Soft pions near the QCD chiral critical point

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SEZIONE DI FIRENZE

E.G., A.Soloviev, D. Teaney, F. Yan PRD (2020)
E.G., A. Soloviev, D. Teaney, F. Yan PRD (2021)
A. Florio, E.G., A. Soloviev, D, Teaney PRD (2022)
A. Florio, E.G., D, Teaney ……. (2023)

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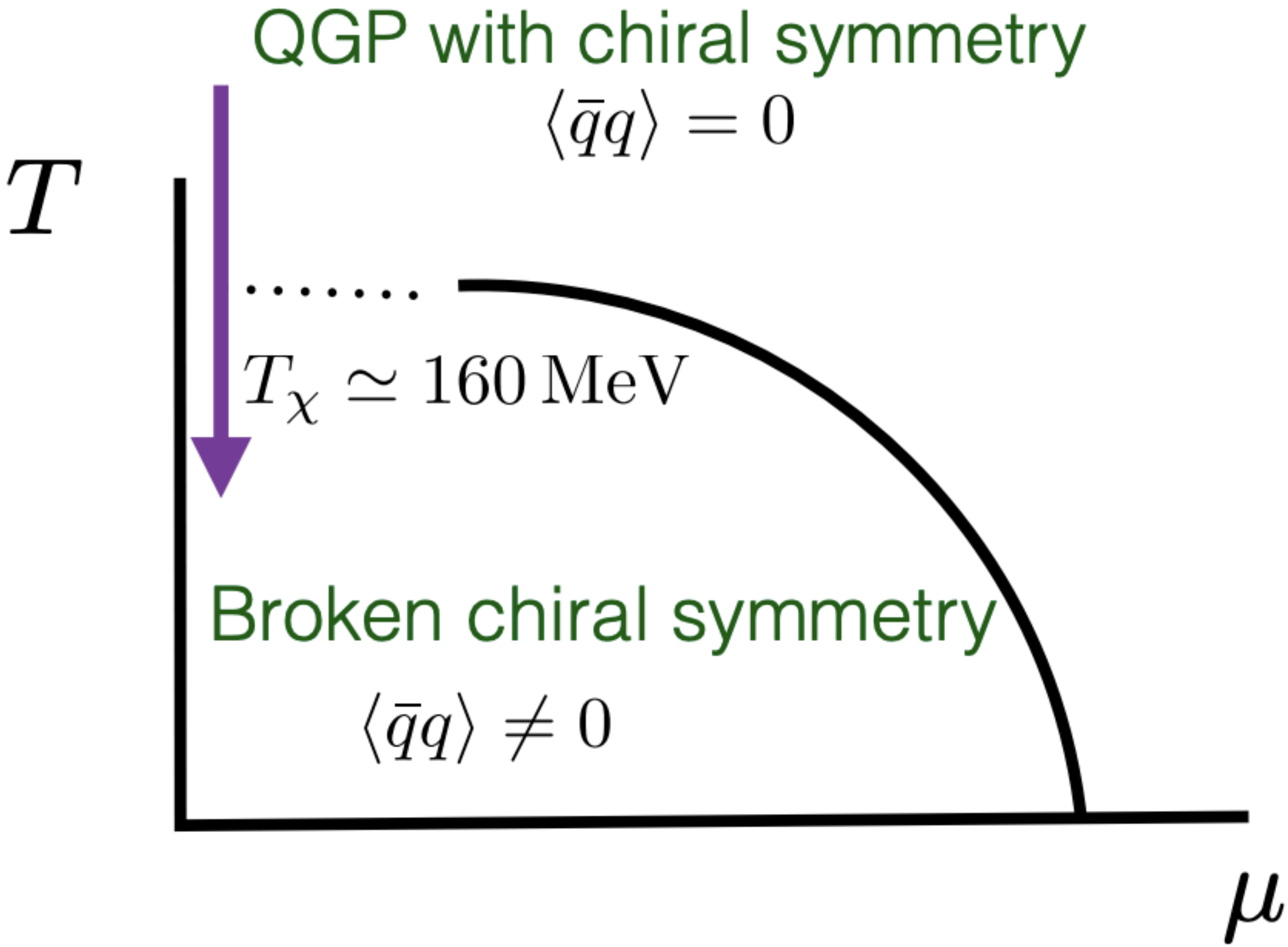
Motivation 1



We are neglecting any hydro-dynamics of the chiral condensate !

Setup: the O(4) phase transition

The (approximated) conserved quantities of 2 flavour QCD are

$T^{\mu\nu}$	J_V^μ	J_A^μ
Stress	Iso-vector (isospin)	Iso-axial
(T, u^μ)	μ_V	μ_A
	$\bar{q}\gamma^0 t_I q$	$\bar{q}\gamma^0\gamma_5 t_I q$

The approximate flavour symmetry $SU(2)_L \times SU(2)_R \sim O(4)$

The order parameter is the chiral condensate

$$\langle \bar{q}q \rangle \sim \phi_\alpha = (\sigma, \varphi_\alpha) = (\text{sigma}, \text{pions})$$

We need the hydrodynamic theory of the charge and the order parameter

Rajagopal Wilczek (93)
Son and Stephanov (02)

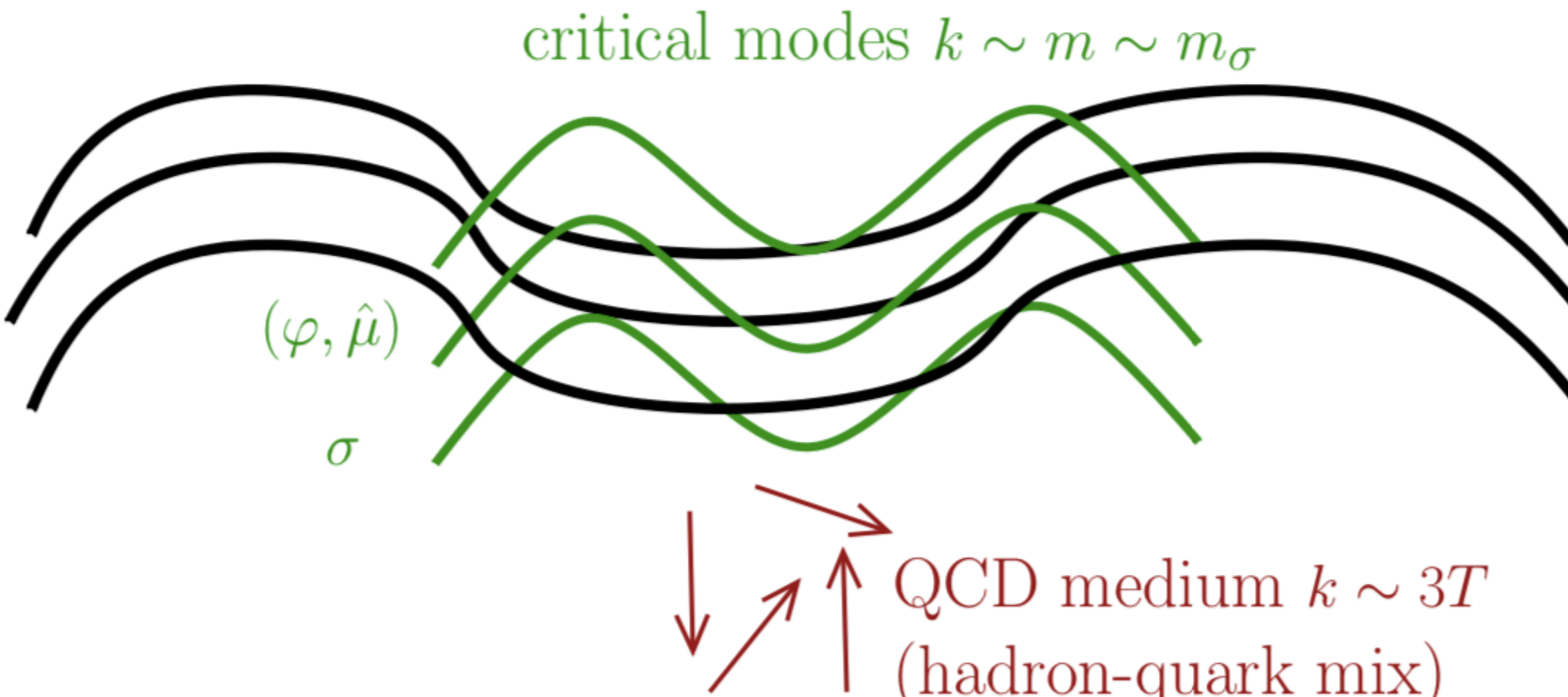
Physical picture T~Tc

Working regime

Rajagopal Wilczek hep-ph/9210253
Son and Stephanov hep-ph/020422

$$k \ll m \sim m_\sigma \ll \pi T_C \sim \pi \Lambda_{QCD}$$

equilbriated hydro modes $k \ll m$



Soft pions+sigma mode on Hydro

Ideal equation

The pressure is the normal plus the potential and a kinetic term of the condensate

$$\mathcal{W}[g_{\mu\nu}, A_\mu, H] = \int_x \sqrt{g} \left\{ p(T) + \frac{\chi_0}{4}\mu_{\alpha\beta}\mu_{\alpha\beta} - \frac{1}{2}\Delta^{\mu\nu}(D_\mu\phi)_\alpha(D_\mu\phi)_\alpha - V(\Phi) + H_\alpha\phi_\alpha \right\}$$

In the ideal case we have a dependence of the field and its spatial derivative (Ginzburg-Landau)

Stress

$$(T^{\mu\nu}) = \frac{2}{\sqrt{g}}\frac{\delta\mathcal{W}}{\delta g_{\mu\nu}}$$

Currents

$$(J^\rho)_{\alpha\beta} = \frac{2}{\sqrt{g}}\frac{\delta\mathcal{W}}{\delta(A_\rho)_{\alpha\beta}}$$

Order parameter

$$\phi_\alpha = \frac{1}{\sqrt{g}}\frac{\delta\mathcal{W}}{\delta H_\alpha}$$

Form symmetry we have

$$\partial_\mu T^\mu_\nu = 0$$

$$\partial_\mu J^\mu_{\alpha\beta} = (\phi_\alpha H_\beta - \phi_\beta H_\alpha)$$

quark mass → H

We need an equation for the condensate

Entropy production

$$\vec{\xi} = \Delta^{\mu\nu}\partial_\mu \phi$$

$$d = u \cdot \partial$$

Entropy

$$s = \frac{1}{T}(e + p - \frac{1}{2}\mu_{\alpha\beta}n_{\alpha\beta})$$

Modified Gibbs relation

$$dp = sdT + \frac{1}{2}n_{\alpha\beta}d\mu_{\alpha\beta} - \frac{1}{2}d\vec{\xi}^2 + \left(-\frac{\partial V}{\partial \phi_\alpha} + H_\alpha\right) d\phi_a$$

Using the equation of motion one can find the entropy production

$$\partial_\mu(su^\mu - \frac{\mu_{\alpha\beta}}{T}(q^\mu)_{\alpha\beta}) = \left[\frac{1}{T}(d\phi)_\alpha + \frac{\mu_{\alpha\beta}}{T}\phi_\beta\right]\left[\partial_\mu \vec{\xi}^\mu_\alpha - \frac{1}{\Phi}\frac{\partial V}{\partial \Phi}\phi_\alpha + H_\alpha\right]$$

$$-\nabla_\mu\left(\frac{u_\nu}{T}\right)\Pi^{\mu\nu} - \partial_\mu\left(\frac{\mu_{\alpha\beta}}{2T}\right)(q^\mu)_{\alpha\beta}$$

Stress

Diffusion

Josephson constrain

In the ideal case the time derivative of the field is "locked" to the chemical potential

$$(d\phi)_\alpha + \mu_{\alpha\beta}\phi_\beta = 0$$

The same equation can be obtain imposing the stability of the condensate

$$[\langle \bar{q}q \rangle, H - \mu N] = 0$$ ← Poisson bracket

In the dissipative case

$$(d\phi)_\alpha + \mu_{\alpha\beta}\phi_\beta = \text{ gradient corrections}$$

such that the entropy is increasing

Dissipative equations-1st order hydro

Relaxation equation for the order parameter

$$u^\mu \partial_\mu \phi_\alpha + \mu_{\alpha\beta}\phi_\beta = \Gamma\left[\partial_\mu(\Delta^{\mu\nu}\partial_\nu\phi_\alpha) - \frac{1}{\Phi}\frac{\partial V}{\partial \Phi}\phi_\alpha + H_\alpha\right] + \zeta^{(1)}\phi_\alpha \nabla\cdot u$$

Dissipative energy momentum tensor

$$T^{\mu\nu} = u^\mu u^\nu(e+p) + pg^{\mu\nu} + (\partial_\mu\phi)_\alpha(\partial_\nu\phi)_\alpha - u^\mu u^\nu u^\sigma u^\rho(\partial_\sigma\phi)_\alpha(\partial_\rho\phi)_\alpha$$

$$-\eta\sigma^{\mu\nu} - \Delta^{\mu\nu}\left[\zeta^{(0)}\nabla\cdot u - \zeta^{(1)}\phi_\alpha\left(\partial_\mu(\Delta^{\mu\nu}\partial_\nu\phi_\alpha) - \frac{1}{\Phi}\frac{\partial V}{\partial \Phi}\phi_\alpha + H_\alpha\right)\right]$$

Current with diffusion

$$(J^\mu)_{\alpha\beta} = n_{\alpha\beta}u^\mu + (J^\mu_\perp)_{\alpha\beta} - T\sigma\Delta^{\mu\nu}\partial_\nu\left(\frac{\mu_{\alpha\beta}}{T}\right)$$

Superfluid component

$$(J^\mu_\perp)_{\alpha\beta} = \Delta^{\mu\nu}[(D_\nu\phi)_\alpha\phi_\beta - (D_\nu\phi)_\beta\phi_\alpha]$$

Wilczek-Rajagopal hep-ph/9210253 + little bit us

Equation of motion (Model G)

Rajagopal Wilczek (93)

Chiral condensate ϕ_a + Axial and Vector charge $n_{ab} = \chi_0 \mu_{ab}$

$$\partial_t \phi_a + g_0\, \mu_{ab}\phi_b = \Gamma_0 \nabla^2 \phi_a - \Gamma_0 (m_0^2 + \lambda \phi^2)\phi_a + \Gamma_0 H_a + \theta_a \,,$$

$$\partial_t n_{ab} + g_0\, \nabla \cdot (\nabla \phi_{[a}\phi_{b]}) + H_{[a}\phi_{b]} = D_0 \nabla^2 n_{ab} + \partial_i \Xi^i_{ab} \,.$$

Ideal part — Dissipative part — Gaussian Noise

- The ideal part is charge conservation and Josephson constraint
- Two dissipative coefficient Γ_0 and D_0 and noise
- The simulation of the stochastic process is done with an ideal step and metropolis update.

Diffusion at high temperature, pion propagation at low temperature as the vev develops

Linearized equation

We linearize the equation around equilibrium (mean field)

$$\phi_\alpha = (\bar\sigma + \delta\sigma, \bar\sigma\varphi_a)$$

Axial charge and pion equation (assuming zero vector chemical potential)

$$\partial_t \varphi = -\mu_A + \Gamma\left(\nabla^2 - m^2\right)\varphi,$$

$$\partial_t \mu_A = v^2(-\nabla^2 + m^2)\varphi + D_0 \nabla^2 \mu_A,$$

Sigma equation

$$\partial_t \delta\sigma = \Gamma\left[\nabla^2 - m_\sigma^2\right]\delta\sigma,$$

The parameter depends on the temperature and on the value of the fields at the minimum

$$v^2 = \frac{\bar\sigma^2}{\chi} \qquad m^2 = \frac{H}{\bar\sigma} \qquad \Gamma = \text{const} \qquad D_0 = \text{const}$$

Solving for the pion we get a dumped Klein Gordon equation

Mean field propagator

The scalar propagator and the vector propagator are trivial

Vector density propagator

$$G_R^{VV}(\omega, k) = \frac{\chi_0 D_0 k^2}{-i\omega + D_0 k^2}.$$

$$G_{\rm sym}^{VV}(t, k) = T\chi_0 e^{-D_0 k^2 t}.$$

Scalar propagator

$$G_R^{\sigma\sigma}(\omega, k) = \frac{\Gamma}{-i\omega + \Gamma(k^2 + m_\sigma^2)}$$

$$G_{\rm sym}^{\sigma\sigma}(t, k) = \frac{T}{k^2 + m_\sigma^2} e^{-\Gamma(k^2 + m_\sigma^2)t}.$$

They both dissipate with two different characteristic rate

Mean field axial channel

$$\partial_t \phi_s \to D_t \phi_s = \partial_t \phi_s - (A_0)_{s0}\bar{\sigma},$$
$$\partial_i \mu_{0s} \to \partial_i \mu_{0s} - (E_i)_{0s},$$

The matrix in of the symmetric correlation is very rich

$$[G_{\rm sym}(t,k)] = \frac{T}{\chi_0} e^{-\frac{1}{2}\Gamma_k t} \begin{pmatrix} \cos(\omega_k t) + \frac{\Delta}{\omega_k}\sin(\omega_k t) & \sin(\omega_k t) \\ -\sin(\omega_k t) & \cos(\omega_k t) - \frac{\Delta}{\omega_k}\sin(\omega_k t) \end{pmatrix}$$

Where

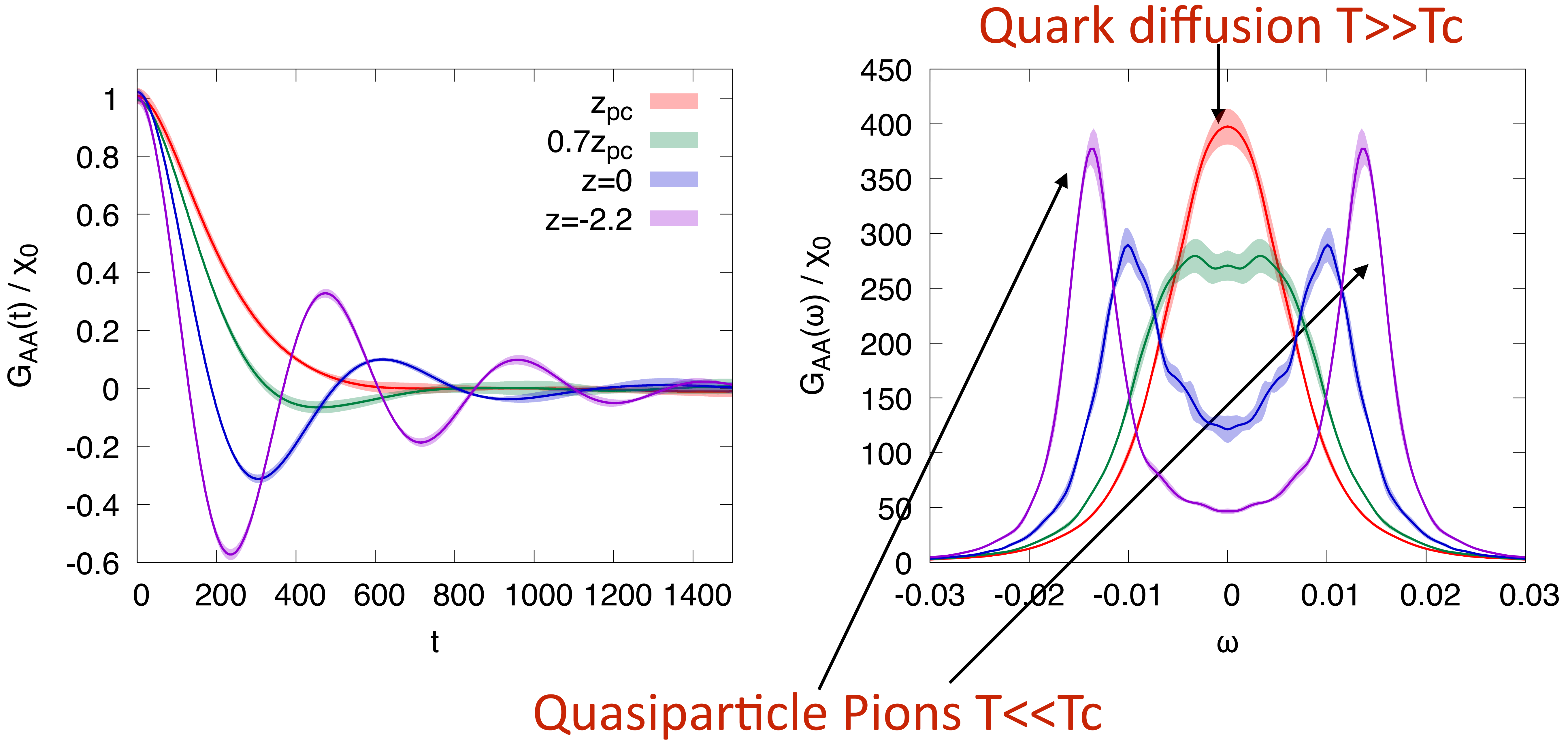
Damping rate $\Gamma_k \equiv \Gamma(k^2 + m^2) + Dk^2 = g_1 + g_2\,,$

Energy dispersion $\omega_k \equiv v^2(k^2 + m^2)$

Phase schift $\Delta \equiv \frac{1}{2}(D_0 k^2 - \Gamma(k^2 + m^2)).$

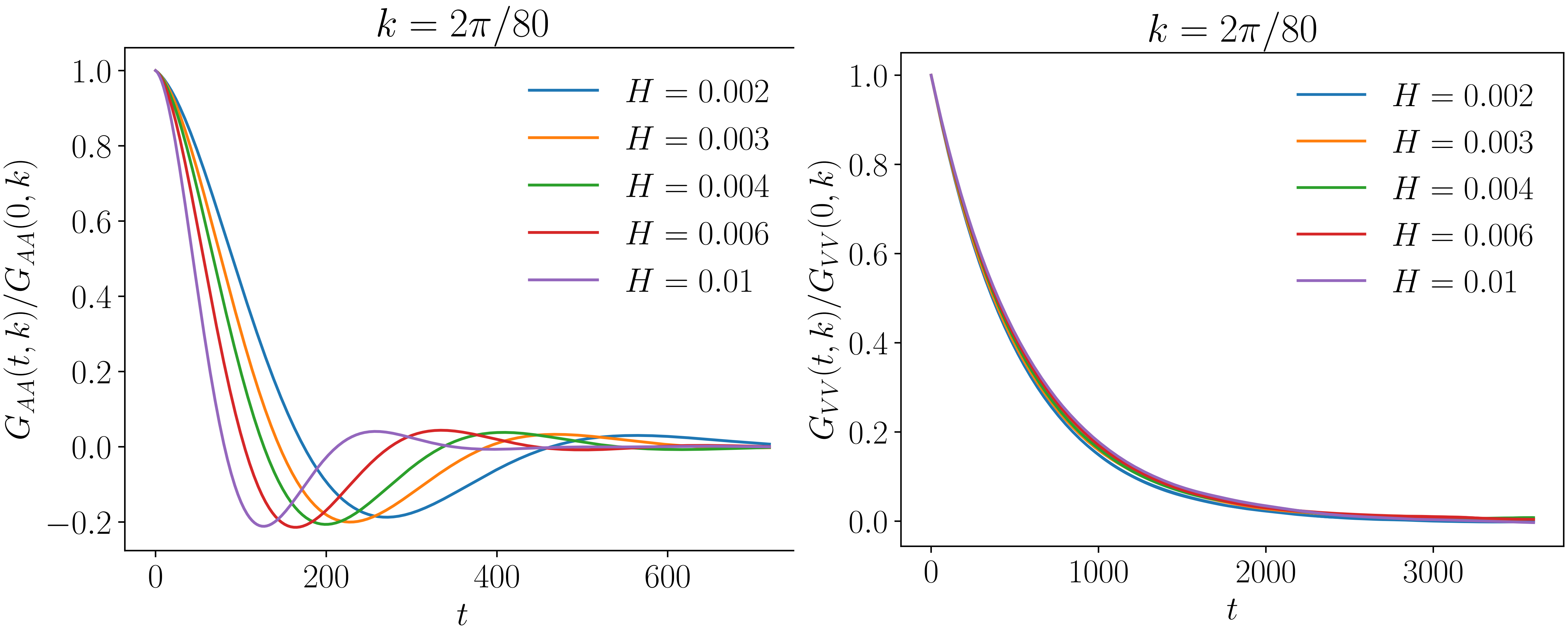
Damped rotating waves !

Propagation of axial charge across the transition



Around T_{pc} the axial charge start changing form a diffusive form to a quasiparticle one

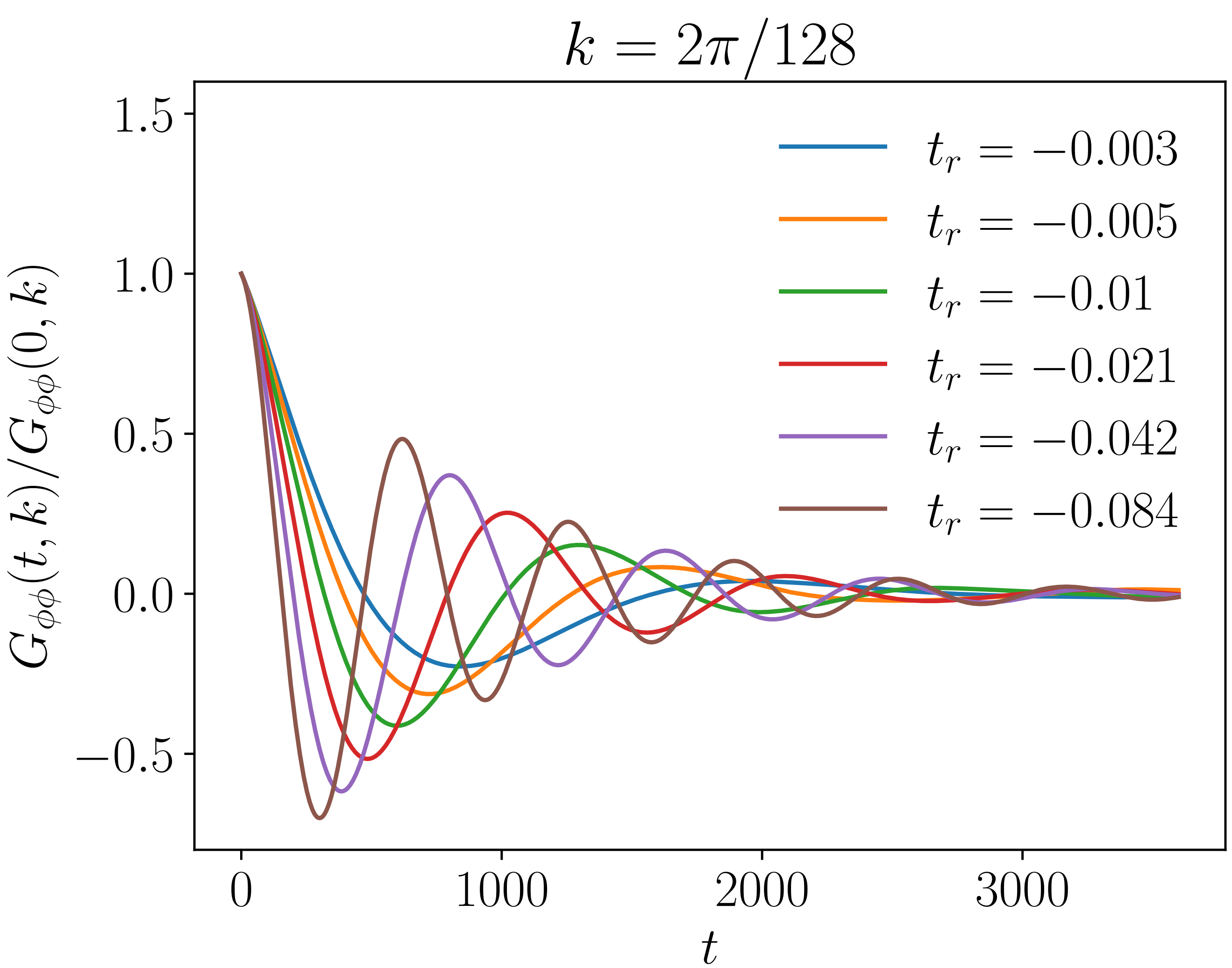
Finite momentum-critical line

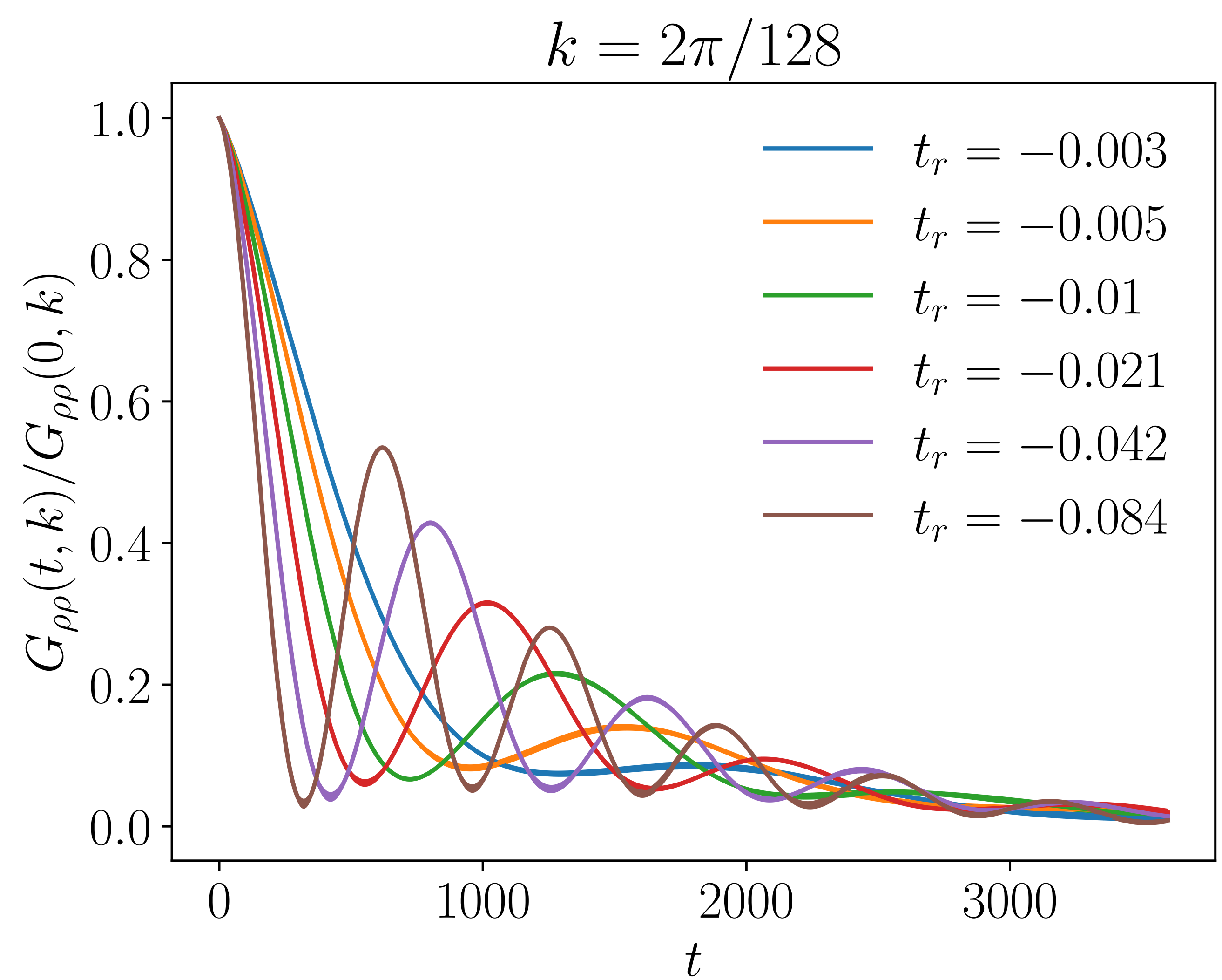


At finite momentum k the axial charge oscillate

The vector current correlator is almost the same for different quark mass !

Finite momentum broken phase H=0



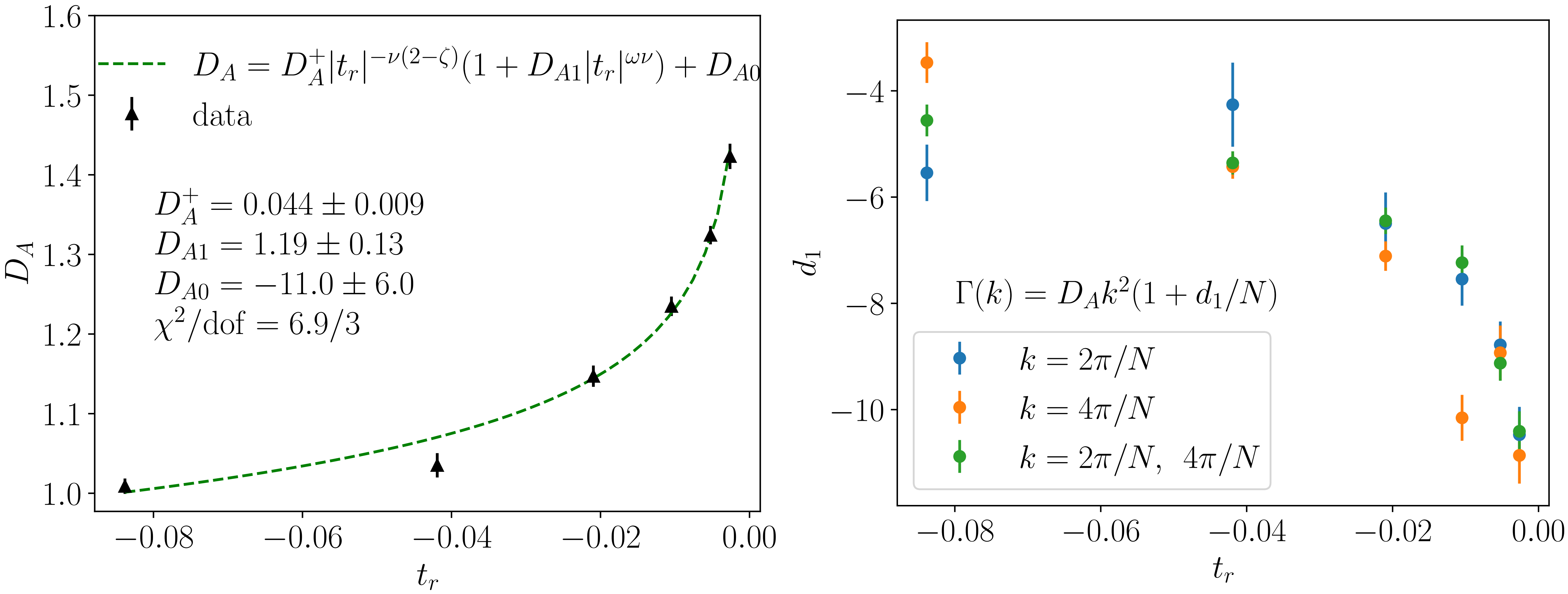


At zero quark mass no preferred direction the vev is wandering

We fit the field with mean field result neglecting the sigma contribution

$$G_{\varphi\varphi}(t,k) = Ce^{-\frac{1}{2}\Gamma(k)t}\left[\cos(\omega(k)t) - \Delta_k \sin(\omega(k)t)\right]$$

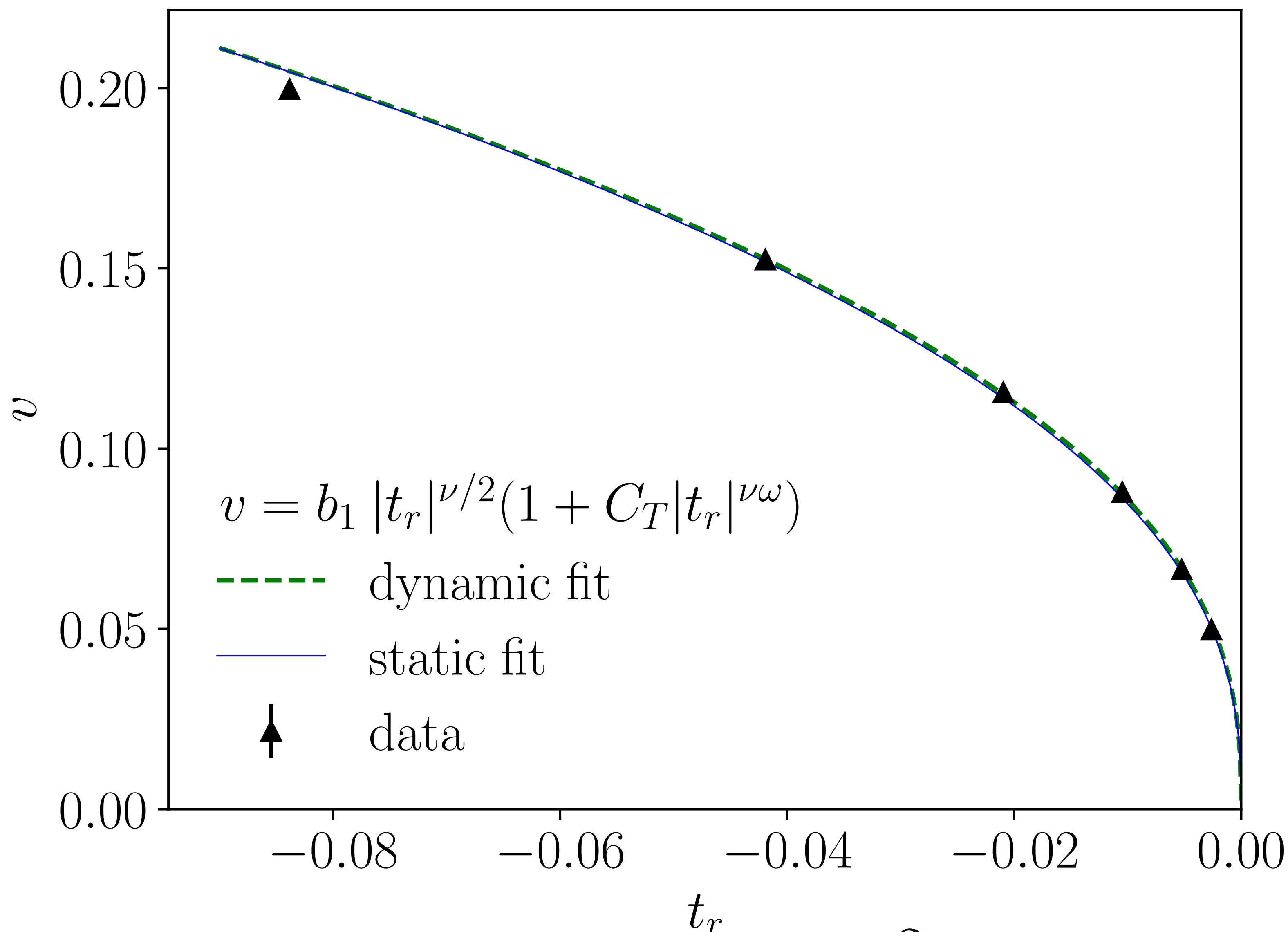
Axial diffusion



The diffusion coefficient scale with according power and sizable finite size effect

$$\Gamma(k) = D_A k^2(1 + d_1/N)$$

Velocity dispersion



$$\omega(k) = vk + v_1 k^2.$$

The dispersion relation of the of the waves actually is determine by the GOR relation

$$v^2 \propto \langle \phi^2 \rangle$$

Effective Boltzmann equation

E.G., A.Soloviev, D. Teaney, F. Yan PRD (2020)

From the linear propagator we can define (using the Wigner transform) an effective kinetic description of the soft pions distribution function

$$\partial_t f_\pi + \frac{\partial E_{\mathbf{p}}}{\partial \mathbf{q}} \frac{\partial f_\pi}{\partial \mathbf{x}} - \frac{\partial E_{\mathbf{p}}}{\partial \mathbf{x}} \frac{\partial f_\pi}{\partial \mathbf{q}} = \text{interaction terms}$$

Well below the phase transition the pions propagate like quasipartilce with a modified energy dispersion from the medium

$$E_{\mathbf{p}} = v^2(p^2 + m^2)$$

Depends on $\bar{\sigma}$

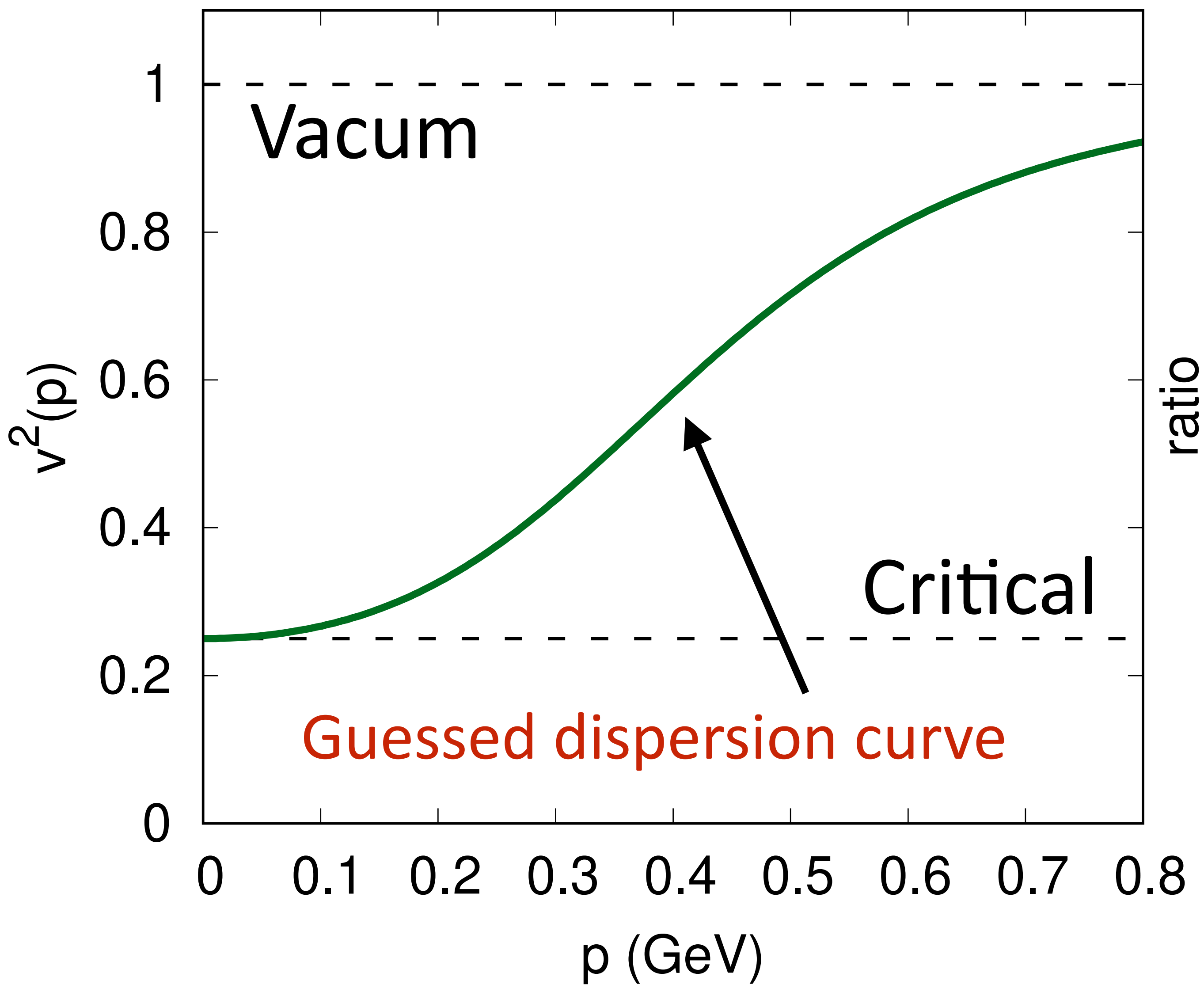
Soft pion enhancement

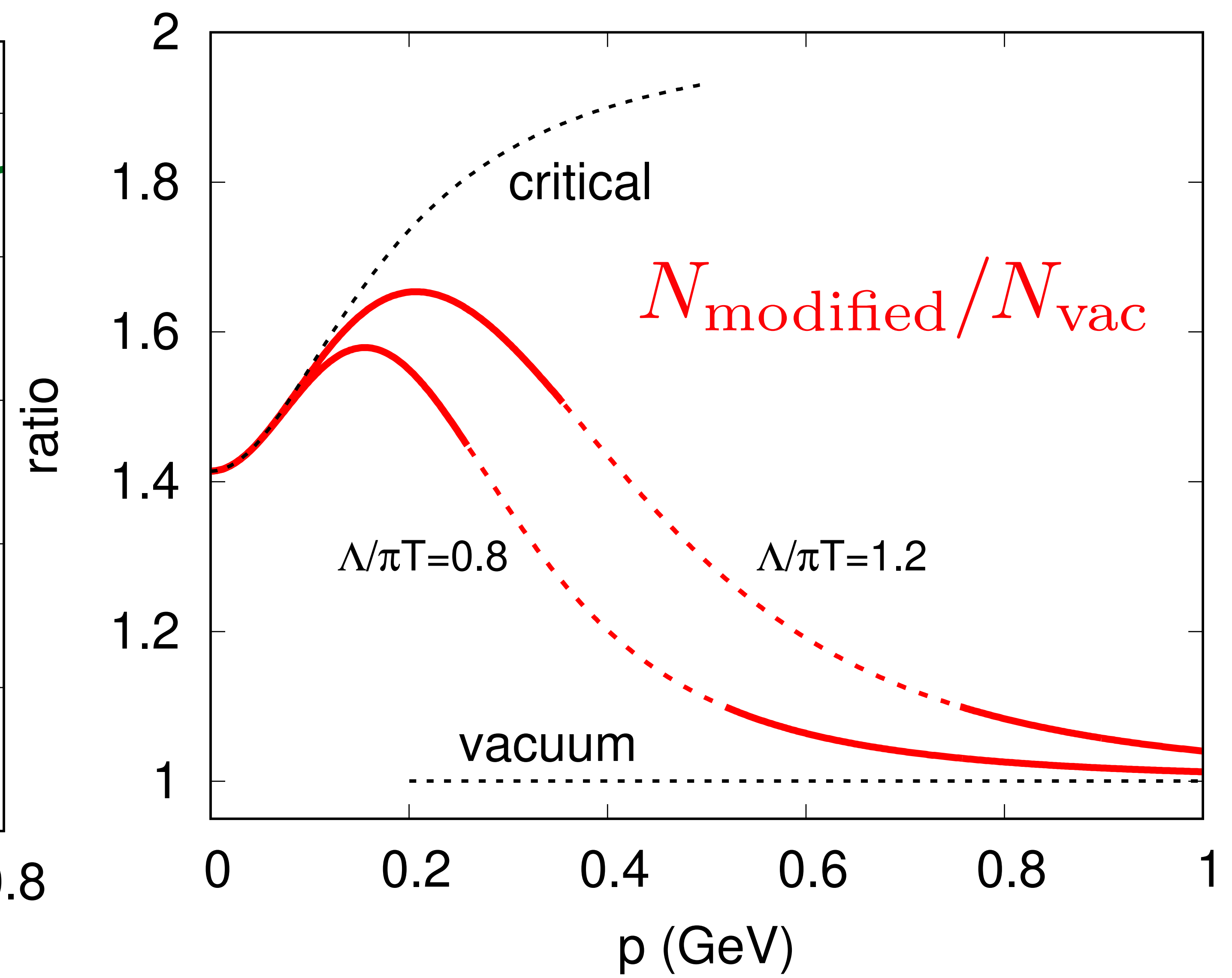
E.G., A. Soloviev, D. Teaney, F. Yan PRD (2021)

The dispersion curve get modified form the phase transition

$$E_{\mathbf{p}} = v^2(p)(p^2 + m^2)$$

$$n(E_p) = \frac{1}{e^{E_p/T} - 1}$$

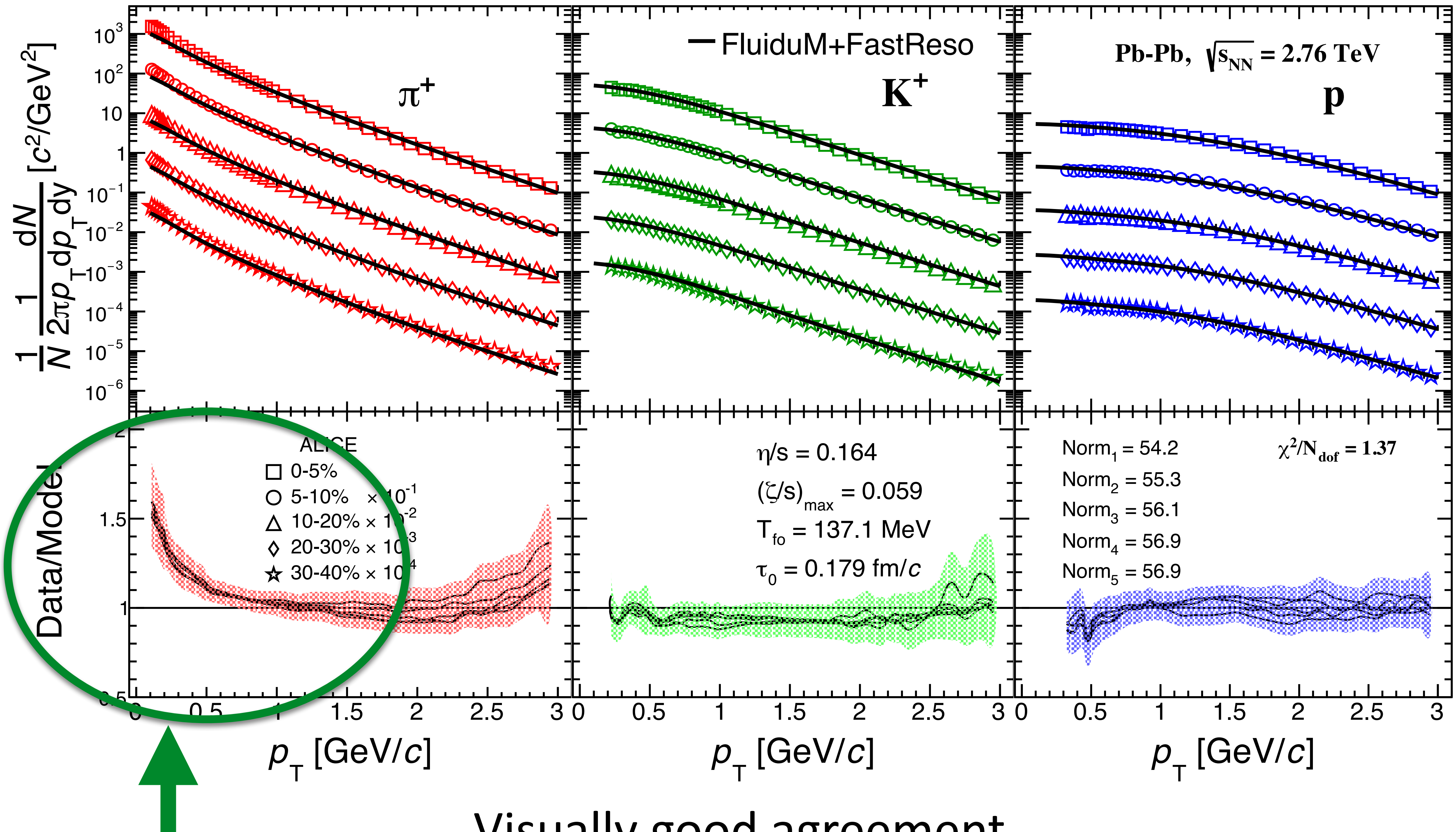




Pion enhanced $p < 0.5$ GeV

Maybe in the data ?

Fit the pt spectra of pions in the first five centralities



Visually good agreement,

but not amazing fit D. Devetak , et al JHEP (2019)

The main discrepancy is for pions at low pt