Quantum Simulation of Nuclear Effective Field Theories

(Manuscript in Progress)

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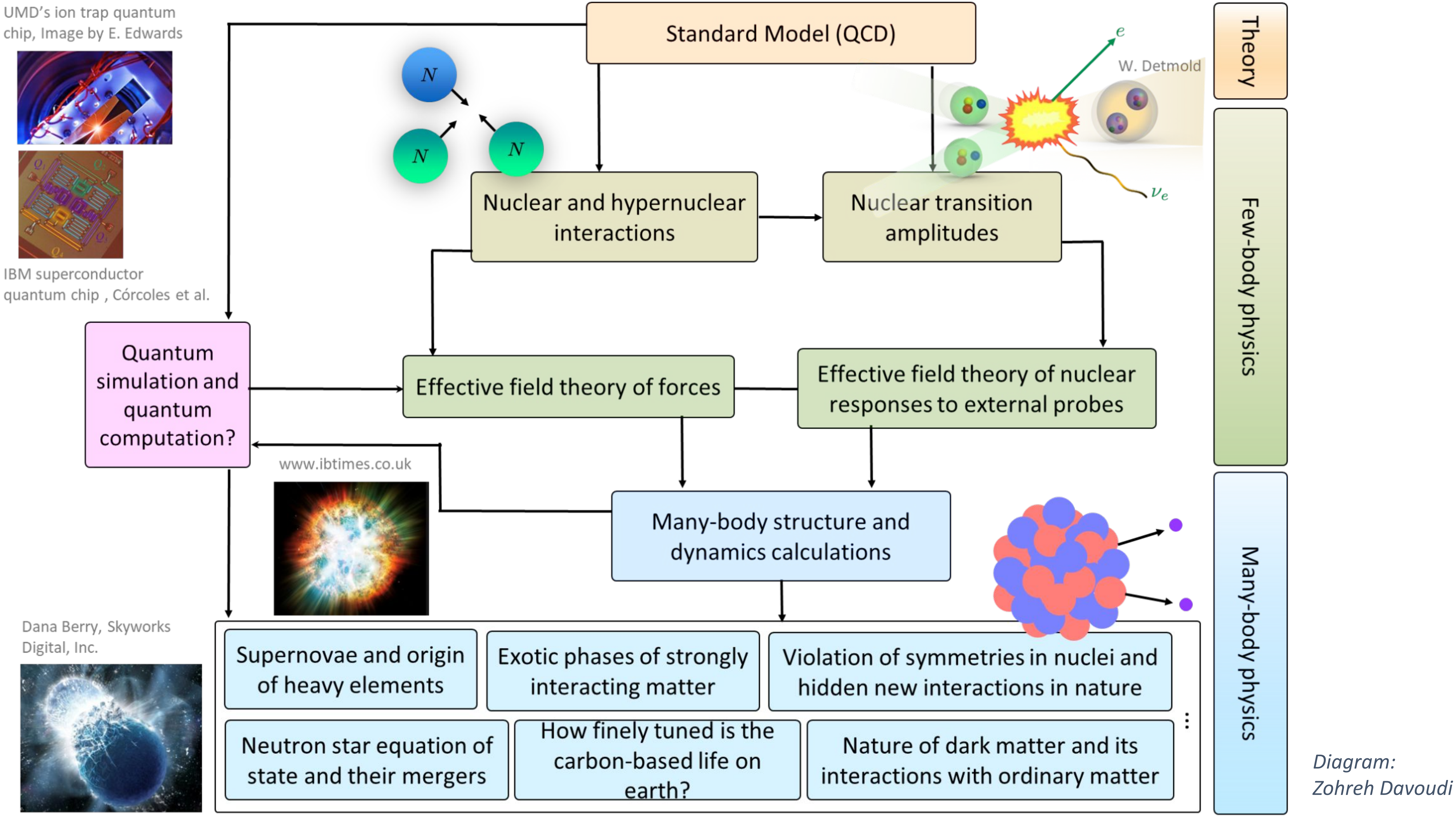
Summary

- Overview of Chiral Effective Field Theory
- Quantum Simulation
- Optimising Quantum Simulation of Chiral Effective Field Theories for Digital Quantum Computers
- Cost Estimates for Nuclear Spectroscopy


UMD's ion trap quantum chip, Image by E. Edwards
IBM superconductor quantum chip , Córcoles et al.
Standard Model (QCD)
Theory
W. Detmold
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N
N
N
Nuclear and hypernuclear interactions
Nuclear transition amplitudes
Few-body physics
Quantum simulation and quantum computation?
Effective field theory of forces
Effective field theory of nuclear responses to external probes
www.ibtimes.co.uk
Many-body structure and dynamics calculations
Many-body physics
Dana Berry, Skyworks Digital, Inc.
Supernovae and origin of heavy elements
Exotic phases of strongly interacting matter
Violation of symmetries in nuclei and hidden new interactions in nature
Neutron star equation of state and their mergers
How finely tuned is the carbon-based life on earth?
Nature of dark matter and its interactions with ordinary matter
Diagram: Zohreh Davoudi

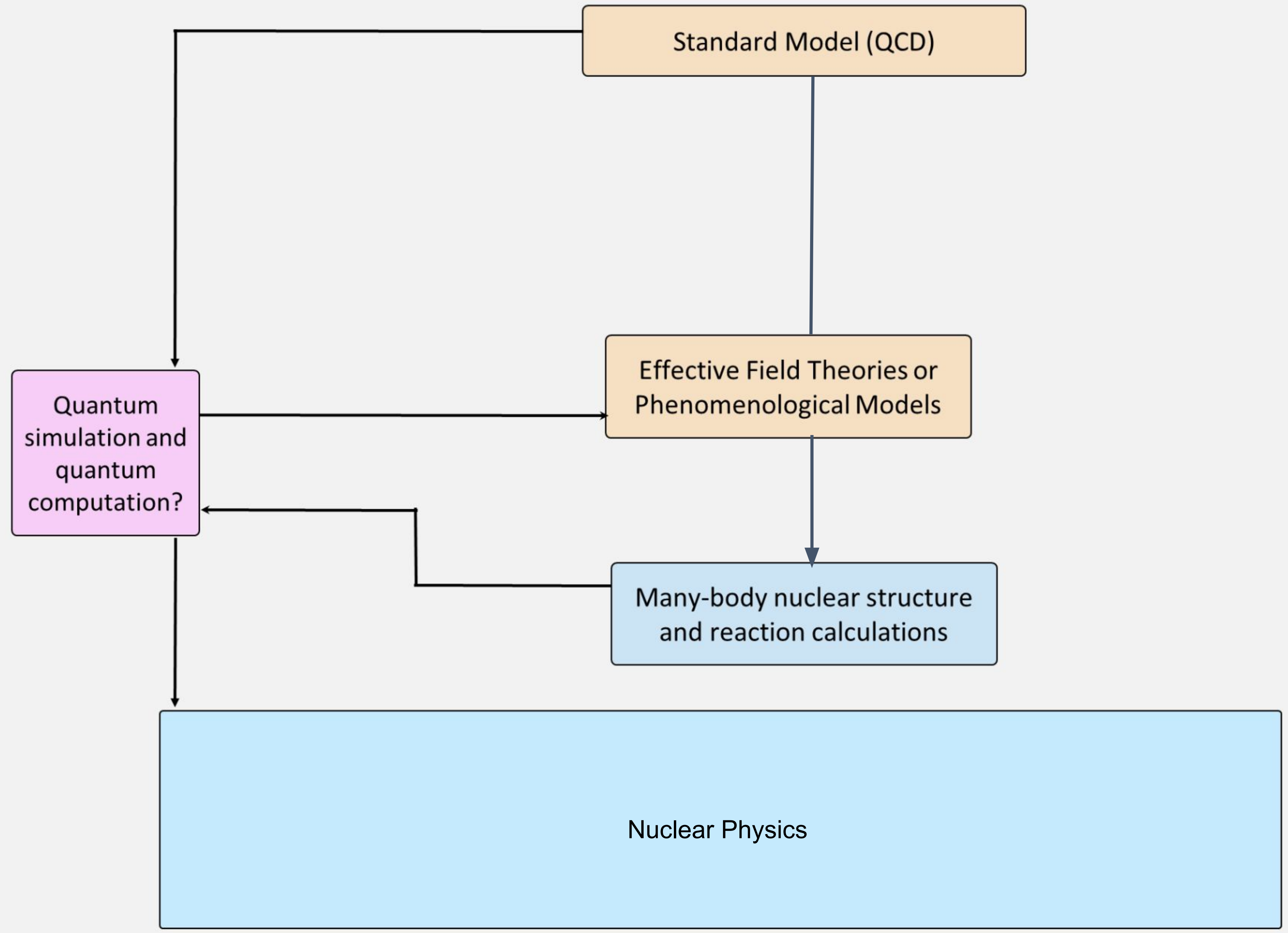

Diagram: Zohreh Davoudi

Nuclear Physics

- Want to calculate nuclear physics quantities:
    - Scattering cross-sections
    - Low-lying spectra
    - etc.

1: 2107.12769

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    - Classical MC can currently just simulate deuterium.
    - Quantumly, first estimates of simple quark transport properties need $>10^{50}$ gates[1].
- Semi-empirical models (e.g. mean field) aren't reliable for large nuclei or theoretically well justified.

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Approaching Nuclear Physics with EFTs: Chiral EFT

- Idea (Weinberg, 1990s): take the most general possible Lagrangian which is consistent with the symmetries of strong interaction, and include all terms up to a given order in momentum expansion.

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- Treats protons, neutrons and pions as the degrees of freedom of the theory.

- Nucleons described by non-relativistic dynamics.

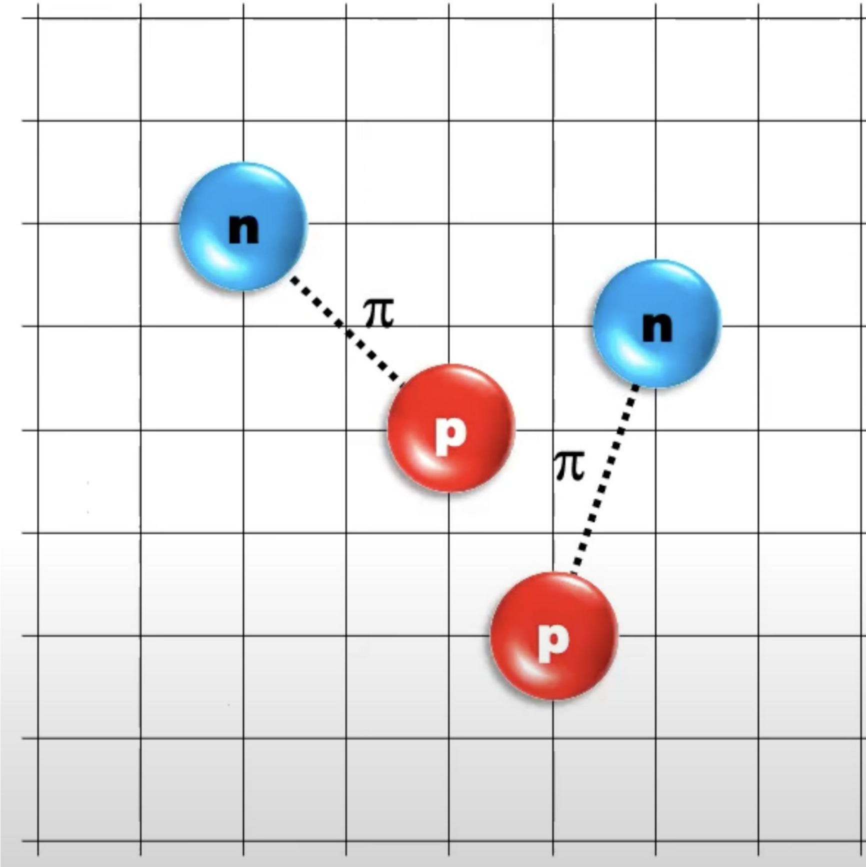


Image: Dean Lee

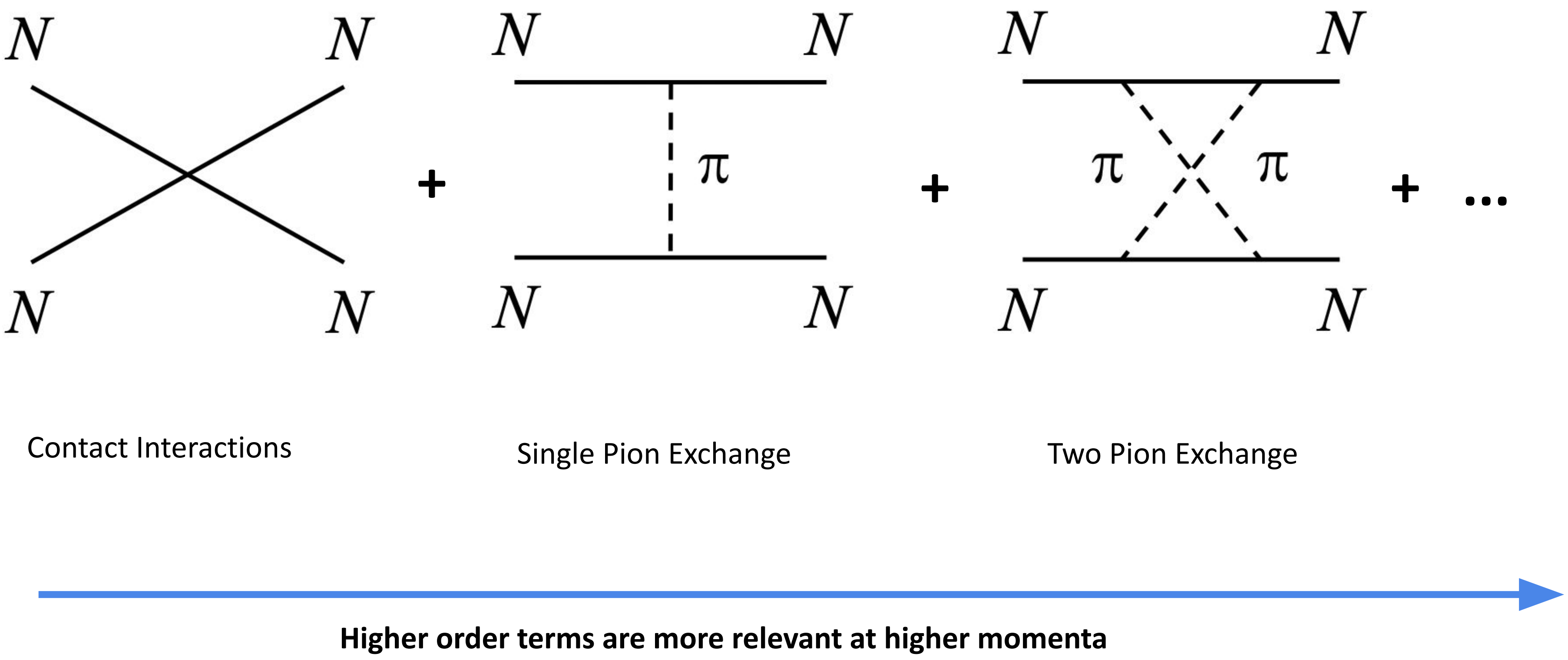
Modelling Nuclear Physics: Chiral EFT

To leading order the Lagrangian looks something like this:

$$
\begin{aligned}
\widehat{\mathcal{L}}^{\Delta=0} \quad = \quad & \frac{1}{2}\partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi} - \frac{1}{2} m_\pi^2 \boldsymbol{\pi}^2 \\
& + \frac{1-4\alpha}{2f_\pi^2}(\boldsymbol{\pi} \cdot \partial_\mu \boldsymbol{\pi})(\boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi}) - \frac{\alpha}{f_\pi^2}\boldsymbol{\pi}^2 \partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi} + \frac{8\alpha - 1}{8 f_\pi^2} m_\pi^2 \boldsymbol{\pi}^4 \\
& + \bar{N}\left[i\partial_0 - \frac{g_A}{2f_\pi}\, \boldsymbol{\tau} \cdot (\vec{\sigma} \cdot \vec{\nabla})\boldsymbol{\pi} - \frac{1}{4f_\pi^2}\, \boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times \partial_0 \boldsymbol{\pi}) \right] N \\
& + \bar{N}\left\{ \frac{g_A(4\alpha - 1)}{4f_\pi^3}\, (\boldsymbol{\tau} \cdot \boldsymbol{\pi}) \left[\boldsymbol{\pi} \cdot (\vec{\sigma} \cdot \vec{\nabla})\boldsymbol{\pi} \right] + \frac{g_A \alpha}{2f_\pi^3}\, \boldsymbol{\pi}^2 \left[\boldsymbol{\tau} \cdot (\vec{\sigma} \cdot \vec{\nabla})\boldsymbol{\pi} \right] \right\} N \\
& - \frac{1}{2} C_S \bar{N} N \bar{N} N - \frac{1}{2} C_T (\bar{N}\vec{\sigma} N) \cdot (\bar{N}\vec{\sigma} N) \; + \; \ldots \, ,
\end{aligned}
$$

Modelling Nuclear Physics: Chiral EFT

The physical model you should have in your head is:



Simulating Chiral EFTs

Classically simulate time evolution:

=> sign problem!

=> huge resource costs

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Quantumly simulate time evolution:

=> provably no sign problem!

=> “efficient”

How feasible is quantum simulation of Chiral EFT?

Our Work

Our Work

Our Work

- We determine gate counts for NISQ and fault-tolerant quantum computers for time evolution and spectroscopy of nuclei.

- For 4 different Hamiltonians corresponding to the leading order terms in the Effective Field Theory expansion.

- Improve on fermionic encodings, bosonic encodings, error analysis, etc. to minimise gate counts and determine which is the most feasible.

- Allows us to compare efficiency of simulating leading order EFT Hamiltonians.

Time Evolution Algorithms

EFTs and Quantum Computing

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$$e^{-iH\delta t} \approx e^{-iH_1\delta t}e^{-iH_2\delta t}\dots e^{-iH_\Gamma\delta t} =: \mathcal{P}(\delta t)$$

$$\left|\left|e^{-iH\delta t} - \mathcal{P}(\delta t)\right|\right| \leq \epsilon$$

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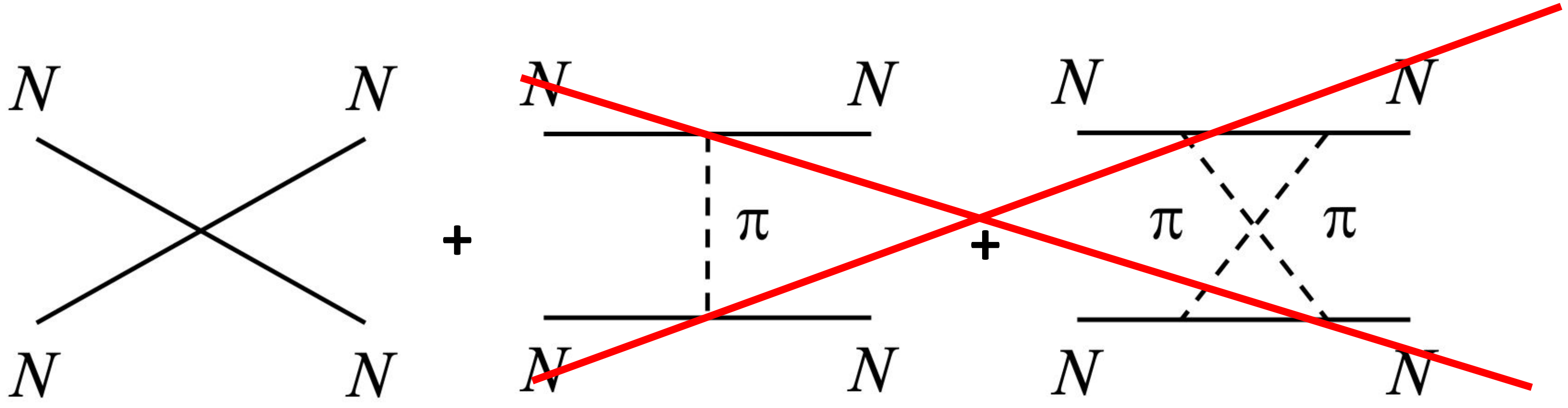
- Product formulae have low overhead and generally perform well!
- *How well can we improve our error bounds and optimise computational resources when simulating Chiral Effective Field Theories?*

Optimising Simulation for the Simplest Theory:
Pionless EFT

Pionless EFT

- Pionless EFT is the simplest Hamiltonian that recreates basic properties of the nucleus[1] for momenta below pion mass.



Contact Interactions

Single Pion Exchange

Two Pion Exchange

[1]See works by: Kaplan, Savage, van Kolck, Bedaque…

Pionless EFT

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- Discretise the theory and put on a 3D lattice rather continuous space:

- We choose a 2nd quantisation and position-space formulation.

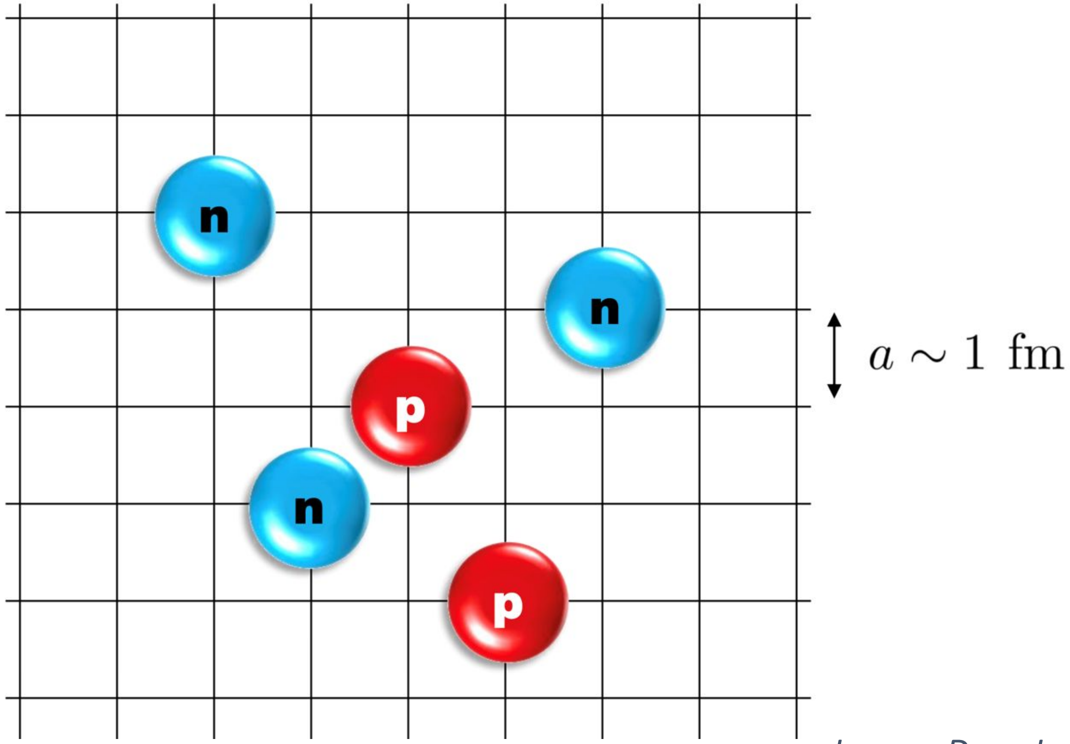

Image: Dean Lee

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Creation and annihilation operators for nucleons.

Nucleon number operator at site *i*.

$$\sigma \in \{p\uparrow, p\downarrow, n\uparrow, n\downarrow\}$$

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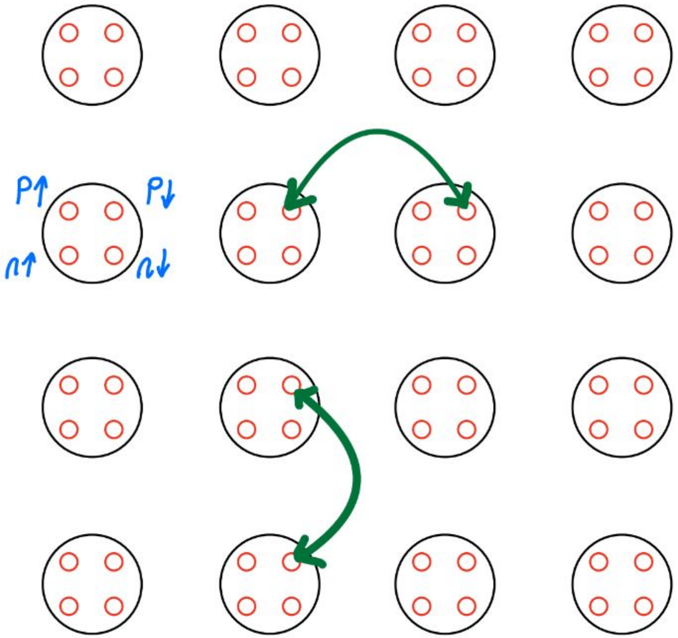

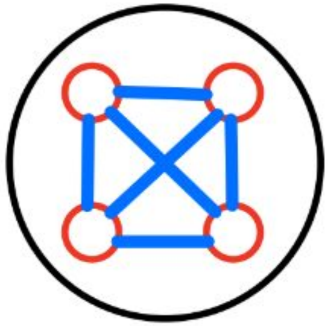

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Take advantage of as many details as possible to reduce simulation costs!

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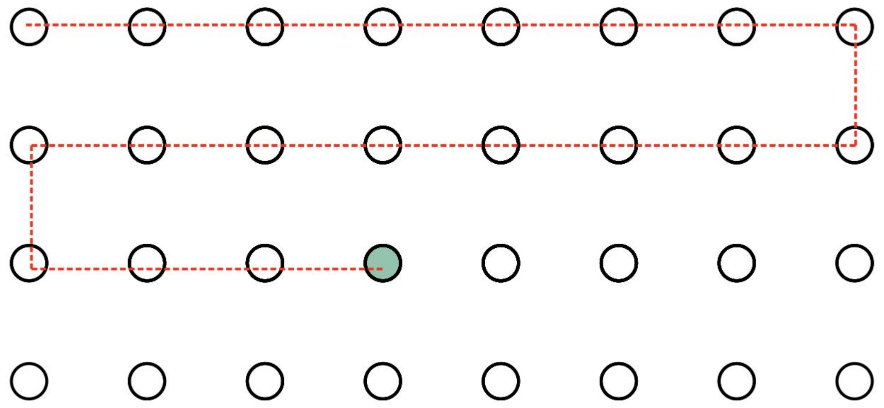

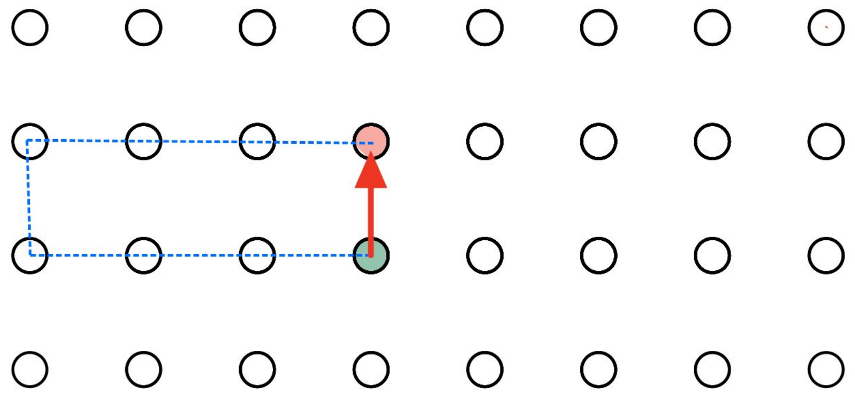

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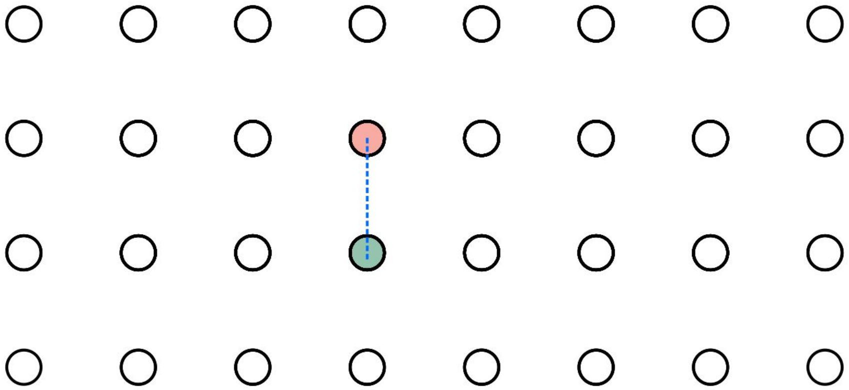

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- Leverage interaction locality + fermion no. conservation: encode fermions using Verstraete-Cirac or Compact encoding.
  - Implement a single hopping operator in $O(1)$ depth

Implementing each Trotter Step

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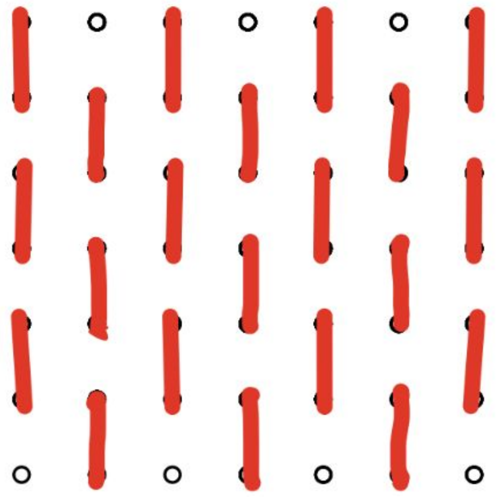

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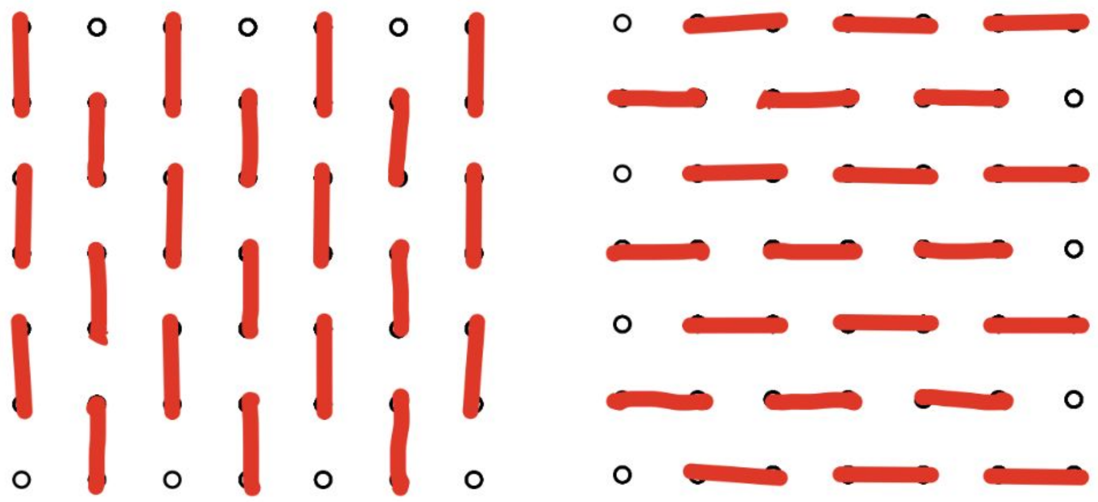

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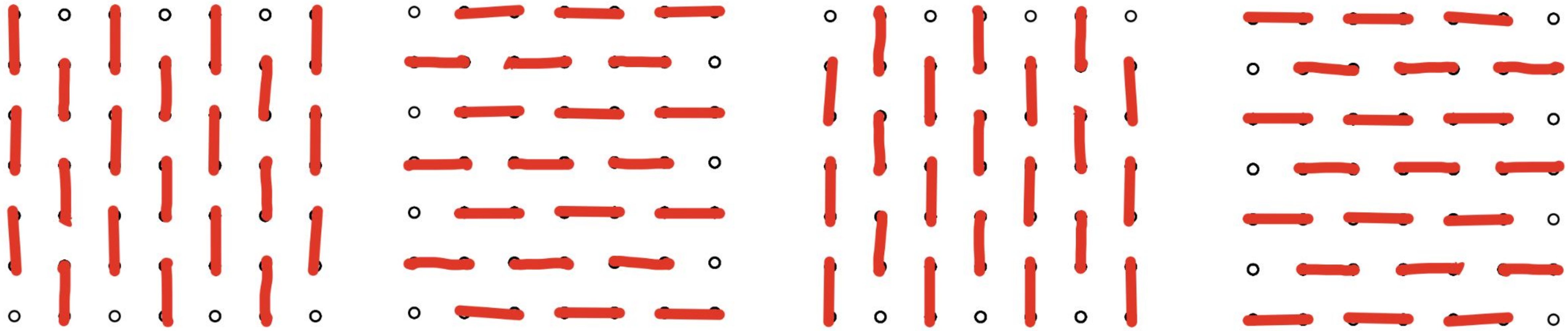

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Fermionic Encoding	Circuit Depth	Number of Qubits
Jordan-Wigner (Naive)	$O(M^2)$	M
FFFT/SWAP Networks	$O(M)$	M
VC or Compact	110	$1.5M$

(M = # fermion modes, M=4,000 for 10x10x10 lattice)

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- Pionless EFT: we can actually do even better than naive VC or Compact.

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- Hamiltonian is number preserving for each type of fermion individually.
- Can encode each fermion “separately”, and “stack” copies of encodings together.

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Better Trotter Bounds

Using Hamiltonian Structure for Better Trotter Error

Better Trotter Bounds

- Childs *et al.* (2019)[1]: improve error bounds to account for commutators: $\left|\left|e^{-iHt} - \mathcal{P}(t)\right|\right| \leq O\left(\alpha t^2 \epsilon^{-1}\right)$

$$H = \sum_{\gamma} H_{\gamma} \qquad \alpha = \sum_{\gamma_2, \gamma_1}^{\Gamma} ||[H_{\gamma_2}, H_{\gamma_1}]||$$

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1: 1912.08854

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- Take advantage of the pionless EFT's number preserving properties

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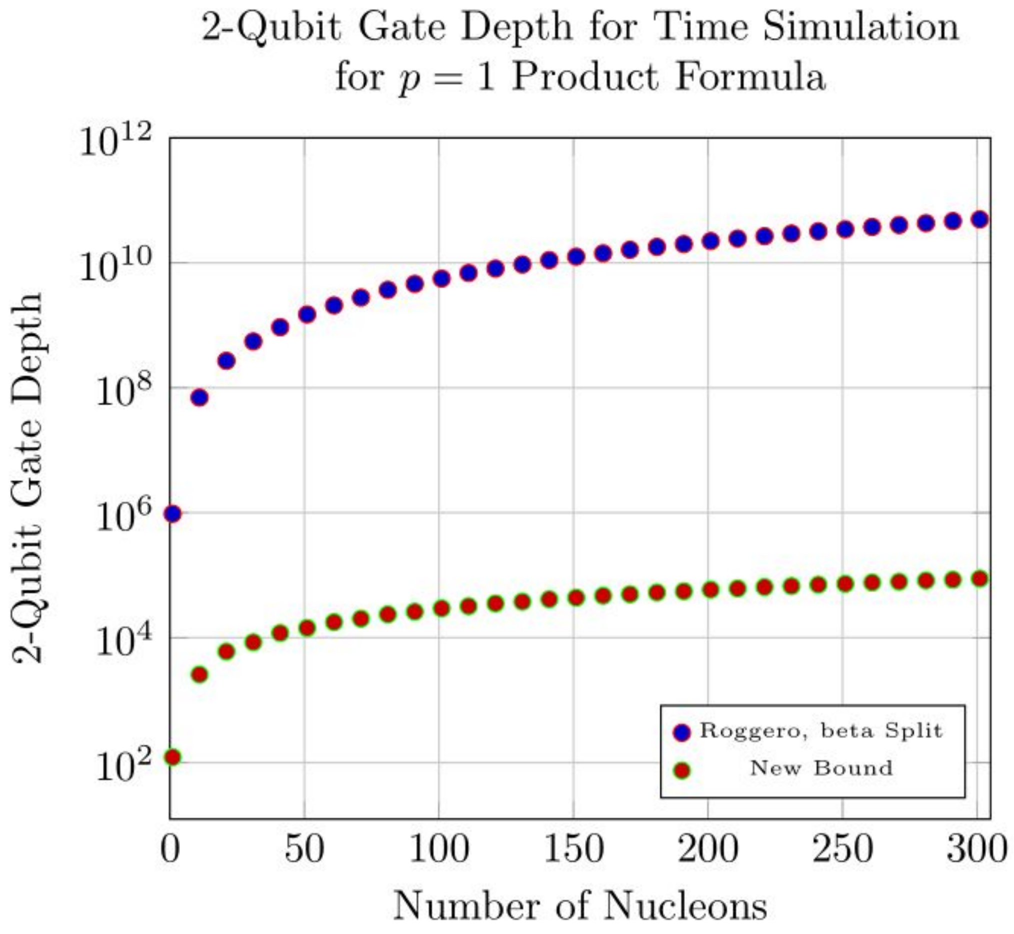
- Combine this with physical constraints on the systems (e.g. preserved particle number as per Su, Huang & Campbell (2020)[2]):

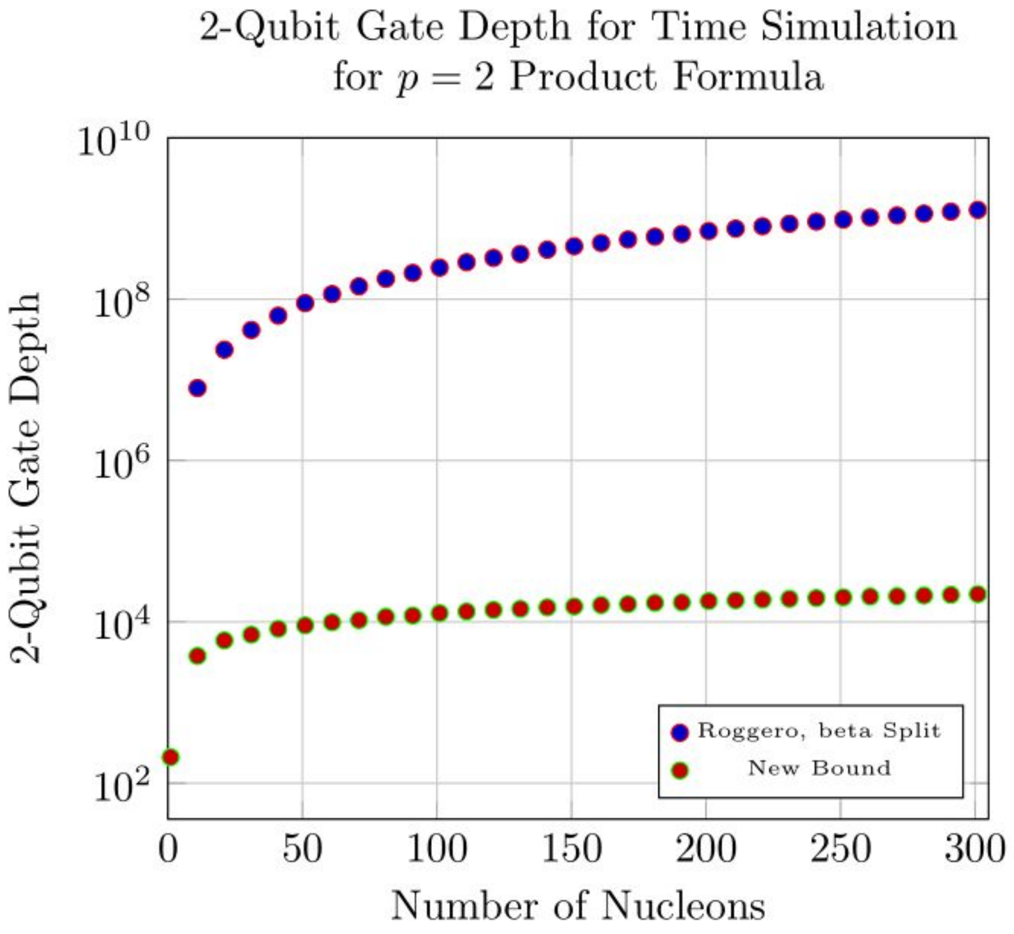
$$\alpha = O(\#\,\text{of particles})$$

1: 1912.08854
2: 2012.09194

Comparisons

- Comparison for p=1,2 Trotter formula having applied these VC encoding + error bounds vs. Roggero *et al.* (2019)[1]:





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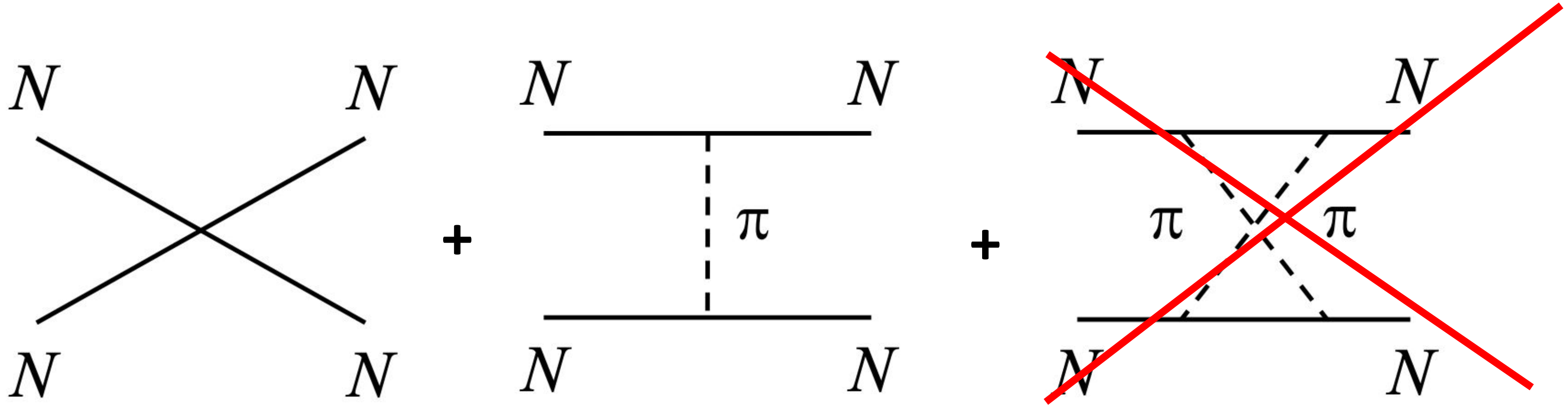
Beyond Pionless EFT

Beyond Pionless EFT

- We now include the first order term



Contact Interactions

Single Pion Exchange

Two Pion Exchange

Beyond Pionless EFT

- Pionless EFT approximates low-energy Hamiltonian.
- We can include higher order interactions:

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu \pi_i \partial^\mu \pi_i - \frac{1}{2} m_\pi^2 \pi_i^2 \qquad \text{Pion Only Terms}$$

$$+N^\dagger \left[\frac{\nabla^2}{2M} - M \right] N$$

$$-\frac{1}{2} C_S (N^\dagger N)(N^\dagger N) - \frac{1}{2} C_T (N^\dagger \sigma_i N)(N^\dagger \sigma_i N) \qquad \text{Nucleon Only Terms}$$

$$+N^\dagger \left[-\frac{1}{4 f_\pi^2} \epsilon_{ijk} \tau_i \pi_j \partial_0 \pi_k - \frac{g_A}{2 f_\pi} \tau_i \sigma_j \partial_j \pi_i \right] N \qquad \text{Nucleon-Pion Interactions}$$

- But what cost do we pay in simulating this?

Beyond Pionless EFT

Models we investigate:

- **Dynamical pions**: include and explicitly simulate pions.

$$\mathcal{H}_0 = \underbrace{\frac{1}{2}\partial_0\pi_i\partial^0\pi_i + \partial_j\pi_i\partial^j\pi_i + \frac{1}{2}m_\pi^2\pi_i^2}_{\text{Pion Kinetic Term}} + \underbrace{N^\dagger\left[\frac{1}{4f_\pi^2}\epsilon_{ijk}\tau_i\pi_j\partial_0\pi_k + \frac{g_A}{2f_\pi}\tau_i\sigma^j\partial_j\pi_j\right]N}_{\text{Nucleon-Pion Interaction}}$$

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- **Instantaneous pions**: include pions, but remove their dynamics.

$$\mathcal{H}_0 = \underbrace{\frac{1}{2}\partial_j\pi_i\partial^j\pi_i + \frac{1}{2}m_\pi^2\pi_i^2}_{\text{Kinetic Ter with }\partial_0\pi_i\text{ removed}} + \underbrace{N^\dagger\left[\frac{g_A}{2f_\pi}\tau_i\sigma^j\partial_j\pi_j\right]N}_{\text{Interaction terms with }\partial_0\pi_i\text{ removed}}$$

(simulation requires additional Monte Carlo runs)

Beyond Pionless EFT

Models we investigate:

- **Dynamical pions**: include and explicitly simulate pions.

$$\mathcal{H}_0 = \underbrace{\frac{1}{2}\partial_0\pi_i\partial^0\pi_i + \partial_j\pi_i\partial^j\pi_i + \frac{1}{2}m_\pi^2\pi_i^2}_{\text{Pion Kinetic Term}} + \underbrace{N^\dagger\left[\frac{1}{4f_\pi^2}\epsilon_{ijk}\tau_i\pi_j\partial_0\pi_k + \frac{g_A}{2f_\pi}\tau_i\sigma^j\partial_j\pi_j\right]N}_{\text{Nucleon-Pion Interaction}}$$

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(simulation requires additional Monte Carlo runs)

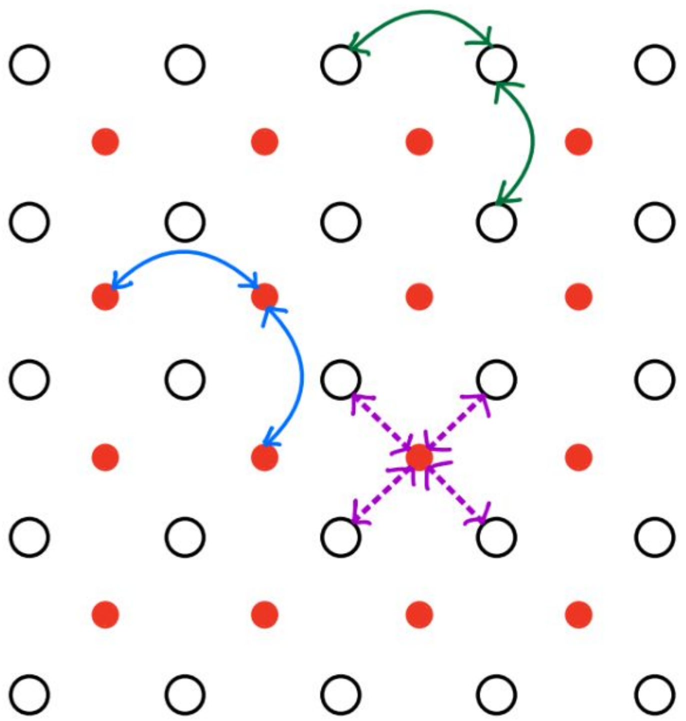
- **One pion exchange:** remove explicit pions and introduce a Yukawa-type potential between fermions.

$$V(r) = \frac{1}{4\pi}\left(\frac{g_A}{2f_\pi}\right)^2 \sum_{x,y}[\tau^\mu]_{ik}[\tau^\mu]_{\alpha\gamma}\left[m_\pi^2\frac{e^{-m_\pi r}}{r}\left\{S_{12}\left(1+\frac{3}{m_\pi r}+\frac{3}{m_\pi^2 r^2}\right)\right.\right.$$
$$\left.\left.+[\sigma^\nu]_{jl}[\sigma^\nu]_{\beta\delta}\right\} - \frac{4\pi}{3}[\sigma^\nu]_{jl}[\sigma^\nu]_{\beta\delta}\delta_{xy}\right]a^\dagger_{ij}(x)a^\dagger_{\alpha\beta}(y)a_{kl}(x)a_{\gamma\beta}(y) \sim \frac{e^{-m_\pi r}}{r}$$

Beyond Pionless EFT

For discretised Hamiltonians, see: Lee (2008)[1] and Madeira *et al.* (2018)[2]

Dynamical Pions



Hollow circles: fermionic sites.
Red circles: bosonic sites.

Instantaneous Pions

Long-Ranged Interaction

1: 0804.3501
2: 1803.10725

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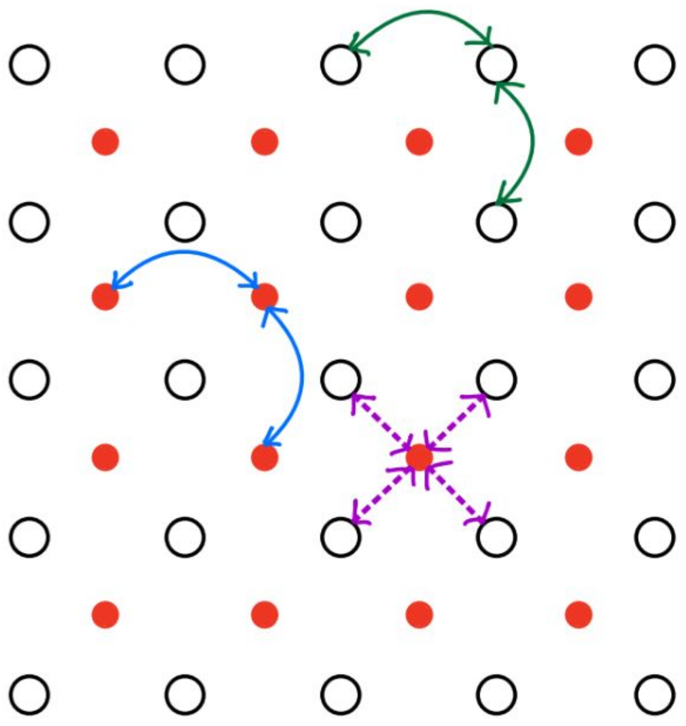

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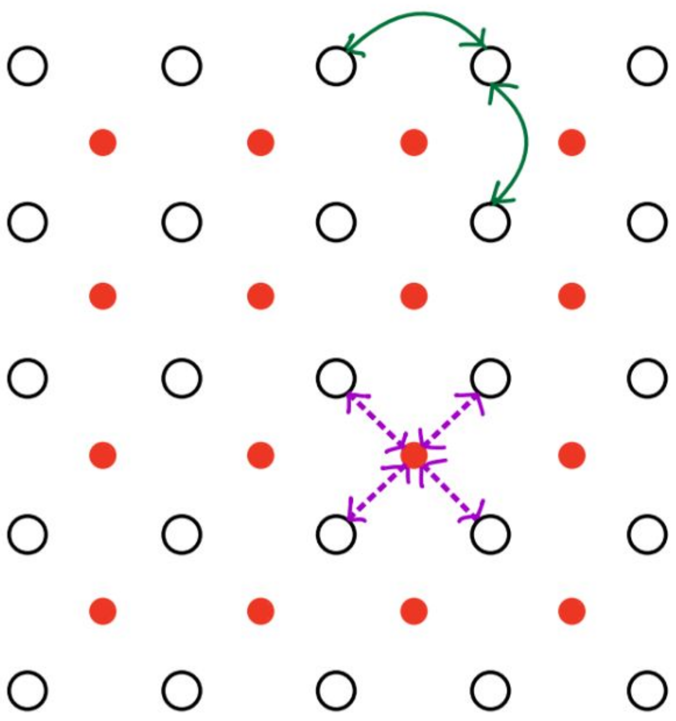

Monte Carlo samples are taken over boson field configurations

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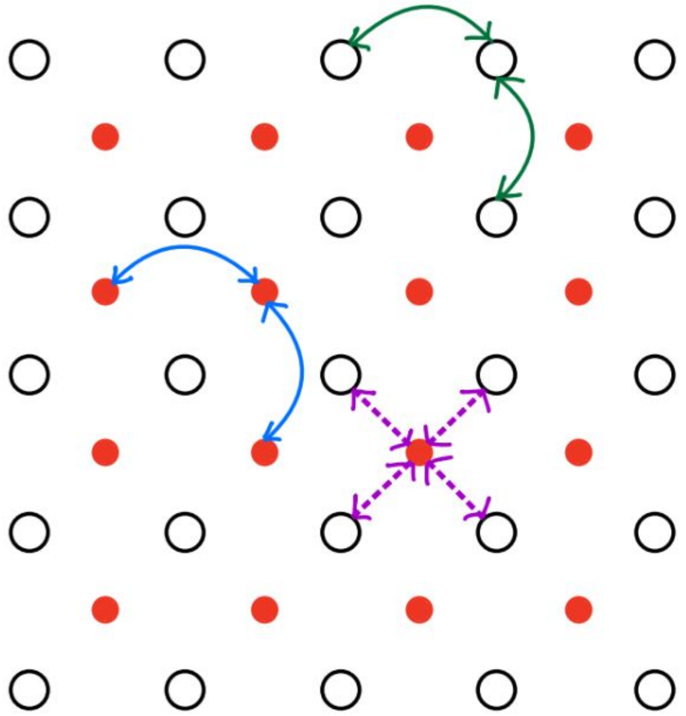

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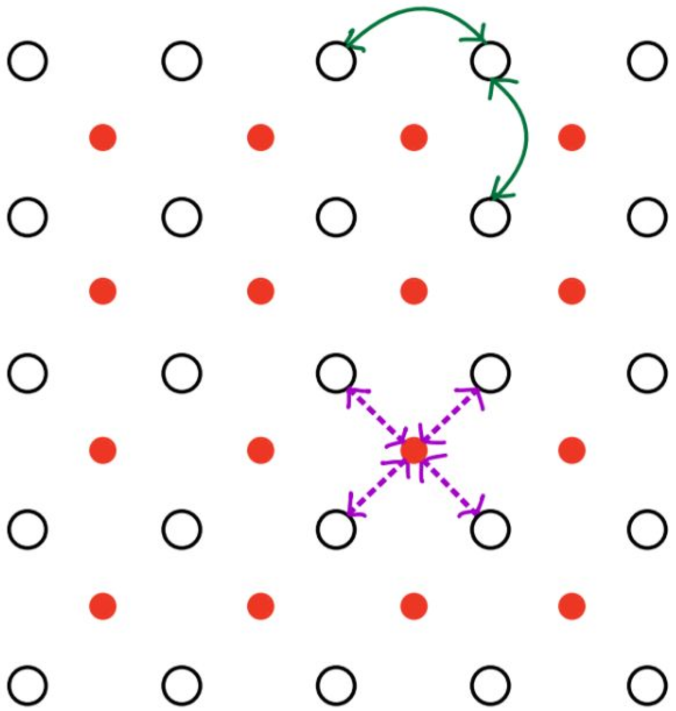

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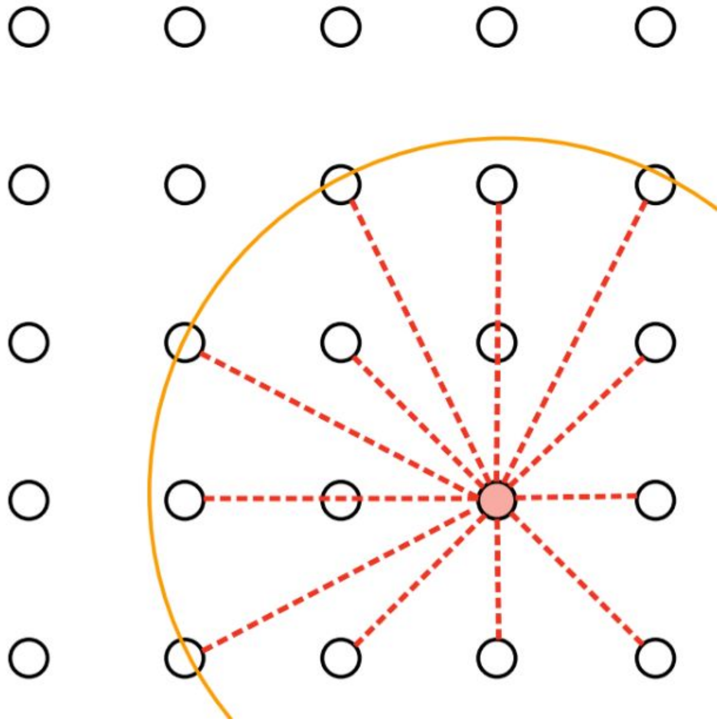

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Monte Carlo samples are taken over boson field configurations

1: 0804.3501
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Beyond Pionless EFT

Models we investigate:

- Dynamical and instantaneous case:
  - Requires explicit encoding of scalar field theory + fermion interactions.
  - See work by Jordan, Lee & Preskill (2012)[1], Klco & Savage (2018)[2] for scalar fields.
  - Need to:
    - Choose pion basis to minimise circuit depth.
    - Choose pion field representation as spin operators.
    - Choose pion field and conjugate momentum cut-off.

- One Pion Exchange case:
  - Determine best representation for interaction given the fermionic encoding.
  - Determining a cut-off length for the long-ranged interaction.
- Both:
  - Circuit decompositions, Hamiltonian decompositions, etc.

1: 1111.3633
2: 1808.10378

Resource Costs for Spectroscopy

Cost Estimates for Quantum Phase Estimation

- Using the standard quantum phase estimation algorithm and p=1 product formulae: 1MeV of energy precision for 6 fermions, on 10x10x10 lattice, with correctness probability p>0.3.

Model	2-Qubit Gate Depth	T-Gate Cost	T-Gate Time Generation
Pionless			
OPE (Long-Ranged)			
Instantaneous Pions			
Dynamical Pions			

T-gate generation time from "Quantum computation with realistic magic state factories", O'Gorman and Campbell, 2016.

- State preparation step not included!

1: 1605.07197

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Model	2-Qubit Gate Depth	T-Gate Cost	T-Gate Time Generation
Pionless	2×10^{7}		
OPE (Long-Ranged)	4×10^{17}		
Instantaneous Pions	3×10^{25}		
Dynamical Pions	1×10^{38}		

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Model	2-Qubit Gate Depth	T-Gate Cost	T-Gate Time Generation
Pionless	$2\text{x}10^{7}$	$1\text{x}10^{17}$	$3\text{x}10^{3}$ - $3\text{x}10^{5}$ years
OPE (Long-Ranged)	$4\text{x}10^{17}$	$2\text{x}10^{28}$	10^{14} - 10^{16} years
Instantaneous Pions	$3\text{x}10^{26}$	$9\text{x}10^{30}$	10^{16} - 10^{18} years
Dynamical Pions	$1\text{x}10^{38}$	$2\text{x}10^{40}$	10^{26} - 10^{28} years

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T-gate generation time from "Quantum computation with realistic magic state factories", O'Gorman and Campbell, 2016.

- Current NISQ devices are nowhere near achieving this:
  - Google's Quantum Supremacy Experiment had depth ~30.
  - IBM currently claims depth ~100.

1: 1605.07197

Cost Estimates for Quantum Phase Estimation

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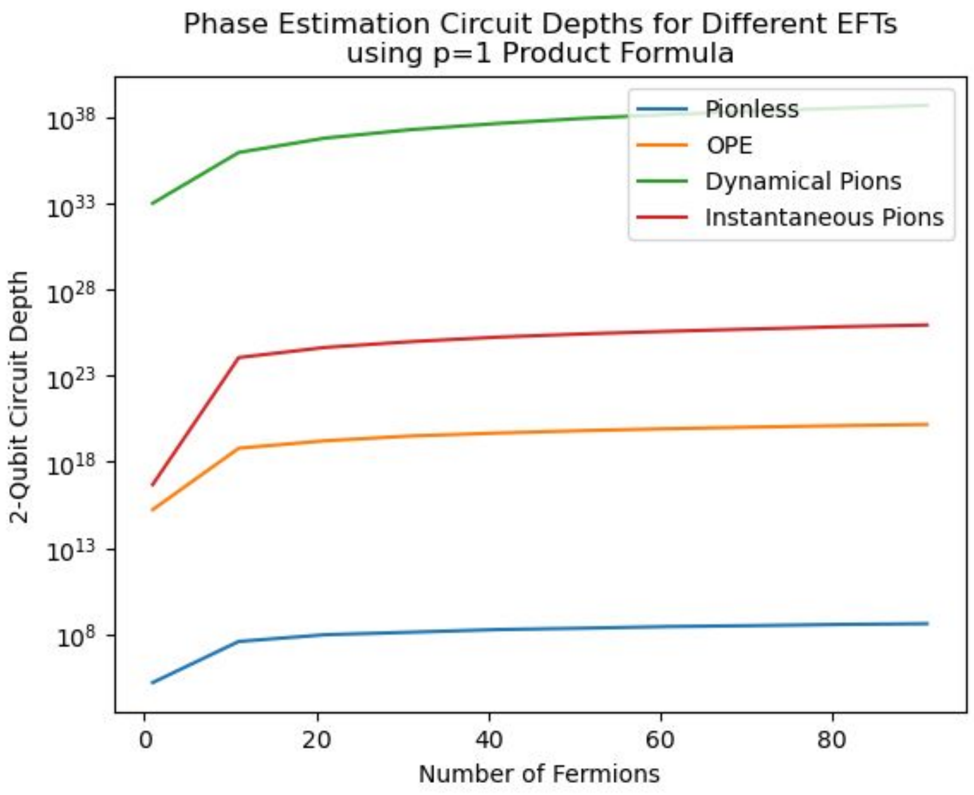

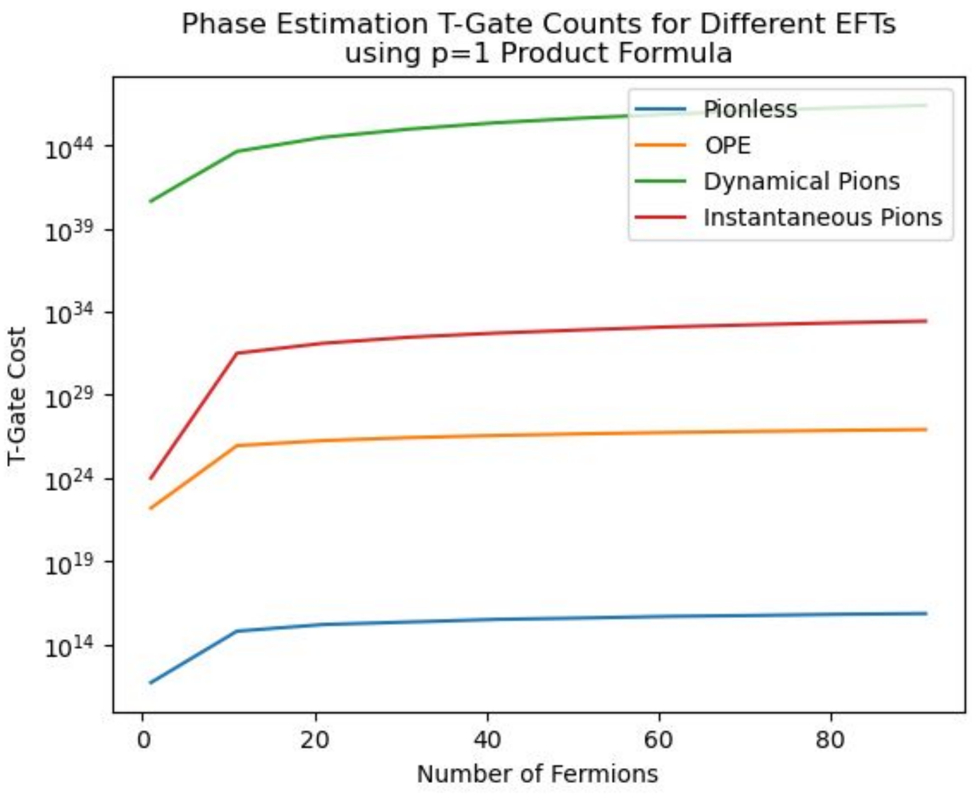

Asymptotic Scaling

- Scaling of resources for time-simulation with p^{th} order product formula **for fixed time**:

η	Number of fermions	E	Energy Scale
L	Lattice size	ε	Precision

Model	2-Qubit Gate Depth	T-Gate Costs	Number of Qubits
Pionless	$O\left(\frac{\eta^{1/p}}{\epsilon^{1/p}}\right)$	$O\left(\frac{\eta^{1/p}L^3}{\epsilon^{1/p}}\log\left(\eta^{1/p}L^3/\epsilon^{1/p}\right)\right)$	$O(L^3)$
OPE (Long-Ranged)	$O\left(\frac{\eta^{1/p}}{\epsilon^{1/p}}\right)$	$O\left(\frac{\eta^{1/p}L^3}{\epsilon^{1/p}}\log\left(\eta^{1/p}L^3/\epsilon^{1/p}\right)\right)$	$O(L^3)$
Instantaneous Pions	$\tilde{O}\left(\left(\frac{E^2L^9\eta^3}{\epsilon^3}\right)^{1/p}\right)$	$\tilde{O}\left(\left(\frac{E^2L^{12}\eta^3}{\epsilon^3}\right)^{1/p}\right)$	$O(L^3\log(\eta^2L^3E/\epsilon^2))$
Dynamical Pions	$\tilde{O}\left(\left(\frac{E^2L^9\eta^4}{\epsilon^3}\right)^{1/p}\right)$	$\tilde{O}\left(\left(\frac{E^2L^{12}\eta^4}{\epsilon^3}\right)^{1/p}\right)$	$O(L^3\log(\eta^3L^3E/\epsilon^2))$

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Conclusions

- Without major improvements to hardware or algorithm, spectroscopy looks unfeasible for NISQ devices.
- OPE and Instantaneous pion models achieve similar efficiencies.
- Dynamical pion models are significantly more expensive than others.

Summary

- Examined the task of time evolution and spectroscopy for 4 Chiral Effective Field Theories.
- For Pionless EFT model, utilising Hamiltonian details allows for 10^4-10^6 better circuit depths using new techniques.
- Provided the first resource estimates for higher order terms in Effective Field Theory.
- Demonstrated a trade-off between different approximations to the first order terms.
- However, all are infeasible for the near/mid-term.

Maybe there's some hope…

Successive improvements have been found for the Fermi-Hubbard model:

Trotter Bounds	Standard Gate Decompositions	Subcircuit Gate Decompositions
Analytic [Ch+18]	976,710	59,830
Analytic [CBC21]	77,236	1,686
Numerical [CBC21]	3,428	259

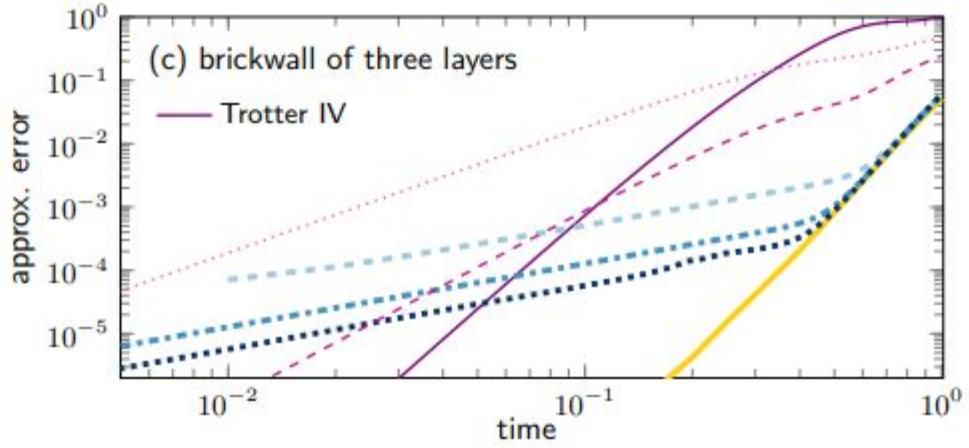
2-qubit gate depth for Fermi-Hubbard model for particular time, error and particle number (Clinton, Bausch, & Cubitt, (2021)[1]).

1: 2003.06886

Routes to Improvement

Further improvements we can make:

- Circuit optimisation algorithms.



Error reduction by factor of ~100, McKeever & Lubasch, (2022)[1]

1: 2205.11427

Routes to Improvement

Further improvements we can make:

- Circuit optimisation algorithms.
- Non-standard gate decompositions.

Trotter Bounds	Standard	Subcircuit
analytic	77,236	1,686

Cost reduction by ~10, Clinton, Bausch & Cubitt, (2020)

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Routes to Improvement

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- New fermionic encodings?

Fermion encoding	Trotter bounds	Standard decomposition	Subcircuit decomposition
VC	analytic	121,478	95,447
compact	analytic	98,339	72,308

Gate cost reduction from developing new fermionic encoding, Clinton, Bausch, & Cubitt (2020)[1]

1: 2003.06886

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Somma (2020)[1], removes need for controlled unitaries – reduces costs by ~20%.

1: 1907.11748

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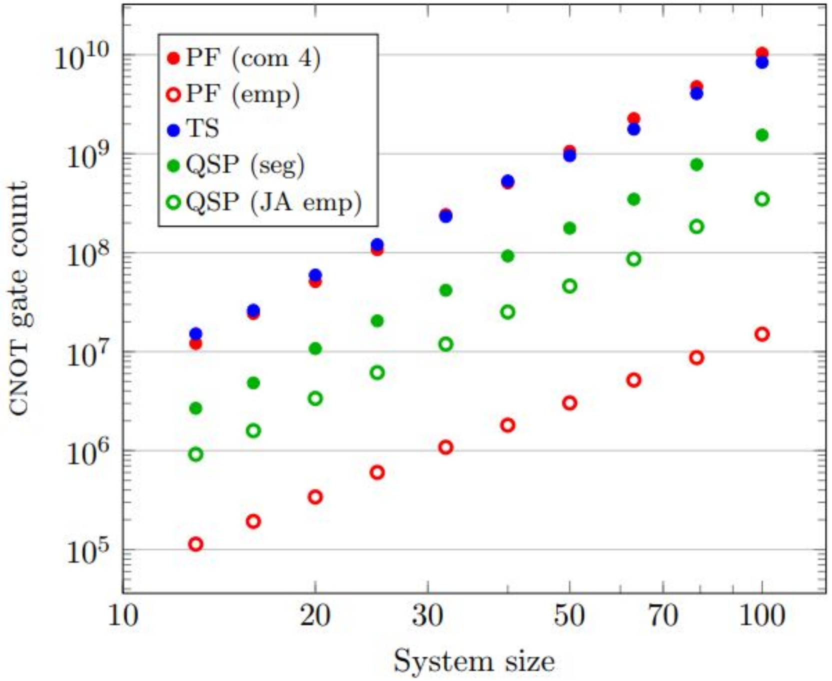

Empirically, the error is often a factor ~10^2 smaller than the analytical bounds Childs *et al.* (2017)[1].

1: 1711.10980

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- Partial error correcting properties of fermionic encodings?
- Randomised simulation/using Hamiltonian symmetries/etc.
- Other ways of simulating nuclear physics besides lattice EFT formulation?

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18 56

Our Work

- Techniques improving quantum algorithms for simulating time evolution of EFTs.

- Beyond pionless EFT & associated costs.

EFTs and Quantum Computing

- First principles nuclear dynamics is difficult – use an effective field theory instead!
- Time evolution is an important primitive, e.g. for quantum phase estimation.

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- Try simulating using product formulae. For Hamiltonian: $H = \sum_{\gamma=1}^{L} H_\gamma$

$$e^{-iH\delta t} \approx e^{-iH_1\delta t} e^{-iH_2\delta t} \ldots e^{-iH_L\delta t} =: \mathcal{P}(\delta t)$$

$$\left|\left|e^{-iH\delta t} - \mathcal{P}(\delta t)\right|\right| \leq \epsilon$$

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- Product formulae have low overhead and generally perform well!

- How well can we improve our error bounds and optimise computational resources?

Previous Work in EFTs and Quantum Computing

Roggero *et al.* (2019)[1] on simulating time evolution and linear response function using pionless effective field theory model.

$$H_{EFT} = \overbrace{\frac{1}{2Ma^2}\sum_{\langle i,j\rangle}\left(a_\sigma^\dagger(i)a_\sigma(j) + a_\sigma^\dagger(j)a_\sigma(i)\right)}^{\text{kinetic term}} + \overbrace{\frac{C}{2}\sum_{i,\sigma\neq\sigma'} n_\sigma(i)n_{\sigma'}(i)}^{\text{2-body onsite interaction}} + \overbrace{\frac{D}{6}\sum_{i,\sigma\neq\sigma'\neq\sigma''} n_\sigma(i)n_{\sigma'}(i)n_{\sigma''}(i)}^{\text{3-body onsite interaction}}$$

Creation and annihilation operators for nucleons.

Nucleon number operator at site *i*.

1: 1911.06368

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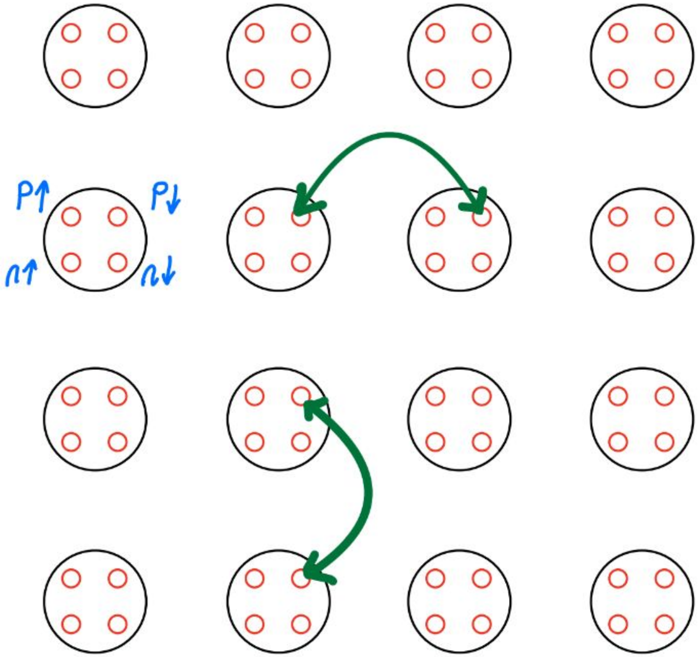

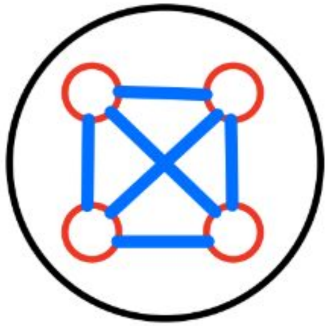

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$$H_{FH} = \underbrace{J\sum_{\langle i,j\rangle}\left(a_\sigma^\dagger(i)a_\sigma(j) + a_\sigma^\dagger(j)a_\sigma(i)\right)}_{\text{kinetic terms}} + \underbrace{U\sum_{i,\sigma\neq\sigma} n_\sigma(i)n_{\sigma'}(i)}_{\text{2-body onsite interaction}}$$

The Fermi-Hubbard model is similar and well studied – can we apply techniques from its analysis + improved Trotter analysis to get bounds?

1: 1911.06368

Better Trotter Bounds

- Childs *et al.* (2019)[1]: improve error bounds to account for commutators: $$\left|\left|e^{-iHt} - \mathcal{P}(t)\right|\right| \leq O\left(\alpha t^2 \epsilon^{-1}\right)$$

$$H = \sum_{\gamma} H_{\gamma} \qquad \alpha = \sum_{\gamma_2, \gamma_1}^{L} ||[H_{\gamma_2}, H_{\gamma_1}]||$$

(we use similar bounds for p=2)

1: 1912.08854

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- Combine this with physical constraints on the systems (e.g. preserved particle number as per Su, Huang & Campbell (2020)[2]):

$$\alpha = O(\#\,\text{of particles})$$

1: 1912.08854
2: 2012.09194

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- Yi & Crosson(2021)[3]: improve error for quantum phase estimation by considering the effective Trotterised Hamiltonian.

1: 1912.08854
2: 2012.09194
3: 2102.12655

Better Circuit Depths

- In simulating the pionless EFT, the most expensive term is the kinetic hopping term.

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  - Alternatives: Fast Fermionic Fourier Transform/SWAP Network/Givens Rotation, need gate depth proportional to number of fermionic modes.

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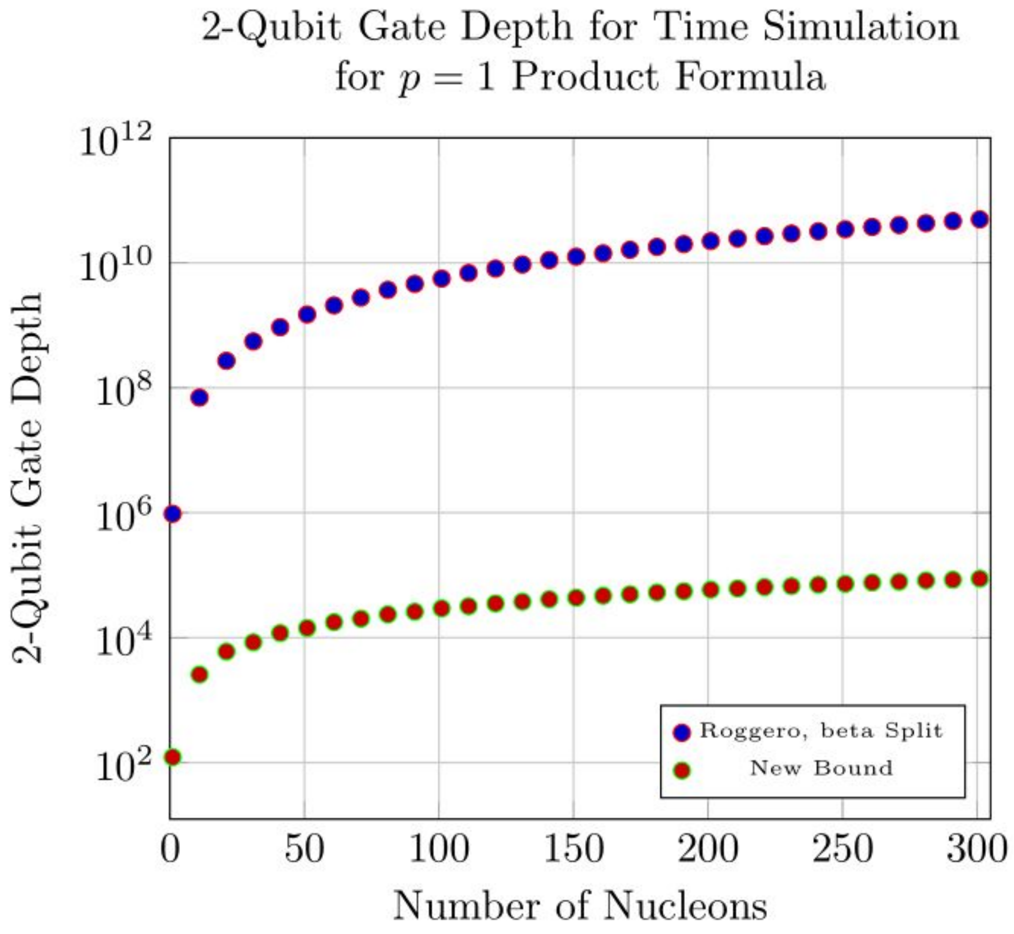
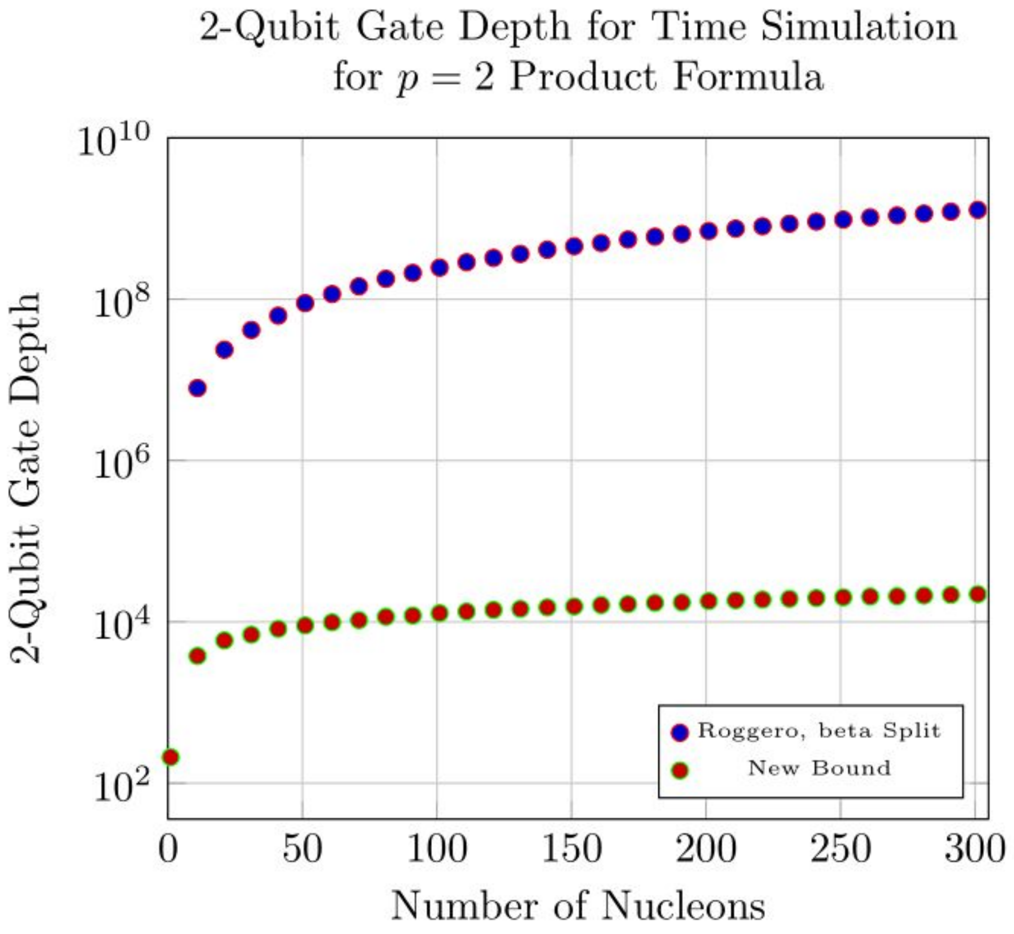
  - Alternatives: Fast Fermionic Fourier Transform/SWAP Network/Givens Rotation, need gate depth proportional to number of fermionic modes.

- Leverage interaction locality: encode fermions using Verstraete-Cirac encoding.
  - Implement in circuit of depth ~100, regardless of number of fermions or lattice size.

Comparisons

- Comparison for p=1,2 Trotter formula having applied these methods vs. bounds from Roggero *et al.* (2019)[1]:





1: 1911.06368

Beyond Pionless EFT

Beyond Pionless EFT

- Pionless EFT approximates low-energy Hamiltonian.
- We can include higher order interactions:

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu \pi_i \partial^\mu \pi_i - \frac{1}{2} m_\pi^2 \pi_i^2$$ Pion Only Terms

$$+N^\dagger \left[\frac{\nabla^2}{2M} - M \right] N$$

$$-\frac{1}{2} C_S (N^\dagger N)(N^\dagger N) - \frac{1}{2} C_T (N^\dagger \sigma_i N)(N^\dagger \sigma_i N)$$ Nucleon Only Terms

$$+N^\dagger \left[-\frac{1}{4 f_\pi^2} \epsilon_{ijk} \tau_i \pi_j \partial_0 \pi_k - \frac{g_A}{2 f_\pi} \tau_i \sigma_j \partial_j \pi_i \right] N$$ Nucleon-Pion Interactions

- But what cost do we pay in simulating this?

Beyond Pionless EFT

Models we investigate:

- **Dynamical pions**: include and explicitly simulate pions.

$$\mathcal{H}_0 = \underbrace{\frac{1}{2}\partial_0\pi_i\partial^0\pi_i + \partial_j\pi_i\partial^j\pi_i + \frac{1}{2}m_\pi^2\pi_i^2}_{\text{Pion Kinetic Term}} + \underbrace{N^\dagger\left[\frac{1}{4f_\pi^2}\epsilon_{ijk}\tau_i\pi_j\partial_0\pi_k + \frac{g_A}{2f_\pi}\tau_i\sigma^j\partial_j\pi_j\right]N}_{\text{Nucleon-Pion Interaction}}$$

Beyond Pionless EFT

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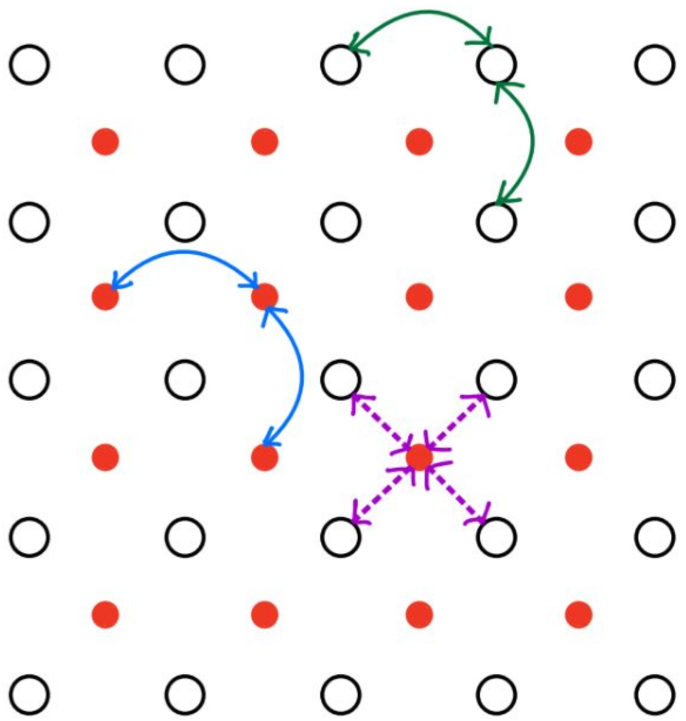
- **One pion exchange:** remove explicit pions and introduce an effective Yukawa-type potential between fermions.

$$V(r) = \frac{1}{4\pi}\left(\frac{g_A}{2f_\pi}\right)^2\sum_{x,y}[\tau^\mu]_{ik}[\tau^\mu]_{\alpha\gamma}\left[m_\pi^2\frac{e^{-m_\pi r}}{r}\left\{S_{12}\left(1+\frac{3}{m_\pi r}+\frac{3}{m_\pi^2 r^2}\right)\right.\right.$$
$$\left.\left.+[\sigma^\nu]_{jl}[\sigma^\nu]_{\beta\delta}\right\} - \frac{4\pi}{3}[\sigma^\nu]_{jl}[\sigma^\nu]_{\beta\delta}\delta_{xy}\right]a_{ij}^\dagger(x)a_{\alpha\beta}^\dagger(y)a_{kl}(x)a_{\gamma\beta}(y) \sim \frac{e^{-m_\pi r}}{r}$$

Beyond Pionless EFT

For discretised Hamiltonians, see: Lee (2008)[1] and Madeira *et al.* (2018)[2]

Dynamical Pions



Hollow circles: fermionic sites.
Red circles: bosonic sites.

Instantaneous Pions

Long-Ranged Interaction

1: 0804.3501
2: 1803.10725

Beyond Pionless EFT

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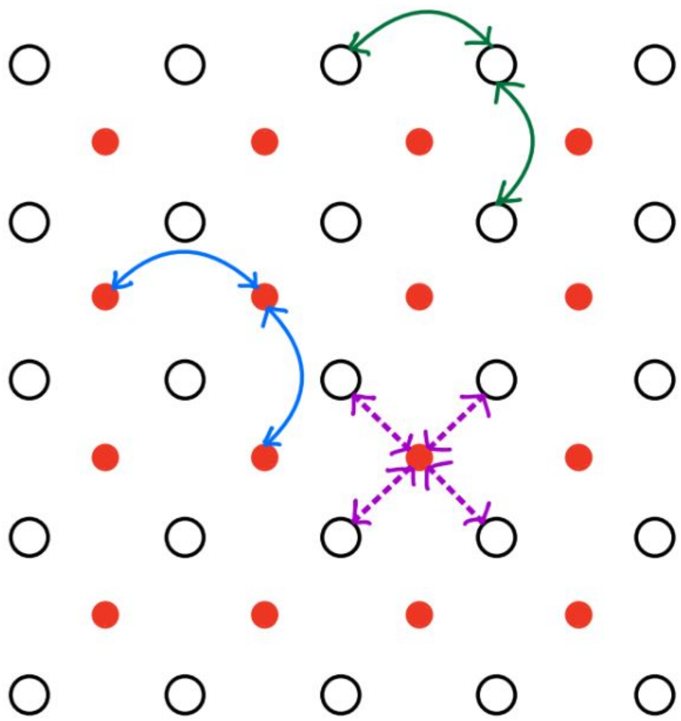

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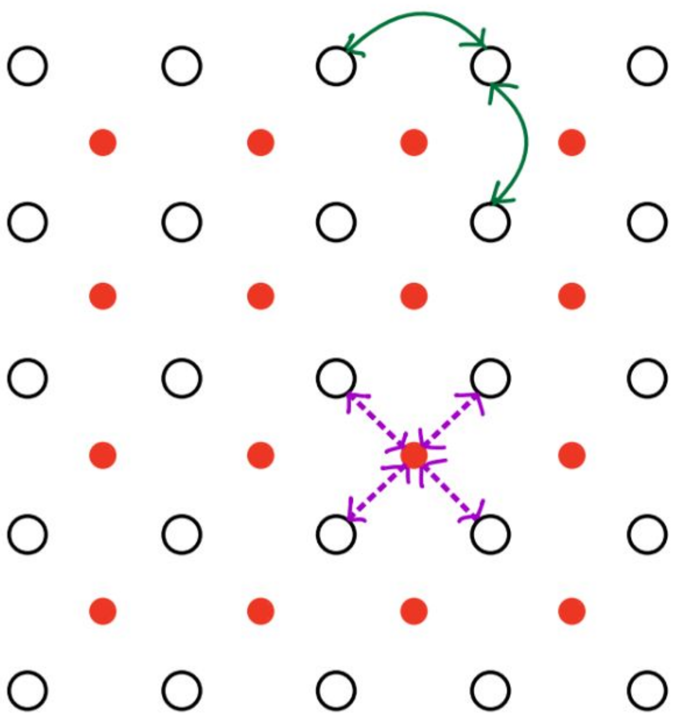

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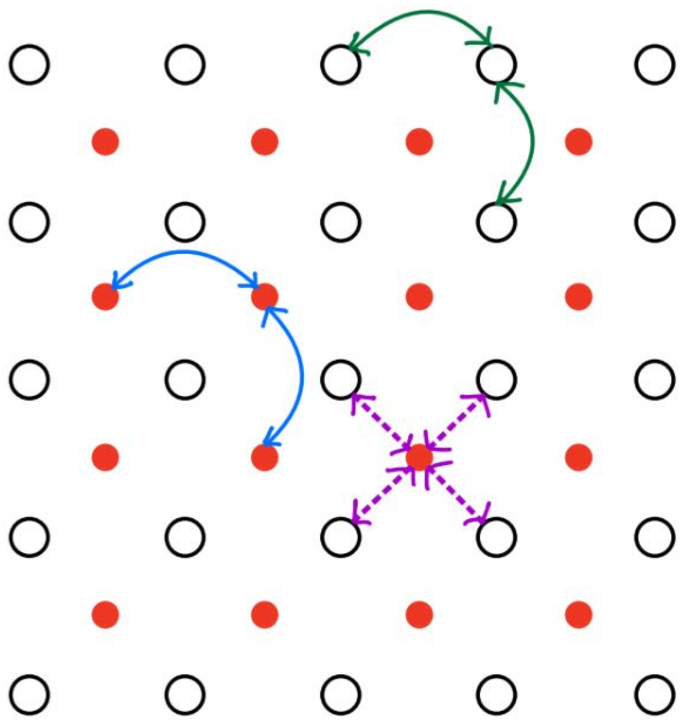

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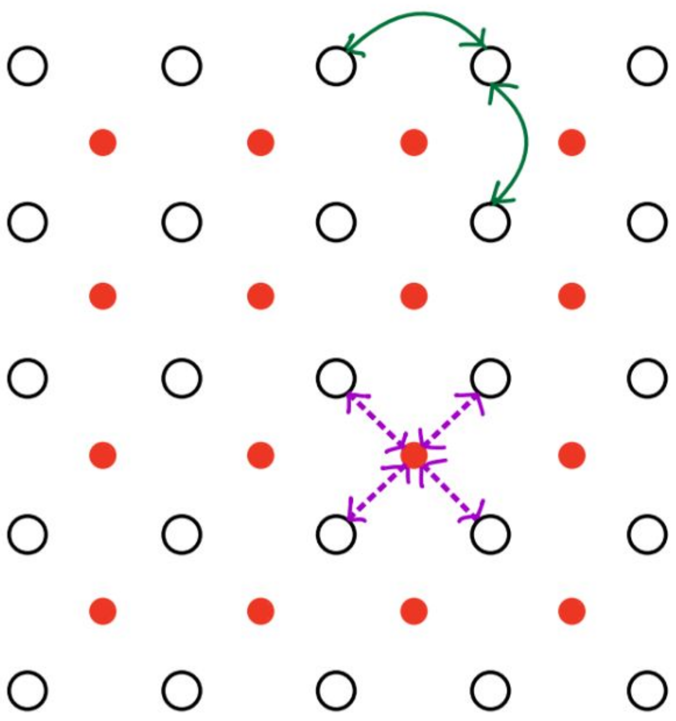

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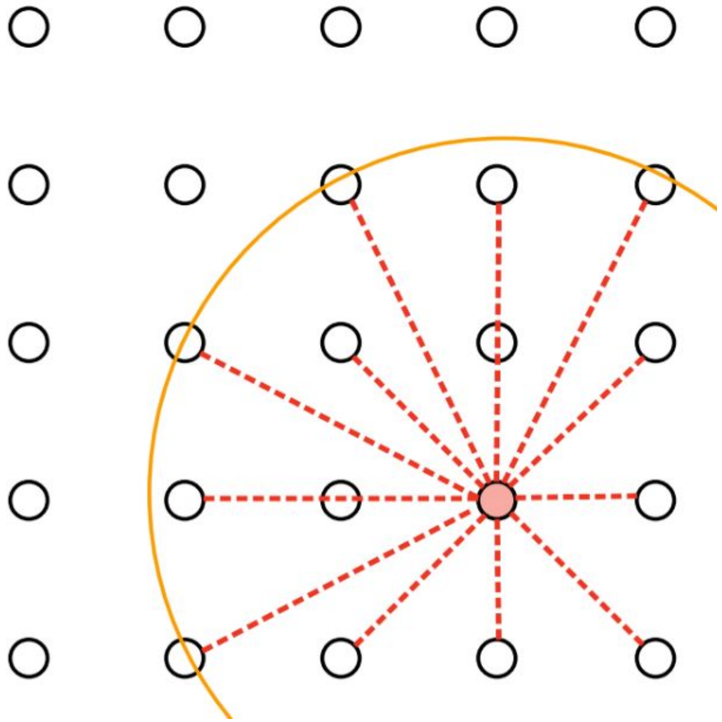

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Beyond Pionless EFT

Models we investigate:

- Dynamical and instantaneous case:
  - Requires explicit encoding of scalar field theory + fermion interactions.
  - See work by Jordan, Lee & Preskill (2012)[1], Klco & Savage (2018)[2] for scalar fields.
  - Need to:
    - Choose pion basis to minimise circuit depth.
    - Choose pion field representation as spin operators.
    - Choose pion field and conjugate momentum cut-off.

1: 1111.3633
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- One Pion Exchange case:
  - Determine best representation for interaction given the fermionic encoding.
  - Determining a cut-off length for the long-ranged interaction.

1: 1111.3633
2: 1808.10378

Cost Estimates for Quantum Phase Estimation

- Using the standard quantum phase estimation algorithm and p=1 product formulae: 1MeV of energy precision for 6 fermions, on 10x10x10 lattice, with correctness probability p>0.9.

Model	2-Qubit Gate Depth	T-Gate Cost	T-Gate Time Generation
Pionless			
OPE			
Instantaneous Pions			
Dynamical Pions			

T-gate generation time from "Quantum computation with realistic magic state factories", O'Gorman and Campbell, 2016.

- State preparation step not included!

1: 1605.07197

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T-gate generation time from "Quantum computation with realistic magic state factories", O'Gorman and Campbell, 2016.

- Current NISQ devices are nowhere near achieving this:
  - Google's Quantum Supremacy Experiment had depth ~30.
  - IBM currently claims depth ~100.

1: 1605.07197

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T-gate generation time from "Quantum computation with realistic magic state factories", O'Gorman and Campbell, 2016.

- Additional question: what is the "correct" precision to work to for each model? High precision pointless if the effective Hamiltonian has larger systematic error.

1: 1605.07197

Maybe there's some hope…

Successive improvements have been found for the Fermi-Hubbard model:

Trotter Bounds	Standard Gate Decompositions	Subcircuit Gate Decompositions
Analytic [Ch+18]	976,710	59,830
Analytic [CBC21]	77,236	1,686
Numerical [CBC21]	3,428	259

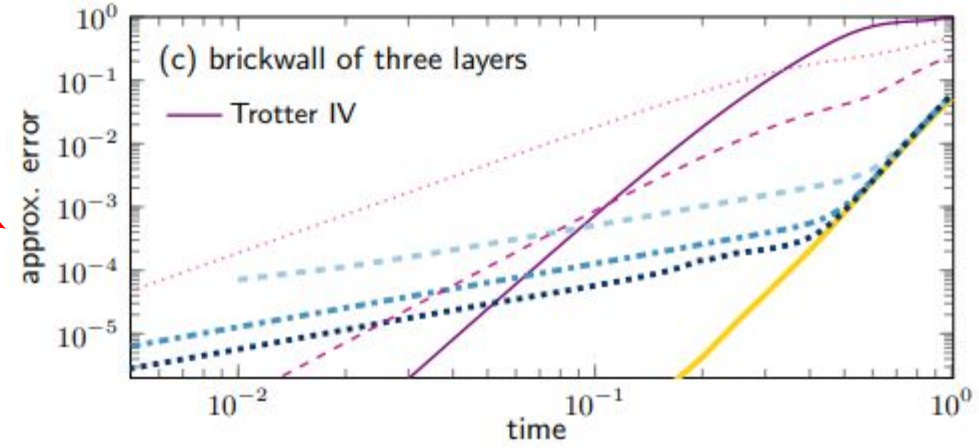
2-qubit gate depth for Fermi-Hubbard model for particular time, error and particle number (Clinton, Bausch, & Cubitt, (2021)[1]).

1: 2003.06886

Routes to Improvement

Further improvements we can make:

- Circuit optimisation algorithms.



Error reduction by factor of ~100, McKeever & Lubasch, (2022)[1]

1: 2205.11427

Routes to Improvement

Further improvements we can make:

- Circuit optimisation algorithms.
- Non-standard gate decompositions.

Trotter Bounds	Standard	Subcircuit
analytic	77,236	1,686

Cost reduction by ~10, Clinton, Bausch & Cubitt, (2020)

1: 2003.06886

Routes to Improvement

Further improvements we can make:

- Circuit optimisation algorithms.
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- New fermionic encodings?

Fermion encoding	Trotter bounds	Standard decomposition	Subcircuit decomposition
VC	analytic	121,478	95,447
compact	analytic	98,339	72,308

Gate cost reduction from developing new fermionic encoding, Clinton, Bausch, & Cubitt (2020)[1]

1: 2003.06886

Routes to Improvement

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- Circuit optimisation algorithms.
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- Better phase estimation algorithms.

Somma (2020)[1], removes need for controlled unitaries – reduces costs by ~20%.

1: 1907.11748

Routes to Improvement

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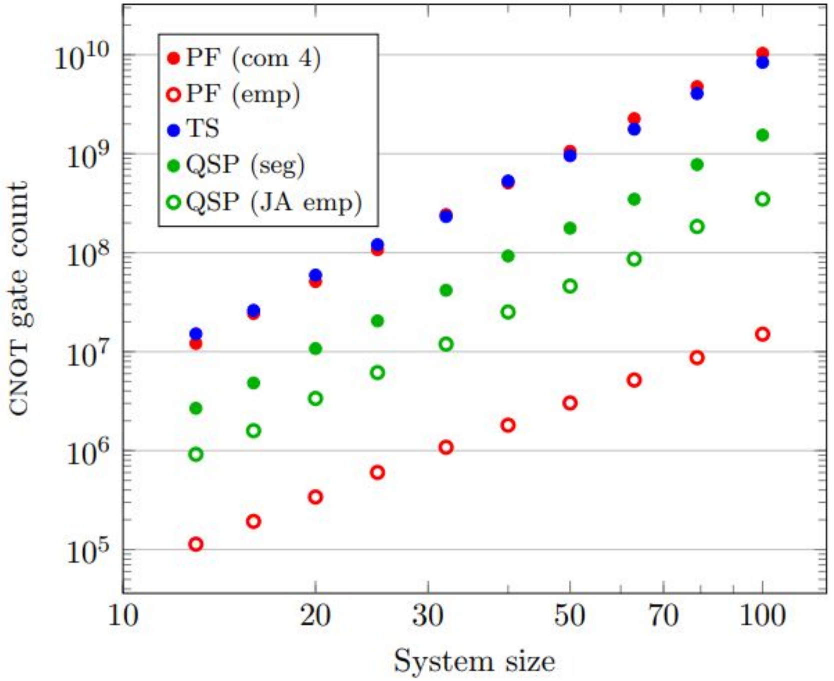
- Empirical vs analytical error?



Empirically, the error is often a factor ~10^2 smaller than the analytical bounds Childs *et al.* (2017)[1].

1: 1711.10980

Routes to Improvement

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- Partial error correcting properties of fermionic encodings?
- Randomised simulation/using Hamiltonian symmetries/etc.
- Other ways of simulating nuclear physics besides lattice EFT formulation?

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Limitations of New Techniques

- Childs et al error improve bounds to account for commutators:

$$\left|\left|e^{-iHt} - \mathcal{P}_p(t)\right|\right| \leq O\left(\alpha t^{1+p}\epsilon^{-p}\right)$$

$$\alpha = \sum_{\gamma_{p+1},\gamma_p \cdots \gamma_1} \left|\left|\left[H_{\gamma_{p+1}}, \ldots \left[H_{\gamma_2}, H_{\gamma_1}\right]\right]\right|\right|$$

- Calculating nested commutator is difficult for p>2.

- Verstraete-Cirac encoding comes with more difficult state preparation.

Future Work

- Different spectroscopy/QPE algorithms.
- Optimising circuit depths and T-gate counts.
- Non-standard gate decompositions.
- Inherent error correction in fermionic encodings
- Lower-cost use cases generally?

Beyond Pionless EFT

- Pionless EFT approximates low-energy Hamiltonian.
- We can include higher order interactions:

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu \pi_i \partial^\mu \pi_i - \frac{1}{2} m_\pi^2 \pi_i \pi_i + N^\dagger \Big[i\partial_0 + \frac{\nabla^2}{2M_0} - \frac{1}{4f_\pi^2}\epsilon_{ijk}\tau_i \pi_j \partial_0 \pi_k - \frac{g_A}{2f_\pi}\tau_i \sigma^j \partial_j \pi_i - M_0 \Big] N - \frac{1}{2} C_S (N^\dagger N)(N^\dagger N) - \frac{1}{2} C_T (N^\dagger \sigma_i N)(N^\dagger \sigma_i N)$$

- But what cost do we pay in simulating this?

Better Trotter Bounds

- Childs *et al.* (2019)[1]: improve error bounds to account for commutators: $\left|\left|e^{-iHt} - \mathcal{P}(t)\right|\right| \leq O\left(\alpha t^2 \epsilon^{-1}\right)$

$$H = \sum_{\gamma} H_{\gamma} \qquad \alpha = \sum_{\gamma_2,\gamma_1}^{L} ||[H_{\gamma_2}, H_{\gamma_1}]||$$

(we use similar bounds for p=2)

- Combine this with physical constraints on the systems (e.g. preserved particle number as per Su, Huang & Campbell (2020)[2]):

$$\alpha = O(\#\,\text{of particles})$$

- Yi & Crosson(2021)[3]: improve error for quantum phase estimation by considering the effective Trotterised Hamiltonian.

1: 1912.08854
2: 2012.09194
3: 2102.12655

Previous Work in EFTs and Quantum Computing

- Pionless EFT is the simplest Hamiltonian that recreates basic properties of the nucleus.

$$H_{EFT} = \overbrace{\frac{1}{2Ma^2}\sum_{\langle i,j\rangle}\Big(a_\sigma^\dagger(i)a_\sigma(j) + a_\sigma^\dagger(j)a_\sigma(i)\Big)}^{\text{kinetic term}} + \overbrace{\frac{C}{2}\sum_{i,\sigma\neq\sigma'} n_\sigma(i)n_{\sigma'}(i)}^{\text{2-body onsite interaction}} + \overbrace{\frac{D}{6}\sum_{i,\sigma\neq\sigma'\neq\sigma''} n_\sigma(i)n_{\sigma'}(i)n_{\sigma''}(i)}^{\text{3-body onsite interaction}}$$

$$H_{FH} = \underbrace{J\sum_{\langle i,j\rangle}(a_\sigma^\dagger(i)a_\sigma(j) + a_\sigma^\dagger(j)a_\sigma(i))}_{\text{kinetic terms}} + \underbrace{U\sum_{i,\sigma\neq\sigma} n_\sigma(i)n_{\sigma'}(i)}_{\text{2-body onsite interaction}}$$

The Fermi-Hubbard model is similar and well studied – can we apply techniques from its analysis + improved Trotter analysis to get bounds?

1: 1911.06368