QUANTUM SIMULATION OF LATTICE GAUGE THEORIES IN MORE THAN 1+1-D:

REQUIREMENTS, CHALLENGES AND METHODS

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Nuclear and Particle Physics on a Quantum Computer: Where do we stand now? ECT*, Trento, June 2023

QUANTUM SIMULATION OF LATTICE GAUGE THEORIES IN MORE THAN 1+1-D: A FEW THOUGHTS ON REQUIREMENTS, CHALLENGES AND METHODS

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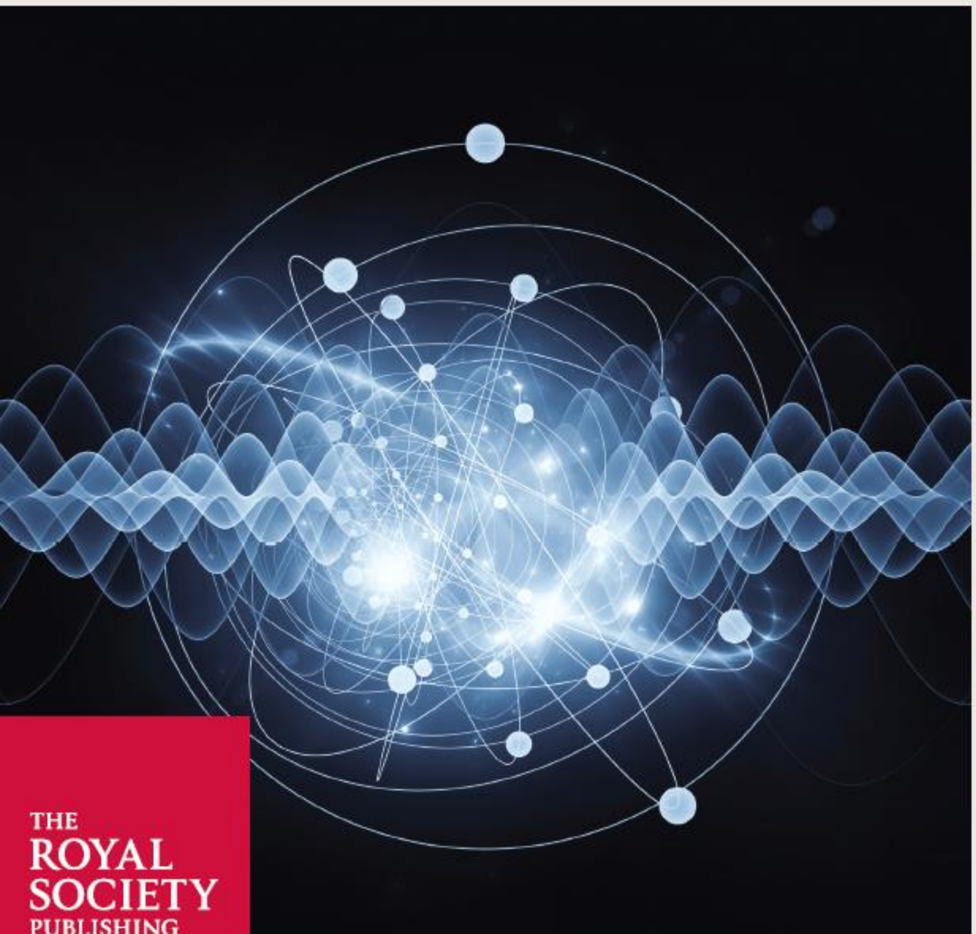
PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A

MATHEMATICAL, PHYSICAL AND ENGINEERING SCIENCES

Quantum technologies in particle physics

Theme issue compiled and edited by Steven D. Bass and Erez Zohar



Quantum simulation of lattice gauge theories in more than one space dimension—requirements, challenges and methods

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Over recent years, the relatively young field of quantum simulation of lattice gauge theories, aiming at implementing simulators of gauge theories with quantum platforms, has gone through a rapid development process. Nowadays, it is not only of interest to the quantum information and technology communities. It is also seen as a valid tool for tackling hard, non-perturbative gauge theory problems by particle and nuclear physicists. Along the theoretical progress, nowadays more and more experiments implementing such simulators are being reported, manifesting beautiful results, but mostly on $1+1$ dimensional physics. In this article, we review the essential ingredients and requirements of lattice gauge theories in more dimensions and discuss their meanings, the challenges they pose and how they could be dealt with, potentially aiming at the next steps of this field towards simulating challenging physical problems in analogue, or analogue-digital ways.

This article is part of the theme issue 'Quantum technologies in particle physics'.

Quantum Simulation of LGT – (Partial?) Summary

- "Step 1" review papers:
  - Wiese, Annalen der Physik 525, 777 (2013)
  - Zohar, Cirac, Reznik, Rep. Prog. Phys. 79, 014401 (2016)
  - Dalmonte, Montangero, Cont. Phys. 57, 388 (2016)

Summarizing the first theoretical proposals for simulating Kogut-Susskind Hamiltonian, using ultracold atoms in optical lattices.

- Experiments (analog / hybrid*, specially built systems):
  - Martinez, Muschik et al, Nature 534, 516 (2016)
  - Bernien et al, Nature 551, 579 (2017)
  - Kokail et al, Nature 569, 365 (2019)
  - Schweizer et al, Nature Physics 15, 1168 (2019)
  - Mil et al, Science 367, 648, 1168 (2020)
  - Yang at al, Nature 587, 392 (2020)
  - Semeghini et al, Science 374, 1242 (2021)
  - Zhou et al, Science 377, 311 (2022)
  - Riechert et al, Phys. Rev. B 105, 205141 (2022)

Implementations of one dimensional / small size Abelian theories, getting more and more scalable

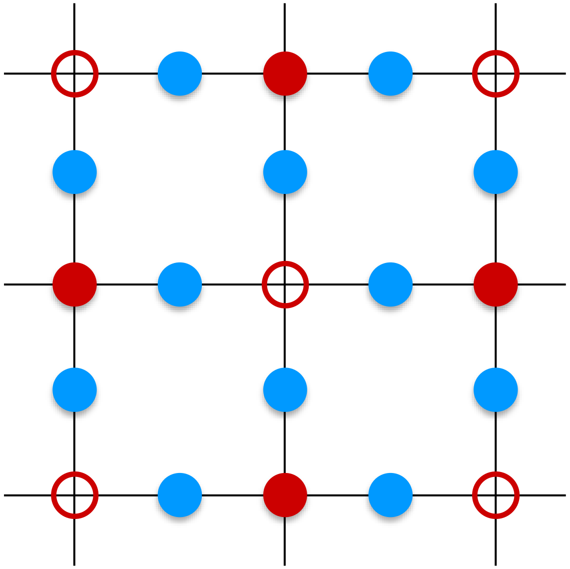
***Add to that:**
many works which implemented things on NISQ cloud devices (e.g. IBMQ, Google, …) and cleverly designed quantum computer algorithms and encodings [Seattle, Trento, Iowa, Fermilab, Maryland, Waterloo, …]

- Contemporary review / "roadmap" papers:
  - Zohar, Nature 534, 7608 (2016)
  - Bañuls et al, Eur. Phys. J. D 74, 165 (2020)
  - Davoudi, Raychowdhury, Shaw, Phys. Rev. D 104, 074505 (2021)
  - Aidelsburger et al, Phil. Trans. R-Soc. A 380, 20210064 (2022)
  - Zohar, Phil. Trans. R-Soc. A 380, 20210069 (2022)
  - Klco, Roggero, Savage, Rep. Prog. Phys. 85, 064301 (2022)
  - Bauer, Davoudi et al, PRX Quantum 4, 027001 (2023)

Higher dimensions and plaquette interactions, dealing with the fermionic matter, dual formulations, quantum computing algorithms, …

Hamiltonian LGTs

- **The lattice is spatial**: time is a continuous, real coordinate.
- **Matter particles** (mostly fermions) – on the **vertices**.
- **Gauge fields** – on the lattice's **links**



Challenge 1:
Our simulated platform needs to describe both fermionic and non-fermionic physics.

1+1d: Jordan-Wigner.

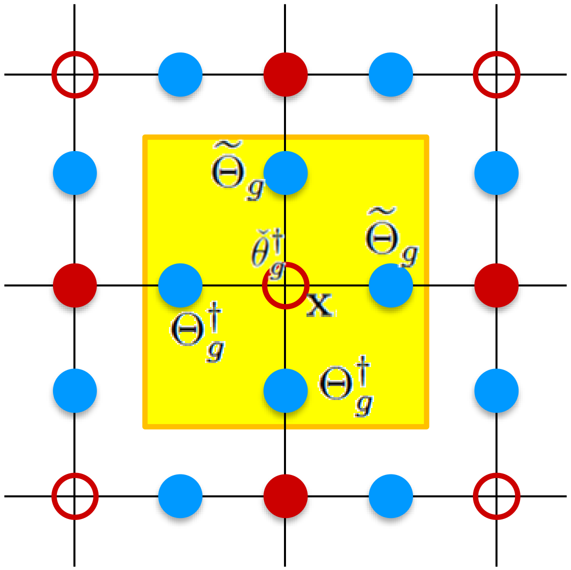
Kogut and Susskind, Phys. Rev. D 11, 395 (1975)

Gauge Transformations

- Act on both the matter and gauge degrees of freedom.

- **Local** : a unique transformation (depending on a unique element of the **gauge group**) may be chosen for each site

- The states are **invariant under each local transformation separately**.



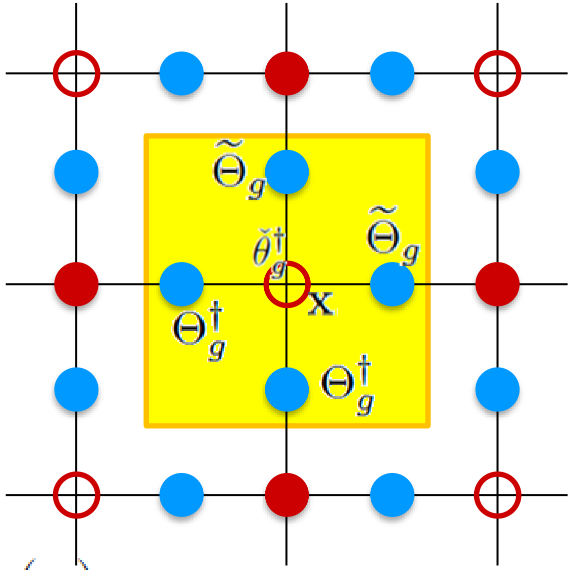
$$\hat{\Theta}_g(\mathbf{x}) = \prod_{k=1\ldots d} \left(\widetilde{\Theta}_g(\mathbf{x},k)\,\Theta_g^\dagger\left(\mathbf{x}-\hat{\mathbf{k}},k\right)\right)\check{\theta}_g^\dagger(\mathbf{x})$$

- **Transformation rules on the links**

$$\{|g\rangle\}_{g\in G}$$

$$\Theta_g |h\rangle = |hg^{-1}\rangle \quad \Theta_g = e^{i\phi_a(g)R_a}$$

$$\widetilde{\Theta}_g |h\rangle = |g^{-1}h\rangle \quad \widetilde{\Theta}_g = e^{i\phi_a(g)L_a}$$



- **Gauge Transformations:**

$$\hat{\Theta}_g(\mathbf{x}) = \prod_{k=1\ldots d}\left(\widetilde{\Theta}_g(\mathbf{x},k)\,\Theta^\dagger_g\left(\mathbf{x}-\hat{\mathbf{k}},k\right)\right)\breve{\theta}^\dagger_g(\mathbf{x})$$

$$\hat{\Theta}_g(\mathbf{x})|\Psi\rangle = |\Psi\rangle \quad \forall \mathbf{x}, g$$

- **Compact Lie Group → Generators → Gauss law, left and right E fields:**

$$G_a(\mathbf{x}) = \sum_{k=1\ldots d}\left(L_a(\mathbf{x},k) - R_a\left(\mathbf{x}-\hat{\mathbf{k}},k\right)\right) - Q_a(\mathbf{x})$$

$$G_a(\mathbf{x})|\Psi\rangle = 0 \quad [G_a(\mathbf{x}), H] = 0 \quad \forall \mathbf{x}, a$$

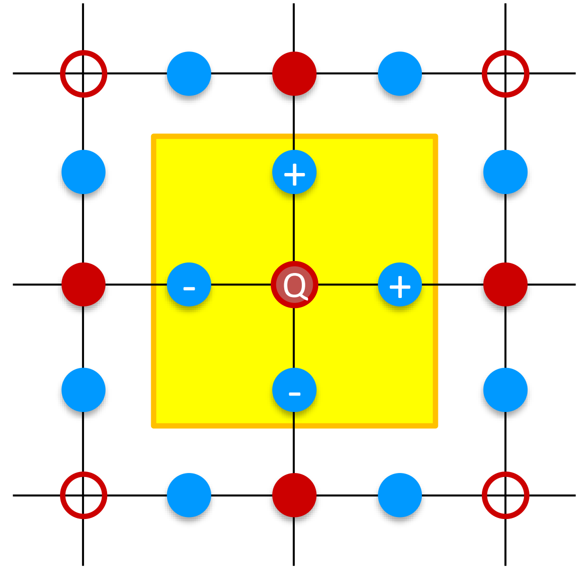
Challenge 2:
Impose / maintain / surpass gauge invariance

Structure of the Hilbert Space

- Generators of gauge transformations (cQED):

$$G(\mathbf{x}) = \mathrm{div} L(\mathbf{x}) - Q(\mathbf{x})$$

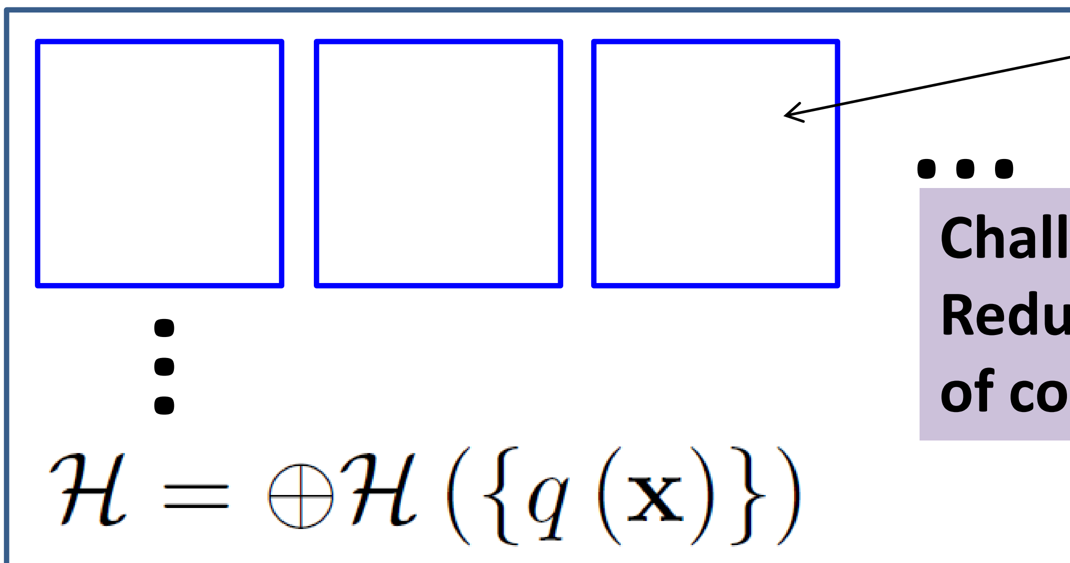
$$\equiv \sum_k \left(L_k(\mathbf{x}) - L_k(\mathbf{x} - \hat{\mathbf{e}}_k)\right) - Q(\mathbf{x})$$



Gauss' Law $G(\mathbf{x})|\psi\rangle = q(\mathbf{x})|\psi\rangle$

$$[G(\mathbf{x}), H] = 0 \quad \forall \mathbf{x}$$

Sectors with fixed
Static charge
configurations



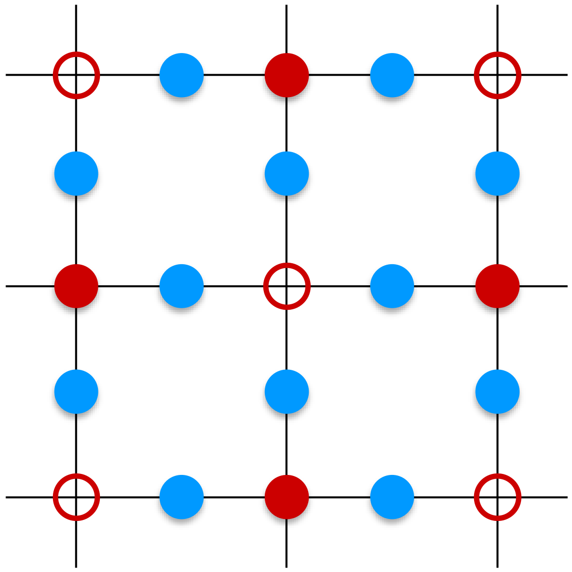
$$\mathcal{H} = \oplus \mathcal{H}(\{q(\mathbf{x})\})$$

Challenge 2.1:
Redundant Hilbert Space – Waste of computational resources.

1+1d:
Solve Gauss's law for the field.

Allowed Interactions

- Must preserve the symmetry – commute with the “Gauss Laws” (generators of symmetry transformations)

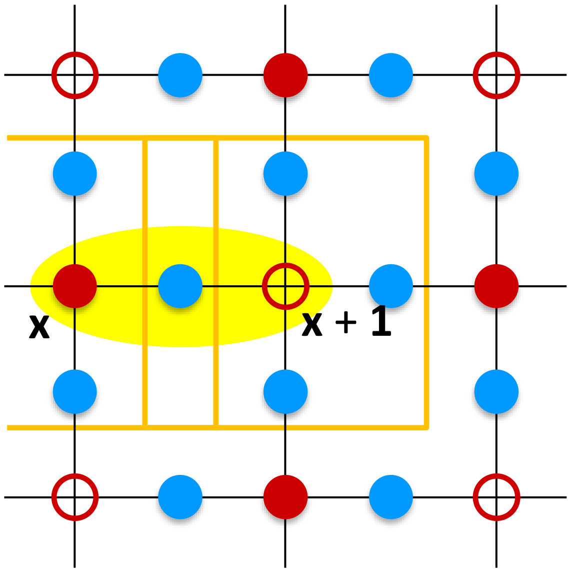


Allowed Interactions

- Must preserve the symmetry – commute with the "Gauss Laws" (generators of symmetry transformations)
- First option: Link (**matter**-**gauge**) interaction:

$$\psi_m^\dagger(\mathbf{x})\,U_{mn}(\mathbf{x},k)\,\psi_n\left(\mathbf{x}+\hat{\mathbf{k}}\right)$$

  – A **fermion** hops to a **neighboring site**, and the **flux on the link in the middle changes** to preserve **Gauss laws on the two relevant sites**

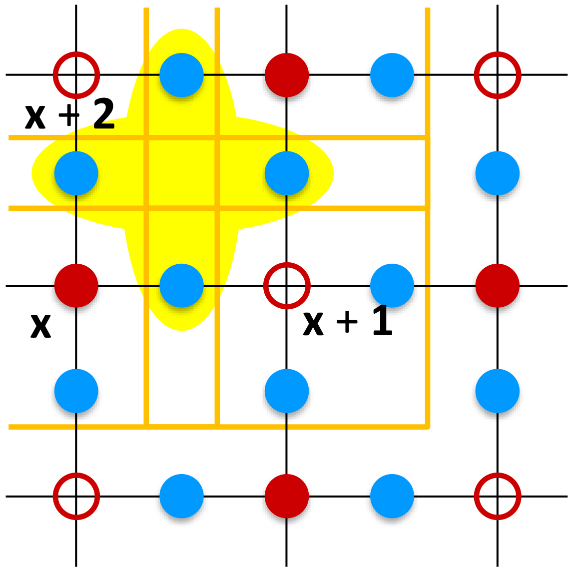


Allowed Interactions

- Must preserve the symmetry – commute with the "Gauss Laws" (generators of symmetry transformations)
- <u>Second option</u>: plaquette interaction:

$$\mathrm{Tr}\left(U(\mathrm{x},1)U(\mathrm{x}+\hat{1},2)U^{\dagger}(\mathrm{x}+\hat{2},1)U^{\dagger}(\mathrm{x},2)\right)$$

  - The **flux on the links of a single plaquette changes** such that the **Gauss laws on the four relevant sites** is preserved.
  - **Magnetic interaction.**



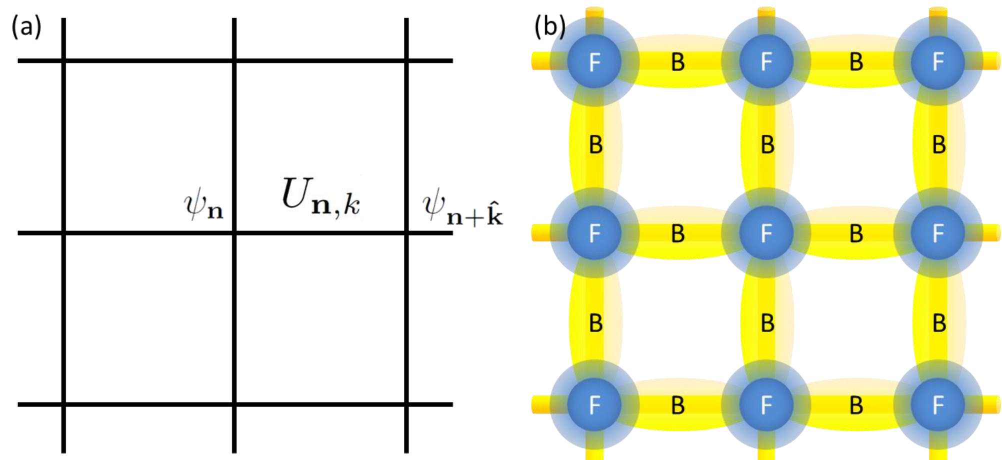
Challenge 3:
Complicated four-body interactions

1+1d: No plaquettes.

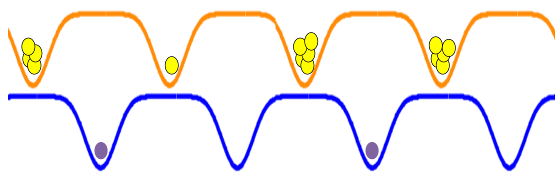
Dealing with the challenges “directly”

Fermions and Bosons → Cold Atoms in Optical Lattices

- **Fermionic** matter fields
- (Bosonic) gauge fields



Super-lattice:



Atomic internal (**hyperfine**) levels

0 (ultracold)

$$\mathbf{F} = \mathbf{I} + \mathbf{L} + \mathbf{S} \qquad \mathbf{F}^2|F, m_{\mathrm{F}}\rangle = F(F+1)|F, m_{\mathrm{F}}\rangle \qquad F_z|F, m_{\mathrm{F}}\rangle = m_{\mathrm{F}}|F, m_{\mathrm{F}}\rangle$$

$$\mathcal{H} = \sum_{\alpha,\beta} \Phi_\alpha^\dagger(\mathbf{x}) \left(\delta^{\alpha\beta} \left(-\frac{\nabla^2}{2m} + V_{\mathrm{op}}^{\alpha}(\mathbf{x}) + V_{\mathrm{T}}(\mathbf{x}) \right) + \Omega^{\alpha\beta}(\mathbf{x}) \right) \Phi_\beta(\mathbf{x})$$

$$+ \sum_{\alpha,\beta,\gamma,\delta} \int \mathrm{d}^3x' \Phi_\alpha^\dagger(\mathbf{x}') \Phi_\beta^\dagger(\mathbf{x}) V_{\alpha\beta\gamma\delta}(\mathbf{x} - \mathbf{x}') \Phi_\gamma(\mathbf{x}) \Phi_\delta(\mathbf{x}')$$

Analog Approach I: Effective Gauge Invariance

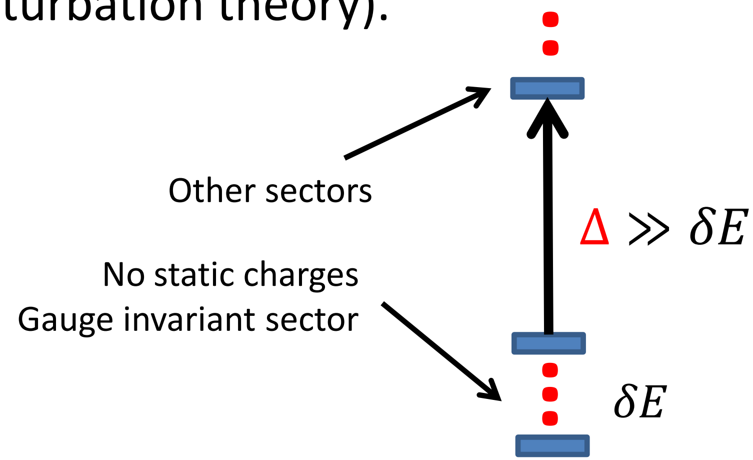
Gauss law is added to the Hamiltonian as a constraint (penalty term, proportional to the square of the symmetry generator).

Leaving a gauge invariant sector of Hilbert space costs too much Energy.

Low energy sector with an effective gauge invariant Hamiltonian.

Emerging plaquette interactions (second order perturbation theory).

- **E. Zohar**, B. Reznik, Phys. Rev. Lett. 107, 275301 (2011)
- **E. Zohar**, J. I. Cirac, B. Reznik, Phys. Rev. Lett. 109, 125302 (2012)
- D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, U.-J. Wiese, P. Zoller, Phys. Rev. Lett. 109, 175302 (2012)
- **E. Zohar**, J. I. Cirac, B. Reznik, Phys. Rev. Lett. 110, 055302 (2013)
- D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, U.-J. Wiese, P. Zoller, Phys. Rev. Lett. 110, 125303 (2013)



Revisited and simplified later:

With dissipation –

K. Stannigel, P. Hauke, D. Marcos, M. Hafezi, M. Dalmonte and P. Zoller, Phys. Rev. Lett. 112, 120406 (2014)

With linear constraints –

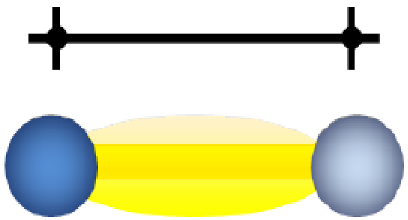
J.C. Halimeh, H. Lang, J. Mildenberger, Z. Jiang, P. Hauke, PRX Quantum 2, 040311 (2021)

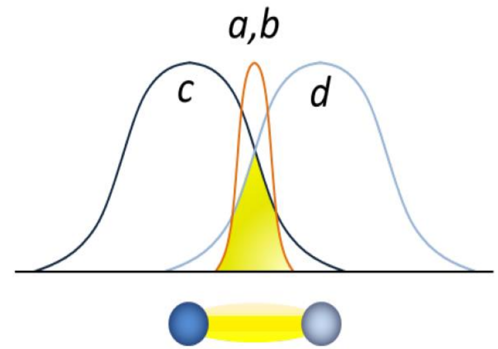
With dynamical decoupling –

V. Kasper, T. V. Zache, F. Jendrzejewski, M. Lewenstein, **E. Zohar**, Phys. Rev. D 107, 014506 (2023)

Analog Approach II:
Atomic Symmetries → Gauge Invariance

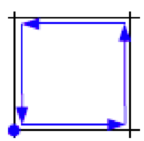
$$\psi_L^\dagger U \psi_R$$

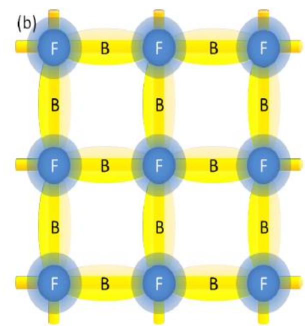




- Links ↔ atomic scattering : gauge invariance is a <u>fundamental</u> symmetry

$$\sum_{\text{plaquettes}} \left(\mathrm{Tr}\left(U_1 U_2 U_3^\dagger U_4^\dagger\right) + h.c.\right)$$





- Plaquettes ↔ gauge invariant links ↔ virtual loops of ancillary fermions.

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. **110**, 125304 (2013)

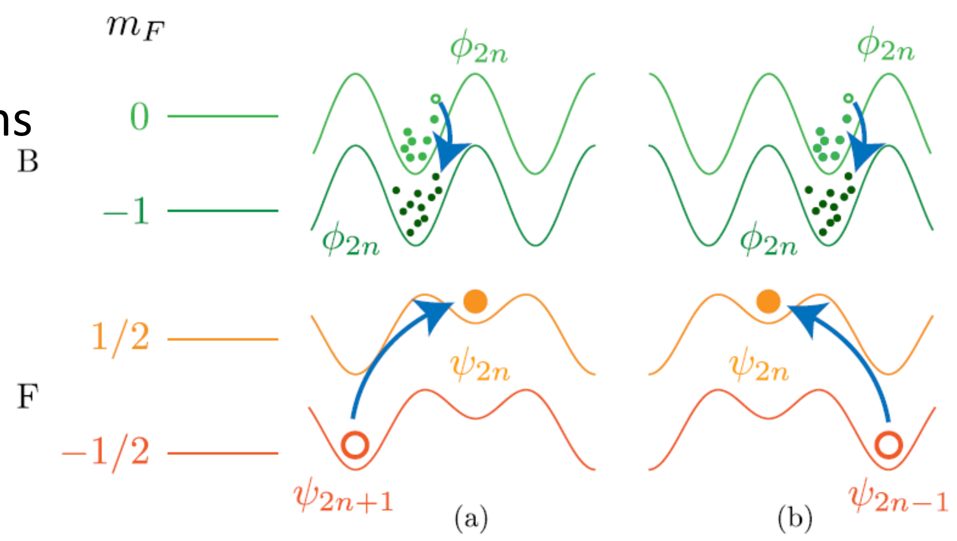
E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. A **88** 023617 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. **79**, 014401 (2016)

D. González Cuadra, **E. Zohar**, J. I. Cirac, New J. Phys. **19** 063038 (2017)

Heidelberg Implementation

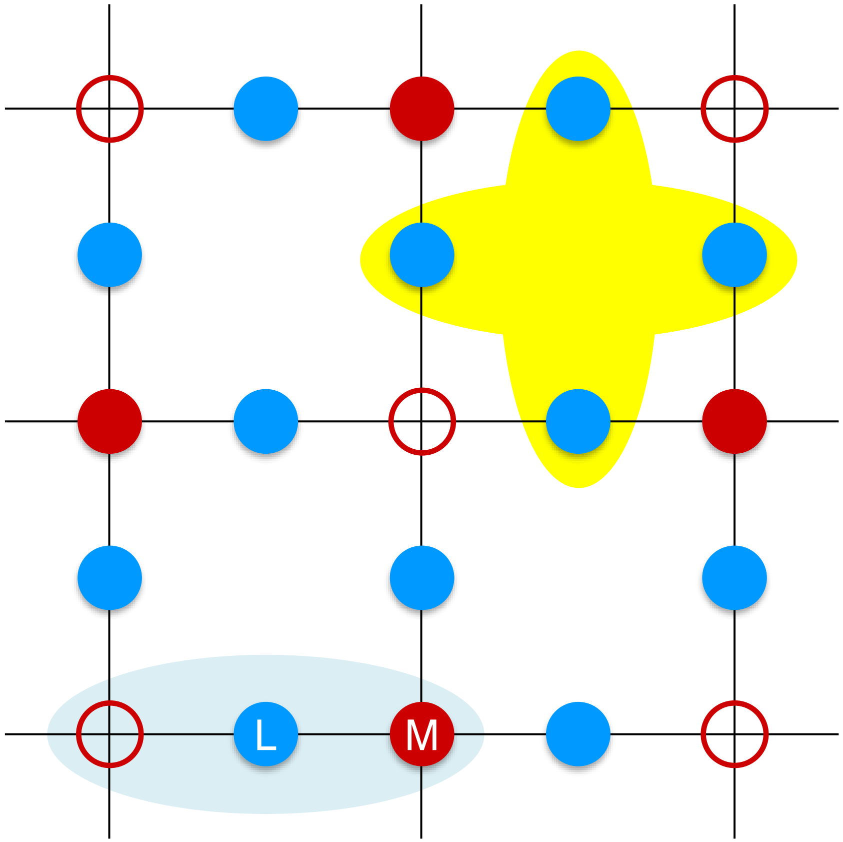
- **Kasper, Hebenstreit, Jendrzejewski, Oberthaler, Berges** NJP 19 023030 (2017):
  - Matter: $F = ½$ ^{6}Li atoms
  - Gauge field: $F = 1$ ^{23}Na atoms
  - No Feshbach resonance!
  - On the links, around 100 atomic bosons – very high electric field truncation (±50)



- **Experimental realization** of a single building block in a similar way: **Mil, Zache, Hegde, Xia, Bhatt, Oberthaler, Hauke, Berges and Jendrzejewski**, Science 367, 6482 (2020)

Digital Plaquette Generation

Combined in hybrid / trotterized evolution



The Z_2 example:

- Plaquette interactions
  $\sigma_x(\mathbf{x},1)\,\sigma_x(\mathbf{x}+\hat{\mathbf{1}},2)\,\sigma_x(\mathbf{x}+\hat{\mathbf{2}},1)\,\sigma_x(\mathbf{x},2)$

- Link interactions
  $\psi^\dagger(\mathbf{x})\sigma_x(\mathbf{x},k)\psi(\mathbf{x}+\hat{\mathbf{k}})$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

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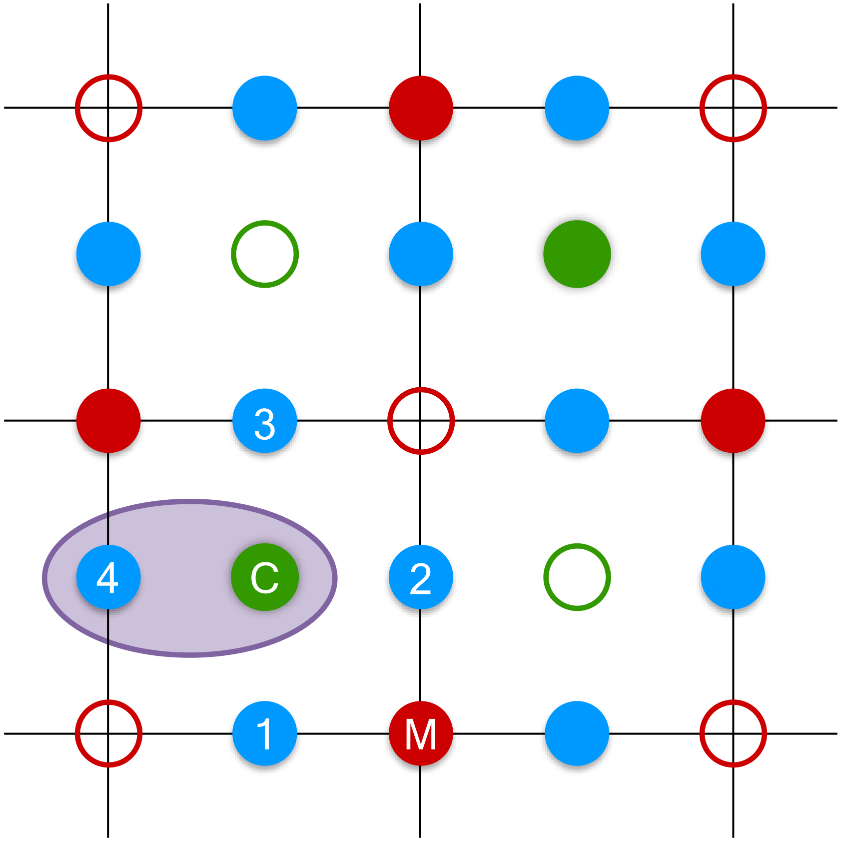
Stators: two-body interactions → four-body interactions

$$\mathcal{U} = \mathcal{U}^\dagger = |\tilde{\uparrow}\rangle\langle\tilde{\uparrow}| + \sigma^x \otimes |\tilde{\downarrow}\rangle\langle\tilde{\downarrow}|$$



$$|\widetilde{in}\rangle = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + |\tilde{\downarrow}\rangle\right)$$

$$\mathcal{U}_4^\dagger |\widetilde{in}\rangle = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + \sigma_4^x \otimes |\tilde{\downarrow}\rangle\right)$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

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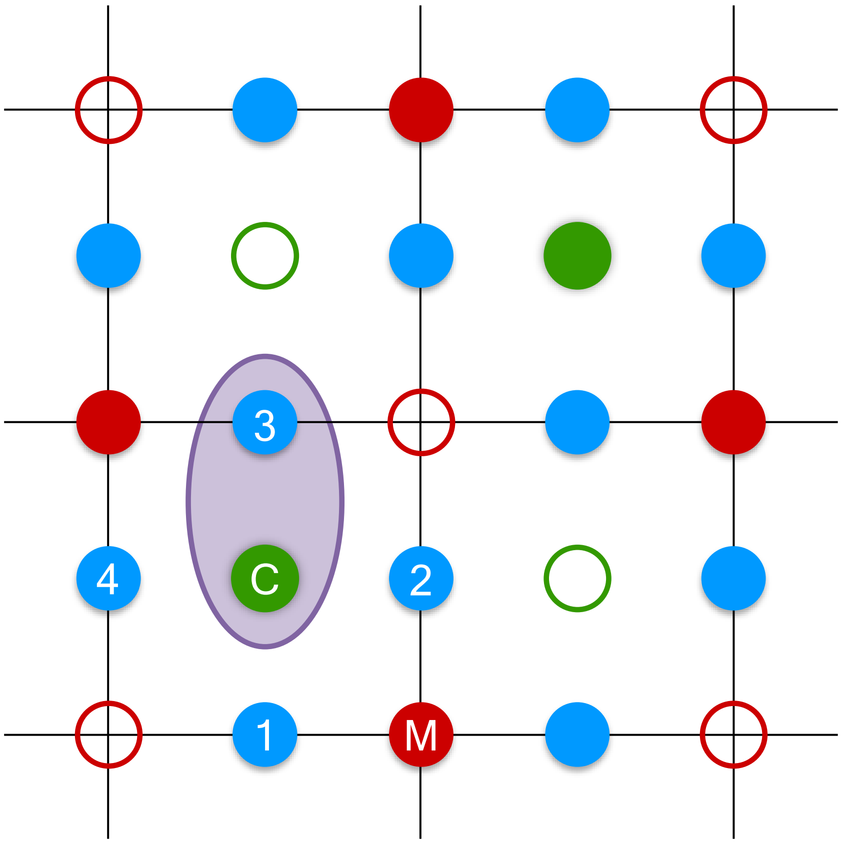

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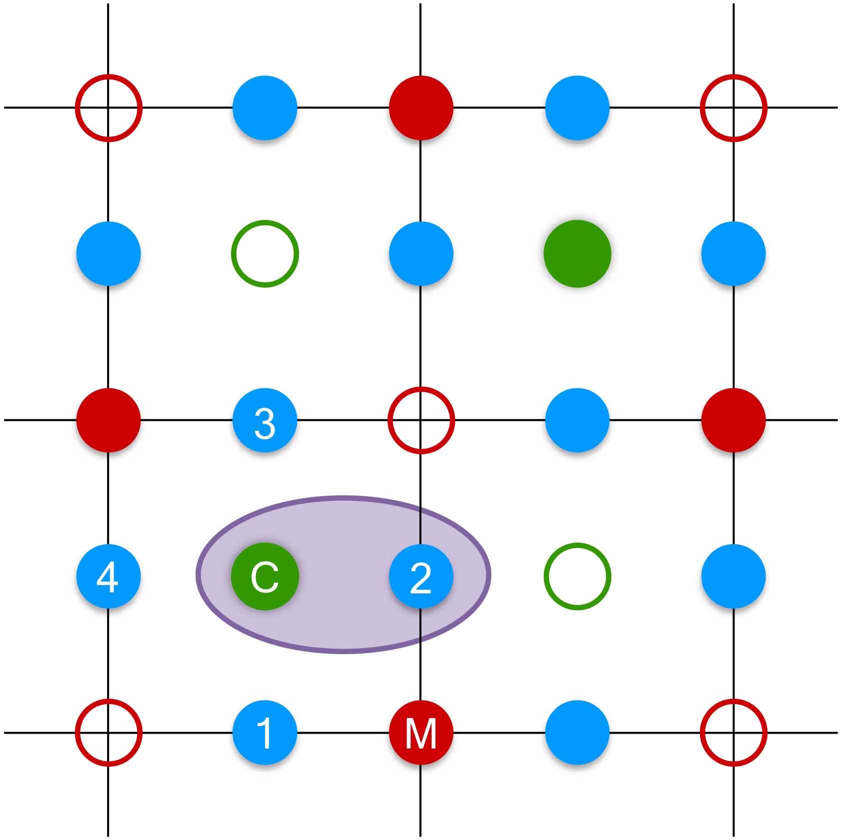

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$$\mathcal{U}_2\mathcal{U}_3^{\dagger}\mathcal{U}_4^{\dagger}\left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \sigma_2^x\sigma_3^x\sigma_4^x \otimes \left|\tilde{\downarrow}\right\rangle\right)$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

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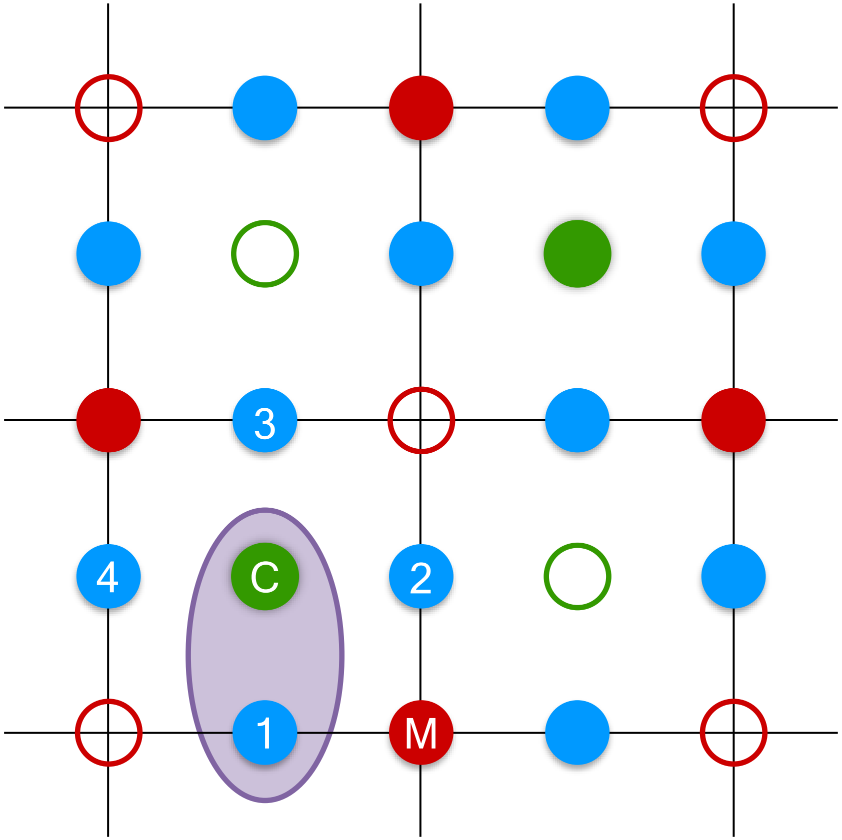

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E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

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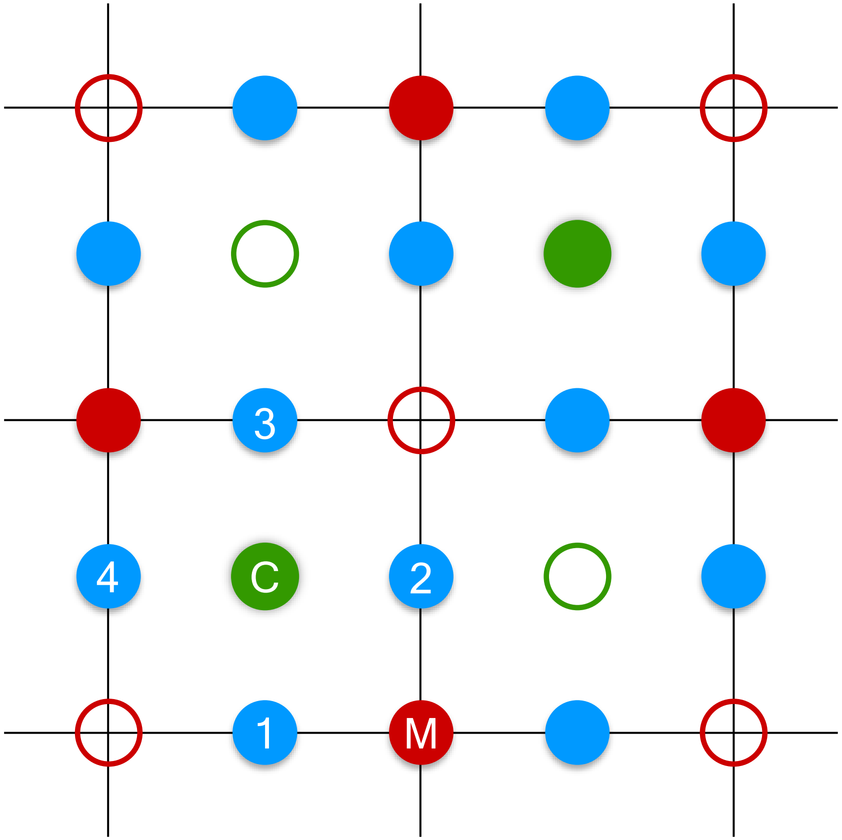

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$$\mathcal{U}_1 \mathcal{U}_2 \mathcal{U}_3^\dagger \mathcal{U}_4^\dagger |\widetilde{in}\rangle = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + \sigma_1^x \sigma_2^x \sigma_3^x \sigma_4^x \otimes |\tilde{\downarrow}\rangle\right)$$

$$S_\square = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + \sigma_\square^x \otimes |\tilde{\downarrow}\rangle\right)$$

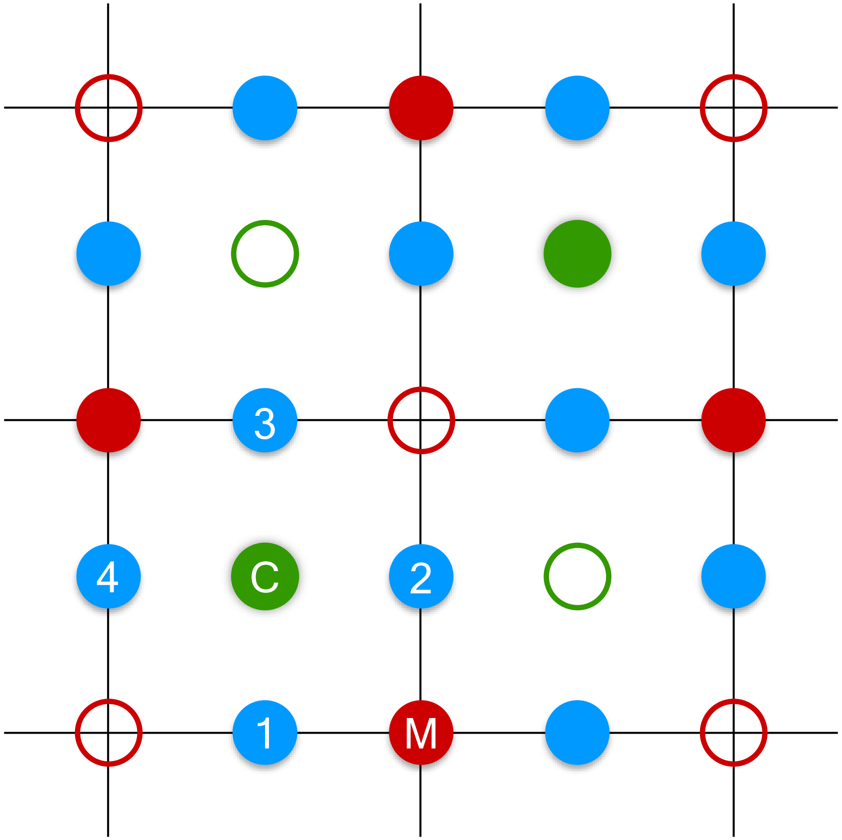
E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Digital Plaquette Generation

Combined in hybrid / trotterized evolution

Stators: two-body interactions → four-body interactions

$$\mathcal{U} = \mathcal{U}^\dagger = |\tilde{\uparrow}\rangle\langle\tilde{\uparrow}| + \sigma^x \otimes |\tilde{\downarrow}\rangle\langle\tilde{\downarrow}|$$



$$S_\square = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + \sigma^x_\square \otimes |\tilde{\downarrow}\rangle\right)$$

$$\tilde{\sigma}^x S_\square = S_\square \sigma^x_\square$$

$$e^{-i\lambda\tilde{\sigma}^x\tau} S_\square = S_\square e^{-i\lambda\sigma^x_\square\tau}$$

$$\mathcal{U}_4\mathcal{U}_3\mathcal{U}_2^\dagger\mathcal{U}_1^\dagger e^{-i\lambda\tilde{\sigma}^x\tau}\mathcal{U}_1\mathcal{U}_2\mathcal{U}_3^\dagger\mathcal{U}_4^\dagger|\widetilde{in}\rangle = |\widetilde{in}\rangle e^{-i\lambda\sigma^x_\square\tau}$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Any gauge group – generalization using stators

$$S = \int dg \left|g_A\right\rangle \left\langle g_A\right| \otimes \left|g_B\right\rangle$$

$$\left(U^j_{mn}\right)_B S = S \left(U^j_{mn}\right)_A$$

$$S_\square = \mathcal{U}_\square \left|\widetilde{in}\right\rangle \equiv \mathcal{U}_1 \mathcal{U}_2 \mathcal{U}_3^\dagger \mathcal{U}_4^\dagger \left|\widetilde{in}\right\rangle \qquad \mathrm{Tr}\left(\widetilde{U^j} + \widetilde{U}^{j\dagger}\right) S_\square = S_\square \mathrm{Tr}\left(U_1^j U_2^j U_3^{j\dagger} U_4^{j\dagger} + H.c.\right)$$

Feasible for finite or truncated infinite groups

E. Zohar, J. Phys. A. 50 085301 (2017) – generalizing Reznik, Aharonov, Groisman, PRA 2002

Common method for implementing plaquette interactions

For example:

Zohar, Farace, Reznik, Cirac, PRL 2017

Zohar, Farace, Reznik, Cirac, PRA 2017

Zohar, J. Phys. A 2017

Bender, Zohar, Farace, New J. Phys. 2018

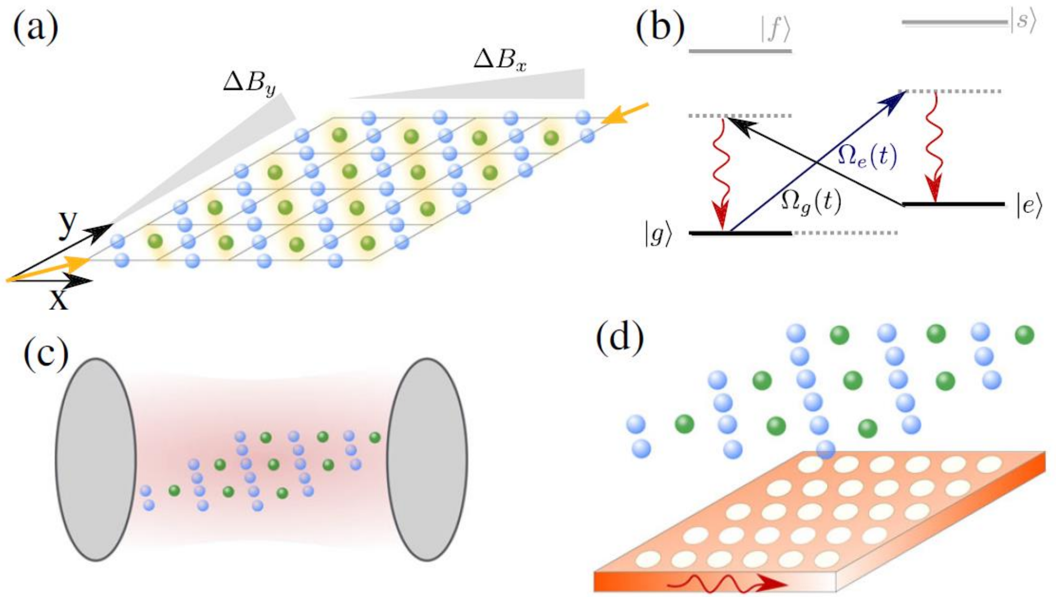
Lamm, Lawrence, Yamauchi, PRD 2019 – Similar approach – no ancillas!

Zohar, PRD 2020

Gonzalez-Cuadra, Zache, Carrasco, Kraus, Zoller, PRL 2023

Recent Proposal – 2+1d Pure Gauge Z_2

First proposal to simulate a Z2 LGT in nanophotonic or cavity QED setups, naturally allowing for long-range interactions, sparing the need for sequential operation: the control interacts with all four links at once, saving experimental run-time.



Armon, Ashkenazi, Garcia-Moreno,
González-Tudela, **Zohar**,
Physical Review Letters 127, 250501 (2021)

More Generally: Duality Transformations

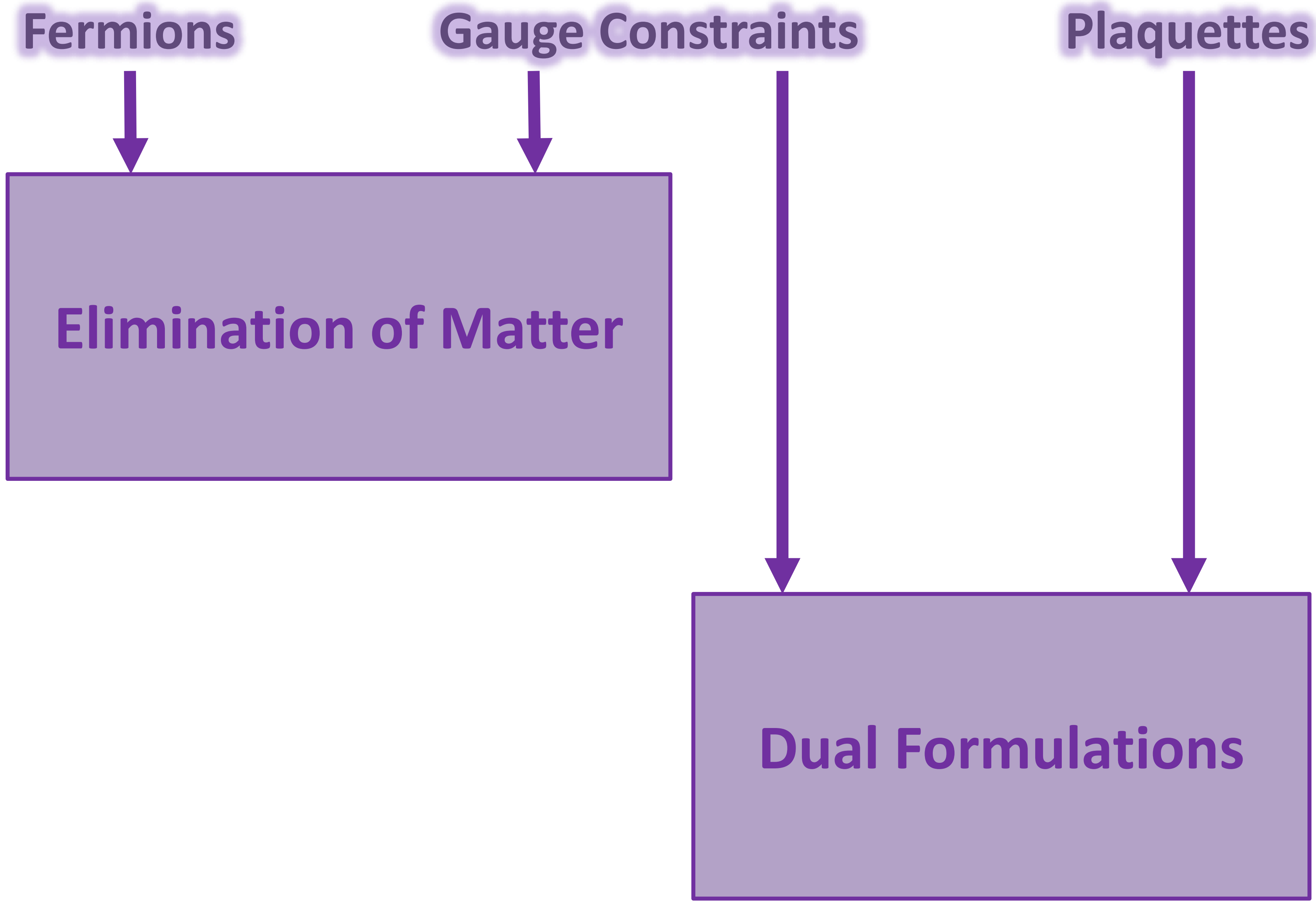
- Theoretically, transferring the information to the ancilla and back may be seen as **switching between two dual formulations**.

- This may be used as a general tool in the context of quantum simulation: make duality transformations feasible, physical transformations which can be implemented in the lab, using local unitaries and measurements.

- The theory behind it: the original system serves as matter, and the dual one – gauge fields, coupled minimally without dynamics.

Shachar Ashkenazi and **Erez Zohar**, Physical Review A 105, 022431 (2022)

Going around the challenges



Other approach: Loop-String-Hadrons (Raychowdhury, Stryker, …) – e.g. PRD 101, 114502 (2020)

Eliminating the fermions

- Fermions are subject to a **global Z_2 symmetry** (parity superselection)
- If this symmetry is **local** (which happens naturally in a lattice gauge theory whose gauge group contains $\mathbf{Z}_2$ as a normal subgroup), it can be used for **locally transferring the statistics information to the gauge field**
- One is left with **hard-core bosonic matter (spins),** with **fermionic statistics taken care of by the gauge field**

$$\psi^{\dagger}(\mathbf{x}) = c(\mathbf{x})\,\eta^{\dagger}(\mathbf{x})$$

Majorana Fermion: Statistics

Hardcore Boson: Physics

E. Zohar, J. I. Cirac, Phys. Rev. B 98, 075119 (2018)

Eliminating the fermions

- With a **local unitary transformation** which is independent of the space dimension, one can remove the fermions from the Hamiltonian, and stay with **hard-core bosonic matter** and **electric field dependent signs** that preserve the fermionic statistics.

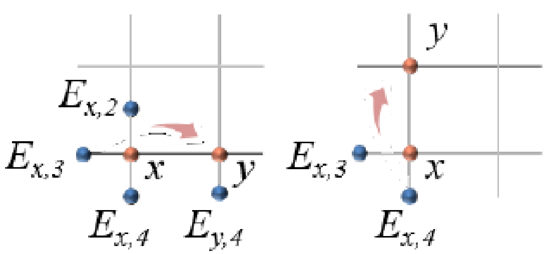
$$\epsilon \sum_{\mathbf{x},i=1,2} \left(\psi^{\dagger}\left(\mathbf{x}\right) U\left(\mathbf{x},i\right) \psi\left(\mathbf{x}+\hat{\mathbf{e}}_i\right) + h.c\right)$$

$$\psi^{\dagger}\left(\mathbf{x}\right) = c\left(\mathbf{x}\right) \eta^{\dagger}\left(\mathbf{x}\right)$$

$$\epsilon \sum_{\mathbf{x},i=1,2} \left(\eta^{\dagger}\left(\mathbf{x}\right) c\left(\mathbf{x}\right) U\left(\mathbf{x},i\right) c\left(\mathbf{x}+\hat{\mathbf{e}}_i\right) \eta\left(\mathbf{x}+\hat{\mathbf{e}}_i\right) + h.c\right)$$

Unitary transformation

$$-i\epsilon \sum_{\mathbf{x},i=1,2} \left(\xi_i \sigma_+\left(\mathbf{x}\right) U\left(\mathbf{x},i\right) \sigma_-\left(\mathbf{x}+\hat{\mathbf{e}}_i\right) + h.c\right)$$



$$\xi_h = e^{i\pi(E_{x,2}+E_{x,3}+E_{x,4}+E_{y,4})}$$

$$\xi_v = e^{i\pi(E_{x,3}+E_{x,4})}$$

E. Zohar, J. I. Cirac, Phys. Rev. B 98, 075119 (2018)

Eliminating the fermions

- This procedure opens the way for **quantum simulation of lattice gauge theories with fermionic matter in 2+1d and more**, even with **simulating systems that do not offer fermionic degrees of freedom**.

- In some cases **the matter can be removed completely**!

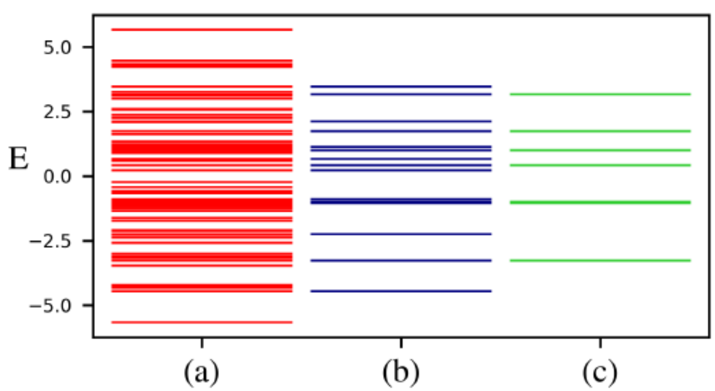
E. Zohar, J. I. Cirac, Phys. Rev. B 98, 075119 (2018)
E. Zohar, J. I. Cirac, Rev. D 99, 114511 (2019)

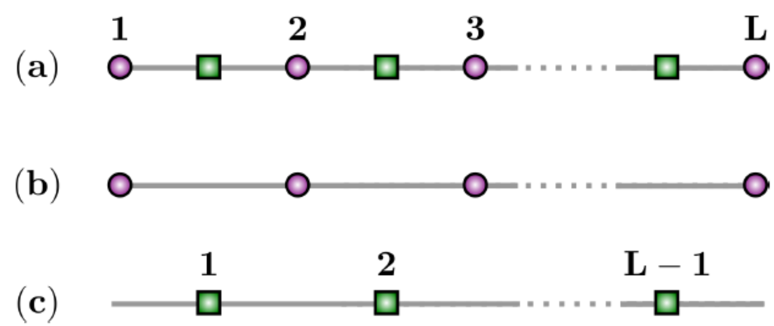
Example: Z_2 quantum simulation

- The entire dynamics of a Z_2 LGT with fermionic matter may be rexpressed as a qubit model, only with gauge field qubits, **without explicit fermions** (or any physical degree of freedom representing the matter), **without gauge constraints**, **using simple local single- and two-qubit unitaries**

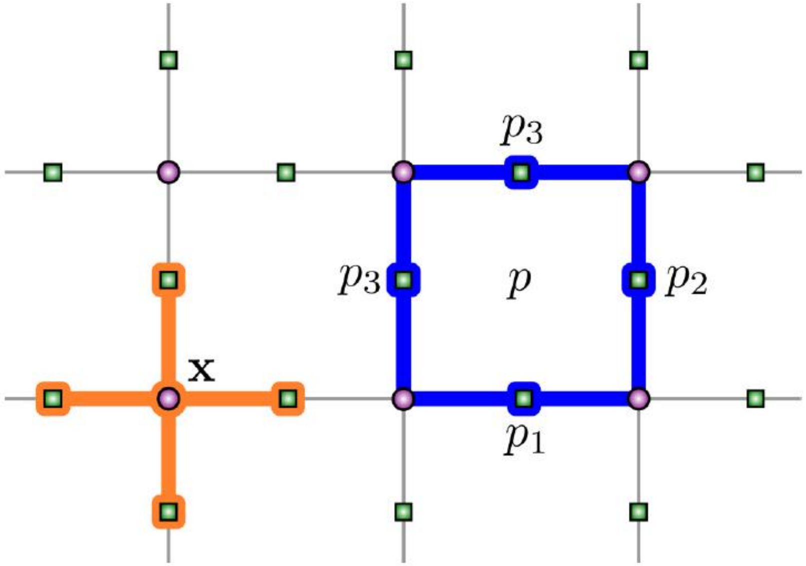
- This can be done in **arbitrary space dimensions**.

- In 1+1d, unlike the other conventional method where the gauge field is eliminated, **each time step is independent of the system size**.





Tomer Greenberg, Guy Pardo, Aryeh Fortinsky, **Erez Zohar**, Phys. Rev. Research 5, 023077 (2023)
See also: Reinis Irmejs, Mari Carmen Bañuls , J. Ignacio Cirac, arXiv:2206.08909 (2022)



$$H = H_E + H_B + H_{GM} + H_m,$$

$$H_E = -h\sum_{\ell} Z(\ell),$$

$$H_B = b\sum_{p} X_1(p)X_2(p)X_3(p)X_4(p)$$

$$H_{GM} = -J\sum_{\mathbf{x},i=1,\ldots,d} \psi^{\dagger}(\mathbf{x})X(\mathbf{x},i)\psi(\mathbf{x}+\mathbf{e}_i) + \text{H.c.}$$

$$H_m = m\sum_{\mathbf{x}}(-1)^{x_1+\ldots+x_d}N(\mathbf{x})$$

$$S(\mathbf{x}) \equiv [\textstyle\prod_{\ell\ni\mathbf{x}} Z(\ell)]$$

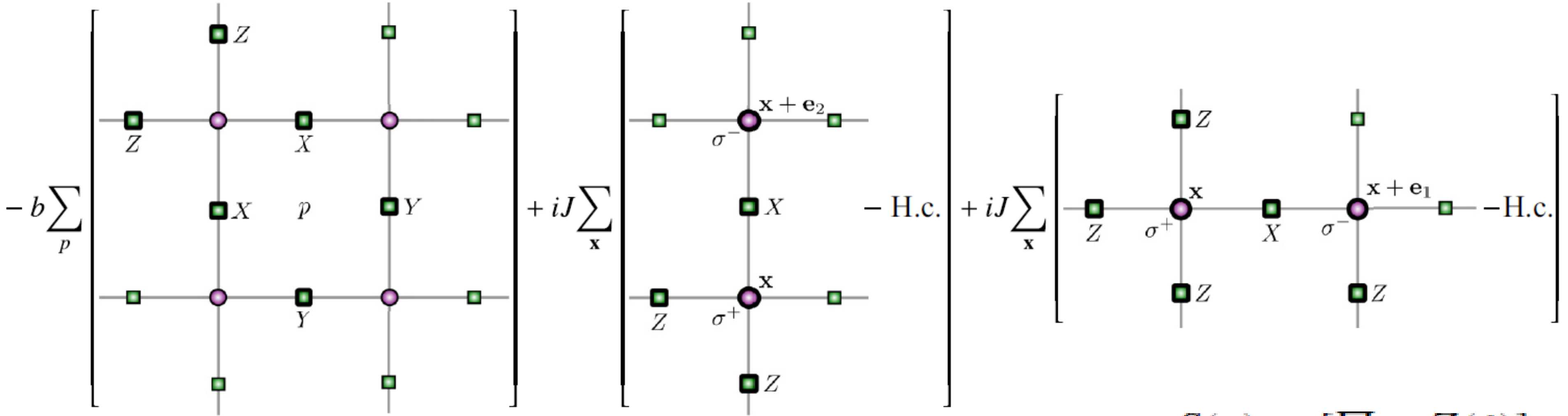
$$\Theta(\mathbf{x}) = S(\mathbf{x})e^{i\pi N(\mathbf{x})}$$

$$\Theta(\mathbf{x})|\psi\rangle = e^{i\pi q(\mathbf{x})}|\psi\rangle, \quad \forall \mathbf{x} \in \mathbb{Z}^d.$$

Tomer Greenberg, Guy Pardo, Aryeh Fortinsky, **Erez Zohar**, Phys. Rev. Research 5, 023077 (2023)

Step 1: Fermions → hard core bosons (spins)

$$H^{(1)} = -h\sum_{\mathbf{x},i} Z(\mathbf{x},i) + m\sum_{\mathbf{x}} (-1)^{x_1+x_2}\sigma^z(\mathbf{x})$$



$$S(\mathbf{x}) \equiv \left[\prod_{\ell \ni \mathbf{x}} Z(\ell)\right]$$

$$\varepsilon(\mathbf{x}) = e^{i\pi q(\mathbf{x})}$$

$$\sigma^z(\mathbf{x})|\psi^{(1)}\rangle = -\varepsilon(\mathbf{x})S(\mathbf{x})|\psi^{(1)}\rangle$$

$$P^{\pm}(\mathbf{x}) = \tfrac{1}{2}(1 \mp \varepsilon(\mathbf{x})S(\mathbf{x}))$$

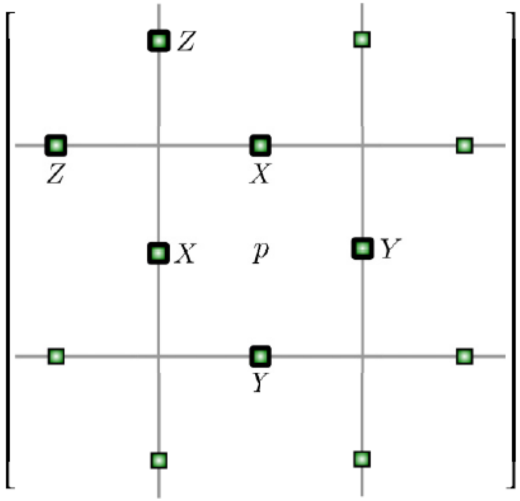
$$(P^{+}(\mathbf{x}) - P^{-}(\mathbf{x}))|\psi^{(1)}\rangle = \sigma^z(\mathbf{x})|\psi^{(1)}\rangle$$

➔ Act on each site with the controlled unitary $\mathcal{U}(\mathbf{x}) = P^{+}(\mathbf{x})\sigma^x(\mathbf{x}) + P^{-}(\mathbf{x})$ to eliminate the matter completely ➔ **Step 2: No matter, no constraints**

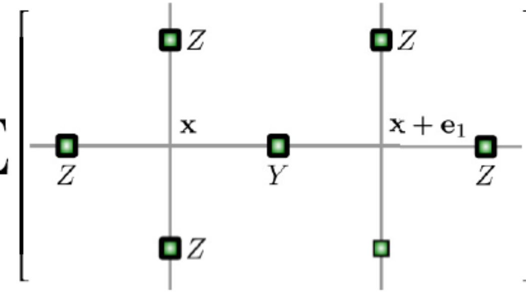
Tomer Greenberg, Guy Pardo, Aryeh Fortinsky, **Erez Zohar**, Phys. Rev. Research 5, 023077 (2023)

Step 2: No matter, no constraints

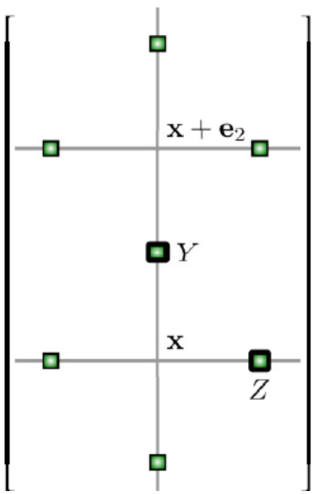
$$\tilde{H}^{(2)} = -h\sum_{\mathbf{x},i} Z(\mathbf{x},i) - \frac{m}{2}\sum_{\mathbf{x}} S(\mathbf{x}) - b\sum_{p}$$



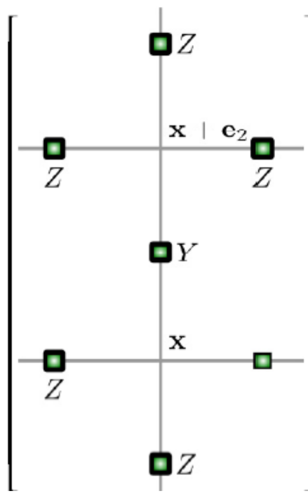
$$+ (-1)^{x_1+x_2}\frac{J}{2}\sum_{\mathbf{x}}$$



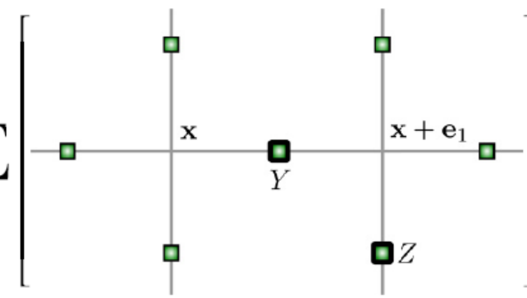
$$+ (-1)^{x_1+x_2}\frac{J}{2}\sum_{\mathbf{x}}$$



$$+ (-1)^{x_1+x_2}\frac{J}{2}\sum_{\mathbf{x}}$$



$$+ (-1)^{x_1+x_2}\frac{J}{2}\sum_{\mathbf{x}}$$



➔ Implement.

Tomer Greenberg, Guy Pardo, Aryeh Fortinsky, **Erez Zohar**, Phys. Rev. Research 5, 023077 (2023)

First example: 4 links, 1+1-d → 3 qubits

$$\hat{H} = \hat{H}_{\mathrm{E}} + \hat{H}_{\mathrm{GM}}$$

$$\hat{H}_{\mathrm{E}} = -\sum_{n=0}^{L-2}\left(hZ_n + \frac{J}{2}Y_n\right),$$

$$-\frac{2}{J}\hat{H}_{\mathrm{GM}} = \sum_{n=1}^{L-3} Z_{n-1}Y_nZ_{n+1} + Y_0Z_1 + Z_{L-3}Y_{L-2}$$

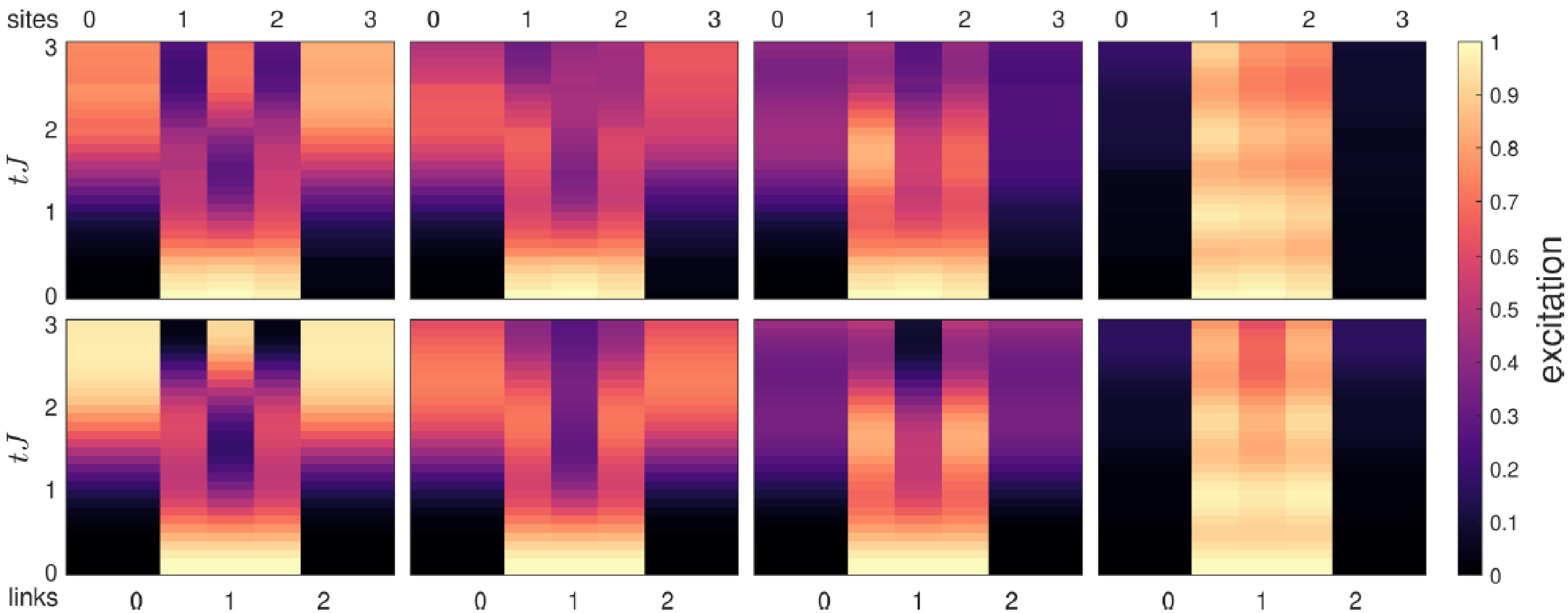


FIG. 4. Time evolution of the $(1+1)d$ model with $L = 4$ sites, from an excitation of the middle $(n = 1)$ link. Measurement on *ibm-lagos* (top) and exact numerical solution (bottom), with different values of h/J (from left to right: 0.1, 0.5, 1, and 3). Plotted is the excitation with respect to the Dirac-sea state: that is, for the links we plot the field $\langle E_n \rangle$ for the even sites (0 and 2) we plot the number of fermions $\langle N_n \rangle$, and for the odd sites (1 and 3) we plot $\langle 1 - N_n \rangle$ that can be thought of as the number of antiparticles. Confinement dynamics is observed for large h/J.

Tomer Greenberg, Guy Pardo, Aryeh Fortinsky, **Erez Zohar**, Phys. Rev. Research 5, 023077 (2023)

First example: 4 links, 1+1-d → 3 qubits

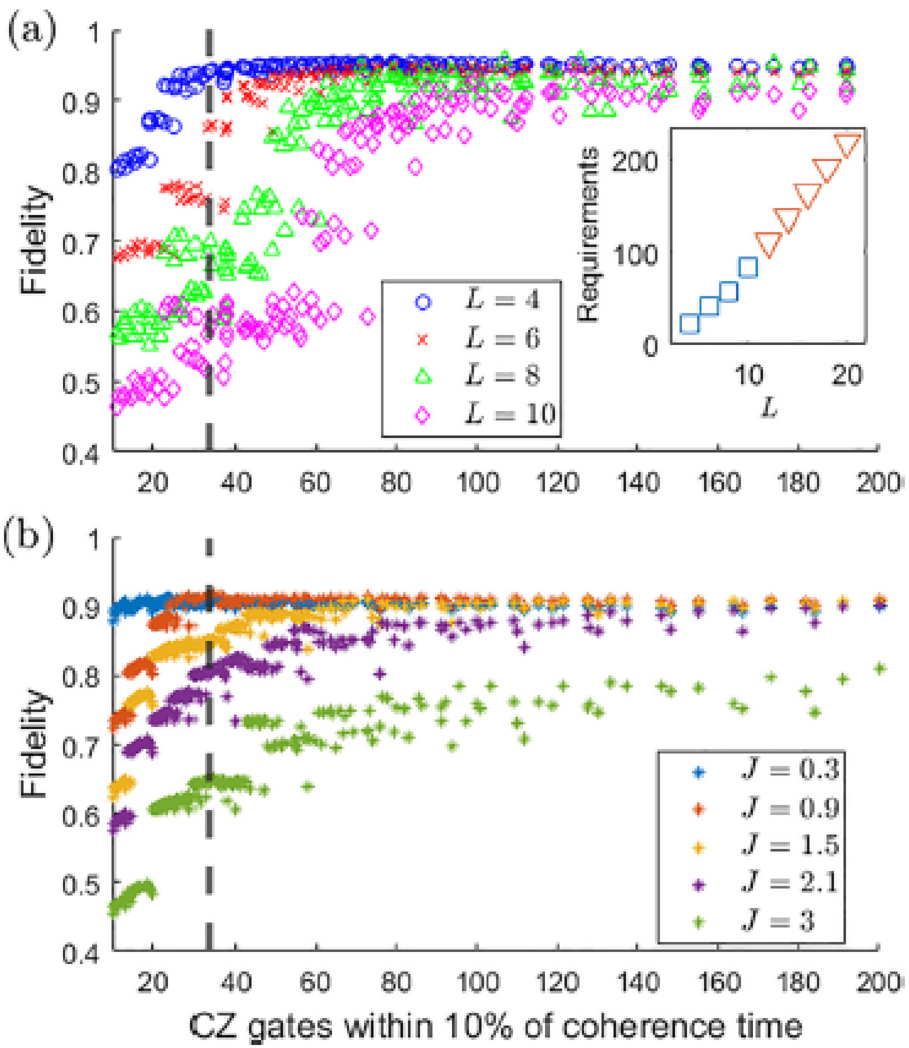


FIG. 8. Fidelity of the ground state for the $(1+1)d$ model obtained via a noisy numerical simulation of the adiabatic ground-state preparation experiment, compared against exact diagonalization, (a) for $J = h = 1$ and different values of the system size L, and (b) for $L = 4$, and different values of J (with $h = 1$). Each data point corresponds to a different value for the coherence time (assuming $T_1 = T_2$) and CZ gate duration, and the horizontal axis is the number of CZ gates that can be performed in 10% of the coherence time (which is a measure of the quality of the quantum hardware). The dashed vertical line represents typical values for IBMQ machines. The inset in (a) shows the hardware requirements, defined as the number of CZ gates in 10% of the coherence time that is required for a ground-state fidelity of at least 90%, for different values of L. The blue squares are derived from the numerical data of (a), and the red triangles are linear extrapolation.

Tomer Greenberg, Guy Pardo, Aryeh Fortinsky, **Erez Zohar**, Phys. Rev. Research 5, 023077 (2023)

Second example: minimal d>1+1

$$Z_n |\psi\rangle = e^{i\pi N_n} |\psi\rangle \quad \text{for} \quad n = 0, 1, 2,$$

$$Z_0 Z_1 Z_2 |\psi\rangle = -e^{i\pi N_3} |\psi\rangle ,$$

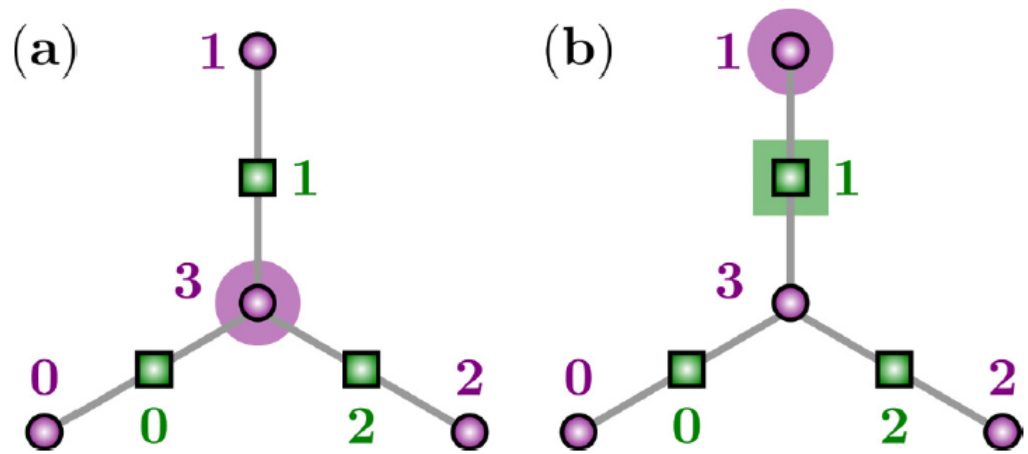
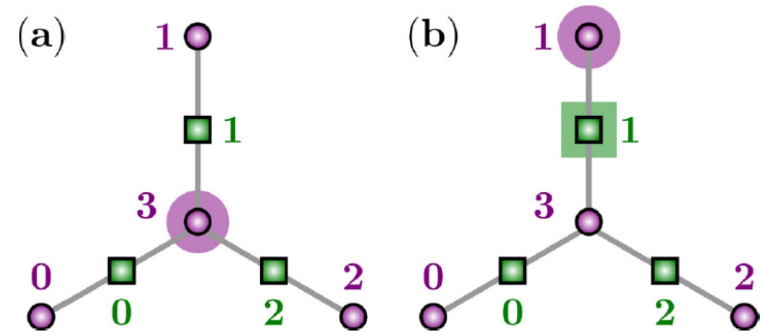


FIG. 6. The quasi-two-dimensional system with four sites, and the indexing convention for the sites (in purple) and for the links (in green). The middle site ($n = 3$) is considered an "odd" site in our chosen sector, which implies that (a) the $J = 0$ ground state is the one where $N_3 = 1$ and all other N_n and E_n equal zero (as indicated by the purple highlighting of the middle node). (b) The initial state of the time-evolution experiment (Fig. 7), with excitation in qubit (link) 1, is the one where $N_1 = E_1 = 1$ and all other N_n and E_n equal zero (as indicated by the highlighting of node 1 and link 1).

Tomer Greenberg, Guy Pardo, Aryeh Fortinsky, **Erez Zohar**, Phys. Rev. Research 5, 023077 (2023)

Second example: minimal d>1+1



$$-\frac{2}{J}\hat{H}_{\mathrm{GM}} = (Y_0 + Y_2) + (Z_0 Y_1 + Y_1 Z_2) + (Y_0 Z_1 Z_2 + Z_0 Z_1 Y_2)$$

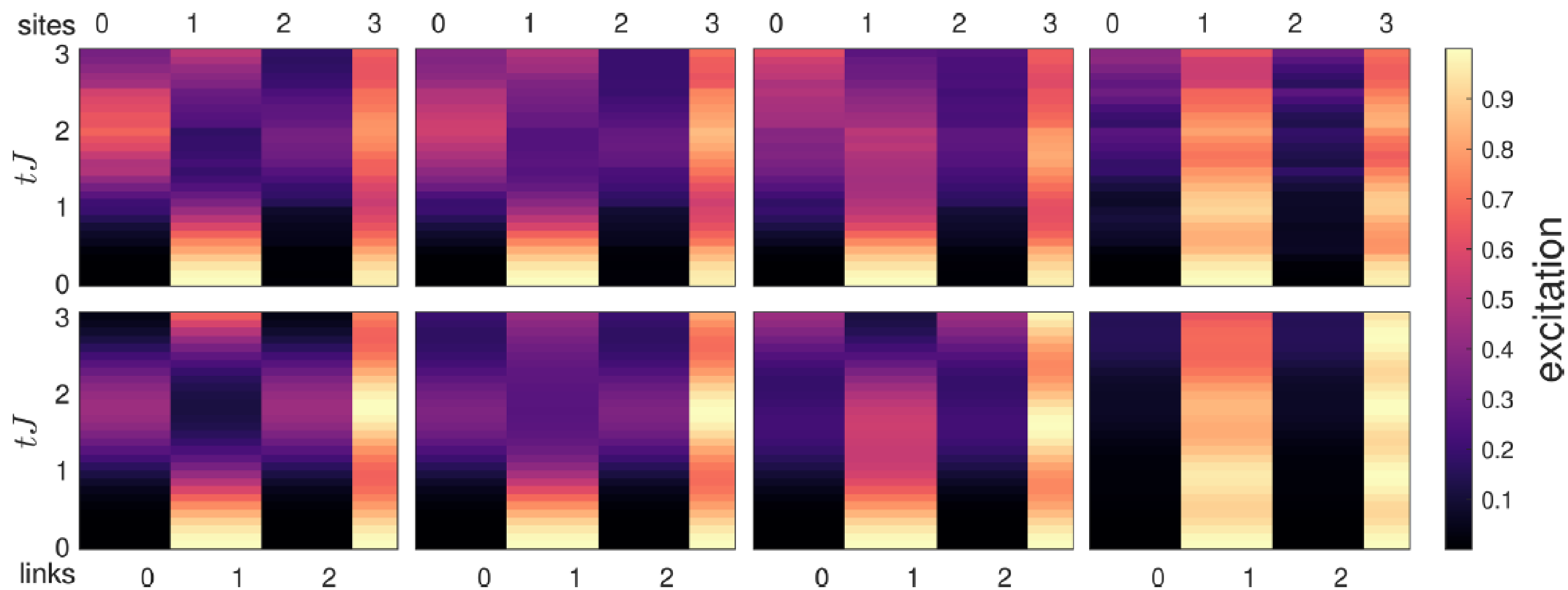
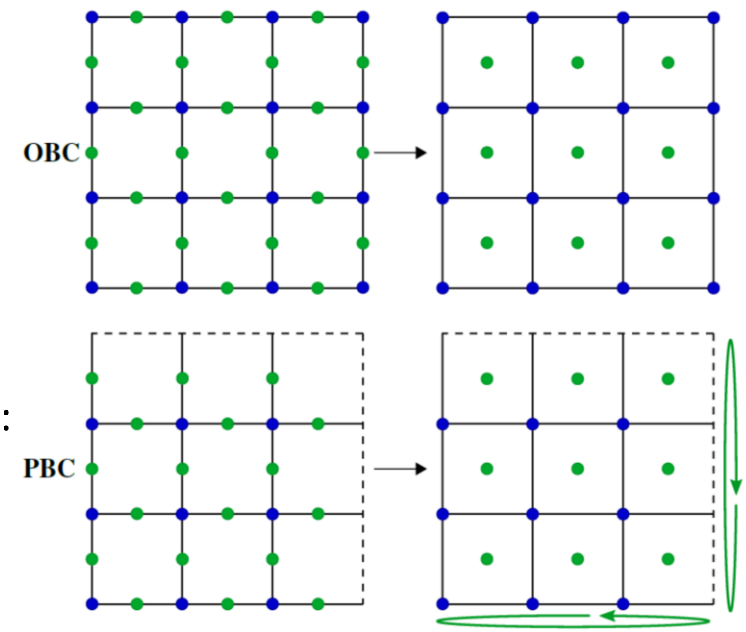


FIG. 7. Time-evolution experiment on the quasi-two-dimensional model depicted in Fig. 6. Measurement on *ibmq-lima* (top) and exact numerical solution (bottom), with different values of h/J (from left to right: 0.1, 0.5, 1, and 3). Plotted is the excitation with respect to the $J=0$ ground state [Fig. 6(a)]; that is, for the links we plot the field $\langle E_n \rangle = \langle \frac{1}{2}(1-Z_n) \rangle$, for sites $n = 0, 1, 2$ we plot $\langle N_n \rangle$, and for the middle site ($n=3$) we plot $\langle 1 - N_n \rangle$ that can be thought of as the number of antiparticles. The initial state is the one where qubit 1 is excited, which corresponds to the original-model state shown in Fig. 6(b). For large h/J the initial state is more robust to the dynamics.

Tomer Greenberg, Guy Pardo, Aryeh Fortinsky, **Erez Zohar**, Phys. Rev. Research 5, 023077 (2023)

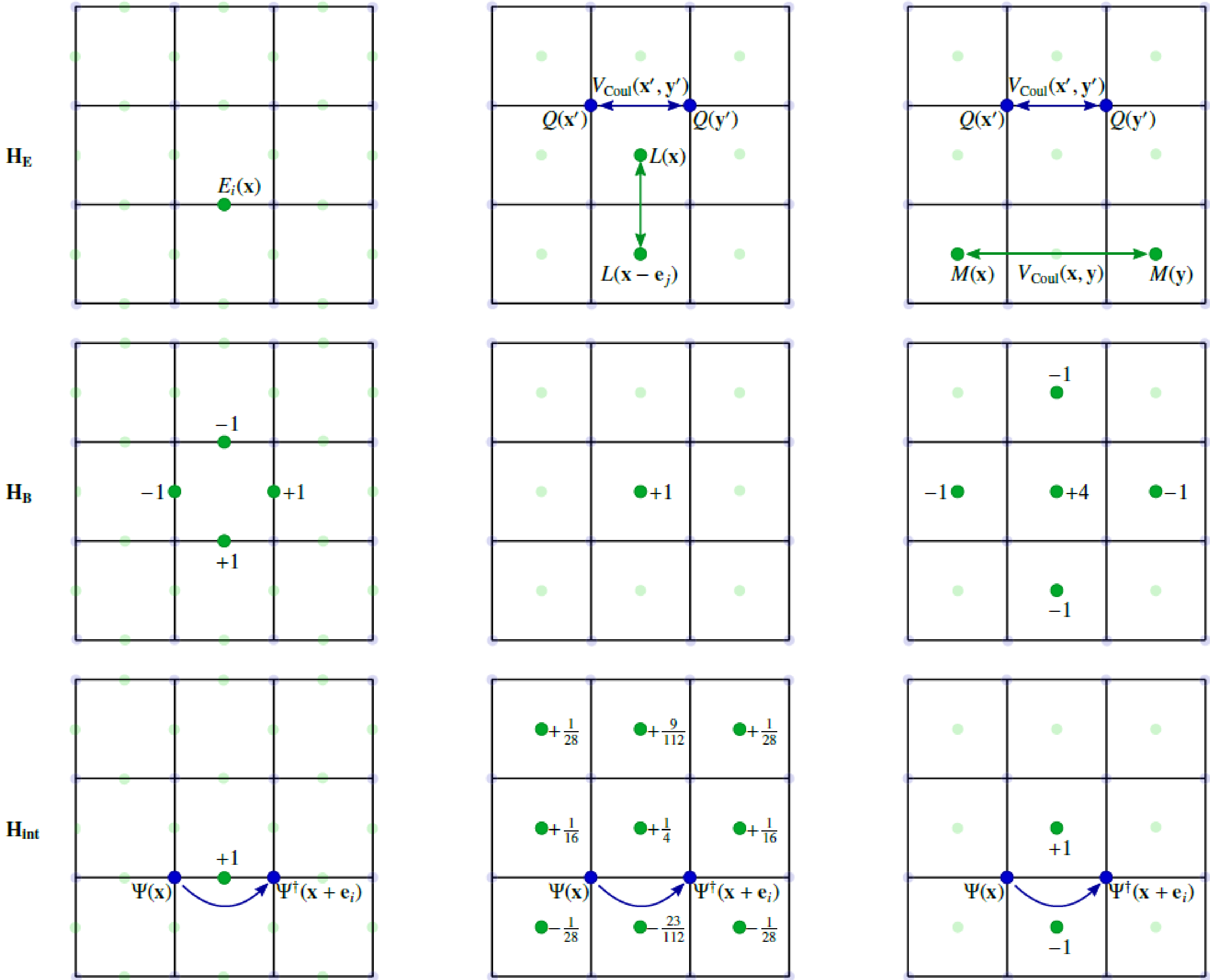
Gauge constraints: resources or redundancies?

- Compact QED in 2+1, dual formalism – no local constraints; In 3+1 – some local constraints are needed in the dual picture too.
- Pure gauge: everything is local:
  - Plaquette, four-body interaction → Non-interacting terms
  - Link terms → Two-body interactions
  - Drell, Quinn, Svetitsky, Weinstein, Phys. Rev. D 19, 619 (1979), Kaplan and Stryker, Phys. Rev. D 102, 094515 (2020)
  - Unmuth-Yockey, Phys. Rev. D 99, 074502 (2019)
  - Bauer & Grabowska, arxiv:2111.08015 (2021)
- With dynamical matter:
  - Haase et al, Quantum 5, 393 (2021), Paulson et al, PRX Quantum 2, 030334 (2021): coupling to matter introduces non-locality in the form of strings (maximal trees)
  - Bender and **Zohar**, Phys Rev. D 102, 114517 (2020): another type of dual formulation, using Green's functions: non-locality does not break spatial symmetries; matter and gauge field Coulomb interactions.



Phys Rev. D 102, 114517 (2020)

Original Formulation | **“Regular” dual formulation** | **Another dual formulation**



Julian Bender and **Erez Zohar**, Phys Rev. D 102, 114517 (2020)

Dualities and Quantum Simulation – General Formulation

- Duality maps may be formulated as <u>physical</u> transformations which are feasible on quantum platforms (including NISQ devices).

- This allows one to build a quantum simulator of both sides of a duality map of models admitting one, enjoying the benefits of both in the same simulator.

- Future generalizations: Matter, non-Abelian.

Shachar Ashkenazi and **Erez Zohar**, Physical Review A 105, 022431 (2022)

In conclusion,

- Quantum Simulation of (lattice) gauge theories is subject to several challenges:
  - Our simulated platform needs to describe both fermionic and non-fermionic physics.
  - Impose / maintain / surpass gauge invariance
    - Redundant Hilbert Space – Waste of computational resources.
  - Complicated four-body interactions

- These can be addressed in various ways, directly and indirectly, in spite of or thanks to the local constraints.
- Quantum simulation of lattice gauge theories is an exponentially growing field; besides the massive experimental progress, there is still room for exciting theoretical study.

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Umberto Borla
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Gertian Roose
(Postdoc – joining soon!)

Ariel Kelman
(Phd)

Matan Feuerstein
(Master)

Jonathan Elyovich
(Master)

- Locally and unitarily mapping fermions to bosons in the presence of lattice gauge fields – general theory + experimental demonstration using IBM qubits.
  - **Zohar**, Cirac, PRB 2018 + PRD 2019
  - **Pardo**, **Greenberg**, Fortinsky, Katz, **Zohar**, Phys. Rev. Research 2023
- Photon-Mediated Quantum Simulation of LGT plaquette interactions.
  - **Armon**, **Ashkenazi**, Garcia-Moreno, Gonzalez-Tudela, **Zohar**, PRL 2021
- Physically implementing Duality Transformations using Local Unitaries and Measurements.
  - **Ashkenazi**, **Zohar**, PRA 2022

- Building non-Abelian LGT quantum simulators using dynamical decoupling (“non-Abelian rotating wave approximation”).
  - Kasper, Zache, Jendrzejewski, Lewenstein, **Zohar**, PRD 2023
- Sign-problem free tensor network construction for studying LGTs with fermionic matter – general theory + benchmarking for $\mathbf{Z}_2$
  - Zohar, Cirac, PRD 2018
  - Emonts, **Kelman**, **Borla**, Moroz, Gazit, **Zohar**, PRD 2023
  - Emonts, **Zohar**, arxiv 2023