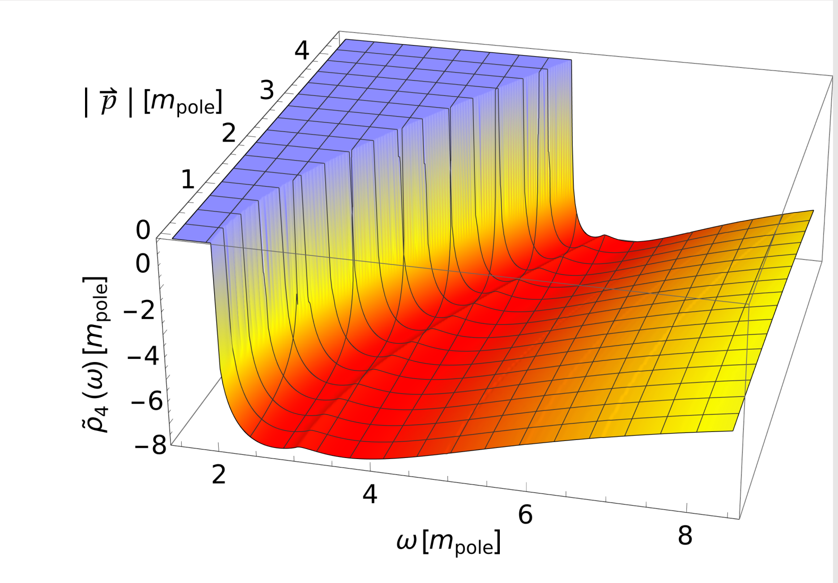
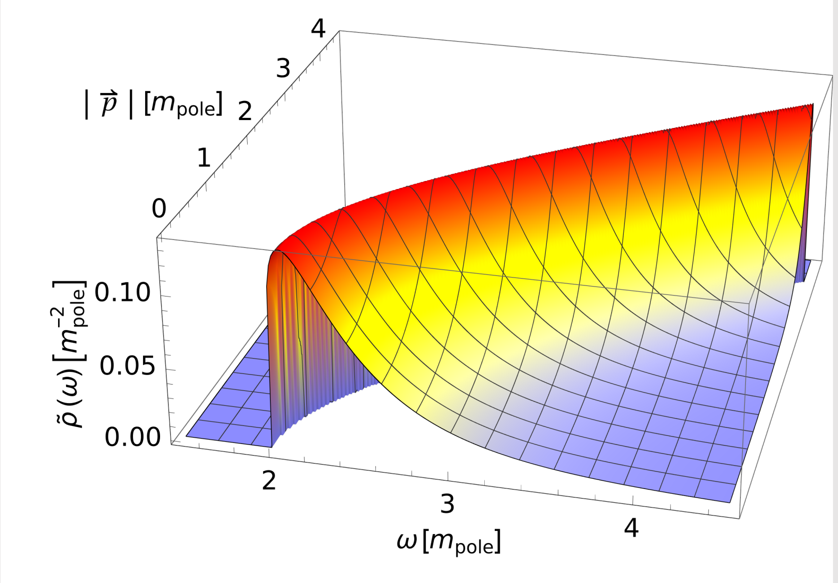
Spectral functions from spectral flows

Jonas Wessely

From first principles QCD to experiments

Trento 23.05.23





In collaboration with **Jan Horak, Friederike Ihssen, Jan M. Pawlowski, Nicolas Wink**

- arXiv:2303.16719

Outline

- Real time correlators with *spectral* functional methods

  (Heavy) Quark diffusion

- Spectral fRG and the Callan-Symanzik cut-off

- Results for real scalar fields in (2+1) dimensions

Real time correlators with spectral functional methods

$$\mathscr{D}_s = \lim_{\omega \to 0} \frac{\sigma(\omega, \vec{p} = 0)}{\omega \chi_q \pi}$$

$$\sigma(\omega, \mathbf{p}) = \frac{1}{\pi} \int \mathrm{d}t\, e^{\mathrm{i}\omega t} \int \mathrm{d}^3x\, e^{\mathrm{i}\mathbf{x}\mathbf{p}} \langle [J_i(t, \mathbf{x}), J_i(0,0)] \rangle$$

- Dynamic observables like transport coefficient require real time correlation functions

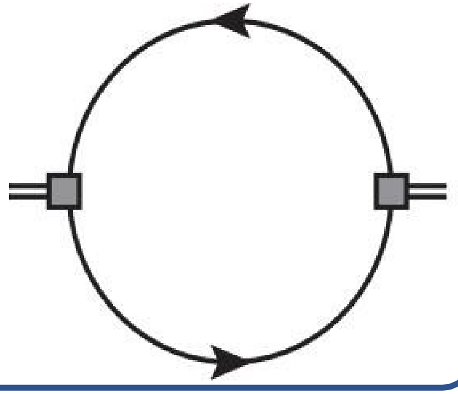
- Large uncertainties on the lattice

Real time correlators with spectral functional methods

$$\mathscr{D}_s = \lim_{\omega \to 0} \frac{\sigma(\omega, \vec{p} = 0)}{\omega \chi_q \pi} \qquad \sigma(\omega, \mathbf{p}) = \frac{1}{\pi} \int \mathrm{d}t \, e^{\mathrm{i}\omega t} \int \mathrm{d}^3x \, e^{\mathrm{i}\mathbf{x}\mathbf{p}} \langle [J_i(t, \mathbf{x}), J_i(0,0)] \rangle$$

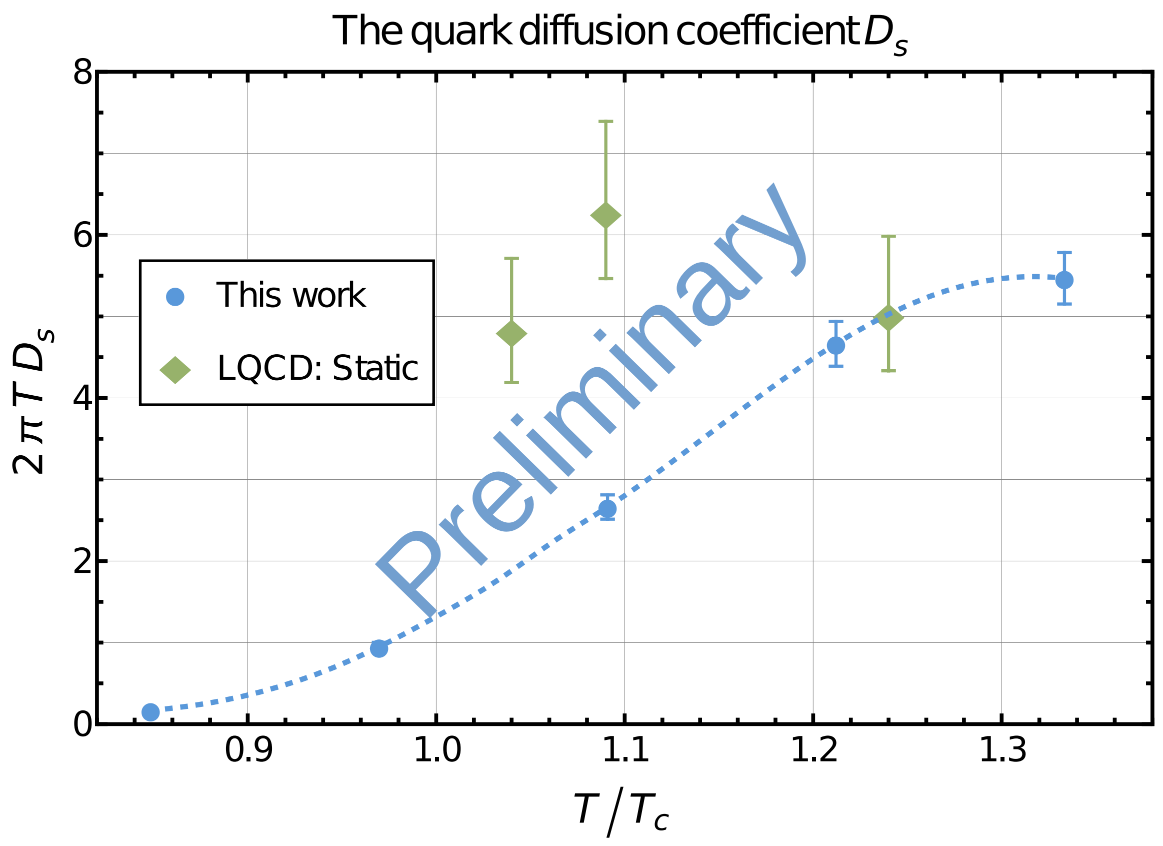
- Dynamic observables like transport coefficient require real time correlation functions
- Large uncertainties on the lattice
- Functional methods: exact diagrammatic expression

$$\sigma(\omega, \mathbf{p}) \propto \mathrm{Im}$$



Need quark propagator in real time

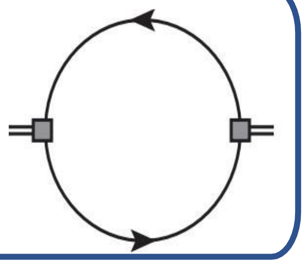
(Not so heavy) quark diffusion



Results from master thesis of Marcel Horstmann

Lattice data: Banerjee et al., Phys. Rev. D 85, 014510

Diffusion channel spectral function

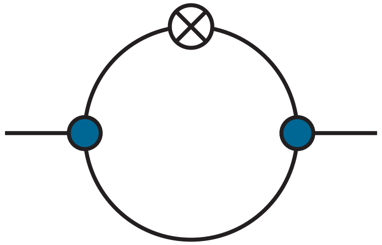
$\sigma(\omega, \mathbf{p}) \propto \mathrm{Im}$ 

- Quark propagator spectral function from improved massive HTL – computation
- Massive HTL:
  N. Haque, Phys. Rev. D 98, 014013
- Non-perturbative input:
  - Full, thermal pole mass and strong coupling from DSE data (light quarks):
    - F. Gao, J. M. Pawlowski,
      Phys. Rev. D 105, 094020
- Quark number susceptibility from lattice data:
  Borsányi et al., JHEP01(2012)138

Spectral functional equations

Spectral diagrams and spectral renormalisation

(Horak, Pawlowski, Wink arXiv: 2006.09778)

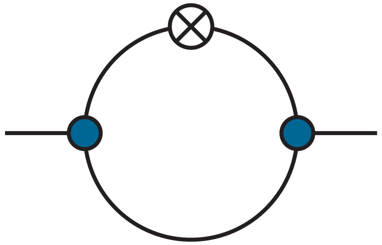


$$\propto \int_q G(q)^2 G(p+q)$$

Spectral functional equations

Spectral diagrams and spectral renormalisation

(Horak, Pawlowski, Wink arXiv: 2006.09778)



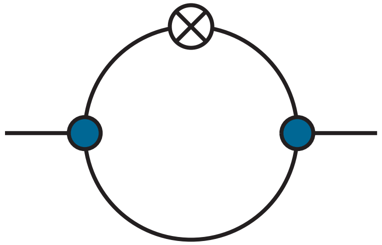
$$\propto \int_q G(q)^2 G(p+q)$$

$$G(p) = \int_\lambda \frac{\rho(\lambda)}{p^2 + \lambda^2}$$

$$= \int_{\lambda_1,\lambda_2,\lambda_3} \rho(\lambda_1)\rho(\lambda_2)\rho(\lambda_2) \int_q \frac{1}{(q^2+\lambda_1^2)(q^2+\lambda_2)((p+q)^2+\lambda_3^2)}$$

Spectral functional equations

Spectral diagrams and spectral renormalisation
(Horak, Pawlowski, Wink arXiv: 2006.09778)



$$\propto \int_q G(q)^2 G(p+q)$$

$$G(p) = \int_\lambda \frac{\rho(\lambda)}{p^2+\lambda^2}$$

$$= \int_{\lambda_1,\lambda_2,\lambda_3} \rho(\lambda_1)\rho(\lambda_2)\rho(\lambda_2) \int_q \frac{1}{(q^2+\lambda_1^2)(q^2+\lambda_2)((p+q)^2+\lambda_3^2)}$$

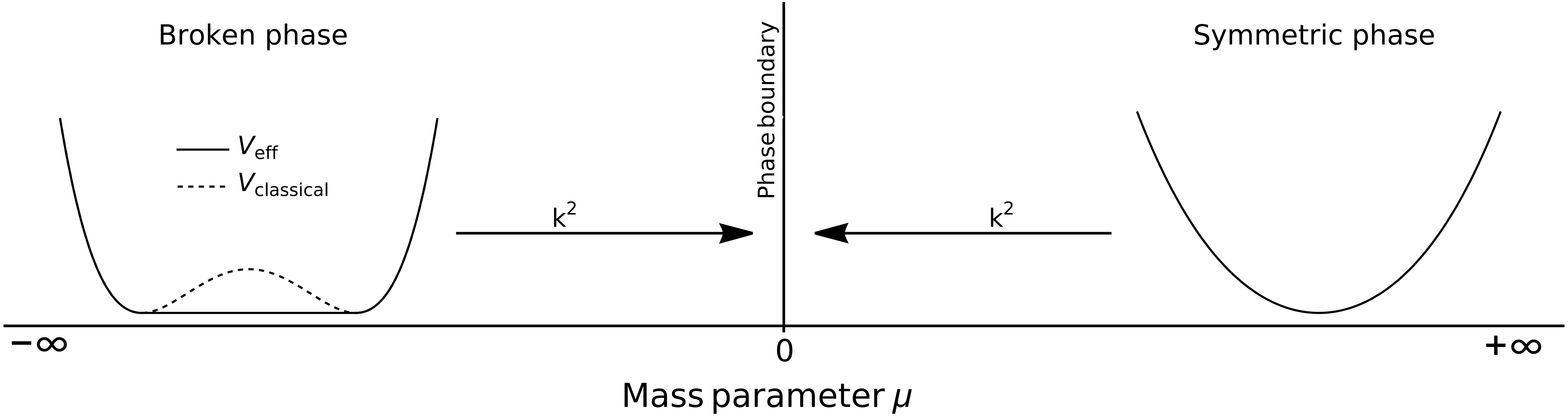
- Loop integrals can be calculated in dimReg
- Access to the full complex plane

- But: additional spectral integrals
- Spectral renormalisation for diverging diagrams

Spectral fRG and the Callan-Symanzik Cutoff

The flowing mass parameter

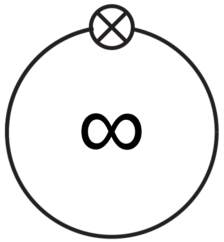
$$S[\phi] = \int \mathrm{d}^3x \left\{ \frac{1}{2}\phi\Big(-\partial^2 + \mu\Big)\phi + \frac{\lambda_\phi}{4!}\phi^4 \right\}$$



Spectral fRG and the Callan-Symanzik Cutoff

The flowing mass parameter

$$S[\phi] = \int \mathrm{d}^3x \left\{ \frac{1}{2}\phi\Big(-\partial^2 + Z_\phi \mu\Big)\phi + \frac{\lambda_\phi}{4!}\phi^4 \right\}$$

$$\mu\partial_\mu \, \Gamma[\phi] \;=\; \tfrac{1}{2}$$


$$+\; \tfrac{1}{2}\,\phi^2$$

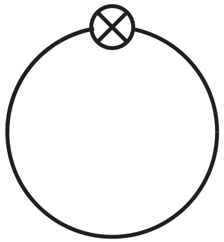

- Without UV-regularisation, divergent diagram!

Spectral fRG and the Callan-Symanzik Cutoff

The flowing mass parameter

$$S[\phi] = \int \mathrm{d}^3x \left\{ \frac{1}{2}\phi\Big(-\partial^2 + Z_\phi \mu\Big)\phi + \frac{\lambda_\phi}{4!}\phi^4 \right\}$$

$$\mu\partial_\mu\, \Gamma[\phi] \;=\; \tfrac{1}{2}$$



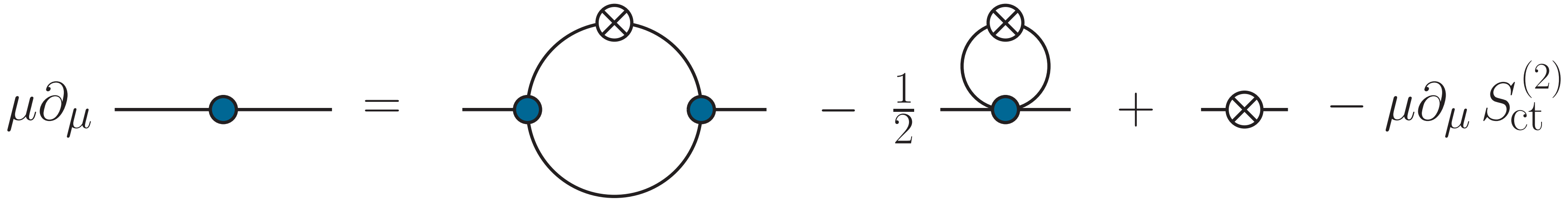
$$+\; \tfrac{1}{2}\,\phi^2 \!-\!\otimes\!-\; -\; \mu\partial_\mu\, S_{\mathrm{ct}}[\phi]$$

- Without UV-regularisation divergent diagram!
- Introduce counter-term flow via limiting prozedure over UV-finite regulators
- Counter-term flow determined by flowing renormalisation condition

fQCD collaboration: arXiv:2206.10232

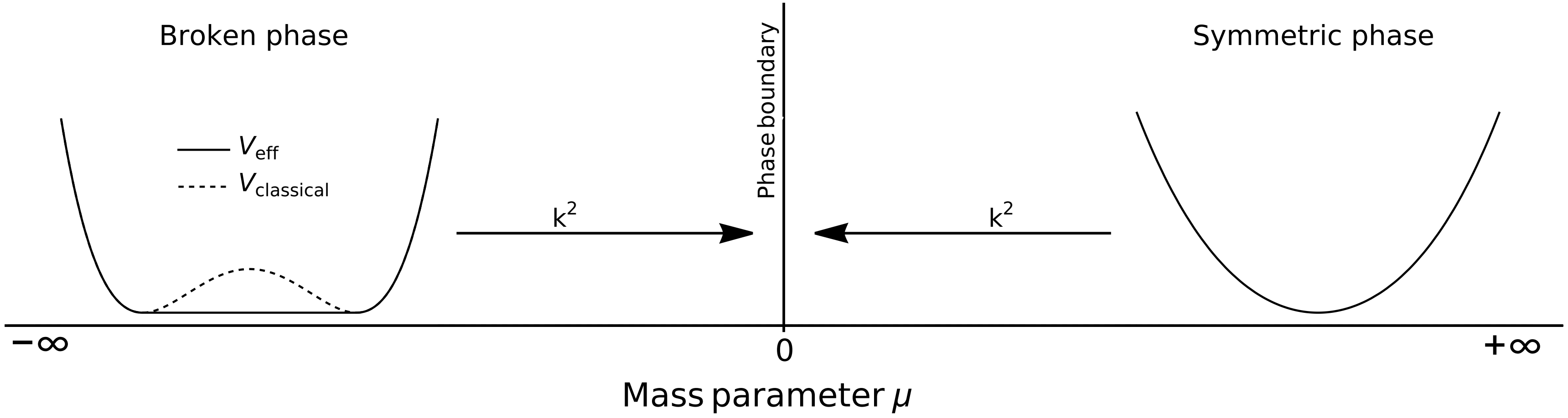
Spectral fRG and the Callan-Symanzik cut-off

flowing renormalisation conditions



- Diagrams in the flow are finite in (2+1) dimensions since the insertion of the cut-off lowers the degree of divergence by 2
- But: initial condition implicitly sets a renormalisation condition
- Exploiting the counter-term gives us the opportunity to control the flow in theory space and eliminates fine-tuning

Real scalar field in 3 dimension
flowing on-shell renormalisation



Classical pole mass on the physical minimum ϕ_0

$$m^2_{\text{pole}} = 2|\mu| = 2k^2$$

Flowing on-shell condition in the broken phase

$$\Gamma^{(2)}[\phi_0]\Big|_{p^2=-2k^2} = 0$$

Classical pole mass at $\phi_0 = 0$

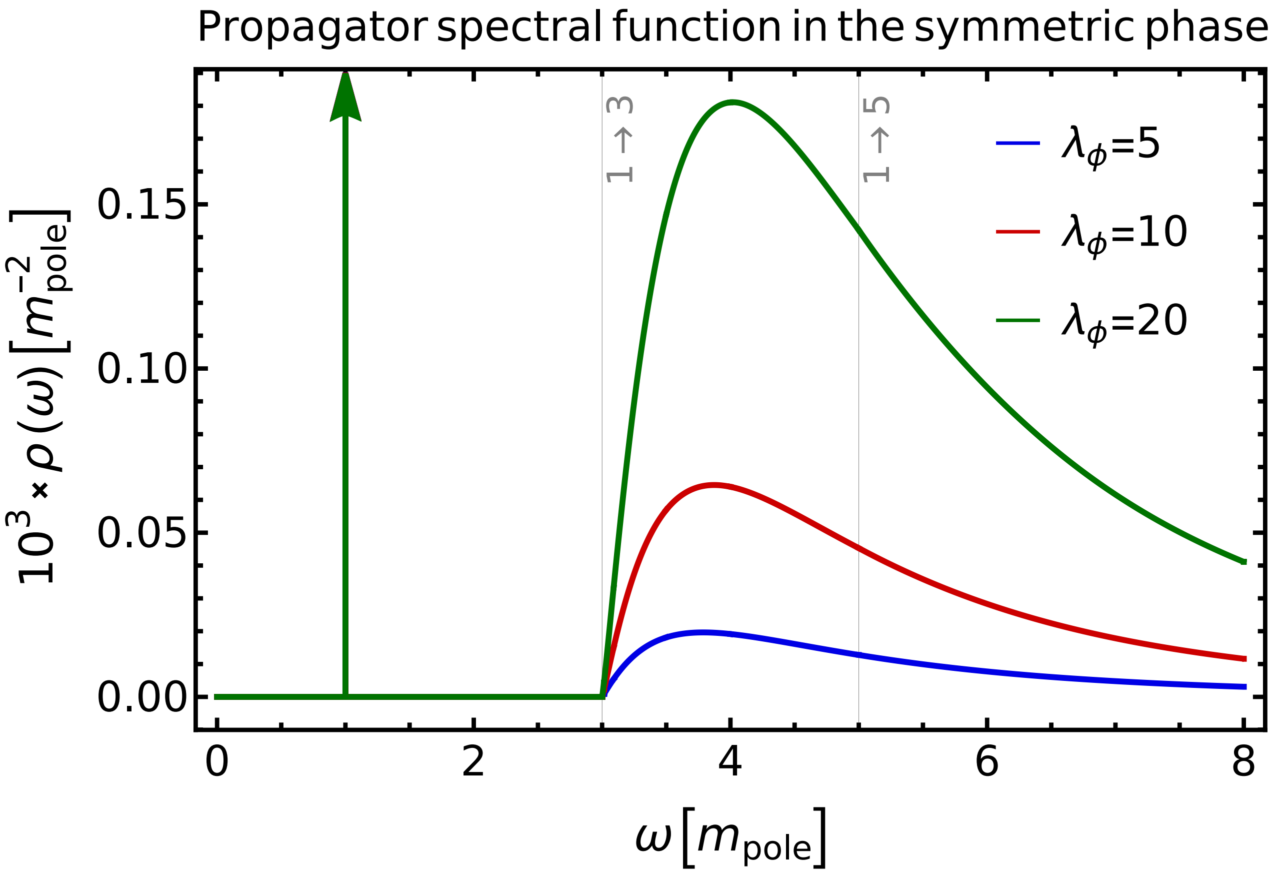
$$m^2_{\text{pole}} = \mu = k^2$$

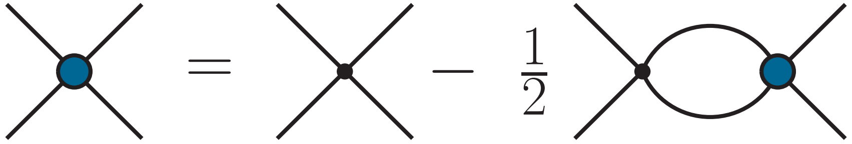
Flowing on-shell condition in the symmetric phase

$$\Gamma^{(2)}[\phi_0 = 0]\Big|_{p^2=-k^2} = 0$$

J. Horak, F. Ihssen, J. M. Pawlowski, JW, N. Wink – arXiv:2303.16719

Results in the symmetric phase

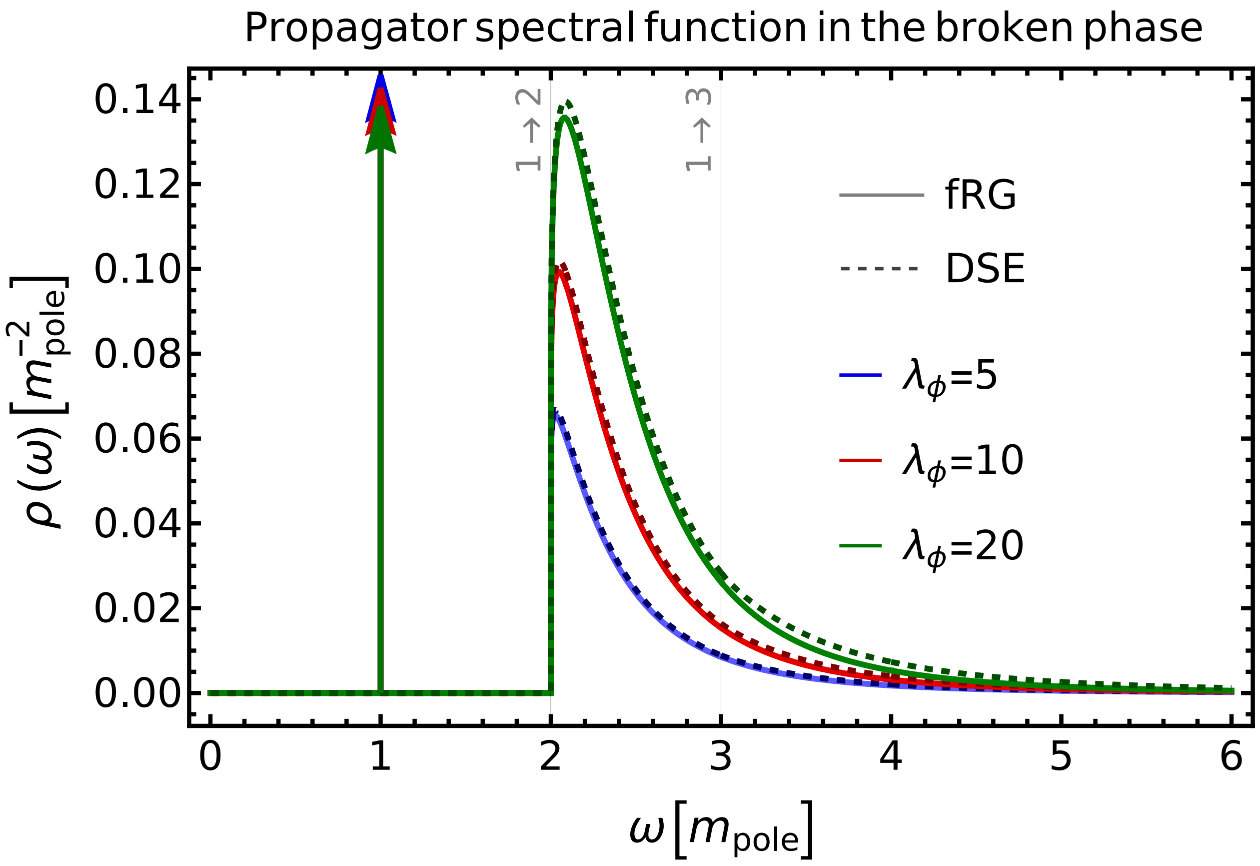


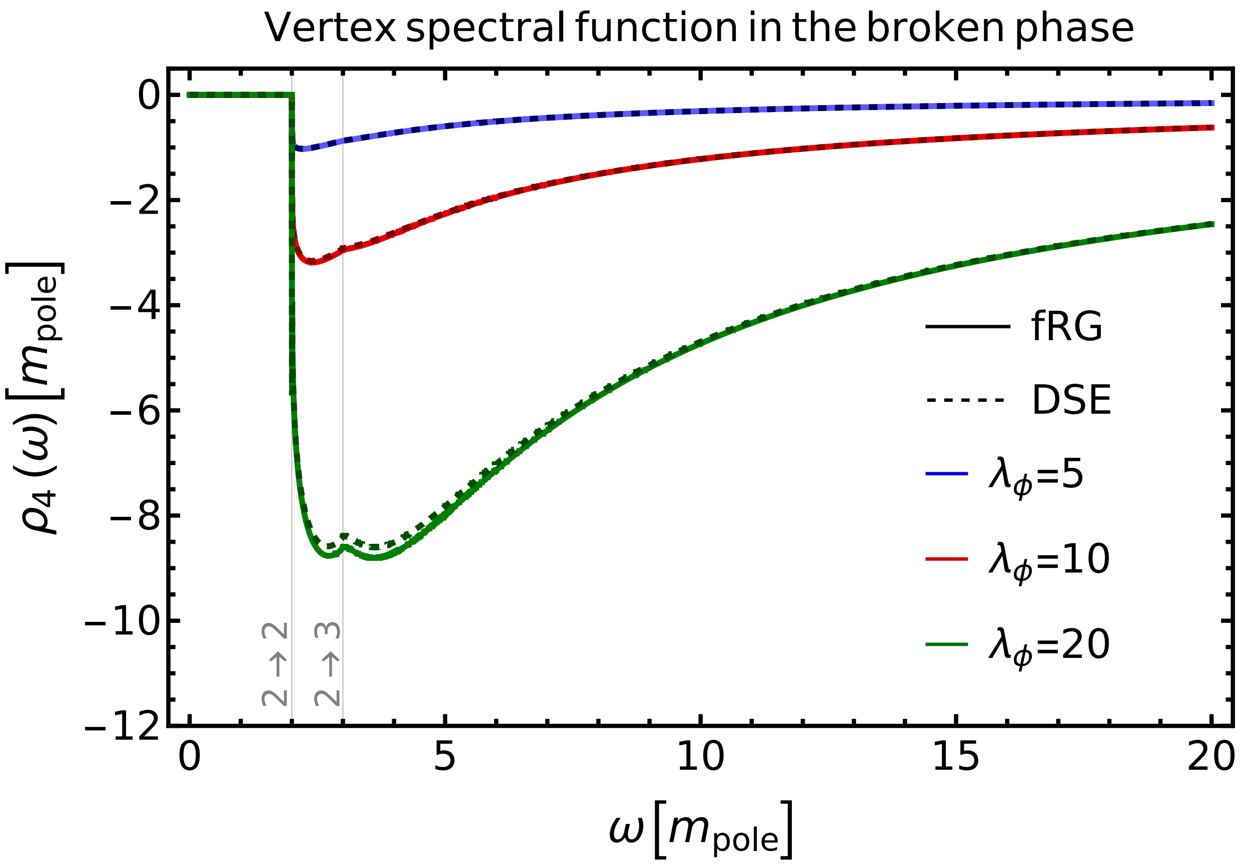


$$\Gamma^{(4)}(p^2 = s) = \lambda_\phi + \int_\lambda \frac{\rho_4(\lambda)}{\lambda^2 + p^2}$$

J. Horak, F. Ihssen, J. M. Pawlowski, JW, N. Wink – arXiv:2303.16719

Results in the broken phase

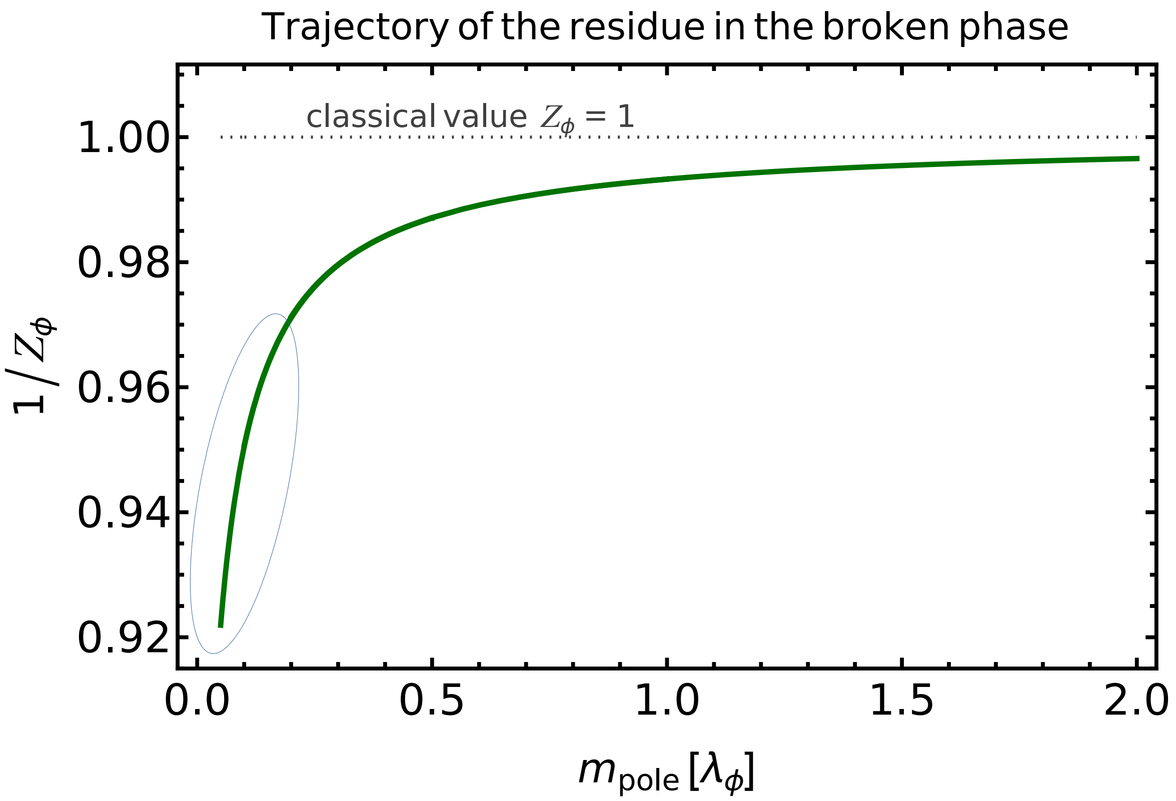


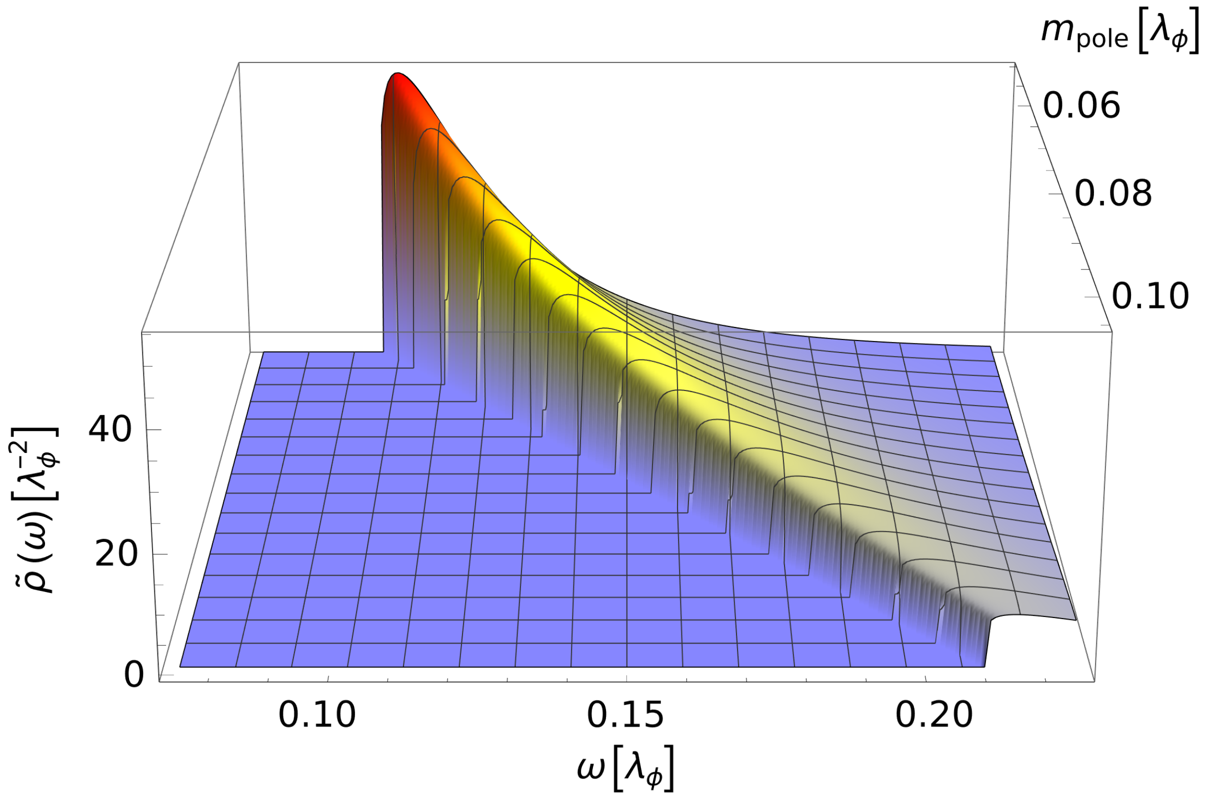


J. Horak, F. Ihssen, J. M. Pawlowski, JW, N. Wink – arXiv:2303.16719

Results in the broken phase

$$\rho(\lambda) = \frac{2\pi}{Z_\phi}\delta(m_\phi^2 - \lambda^2) + \tilde{\rho}(\lambda)$$





Wrap-up

- Spectral functional equations are powerful tool to calculate self-consistent spectral functions

- The spectral, functional Callan-Symanzik equation connects correlation functions in the limit of high masses with their massless limit.

- Flowing (on-shell) renomalisation controls trajectory in theory space and can eliminate fine-tuning problems

- TODO: extend framework to finite temperature and chemical potential

- FRG specific: include a field-dependent effective potential

- Next goal: self-consistent quark spectral functions at finite T

  Diffusion coefficients and electric conductivity

Back up

Application to a real scalar field in 3 dimension

local feedback

Start with

$\varrho_k \Leftrightarrow \Gamma_k^{(2)}$

$\partial_t \Gamma_k^{(2)}$

for real and imaginary frequencys

$k \to k - \Delta k$ Integration Step

And Renormalize

$\Gamma_{k-\Delta k}^{(2)}$

For real and imaginary frequency

$$\rho(w) = \frac{-2 Im \Gamma^{(2)}}{(Re\Gamma^{(2)} + R_k)^2 + (Im\Gamma^{(2)})^2}\Big|_{p^0 = -i(w + i0^+)}$$

Calculate

$\varrho_{k-\Delta k} \Leftrightarrow \Gamma_{k-\Delta k}^{(2)}$

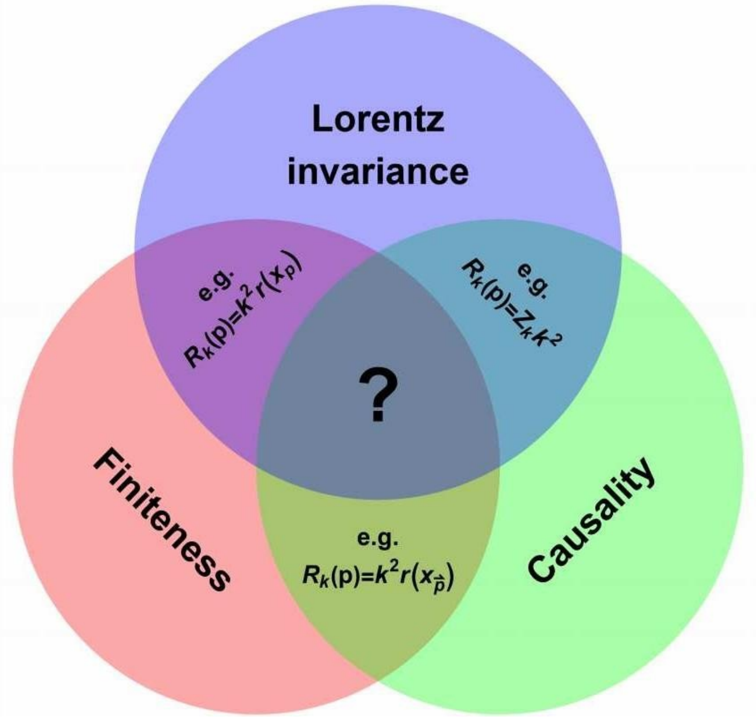
Repeat with updated parameters at $k - \Delta k$

J. Horak, F. Ihssen, J. M. Pawlowski, JW, N. Wink – in preparation

Spectral fRG and the Callan-Symanzik cut-off

arXiv:2206.10232

$$S[\phi] \to S[\phi] + \frac{1}{2}\int_q \phi(q) R_k(q^2)\phi(-q) \quad \Longrightarrow \quad G_(p) = \frac{1}{\Gamma_k^2(p^2) + R_k(p^2)}$$

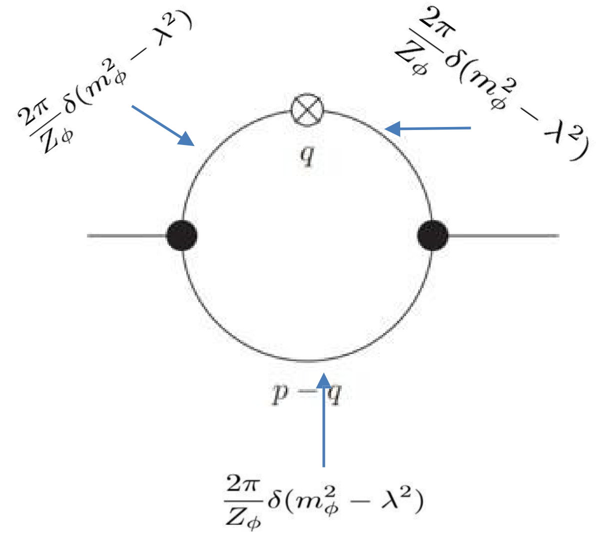


- Have to choose 2 out of 3 properties:
    - UV-regularisation
    - Lorentz invariance
    - Causal propagator at finite k

Backup

$$\rho(\lambda) = \frac{2\pi}{Z_\phi}\delta(m_\phi^2 - \lambda^2) + \tilde{\rho}(\lambda)$$

$$I_{\text{pol}}(\lambda_1, \lambda_2, \lambda_3, p^2)$$
$$= \int \frac{d^3q}{(2\pi)^3}\left(\prod_{i=1}^{2} \frac{1}{(q^2+\lambda_i^2)}\right)\frac{1}{((q+p)^2+\lambda_3^2}$$



Pole-contributions:

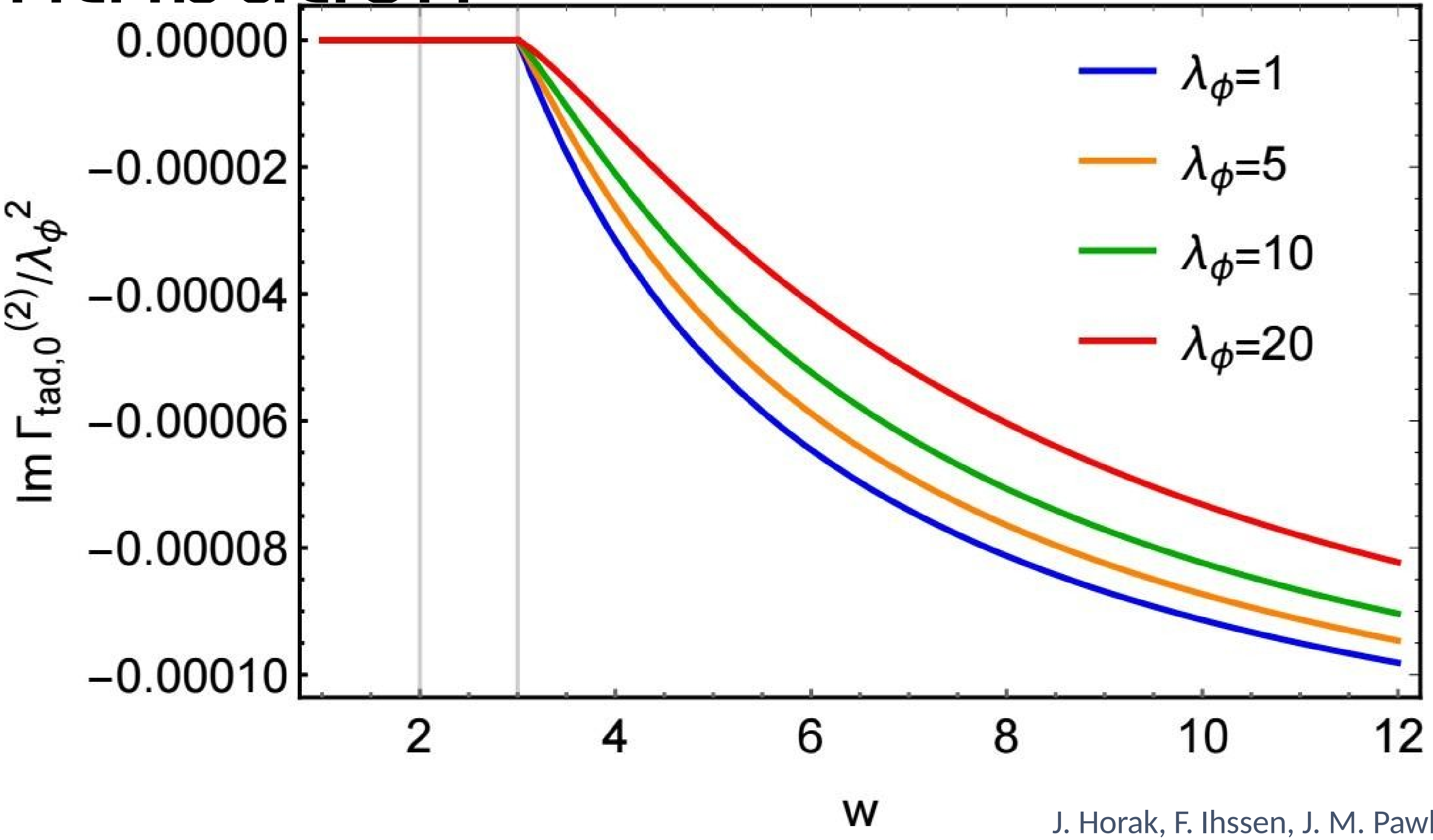
$\lambda_1^2 = \lambda_2^2 = \lambda_3^2 = k^2$

$$\frac{1}{(\lambda^2+q^2)^2} = \frac{-1}{2\lambda}\partial_\lambda \frac{1}{(\lambda^2+q^2)}$$

$$\partial_k \Gamma^{(2)}_{k,1^{st}-order}(p^2)$$
$$= -\frac{1}{2}\frac{(\Gamma_k^{(3)})^2}{Z_k^3}\frac{1}{4\pi p}\partial_k I_{pol}^{DSE}(m_k, m_k, p^2)$$

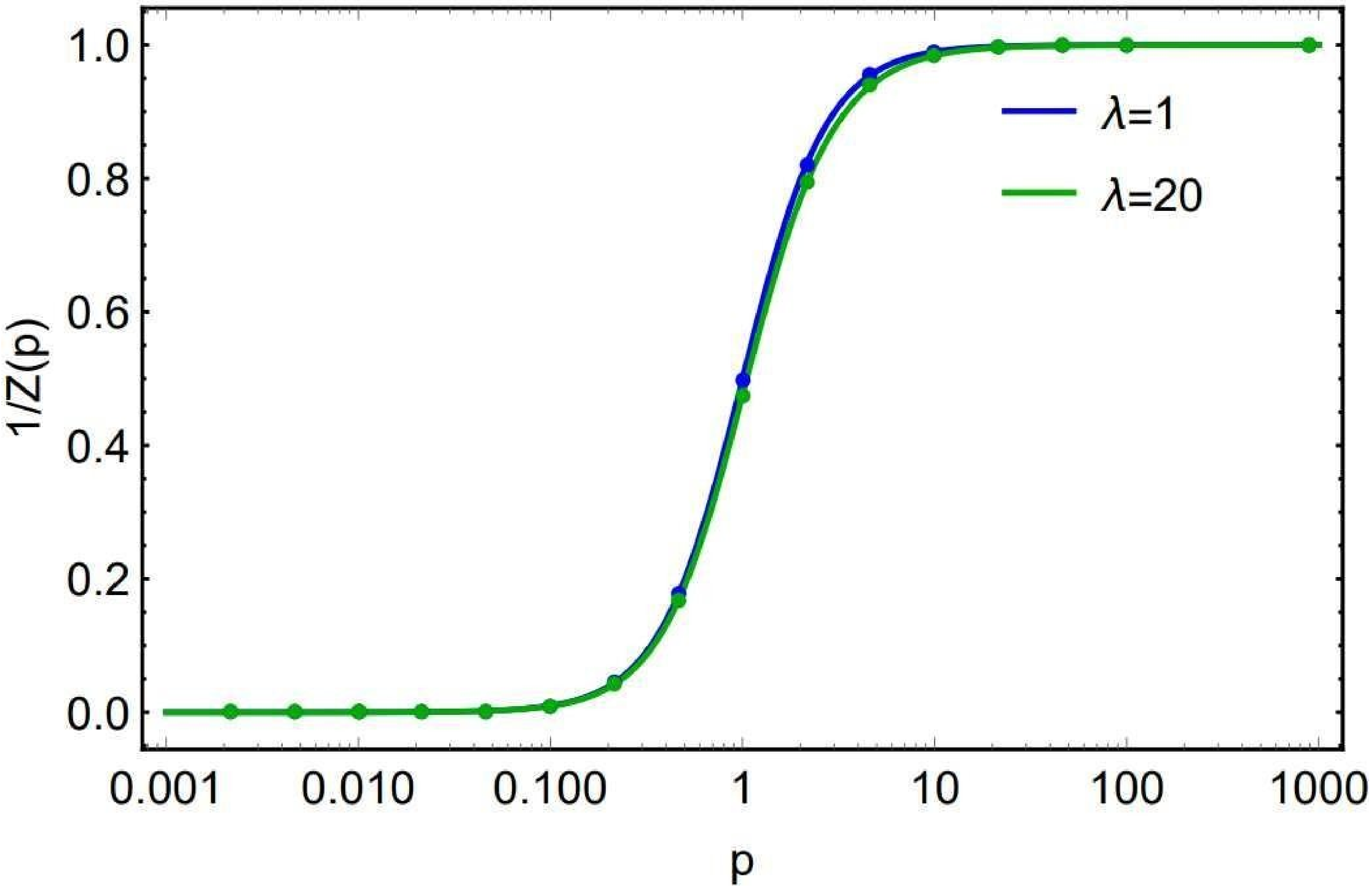
$$\int_k^{k_0} \partial_k \Gamma_k^{(2)}(p^2) = -\frac{1}{2}\left[\mathcal{F}(k) I_{\text{pol}}^{DSE}(m_k, m_k, p^2)\right]_k^{k_0}$$
$$+ \frac{1}{8\pi p}\int_k^{k_0} dk\, \partial_k \mathcal{F}(k)\, \text{Arctan}[\frac{p}{2m_k}]$$

Results: 1-cut (dynamical) Tadpole contribution



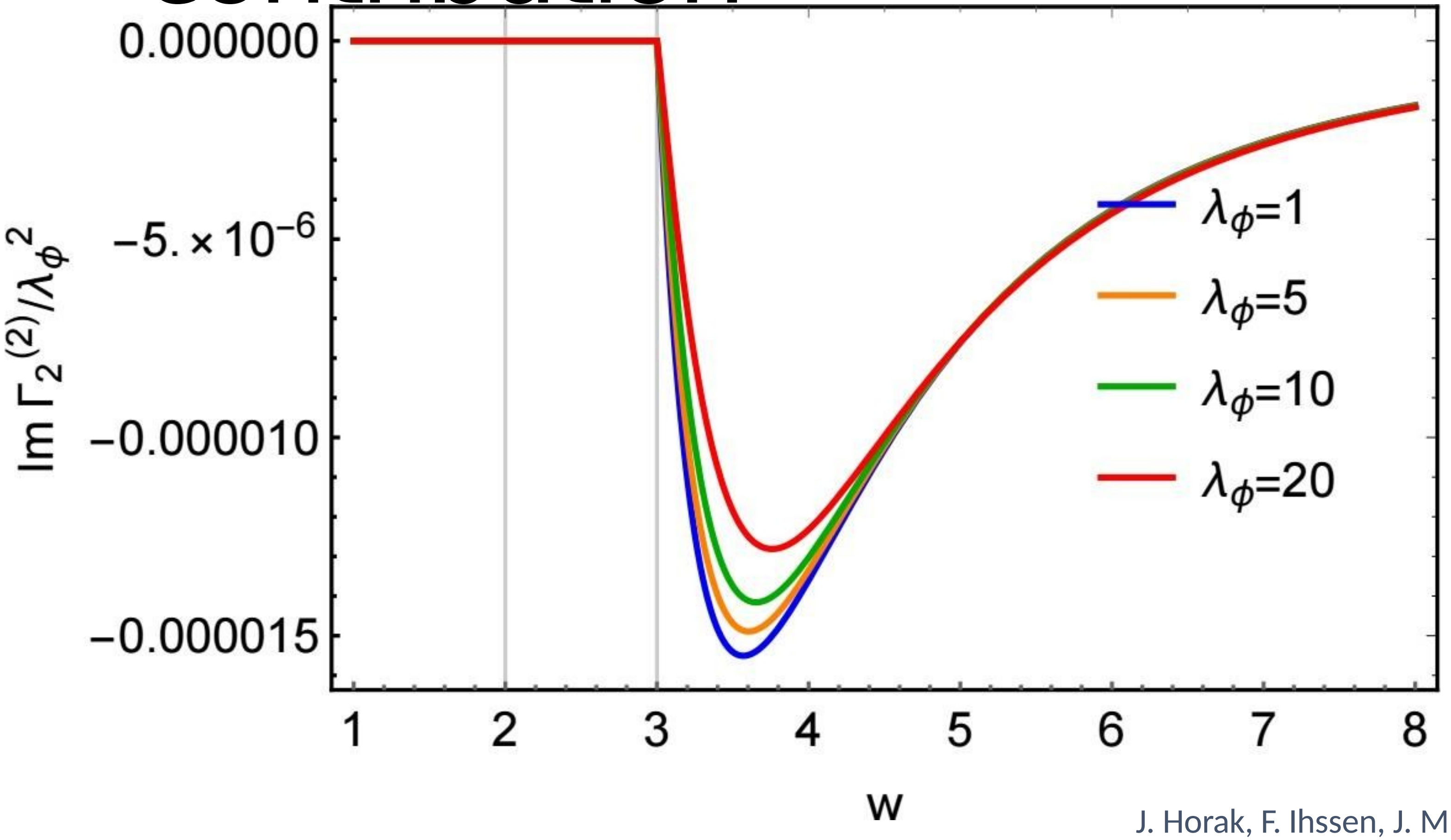
J. Horak, F. Ihssen, J. M. Pawlowski, JW, N. Wink – in preparation

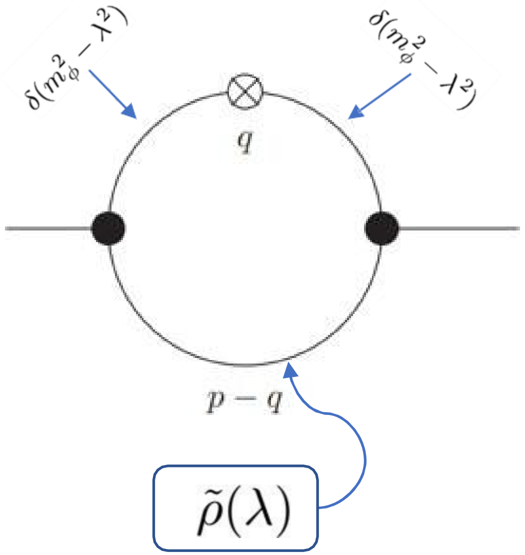
Results: Propagator dressing on the euclidean Axis



J. Horak, F. Ihssen, J. M. Pawlowski, JW, N. Wink – in preparation

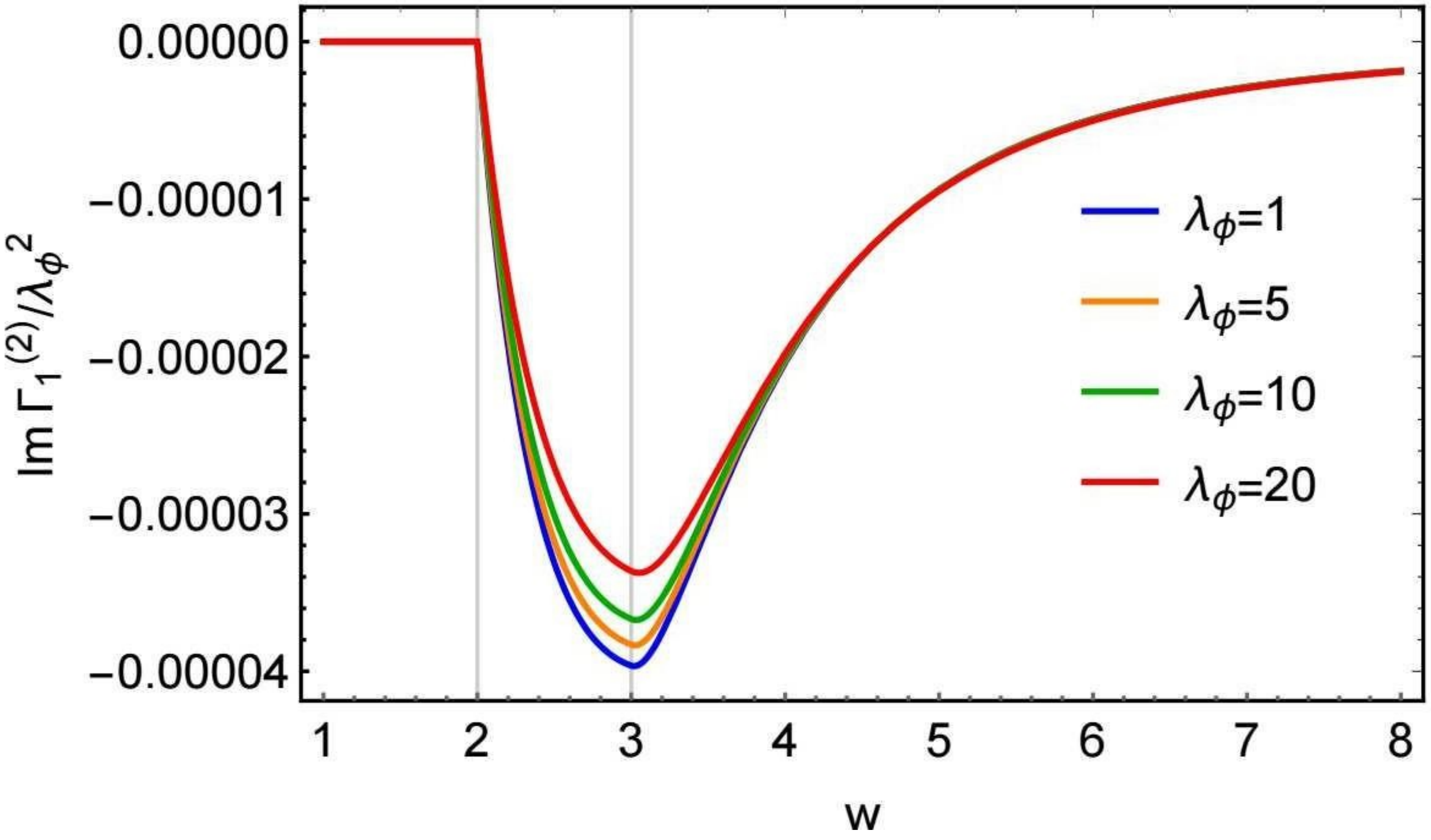
Results: 1-cut contribution





J. Horak, F. Ihssen, J. M. Pawlowski, JW, N. Wink – in preparation

Results: 1-Cut contribution (1)



J. Horak, F. Ihssen, J. M. Pawlowski, JW, N. Wink – in preparation