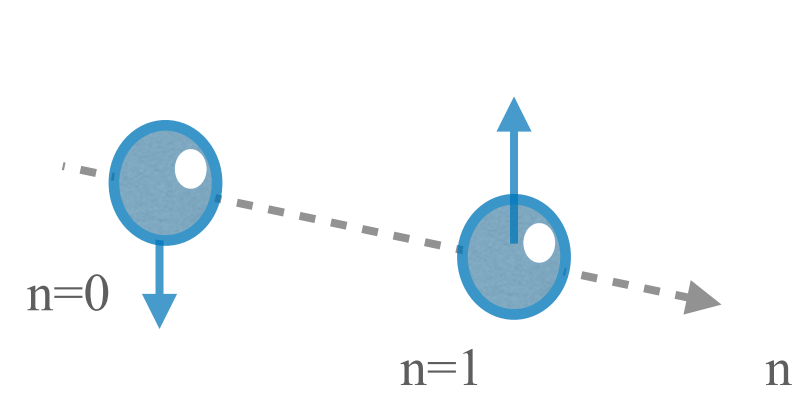
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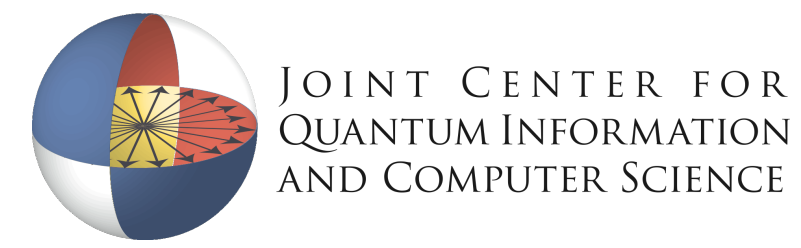
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Symmetry ignorant versus conscious



$$\rho = \begin{pmatrix} \rho_2 & & \\ & [\rho_1] & \\ & & \rho_0 \end{pmatrix} \begin{matrix} \leftarrow 2 \text{ particle} \\ \leftarrow 1 \text{ particle} \\ \leftarrow 0 \text{ particle} \end{matrix}$$

$$u = \begin{pmatrix} e^{i\theta} & & \\ & [u_1] & \\ & & e^{i\phi} \end{pmatrix} \quad u_1 = \begin{bmatrix} e^{i(\alpha+\gamma)} \cos(\beta) & e^{-i(\alpha-\gamma)} \sin(\beta) \\ -e^{i(\alpha-\gamma)} \sin(\beta) & e^{-i(\alpha+\gamma)} \cos(\beta) \end{bmatrix}$$



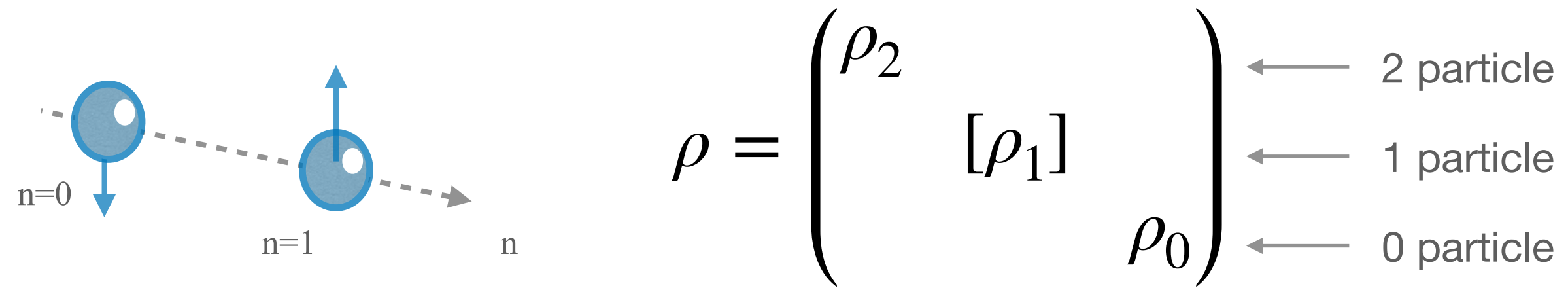
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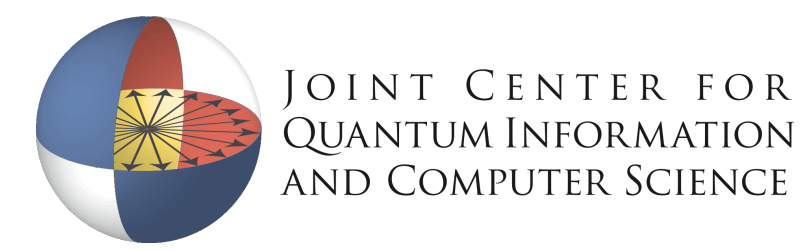
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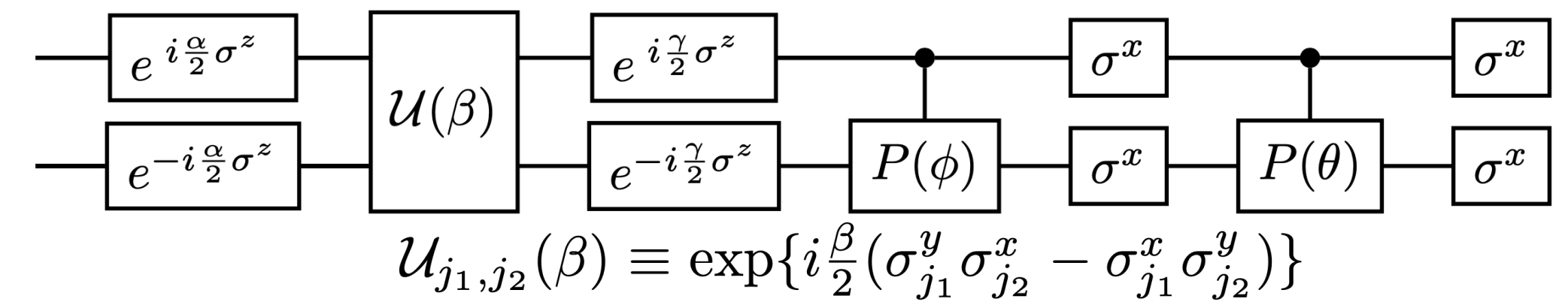
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Jake Bringewatt

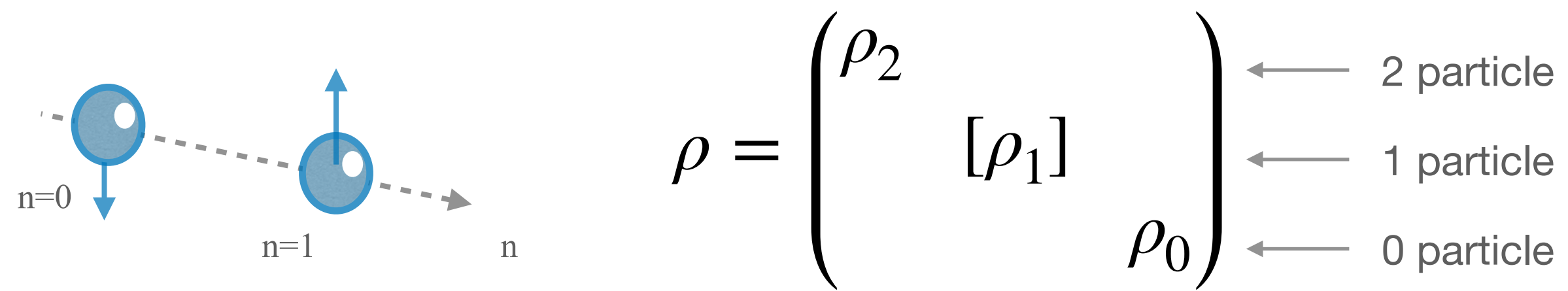


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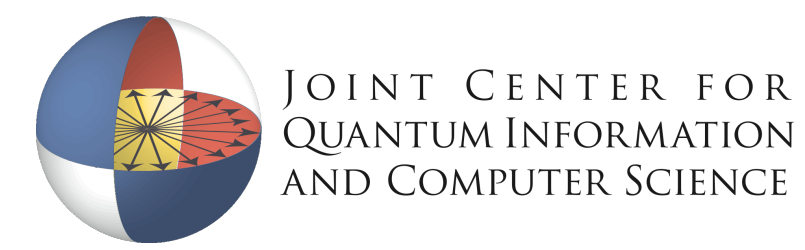


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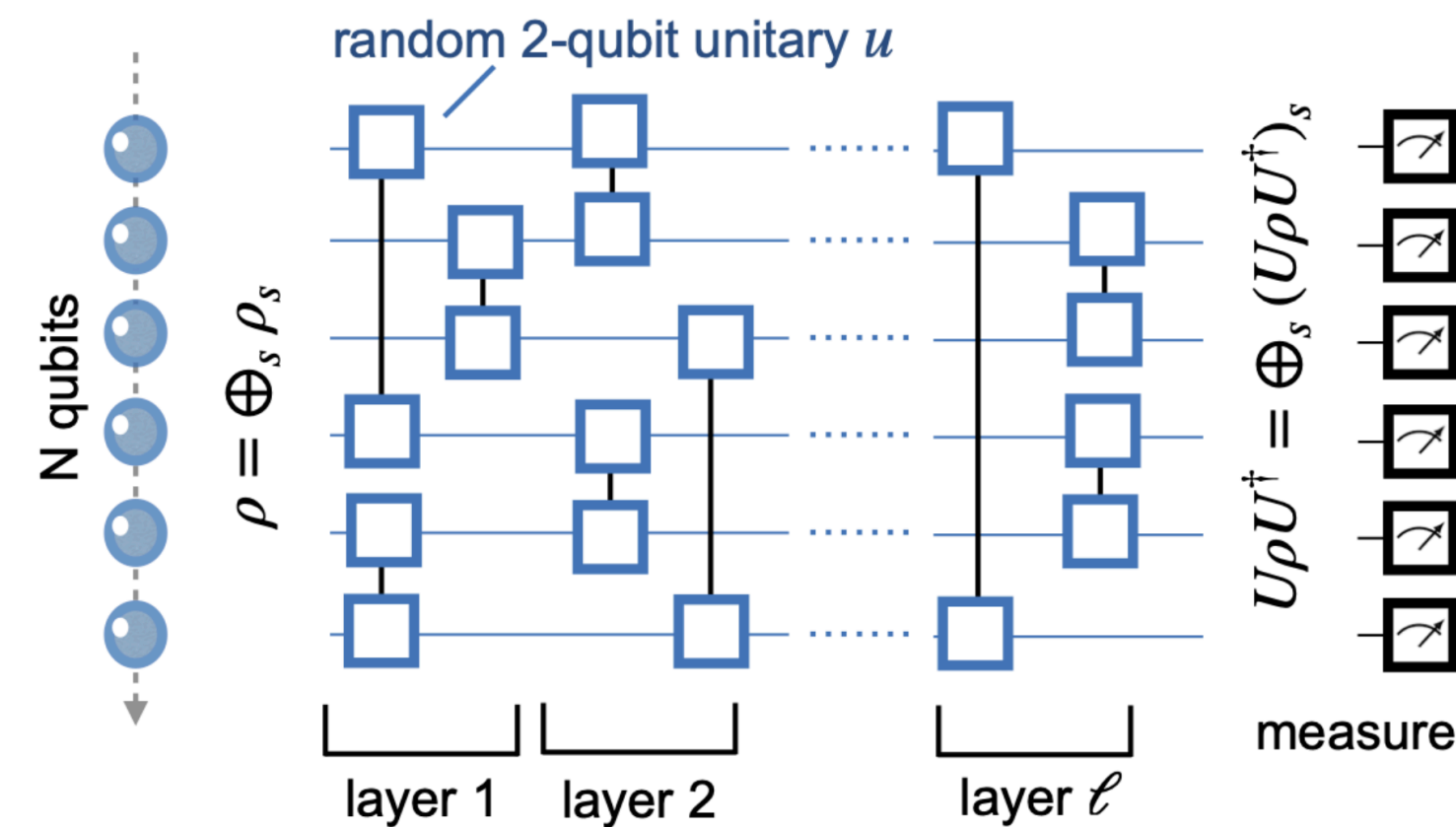
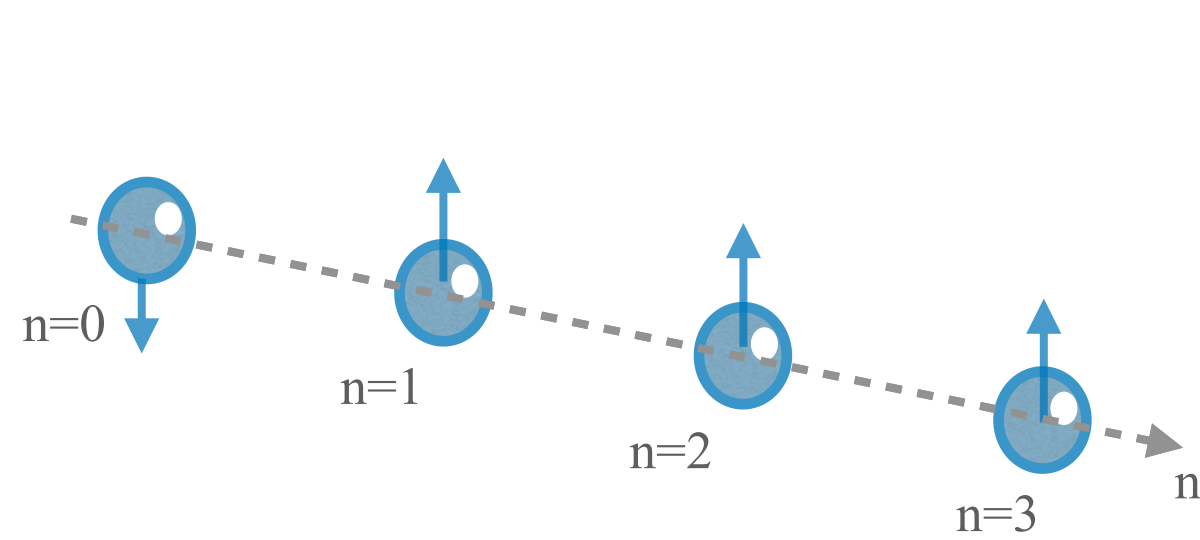
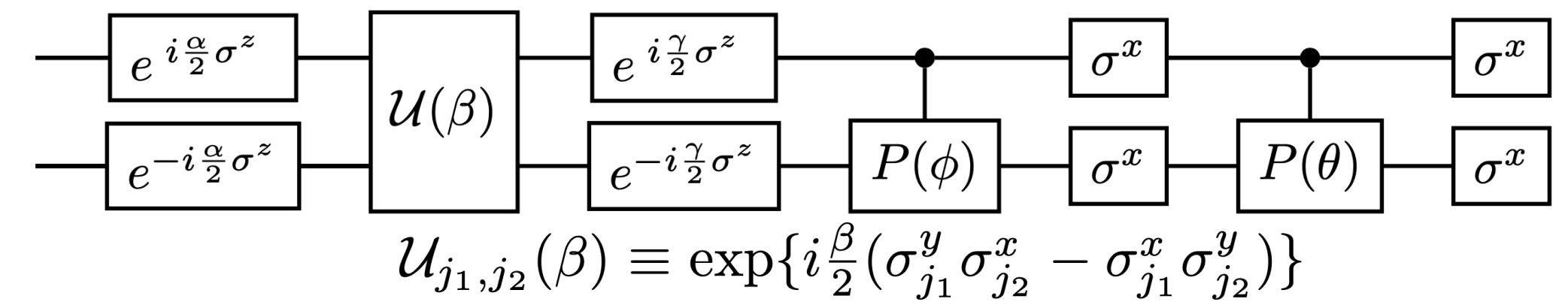
$$u = \begin{pmatrix} e^{i\theta} & & \\ & [u_1] & \\ & & e^{i\phi} \end{pmatrix} \quad u_1 = \begin{bmatrix} e^{i(\alpha+\gamma)} \cos(\beta) & e^{-i(\alpha-\gamma)} \sin(\beta) \\ -e^{i(\alpha-\gamma)} \sin(\beta) & e^{-i(\alpha+\gamma)} \cos(\beta) \end{bmatrix}$$



Jake Bringewatt

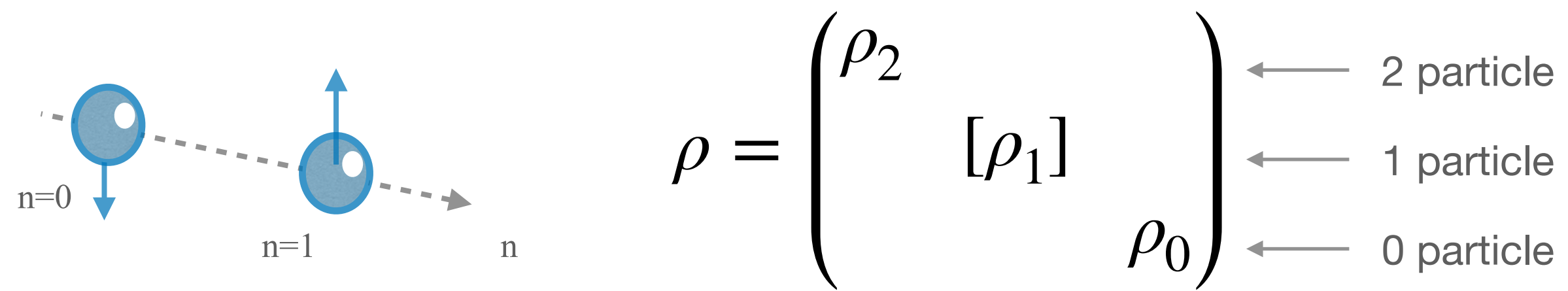


Jon Kunjummen



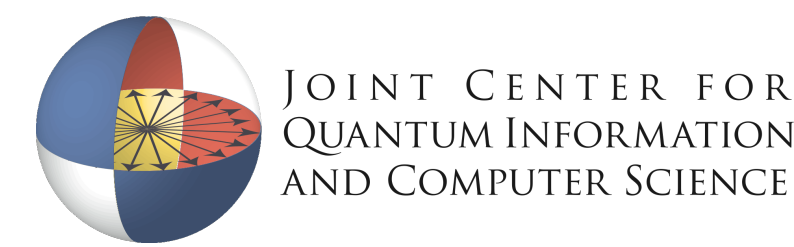
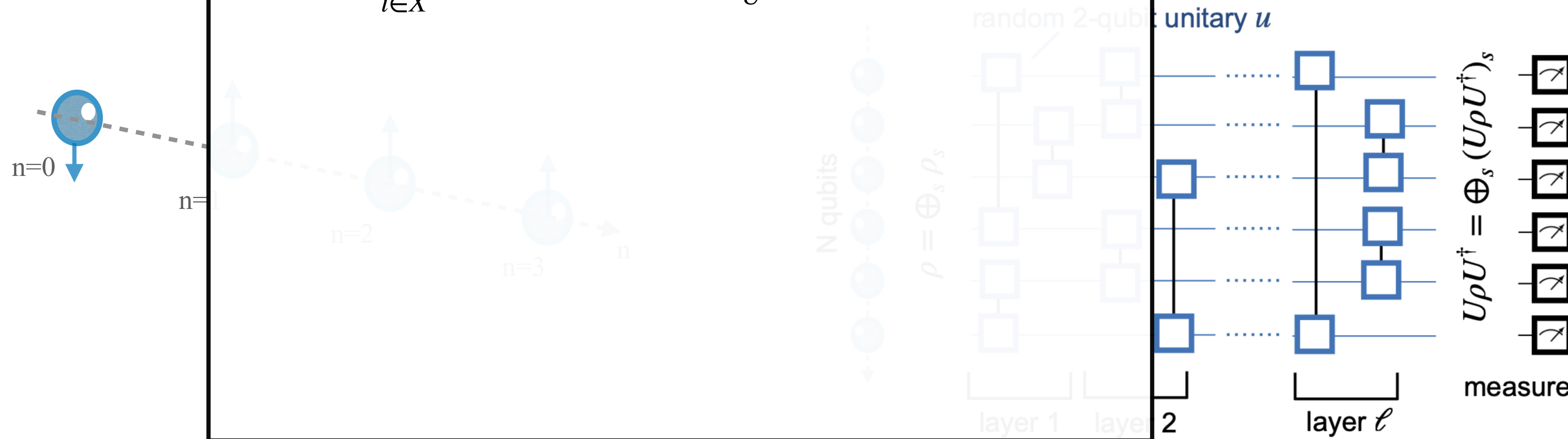
Random Measurement

Symmetry ignorant versus conscious



$$u = \begin{pmatrix} e^{i\theta} \\ [u_1] \end{pmatrix} \quad u_1 = \begin{bmatrix} e^{i(\alpha+\gamma)} \cos(\beta) & e^{-i(\alpha-\gamma)} \sin(\beta) \\ -e^{i(\alpha-\gamma)} \sin(\beta) & e^{-i(\alpha+\gamma)} \cos(\beta) \end{bmatrix}$$

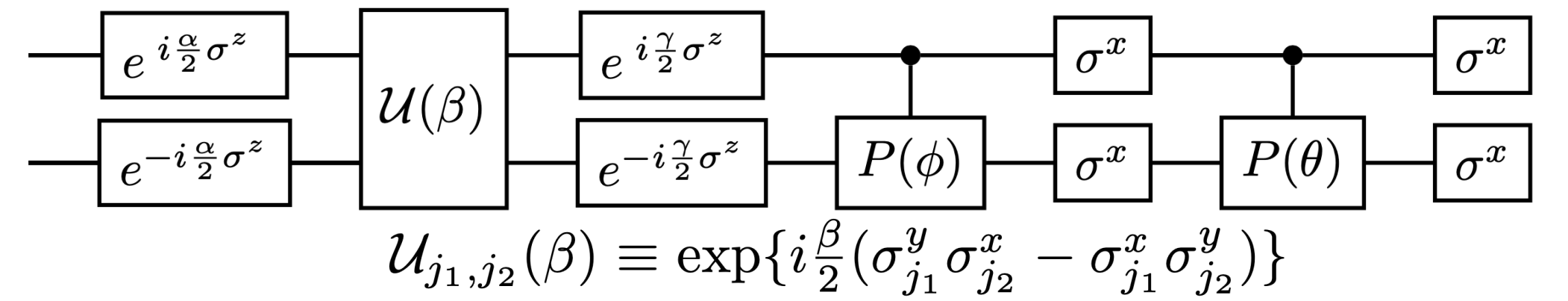
$$\frac{1}{|X|} \sum_{i \in X} U^{\otimes k} \otimes (U^*)^{\otimes k} = \int_U U^{\otimes k} \otimes (U^*)^{\otimes k} dU$$



Jake Bringewatt

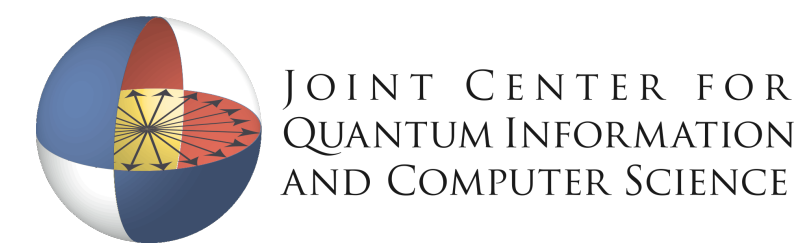
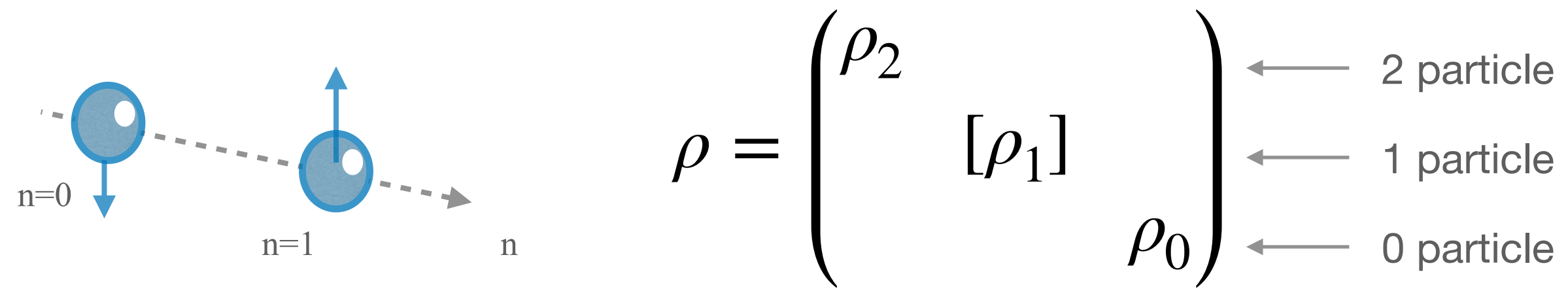


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Random Measurement

Symmetry ignorant versus conscious

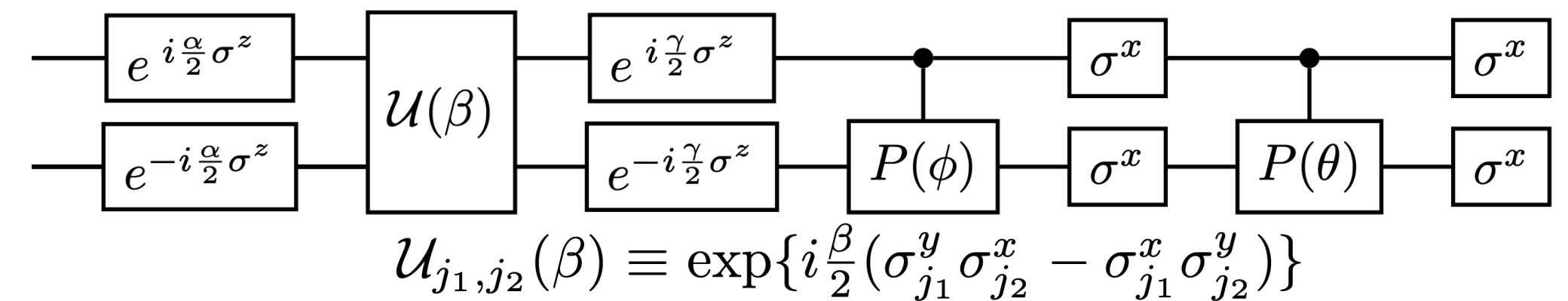


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$$u = \begin{pmatrix} e^{i\theta} \\ [u_1] \end{pmatrix} \quad u_1 = \begin{bmatrix} e^{i(\alpha+\gamma)} \cos(\beta) & e^{-i(\alpha-\gamma)} \sin(\beta) \\ -e^{i(\alpha-\gamma)} \sin(\beta) & e^{-i(\alpha+\gamma)} \cos(\beta) \end{bmatrix}$$



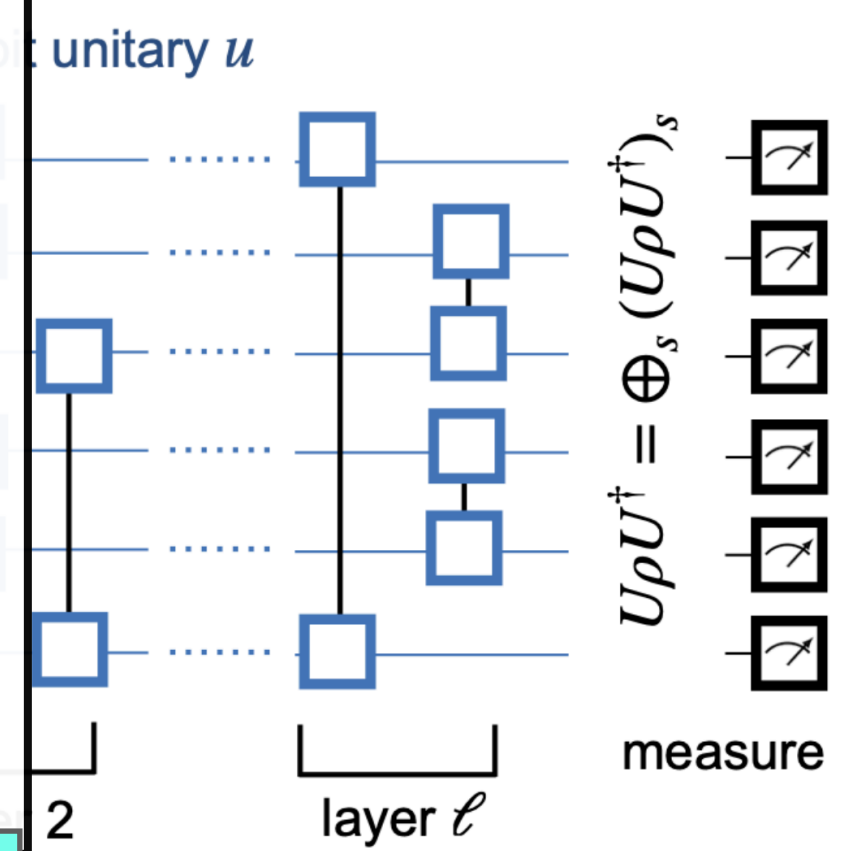
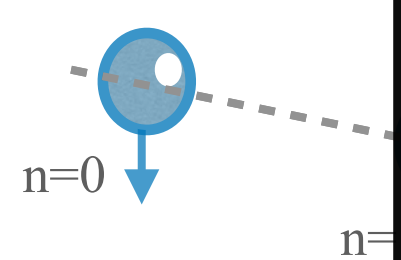
$$U_{j_1, j_2}(\beta) \equiv \exp\left\{i\frac{\beta}{2}(\sigma_{j_1}^y \sigma_{j_2}^x - \sigma_{j_1}^x \sigma_{j_2}^y)\right\}$$

$$\frac{1}{|X|} \sum_{i \in X} U^{\otimes k} \otimes (U^*)^{\otimes k} = \int_U U^{\otimes k} \otimes (U^*)^{\otimes k} dU$$

$$(\mathcal{B}^s)^{i'j'k'l'} \equiv \langle U_{ij}^s U_{i'j'}^{s*} U_{kl}^s U_{k'l'}^{s*} \rangle \quad (\mathcal{A}^s)^{kl} = \delta_{ij} \delta_{kl} / d_s$$

$$-\frac{d_s^2}{d_s^2 - 1} \left[(\mathcal{A}^s)^{i'j'} (\mathcal{A}^s)^{k'l'} + (\mathcal{A}^s)^{k'l'} (\mathcal{A}^s)^{i'j'} \right]$$

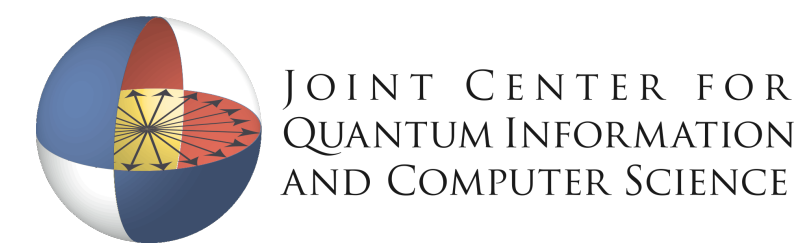
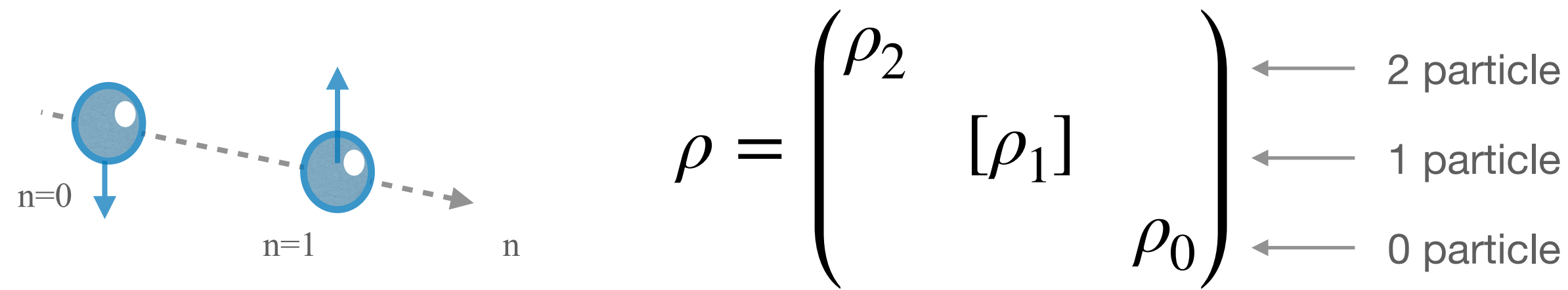
$$(\mathcal{B}^s)^{i'j'k'l'} = -\frac{\delta_{ii'} \delta_{kk'} \delta_{jj'} \delta_{ll'} + \delta_{ik'} \delta_{k'i'} \delta_{jj'} \delta_{ll'}}{d_s(d_s^2 - 1)}$$



Elben, Vermersch, Dalmonte, Cirac, Zoller, PRL 120, 050406 (2018)
 Vermersch, Elben, Dalmonte, Cirac, Zoller, PRA 97, 023604 (2018)

Random Measurement

Symmetry ignorant versus conscious



Jake Bringewatt



Jon Kunjummen

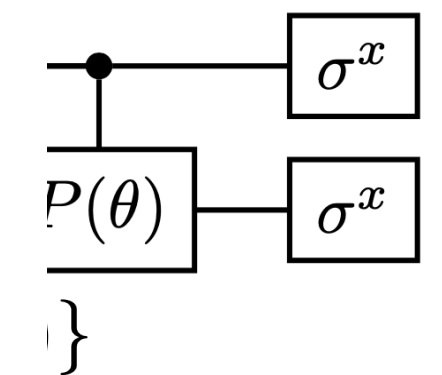
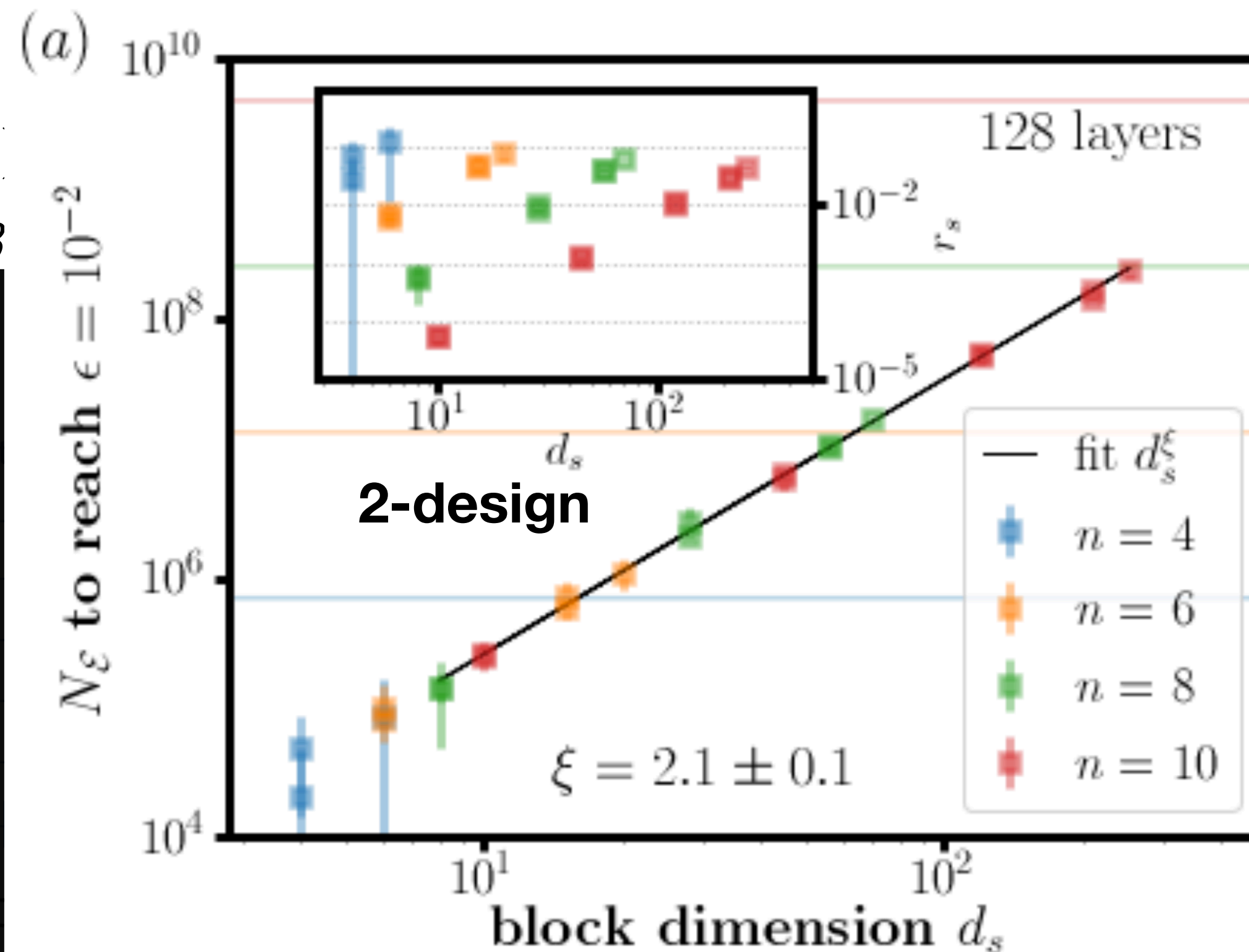
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$$-\frac{d_s^2}{d_s^2 - 1} \left[(\mathcal{A}^s)^{i'j'} (\mathcal{A}^s)^{k'l} + (\mathcal{A}^s)^{k'l} (\mathcal{A}^s)^{i'j'} \right]$$

$$(\mathcal{B}^s)^{i'j'k'l} = -\frac{\delta_{ii'} \delta_{kk'} \delta_{jj'} \delta_{ll'} + \delta_{ik'} \delta_{ki'} \delta_{jj'} \delta_{ll'}}{d_s(d_s^2 - 1)}$$



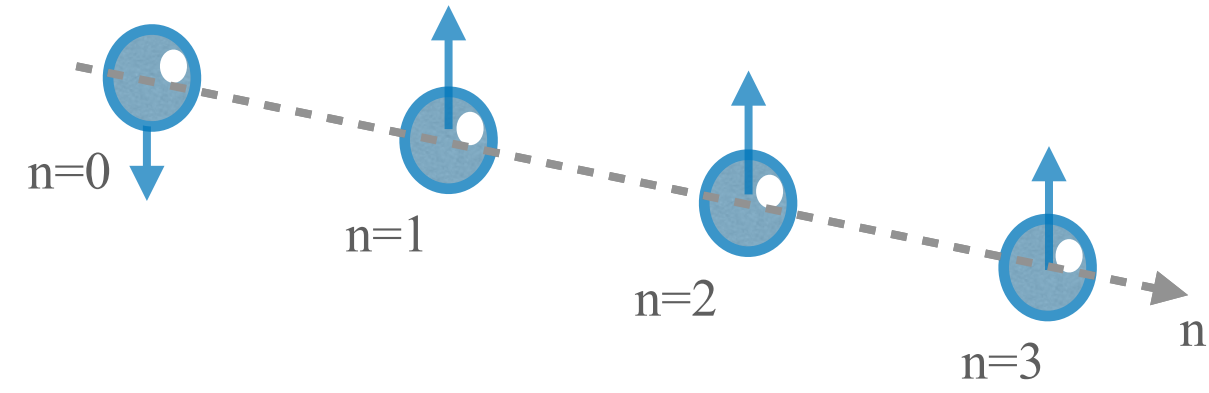
Elben, Vermersch, Dalmonte, Cirac, Zoller, PRL 120, 050406 (2018)
 Vermersch, Elben, Dalmonte, Cirac, Zoller, PRA 97, 023604 (2018)

Symmetry-conscious Random Measurement

Lattice Gauge Theories

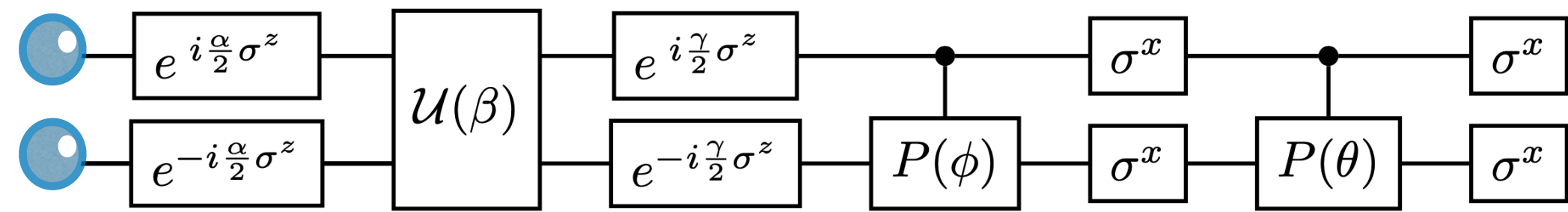
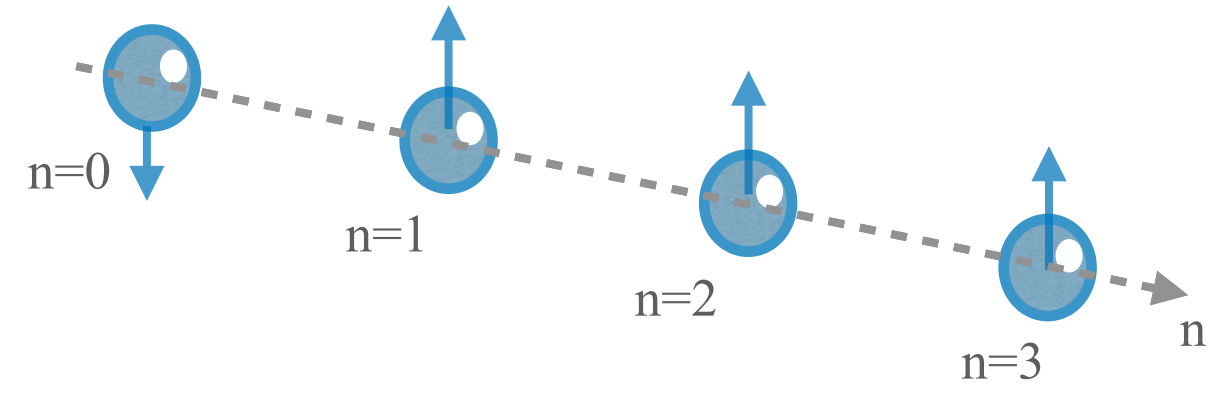
Symmetry-conscious Random Measurement

Lattice Gauge Theories



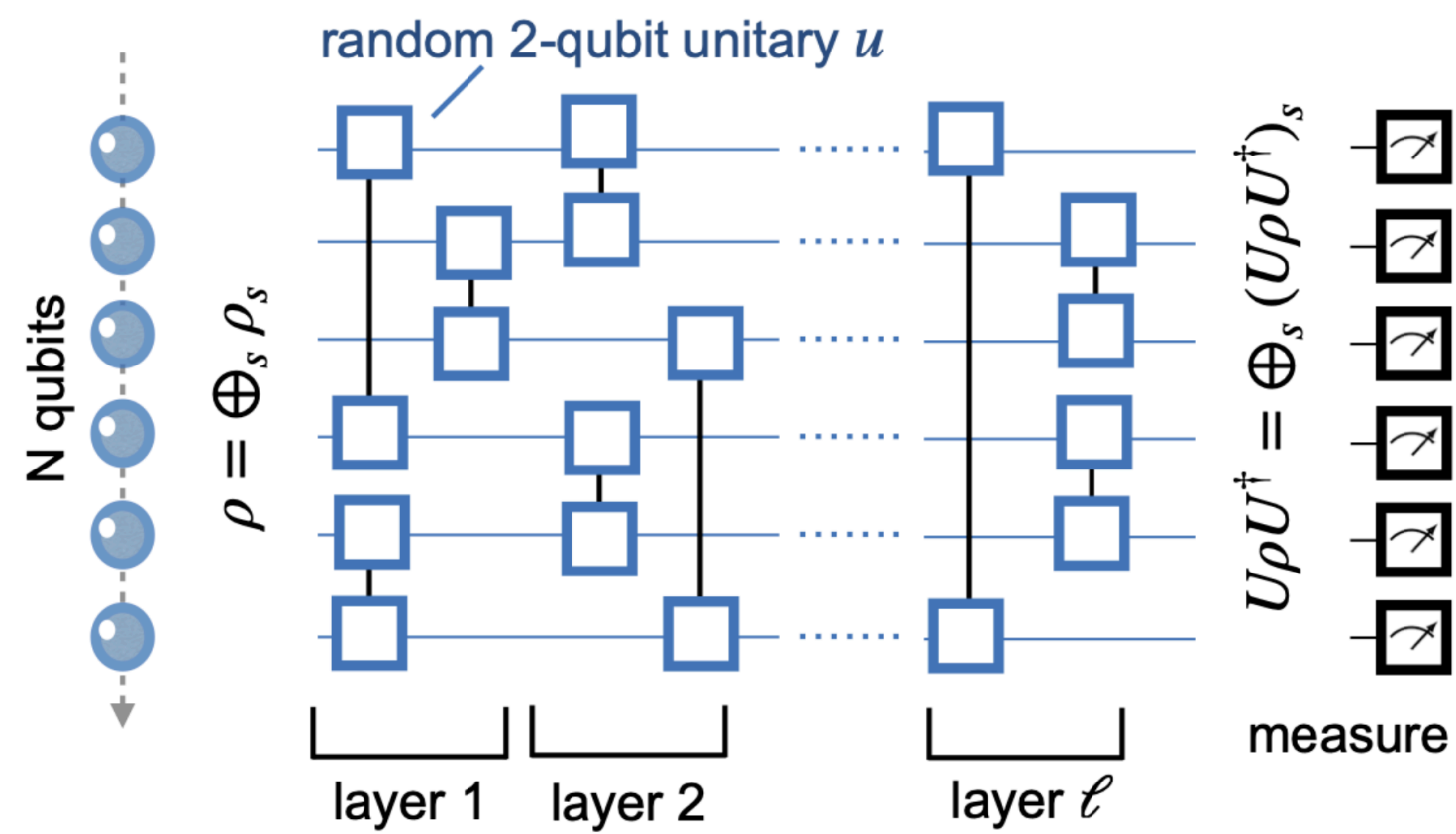
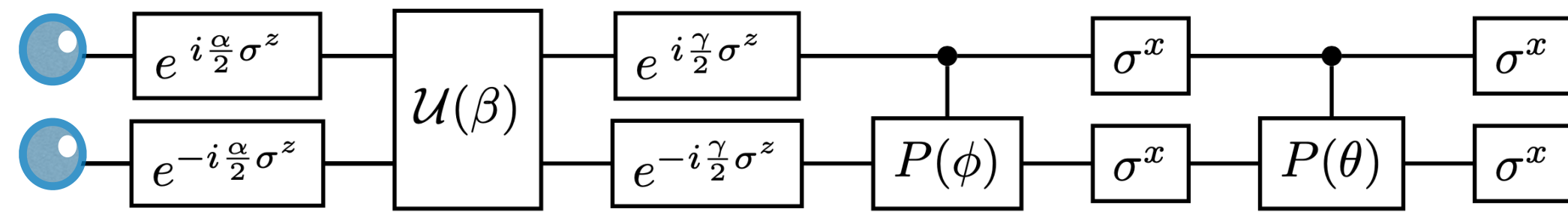
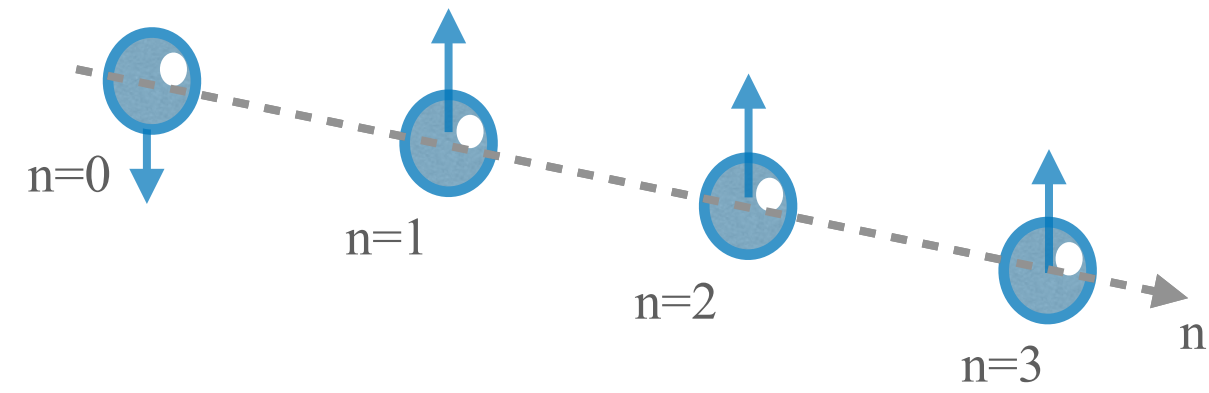
Symmetry-conscious Random Measurement

Lattice Gauge Theories



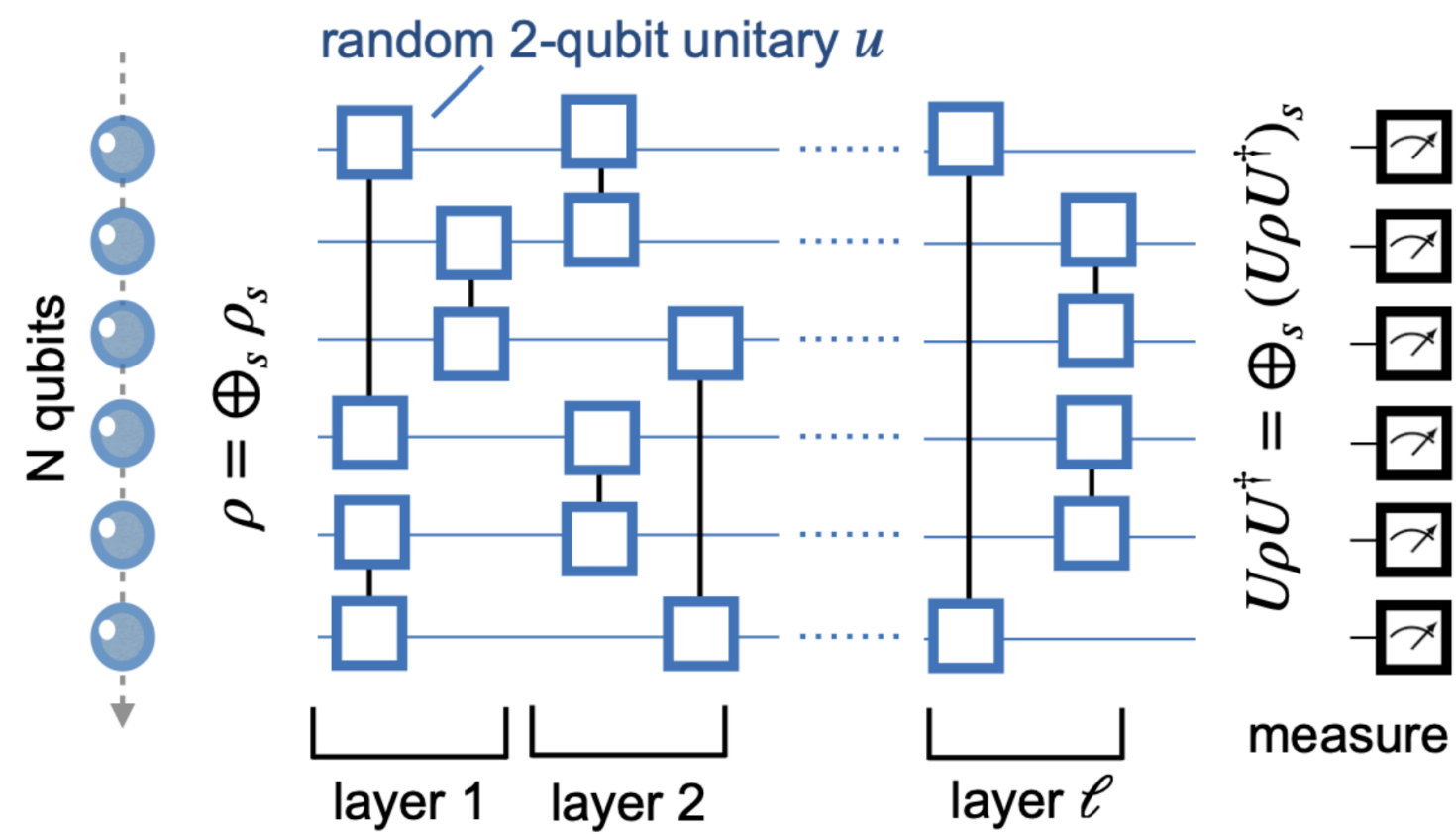
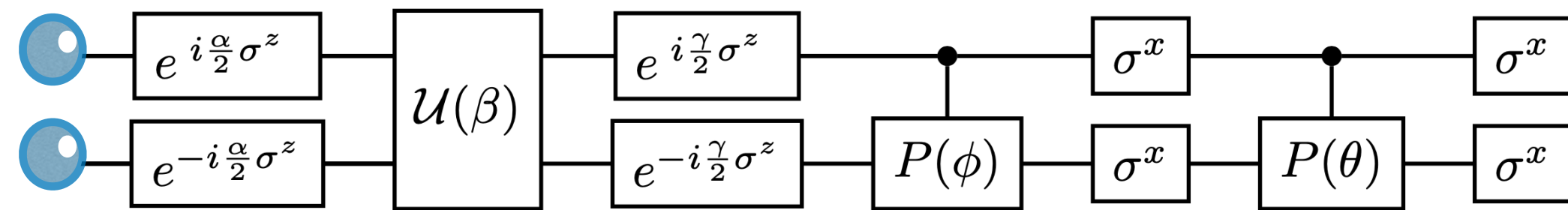
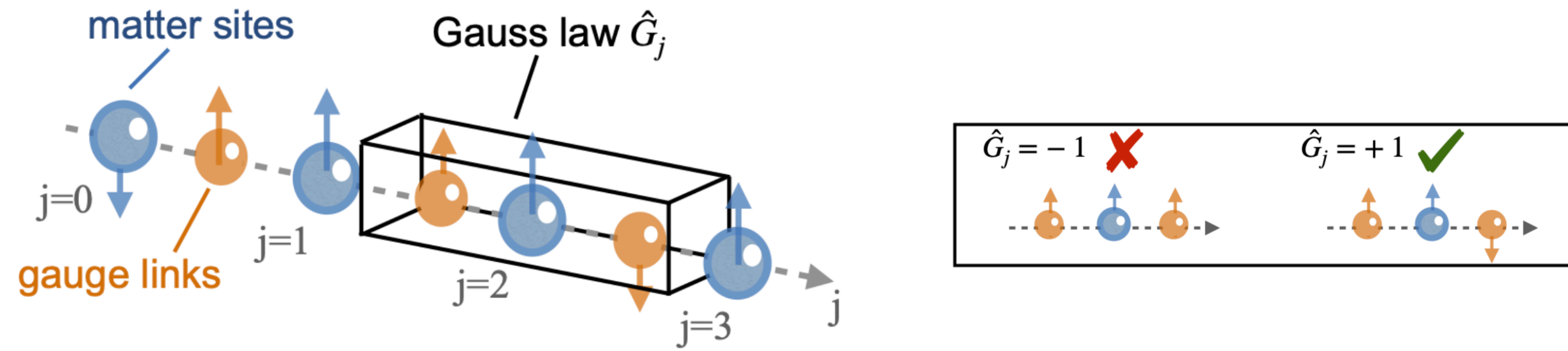
Symmetry-conscious Random Measurement

Lattice Gauge Theories



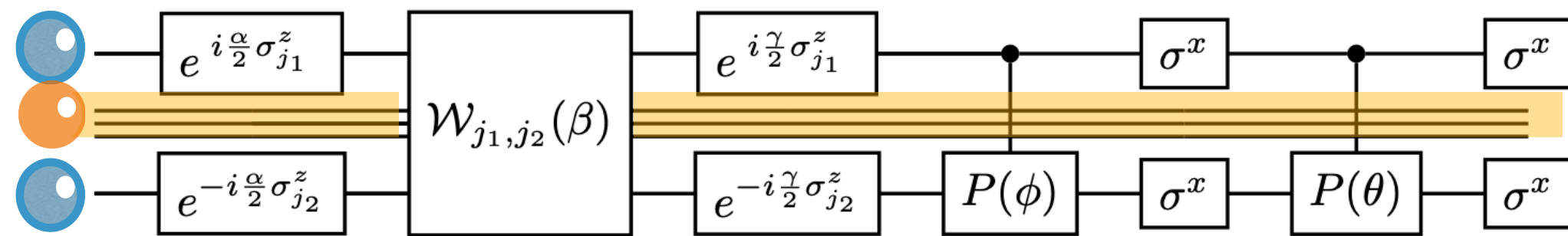
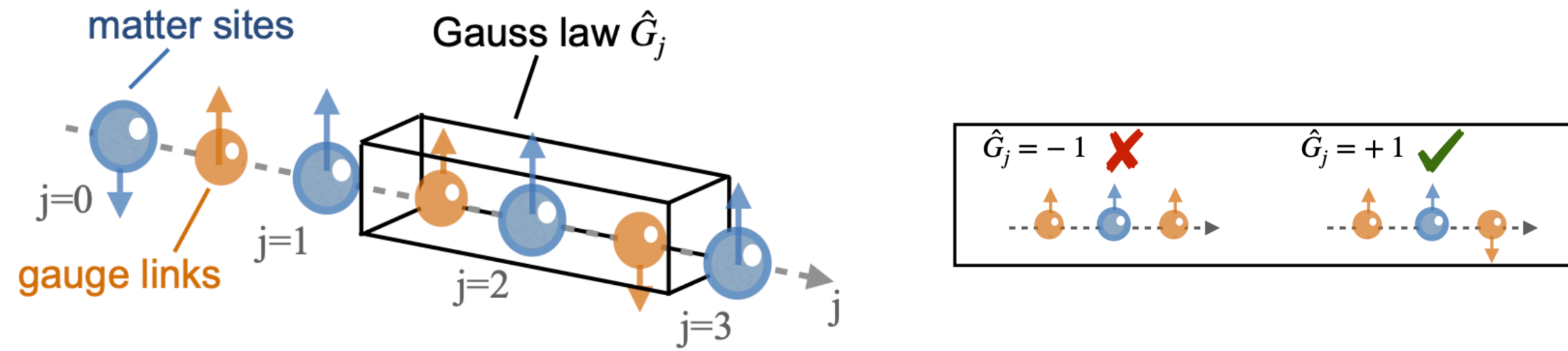
Symmetry-conscious Random Measurement

Lattice Gauge Theories



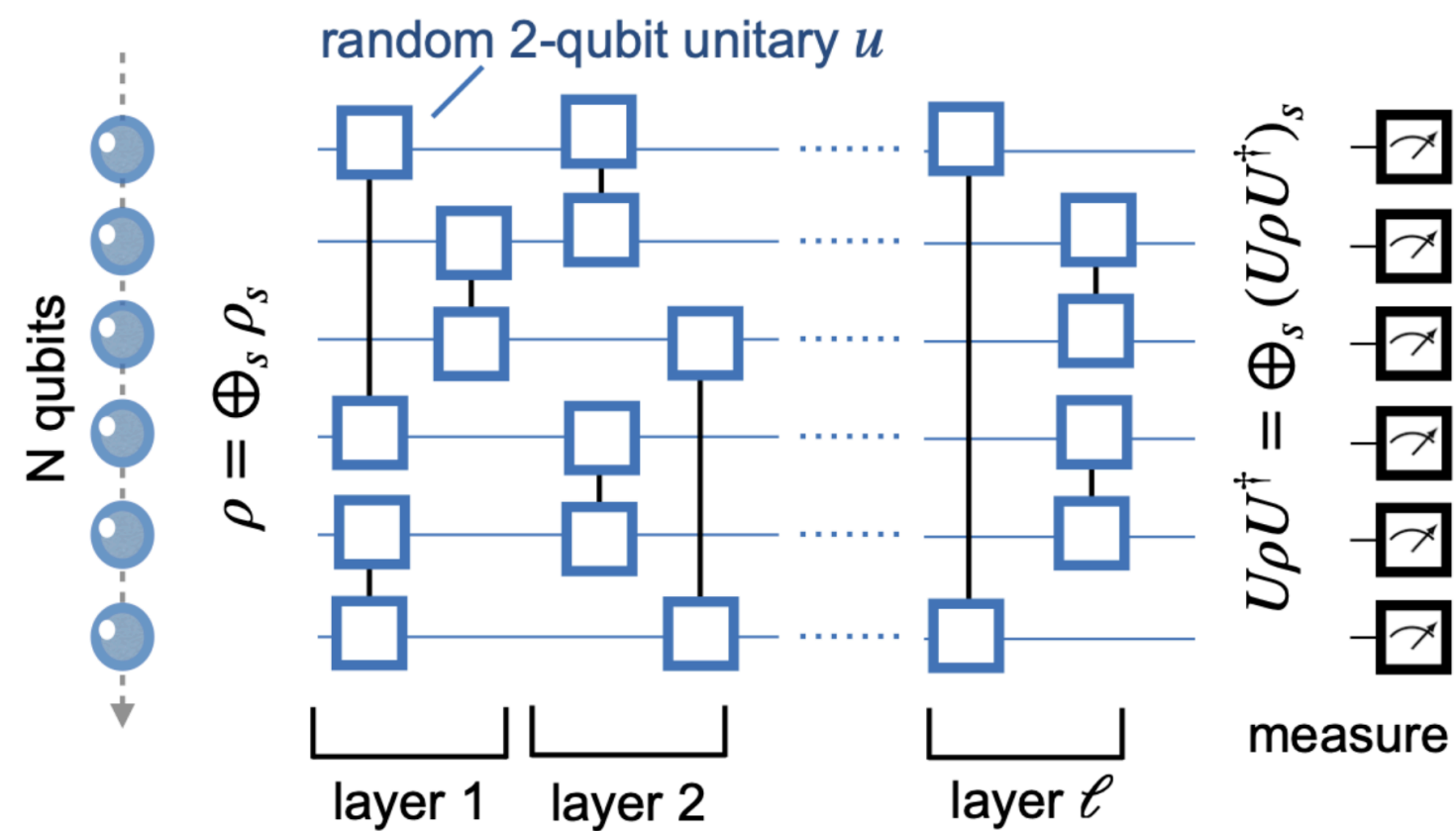
Symmetry-conscious Random Measurement

Lattice Gauge Theories



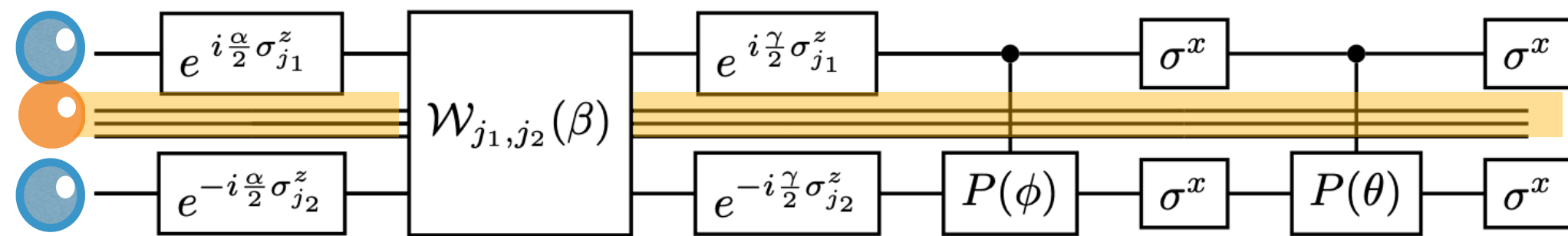
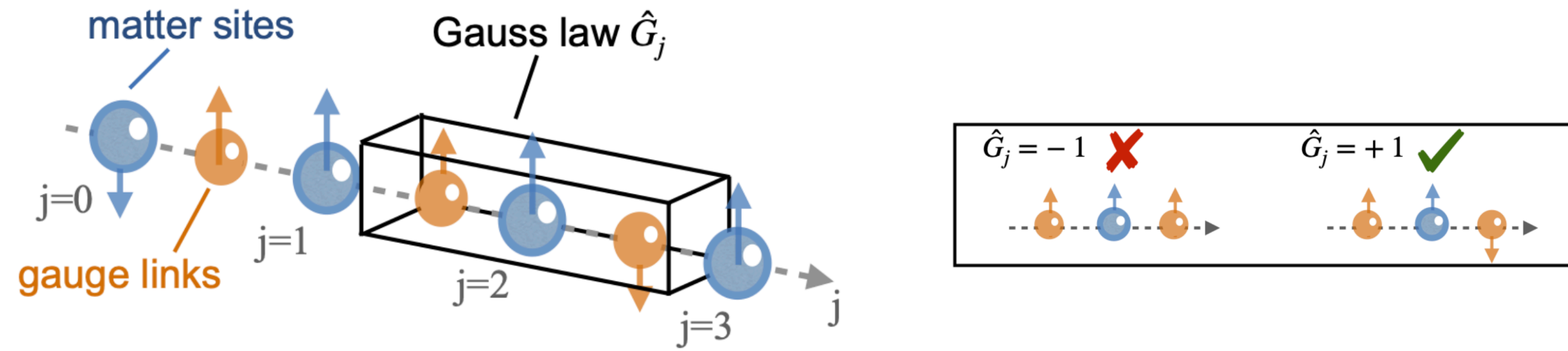
$$\mathcal{W}_{j_1, j_2}(\beta) \equiv \exp\left\{i\frac{\beta}{2}(\sigma_{j_1}^y W_{j_1, j_2} \sigma_{j_2}^x - \sigma_{j_1}^x W_{j_1, j_2} \sigma_{j_2}^y)\right\}$$

$$W_{j_1, j_2} \equiv \prod_{j=j_1}^{j_2-1} \tilde{\sigma}_{j, j+1}^x$$



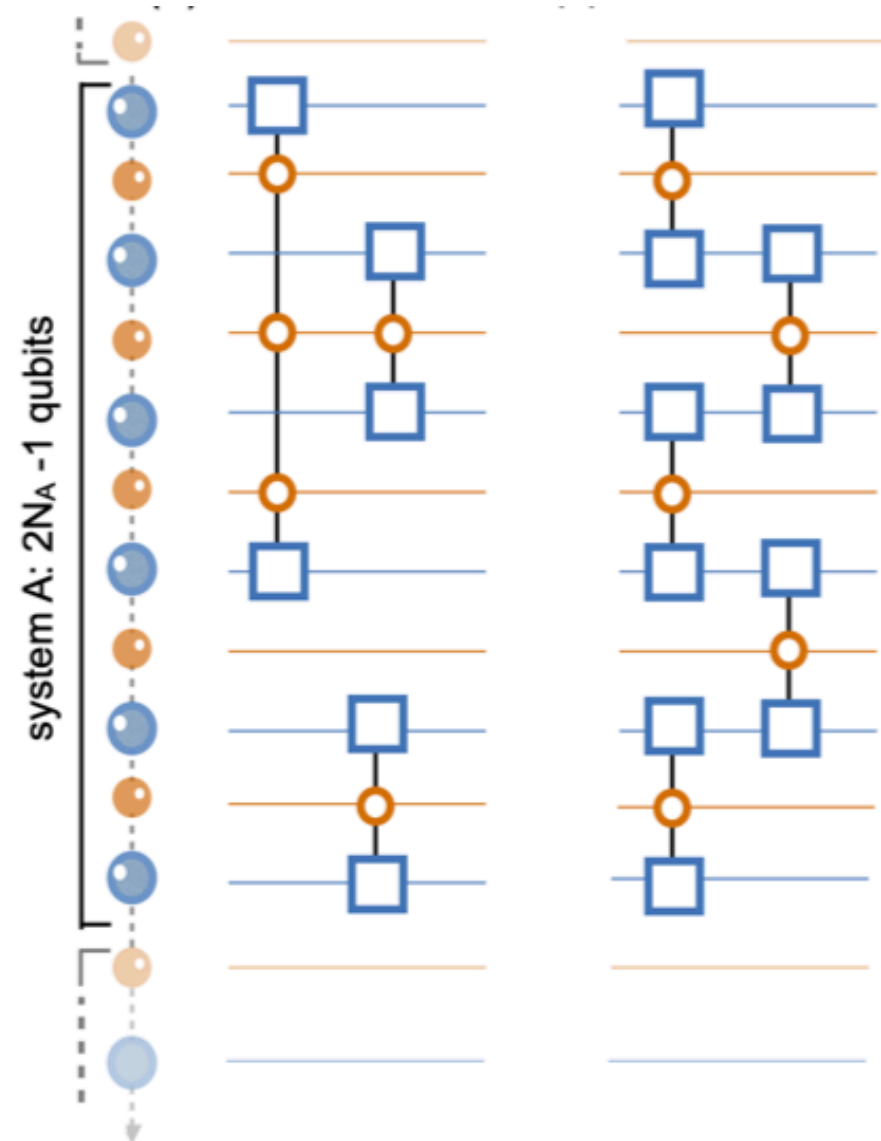
Symmetry-conscious Random Measurement

Lattice Gauge Theories



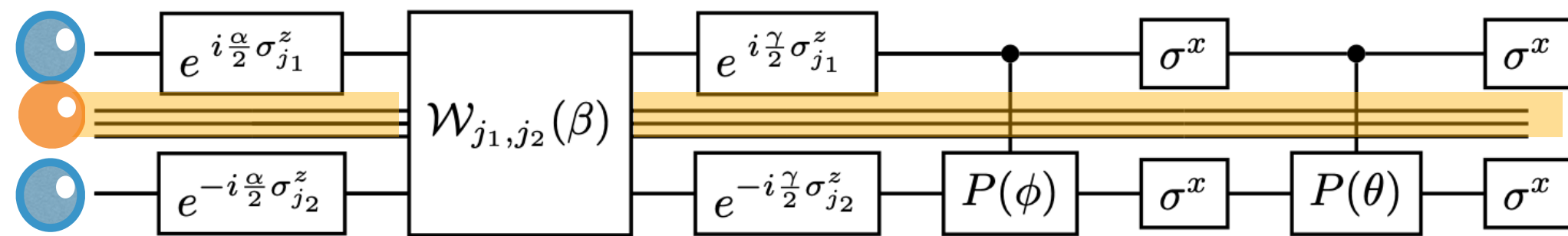
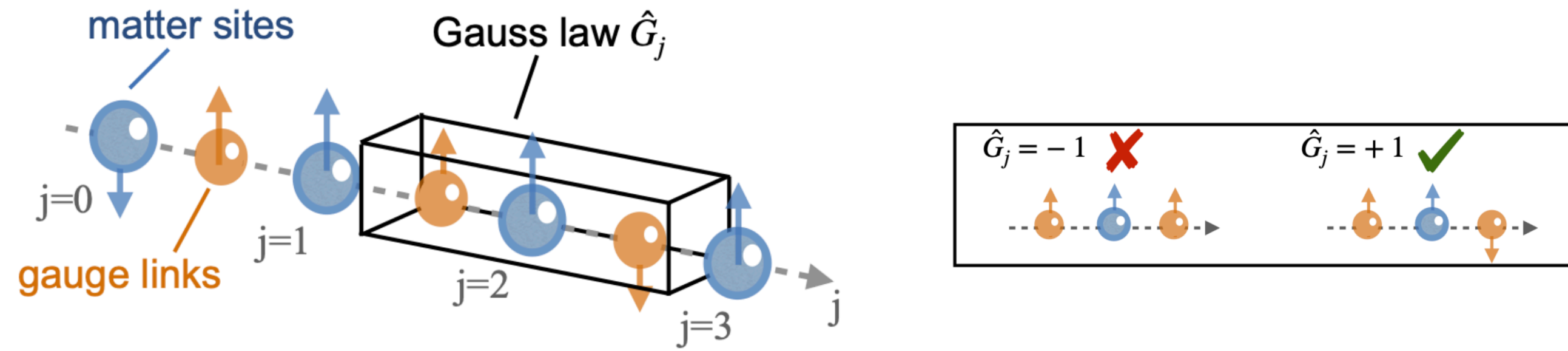
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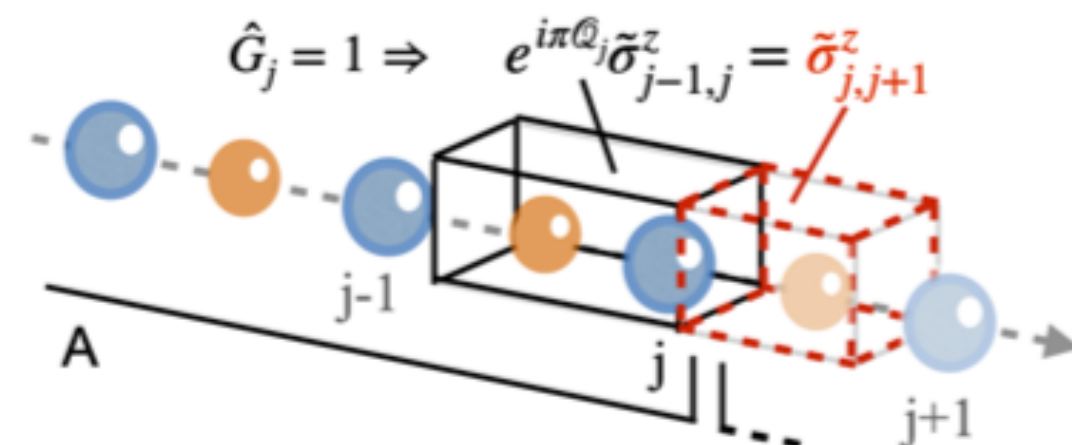
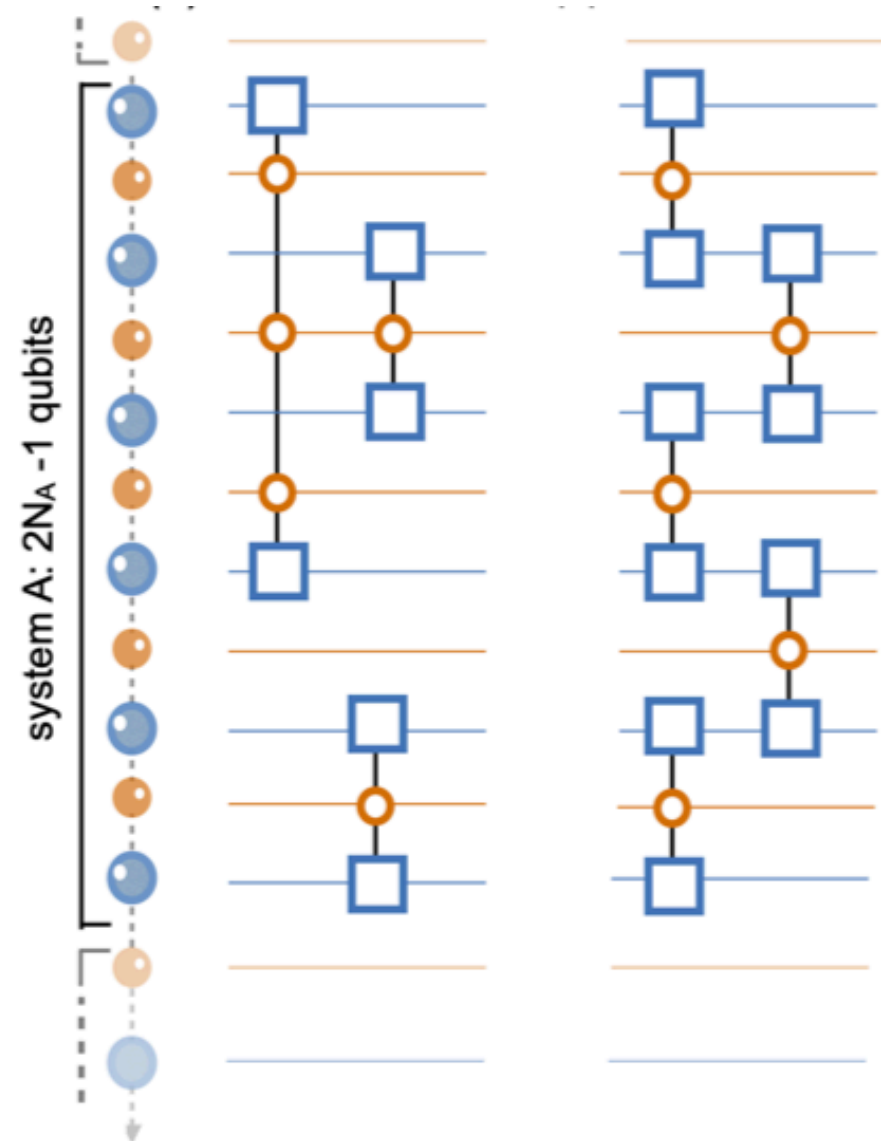
Symmetry-conscious Random Measurement

Lattice Gauge Theories



$$\mathcal{W}_{j_1, j_2}(\beta) \equiv \exp\left\{i\frac{\beta}{2}(\sigma_{j_1}^y W_{j_1, j_2} \sigma_{j_2}^x - \sigma_{j_1}^x W_{j_1, j_2} \sigma_{j_2}^y)\right\}$$

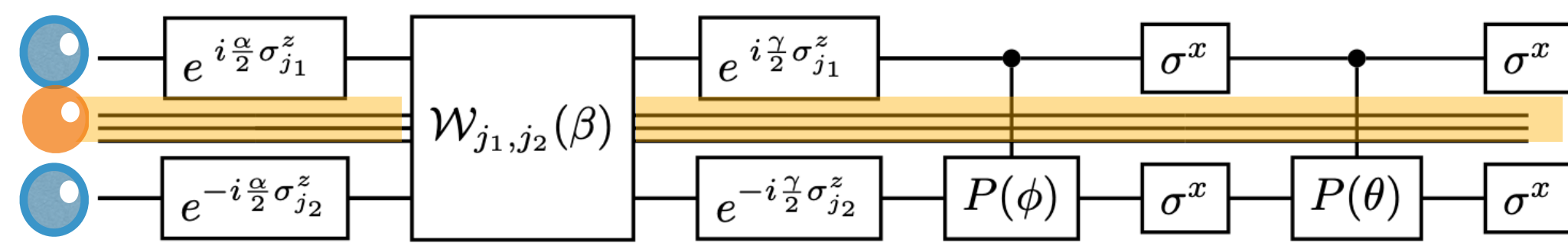
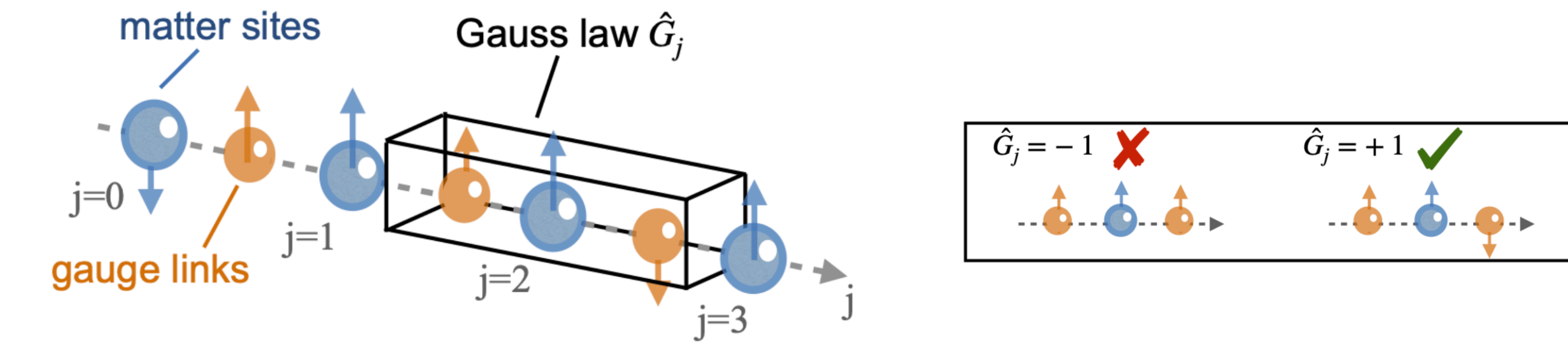
$$W_{j_1, j_2} \equiv \prod_{j=j_1}^{j_2-1} \tilde{\sigma}_{j, j+1}^x$$



$$\rho = \bigoplus_s \rho_s$$

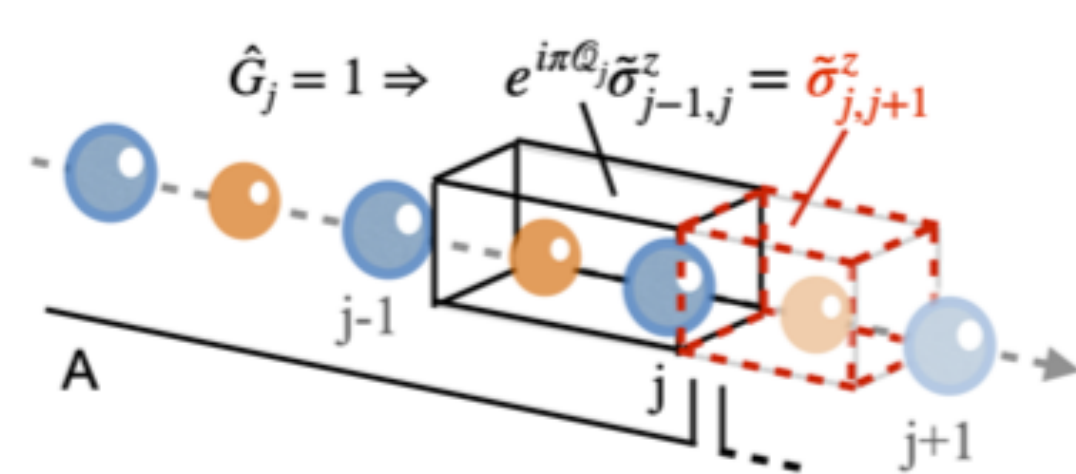
Symmetry-conscious Random Measurement

Lattice Gauge Theories

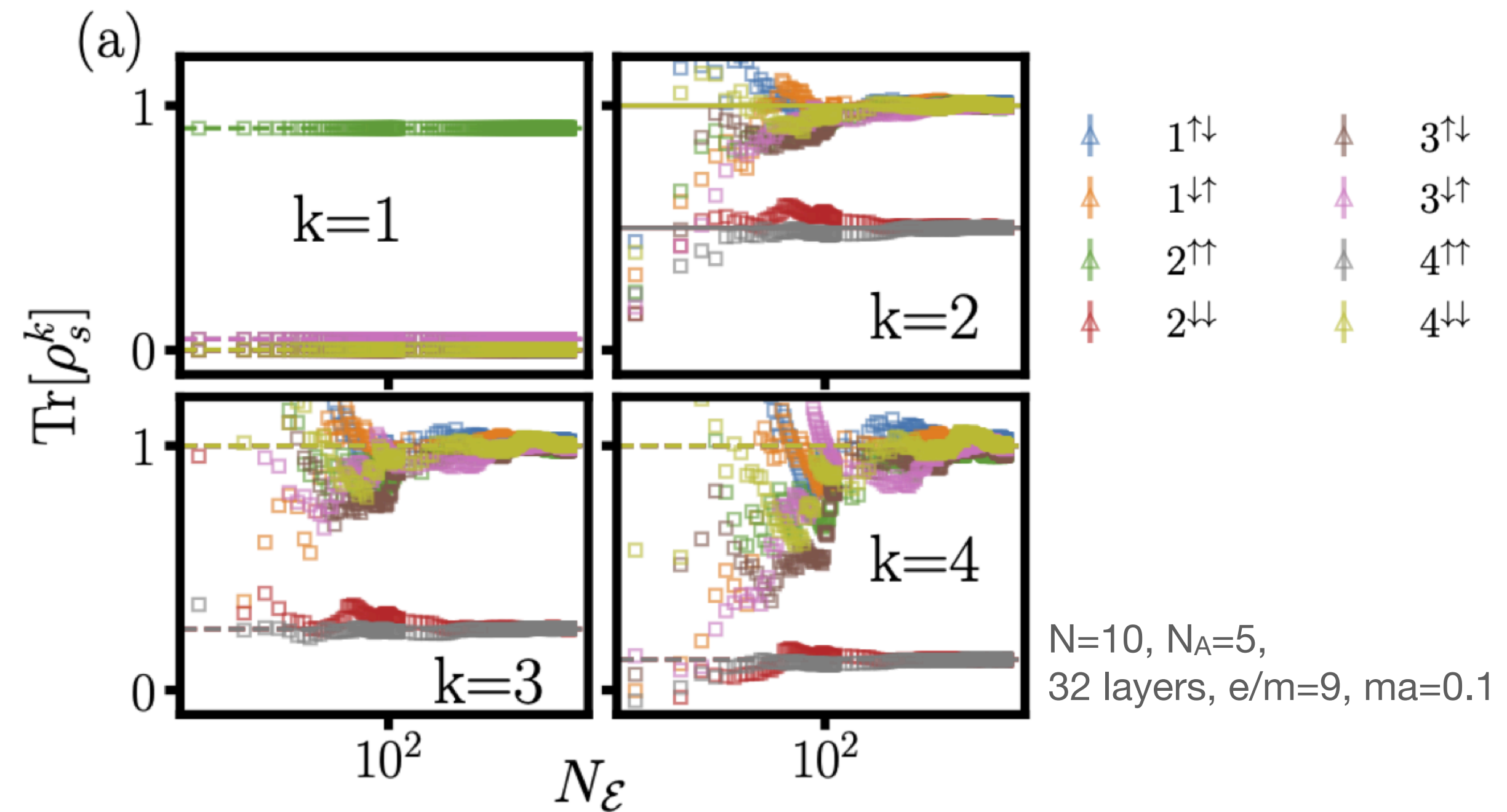
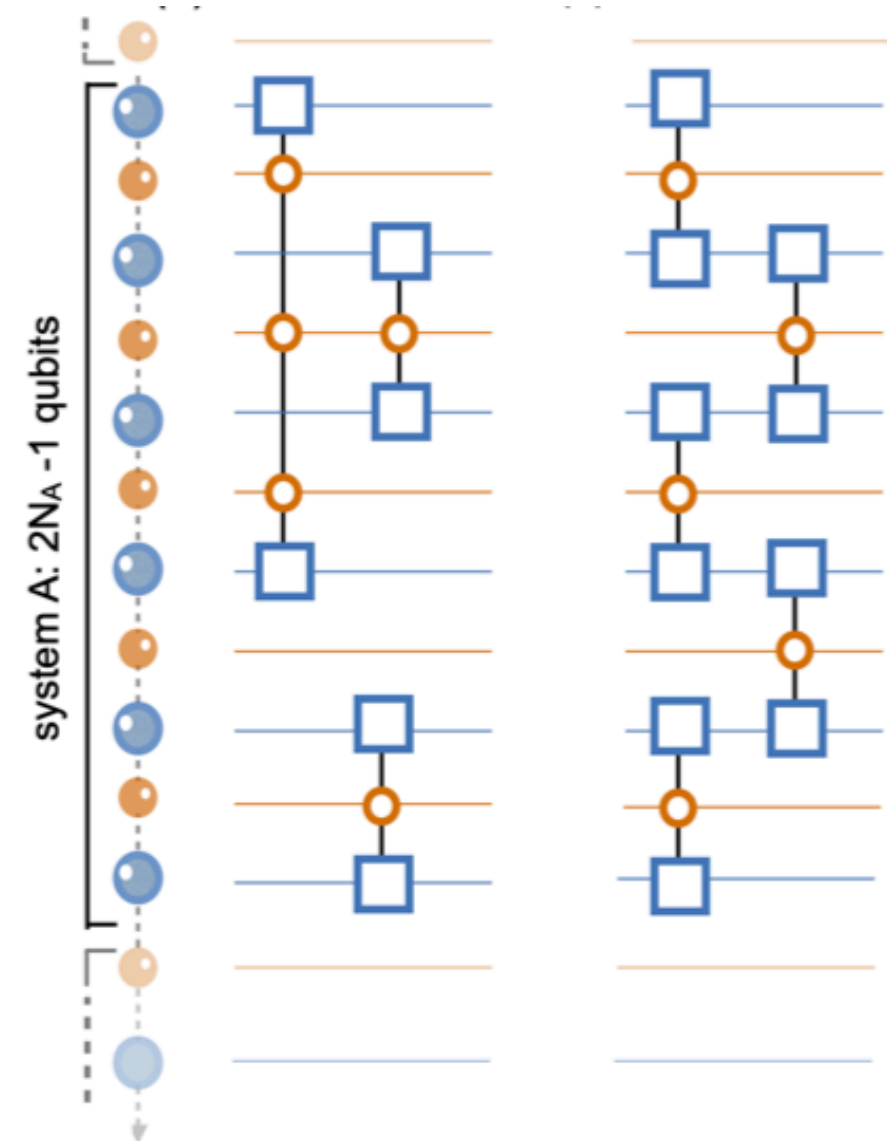


$$\mathcal{W}_{j_1, j_2}(\beta) \equiv \exp\left\{i\frac{\beta}{2}(\sigma_{j_1}^y W_{j_1, j_2} \sigma_{j_2}^x - \sigma_{j_1}^x W_{j_1, j_2} \sigma_{j_2}^y)\right\}$$

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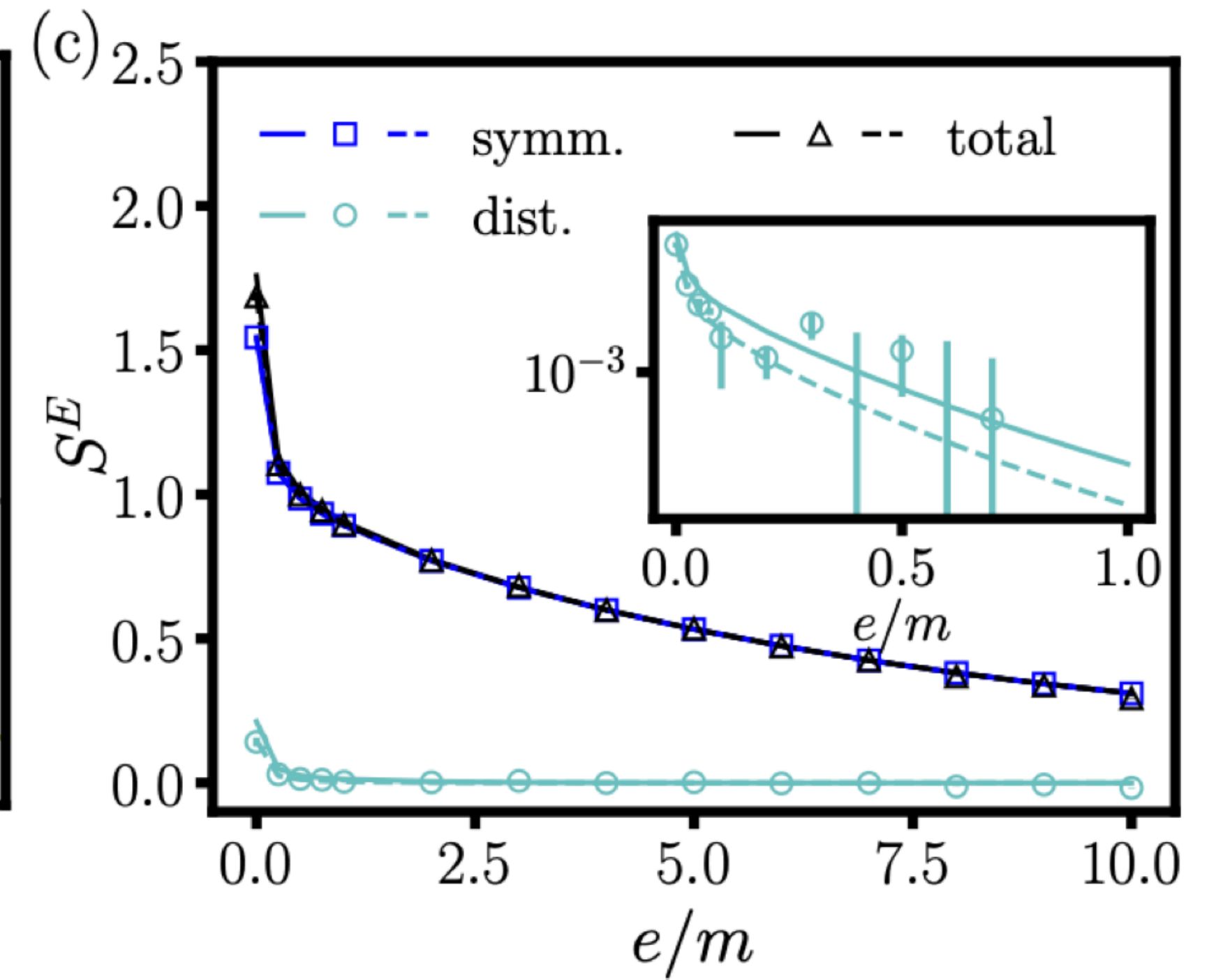
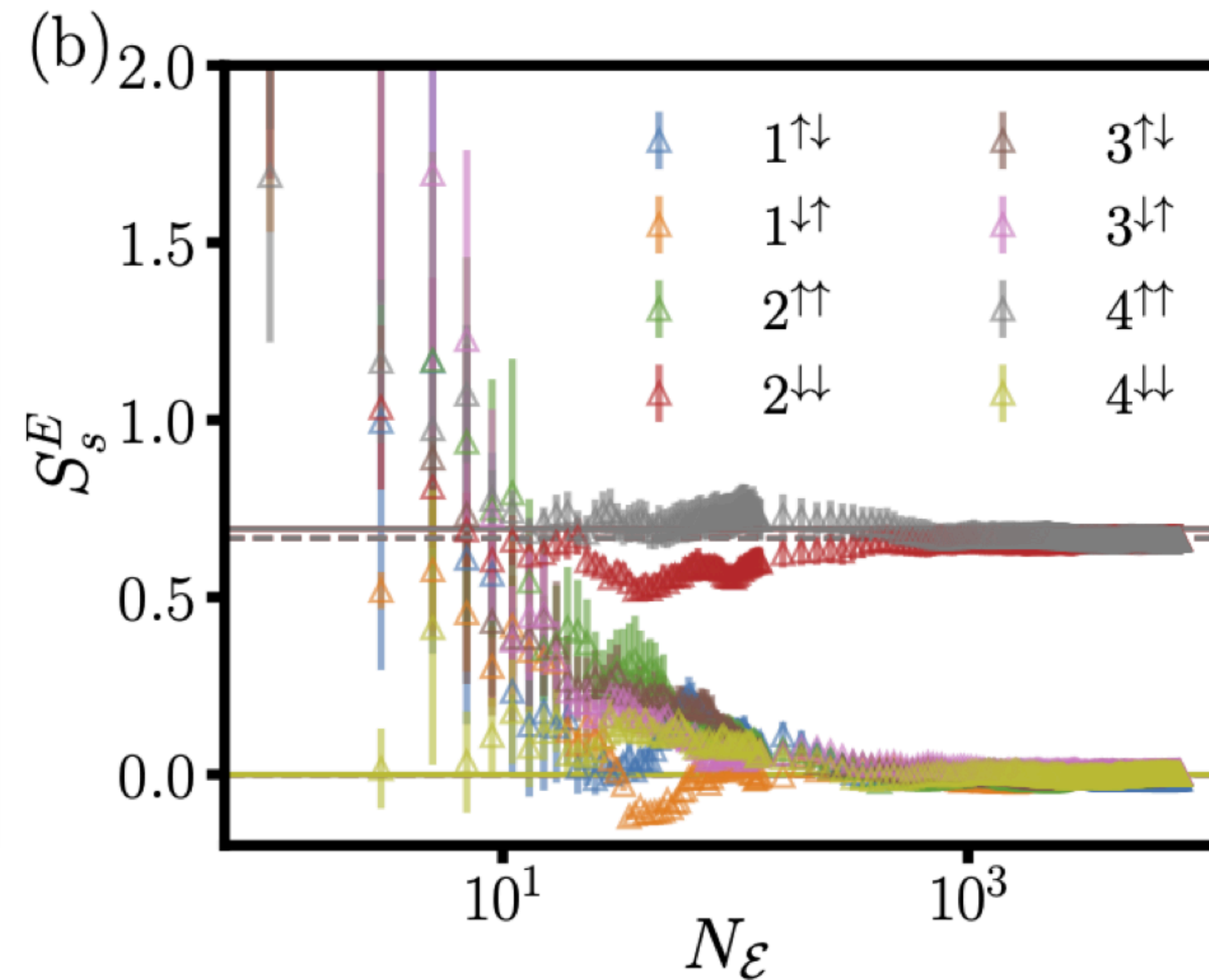


Symmetry-conscious Random Measurement

Lattice Gauge Theories

Measure distillable Entanglement Entropy

$$S^E = - \sum_s p_s \log(p_s) + \sum_s p_s S_s^E \quad p_s \equiv \text{Tr}_s(\rho_s)$$



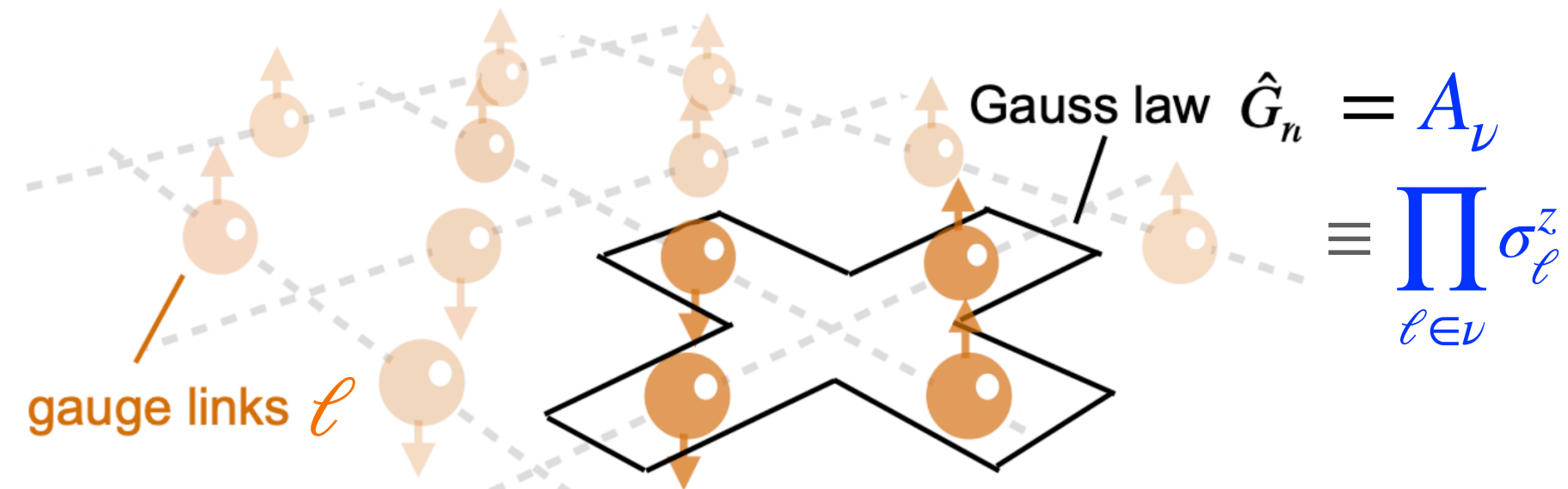
Symmetry-conscious Random Measurement

Lattice Gauge Theories

Symmetry-conscious Random Measurement

Lattice Gauge Theories

- **Z₂ Lattice Gauge Theory**



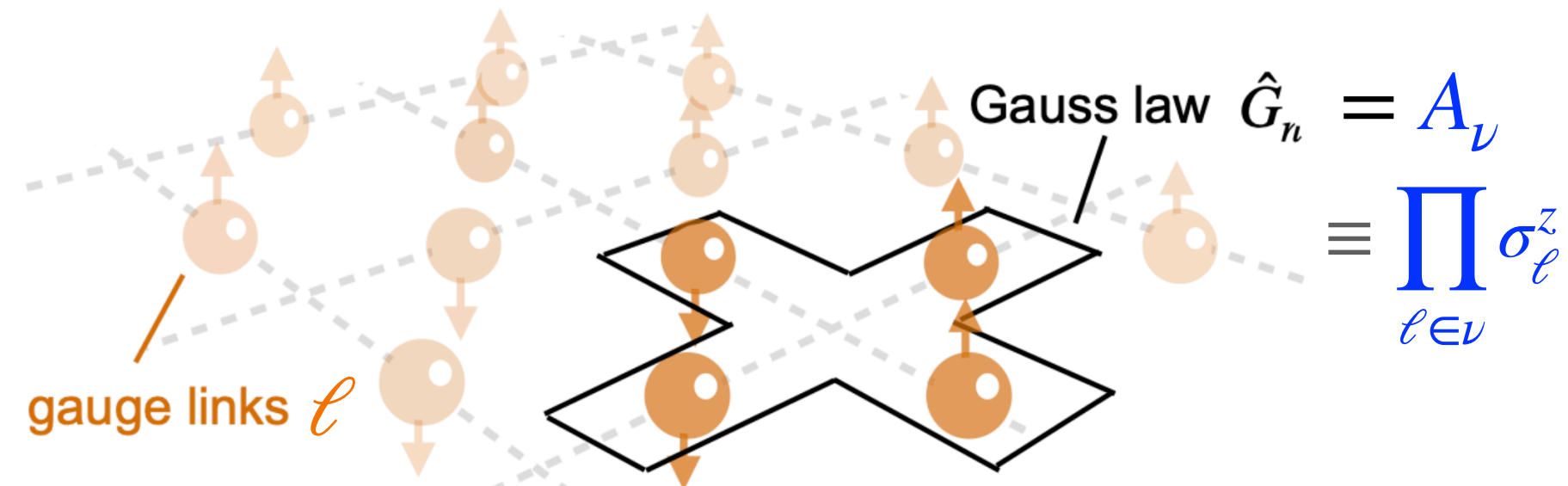
$$H = -\epsilon \sum_{\ell} \sigma_{\ell}^z - \sum_p B_p$$

$p \equiv \prod_{\ell \in p} \sigma_{\ell}^x$

Symmetry-conscious Random Measurement

Lattice Gauge Theories

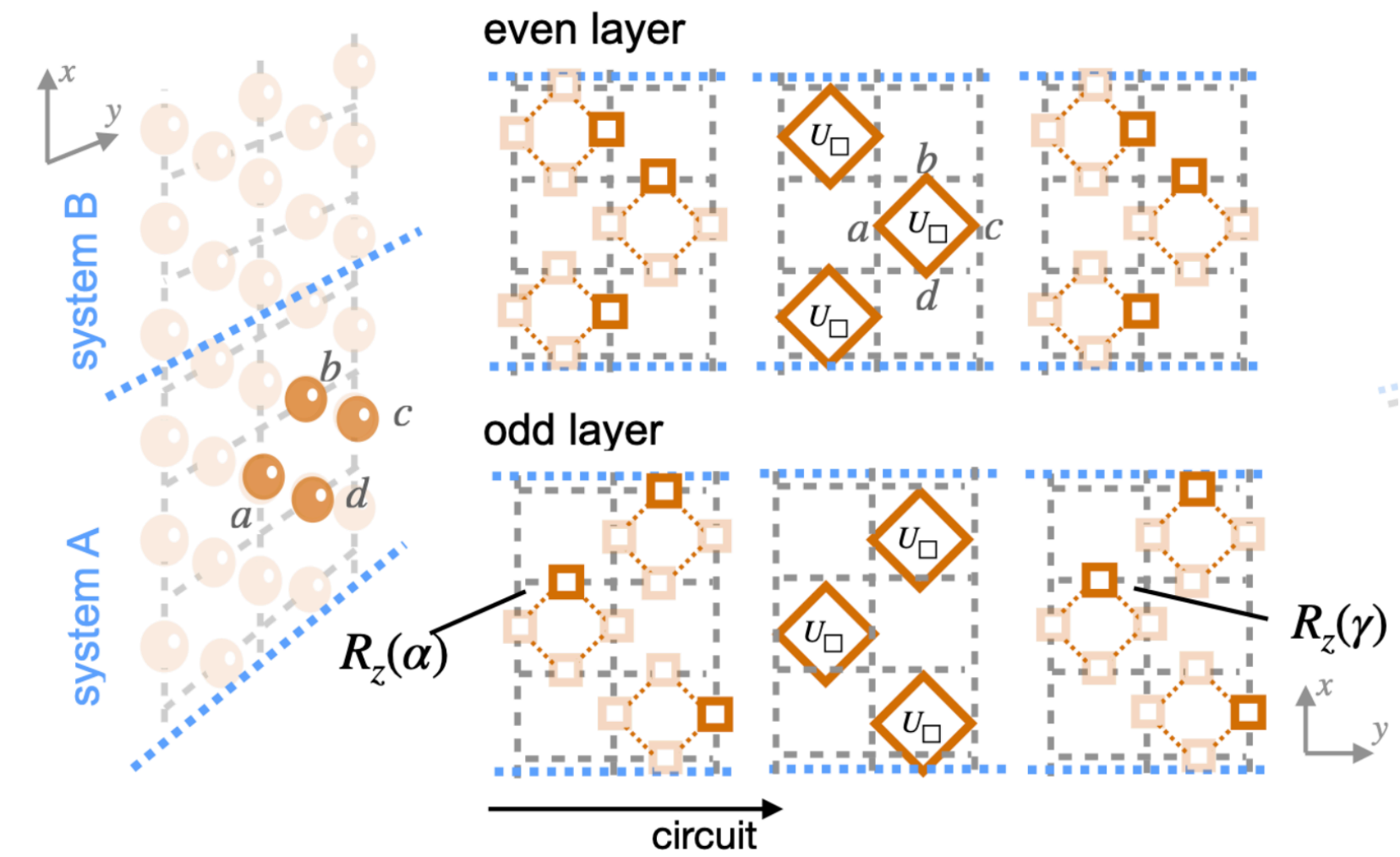
- **Z₂ Lattice Gauge Theory**



$$H = -\epsilon \sum_{\ell} \sigma_\ell^z - \sum_p B_p$$

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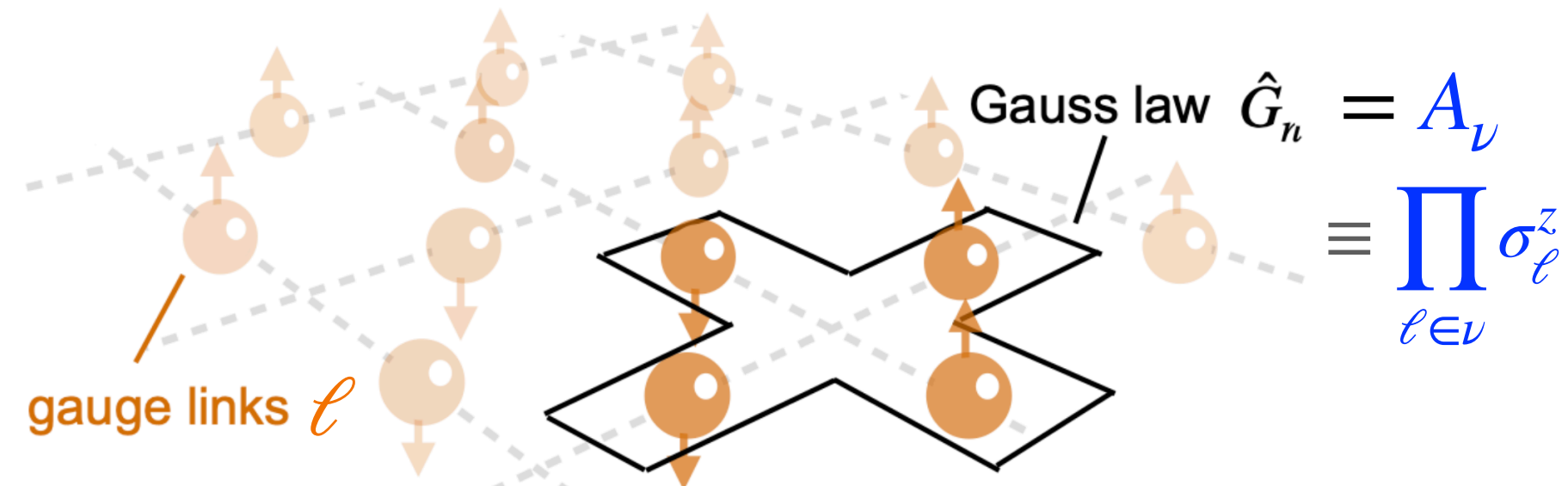
Recipe: symmetry-conscious k-design



Symmetry-conscious Random Measurement

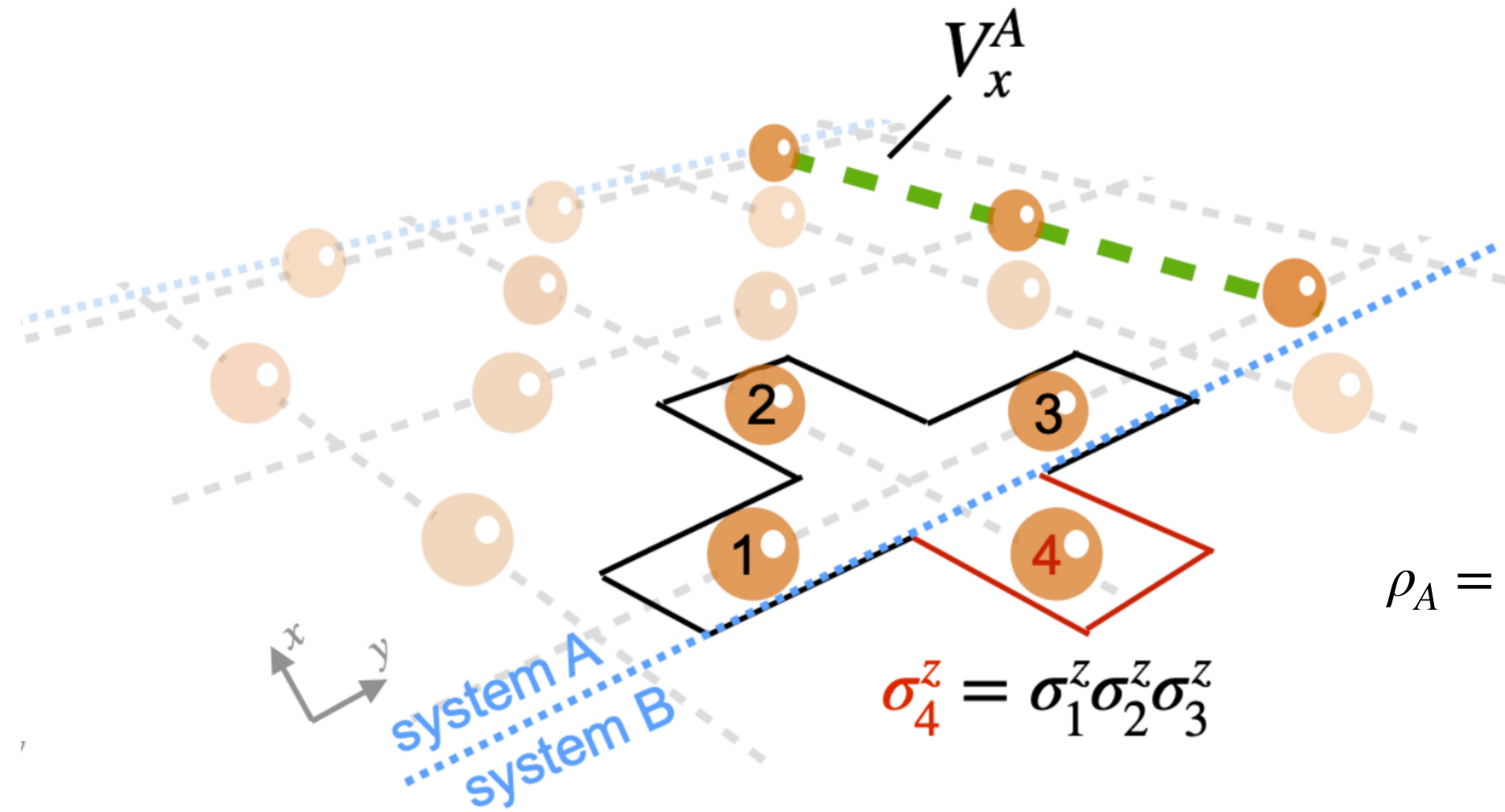
Lattice Gauge Theories

- Z₂ Lattice Gauge Theory**



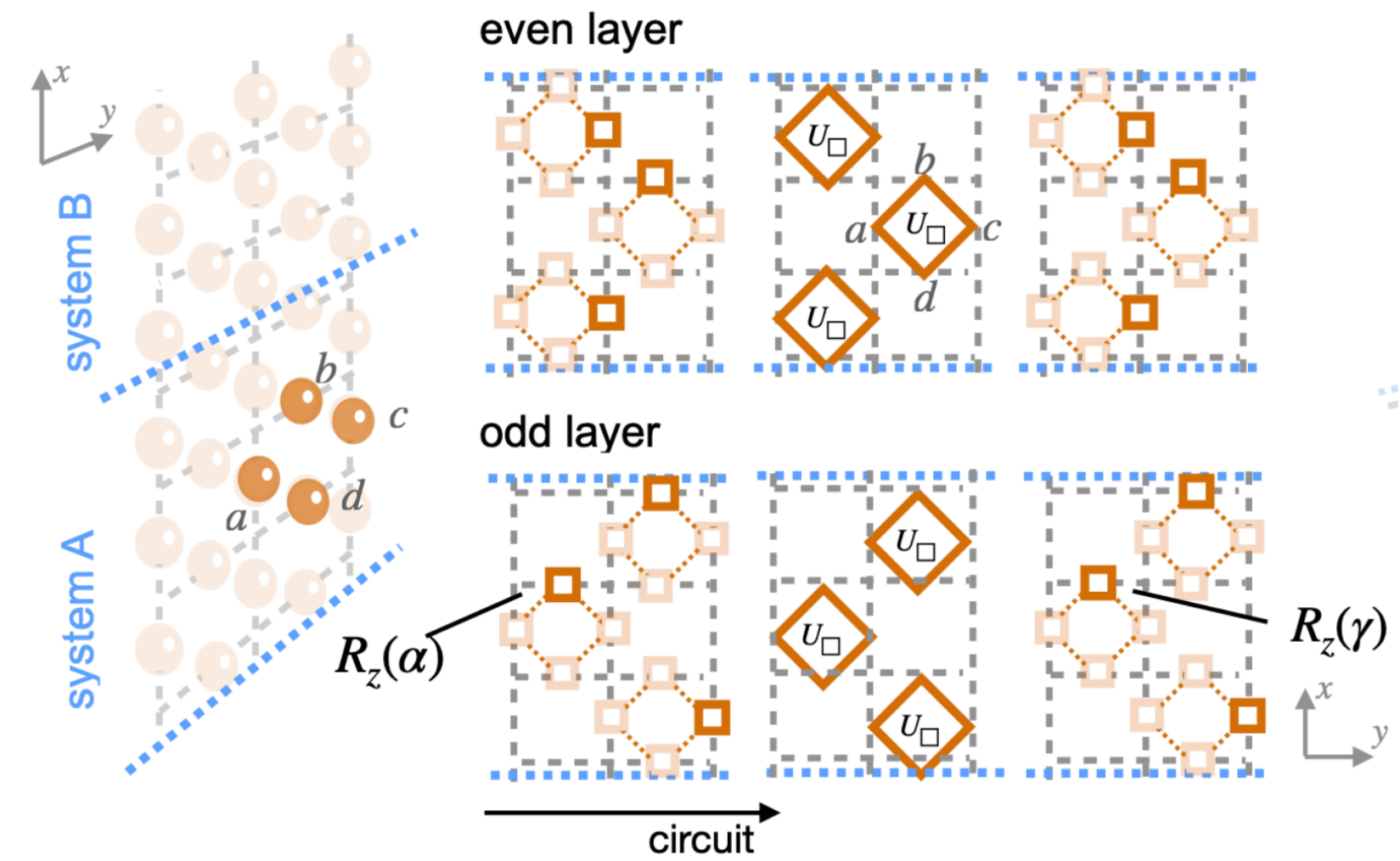
$$H = -\epsilon \sum_{\ell} \sigma_{\ell}^z - \sum_p B_p$$

$$B_p \equiv \prod_{\ell \in p} \sigma_{\ell}^x$$



$$\rho_A = \begin{pmatrix} \square & \dots & \dots \\ \vdots & \square & \dots \\ \dots & \dots & \ddots \end{pmatrix}$$

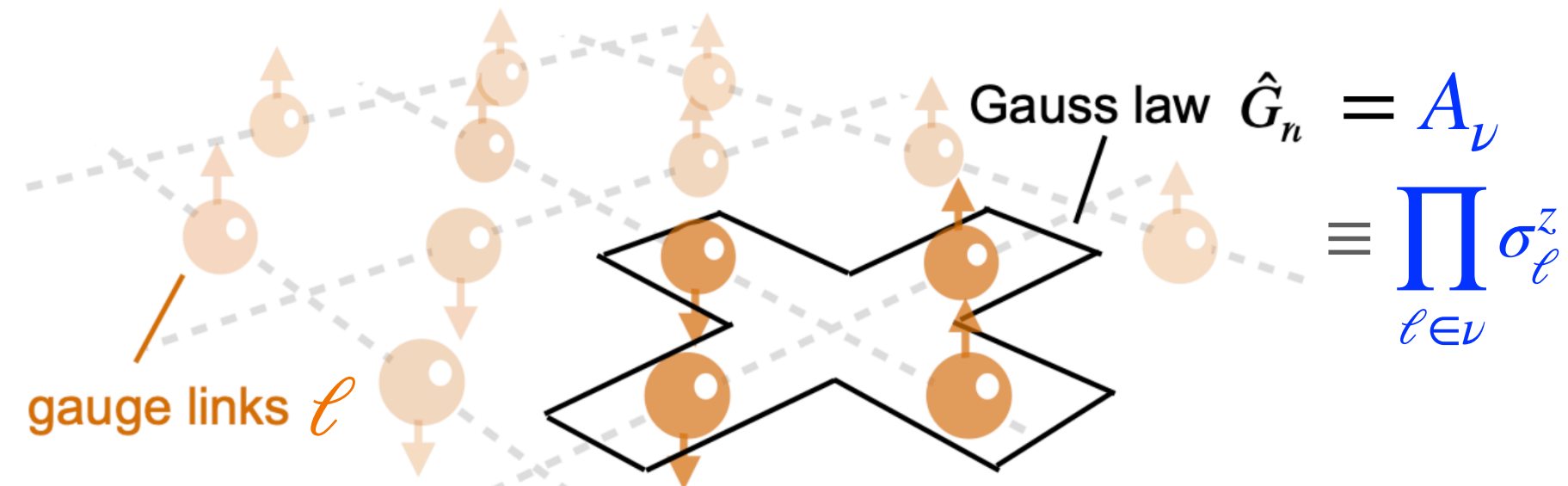
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Symmetry-conscious Random Measurement

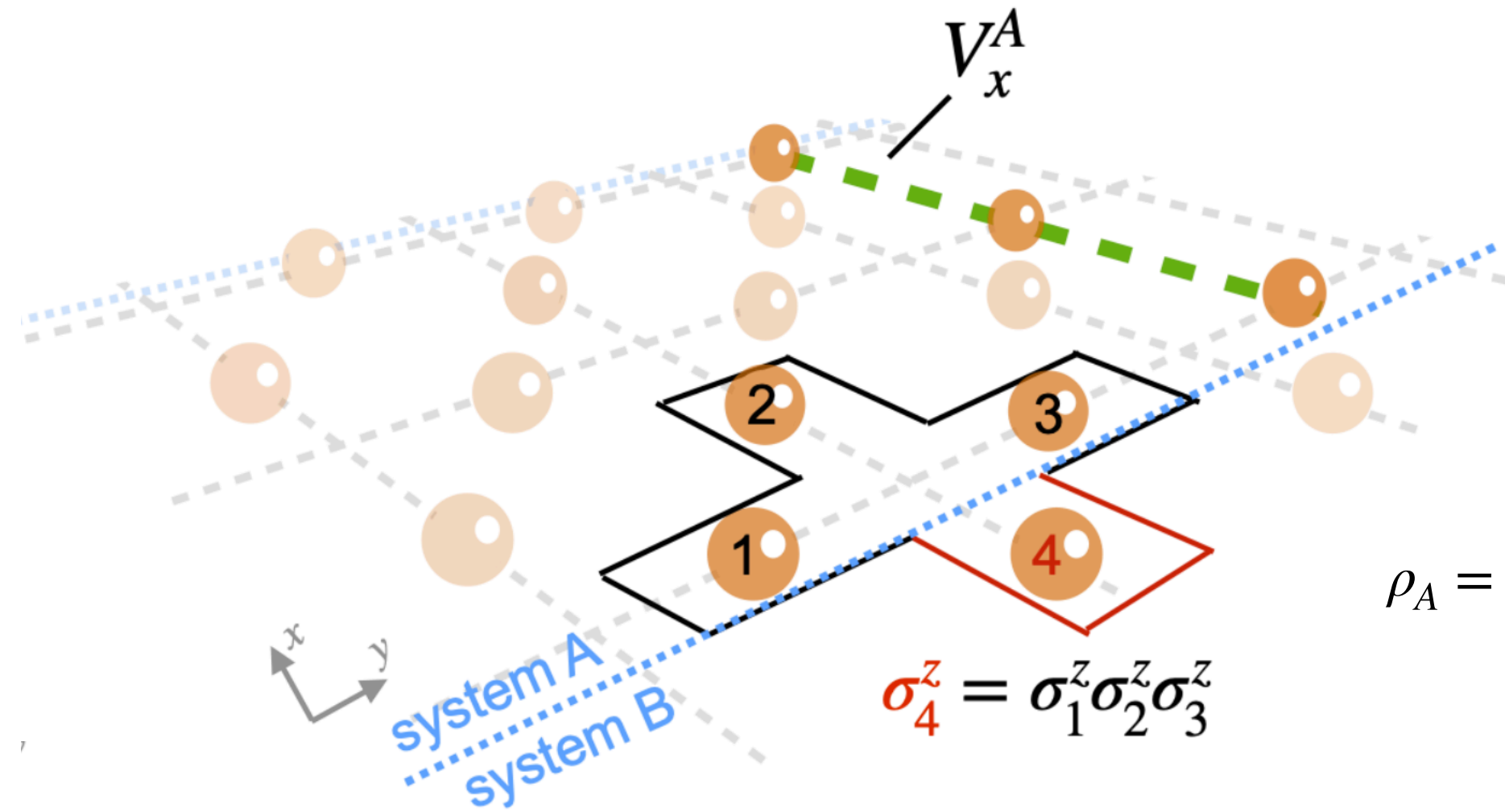
Lattice Gauge Theories

- Z₂ Lattice Gauge Theory**



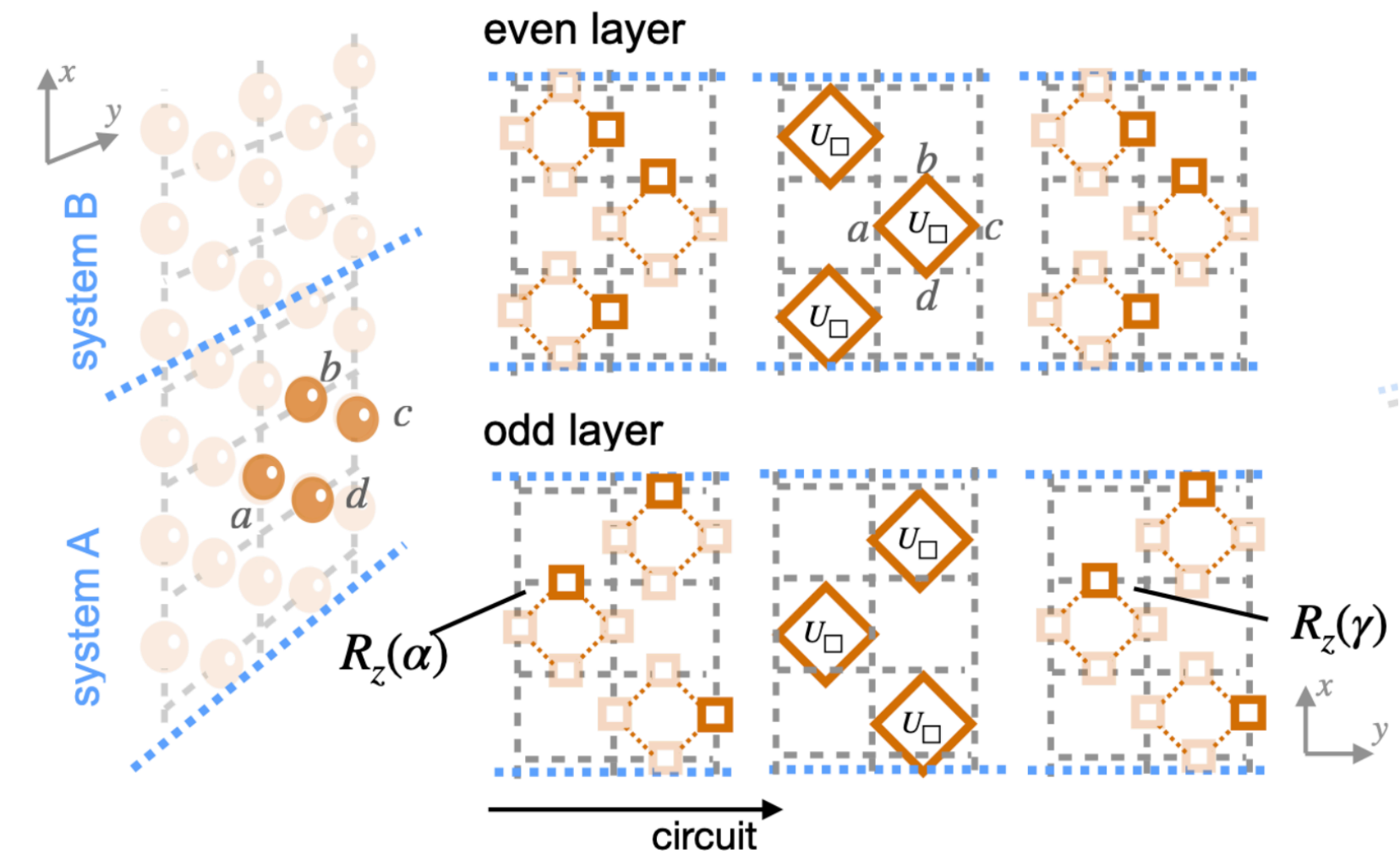
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Recipe: symmetry-conscious k-design



(for every plaquette p)

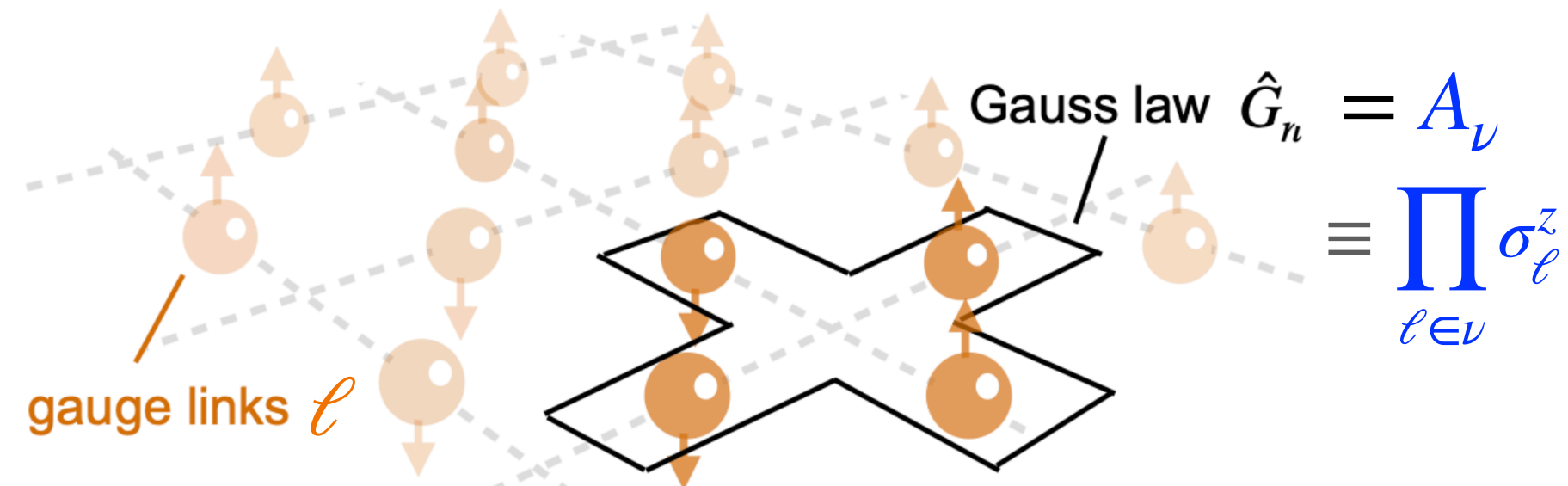
1. Draw α, β, γ randomly from 1-qubit CUE

$$u_1 = \exp\{i\gamma\sigma^z\} \exp\{i\beta\sigma^x\} \exp\{i\alpha\sigma^z\}$$

Symmetry-conscious Random Measurement

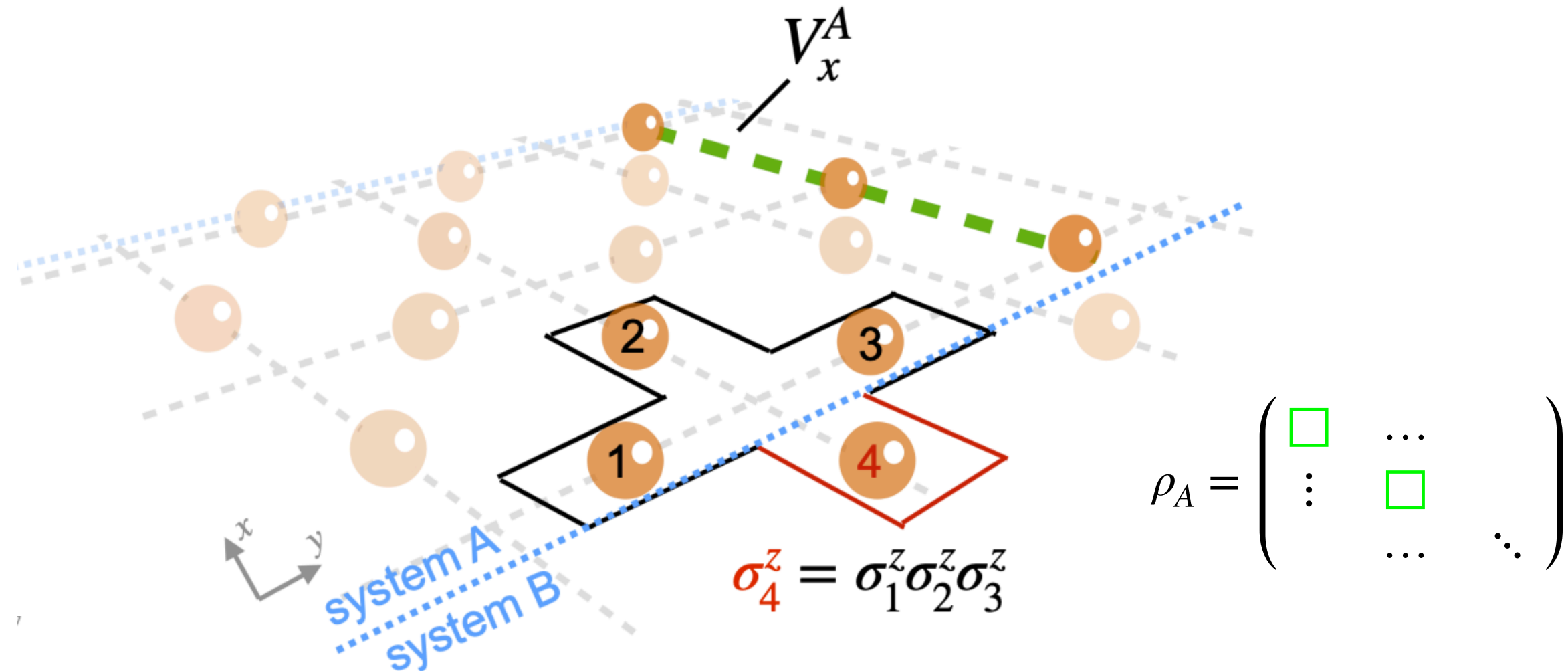
Lattice Gauge Theories

- Z₂ Lattice Gauge Theory**

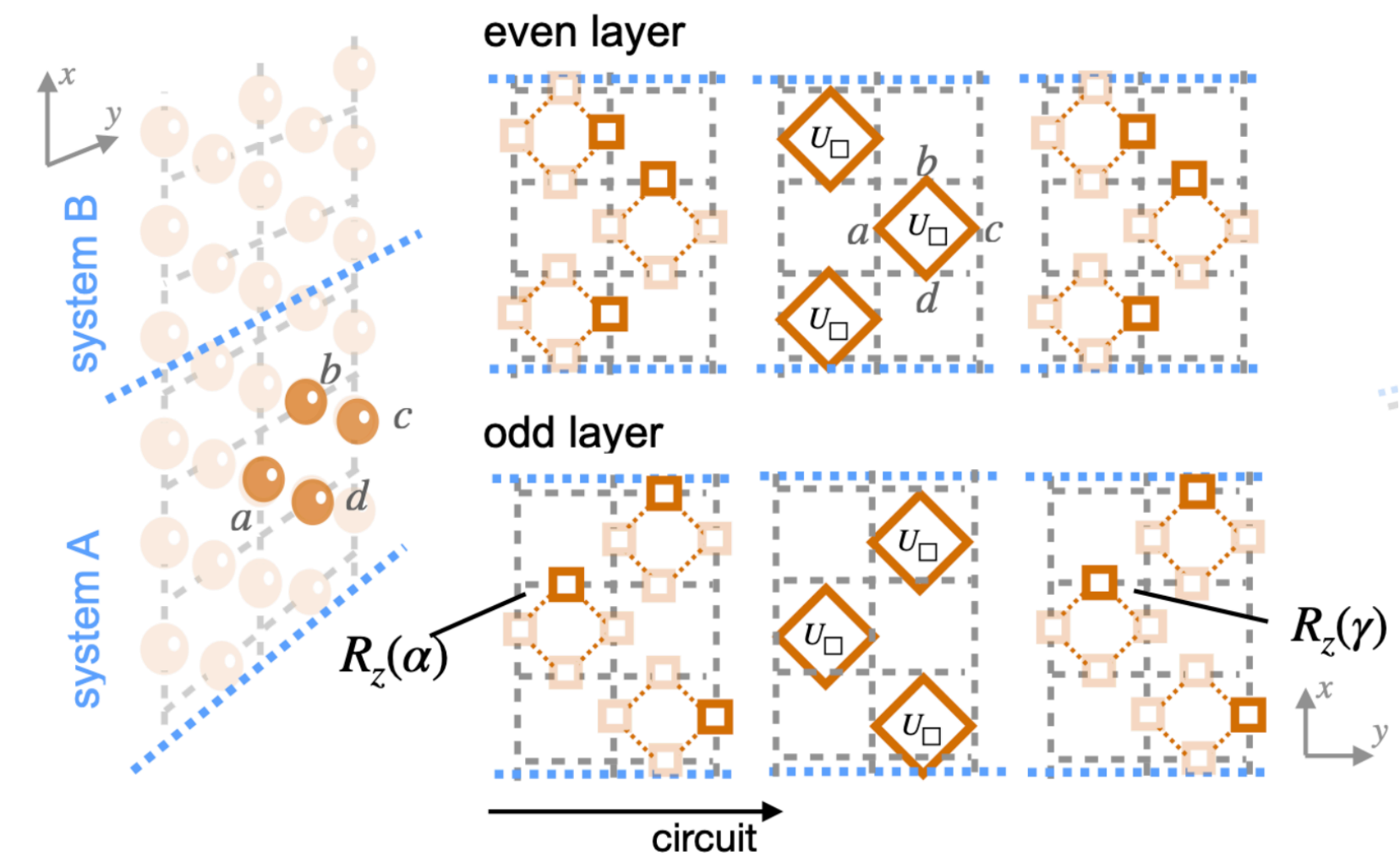


$$H = -\epsilon \sum_{\ell} \sigma_\ell^z - \sum_p B_p$$

$$B_p \equiv \prod_{\ell \in p} \sigma_\ell^x$$



Recipe: symmetry-conscious k-design



(for every plaquette p)

1. Draw α, β, γ randomly from 1-qubit CUE

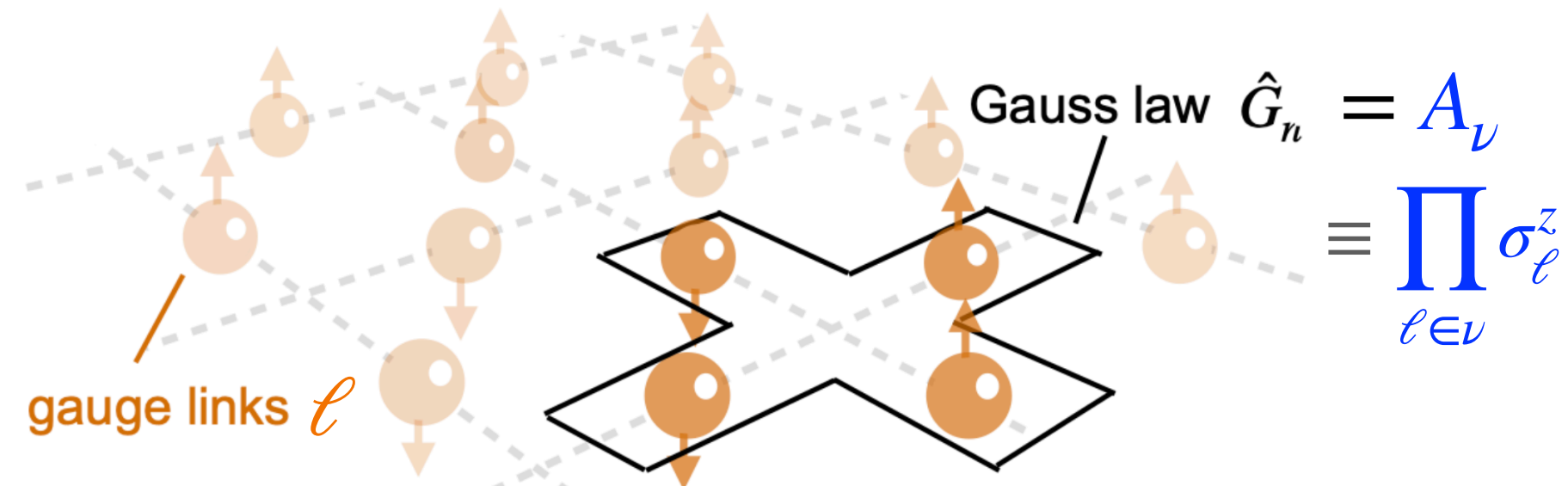
$$u_1 = \exp\{i\gamma\sigma^z\} \exp\{i\beta\sigma^x\} \exp\{i\alpha\sigma^z\}$$

2. Apply 1-qubit $R_z(\alpha) = e^{i\alpha\sigma^z}$ to one randomly chosen link $\ell \in p$

Symmetry-conscious Random Measurement

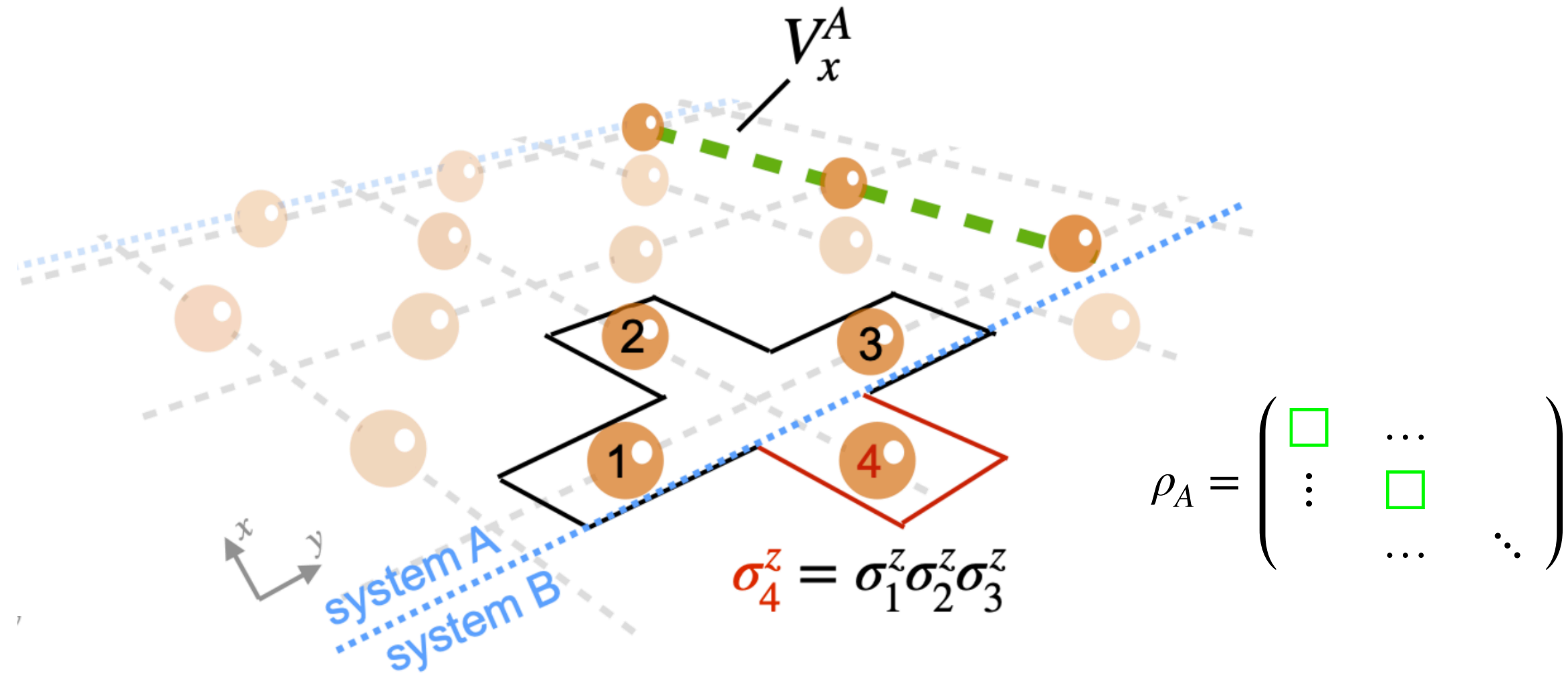
Lattice Gauge Theories

- Z₂ Lattice Gauge Theory**



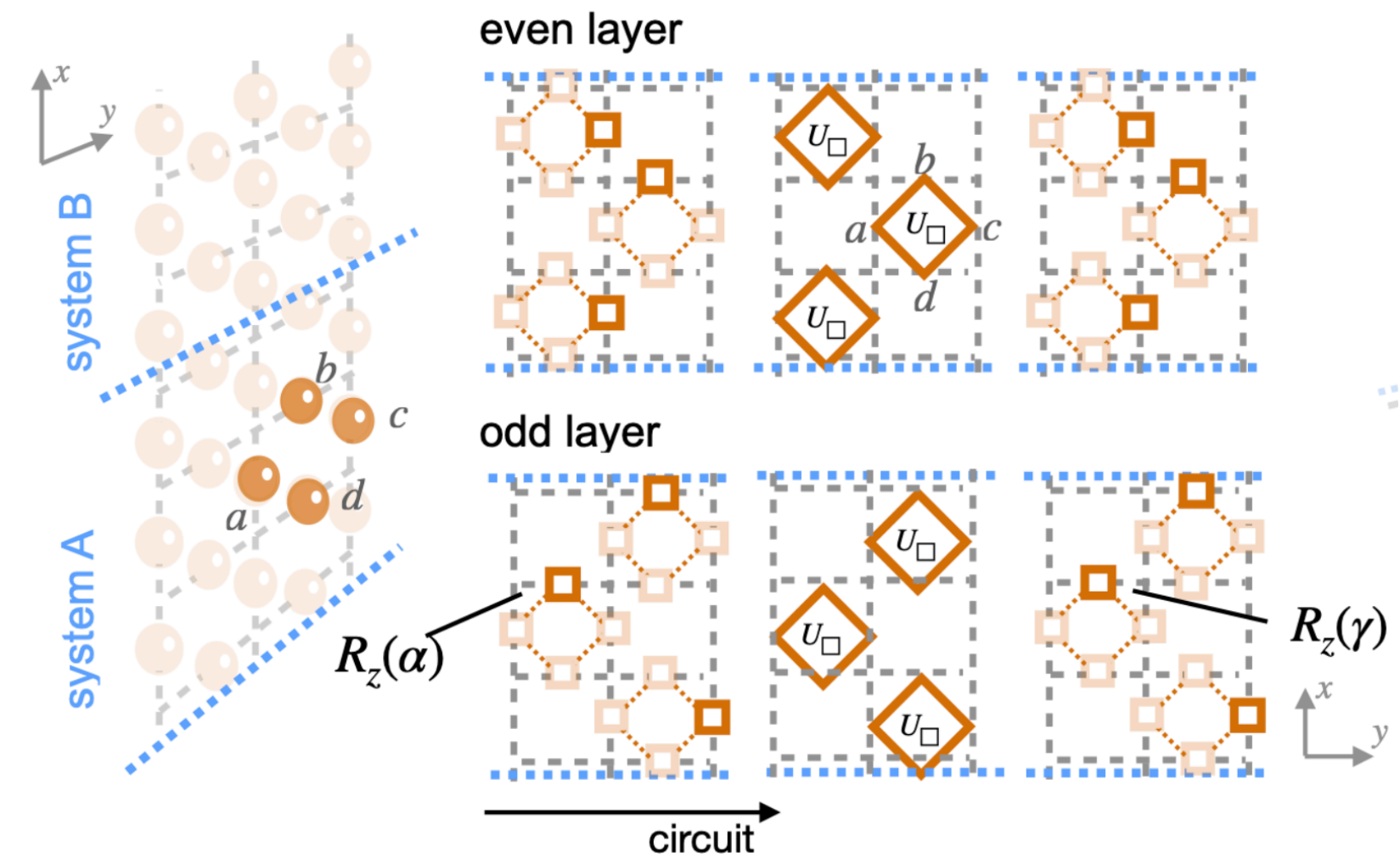
$$H = -\epsilon \sum_{\ell} \sigma_\ell^z - \sum_p B_p$$

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$$\rho_A = \begin{pmatrix} \square & \dots & \dots \\ \vdots & \square & \dots \\ \dots & \dots & \ddots \end{pmatrix}$$

Recipe: symmetry-conscious k-design



(for every plaquette p)

1. Draw α, β, γ randomly from 1-qubit CUE

$$u_1 = \exp\{i\gamma\sigma^z\} \exp\{i\beta\sigma^x\} \exp\{i\alpha\sigma^z\}$$

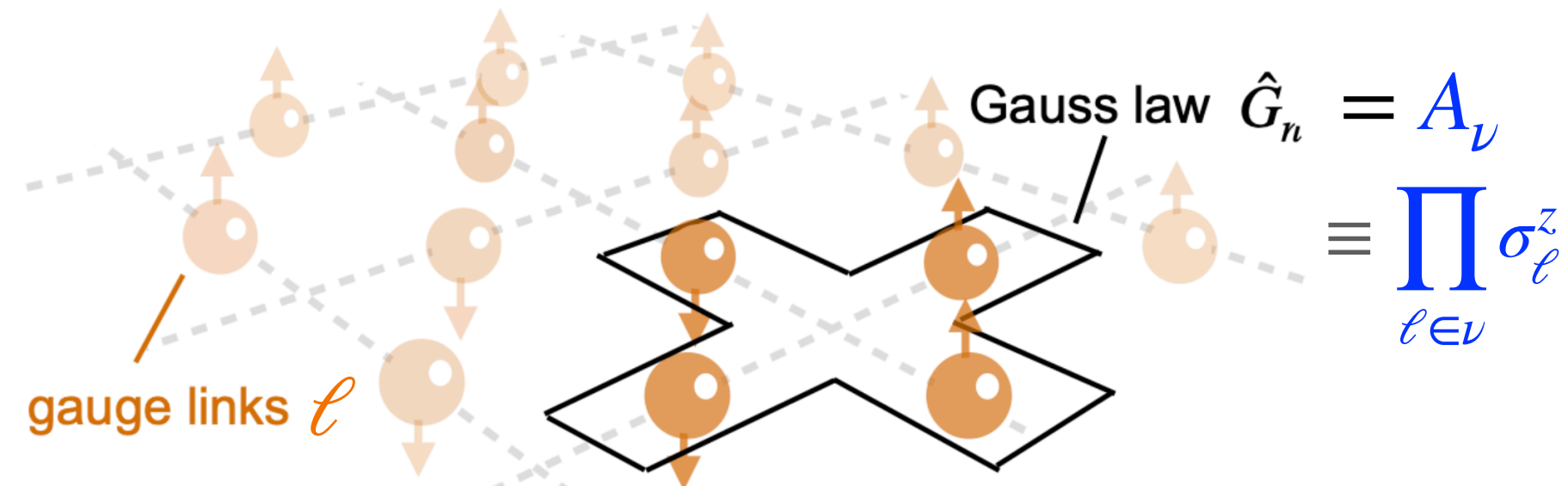
2. Apply 1-qubit $R_z(\alpha) = e^{i\alpha\sigma^z}$ to one randomly chosen link $\ell \in p$

3. Apply 4-qubit $U_\square = e^{i\beta\sigma^x\sigma^x\sigma^x\sigma^x}$

Symmetry-conscious Random Measurement

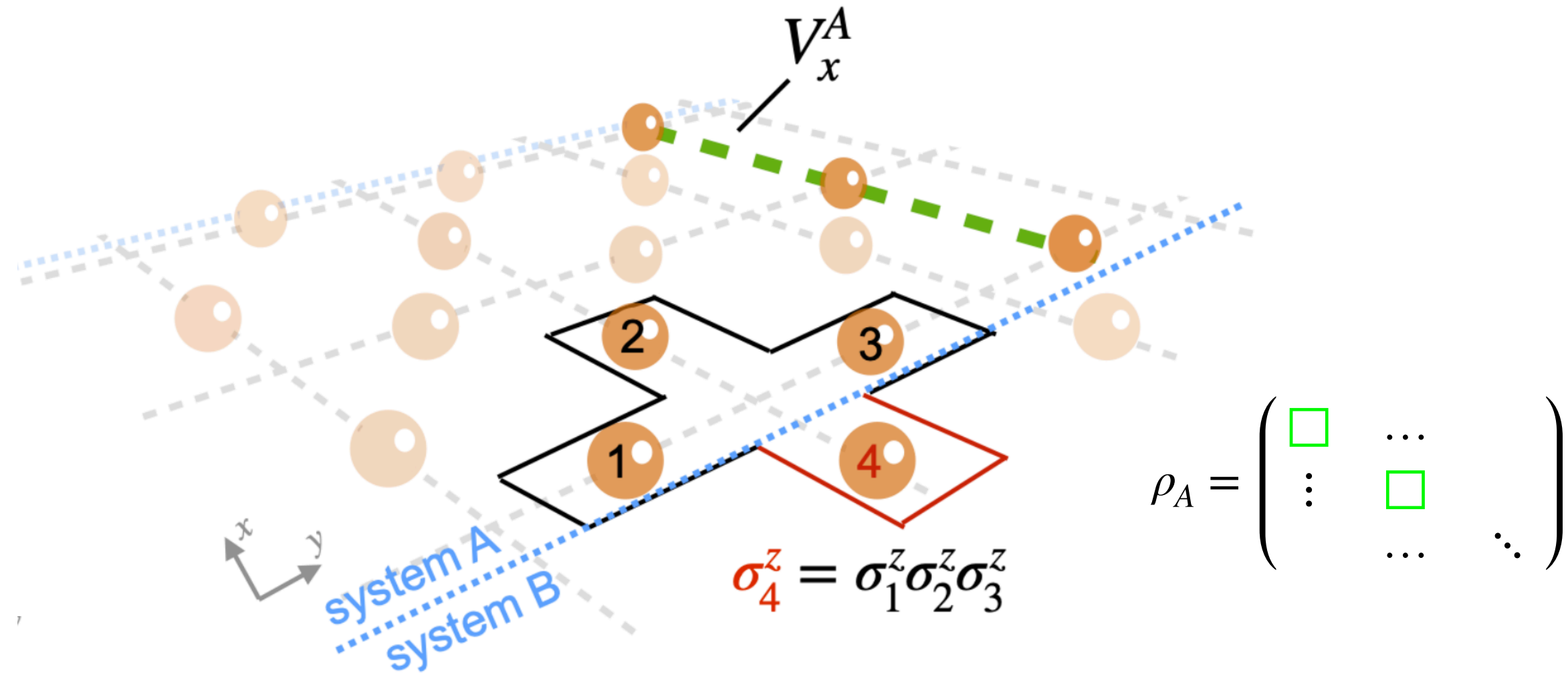
Lattice Gauge Theories

- Z₂ Lattice Gauge Theory**

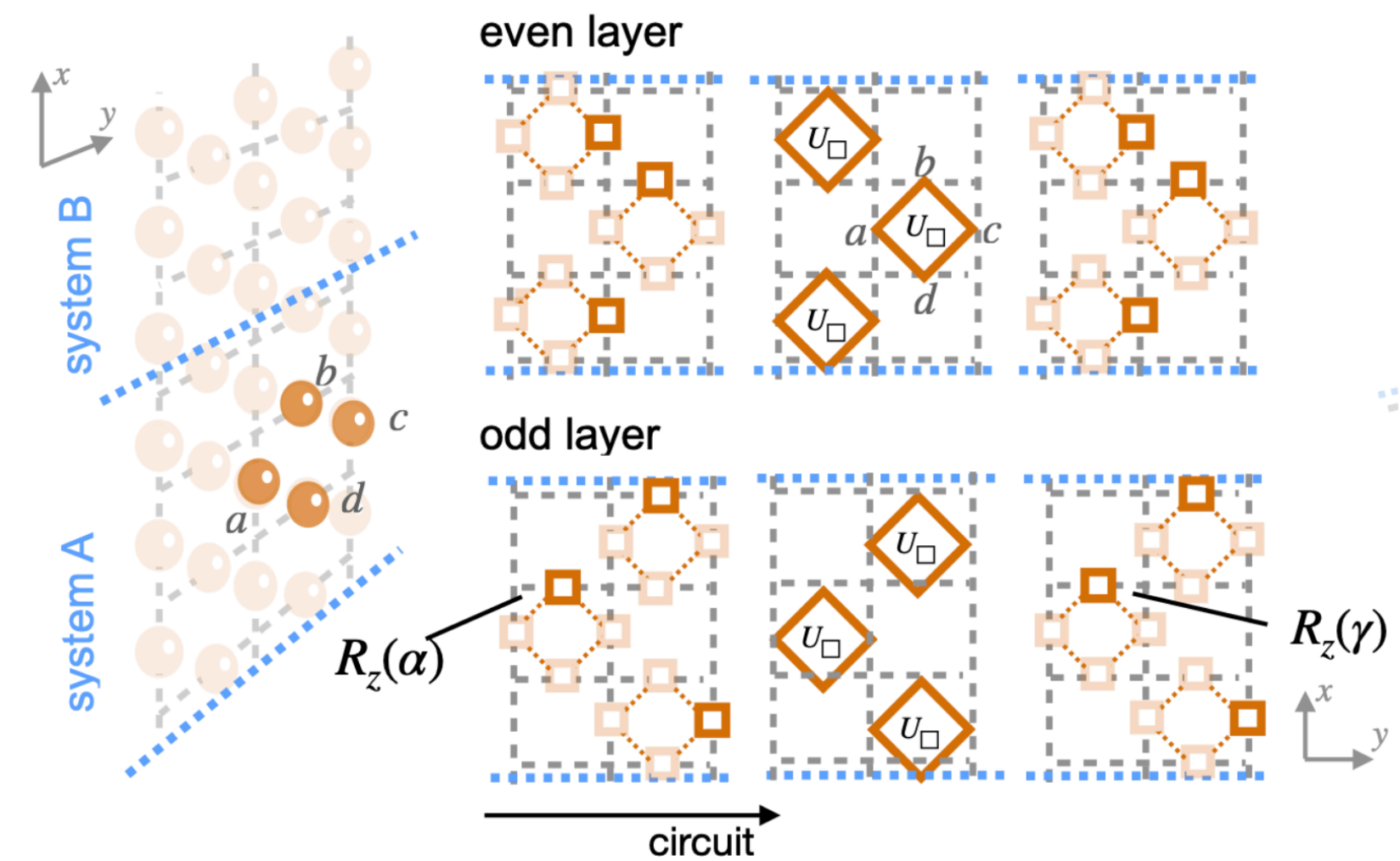


$$H = -\epsilon \sum_{\ell} \sigma_\ell^z - \sum_p B_p$$

$$B_p \equiv \prod_{\ell \in p} \sigma_\ell^x$$



Recipe: symmetry-conscious k-design



(for every plaquette p)

1. Draw α, β, γ randomly from 1-qubit CUE

$$u_1 = \exp\{i\gamma\sigma^z\} \exp\{i\beta\sigma^x\} \exp\{i\alpha\sigma^z\}$$

2. Apply 1-qubit $R_z(\alpha) = e^{i\alpha\sigma^z}$ to one randomly chosen link $\ell \in p$

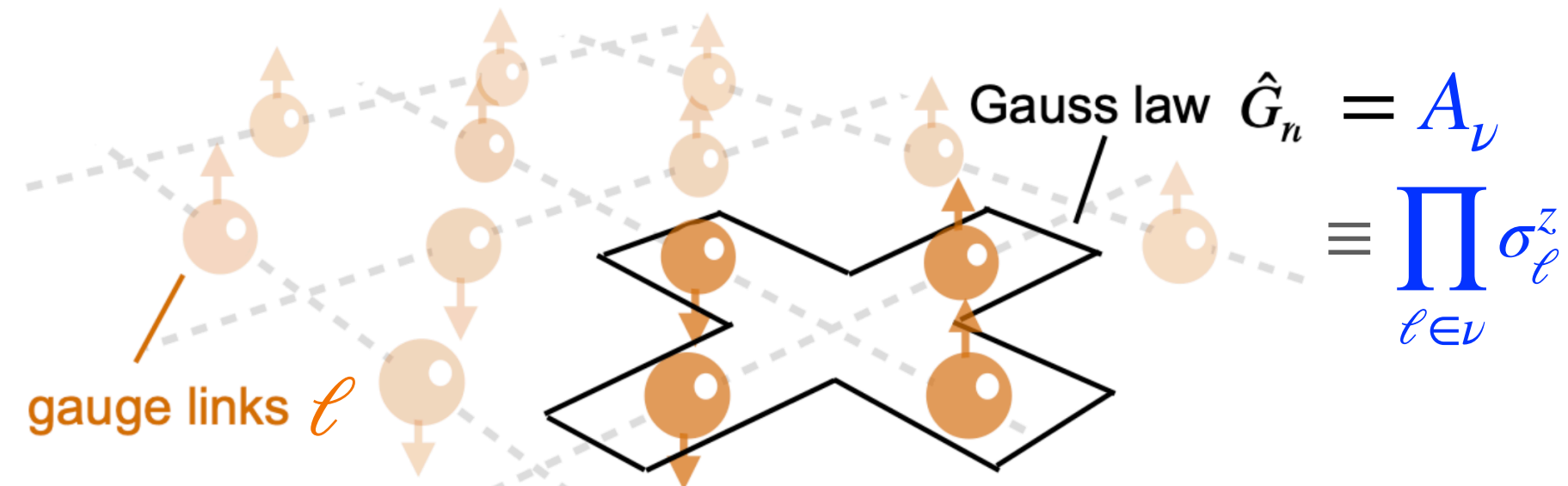
3. Apply 4-qubit $U_\square = e^{i\beta\sigma^x\sigma^x\sigma^x\sigma^x}$

4. Apply 1-qubit $R_z(\alpha) = e^{i\gamma\sigma^z}$ to same $\ell \in p$

Symmetry-conscious Random Measurement

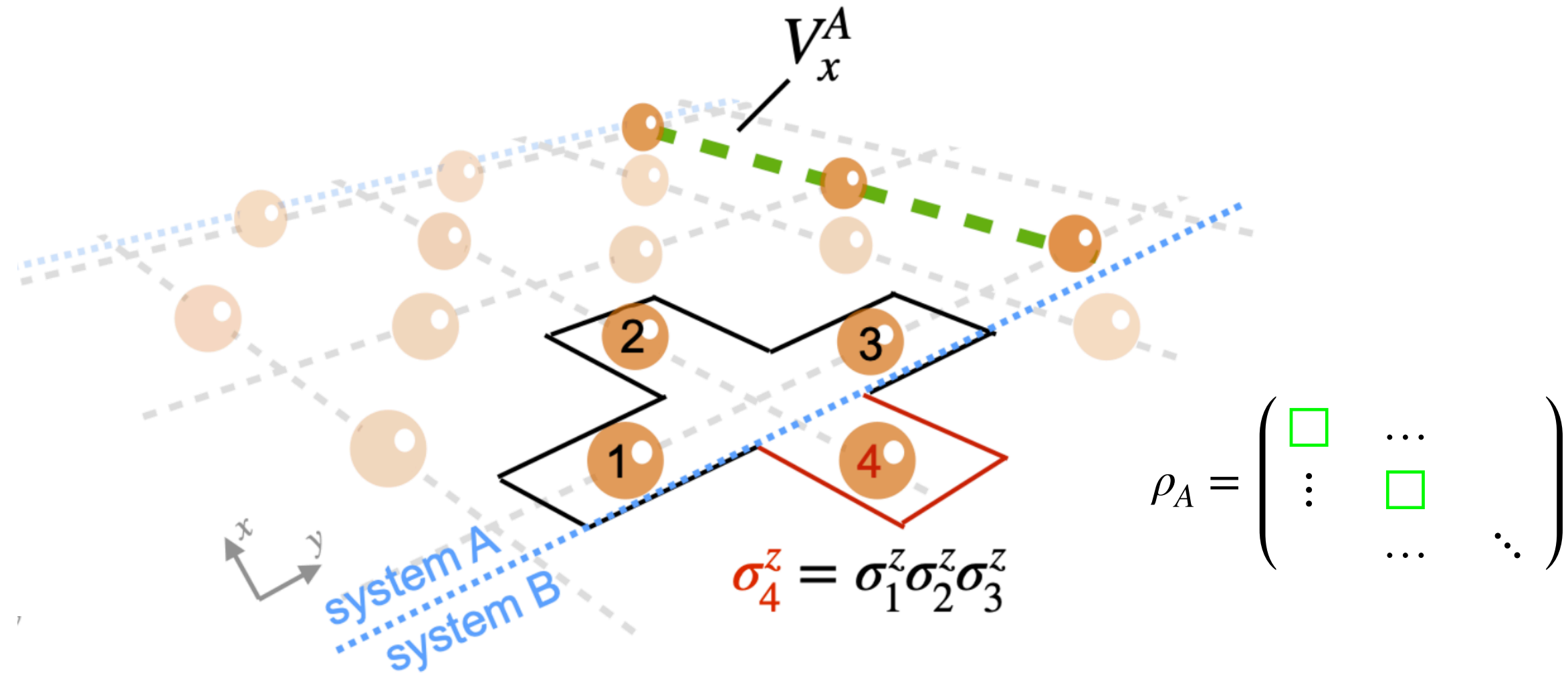
Lattice Gauge Theories

- Z₂ Lattice Gauge Theory**

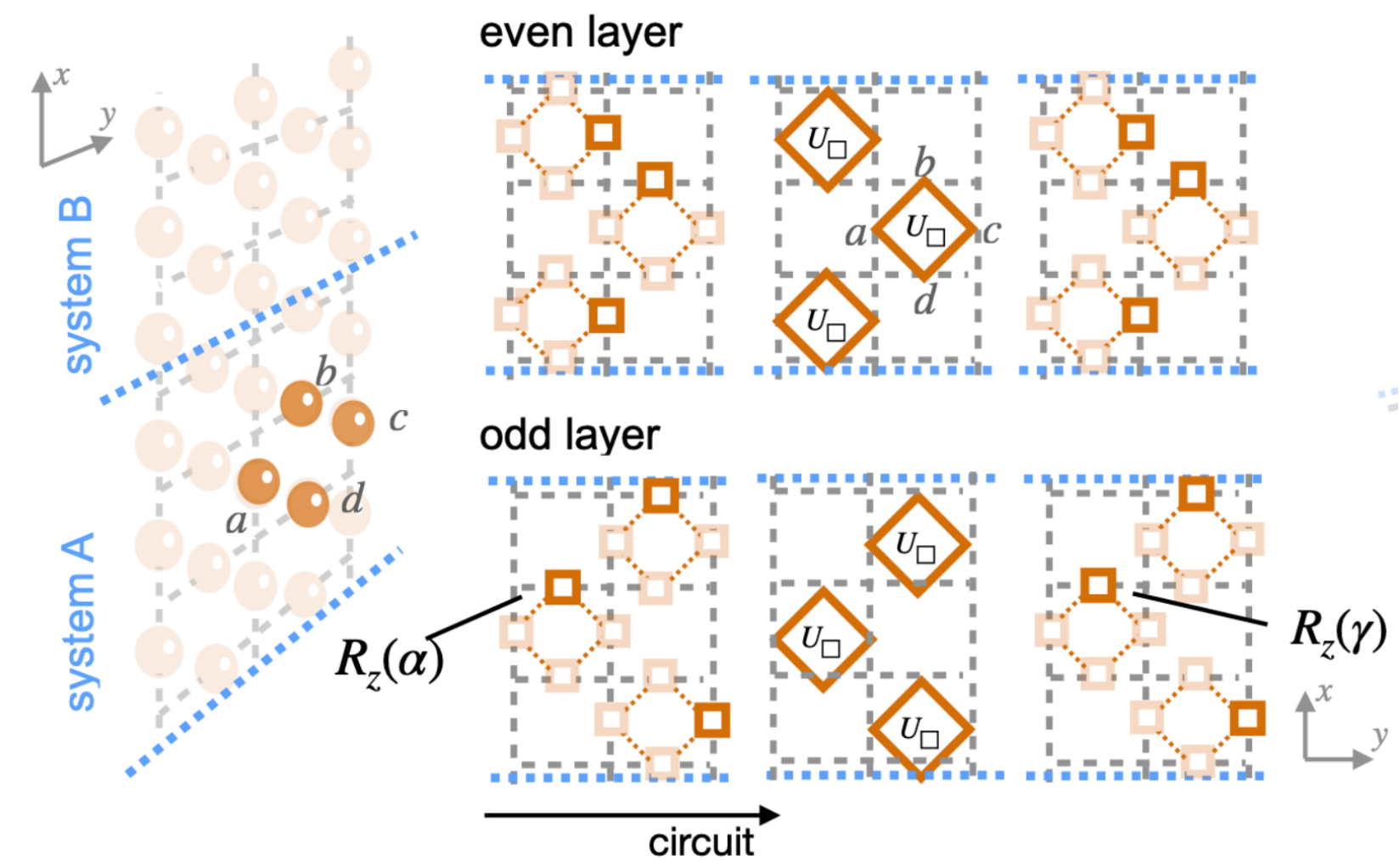


$$H = -\epsilon \sum_{\ell} \sigma_\ell^z - \sum_p B_p$$

$$p \equiv \prod_{\ell \in p} \sigma_\ell^x$$



Recipe: symmetry-conscious k-design



(for every plaquette p)

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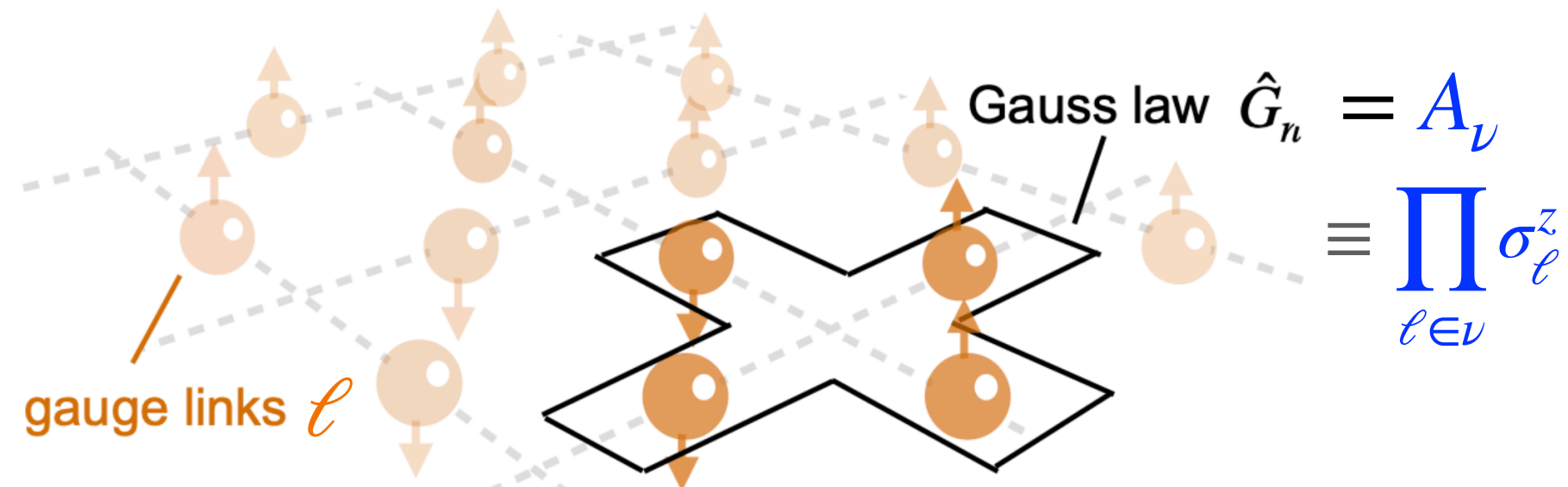
4. Apply 1-qubit $R_z(\alpha) = e^{i\gamma\sigma^z}$ to same $\ell \in p$

5. Repeat (odd and even layers)

Symmetry-conscious Random Measurement

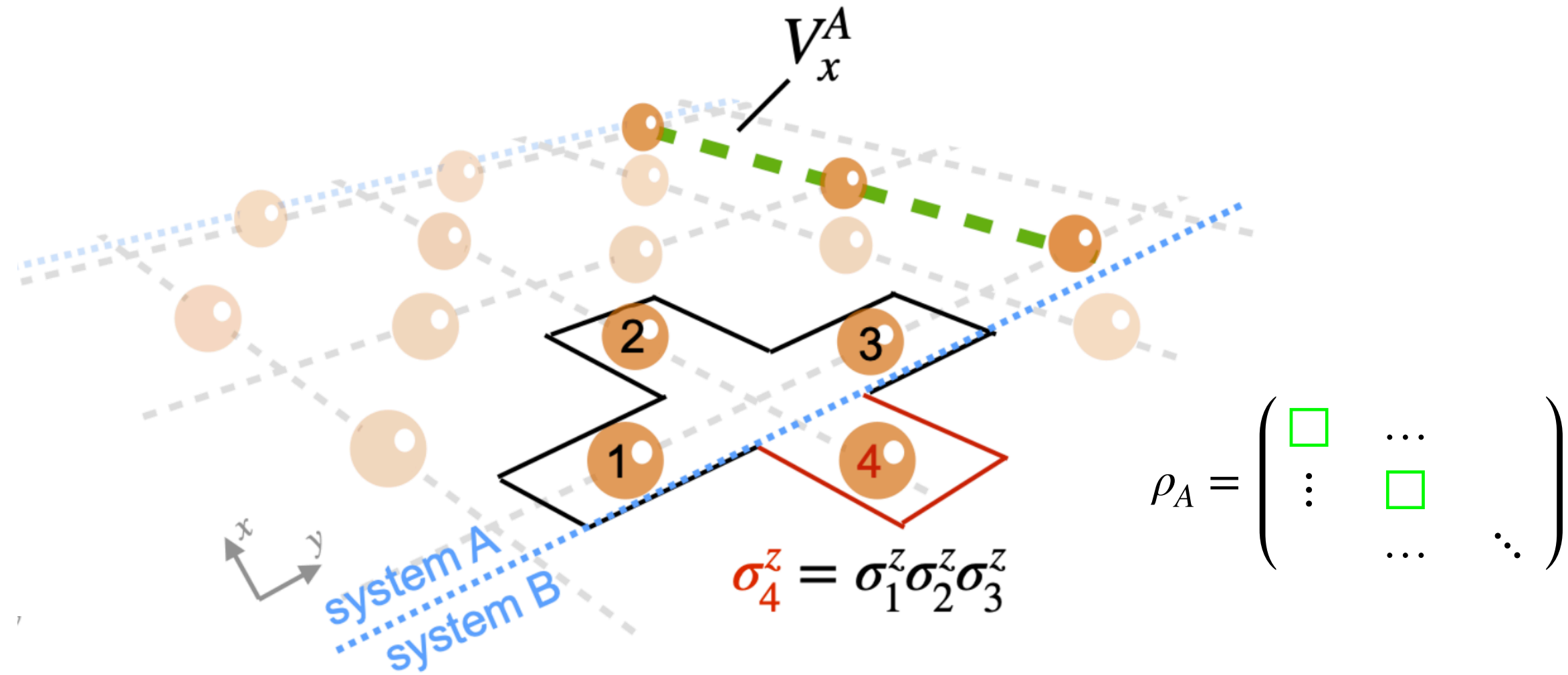
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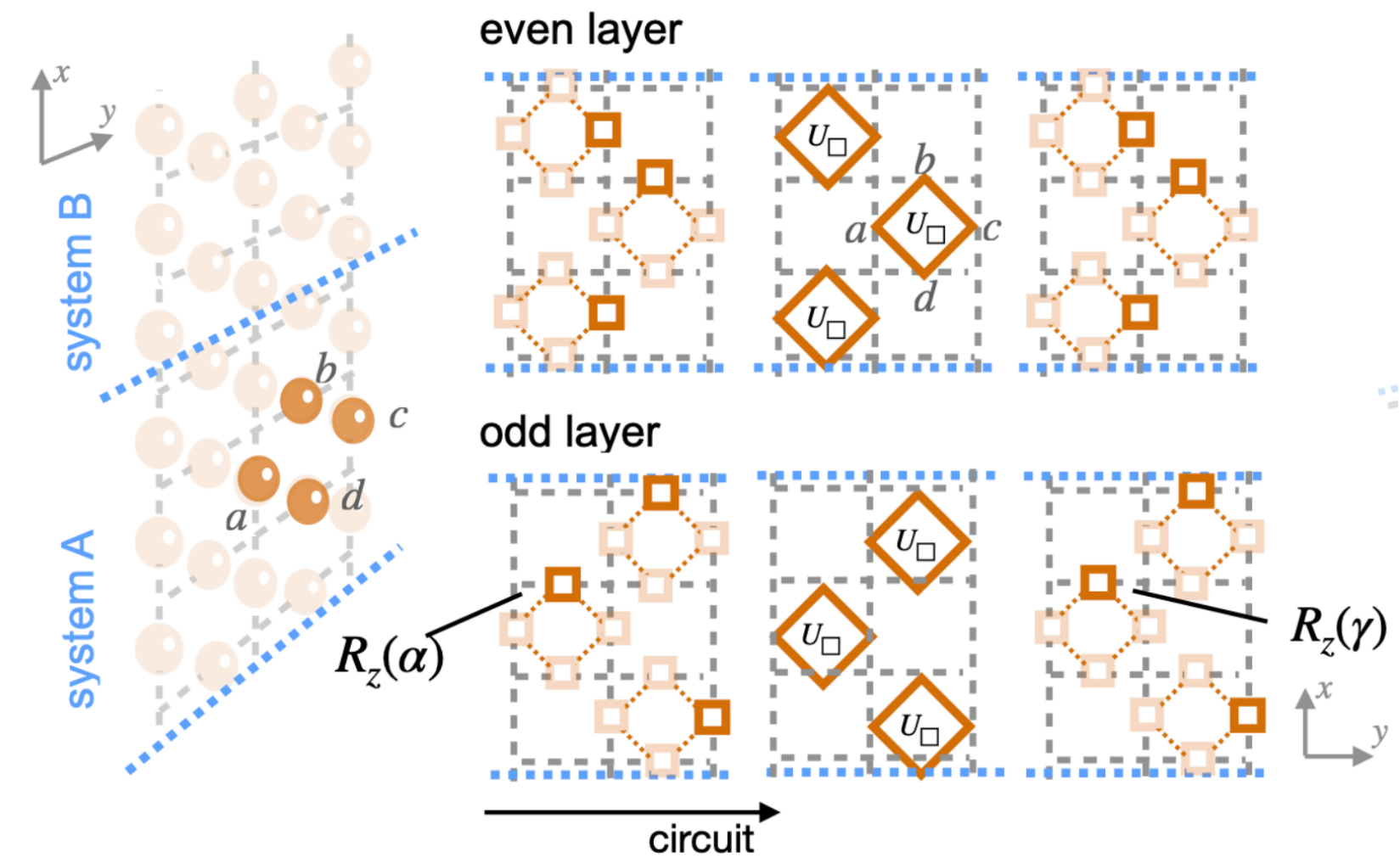


$$H = -\epsilon \sum_{\ell} \sigma_\ell^z - \sum_p B_p$$

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Recipe: symmetry-conscious k-design



(for every plaquette p)

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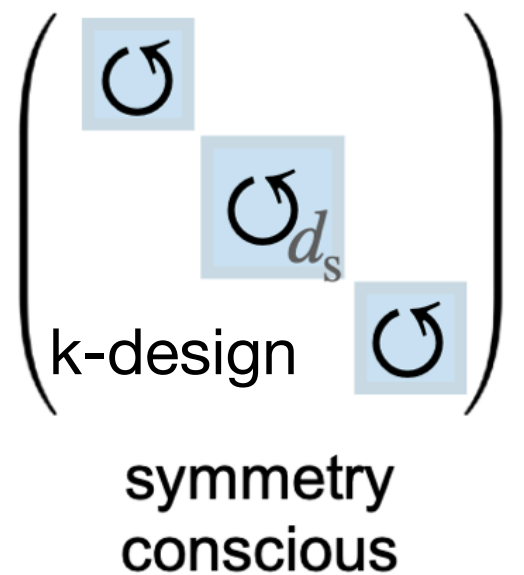
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2. Apply 1-qubit $R_z(\alpha) = e^{i\alpha\sigma^z}$ to one randomly chosen link $\ell \in p$

3. Apply 4-qubit $U_\square = e^{i\beta\sigma^x\sigma^x\sigma^x\sigma^x}$

4. Apply 1-qubit $R_z(\gamma) = e^{i\gamma\sigma^z}$ to same $\ell \in p$

5. Repeat (odd and even layers)

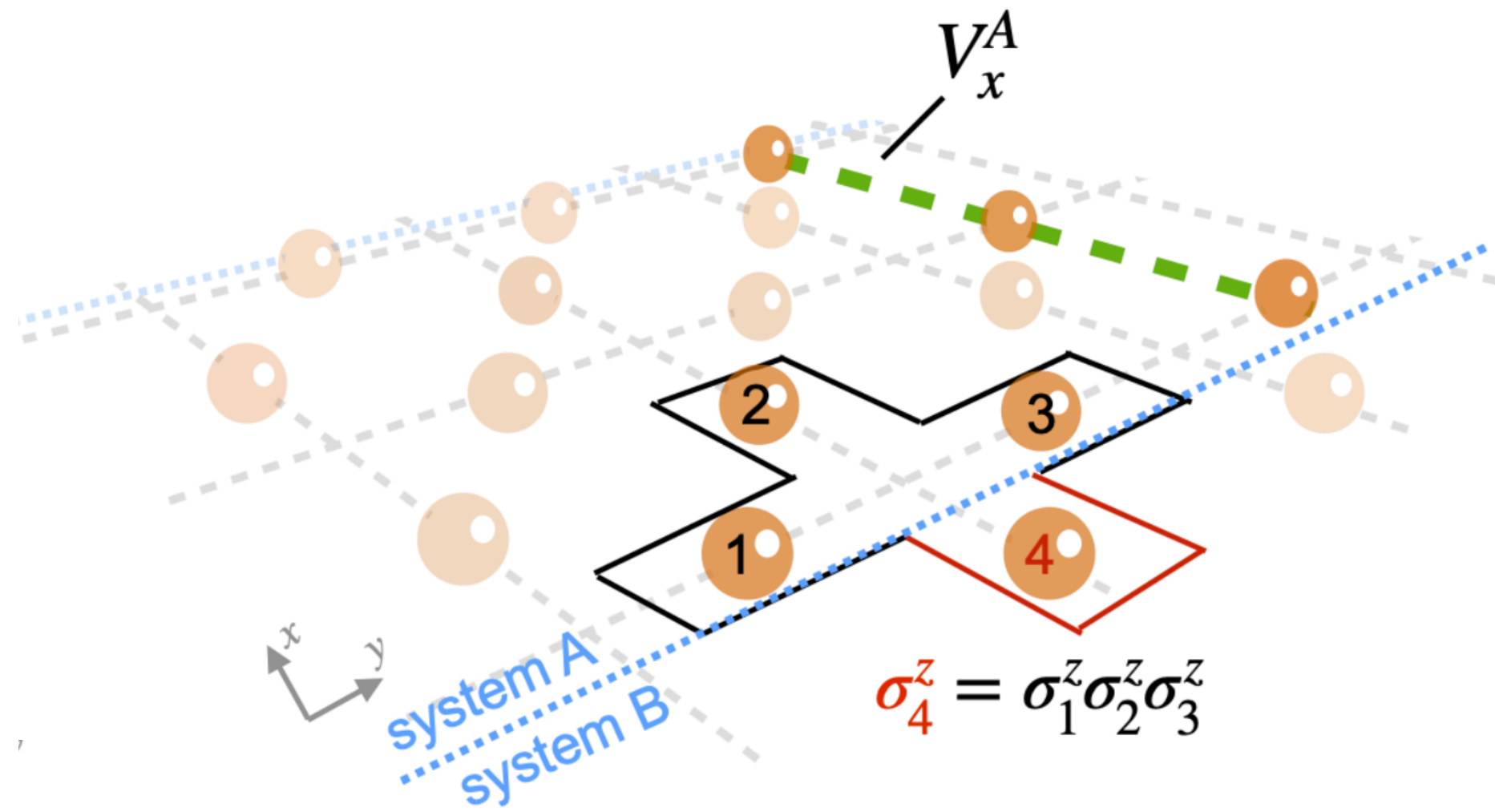


Symmetry-conscious Random Measurement

Lattice Gauge Theories

Symmetry-conscious Random Measurement

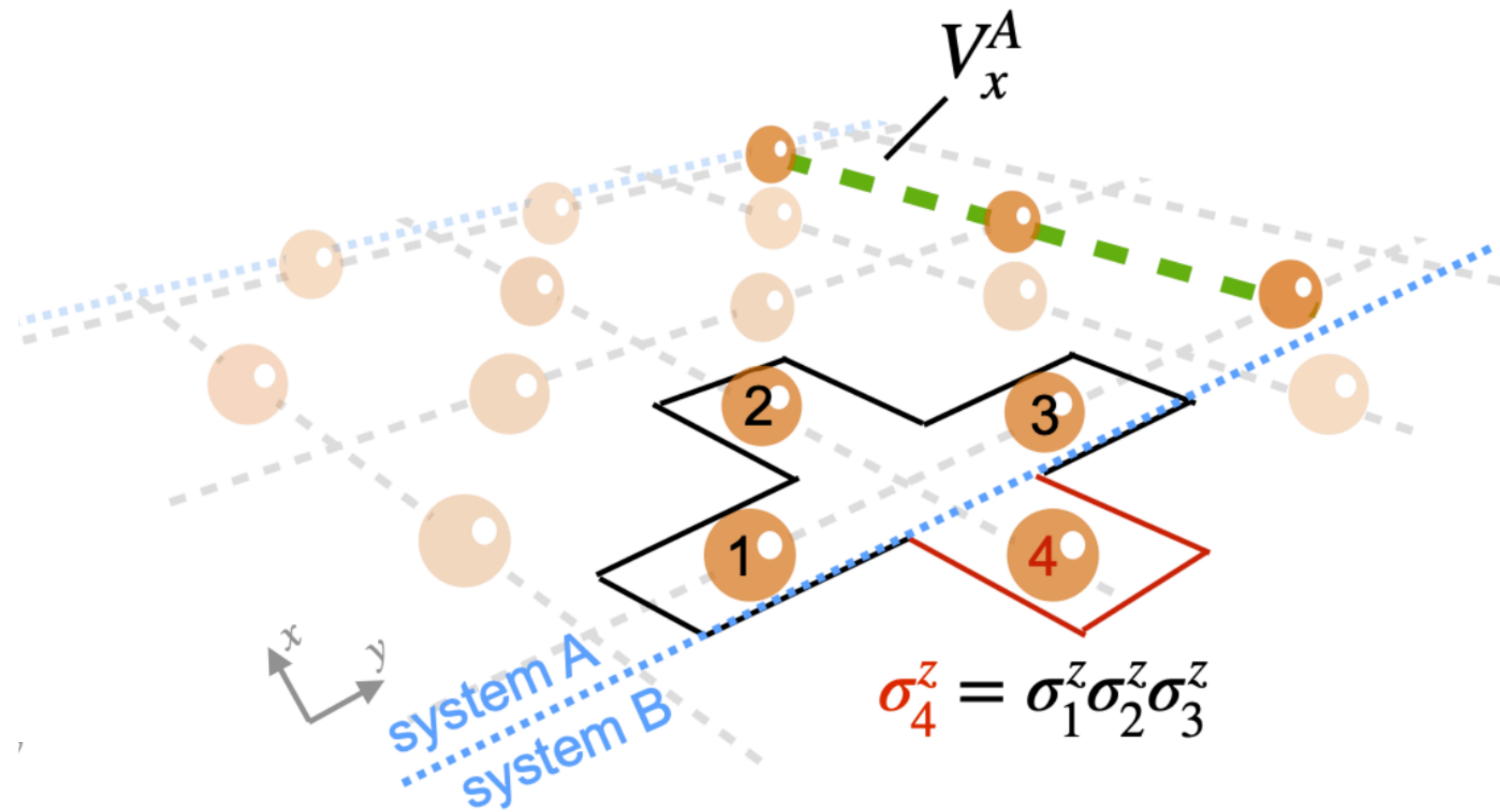
Lattice Gauge Theories



$$\rho_A = \begin{pmatrix} \square & \dots & \\ \vdots & \square & \\ \dots & \dots & \ddots \end{pmatrix}$$

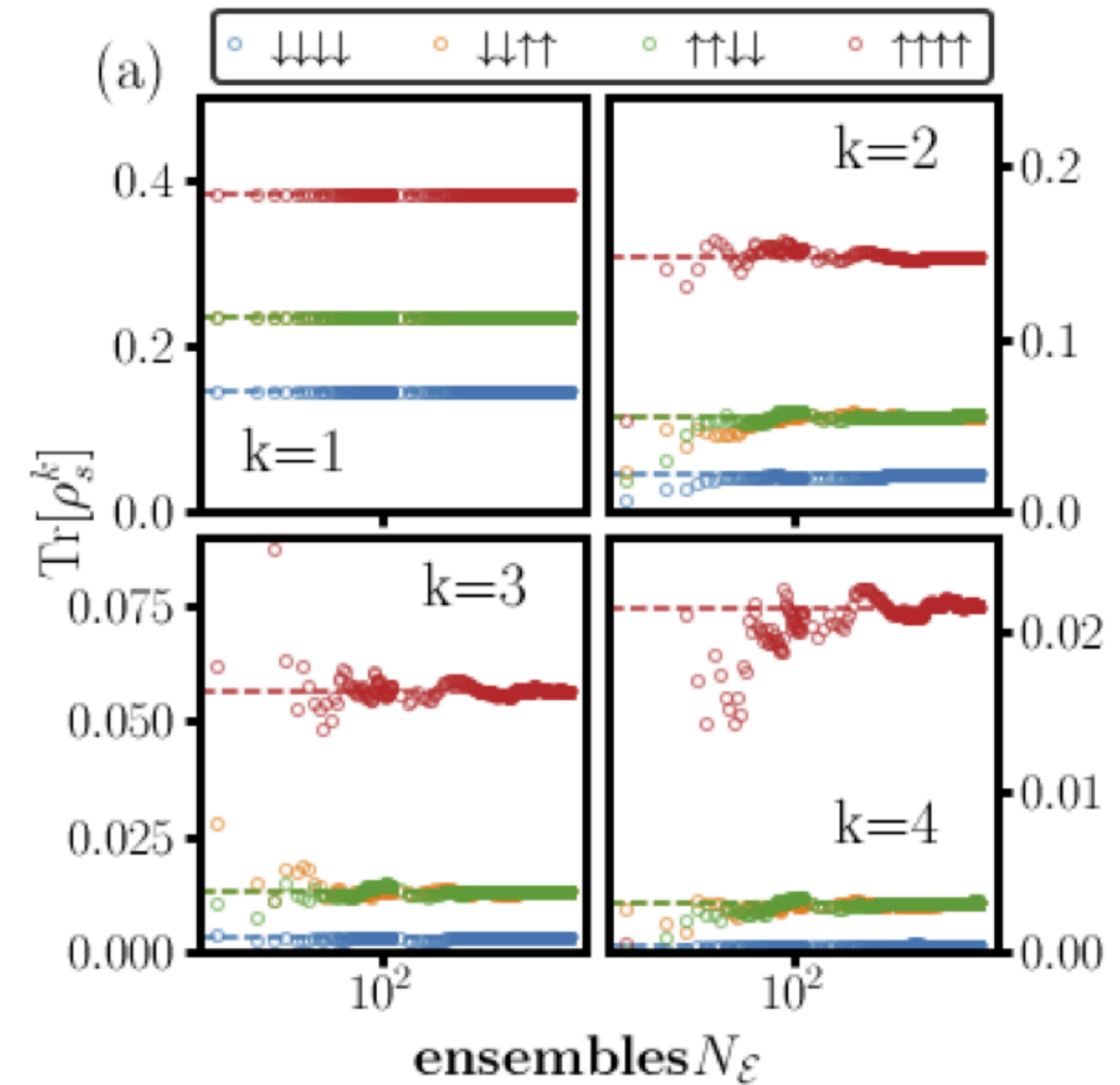
Symmetry-conscious Random Measurement

Lattice Gauge Theories



$$\rho_A = \begin{pmatrix} \square & \dots & & \\ \vdots & \square & & \\ & \dots & \ddots & \\ & & & \ddots \end{pmatrix}$$

k-designs give k-Renyi entropies



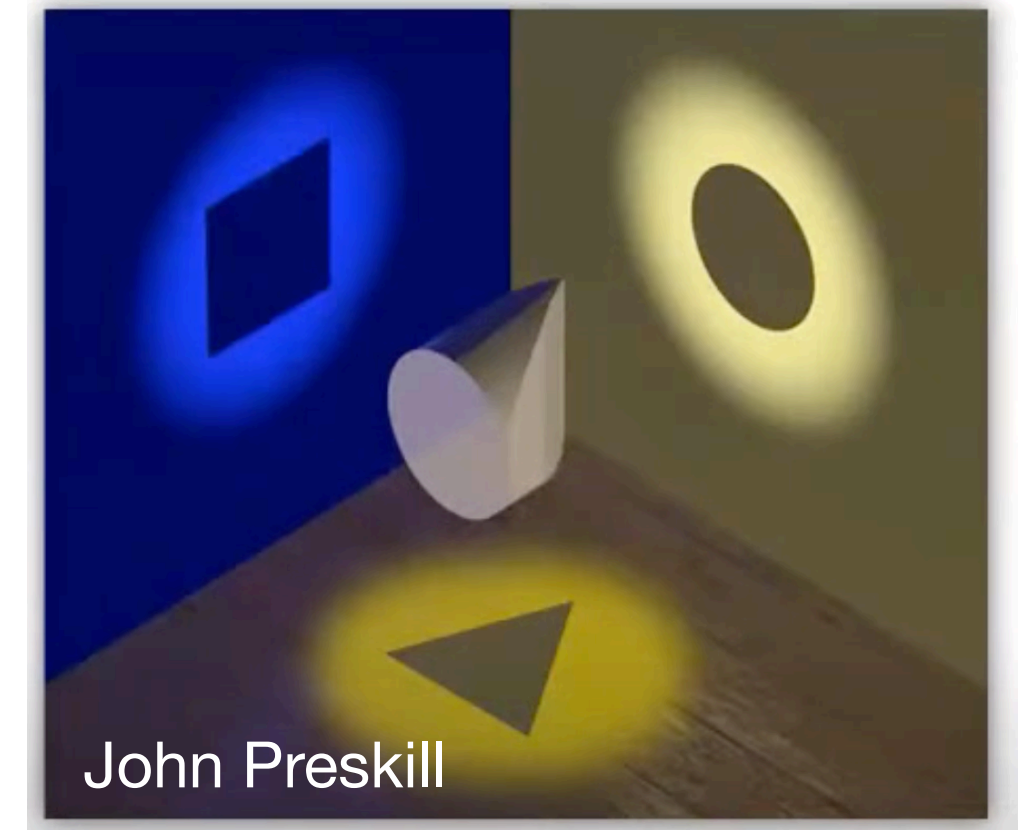
Symmetry-conscious Random Measurement

Lattice Gauge Theories

Symmetry-conscious Random Measurement

Lattice Gauge Theories

- **Shadows**

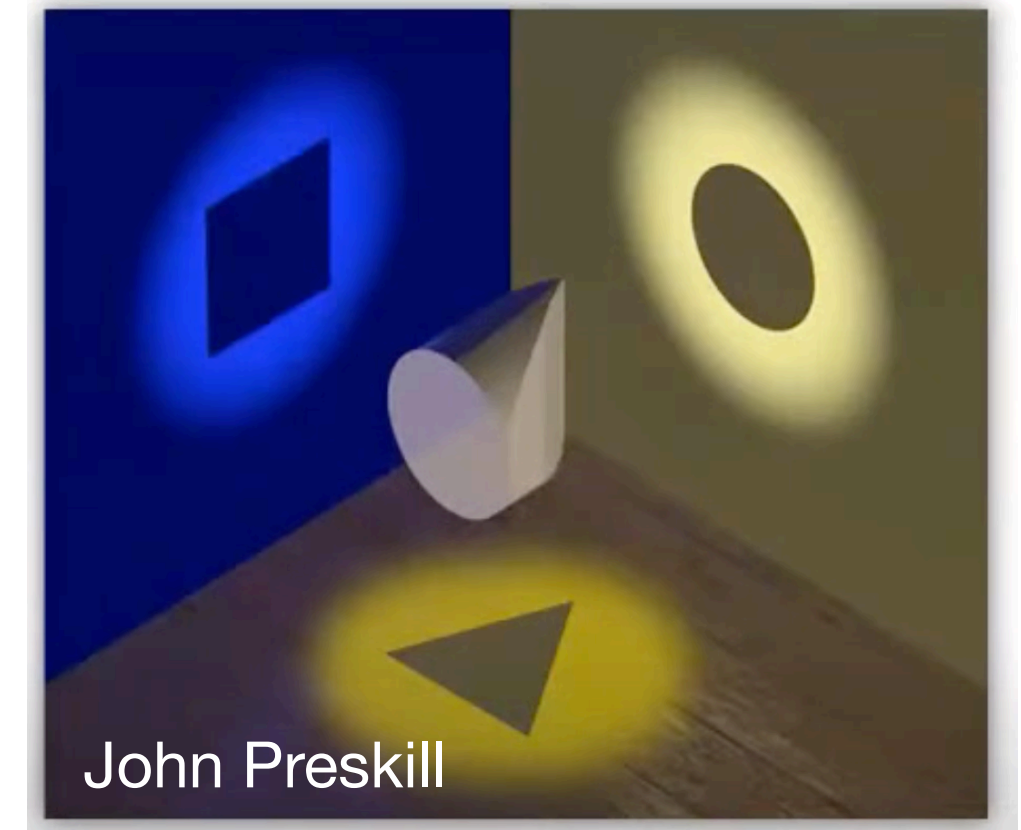
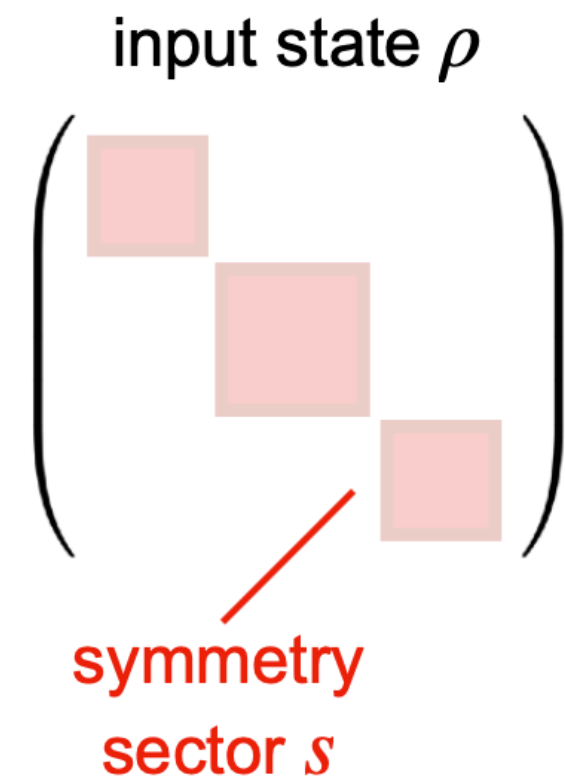


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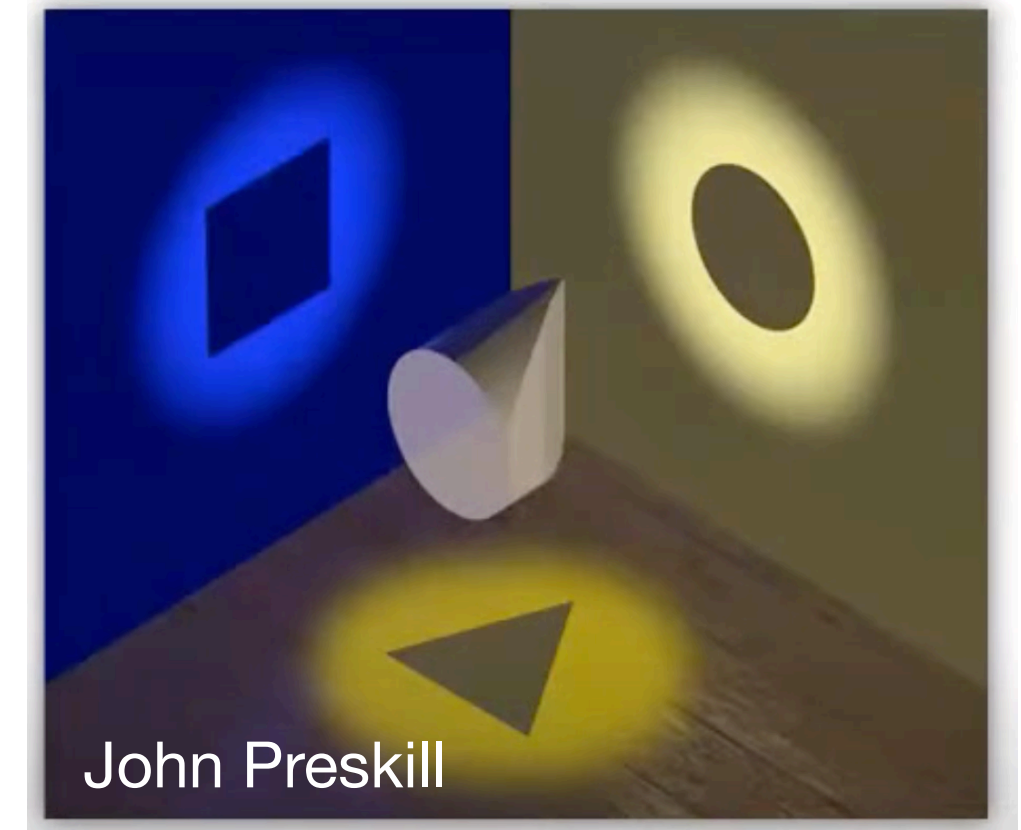
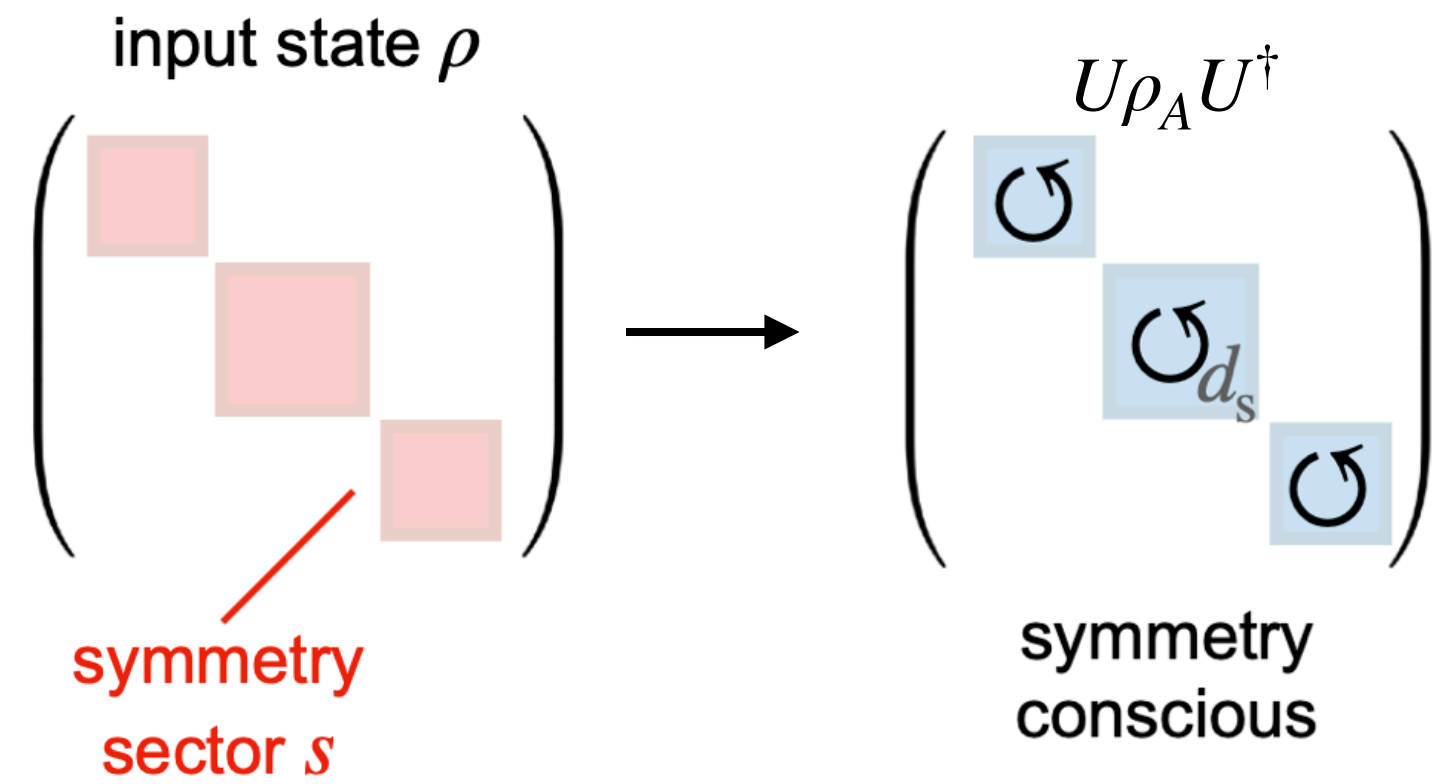


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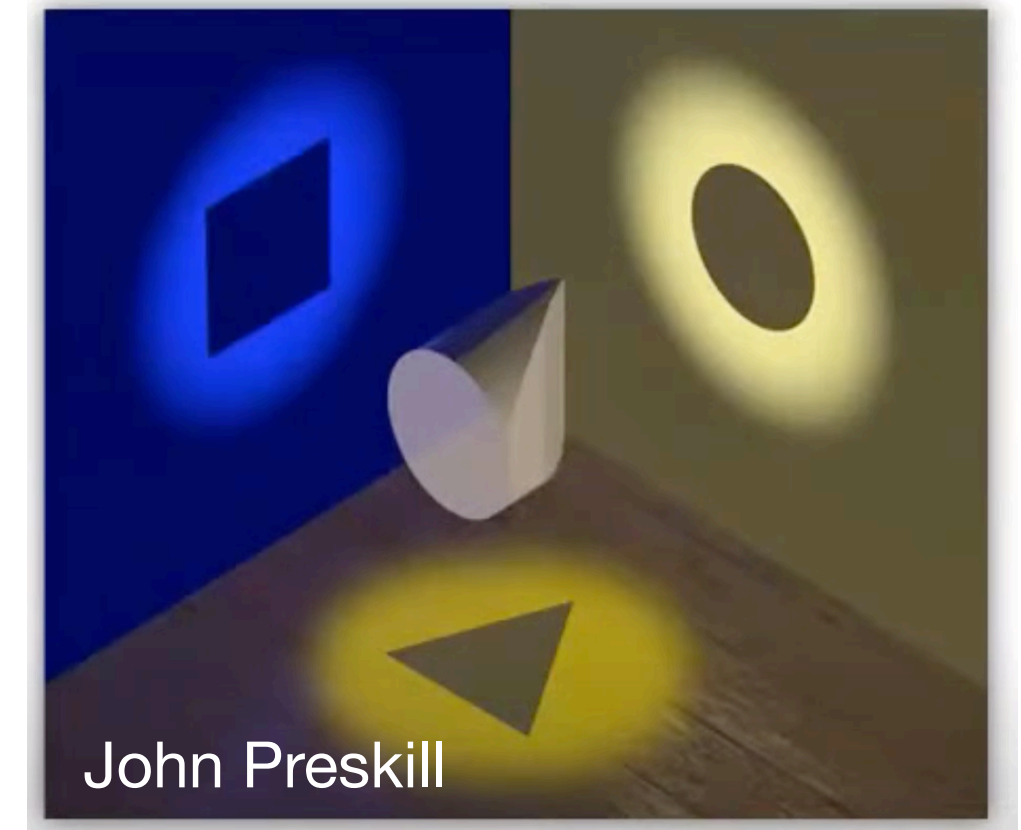
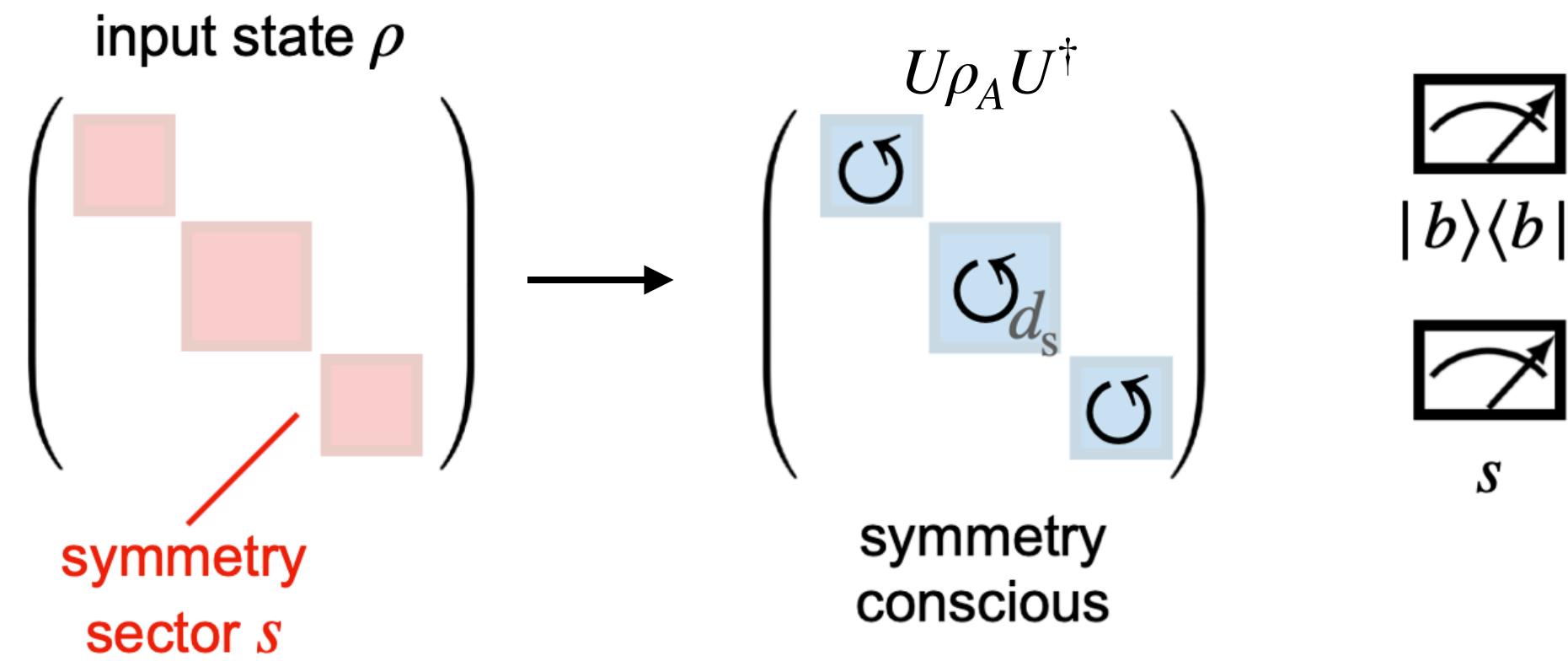


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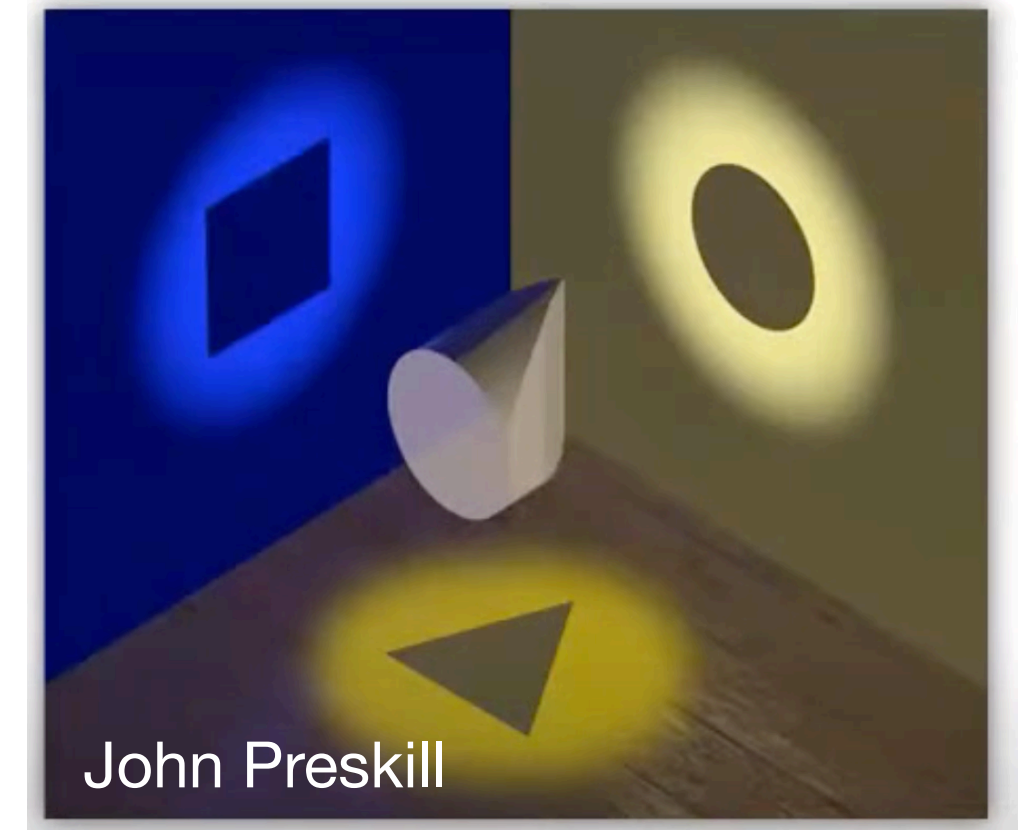
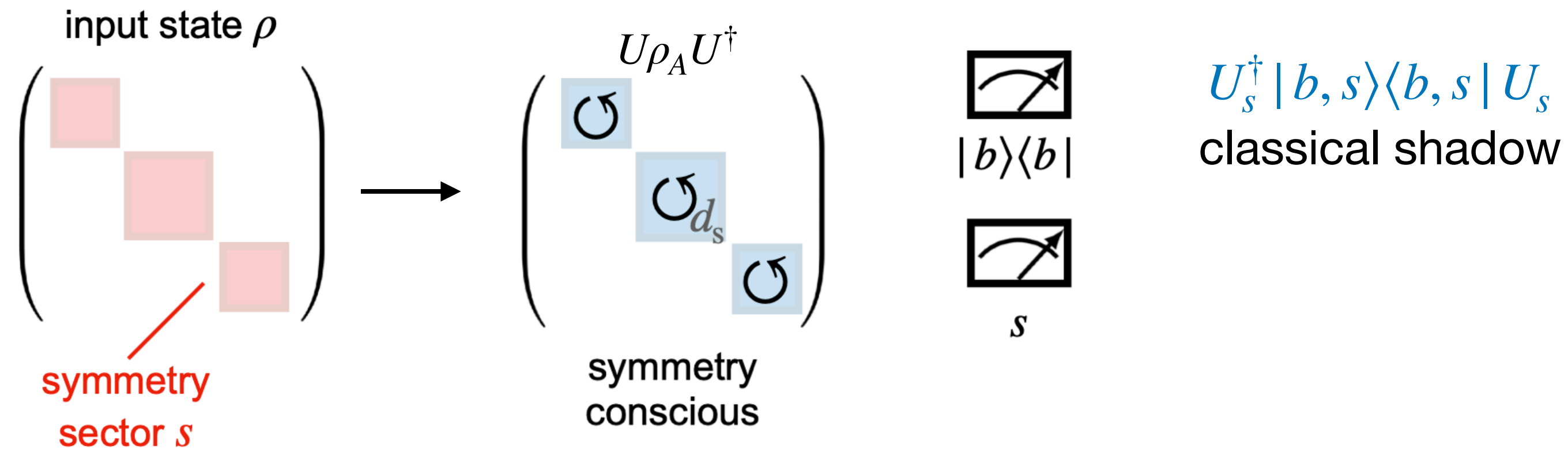
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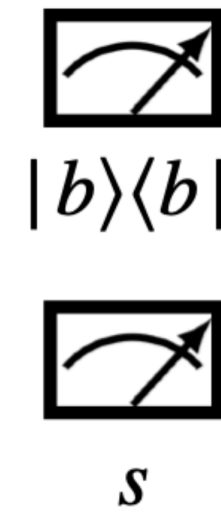
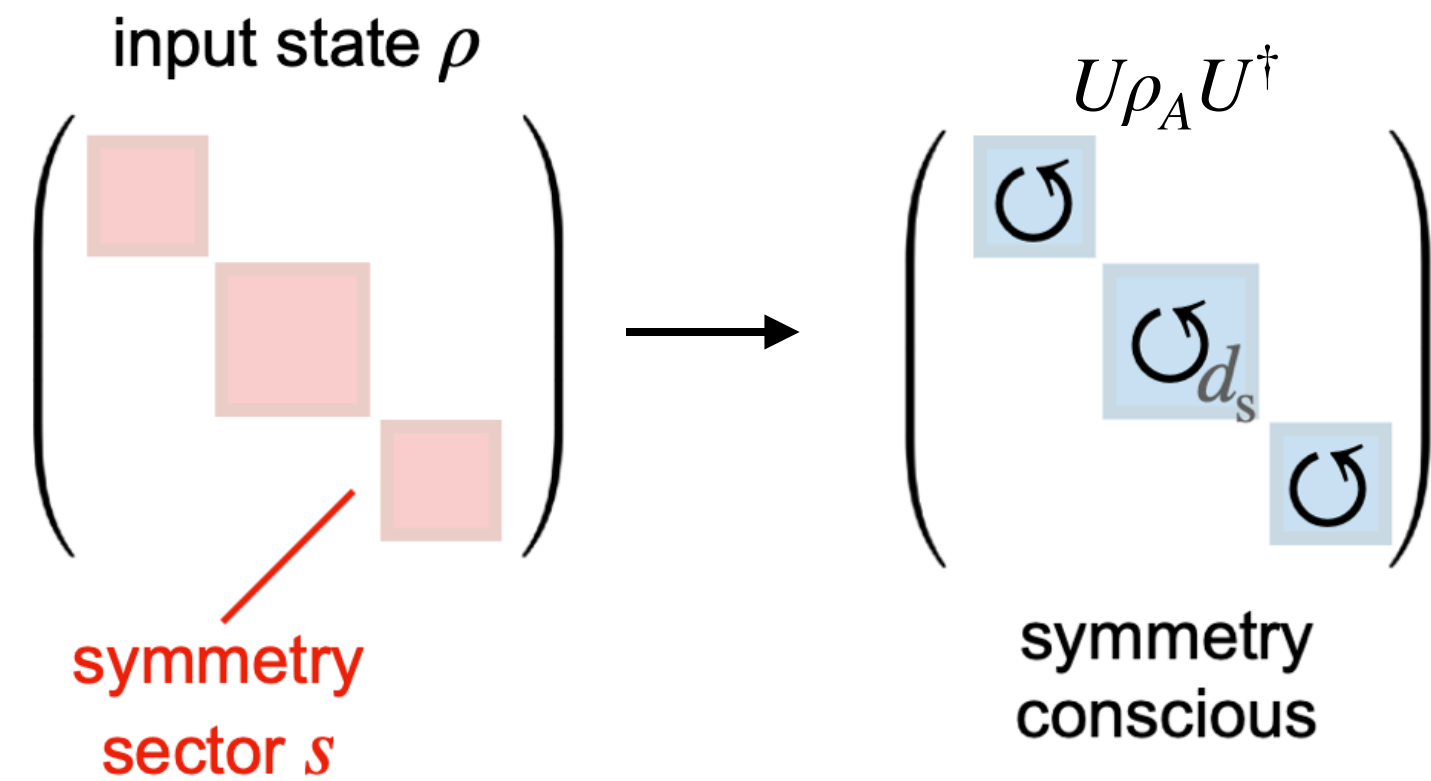
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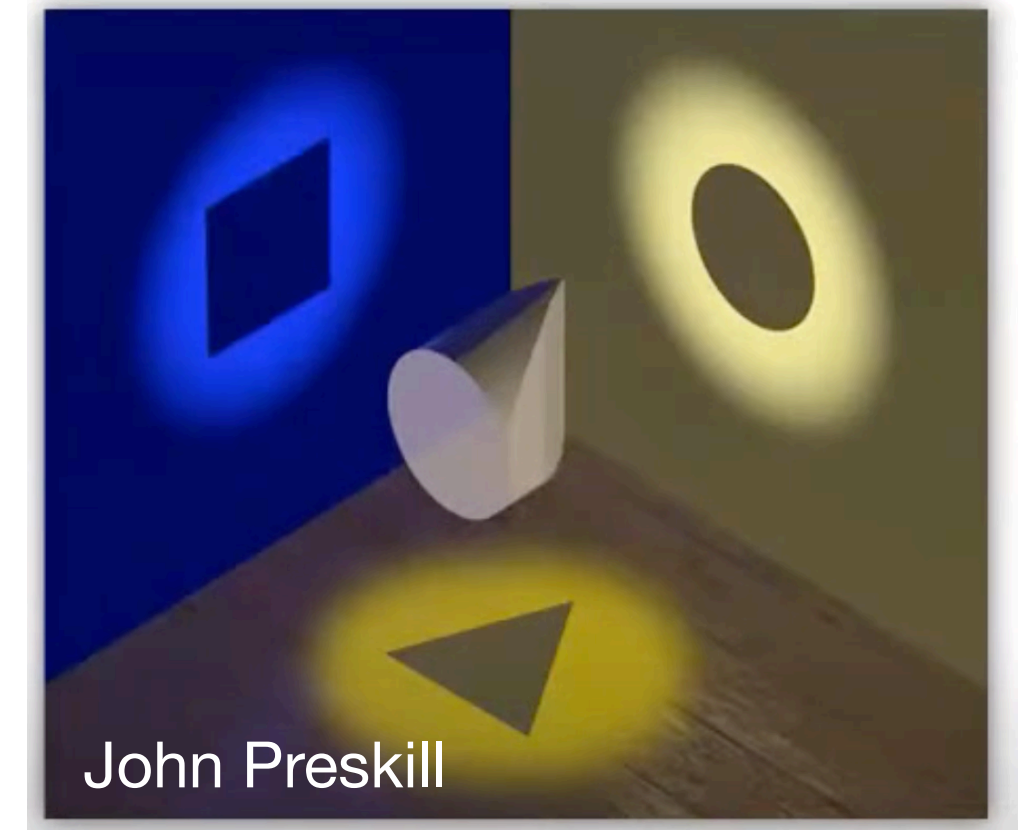


$U_s^\dagger |b, s\rangle\langle b, s| U_s$
classical shadow

$$\bar{\rho}_s = \mathbb{E}[\mathcal{M}^{-1}(U_s^\dagger |b, s\rangle\langle b, s| U_s)]$$

$$\bar{\rho}_s = \rho_s / \text{Tr} \rho_s$$

$$\mathcal{M}^{-1}(X) = (d_s + 1)X - \text{Tr}_s[S] \mathbb{1}_s$$



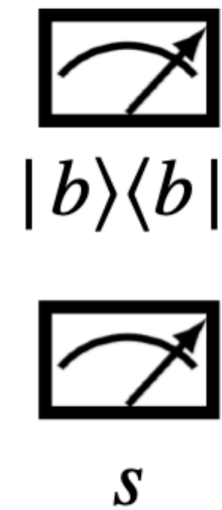
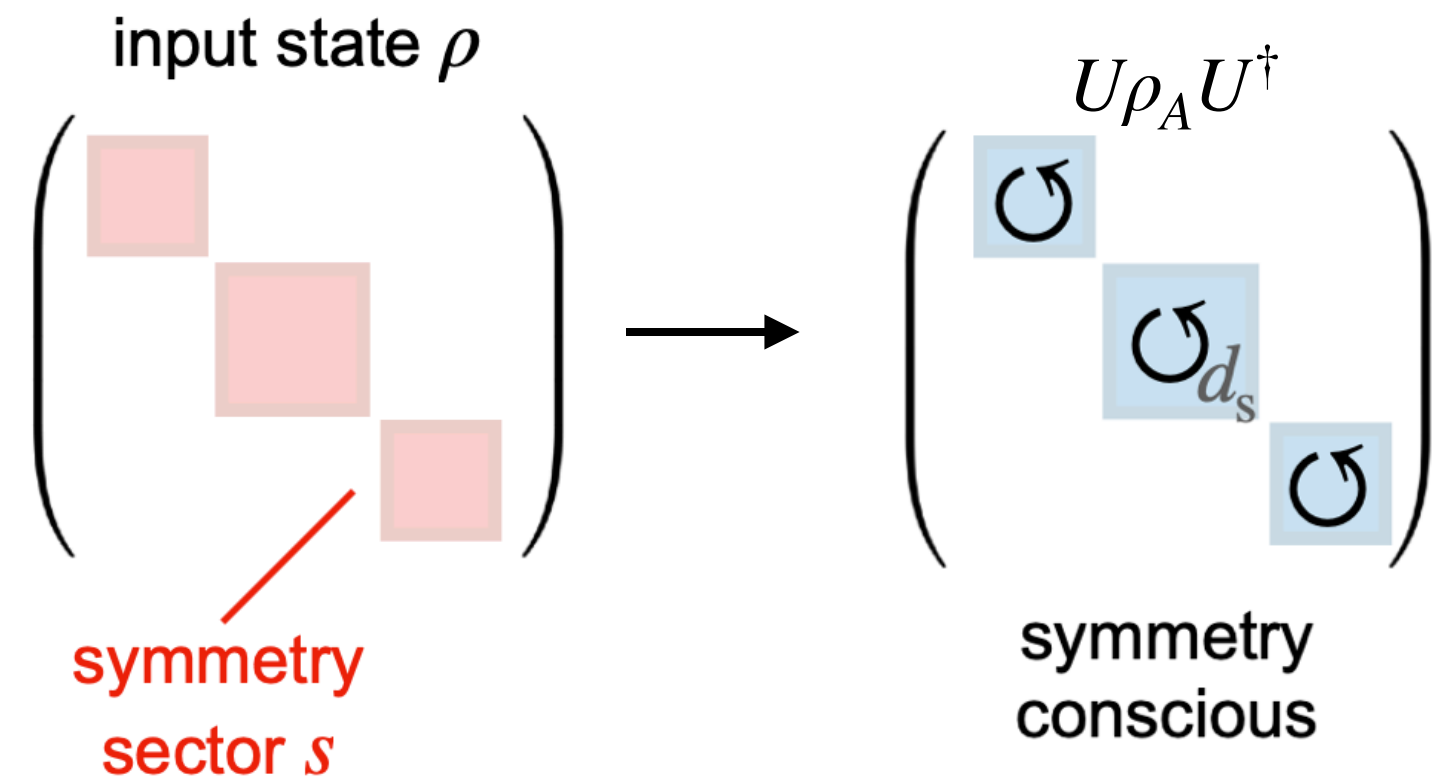
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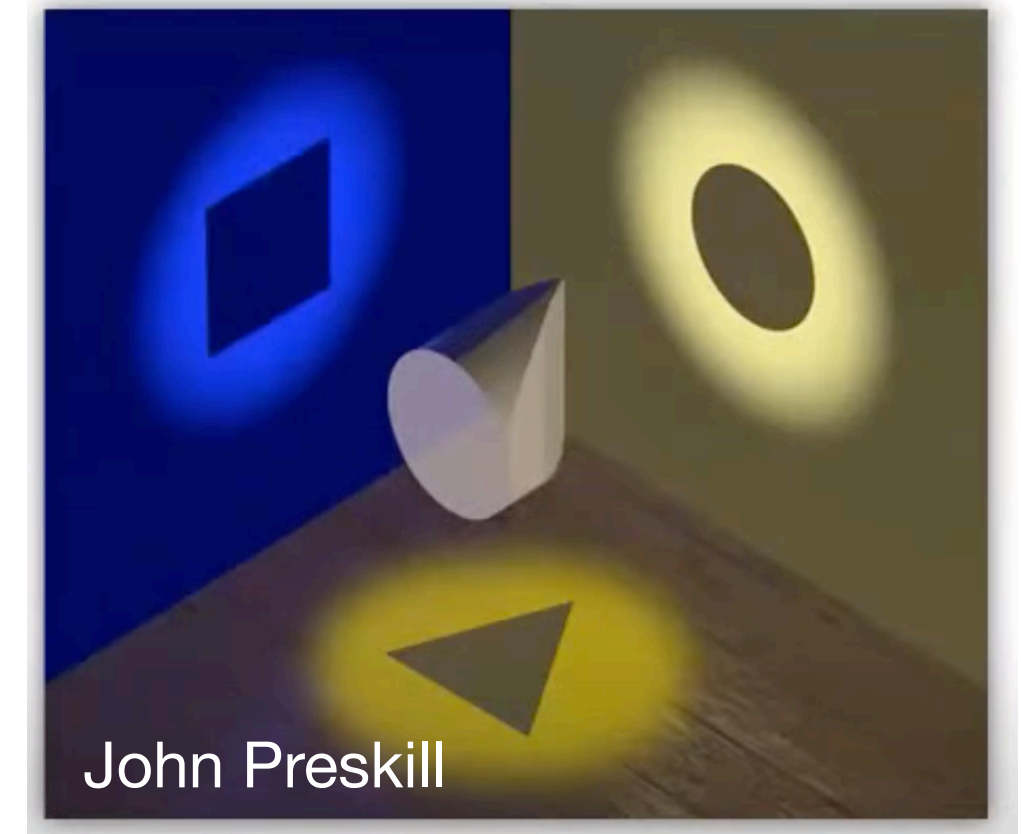


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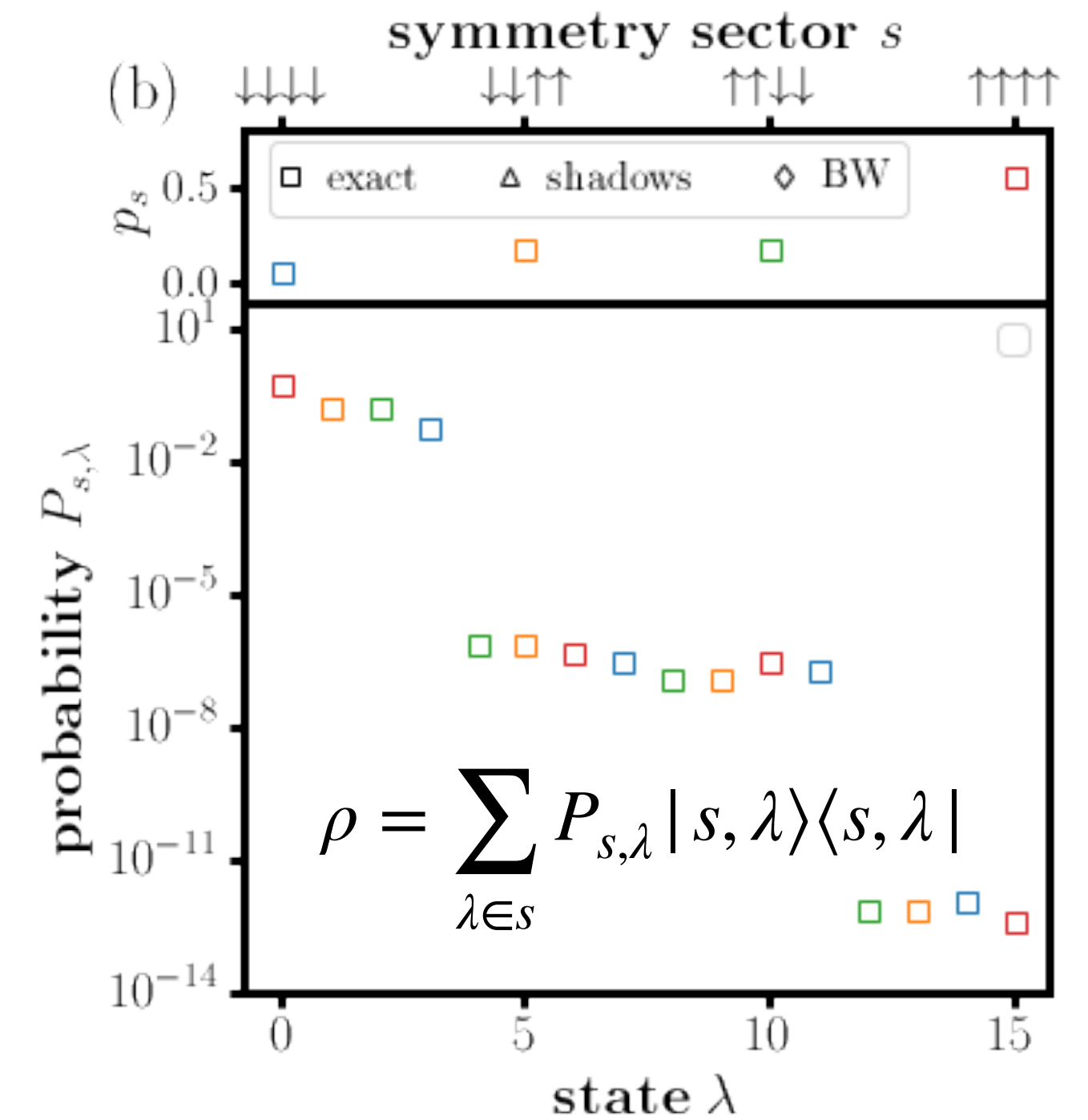
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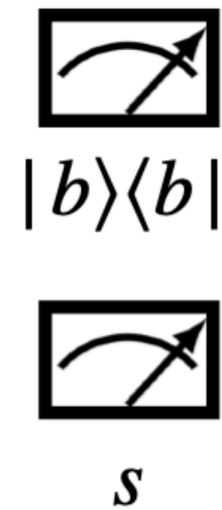
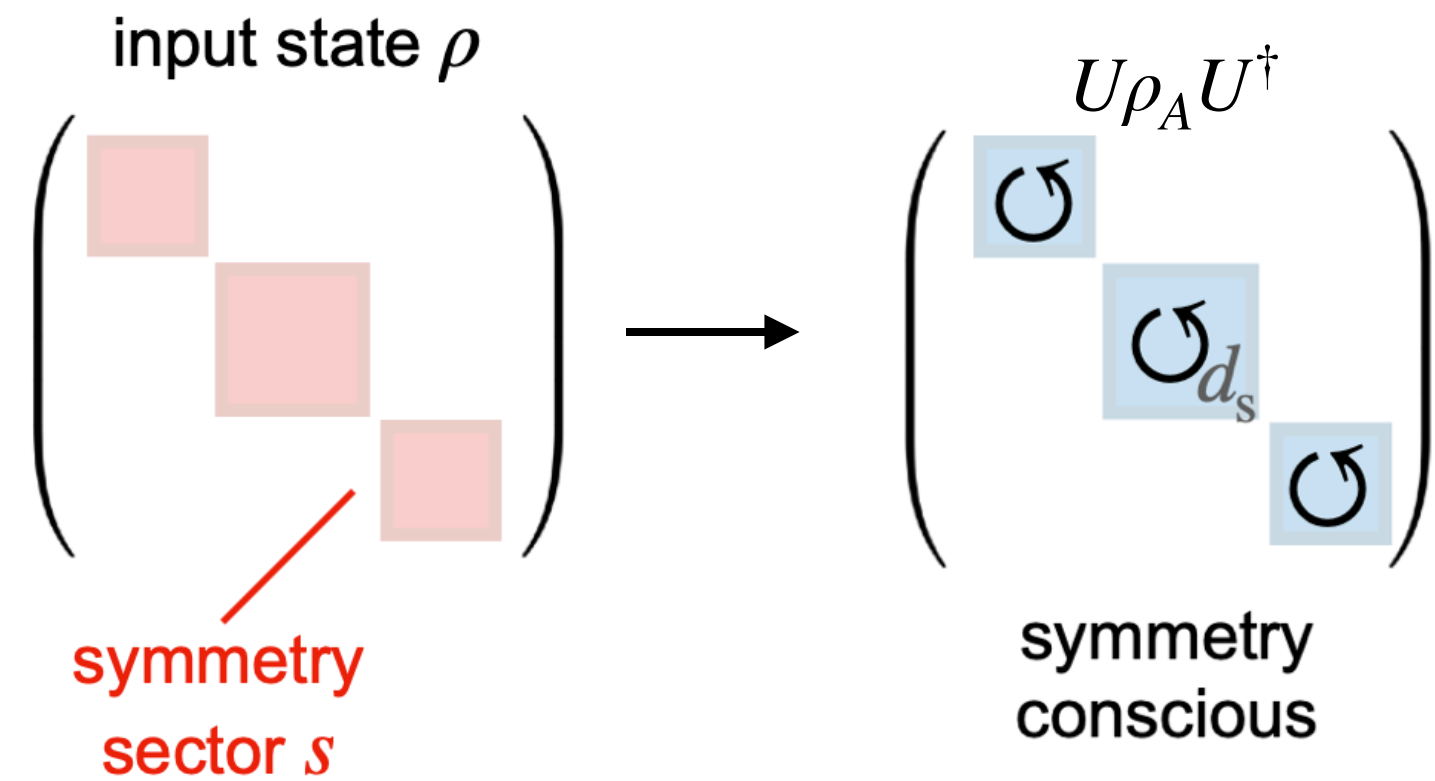
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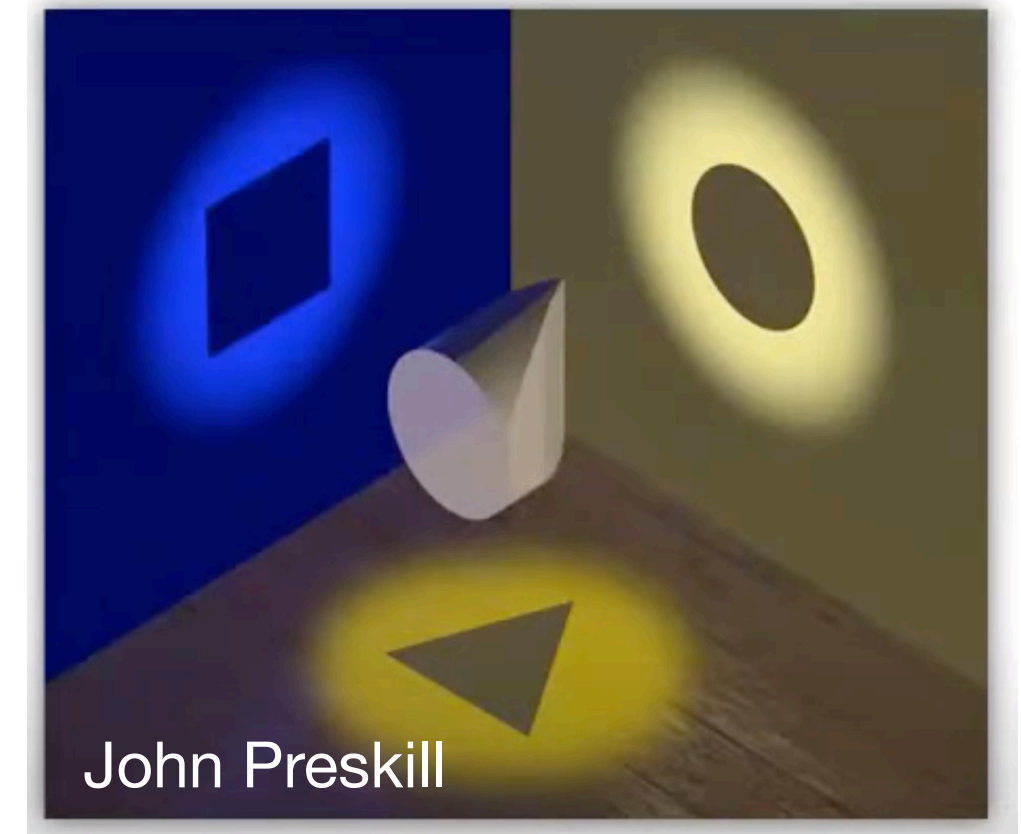


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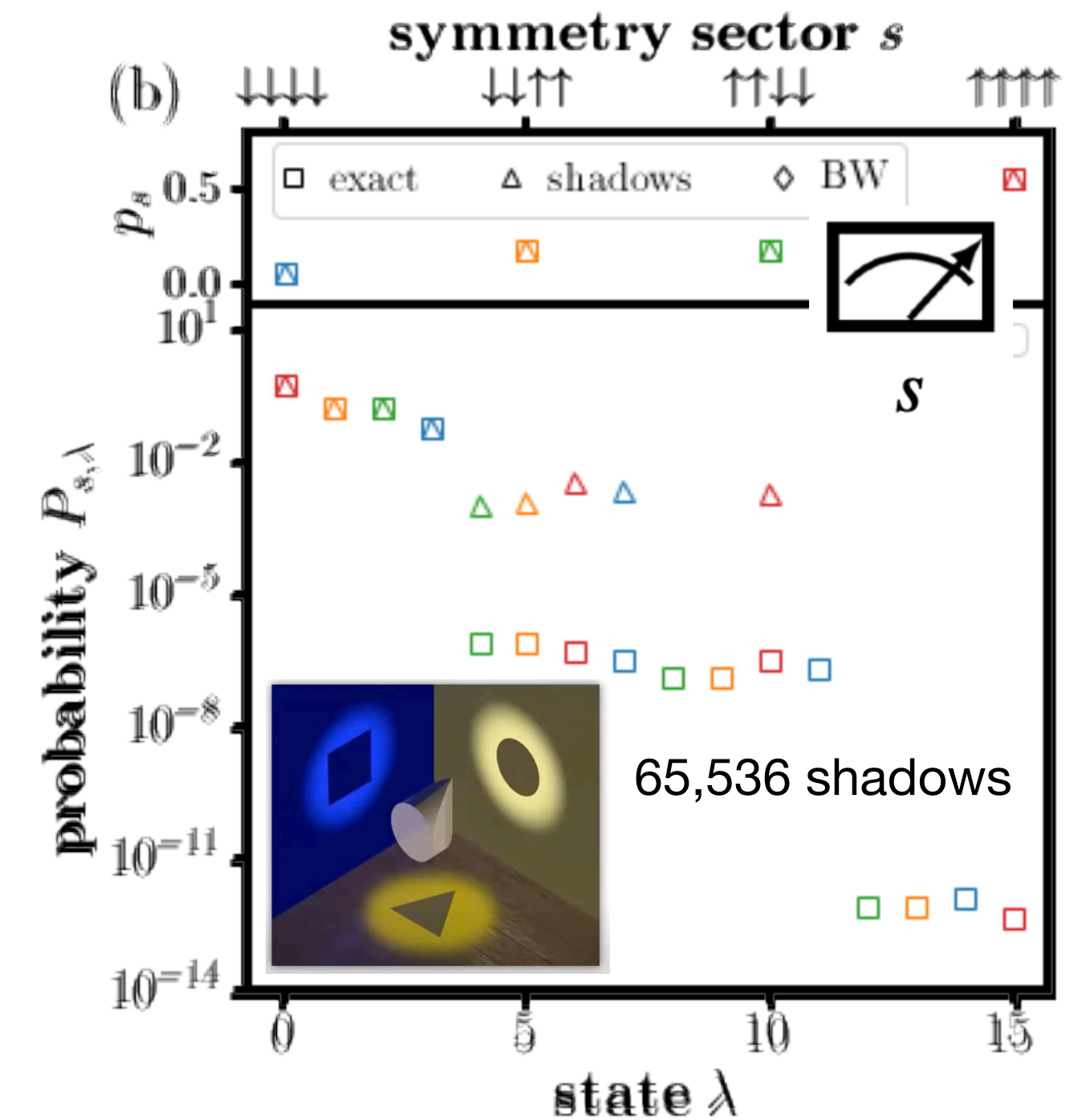
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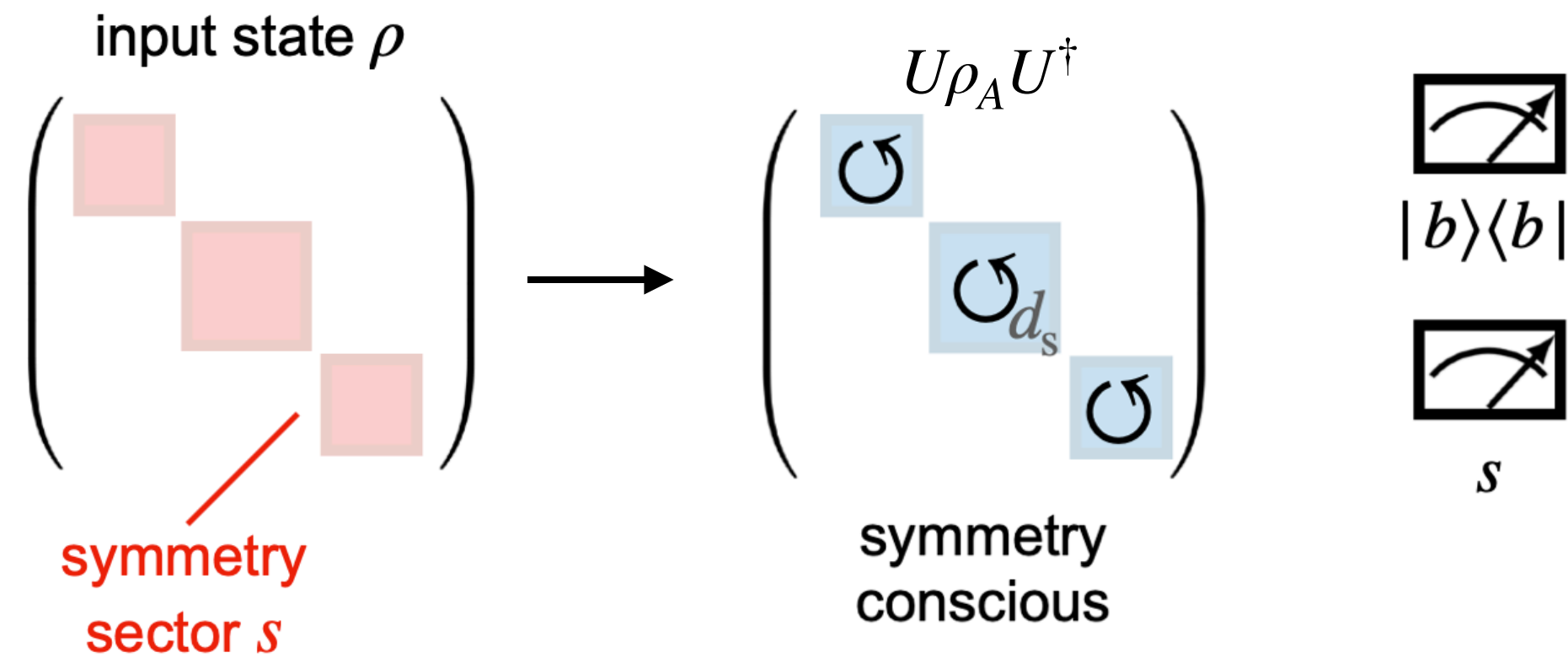
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Symmetry-conscious Random Measurement

Lattice Gauge Theories

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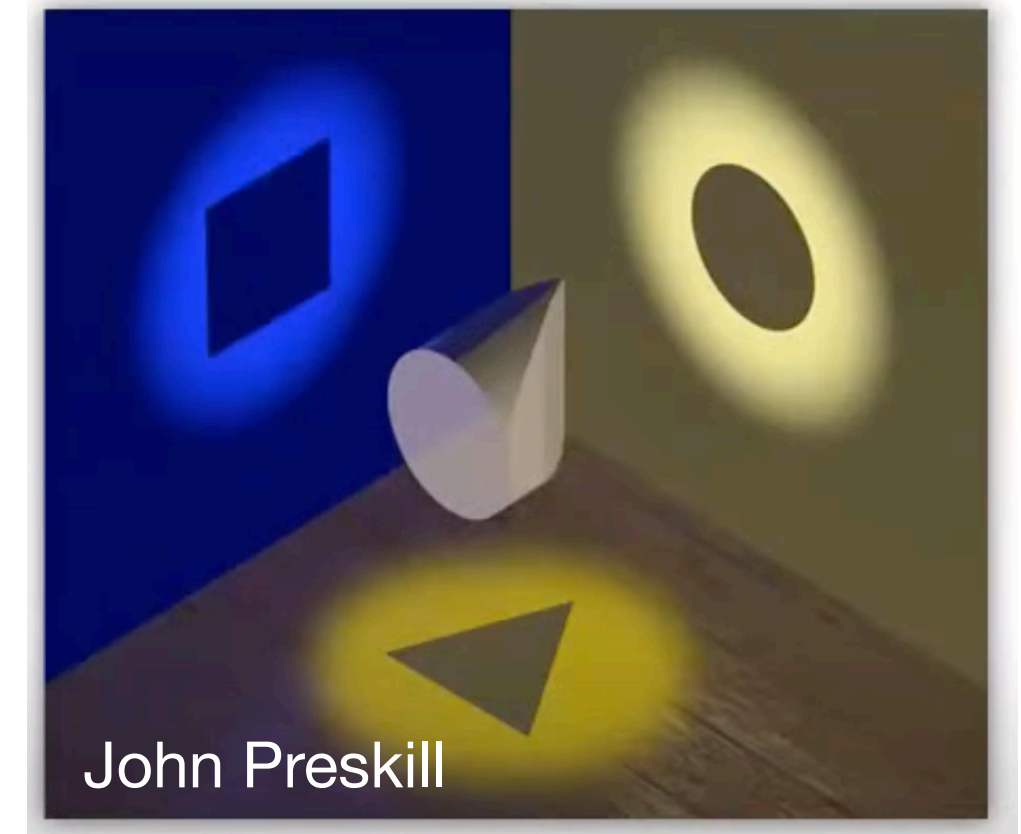
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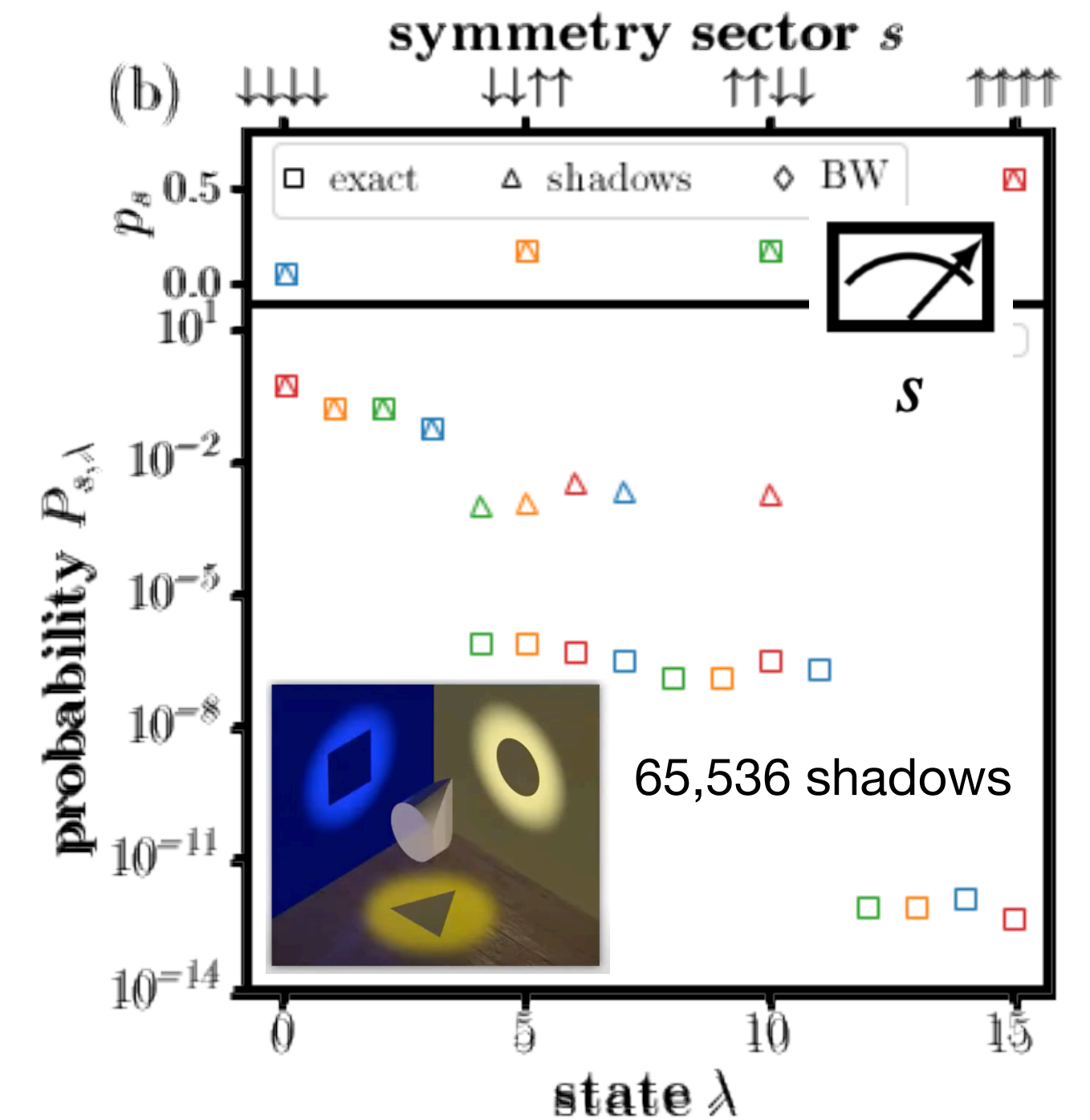
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- **Entanglement Hamiltonian Tomography: Bisognano Wichmann**

Bisognano, Wichmann, J Math Phys 16, 985 (1975) & 17, 303 (1976)
 Dalmonte, Vermersch, Zoller, Nature 14 2018 827-831



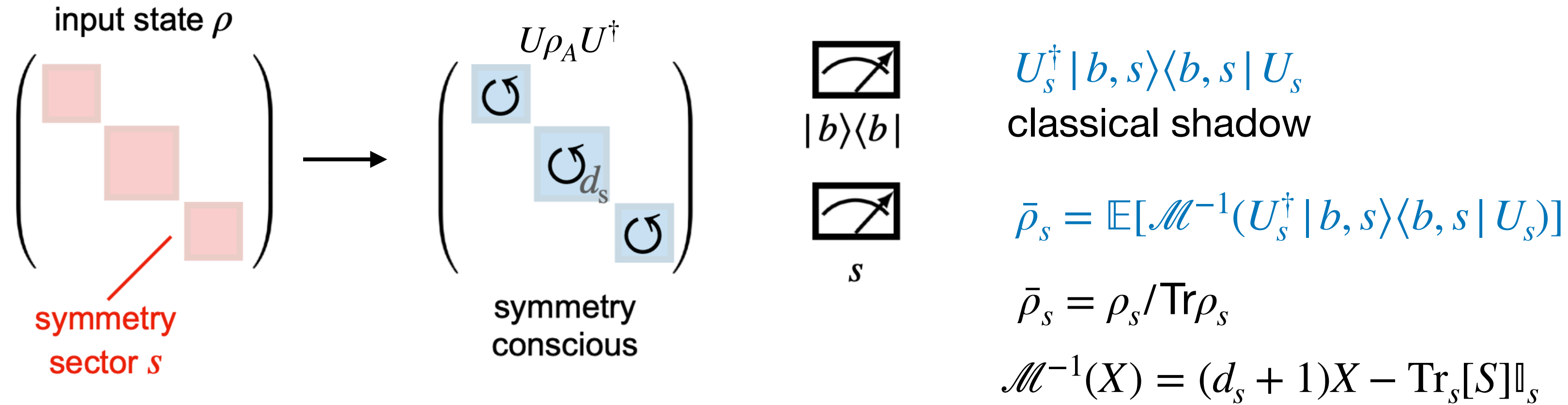
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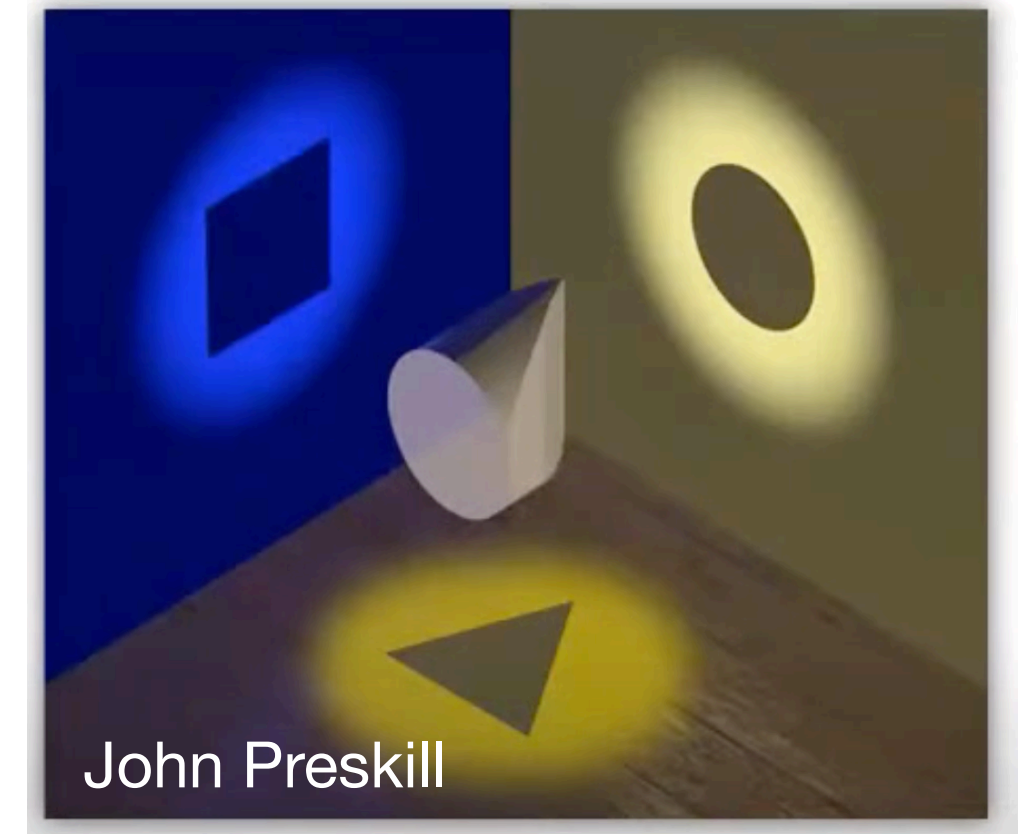


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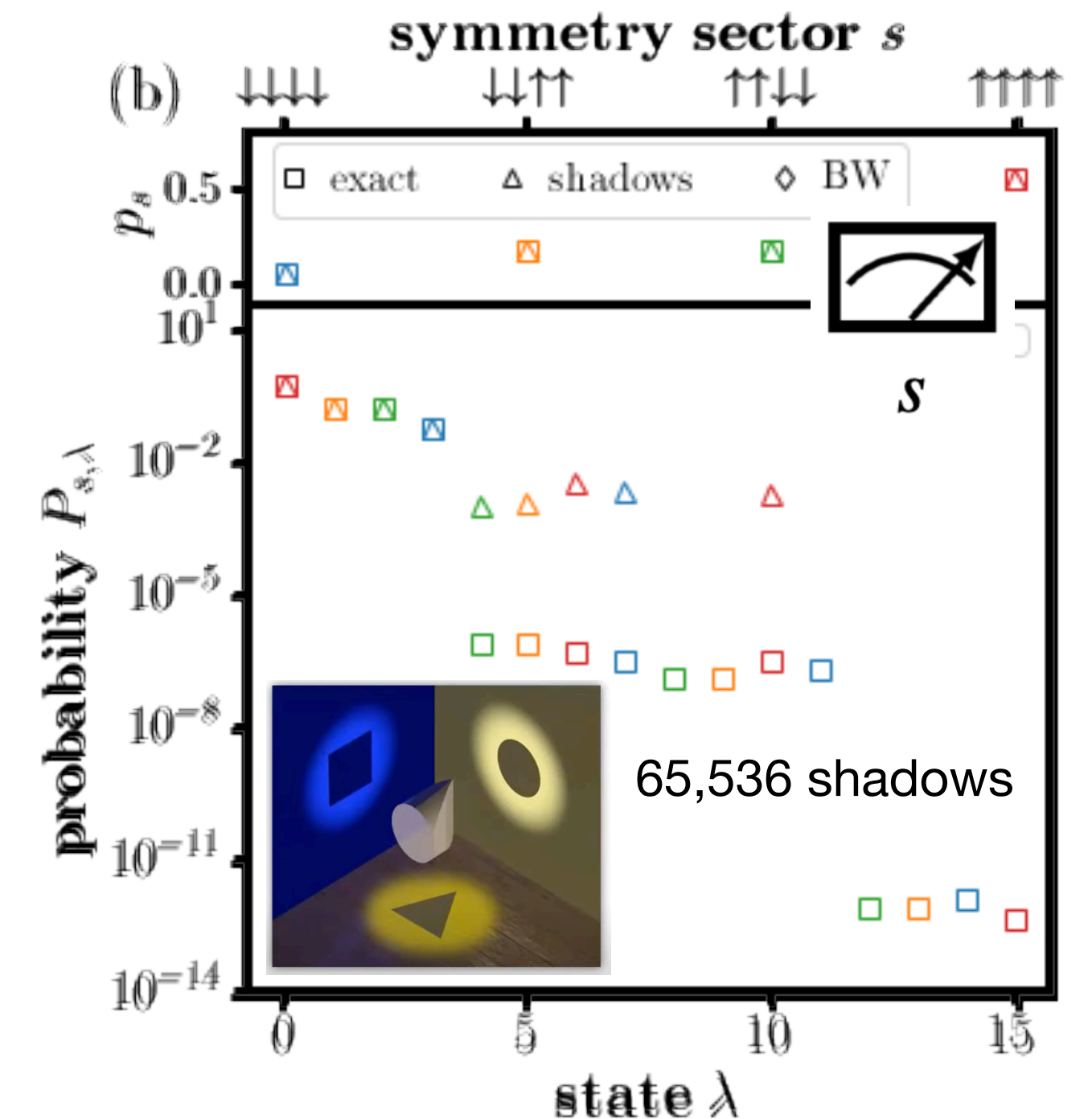
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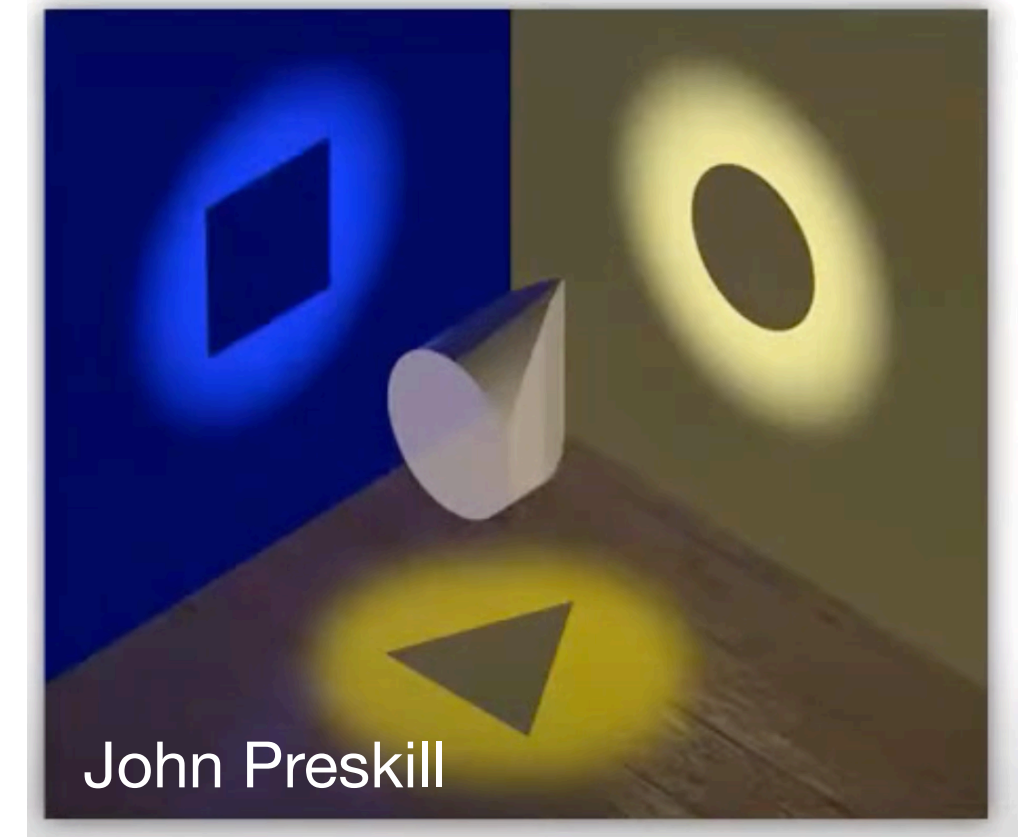
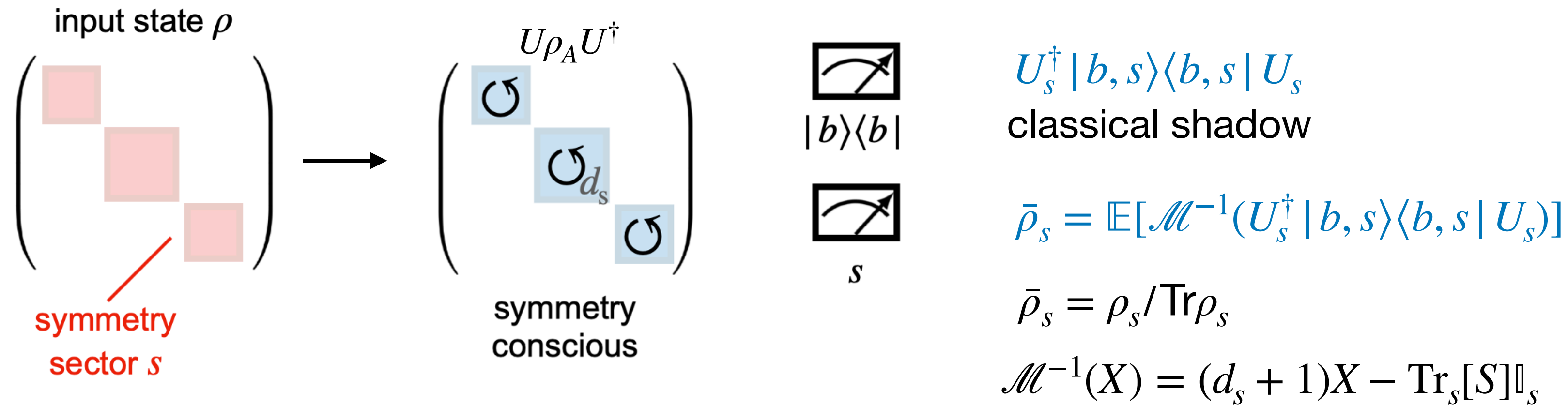


Jake Bringewatt, Jon Kunjummen, NM, arXiv:2303.15519

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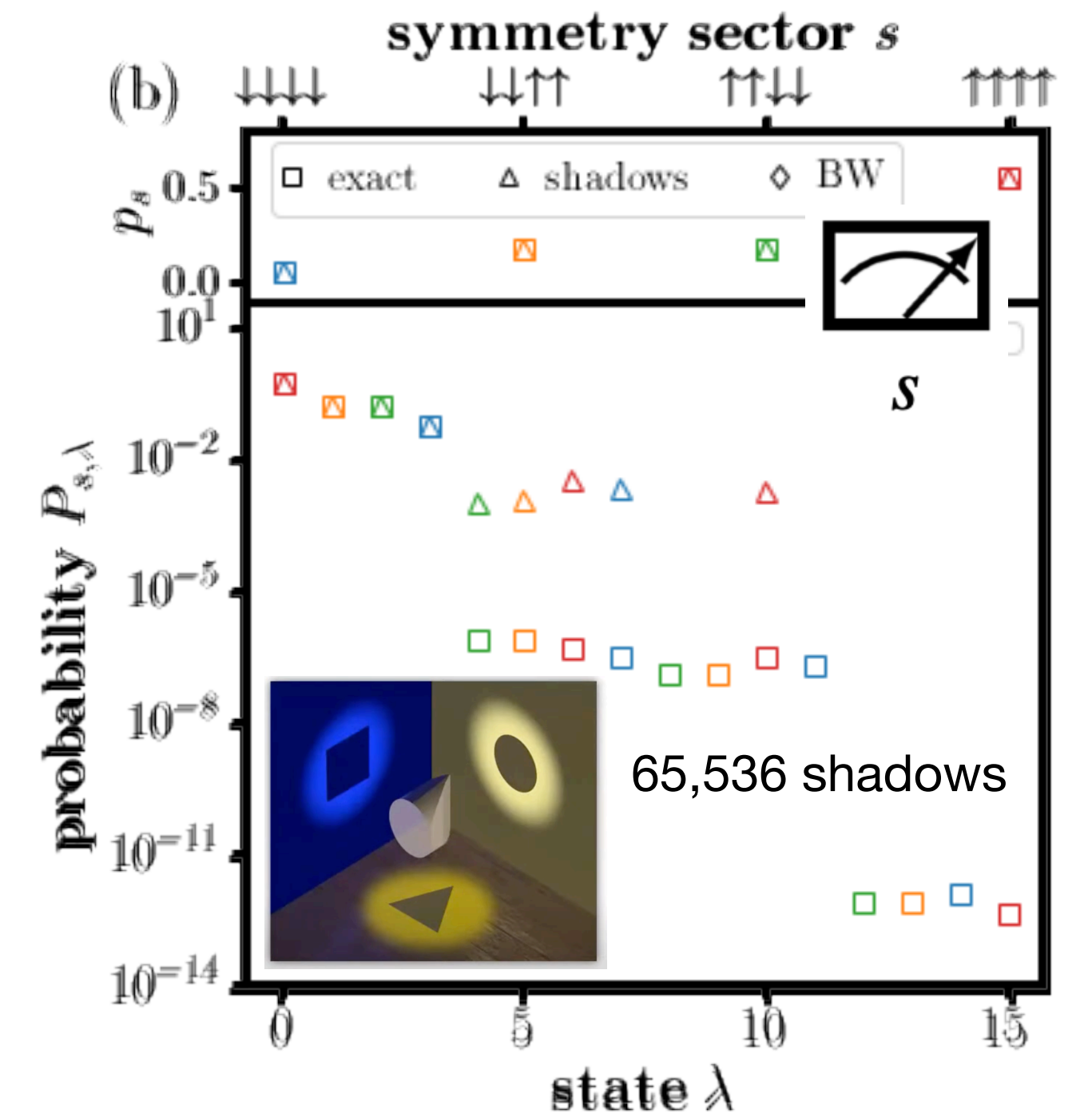
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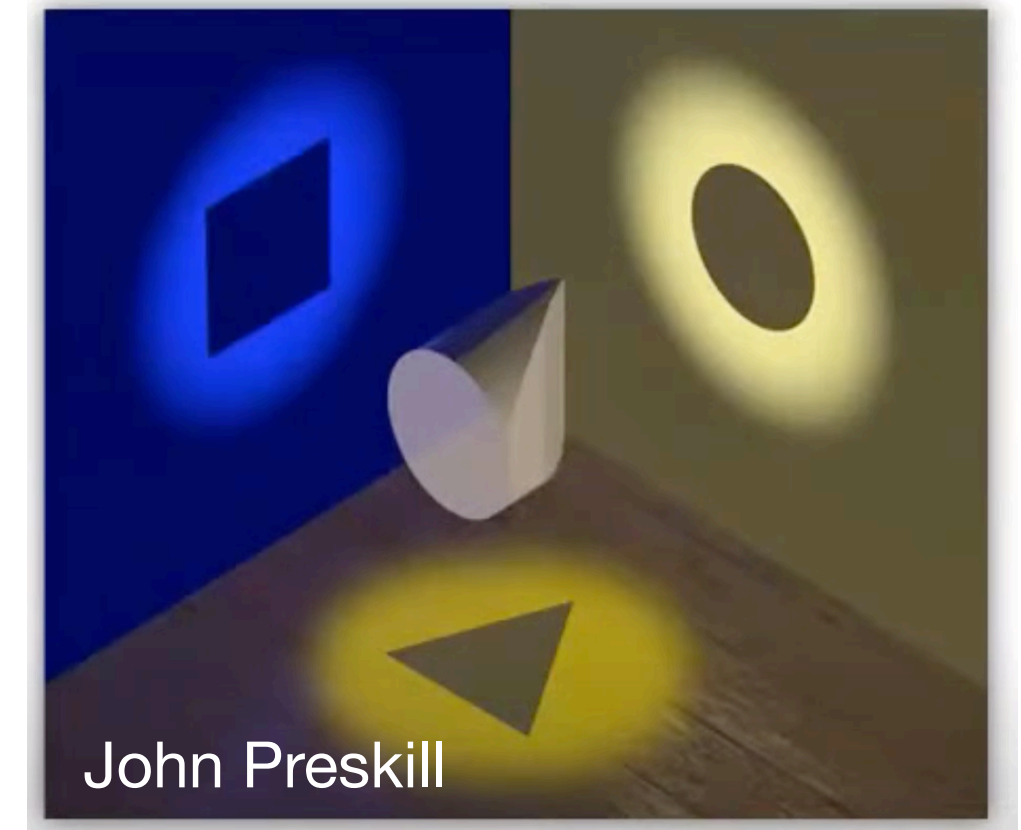
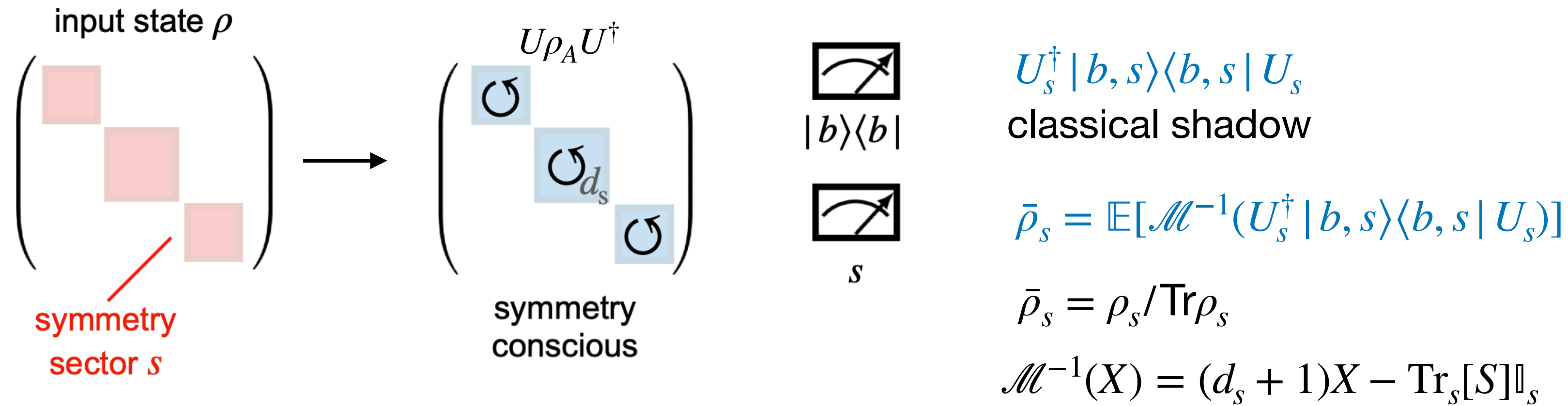


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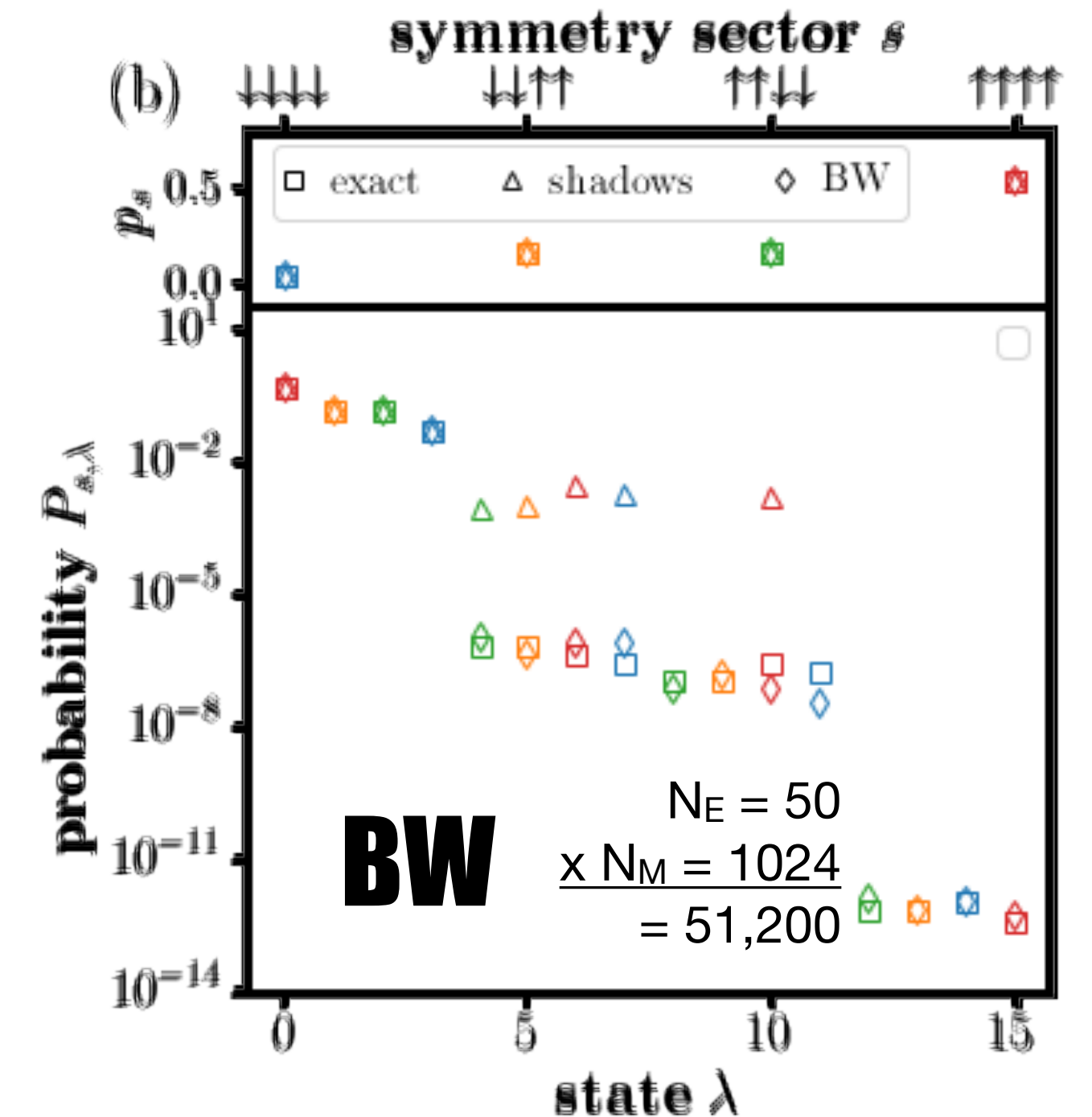
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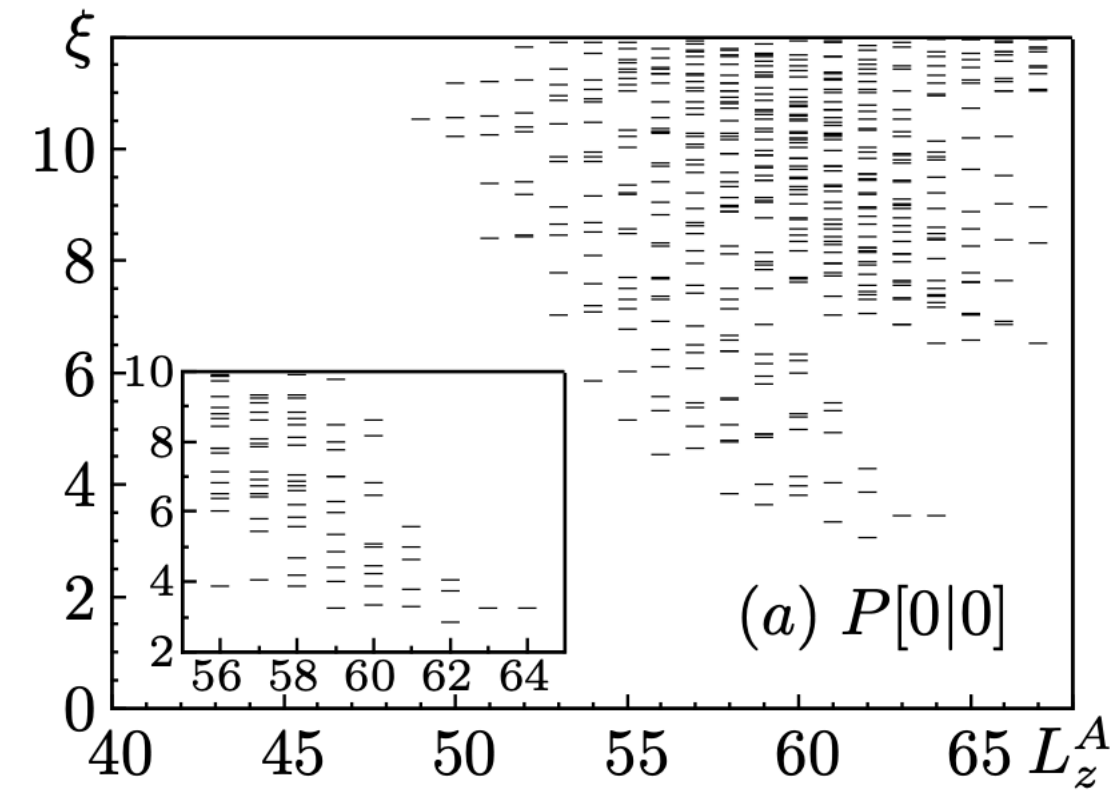
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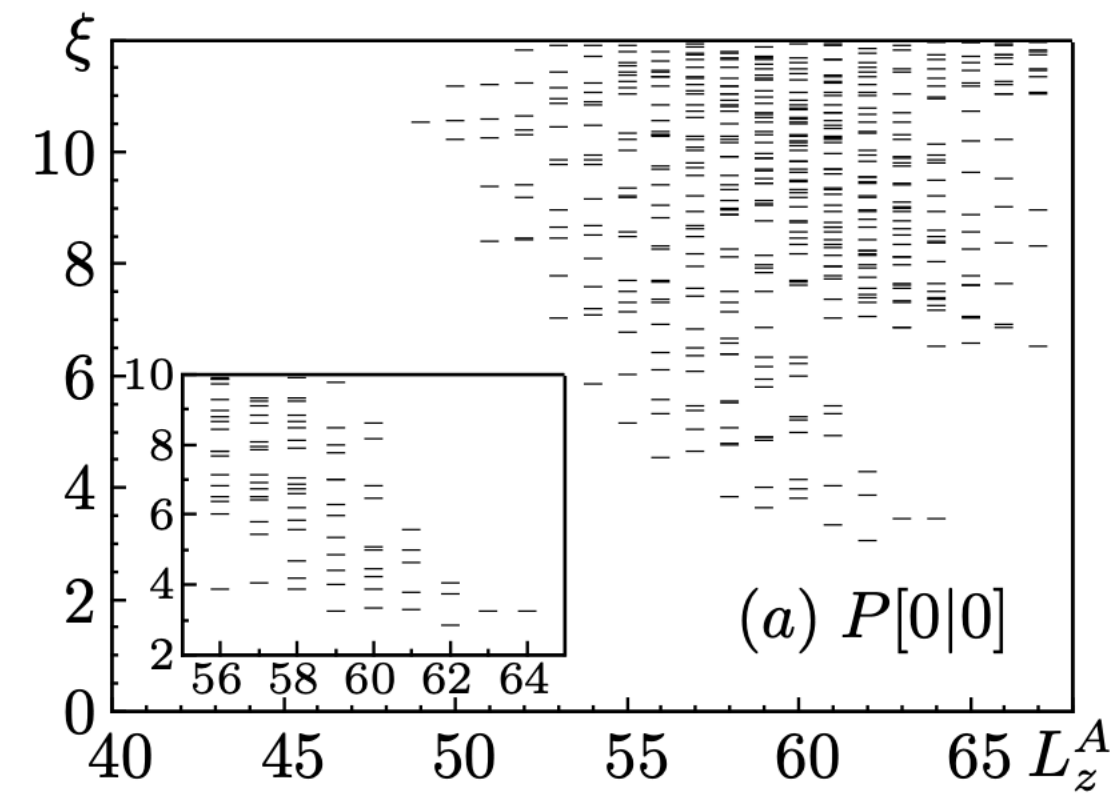
Detecting Topological Order



Li Haldane, PRL 101 01504 (2008)

Symmetry-conscious Random Measurement

Detecting Topological Order



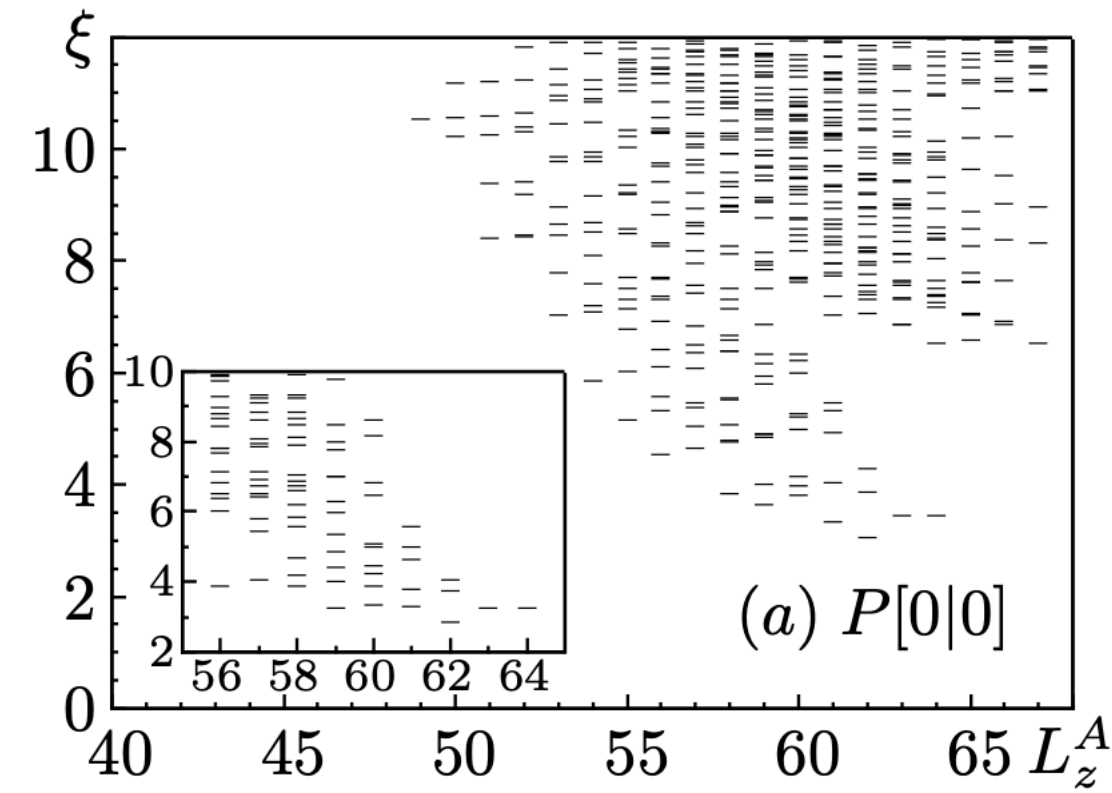
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Topological Order (Deconfined) ϵ_c Trivial (Confined) ϵ

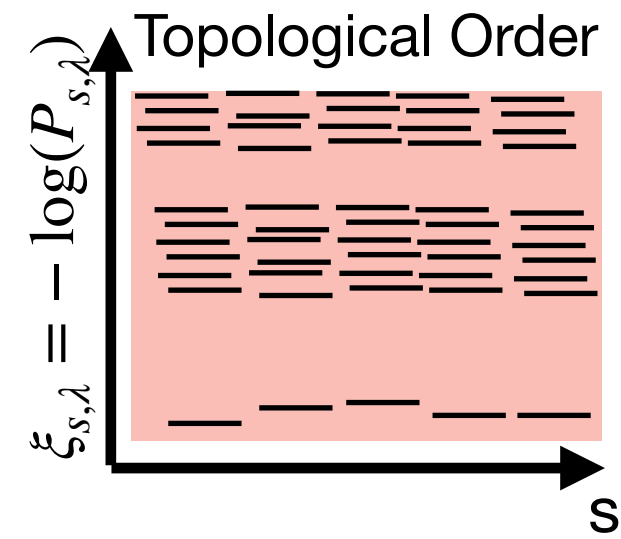
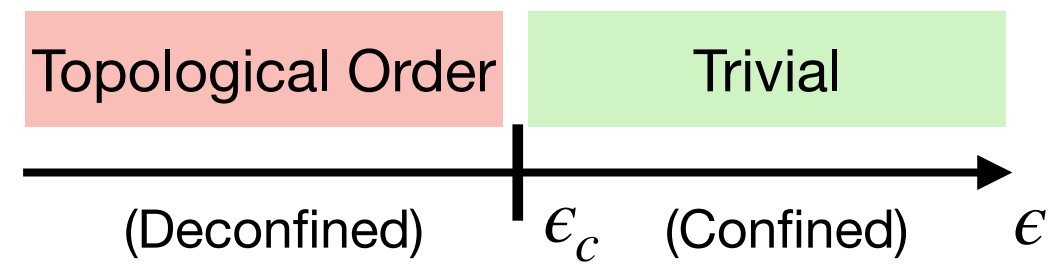
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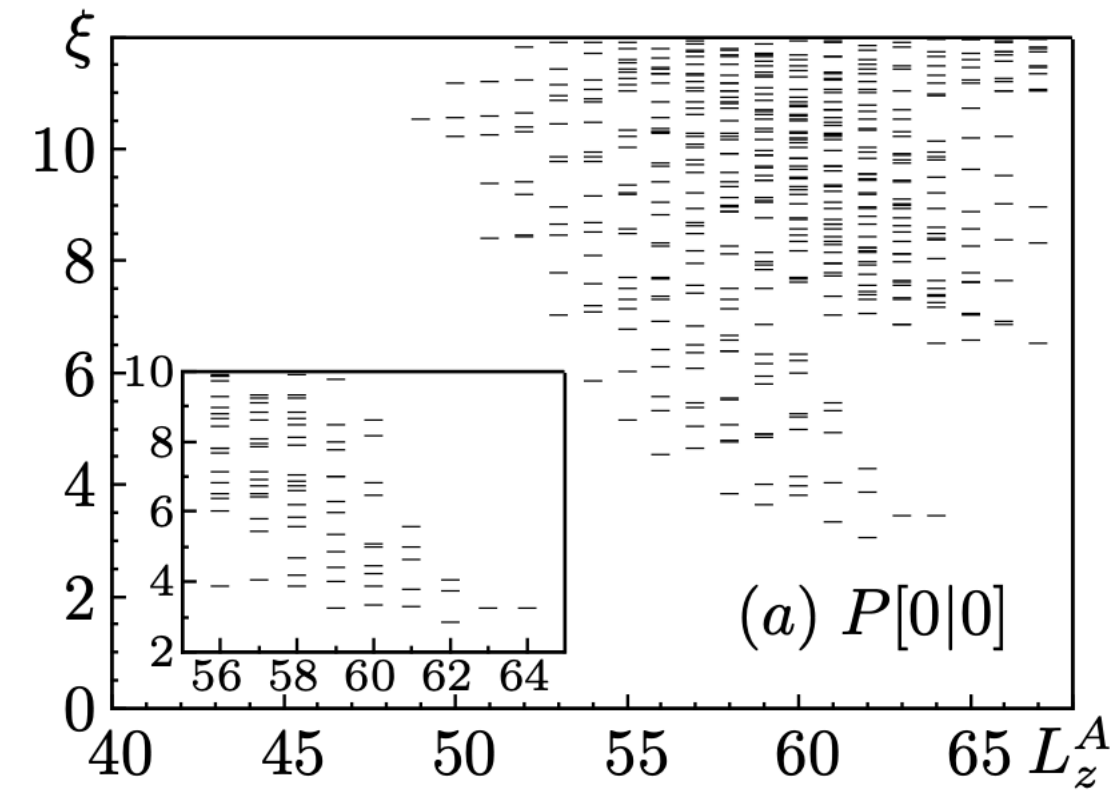
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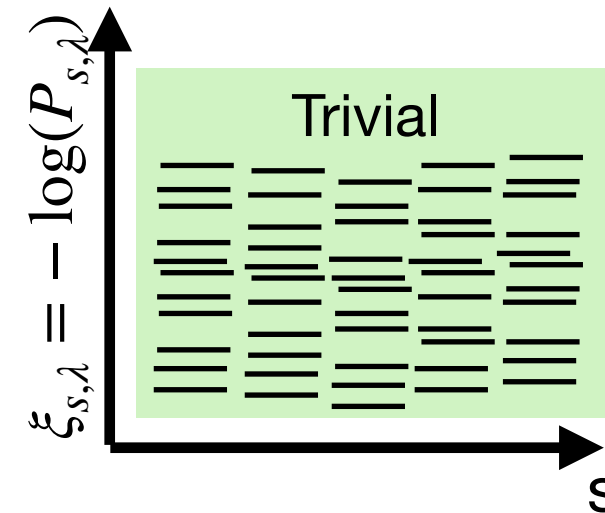
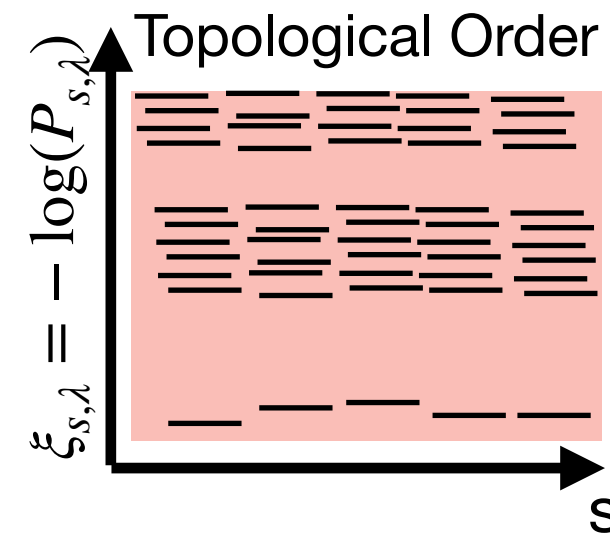
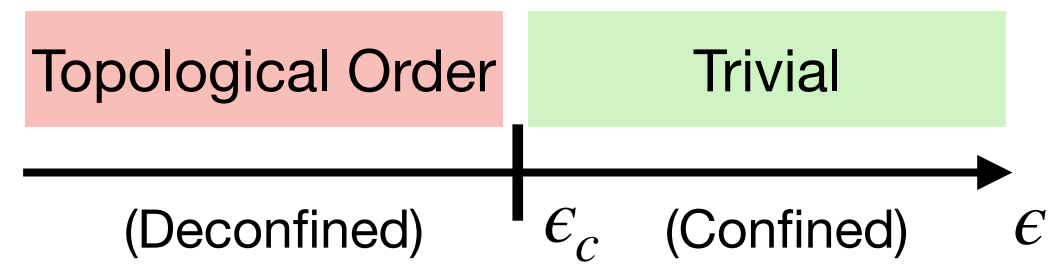
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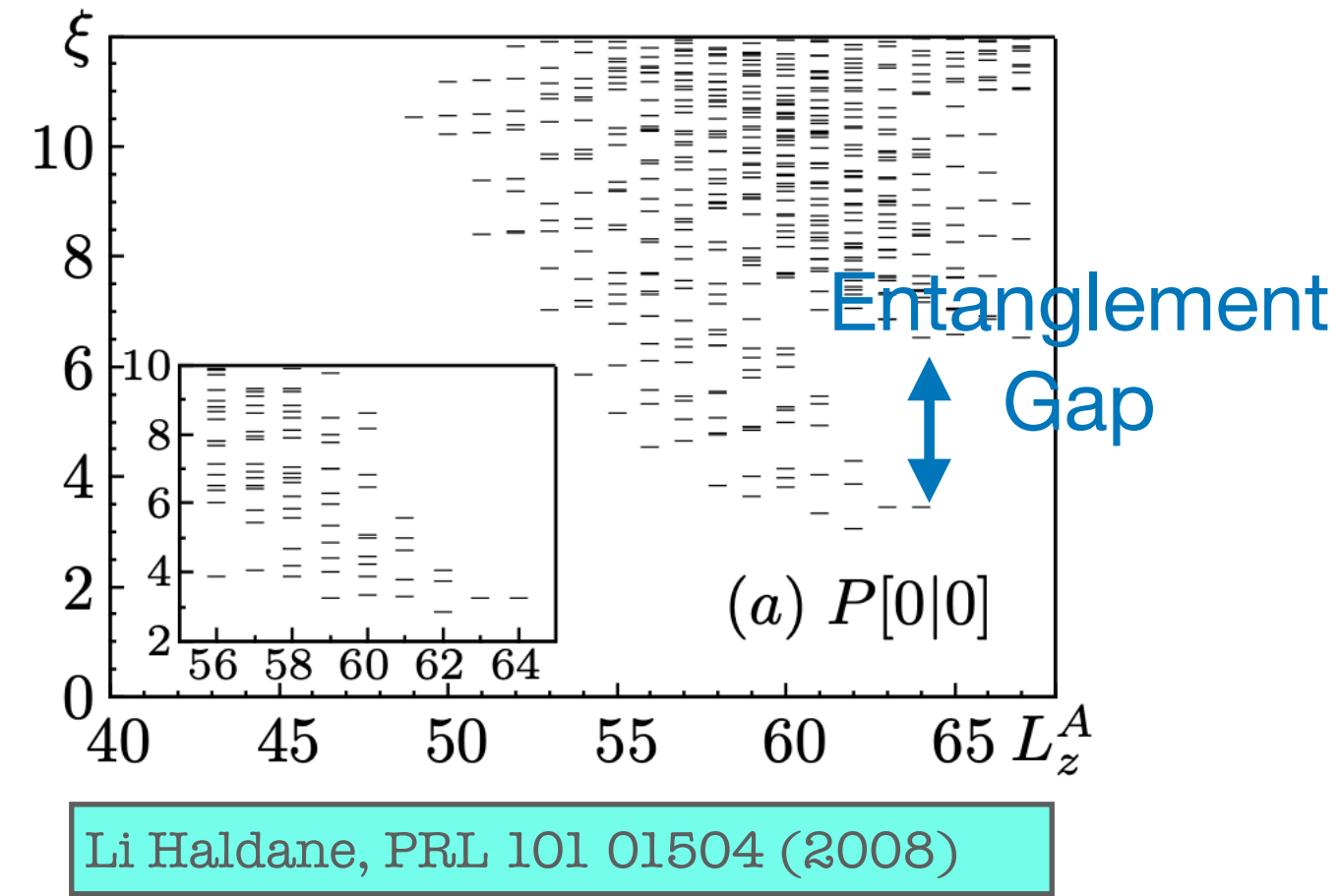
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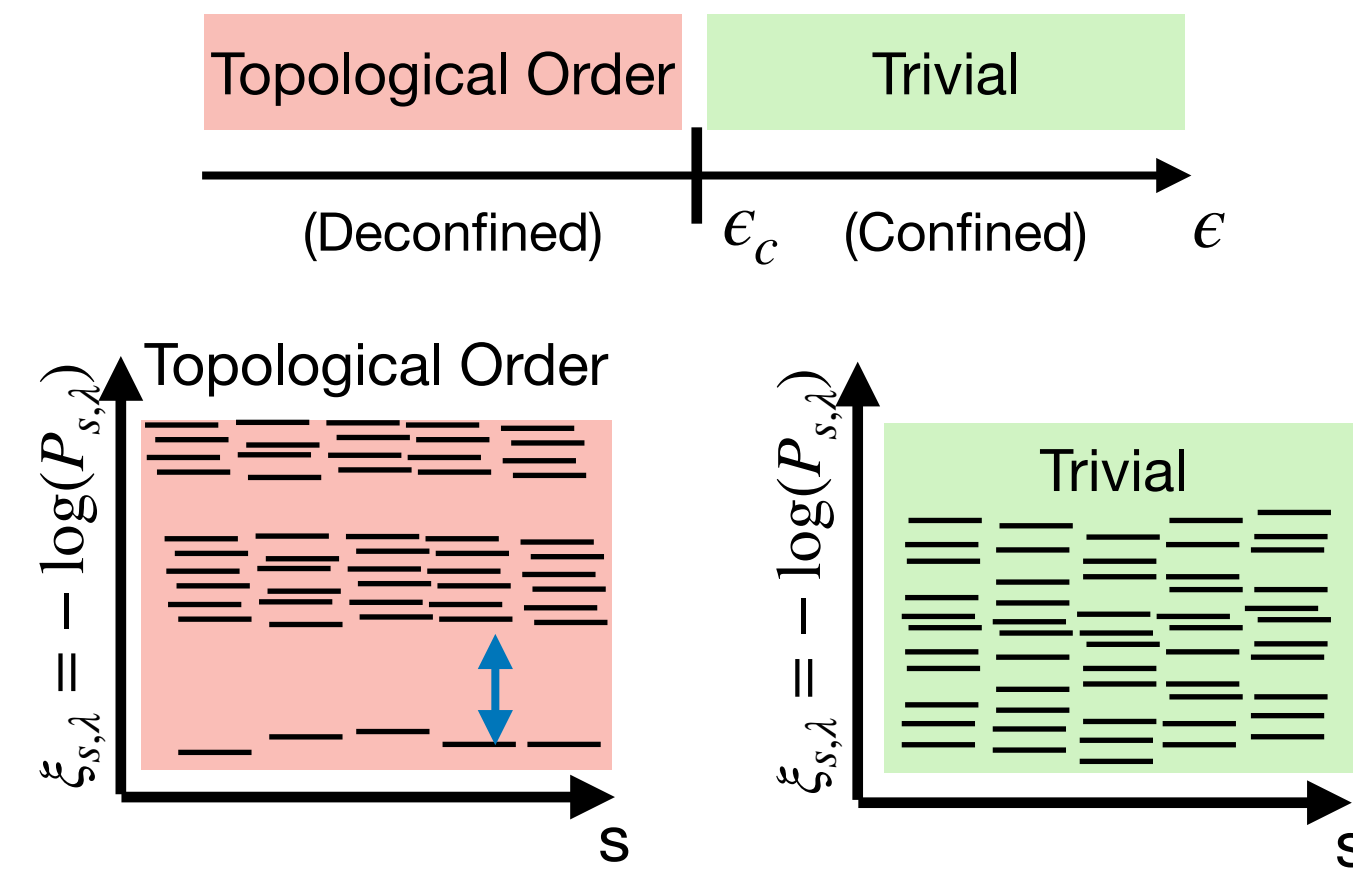


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Detecting Topological Order

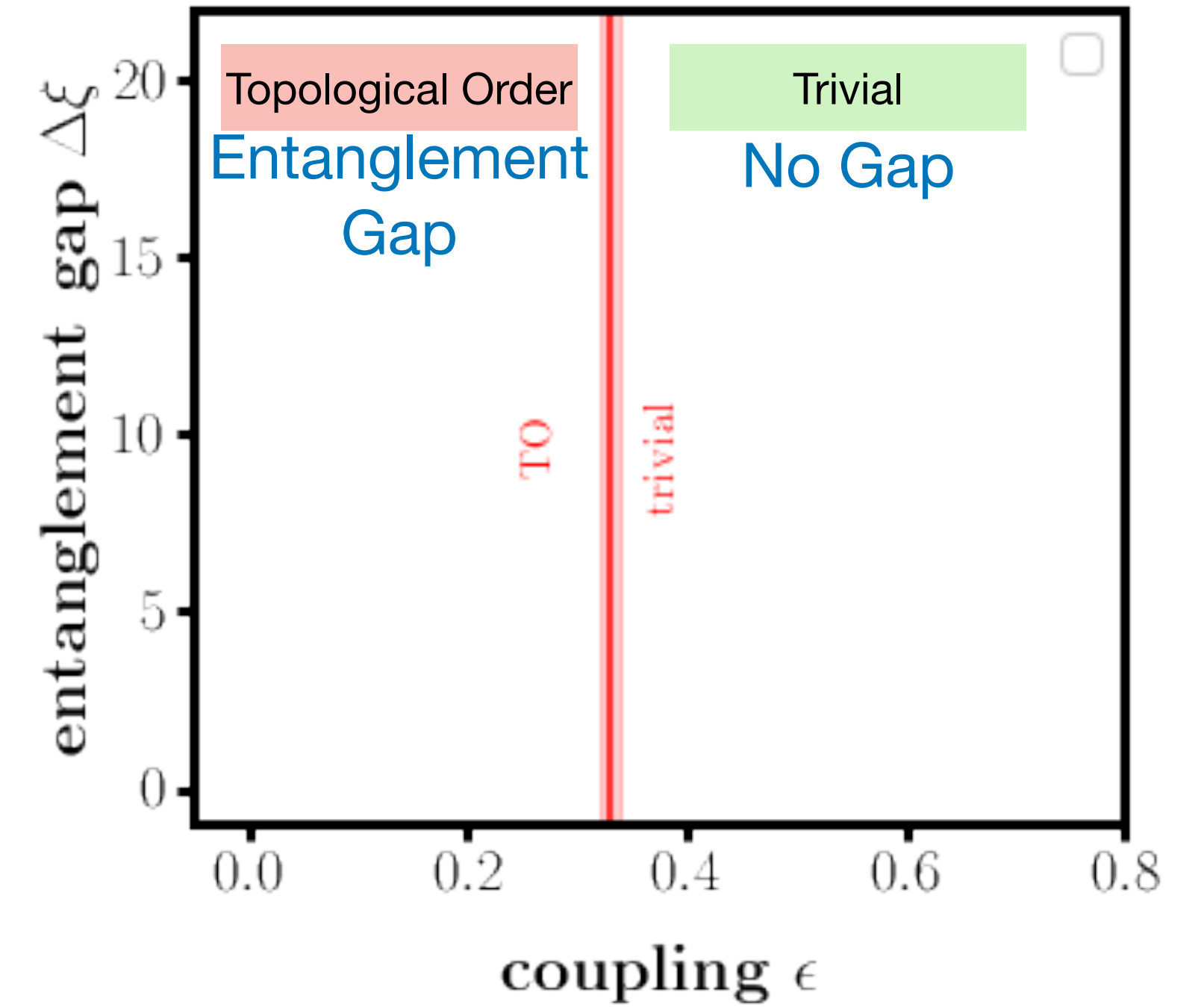
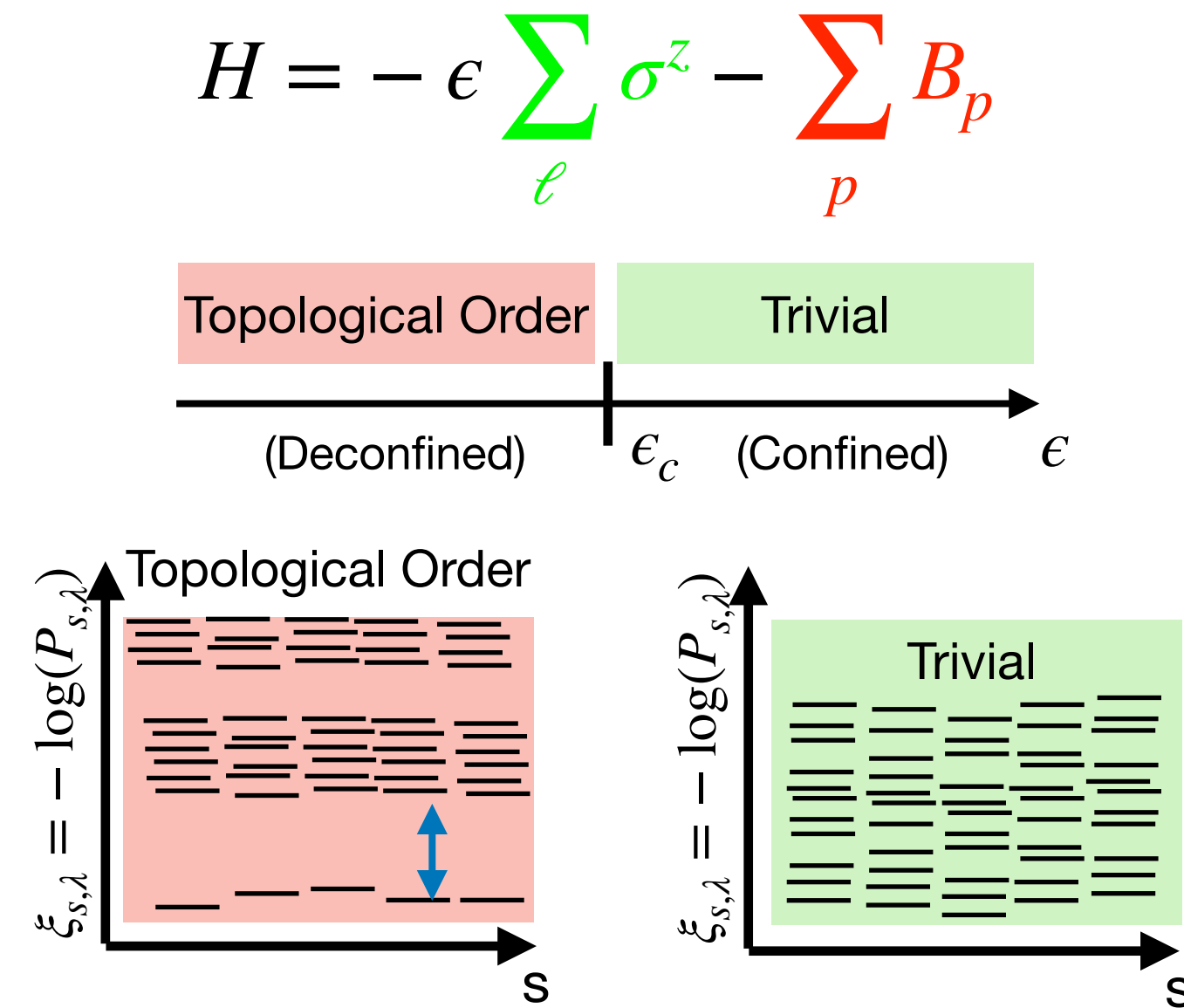
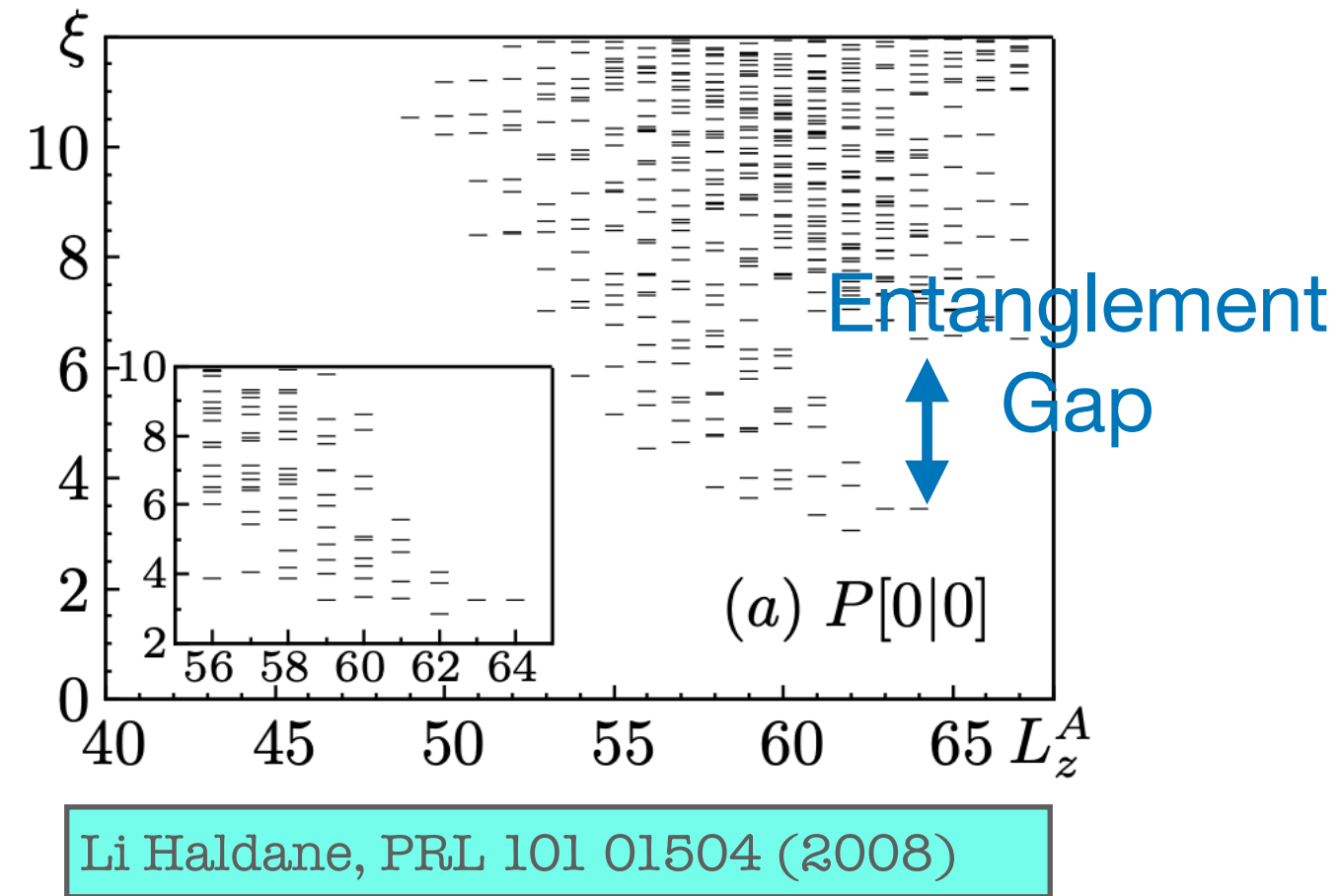


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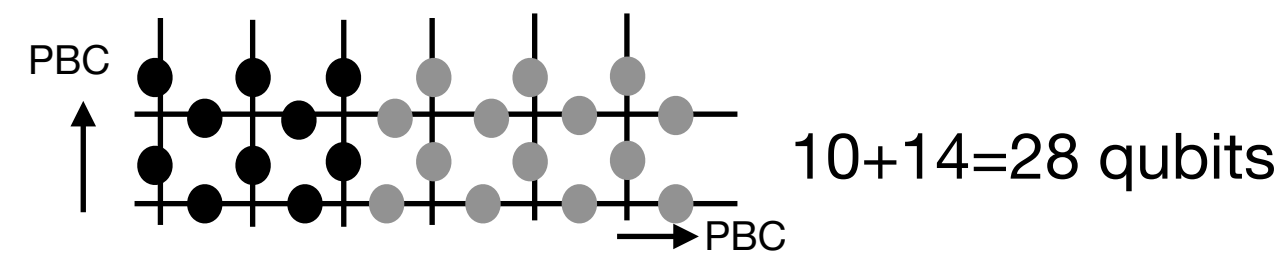
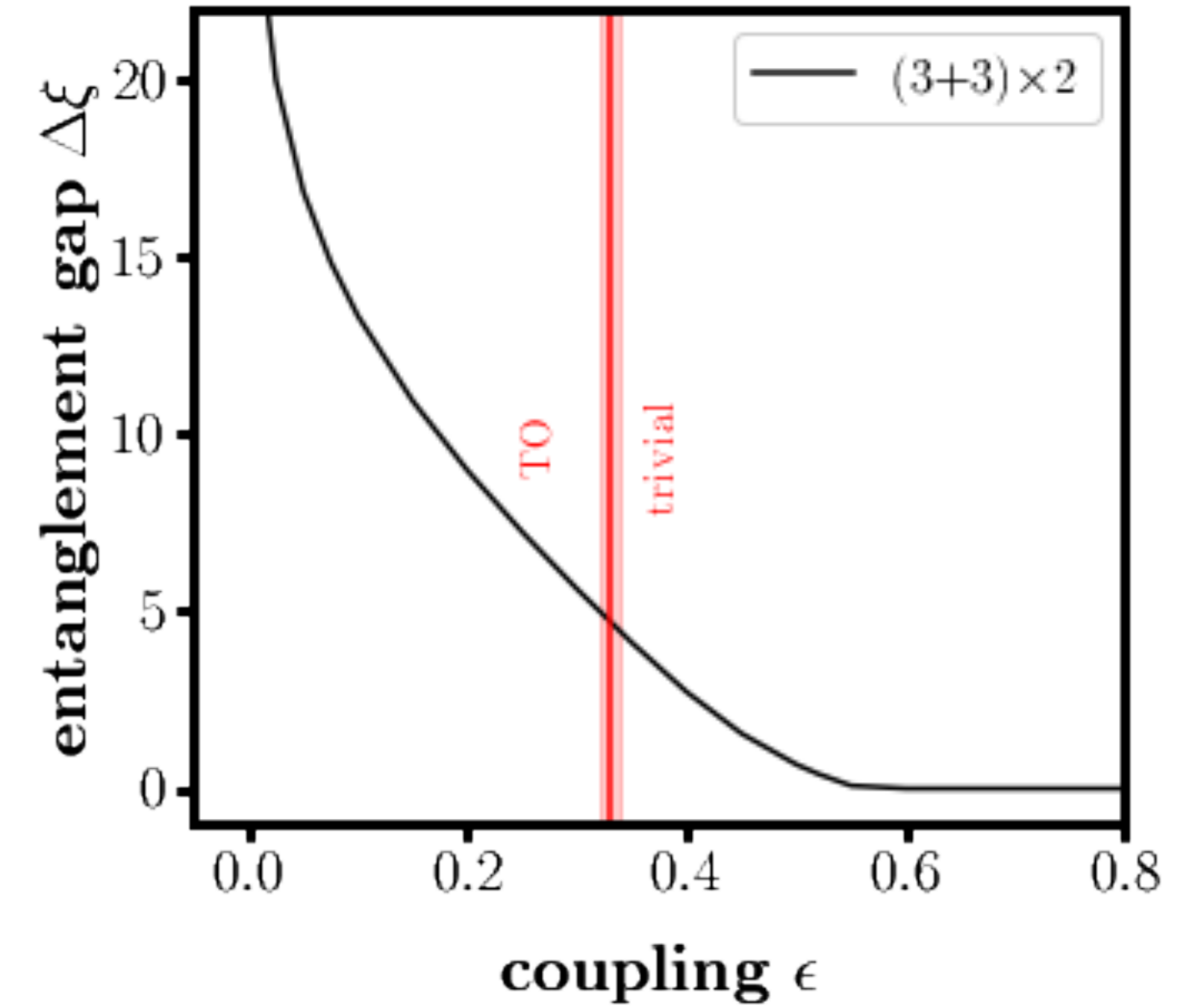
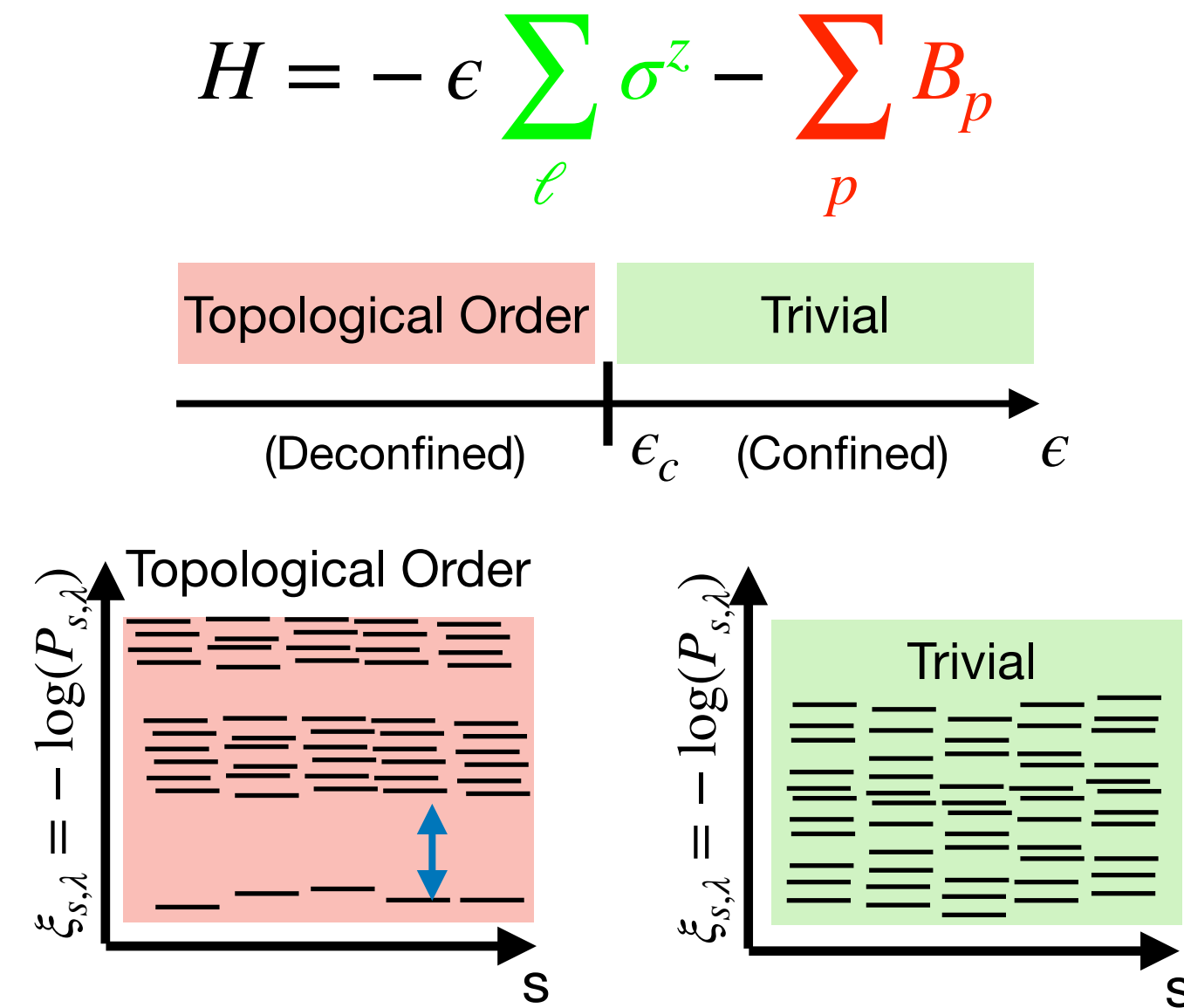
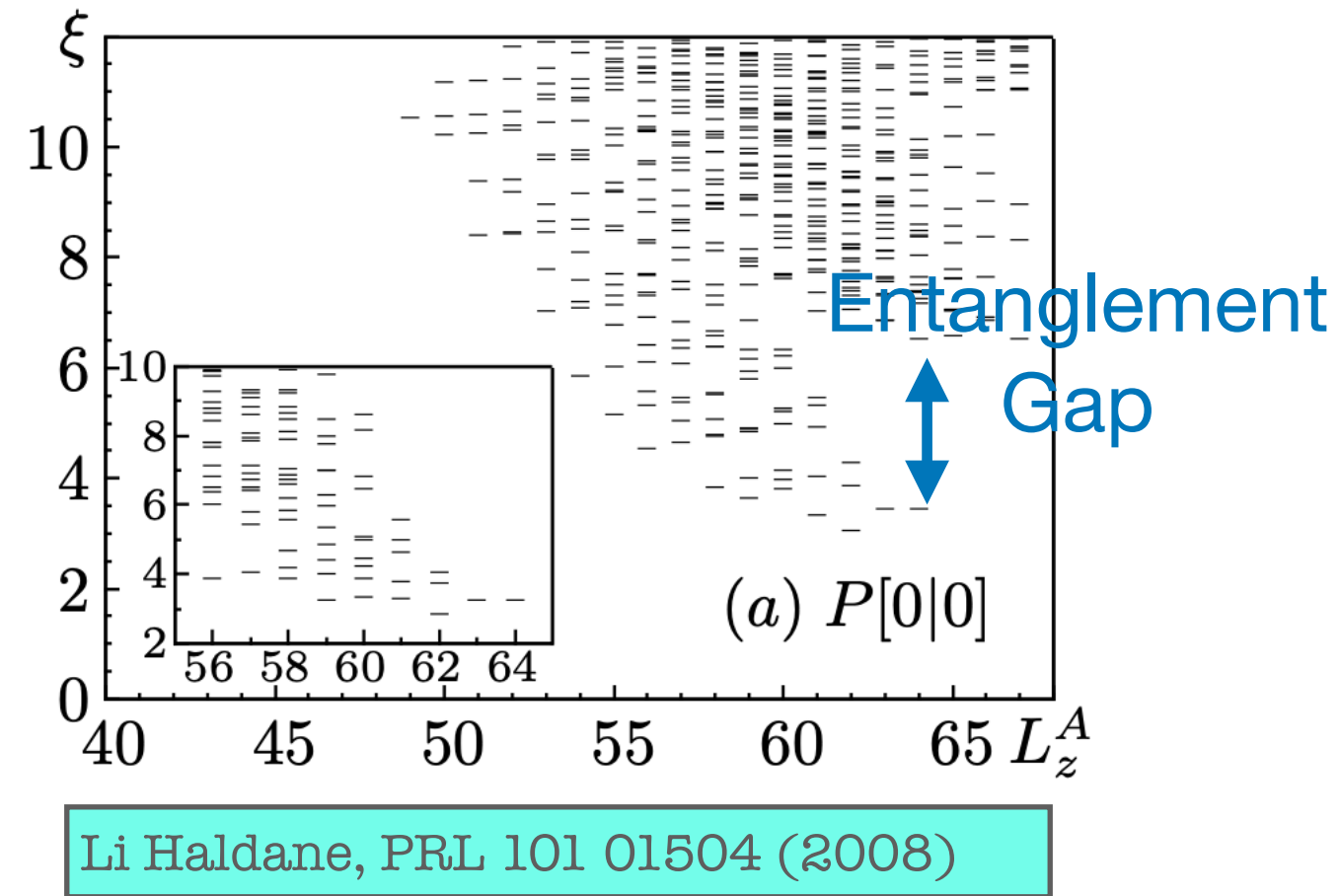
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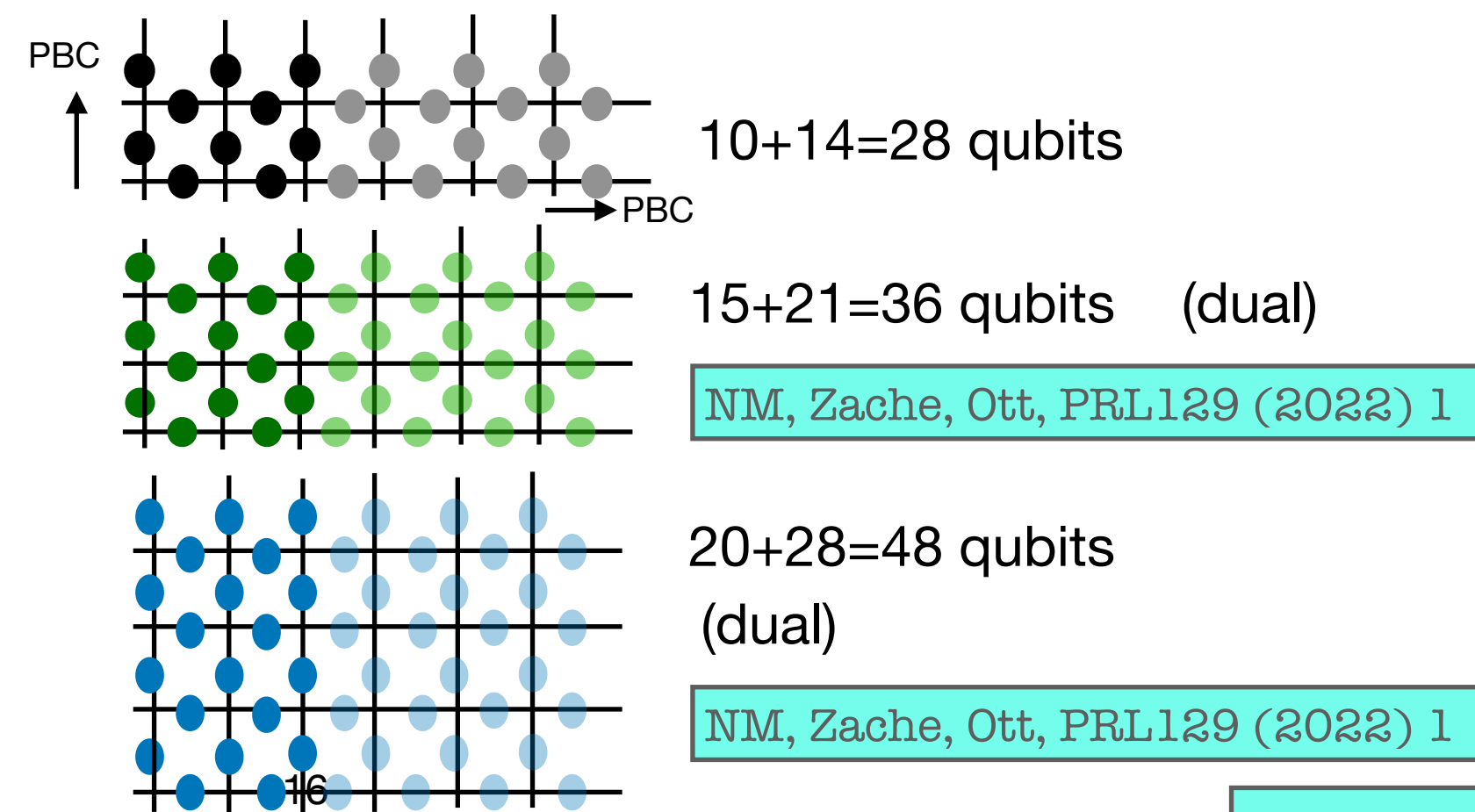
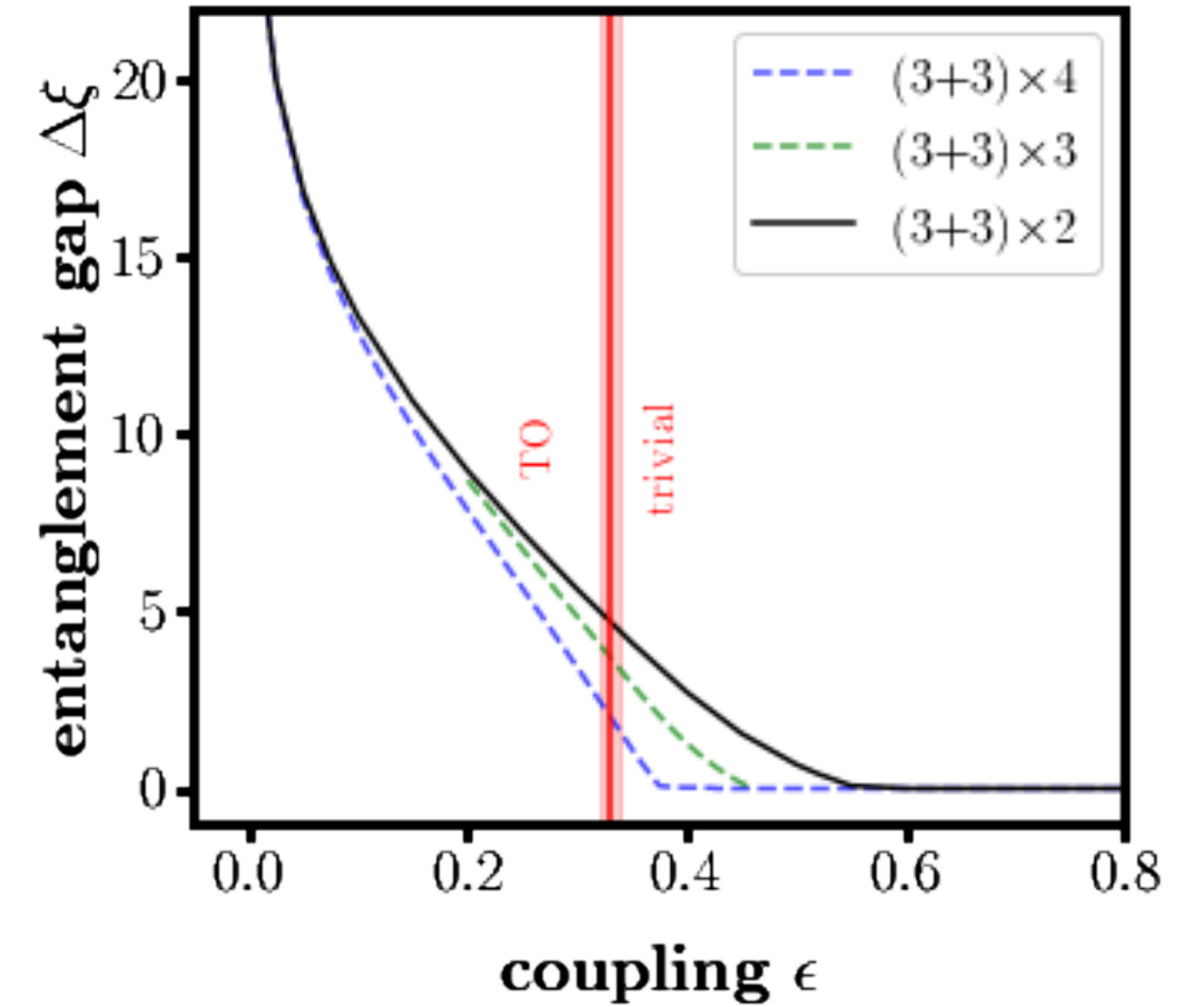
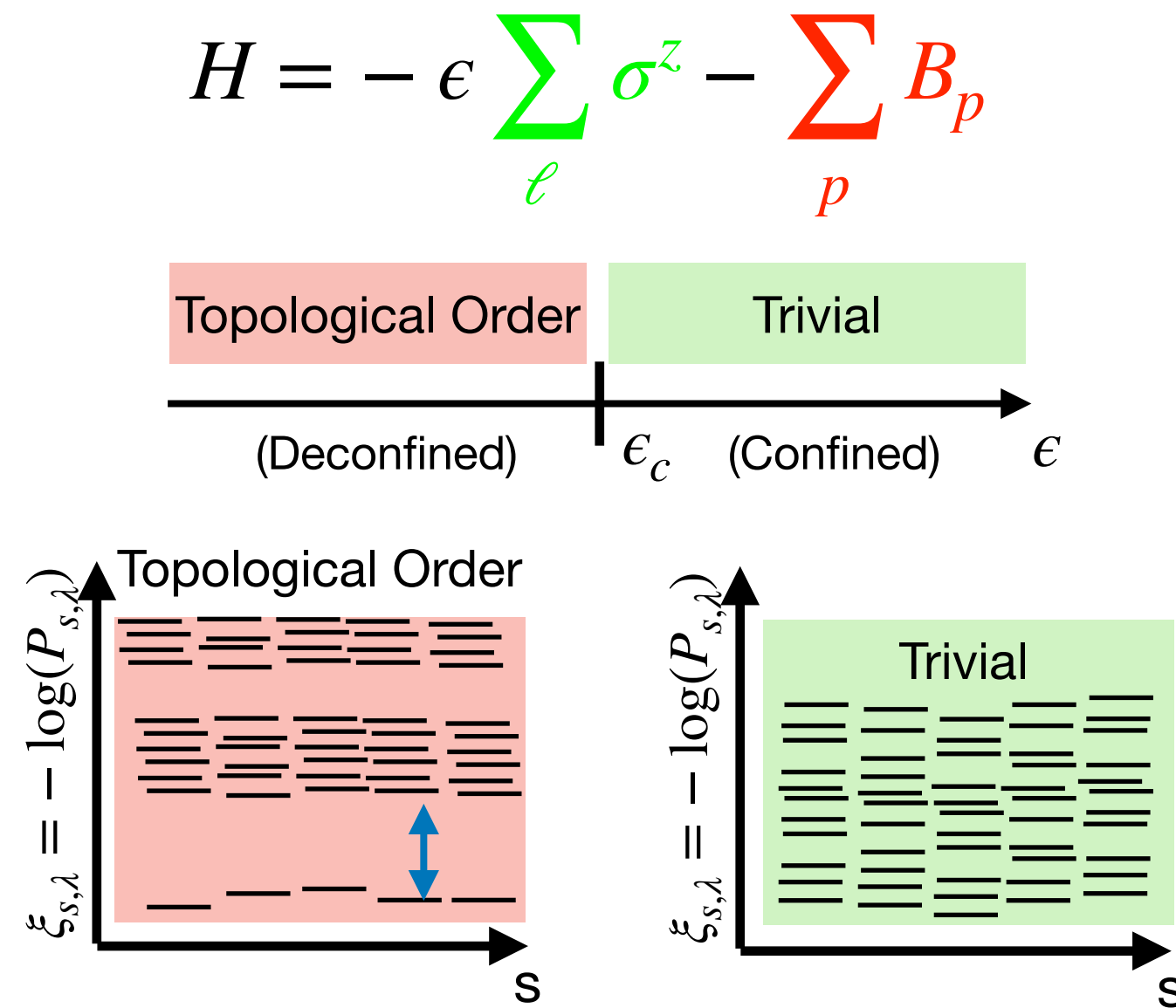
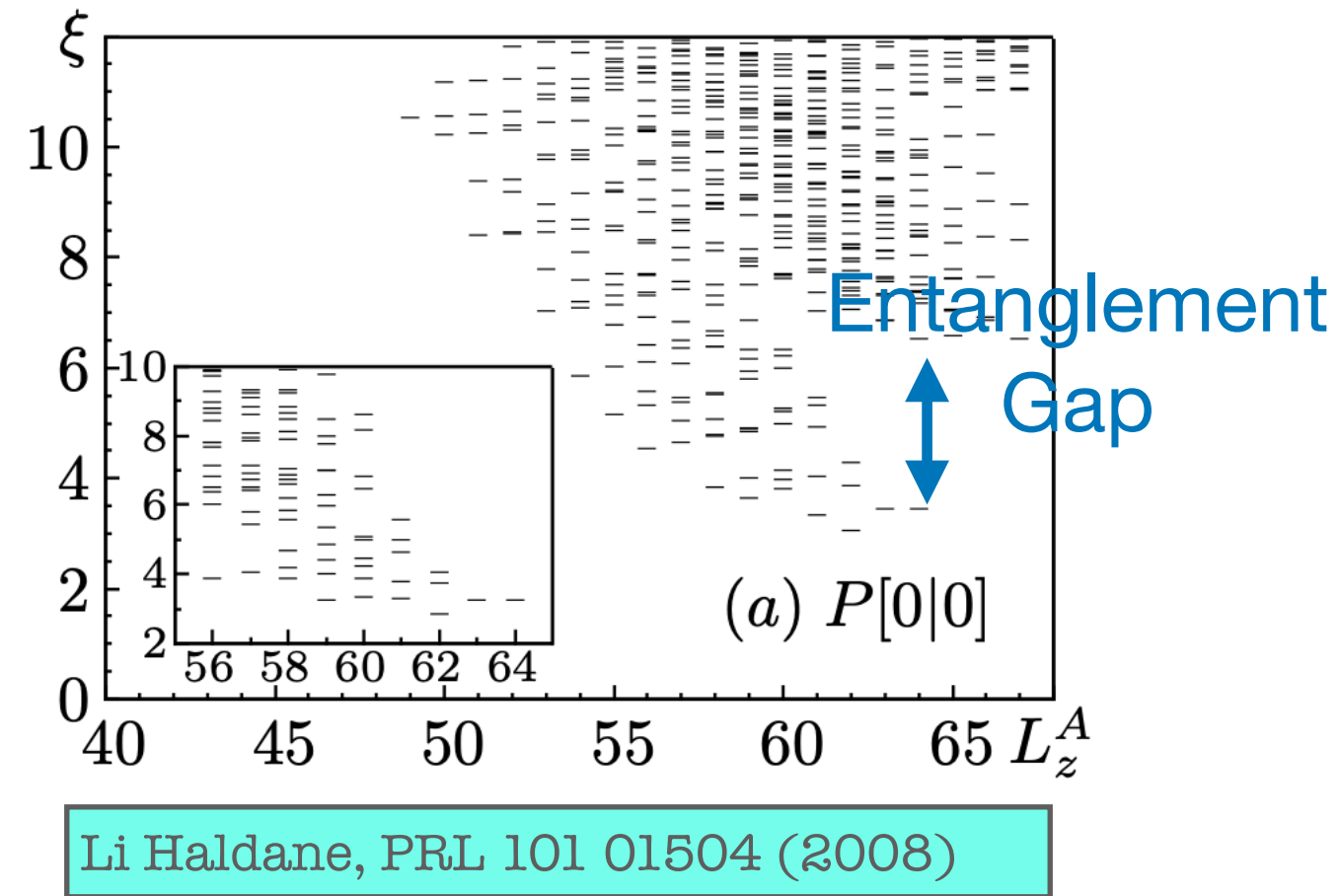
Symmetry-conscious Random Measurement

Detecting Topological Order



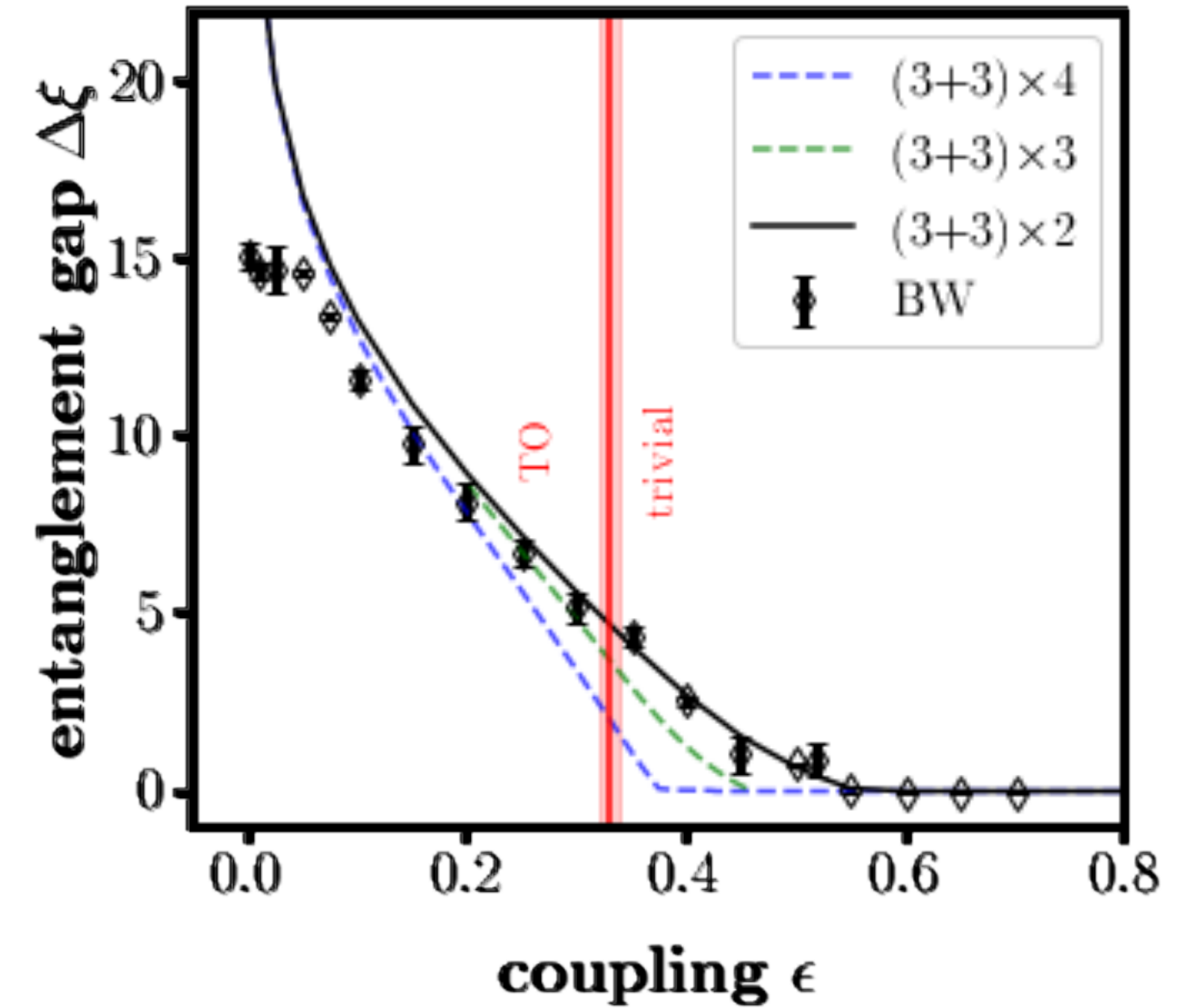
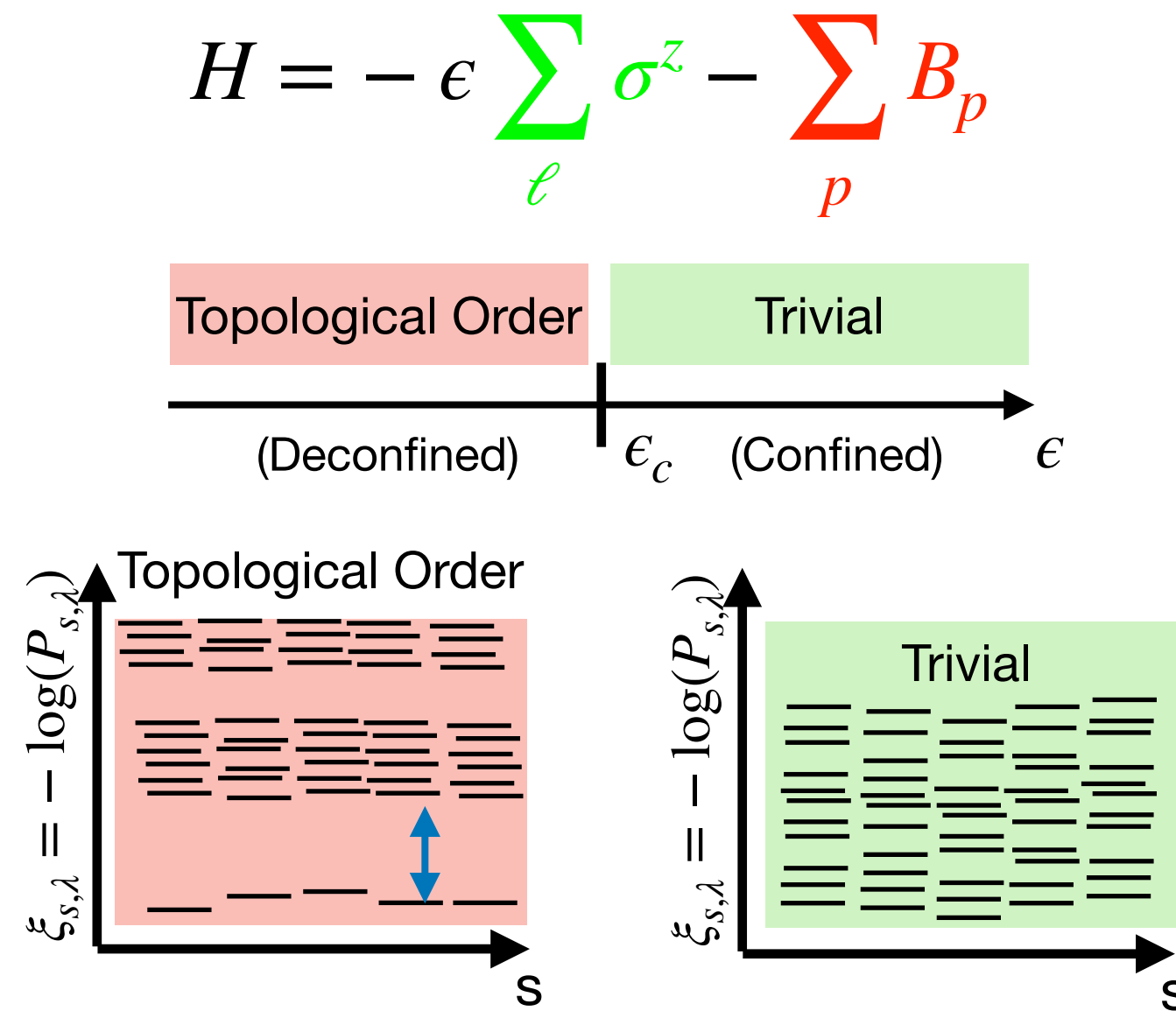
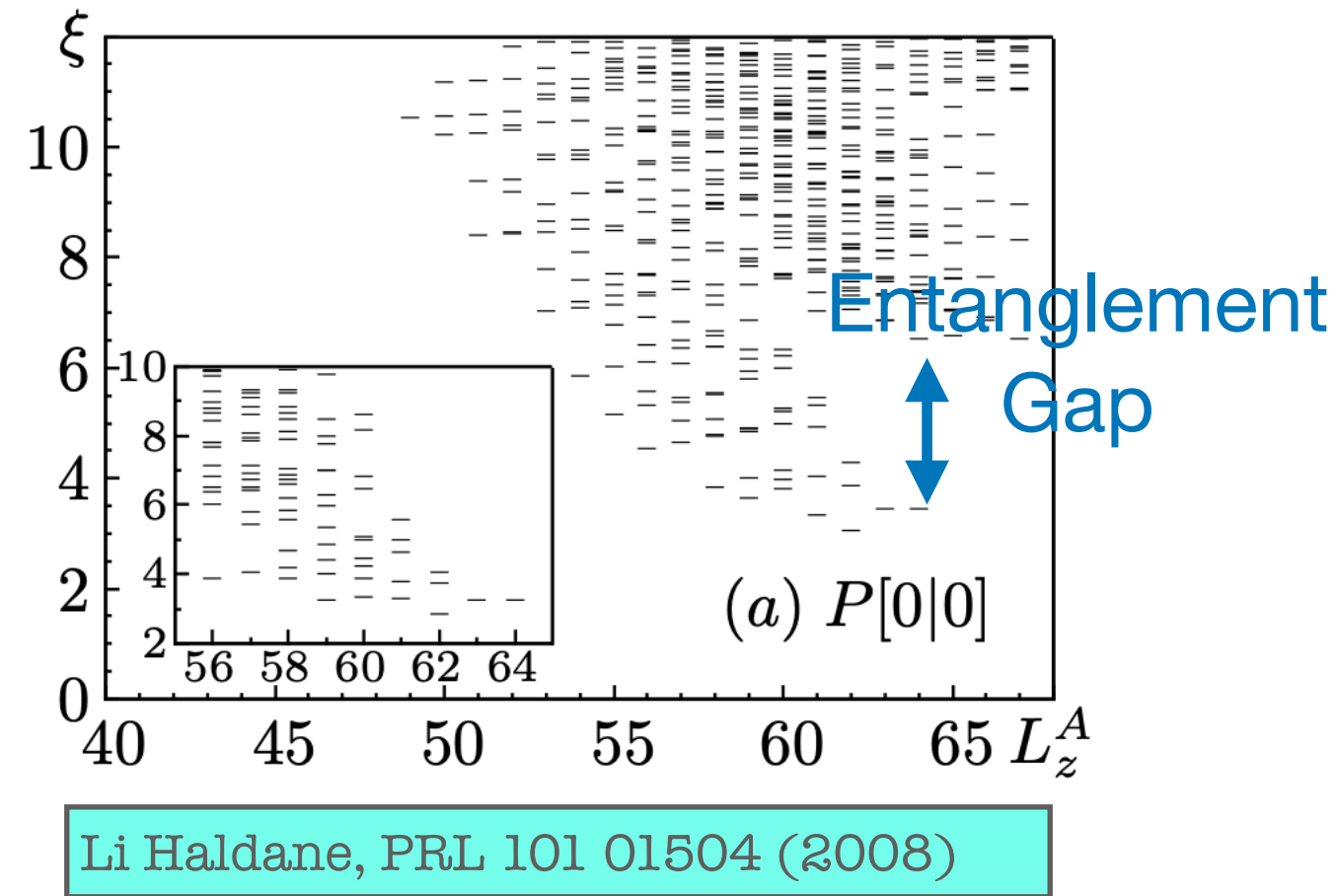
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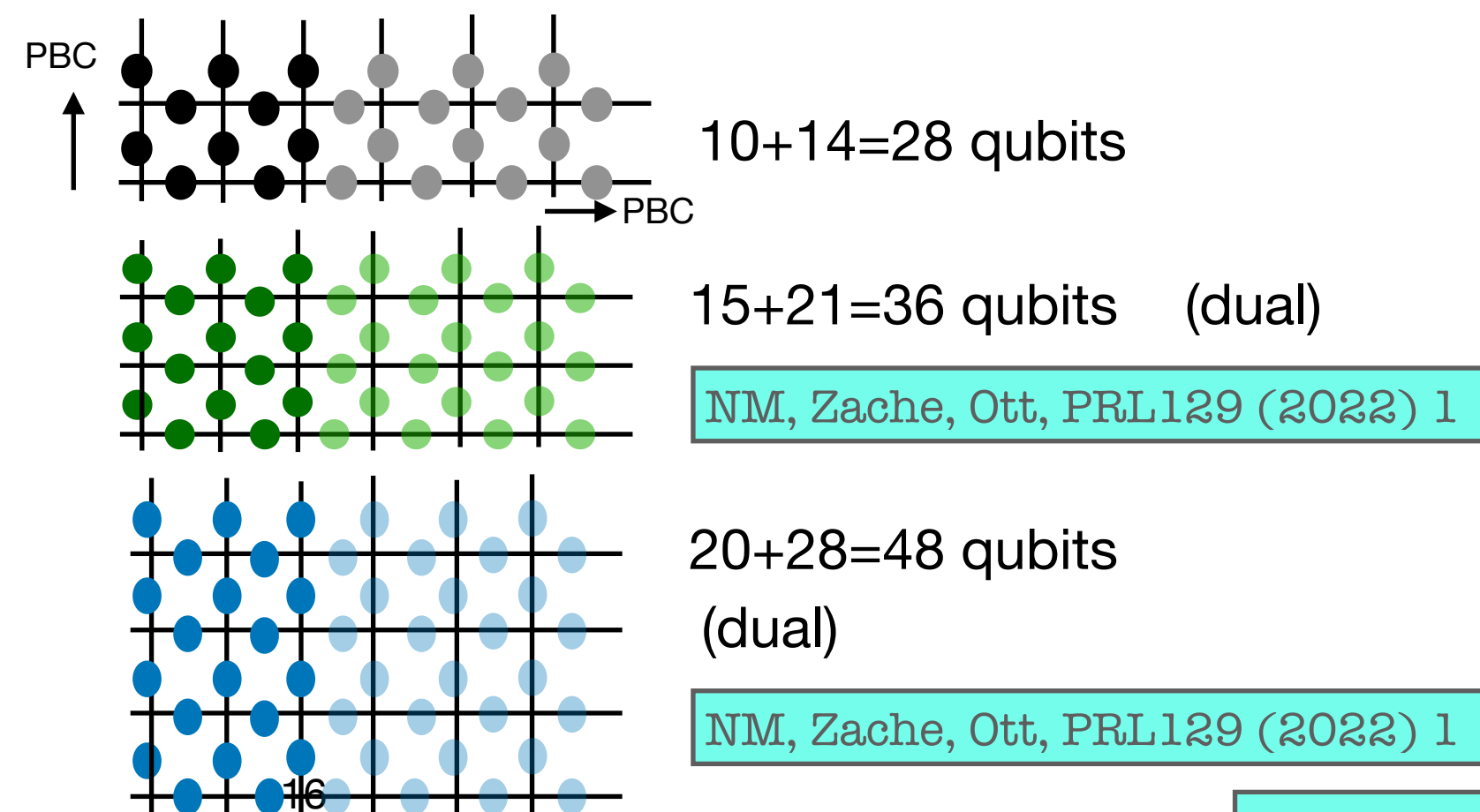
Detecting Topological Order



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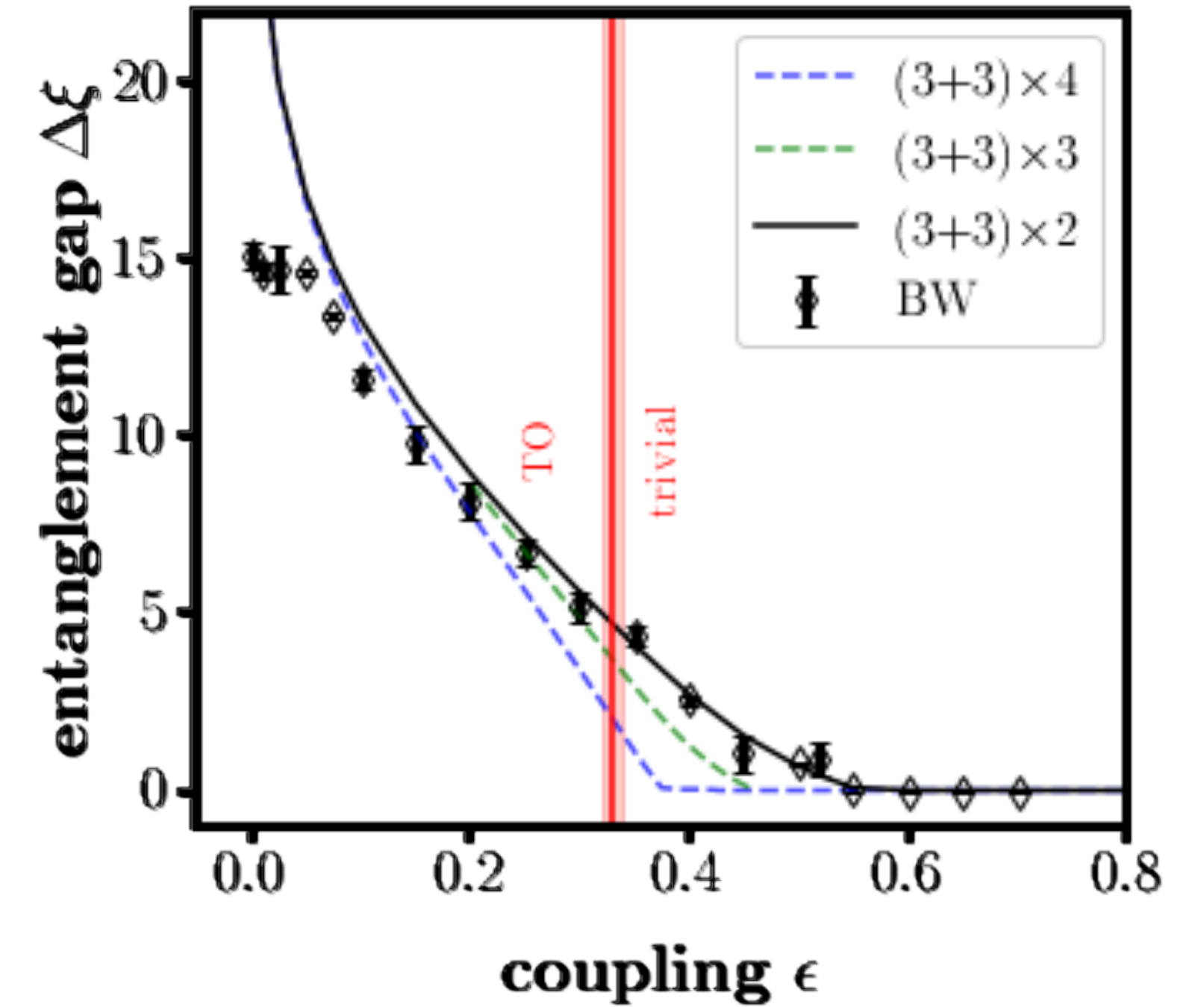
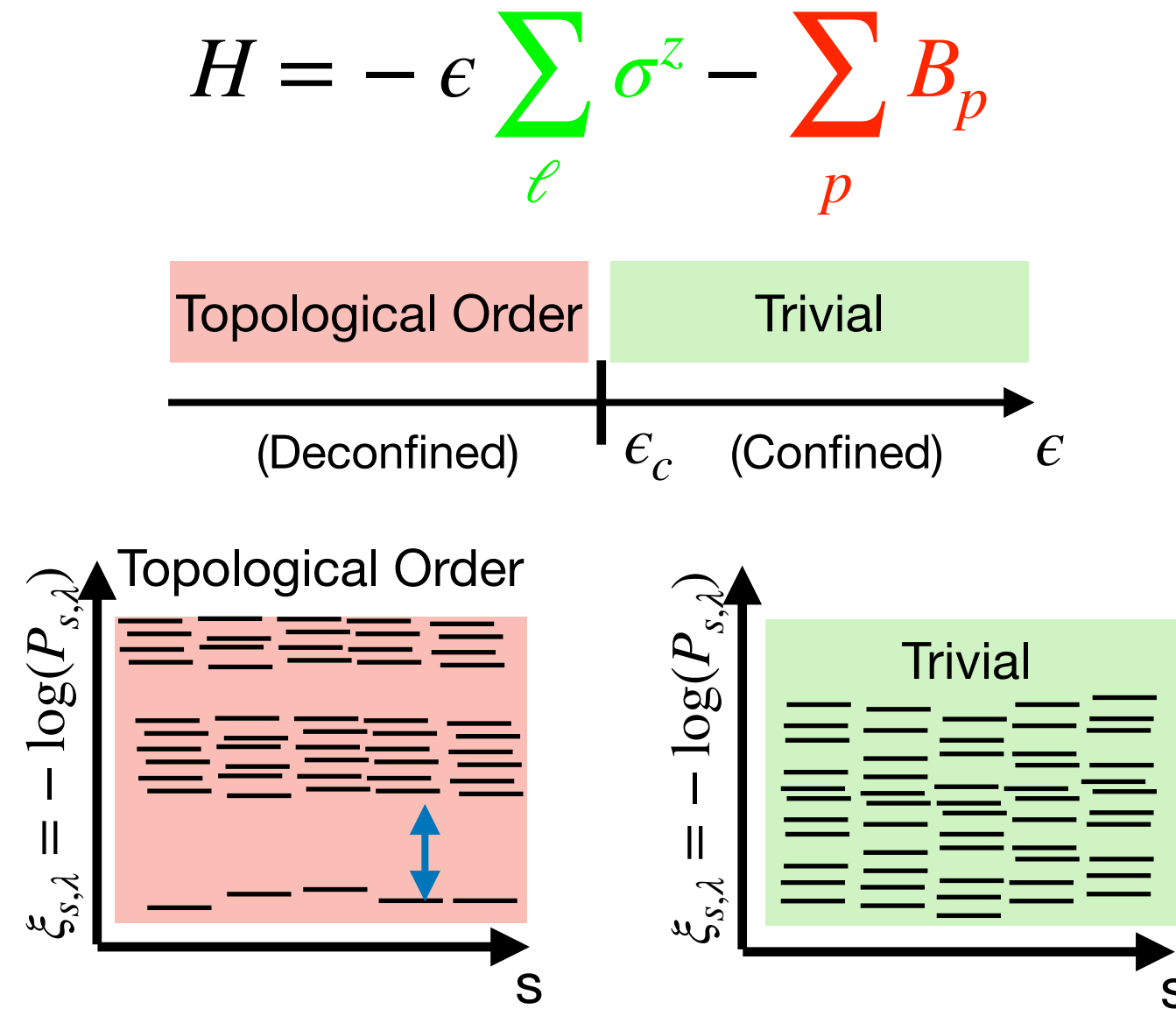
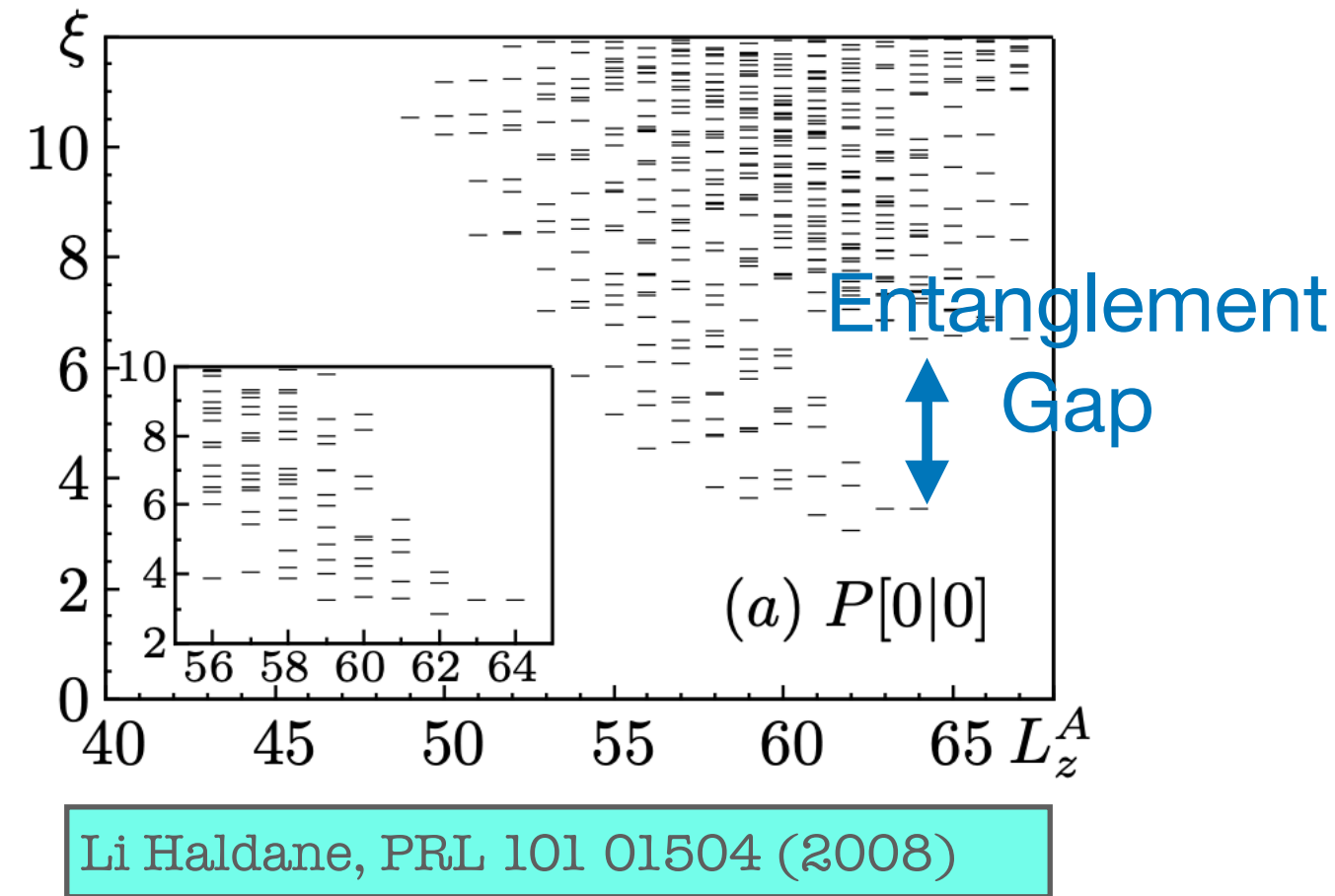
BW	sampling cost	circuit layer
	$N_E = 50$	64 layers
	$\times N_M = 1024$	
	$= 51,200$	

(overkill, because we can)



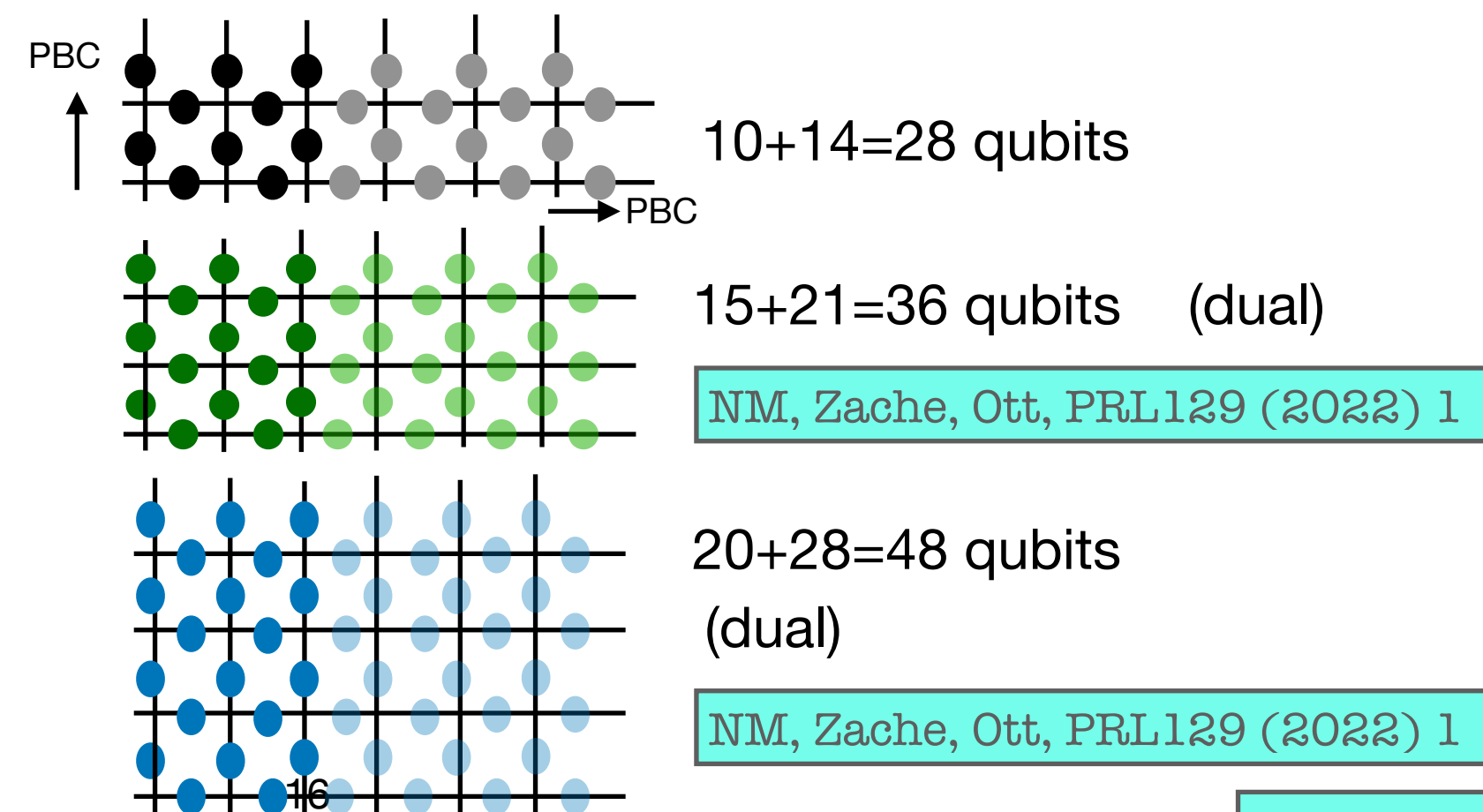
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$$d_A = 2^{20} = 1,048,576$$

$$d_s = 2^8 = 256$$

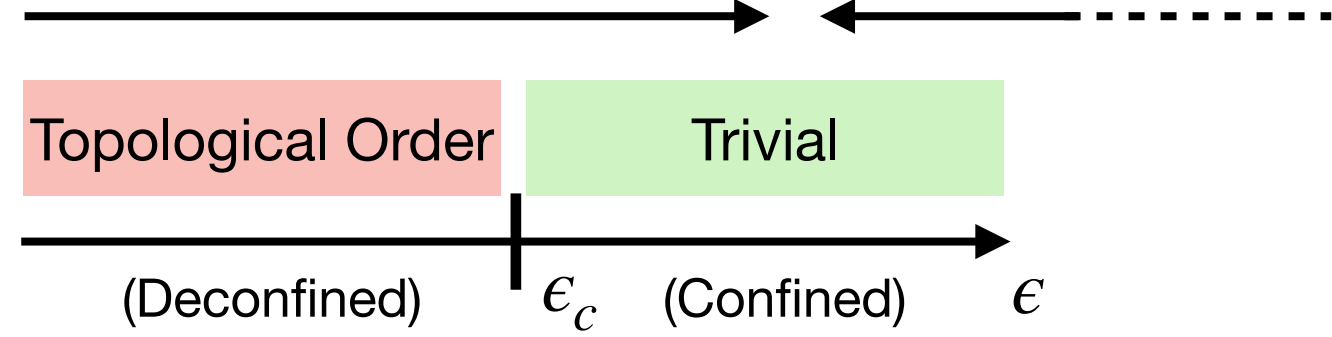
Thermalization of Gauge Theories

 universität
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Thermalization of Gauge Theories

$$H = -\epsilon \sum_{\ell} \sigma^z - \sum_p B_p$$



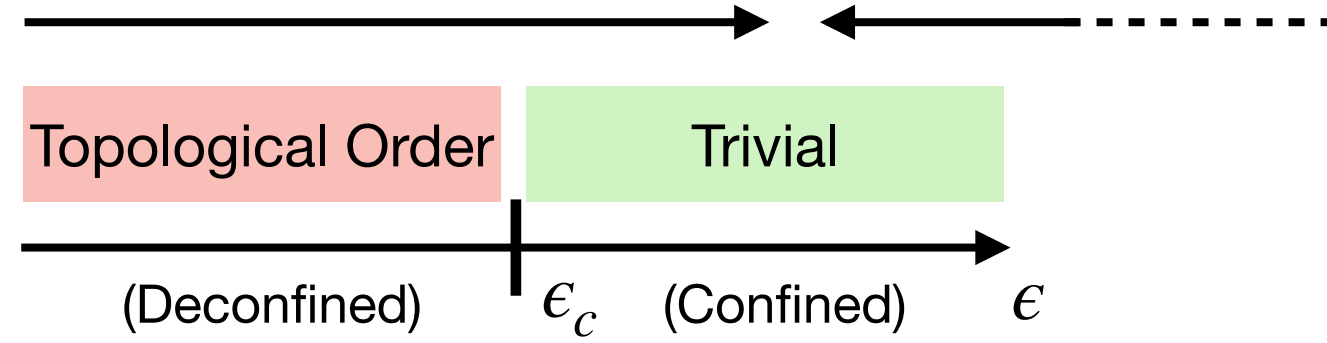
Quantum Quench

universität
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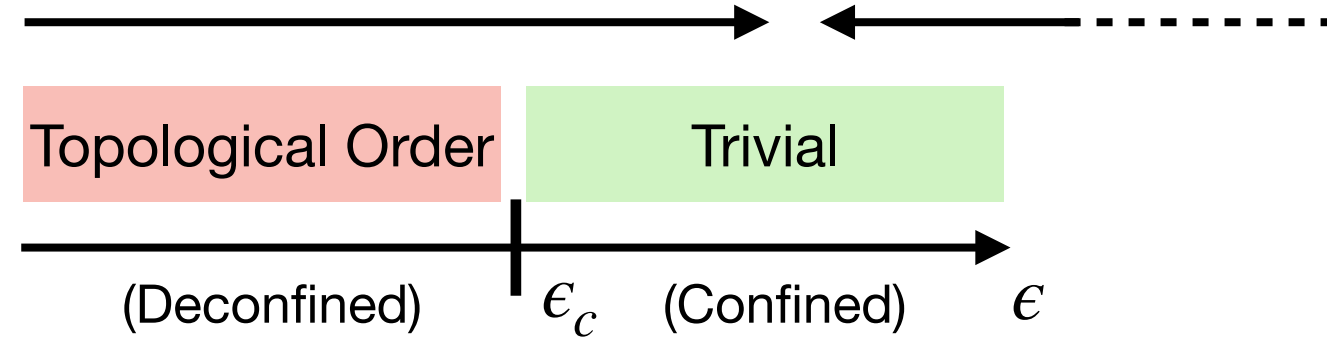
from Entanglement Structure

universität
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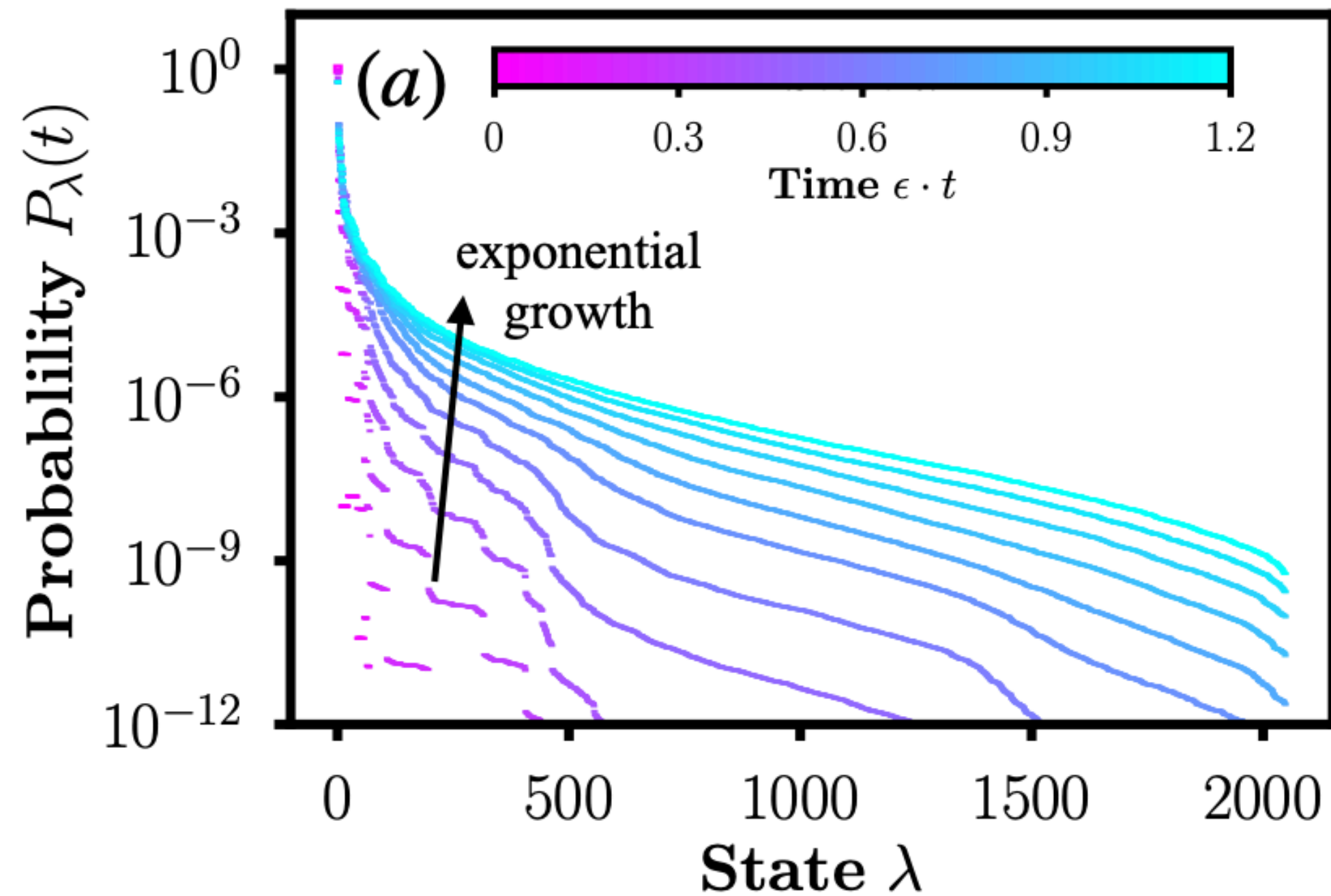
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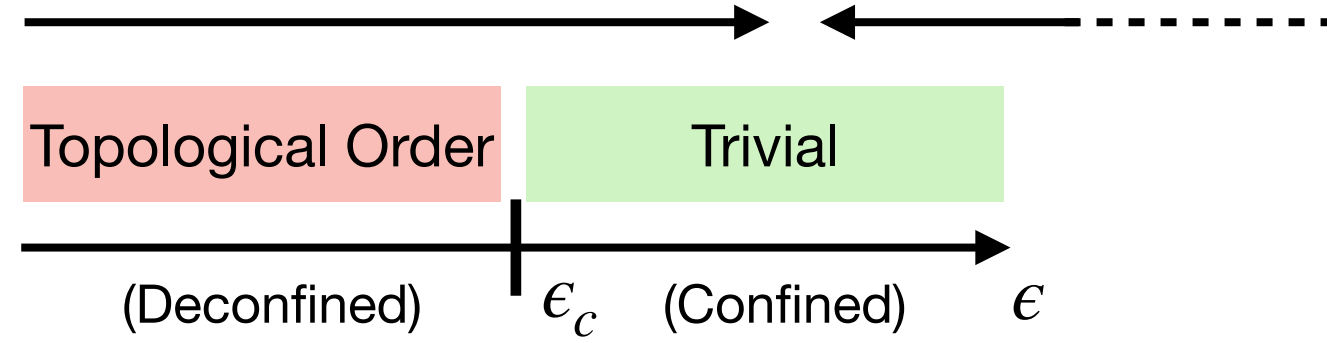


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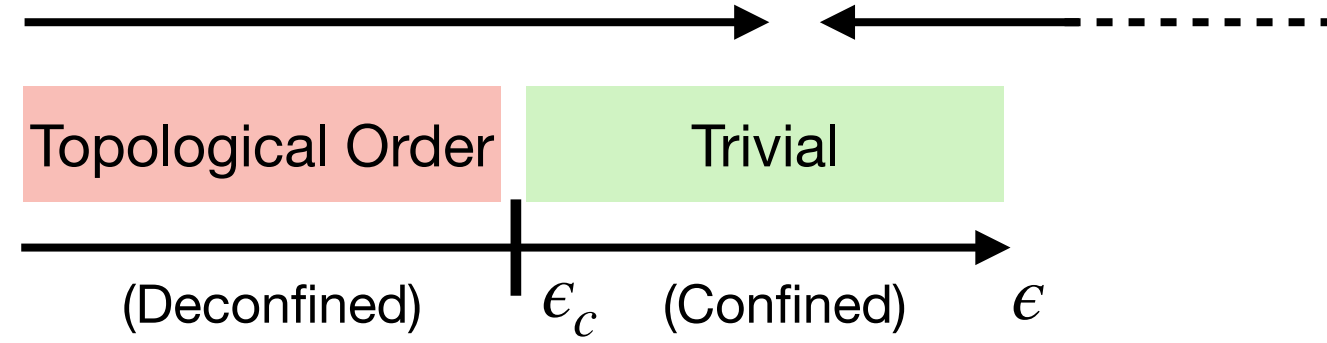
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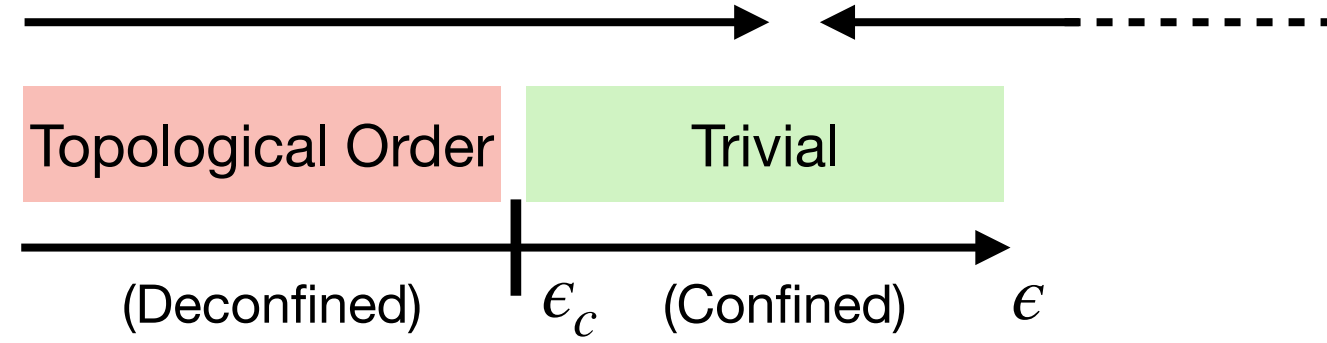
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from Entanglement Structure

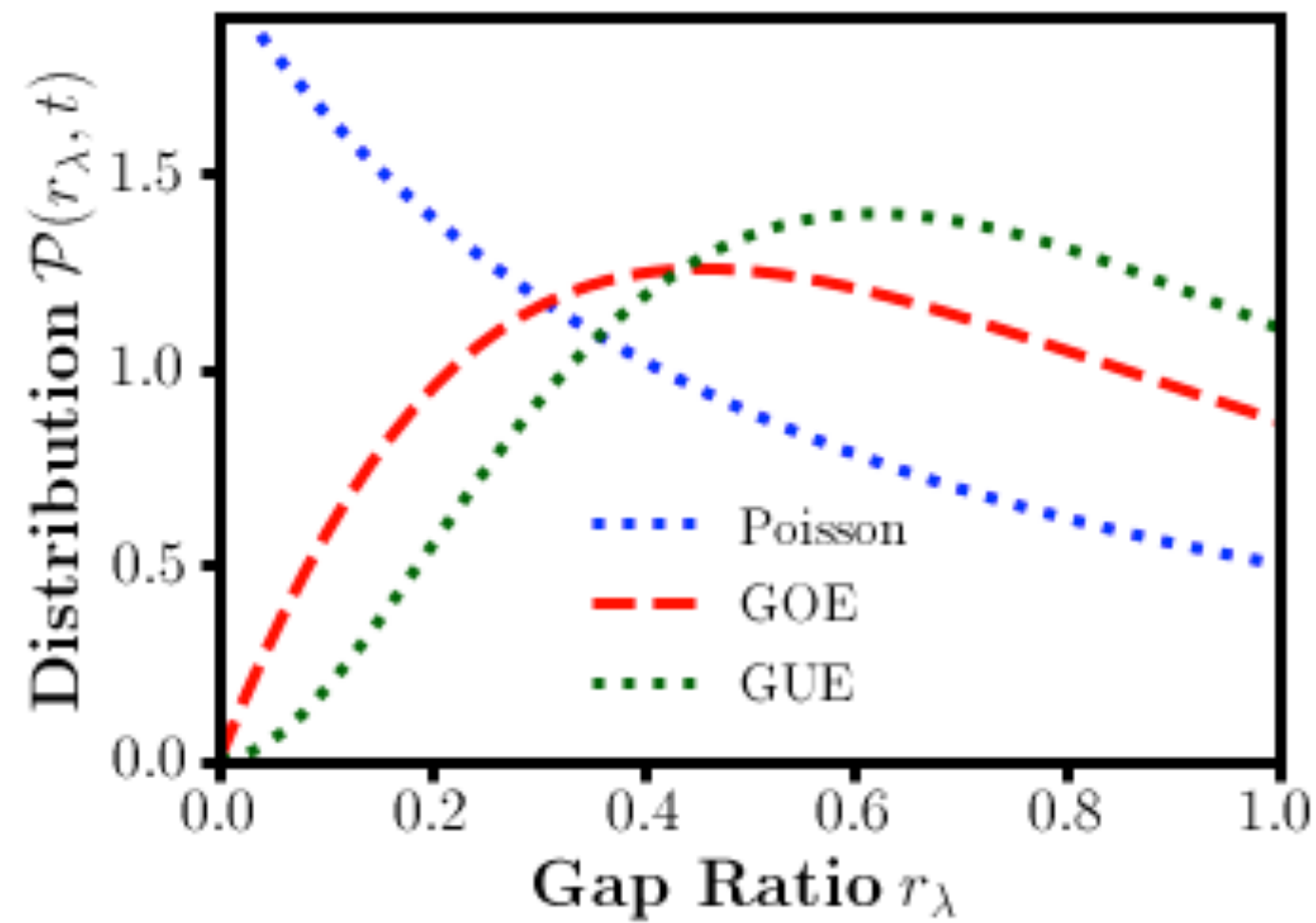
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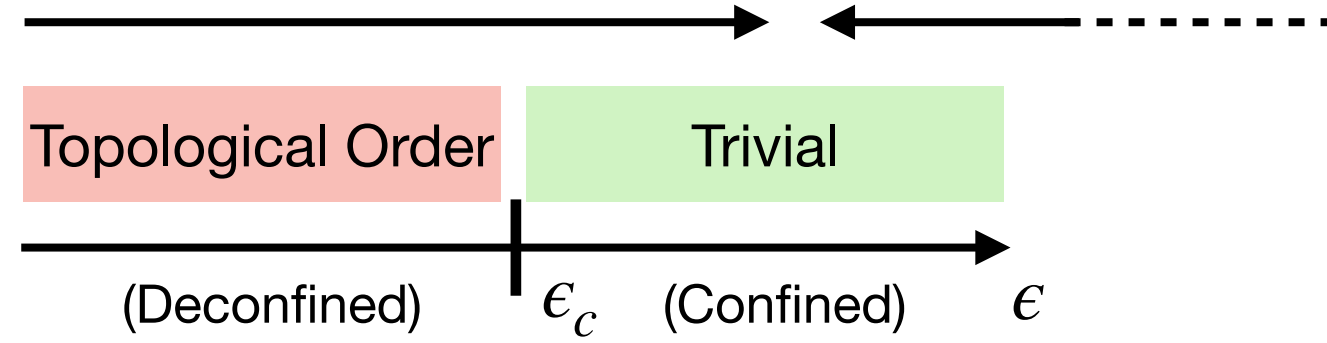
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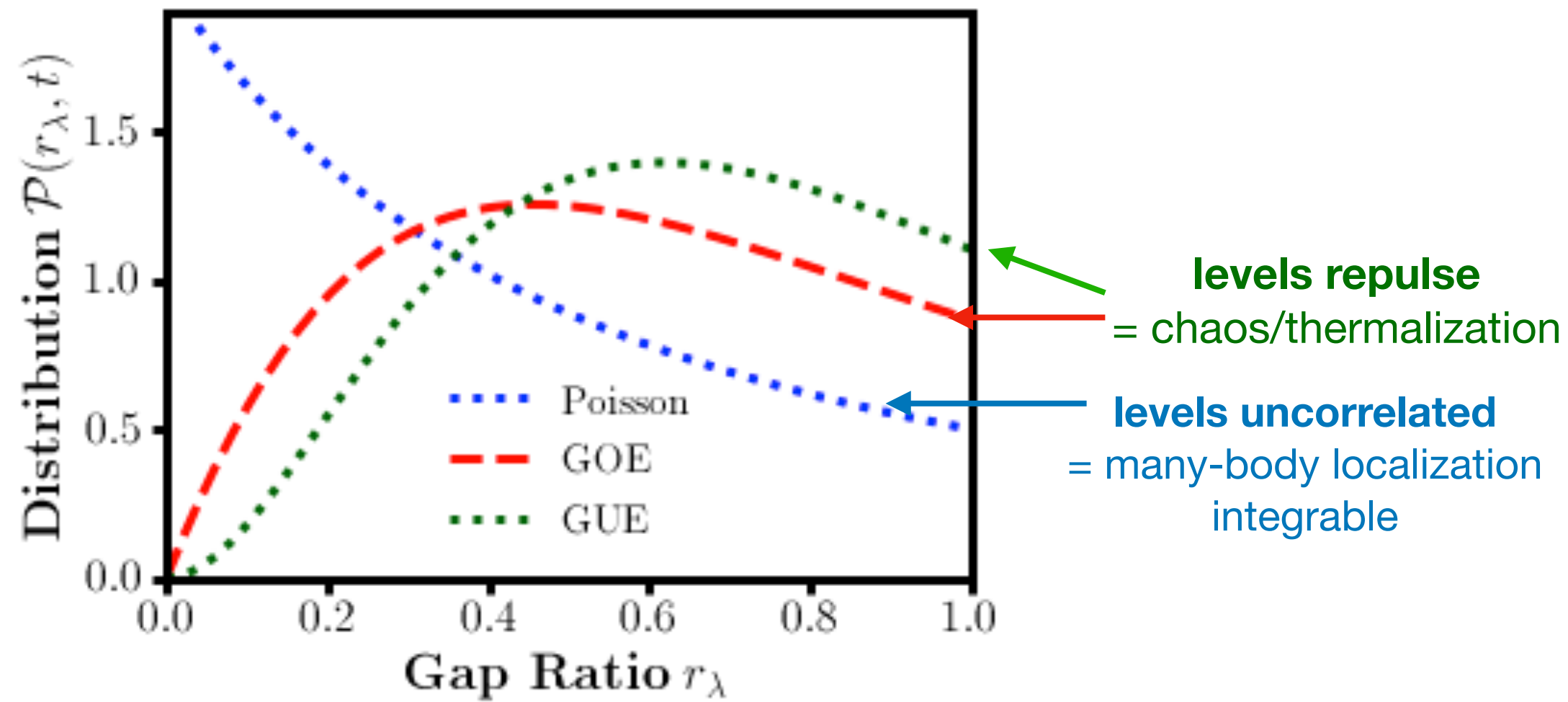
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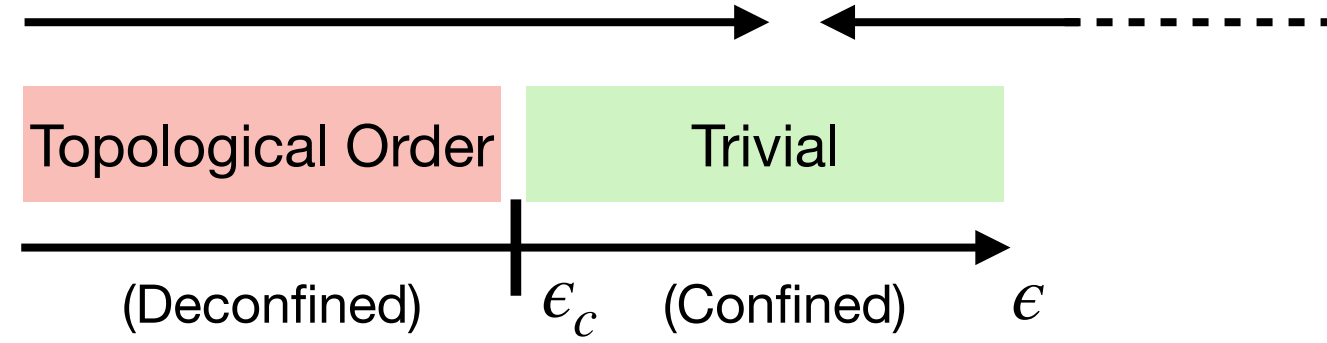
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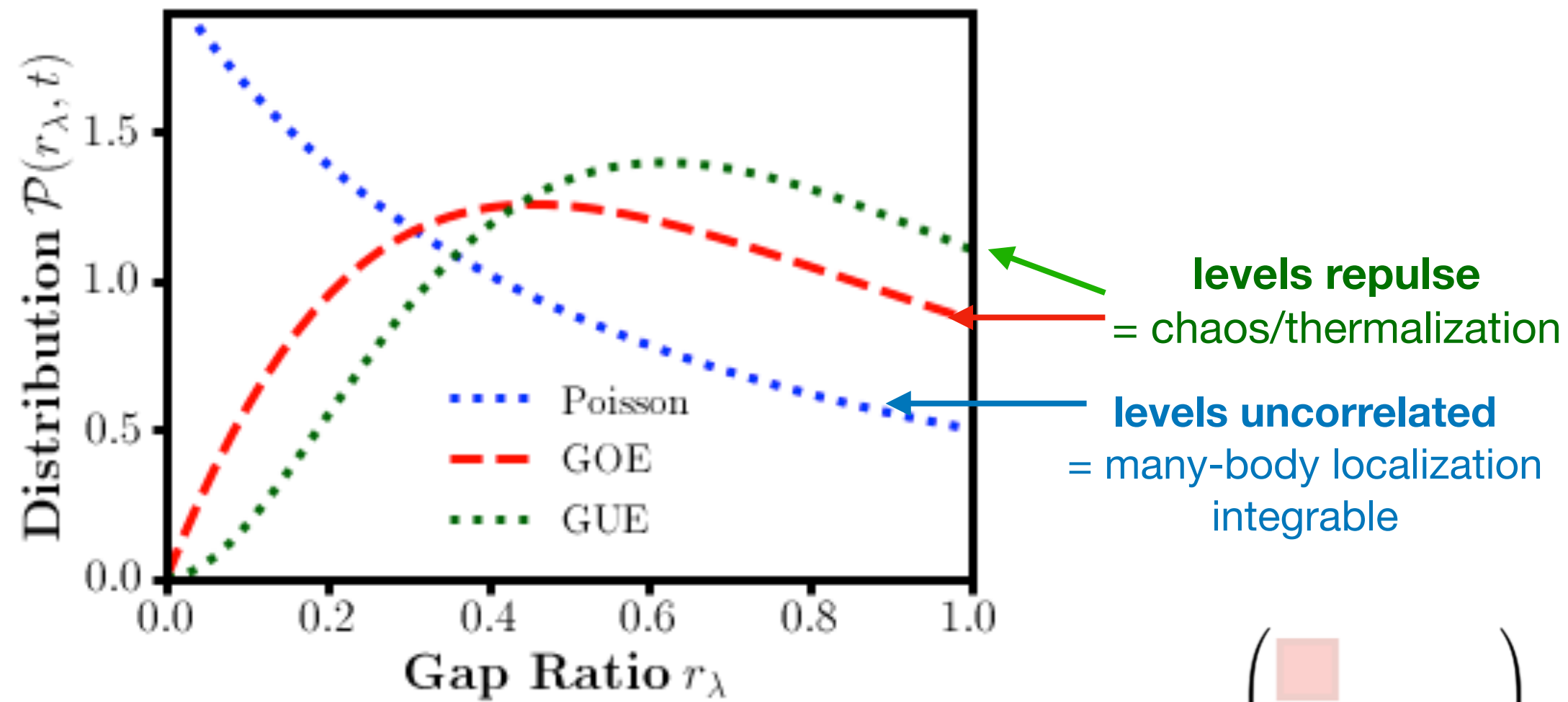
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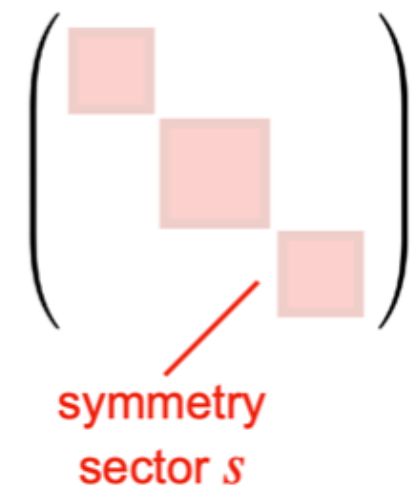
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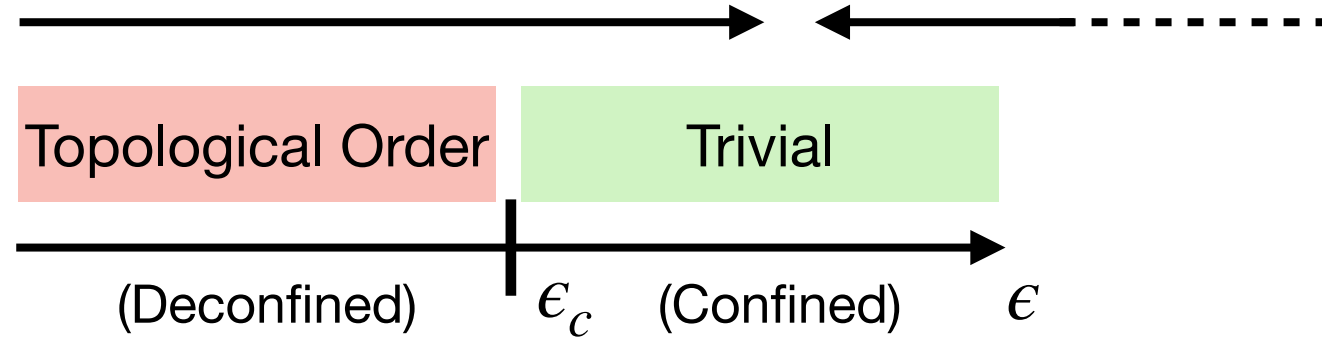
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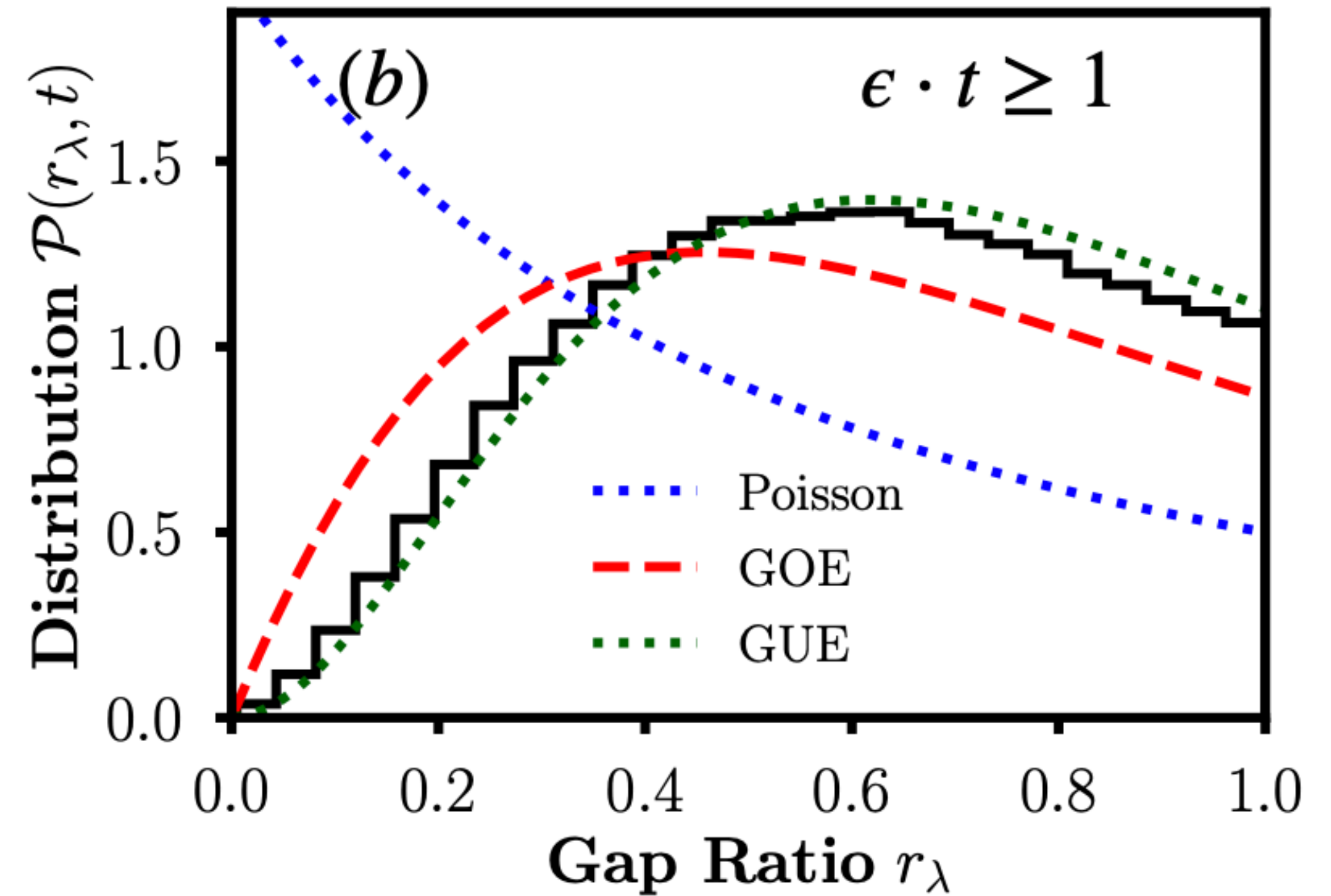
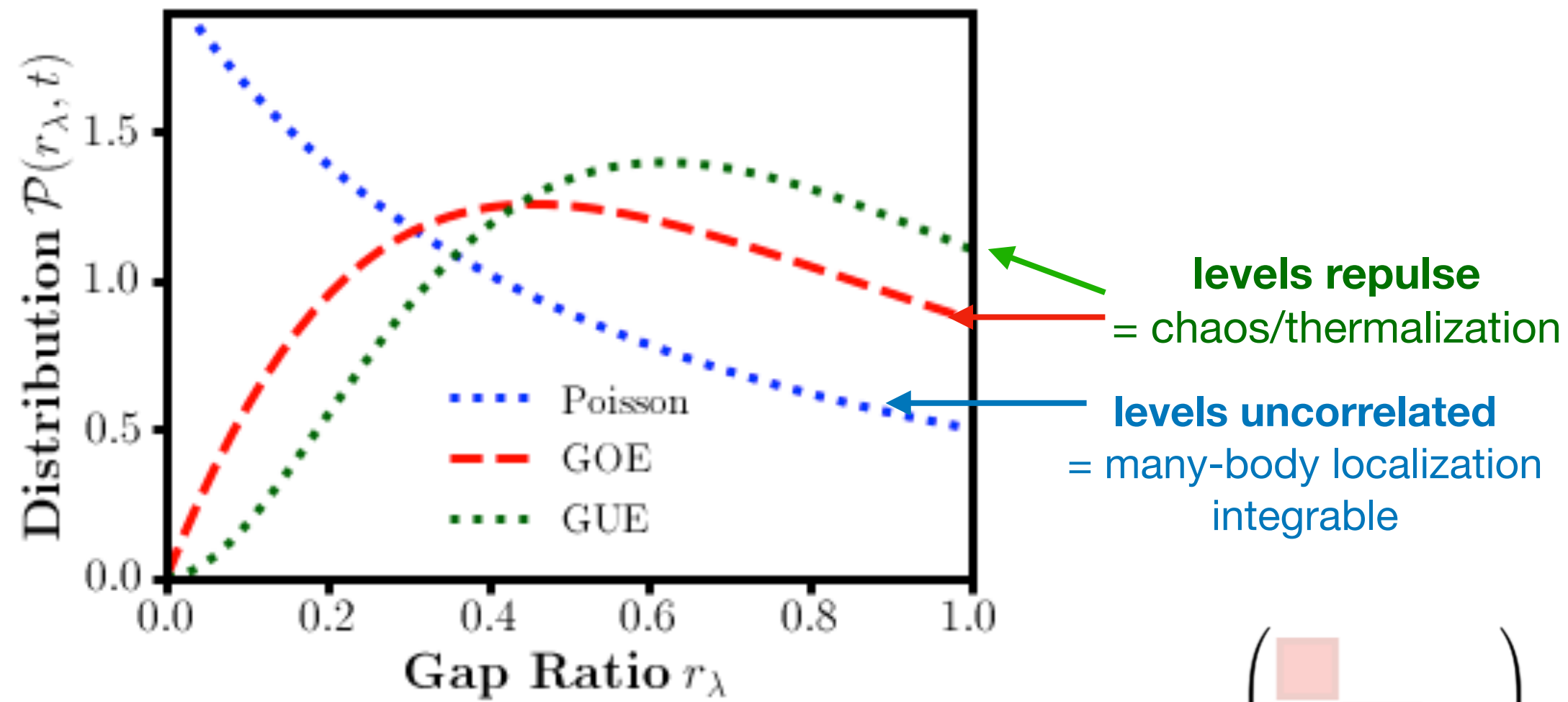
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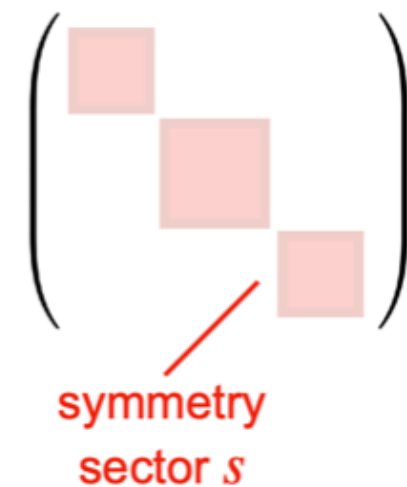
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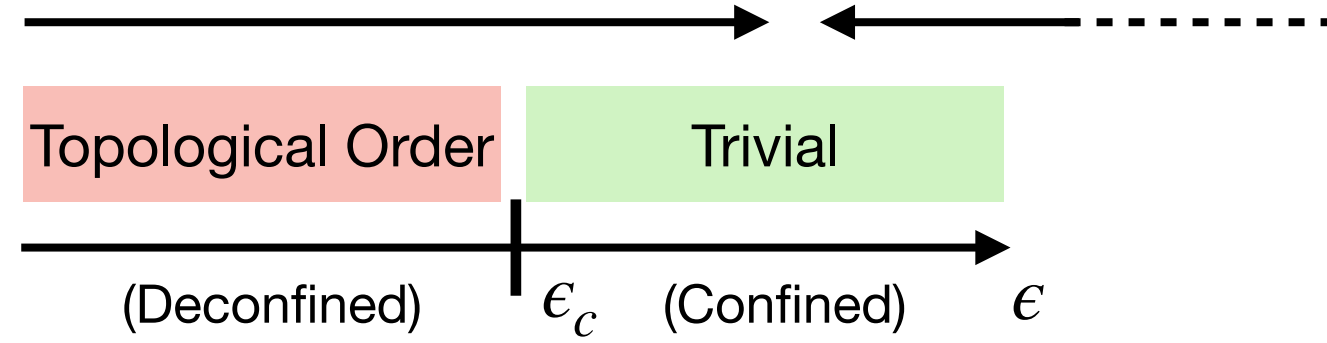
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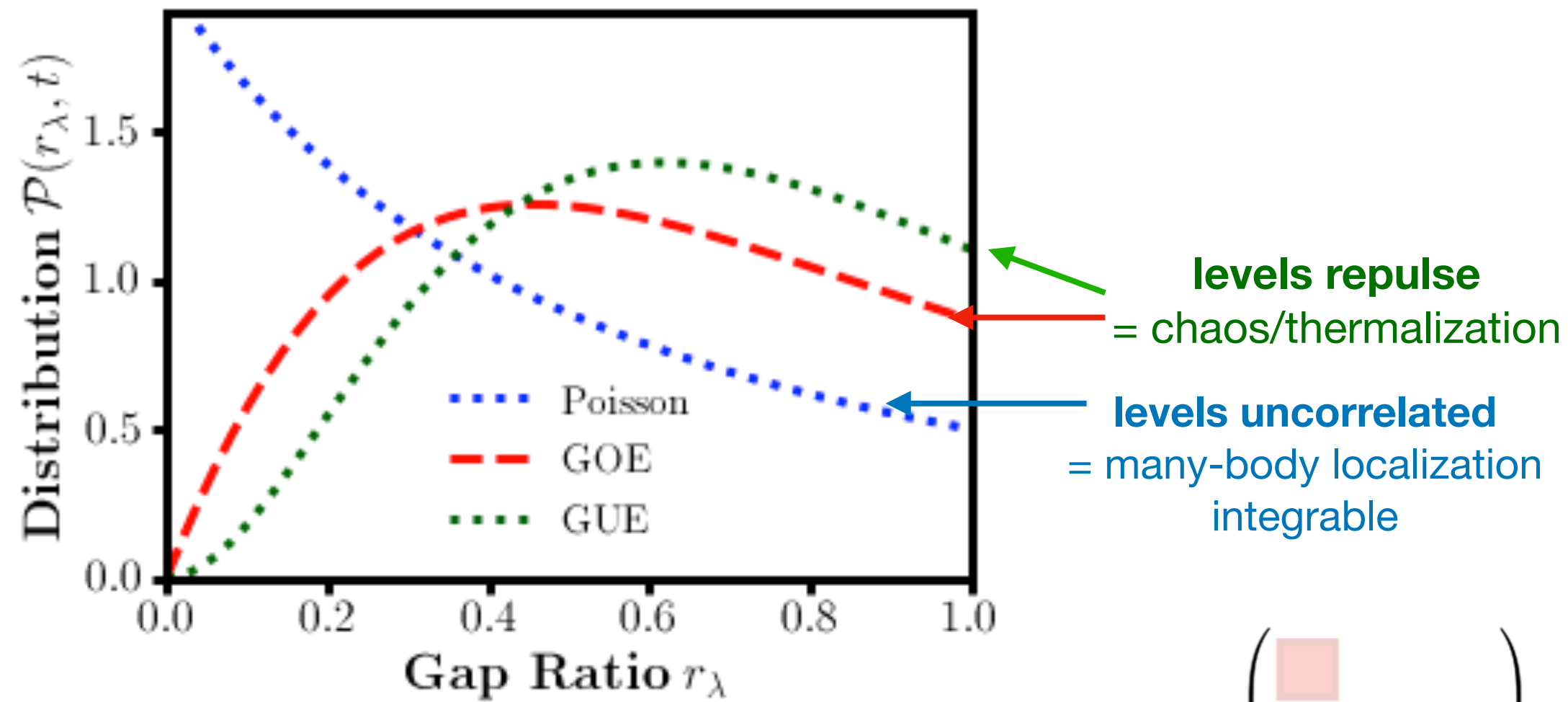
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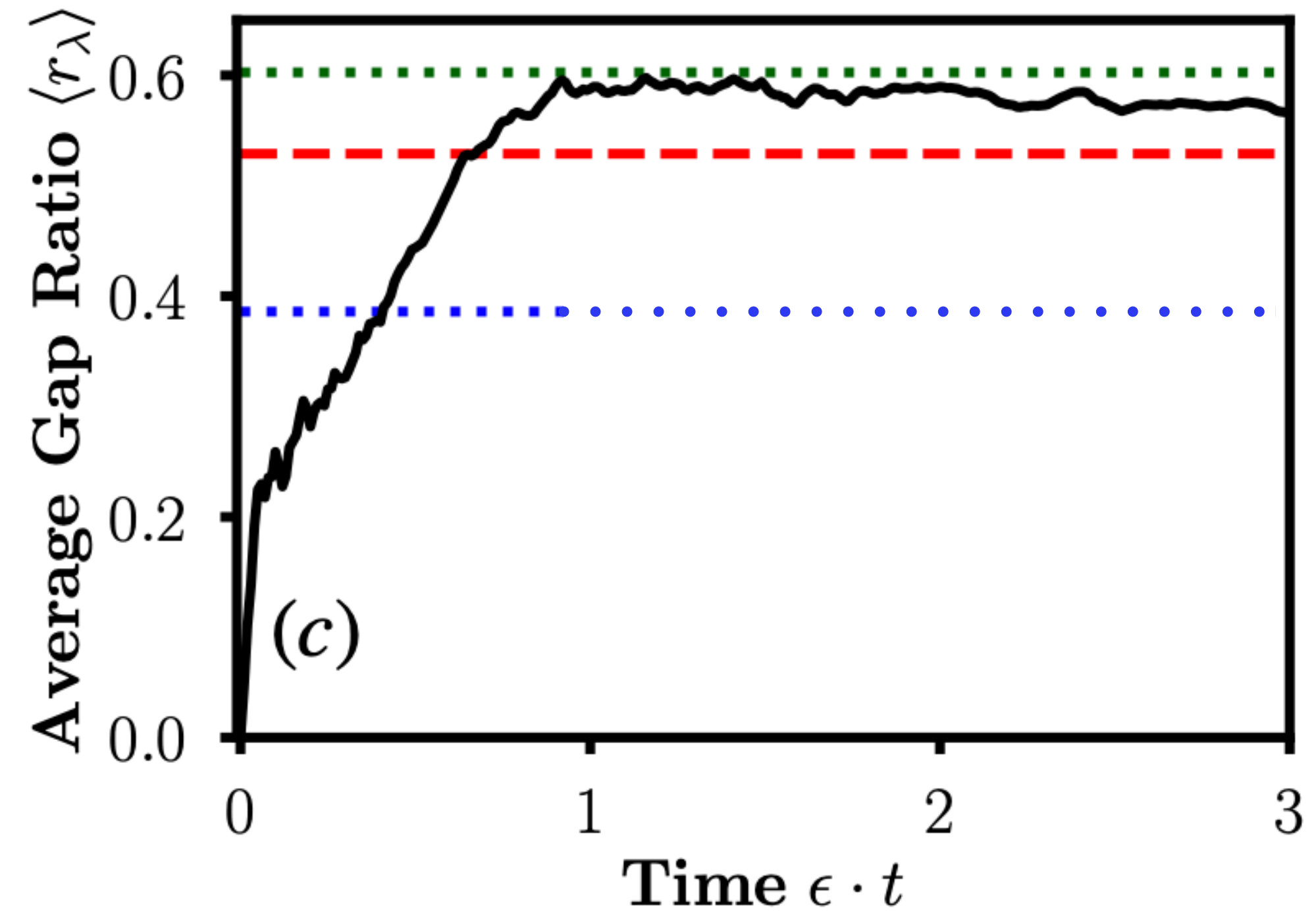
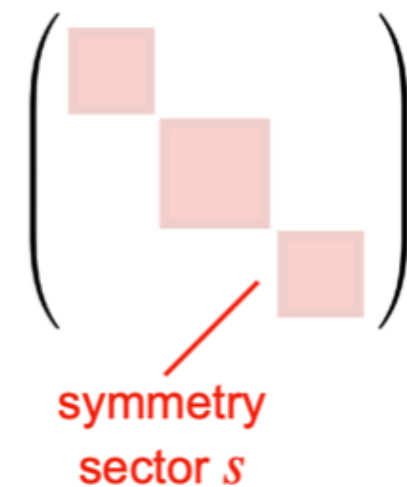
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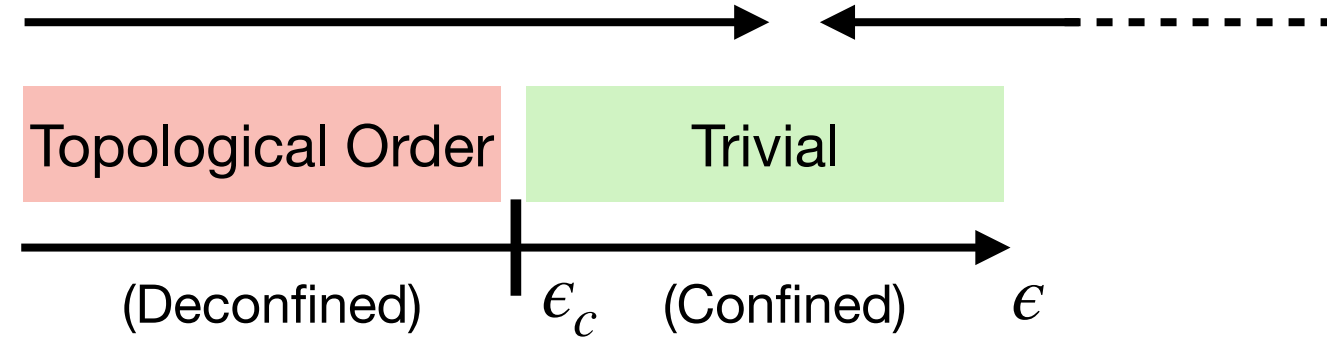
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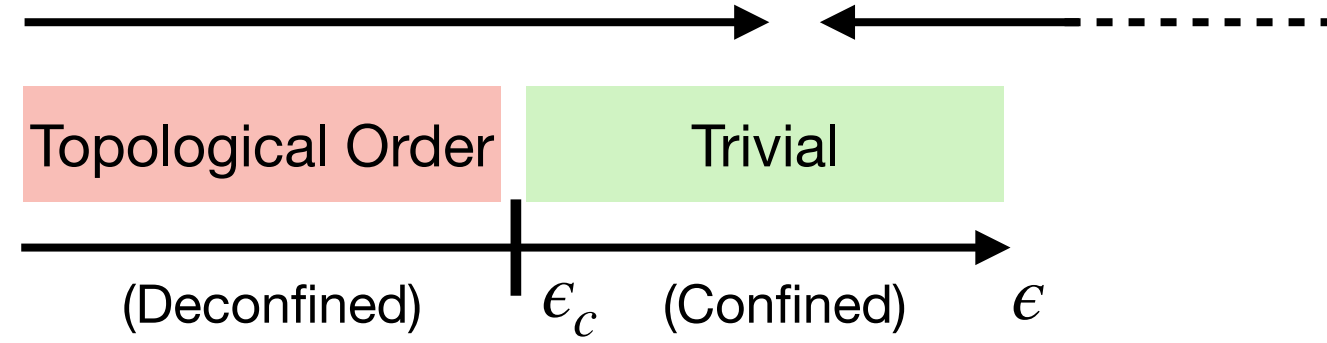
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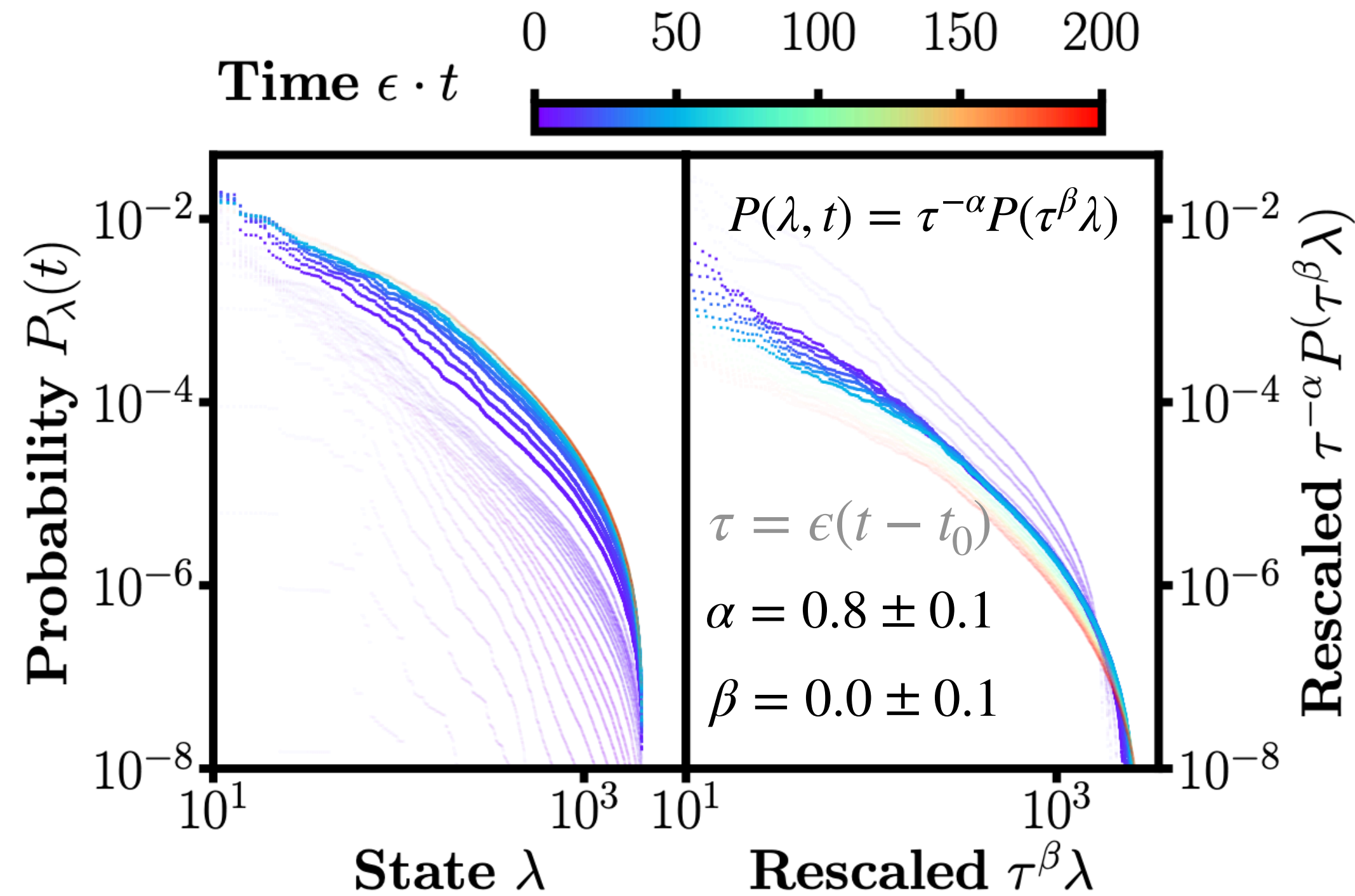
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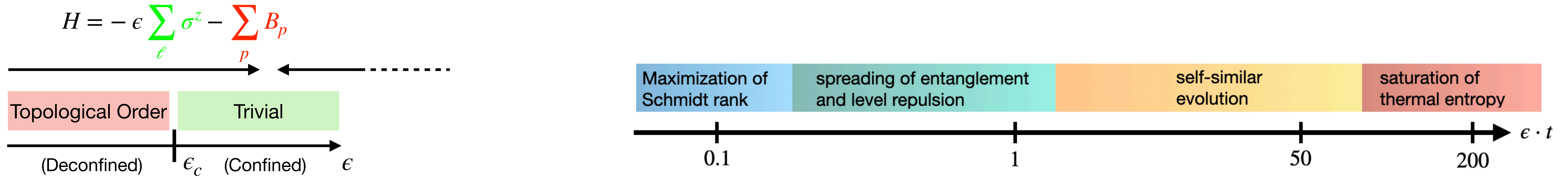
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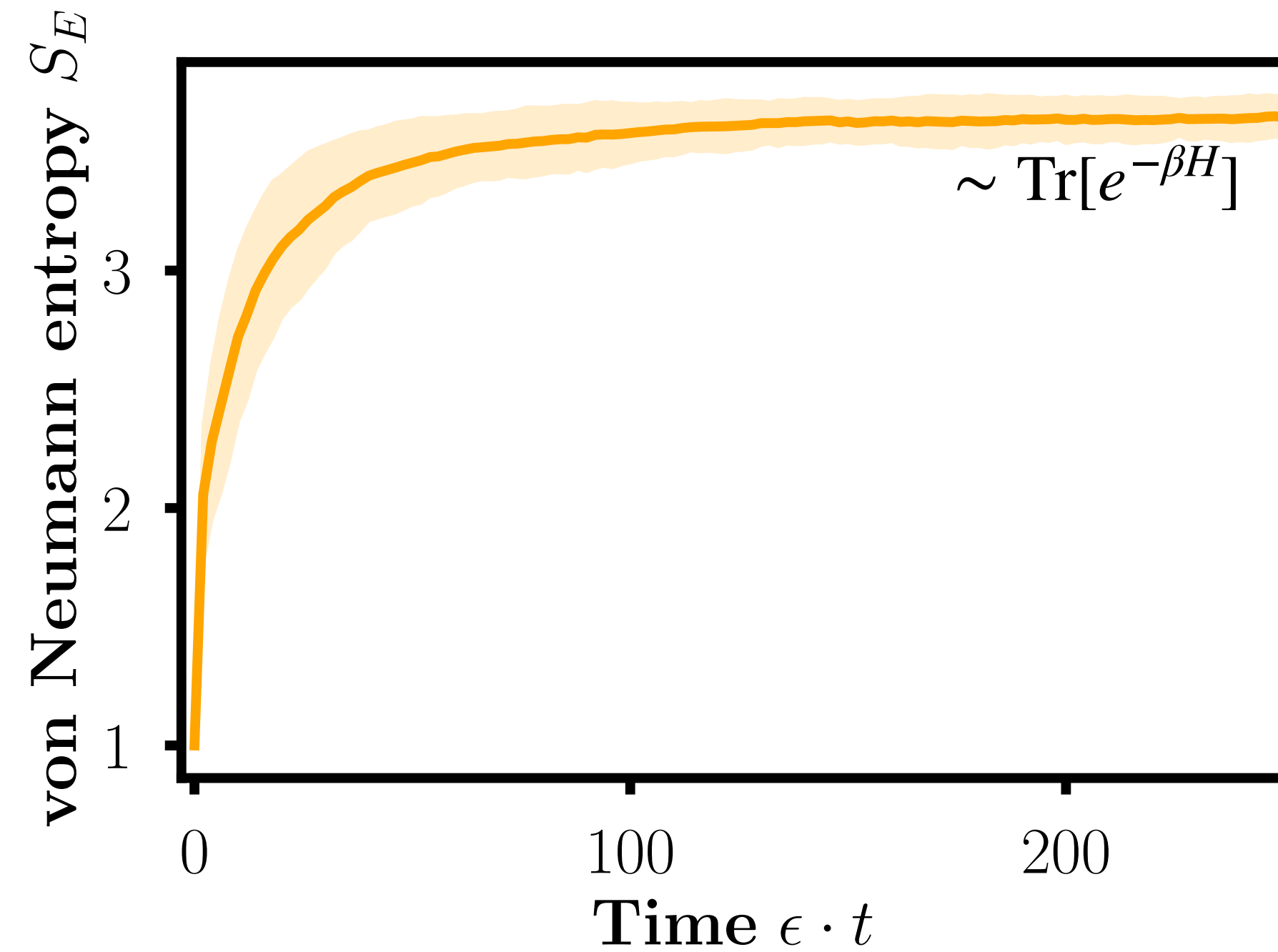
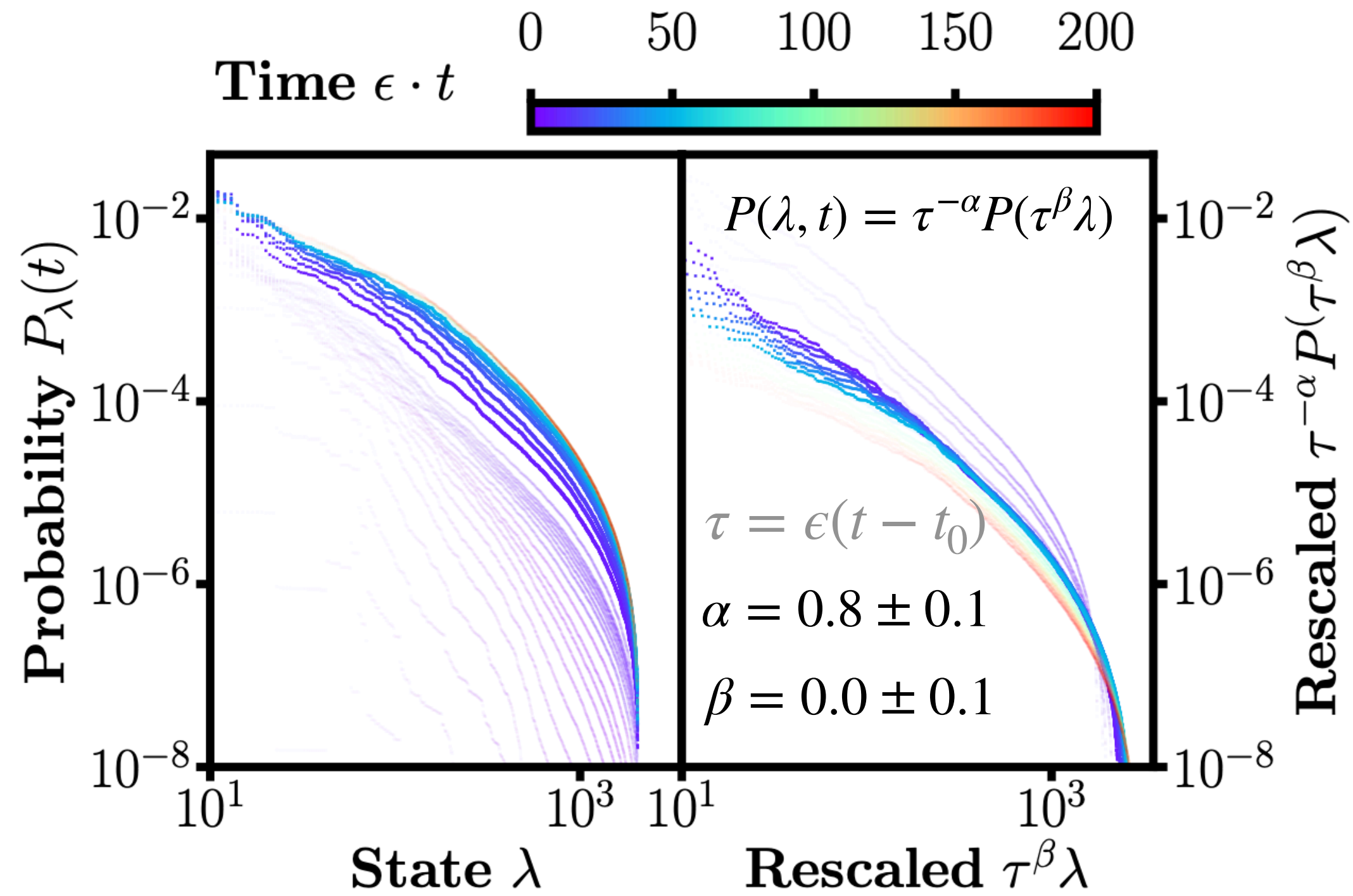
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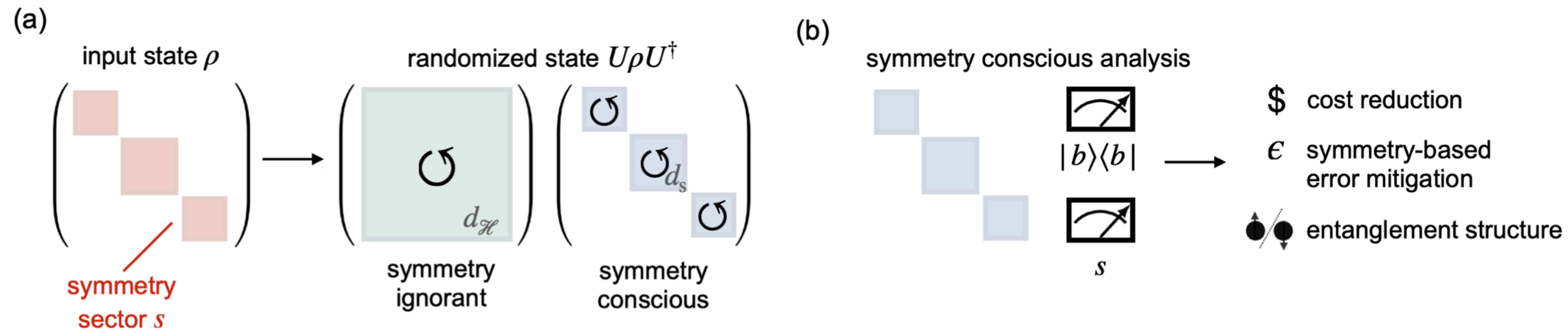
Conclusions

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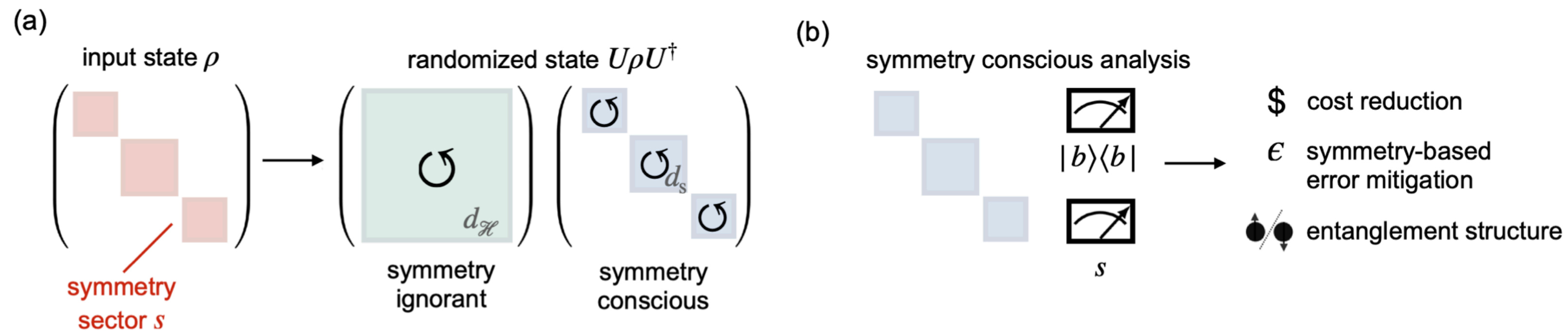
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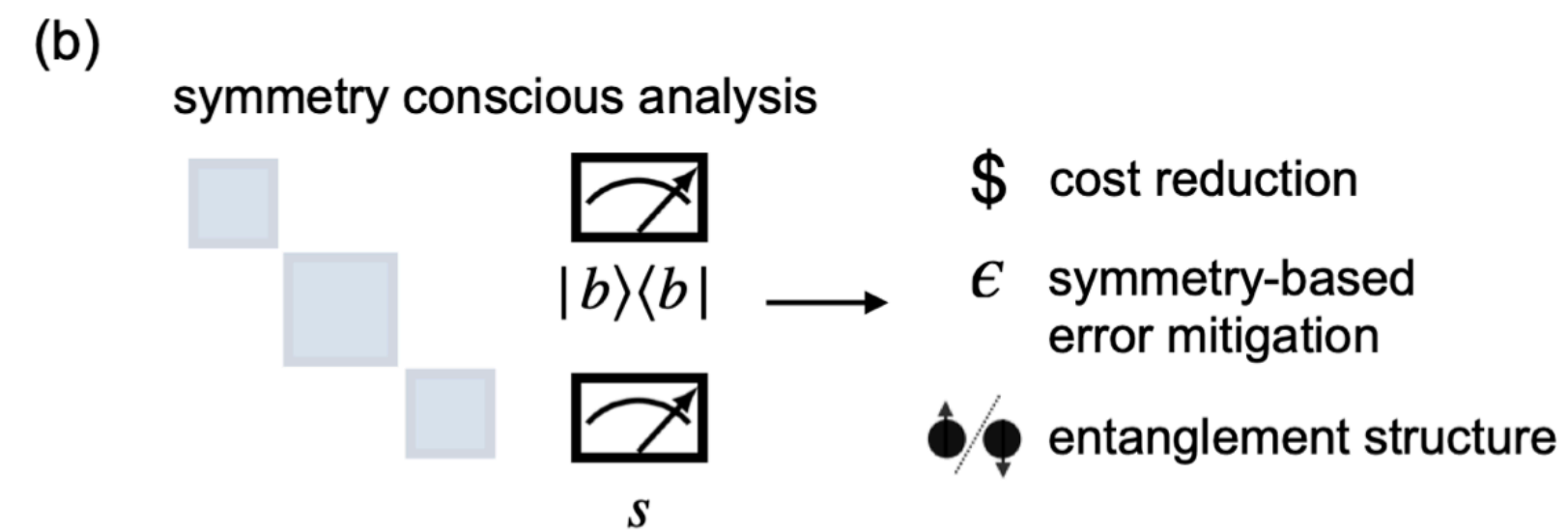
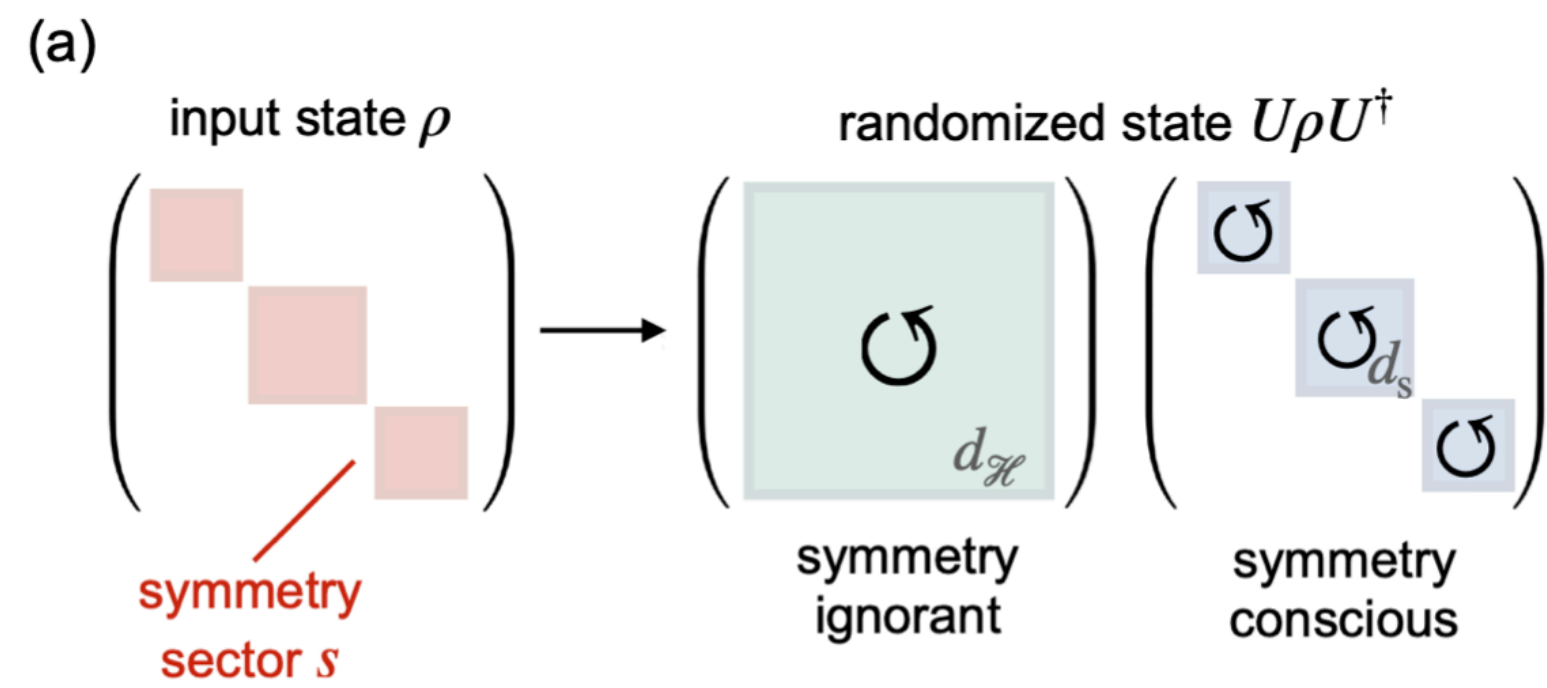


If you can implement time evolution,
you can implement
symmetry-conscious randomization!

Elben, Vermersch, Dalmonte, Cirac, Zoller, PRL 120, 050406 (2018)
Vermersch, Elben, Dalmonte, Cirac, Zoller, PRA 97, 023604 (2018)

Conclusions

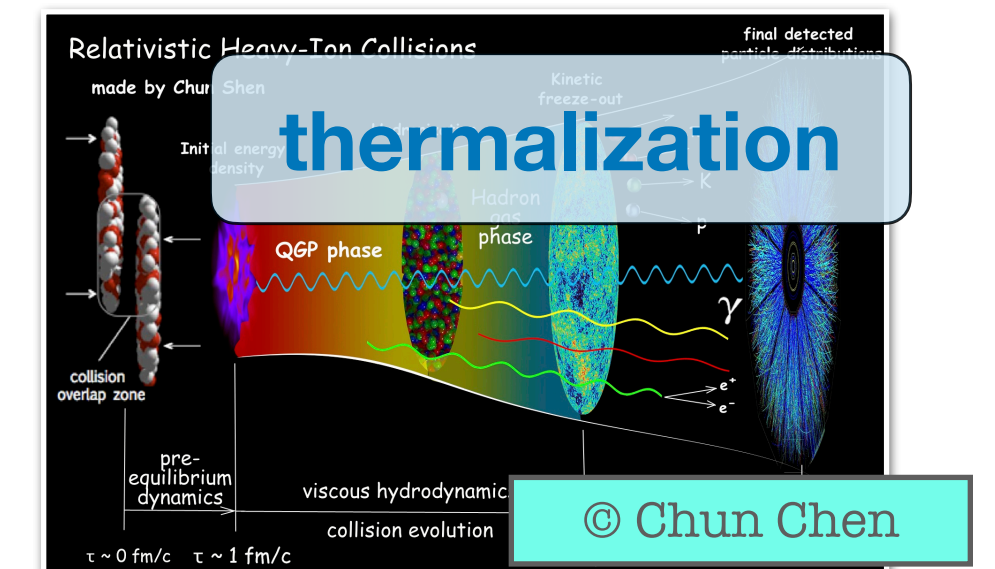
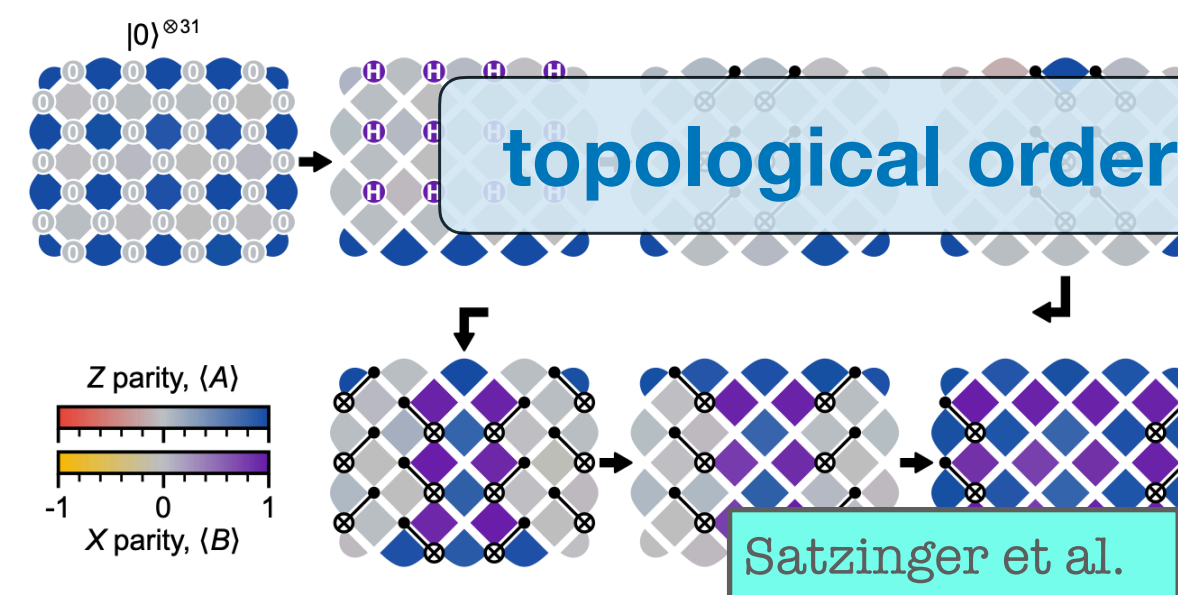
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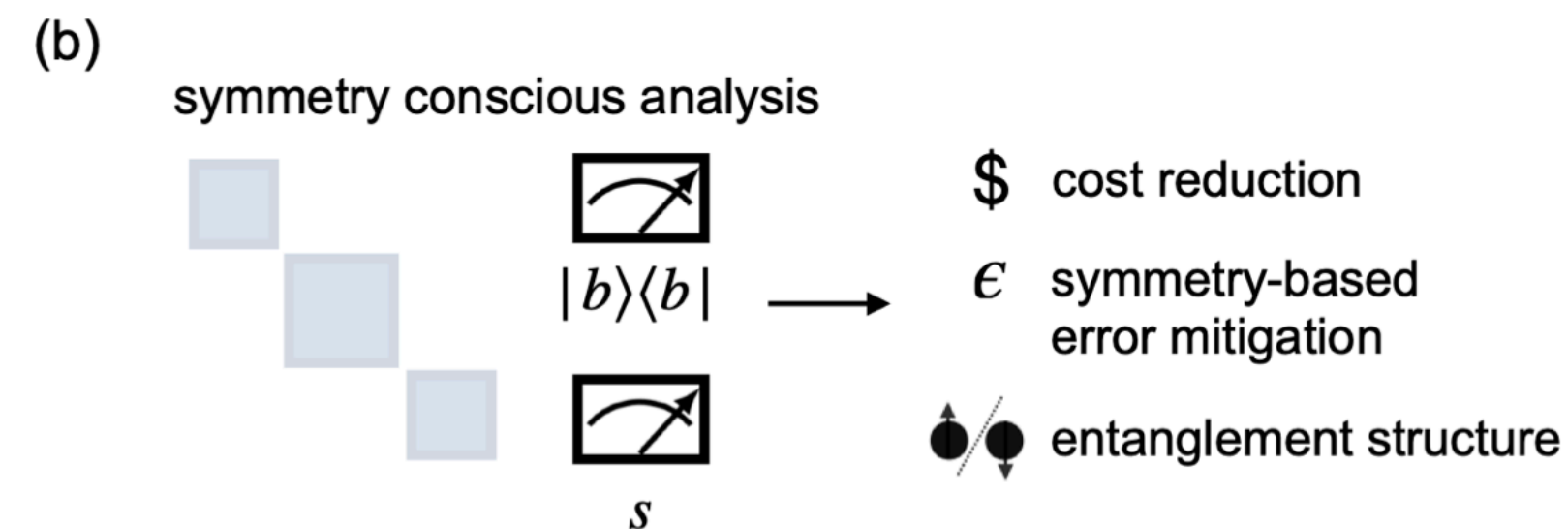
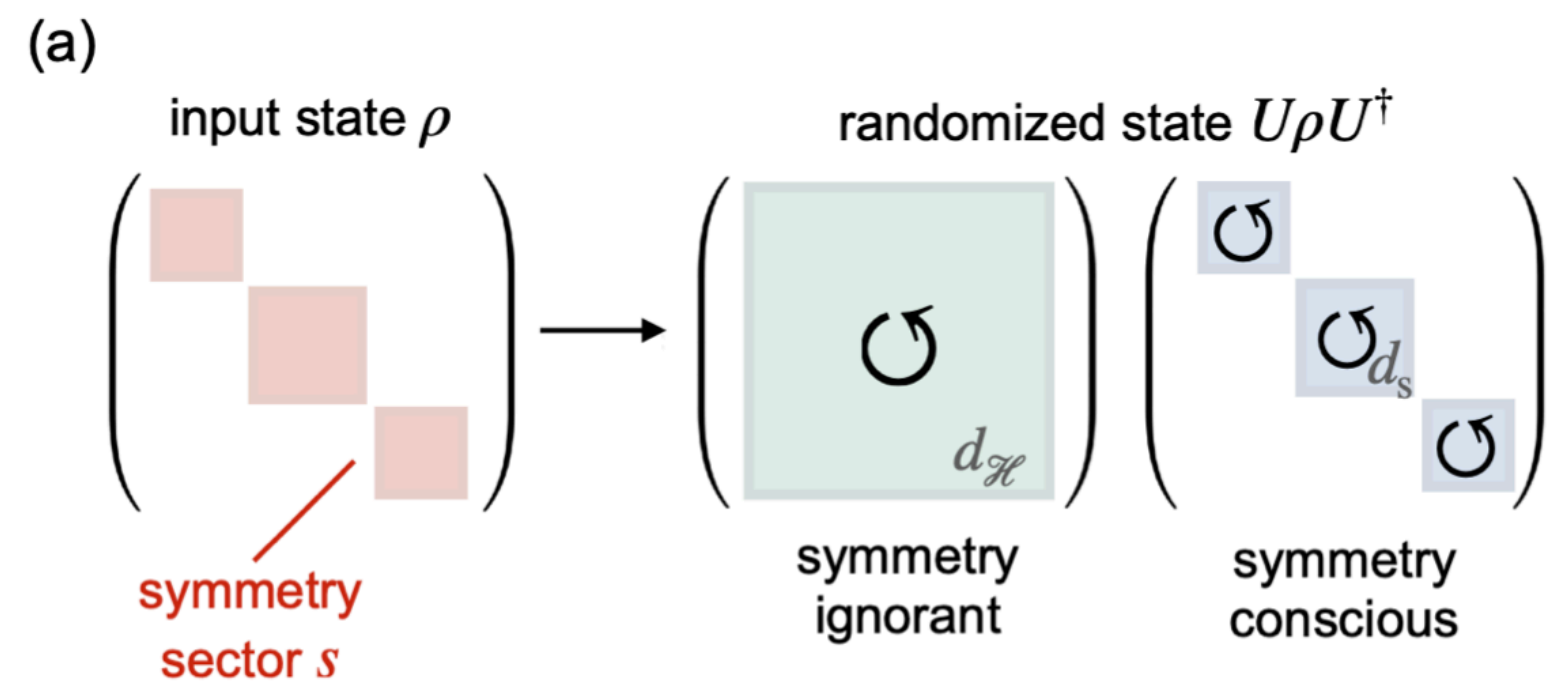
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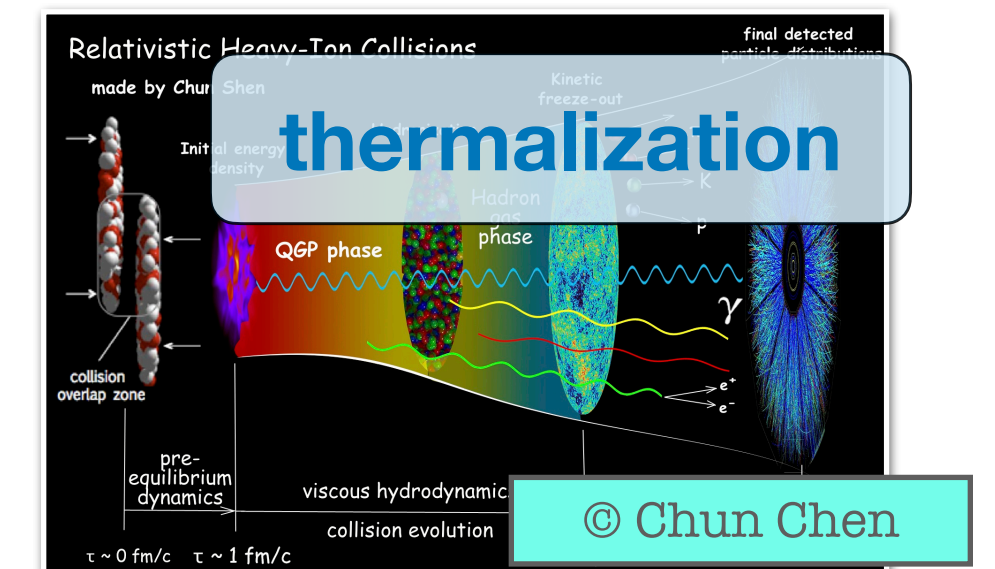
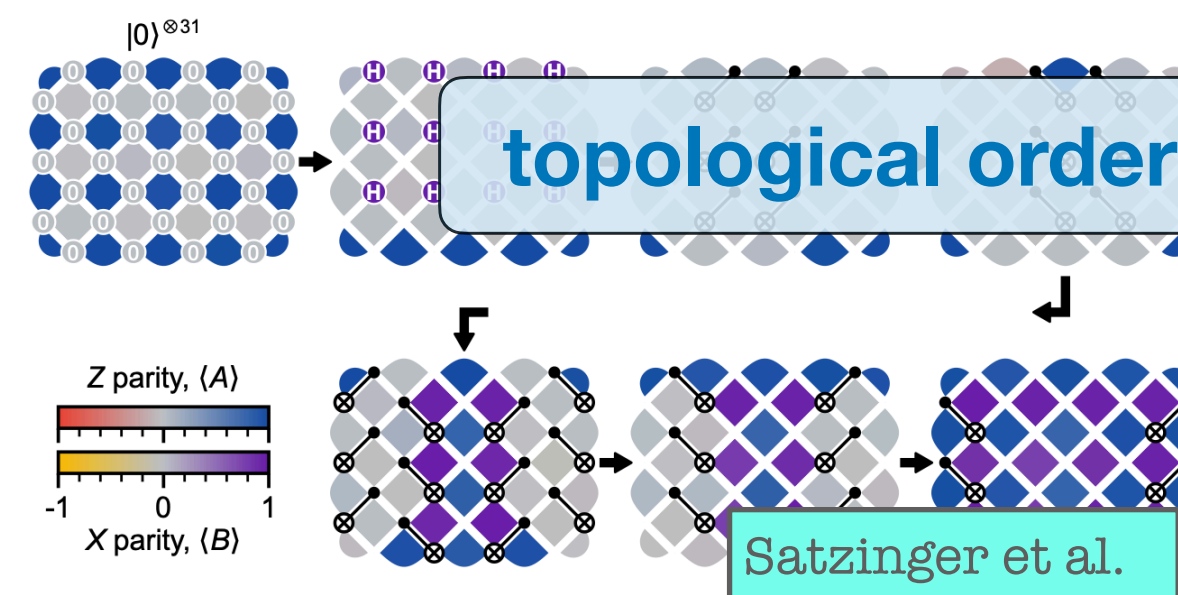
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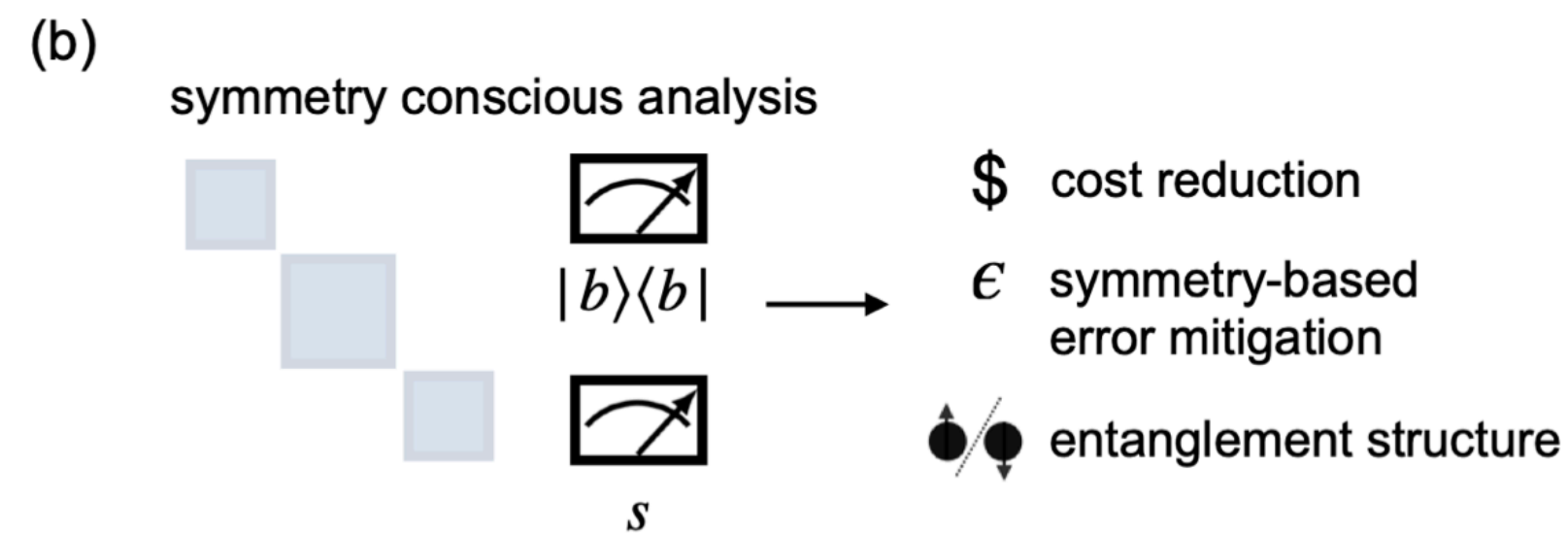
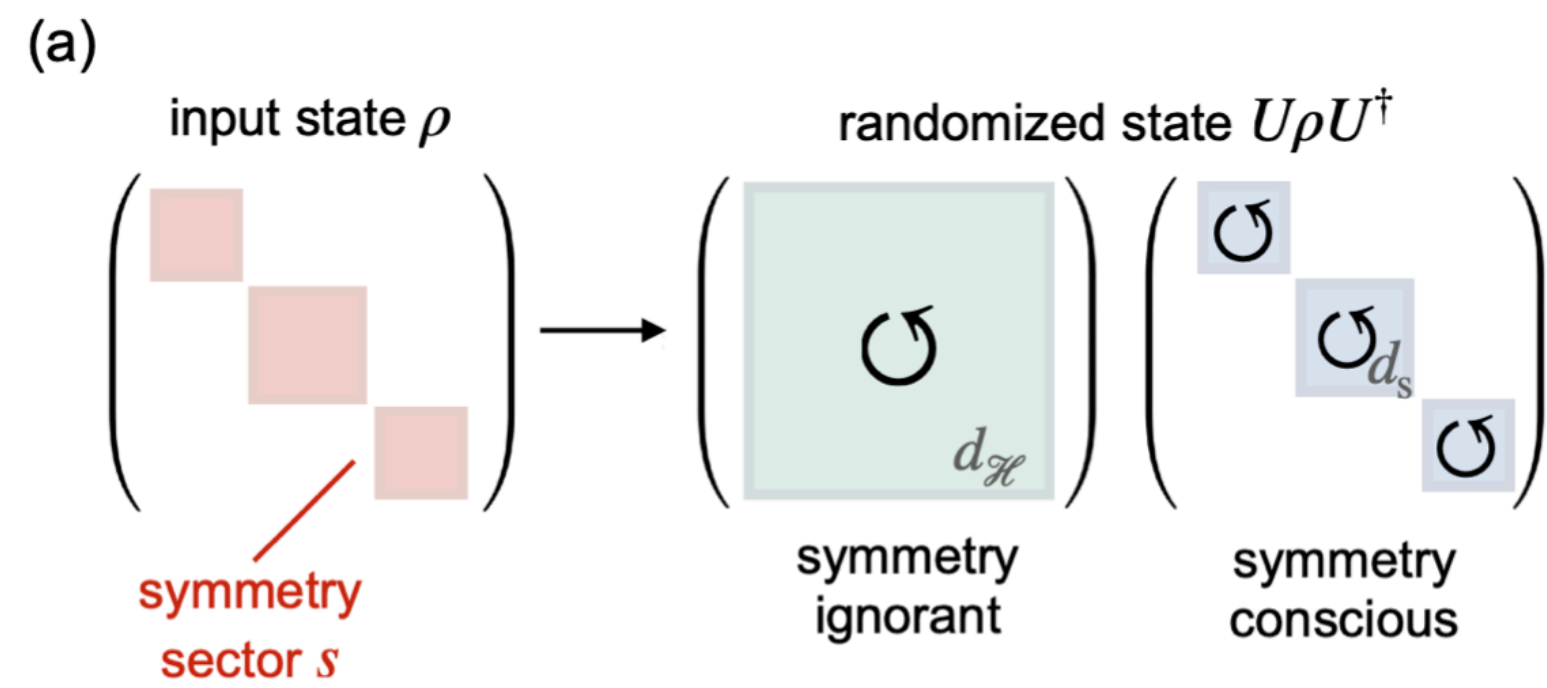
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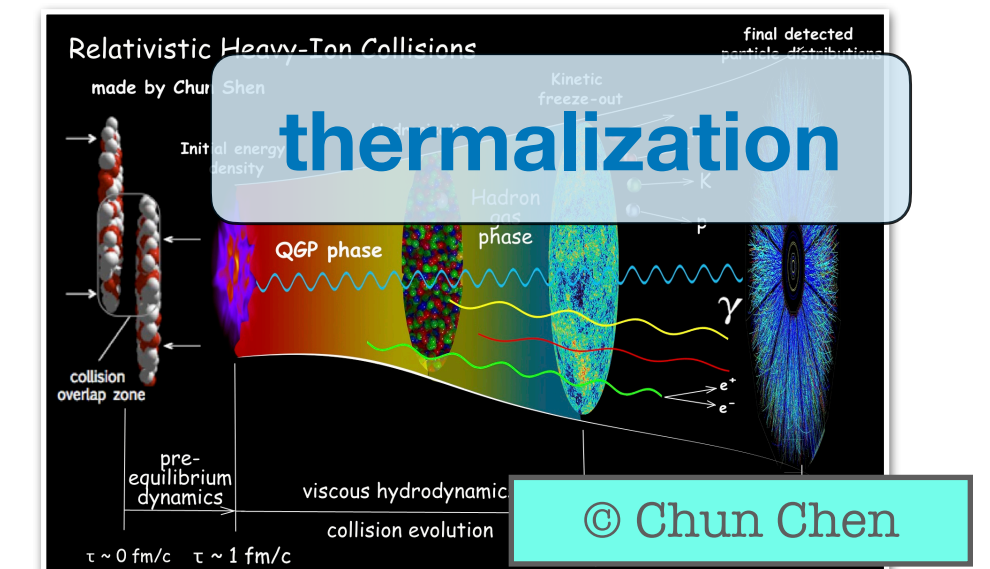
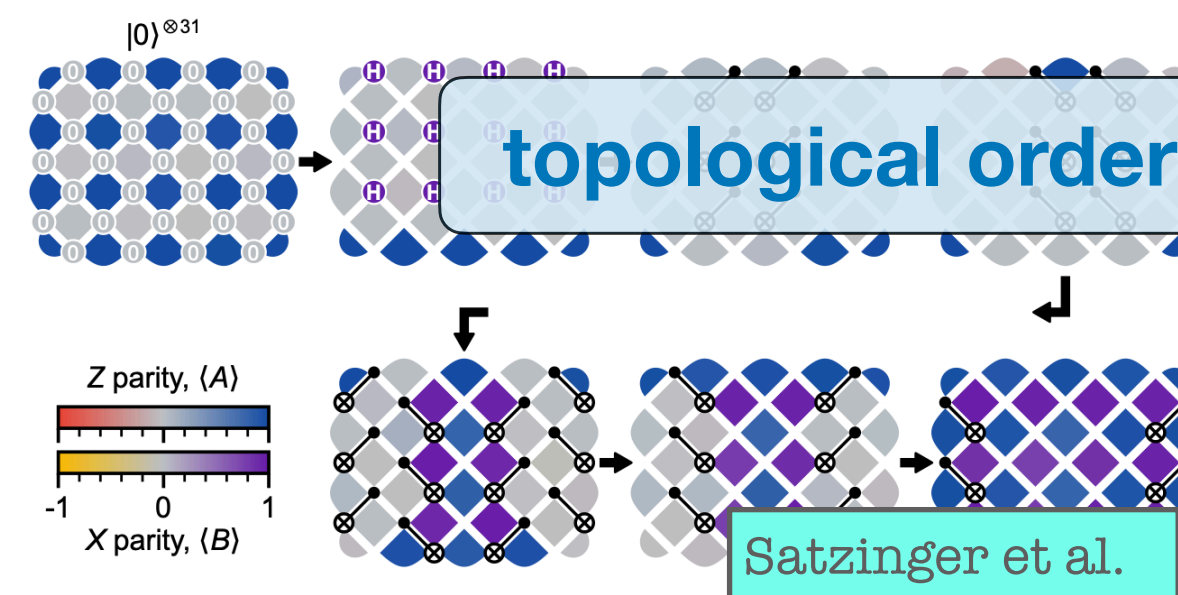
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- Deep scrambling algorithms soon feasible. A lot more work. Robustness? Performance guarantees?
- Explore and combine with smart ideas: quantum variational & machine learning

Kokail, et al, Phys. Rev. Lett. 127, 170501 (2021)
 Huang et al., Science 376 1182 (2022)
 Huang, Nat. Rev. 4 (2022)

Thanks to a lot of smart collaborators



Ron Belyansky



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Maryland



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(NCSU)

Innsbruck



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Torsten Zache



Jake Bringewatt



Zohreh Davoudi



Alexey Gorshkov



Joe Carolan
(UIUC → UMD)



Eugene Dumitrescu
(ORNL)

Duke



Marko Cetina



Lei Feng



Or Katz



Chris Jarzynski



Jon Kunjummen

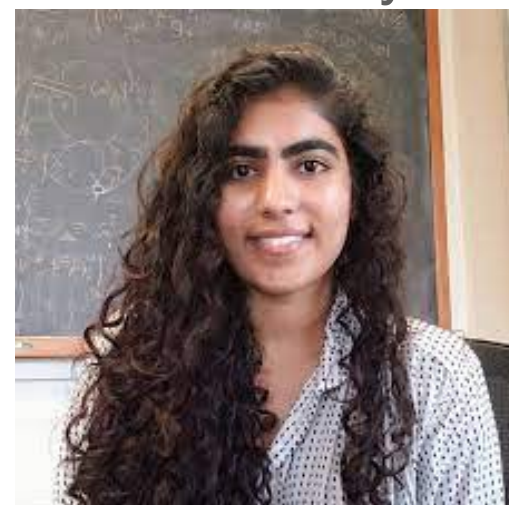


Connor Powers



Anthony Ciavarella

Seattle



Greeshma Oruganti



Alex Schuckert



Seth Whitsitt



Nicole Yunger Halpern



Martin Savage



Henry Froland



Marc Illa Subina



Jeremy Hartse