Lattice computation of the Kugo-Ojima function

Nuno Brito[1], Orlando Oliveira [1], Paulo Silva [1], Joannis Papavassiliou [2], Maurício Ferreira [2], Arlene C. Aguilar [3]

[1] CFisUC, Department of Physics, University of Coimbra, Portugal
[2] University of Valencia and IFIC, Spain
[3] University of Campinas, Brazil

May 24, 2023

UNIVERSIDADE D COIMBRA

Outline

Faddeev-Popov quantization procedure

- effective Lagrangian

$$\mathcal{L}_{eff} = \mathcal{L}_{QCD} + \mathcal{L}_{GF}^{\xi} + \mathcal{L}_{FP}$$

- not gauge invariant anymore!
- however, invariant under BRST transformations
- BRST charge

$$Q_B = \int d^3x J^0(x)$$

- J^μ is the BRST Noether current

Kugo-Ojima confinement mechanism

- assumes a unbroken BRST charge Q_B
  - allows to define the subspace of the physical states $|phys\rangle$:

$$\mathcal{V}_{phys} = \{|phys\rangle : Q_B|phys\rangle = 0\}$$

- total space $\mathcal{V}$ has indefinite metric and contains physical states (like baryons and mesons) as well as non-physical states (e.g. free gluons and ghosts)
- $\mathcal{V}_{phys}$ only contains color singlet states, if the charge Q^a of global gauge symmetry is unbroken and BRST-exact

$$\langle \Phi | Q^a | \Phi' \rangle = 0$$

for any physical states $|\Phi\rangle$ in $\mathcal{V}_{phys}$.

Kugo-Ojima confinement mechanism

- in such scenario, the **Kugo-Ojima confinement parameter** u^{ab} should satisfy

$$\mathrm{u}^{ab} = -\delta^{ab}.$$

- infrared limit of the function $u^{ab}(p^2)$:

$$\mathrm{u}^{ab} = \lim_{p^2 \longrightarrow 0} u^{ab}(p^2)$$

- $u^{ab}(p^2)$ defined from

$$\int d^4x e^{ip(x-y)} \left\langle D_\mu^{ae} c^e(x) g_0 f^{bcd} A_\nu^d(y) \bar{c}^c(y) \right\rangle = \left(\delta^{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) u^{ab} \left(p^2\right).$$

T. Kugo, I. Ojima, *Prog. Theor. Phys. Suppl* **66** (1979) 1; T. Kugo, arXiv:hep-th/9511033.

Lattice calculation of the Kugo-Ojima function

$$\mathcal{U}^{ab}_{\mu\nu}(k) = \left\langle \sum_{x,y} \sum_{c,d,e} e^{-ik\cdot(x-y)} D^{ae}_{\mu} \left(M^{-1} \right)^{ec}_{xy} f^{bcd} A^{d}_{\nu}(y) \right\rangle_U$$

$$u(k) = \frac{1}{(N_d-1)(N_c^2-1)} \sum_{\mu,a} \mathcal{U}^{aa}_{\mu\mu}(k)$$

- for practical reasons, we use a point source in the inversion

$$\mathcal{U}^{ab}_{\mu\nu}(k) = \left\langle \sum_{x} \sum_{c,d,e} e^{-ik\cdot(x-y_0)} D^{ae}_{\mu} \left(M^{-1} \right)^{ec}_{xy} f^{bcd} A^{d}_{\nu}(y_0) \right\rangle_U$$

Lattice Kugo-Ojima cooking recipe

1. prepare the source

$$f_{abc}A^c_\mu(x) = -\frac{1}{2}\mathrm{Tr}\left[\left\{\left(U^\dagger_{x,-\mu} + U_{x,\mu}\right) - \left(U^\dagger_{x,-\mu} + U_{x,\mu}\right)^\dagger\right\}[t^a, t^b]\right]$$

2. Solve the system, taking care of zero modes

$$MY = M\phi_{b,\nu} \quad ; \quad M\psi_{b,\nu} = Y$$

3. apply the covariant derivative

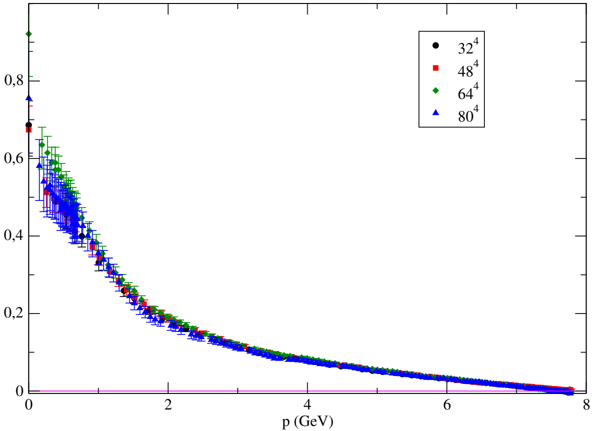
$$(D_\mu[U])^{ab}_{xy} = 2\ \mathrm{Re}\,\mathrm{Tr}\left[t^b t^a U_{x,\mu}\right]\delta_{x+\hat{\mu},y} - 2\ \mathrm{Re}\,\mathrm{Tr}\left[t^a t^b U_{x,\mu}\right]\delta_{x,y}$$

4. apply FFT, including correction due to point source

Lattice setup

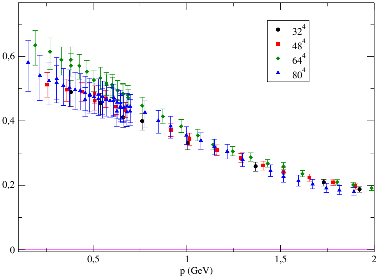
- Wilson gauge action, $\beta = 6.0$ ($a \sim 0.1$fm)
- 32^4, 48^4, 64^4, and 80^4 ($(3fm)^4 < V < (8fm)^4$)
- 100 configurations, 1 point source
  - 50 configurations for the largest volume
  - several point sources for the smallest volume
- Chroma and PFFT libraries
- simulations performed on Navigator supercomputer Coimbra
- single configuration, point source: 32 (double) inversions

Results

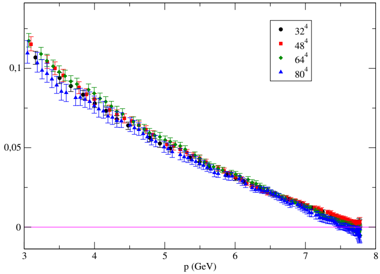


Results

Low momenta

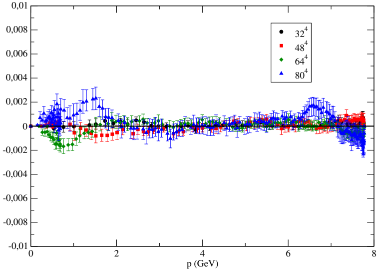


High momenta

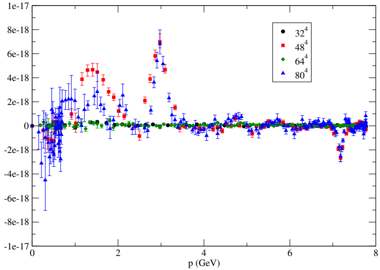


Checking consistency

Longitudinal component

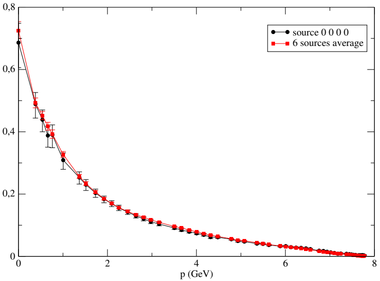


Imaginary part

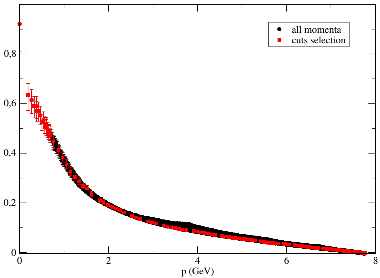


Statistics issues, Lattice artifacts

Adding more point sources

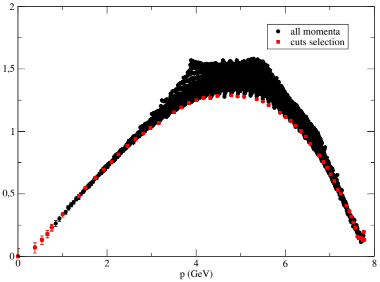


Without momentum cuts

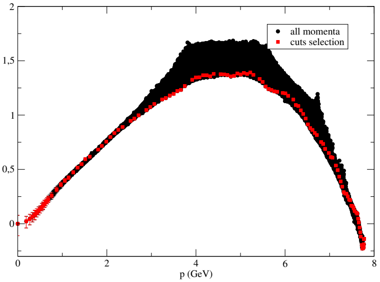


Lattice artifacts

Without momentum cuts ("dressing function") 32^4



Without momentum cuts ("dressing function") 64^4



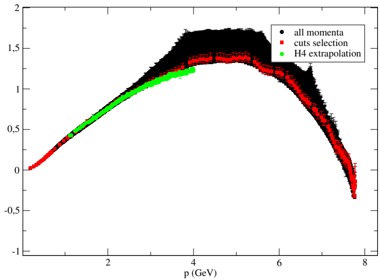
Results — H(4) extrapolation

- lattice scalar quantity F function of H(4) invariants

$$p^{[n]} = \sum_{\mu} p_{\mu}^{n}\,, \; n = 2, 4, 6, 8$$

- small lattice corrections:

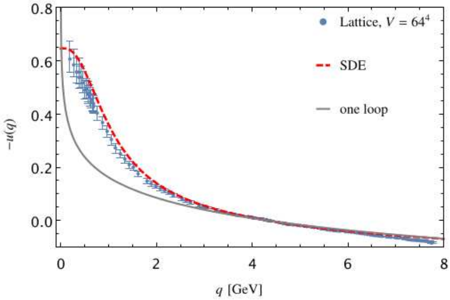
$$\begin{aligned} F_{Lat} &= F(p^{[2]}, p^{[4]}, p^{[6]}, p^{[8]}) \\ &\sim F(p^{[2]}, 0, 0, 0) + \dots \end{aligned}$$



1 2 9 0 UNIVERSIDADE D COIMBRA

Comparison with SDE and perturbative results

- Renormalized at μ=4.3 GeV ($u(\mu) = 0$)
- good agreement with 1-loop for high q
- non-perturbative effects below 3GeV



1 2 9 0 UNIVERSIDADE D COIMBRA

Conclusions and outlook

- Lattice computation of the Kugo-Ojima function
- several lattice volumes up to $(8\mathrm{fm})^4$
- checked tensor structure, lattice artifacfs
- good agreement with outcome from SDE
- does not seem to be compatible with Kugo-Ojima scenario $(u(0) = -1)$
- Outlook:
  - increase statistics
  - larger lattice volumes

1 2 9 0 UNIVERSIDADE D COIMBRA

Acknowledgements

CFisUC
LCA Laboratory for Advanced Computing
FCT Fundação para a Ciência e a Tecnologia
REPÚBLICA PORTUGUESA

Work supported by national funds from FCT – Fundação para a Ciência e a Tecnologia, I.P., Portugal, within projects UIDB/04564/2020, UIDP/04564/2020 and CERN/FIS-PAR/0023/2021. P. J. S. acknowledges financial support from FCT (Portugal) under contract CEECIND/00488/2017. Simulations performed in supercomputers Navigator, managed by LCA – University of Coimbra [url: www.uc.pt/lca], Lindgren, Sisu (through PRACE projects COIMBRALATT [DECI-9] and COIMBRALATT2 [DECI-12]) and Bob through FCT project CPCA/A2/6816/2020.