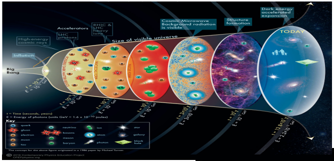
Riffing on the QCD thermal transition in the chiral limit

Francesca Cuteri

May 25th, 2023 - ECT*

HOT DENSE & MAGNETIZED STATES OF QCD MATTER

Hot/dense/magnetized states of QCD matter - The sketch



Early Universe (QCD epoch)

$100\text{MeV} \lesssim T \lesssim 200\text{MeV}$

$b = (8.60 \pm 0.06) \times 10^{-11}$, $q = 0$

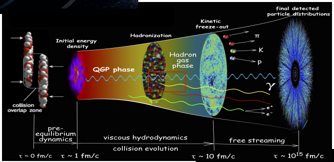
$|l| = |l_e + l_\mu + l_\tau| < 0.012$

$B? \longrightarrow 10^{-16} \lesssim B \lesssim 10^{-9}\text{Gauss}$ (EGMFs)

Hotter

“diluter”

$1\text{eV} \approx 12 \times 10^3 K$; $B_{\text{Earth}} \approx 0.5\text{Gauss}$



Heavy ion collisions (HIC)

$50\text{MeV} \lesssim T \lesssim 200\text{MeV}$

$n \lesssim 0.12\text{fm}^{-3}$

$\delta = N - Z/(N+Z) \lesssim 0.25$

$B \lesssim 10^{19}\text{Gauss} \sim 10^{20} B_{\text{Earth}}$

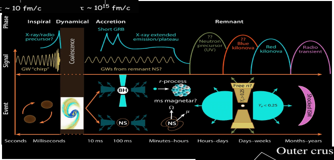
$T \in [50, 80]\text{MeV}$

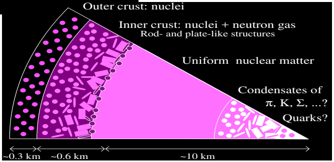
$n \sim 2 n_0, \quad n_0 = 0.16\text{fm}^{-3}$

$n_{\text{quark}} \neq 0$

$10^{10} \lesssim B \lesssim 10^{12}\text{Gauss} \rightarrow B \gtrsim 10^{16}\text{Gauss}$

Binary Neutron Star (BNS) mergers





$T \lesssim 1\text{KeV}$

$0.3 n_0 \lesssim n \lesssim 15 n_0, \quad n_0 = 0.16\text{fm}^{-3}$

$n_p / n \sim 0.04$, at n_0

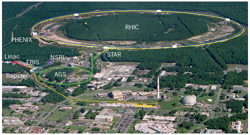
$10^8 \lesssim B \lesssim 10^{15}\text{Gauss}$

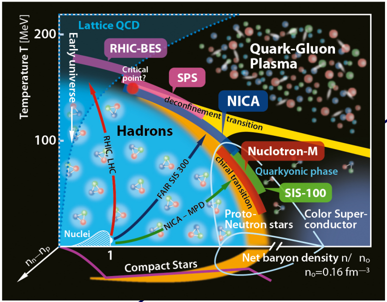
Neutron star interior

Colder

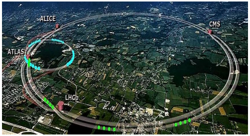
denser

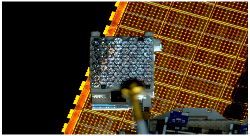
Major experimental and observational campaigns





Observational astronomy





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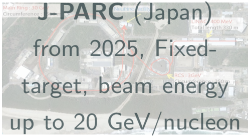


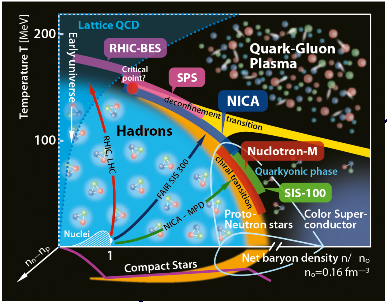
J-PARC (Japan)
from 2025, Fixed-target, beam energy up to 20 GeV/nucleon

NICA (Russia)
from 2022, collisions up to 11 GeV/nucleon

FAIR (Germany)
from 2025, high statistics at highest baryon density

RHIC (USA)
through 2025, more high-energy collisions (focus, hard probes)

LHC (CERN)
from 2022, in run 3 highest-energy collisions available



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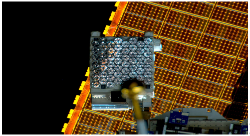
Heavy ion collisions



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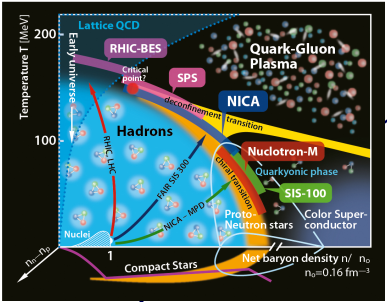


Observational astronomy

Heavy ion collisions

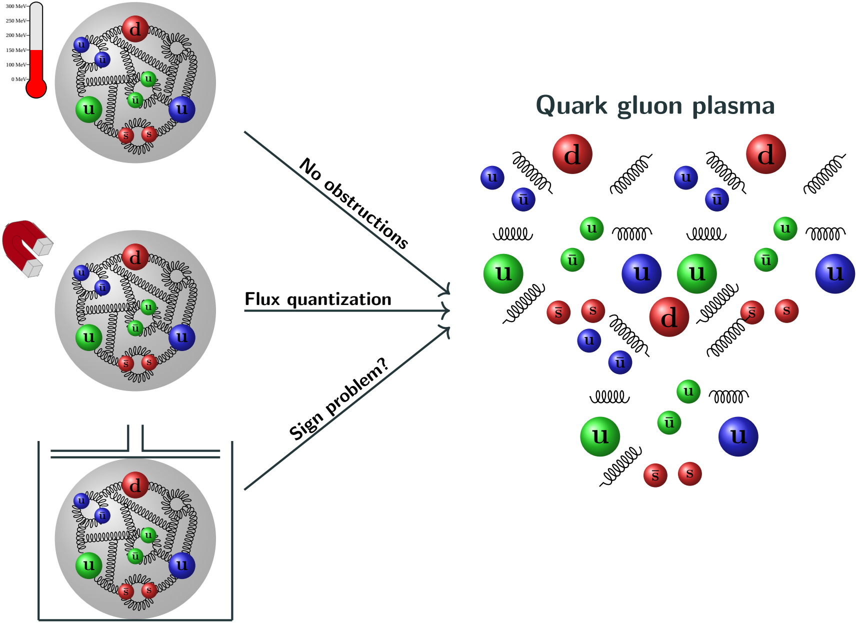
SKA (AU, RSA)
from 2027, origin/evolution of cosmic magnetic fields

VIRGO (IT-FR)
since 2016, detecting GW in collaboration with **LIGO**

NICER (ISS)
since 2017, Neutron Star Interior Composition Explorer

STRONGLY INTERACTING MATTER
ON THE LATTICE
AT NONZERO TEMPERATURE/DENSITY

QCD matter under extreme conditions on the lattice



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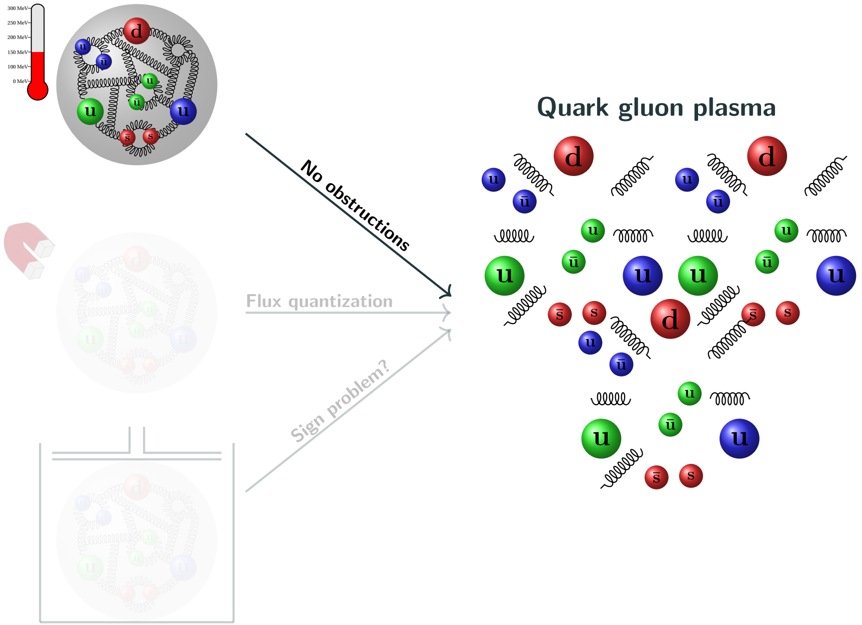


Thermal lattice QCD

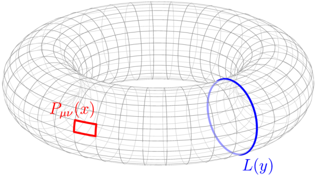
Quantum mechanical statistical system in heatbath

$$Z(T) = \mathrm{Tr}\left[e^{-\hat{H}/T}\right]$$

Euclidean quantum field theory

$$Z(T) = \int \mathcal{D}A\,\mathcal{D}\psi\,\mathcal{D}\bar{\psi}\; e^{-\mathcal{S}_E[A,\psi,\bar{\psi}]}$$

$$\mathcal{S}_E[A,\psi,\bar{\psi}] = \int_0^{\frac{1}{T}} \mathrm{d}x_4 \int_V \mathrm{d}^3x \mathcal{L}_E\left[A,\psi,\bar{\psi}\right]$$



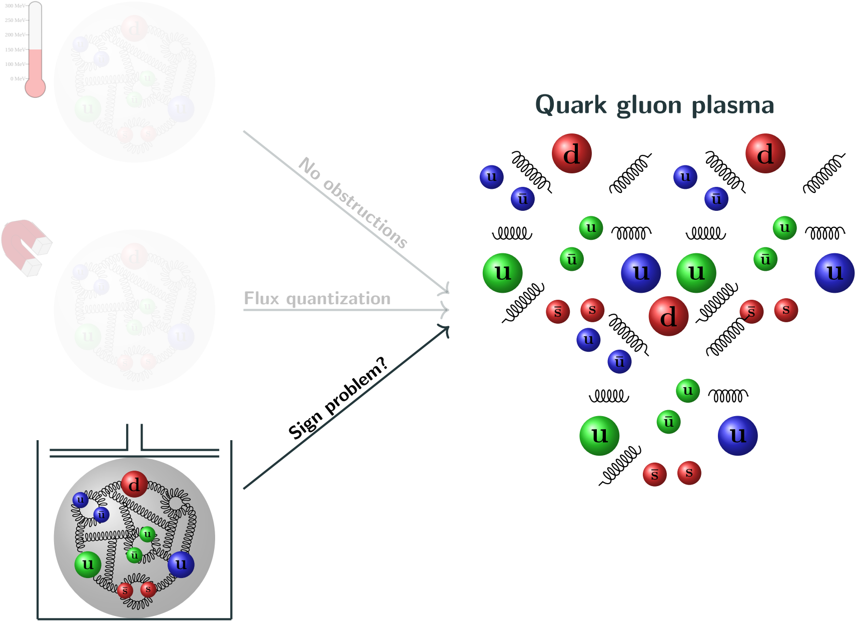
$$T = \frac{1}{a(\beta)N_\tau}$$

$$a \ll \xi \ll aN_\sigma$$

$$a \ll aN_\tau \ll aN_\sigma$$

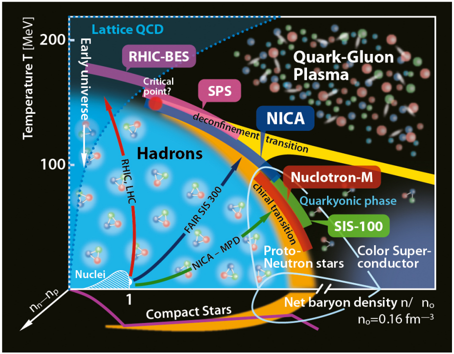
- Bosonic (fermionic) fields periodic (anti-periodic) in the finite time direction to ensure Bose-Einstein (Fermi-Dirac) statistics
- Continuum limit at fixed T: $a \to 0 \Longleftrightarrow N_\tau \to \infty$

QCD matter under extreme conditions on the lattice



Then $T - n_B$ phase diagram via Lattice QCD? Not really...

Nonzero **baryon density** $n = \frac{n_u + n_d}{3}$
$\rightarrow$ excess of matter over antimatter



Sign problem

Nonzero **isospin density** $n_I = n_u - n_d$
$\rightarrow$ excess of neutrons over protons

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Chemical potentials basis: $\mu_u = \mu_\ell + \mu_I, \quad \mu_d = \mu_\ell - \mu_I, \quad \mu_s$

⚠ $\mu_\ell \neq 0$ and/or $\mu_s \neq 0$
$\longrightarrow$ unsuited for lattice simulations!

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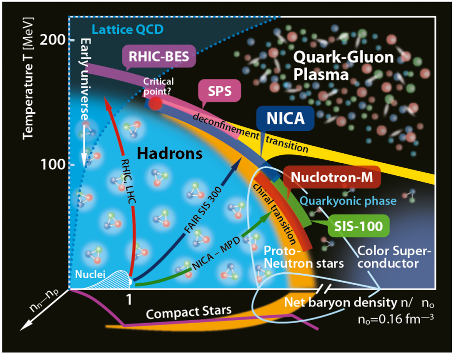

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❗ $\mu_I = \mu_u = -\mu_d \neq 0; \quad \mu_\ell = \mu_s = 0$
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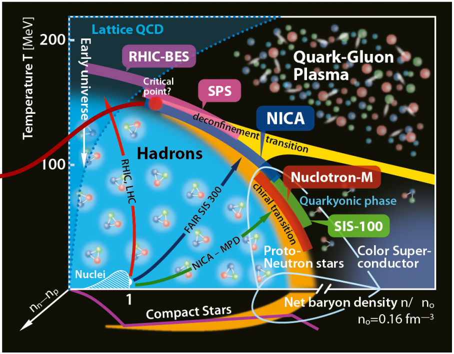
Critical endpoint conjectured, but out of reach for lattice investigations



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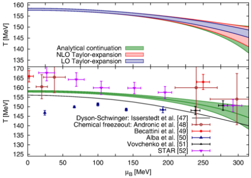
"Coping" with the sign problem

🏁 Constrain sign-problem-affected regions in the phase diagram on the basis of results obtained in calculable domains

1. Depart from point/region where QCD matter undergoes smooth crossover

- Taylor expansion or analytic continuation
  🔗 Borsanyi et al. (2020)



2. Locate and follow critical boundaries bending into tricritical points in the **QCD phase diagram in the chiral limit**

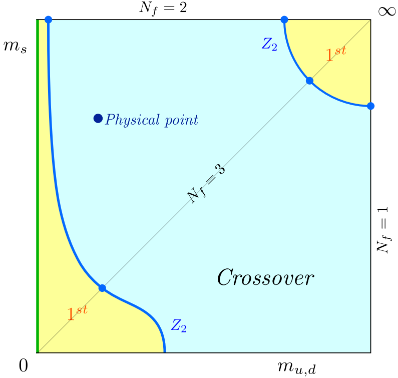
- Extrapolating according to known critical exponents

THE QCD PHASE DIAGRAM IN THE CHIRAL LIMIT AT ZERO DENSITY

Standard $(m_s, m_{u,d})$ Columbia plot

The order of the QCD thermal phase transition depends on the quark masses

- State of system is defined by the set of parameters $(m_s, m_{u,d}, \beta, N_\tau)$
- β tuned at $\beta_c \; \forall \, (m_s, m_{u,d}, N_\tau)$, and the order of the transition is plotted

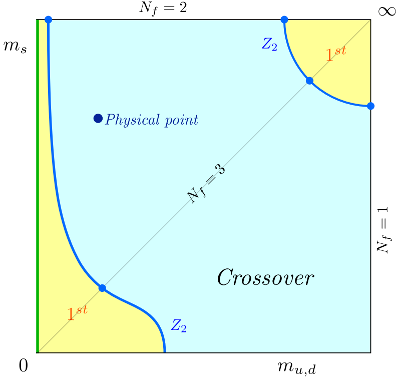


Standard $(m_s, m_{u,d})$ Columbia plot

The order of the QCD thermal phase transition depends on the quark masses

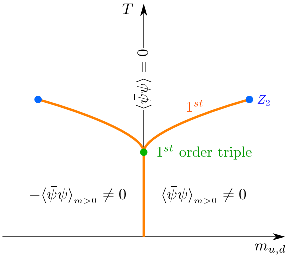
$m_{u,d}, m_s \to \infty$ Breaking of global $Z(3)$, order parameter $|L|$

$m_{u,d}, m_s \to 0$ Restoration of global $SU_L(N_f) \times SU_R(N_f)$, order parameter $\langle\bar{\psi}\psi\rangle$



$m_{u,d} \to 0$ Effective restoration of global $U_A(1)$ for large enough temperatures.

$m_{u,d} \to 0$ Line of triple points



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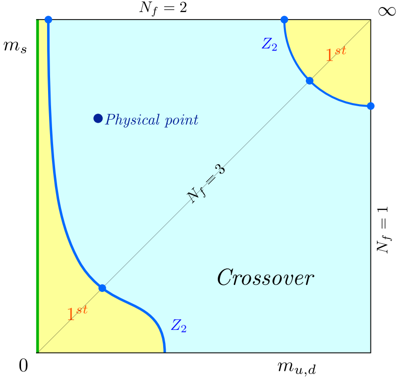

- Chiral 1^{st} order region wider for larger N_f, until $N_f = 4$ de Forcrand, D'Elia (2017)

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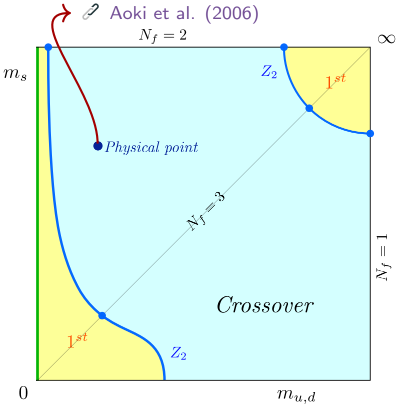

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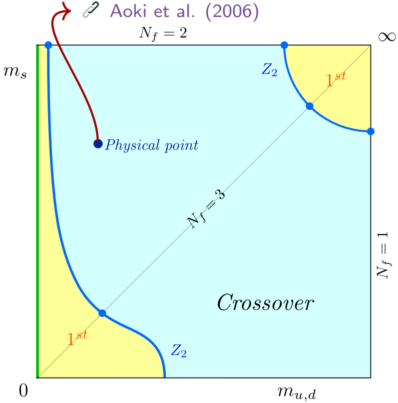

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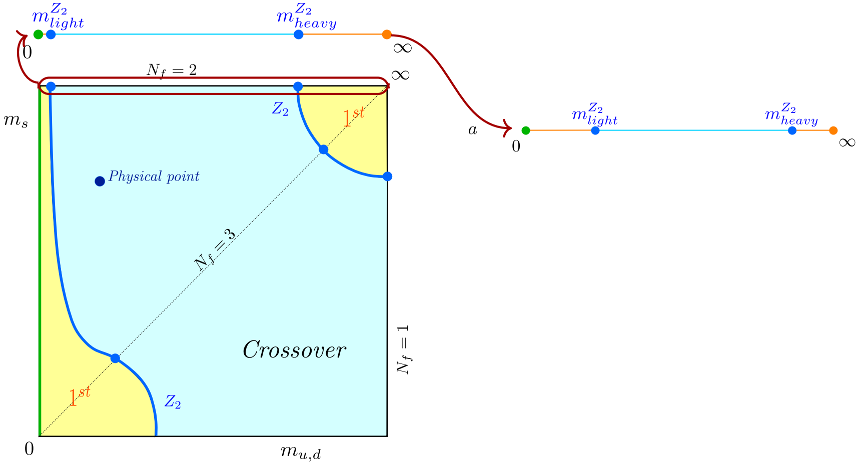
- Continuum extrapolated results @m_{phys}
- $m_{u,d}$ very small. Transition affected by remnants of the chiral universality class?

$(m_s, m_{u,d})$ Columbia plot in the continuum

Columbia plot from the "unimproved viewpoint", for $N_f = 2$, $m_{u,d} \to 0, \infty$

- Light/heavy 1^{st} order region does shrink/enlarge as $a \to 0$



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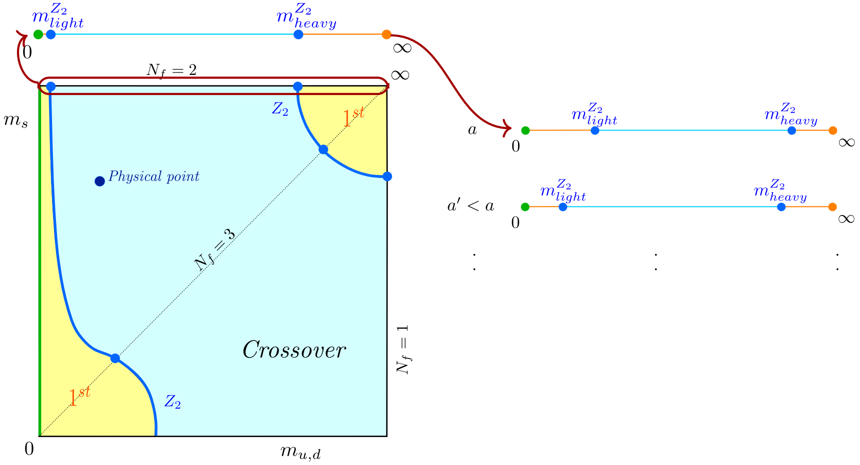

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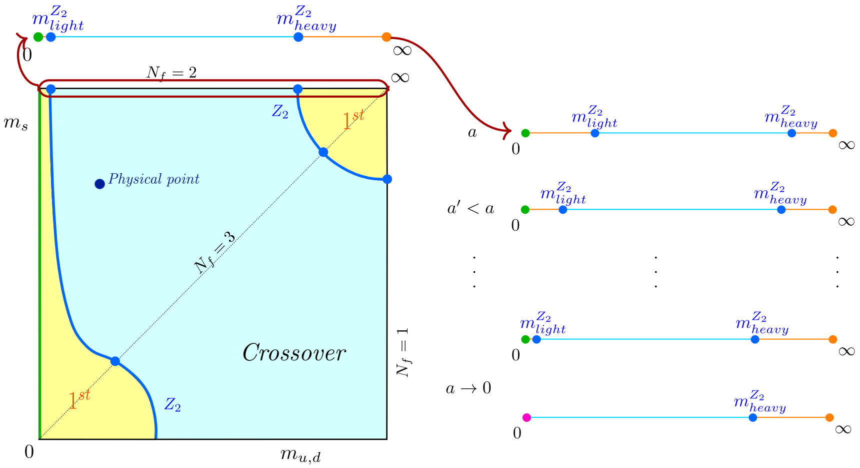
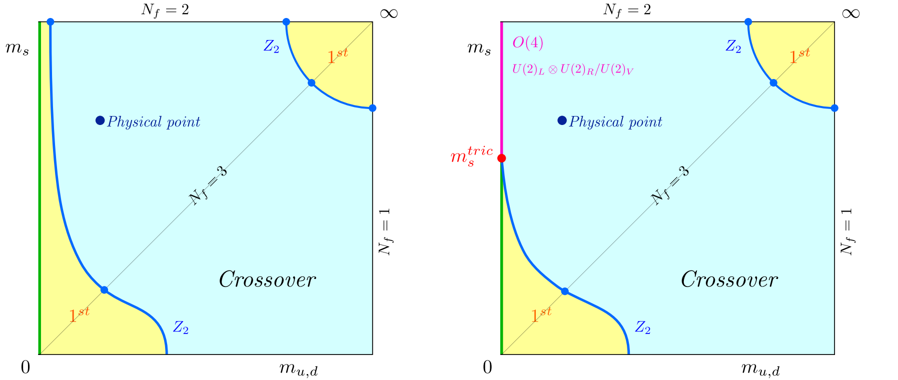


$(m_s, m_{u,d})$ Columbia plot in the continuum

Possible scenarios for the nature of $N_f = 2$ chiral transition Pisarski, Wilczek (1984)



- Two scenarios $\leftrightarrow$ renormalization group flow in 3D sigma models, augmented by 't Hooft term for the axial anomaly, using the epsilon expansion
    - Transition in the chiral limit predicted to be of first order for $N_f \geq 3$
    - Relevance of the strength of the $U(1)_A$ anomaly at T_c for $N_f = 2$

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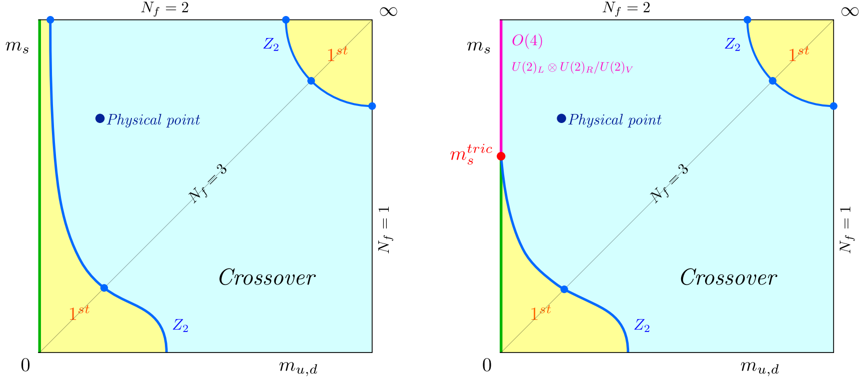
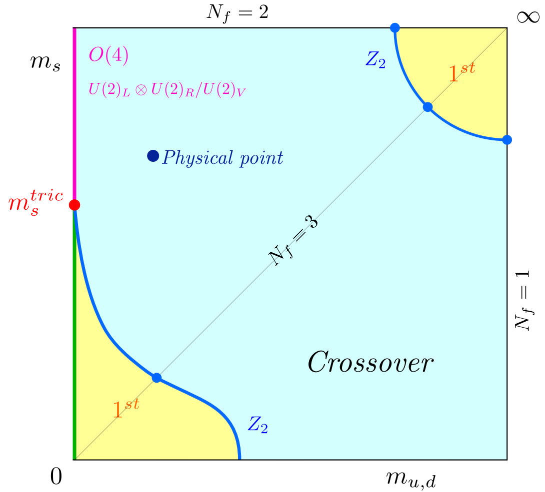


- "Direct approach": $\mu = 0$, $N_f = 2$ and $m_{u,d} \to 0$ proved to be too expensive
- "Indirect approaches": tricritical scaling laws for extrapolations to $m_{u,d} \to 0$

Columbia plot - Locating and mapping Z_2 boundaries



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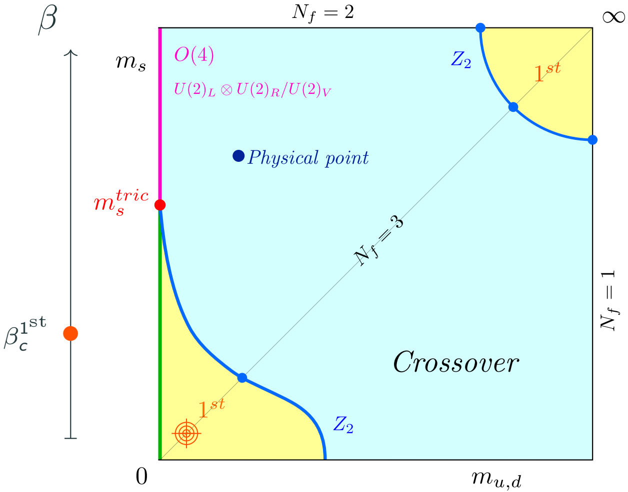

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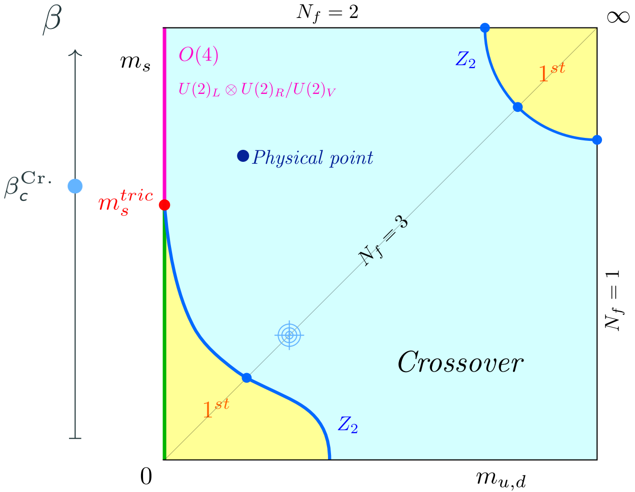

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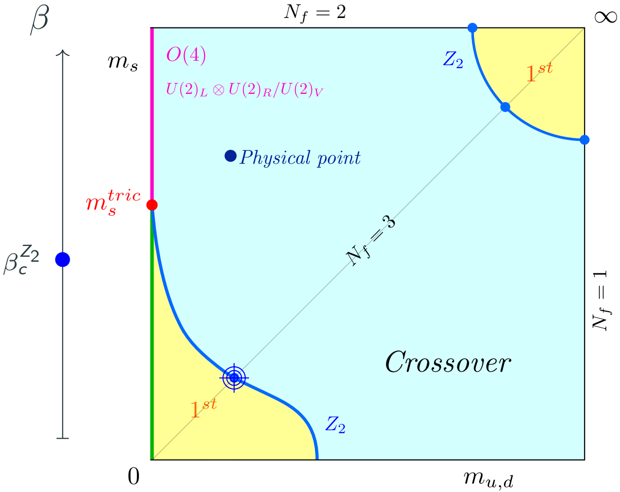

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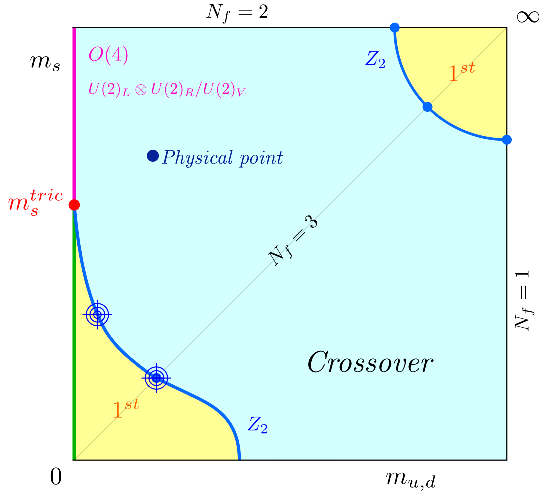

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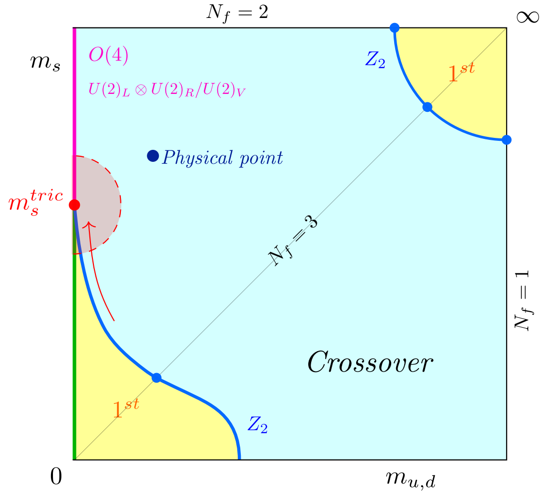


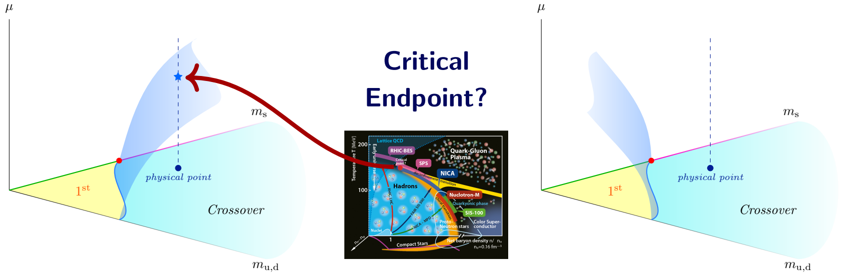
Towards real-μ Columbia plot

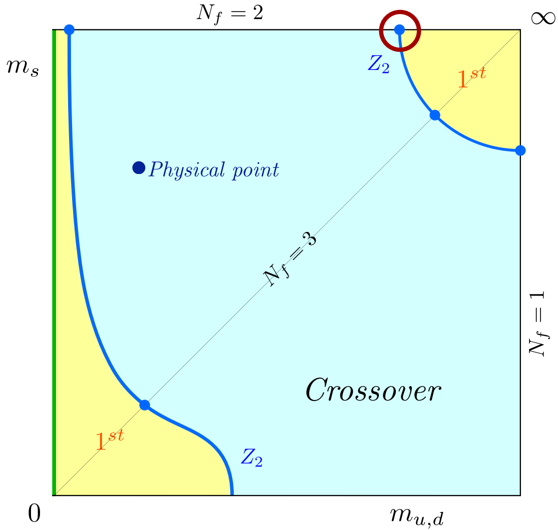
Is the first order chiral region only a lattice artifact?

- Tricritical points in the chiral limit useful targets for extrapolations...
- ...one important source of systematic uncertainty is eliminated if a tricritical point is guaranteed to exist!

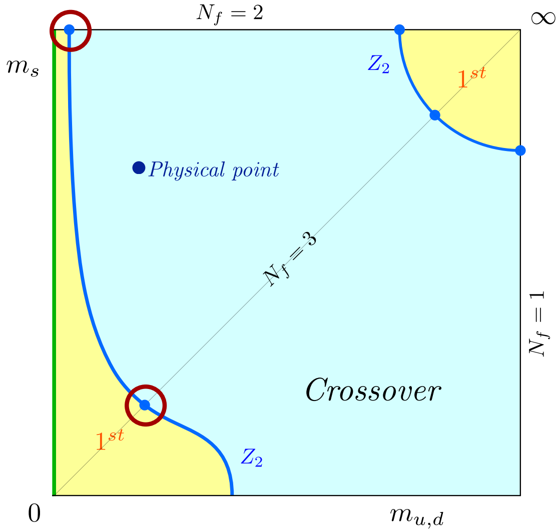
Phenomenological motivation

🏁 Constrain the sign-problem-affected regions in the QCD phase diagram





Cuteri, Philipsen, Schön, Sciarra (2020)

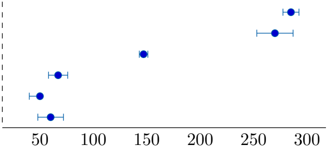

$N_f = 2$
∞
m_s
Z_2
1^{st}
Physical point
$N_f = 3$
$N_f = 1$
Crossover
1^{st}
Z_2
0
$m_{u,d}$

Columbia plot - Results for $m_{\text{light}}^{Z_2}$

Staggered fermion discretization

N_f	Action	N_τ	Ref.
	std	4	Karsch, Laermann, Schmidt (2001)
	std	4	Christ, Liao (2003)
3	std	6	de Forcrand, Kim, Philipsen (2007)
	p4	4	Karsch et al. (2004)
	HISQ	6	Bazavov et al. (2017)
2	std-μ_i	4	Bonati et al. (2014)

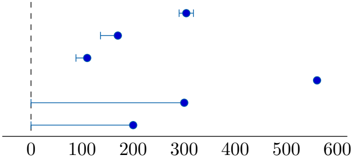
$m_\pi^{Z_2}$ [MeV] at $\mu = 0$



Wilson fermion discretization

N_f	Action	N_τ	Ref.
3	clover	6-8	Jin et al. (2015)
	clover	8-10	Jin et al. (2017)
	clover	10-12	Kuramashi et al. (2020)
2	std	4	Philipsen, Pinke (2016)
	tm	12	tftM coll. (2013)
	clover	16	Brandt et al. (2017)

$m_\pi^{Z_2}$ [MeV] at $\mu = 0$



⚠ Rather strong cutoff and discretization effects, but an alternative way to analyze cutoff effects suggests a unified description of all available results!

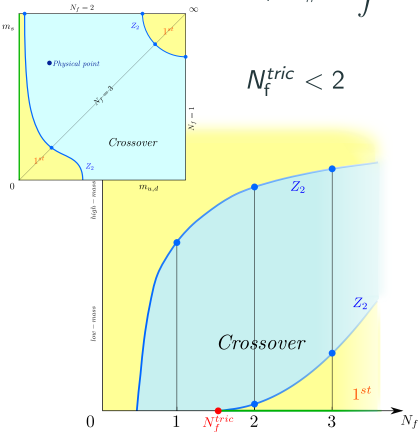
CONTROLLED EXTRAPOLATIONS TO THE CHIRAL LIMIT

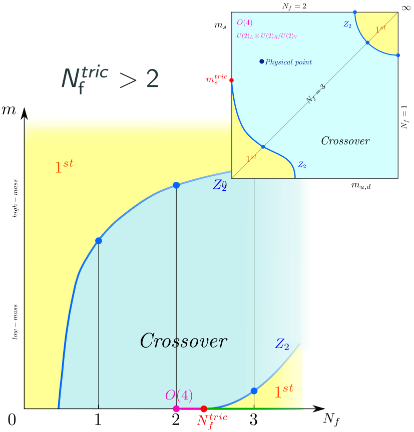
Cuteri, Philipsen, Sciarra (2017) Cuteri, Philipsen, Sciarra (2021)

(m, N_f) Columbia plot for N_f degenerate quarks

Yet another "indirect approach": promoting N_f to continuous real parameter

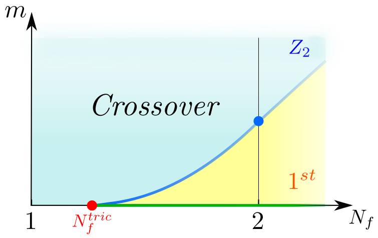
$$Z_{N_f=2.\#} = \int \mathcal{D}U \, [\det M(U,m)]^{2.\#} \, e^{-S_G}$$





1^{st} order for $N_f \geq 3 \implies 2^{\text{nd}}$ order for $N_f = 2$ requires $N_f^{tric} \in (2,3)$

A tricritical point is guaranteed to exist in this case



- No chiral phase transition for $N_\mathrm{f} = 1 \Longrightarrow 1^{\mathrm{st}}$ order for $N_\mathrm{f} > 1$ must weaken with decreasing N_f until vanishing in a tricritical point!
- Z_2 line bends into tricritical point with known critical exponents

$$N_\mathrm{f}^c\big(am(N_\tau), N_\tau\big) = N_\mathrm{f}^{\mathrm{tric}}(N_\tau) + \mathcal{B}_1(N_\tau)(am)^{2/5} + \mathcal{B}_2(N_\tau)(am)^{4/5} + \mathcal{O}\big((am)^{6/5}\big)$$

The full bare picture in $\{\beta, am, N_\mathrm{f}, N_\tau\}$

To complete the picture, a lattice spacing axis should be added to all sketches!

- We vary it via N_τ, that like N_f we may promote to a real parameter
- Then all parameters in $\{\beta, am, N_\mathrm{f}, N_\tau\}$ can be treated on equal footing

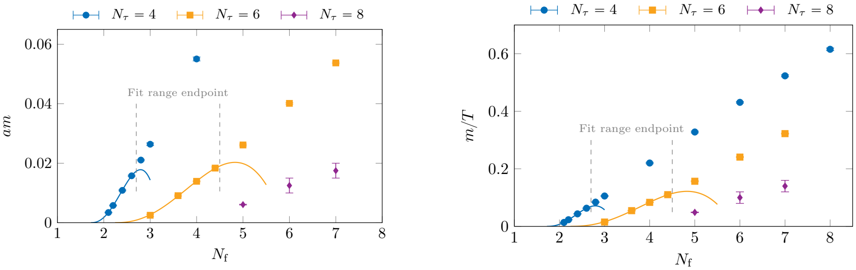
Useful subspaces and projections in the hyperspace

$B_3 = 0$ phase boundary for chiral symmetry restoration: 3D subspace.

$B_4 = 1.604$ critical surface separating crossovers from first order regions

The critical surface can be projected on all possible planes & Ansätze based on tricritical scaling can be constructed for all those planes!

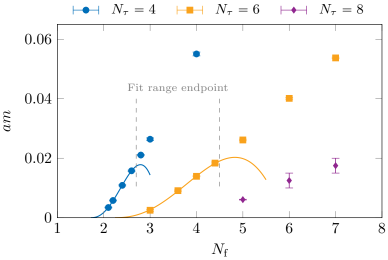
Plane of quark mass and number of flavors (β implicit)

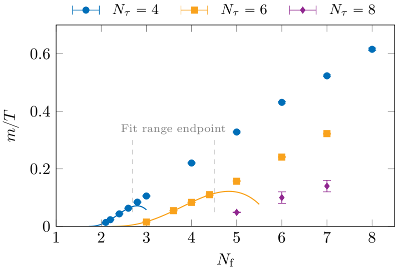


- Strengthening of transitions with increasing N_f, weakening with increasing N_τ
  - No sign of convergence towards a continuum limit
- Monitoring the intercepts $N_f^{tric}(N_\tau)$, with fits of the form

$$am_c\big(N_f(N_\tau), N_\tau\big) = \mathcal{D}_1(N_\tau)\big(N_f - N_f^{tric}(N_\tau)\big)^{5/2} + \mathcal{D}_2(N_\tau)\big(N_f - N_f^{tric}(N_\tau)\big)^{7/2} + \mathcal{O}\Big(\big(N_f - N_f^{tric}(N_\tau)\big)^{9/2}\Big)$$

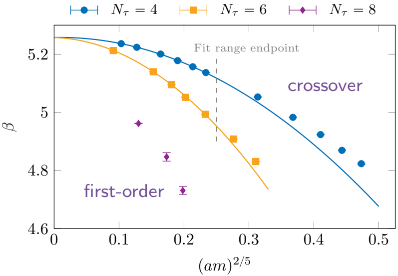
Plane of quark mass and number of flavors (β implicit)





- Strengthening of transitions with increasing N_f, weakening with increasing N_τ
  - No sign of convergence towards a continuum limit
- Monitoring the intercepts $N_\mathrm{f}^\mathrm{tric}(N_\tau)$, with fits of the form
  - Very narrow scaling region even for next-to-leading order fits
  - Difficult to determine $N_\mathrm{f}^\mathrm{tric}(N_\tau)$ reliably
  - Further sliding to the right of $N_\mathrm{f} = 3$?

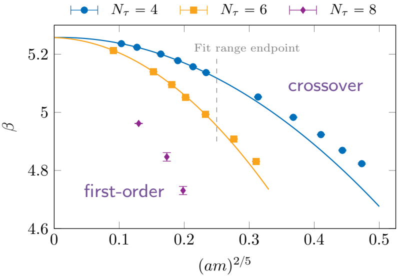
Plane of gauge coupling and quark mass (N_f implicit)



- Lower β axis is a triple line of first order transitions up until $\beta_{\text{tric}}(N_\tau)$
- Here monitoring the intercepts $\beta_{\text{tric}}(N_\tau)$ obtained by fits to

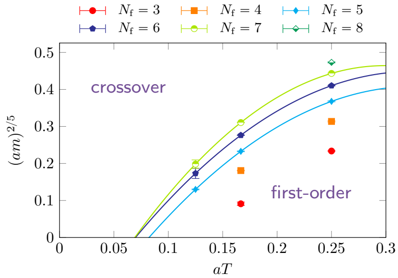
$$\beta_c(am, N_f(N_\tau), N_\tau) = \beta_{\text{tric}}(N_\tau) + \mathcal{C}_1(N_\tau)(am)^{2/5} + \mathcal{C}_2(N_\tau)(am)^{4/5} + \mathcal{O}\left((am)^{6/5}\right)$$

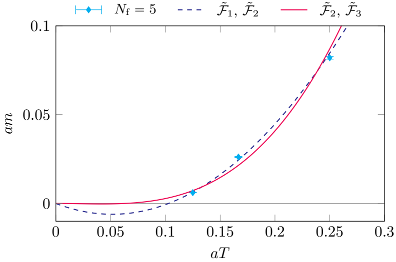
Plane of gauge coupling and quark mass (N_f implicit)



- Lower β axis is a triple line of first order transitions up until $\beta_{\text{tric}}(N_\tau)$
- Here monitoring the intercepts $\beta_{\text{tric}}(N_\tau)$ obtained by fits to
  - Convincing picture of tricritical scaling!
  - Chiral critical surface terminating in a tricritical line in the chiral limit!

Plane of quark mass and temperature $aT = N_\tau^{-1}$ (β implicit)



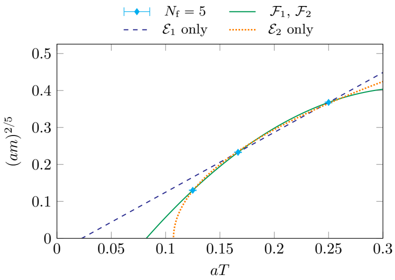


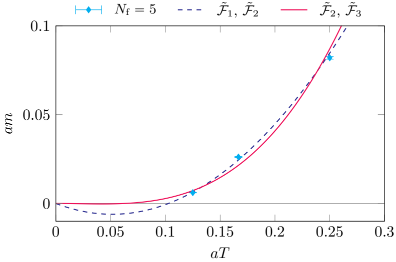
- Tricritical point not guaranteed for any N_f: Need to test functional behavior!
- continuum 1st-order transitions $\forall\, N_f \geq 3$?! Then, polynomial cutoff effects

$$am_c(N_\tau, N_f) = \tilde{\mathcal{F}}_1(N_f)\, aT + \tilde{\mathcal{F}}_2(N_f)\, (aT)^2 + \tilde{\mathcal{F}}_3(N_f)\, (aT)^3 + \mathcal{O}\big((aT)^4\big)$$

$N_f = 5$ (and $N_f = 6$ and $N_f = 7$) data, inconsistent with this scenario!

Plane of quark mass and temperature $aT = N_\tau^{-1}$ (β implicit)



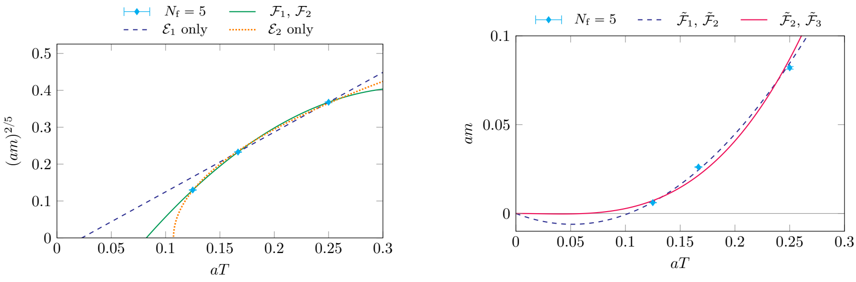


- Tricritical point not guaranteed for any N_f: Need to test functional behavior!
- Or, critical lines might end on the aT axis in a tricritical point according to

$$aT_c(am, N_\mathrm{f}) = aT_\mathrm{tric}(N_\mathrm{f}) + \mathcal{E}_1(N_\mathrm{f})(am)^{2/5} + \mathcal{E}_2(N_\mathrm{f})(am)^{4/5} + \mathcal{O}\big((am)^{6/5}\big)$$

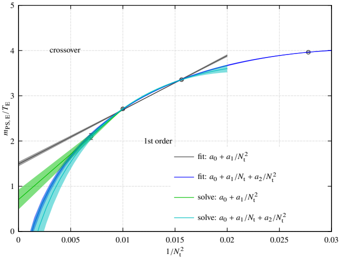
$$\Big(am_c(N_\tau, N_\mathrm{f})\Big)^{2/5} = \mathcal{F}_1(N_\mathrm{f})\big(aT - aT_\mathrm{tric}(N_\mathrm{f})\big) + \mathcal{F}_2(N_\mathrm{f})\big(aT - aT_\mathrm{tric}(N_\mathrm{f})\big)^2 + \mathcal{O}\Big(\big(aT - aT_\mathrm{tric}(N_\mathrm{f})\big)^3\Big)$$

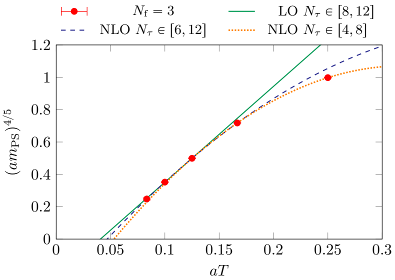
Plane of quark mass and temperature $aT = N_\tau^{-1}$ (β implicit)



- Tricritical point not guaranteed for any N_f: Need to test functional behavior!
- We only have three data points, but we observe...
  - only slight deviations from LO scaling,
  - compatibility with the existance of a $N_\tau^{\text{tric}}(N_f)$ we can bound, and
  - consistency with $N_\tau^{\text{tric}}(N_f = 5) \approx 12.5$ and a monotonically rising $N_\tau^{\text{tric}}(N_f)$

Plane of quark mass and temperature $aT = N_\tau^{-1}$ (β implicit)





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- We only have three data points, but we observe. . .
  - only slight deviations from LO scaling,
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  - consistency with $N_\tau^{\mathrm{tric}}(N_f = 5) \approx 12.5$ and a monotonically rising $N_\tau^{\mathrm{tric}}(N_f)$
- We successfully reanalyzed (using $am_{PS}^2 \propto am_q$) published clover-improved Wilson data for the critical pseudo-scalar mass Kuramashi *et al.* '20

What do we conclude?

- We conclude that the first order chiral phase transitions observed for:
  - $N_\mathrm{f} = 3$ $\mathcal{O}(a)$-improved Wilson fermions
  - $N_\mathrm{f} \leq 6$ standard staggered fermions

  aren't connected to the continuum chiral limit, where transition is 2nd order
- We conjecture the transition in the chiral limit to stay 2nd order up to the conformal window