

# Heavy Quark Spectral Functions and Momentum Broadening in a highly occupied non-abelian plasma off-equilibrium

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May 24, 2023



Work in preparation in collaboration with Sören Schlichting & Sayantan Sharma  
From first-principles QCD to experiments, ECT\* Trento



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- Initial stages of a heavy-ion collisions (HIC) constitute of an out-of-equilibrium highly-occupied plasma of gauge fields.
- Studying the dynamics of heavy quarks in presence of such highly occupied gauge fields in the pre-equilibrium stage could be an excellent probe in understanding the approach to thermalization in such systems.

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- Studies involving charm quarks inside a glasma have been performed primarily in the **infinite-mass** limit using electric field (EE) correlators until now.
- Our goal is to study heavy quark dynamics using relativistic fermion fields and perform a comparative analysis with light quarks under **out-of-equilibrium conditions**.

# Summary of our main results

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- We observed that in-medium effects on the quark dynamics dictate the validity of various approximation schemes to study quark momentum broadening and diffusion.
- Reliable results for heavy quark momentum diffusion coefficient from our analysis to appear soon.

# Initial Conditions: Non-abelian plasma in the self-similar regime

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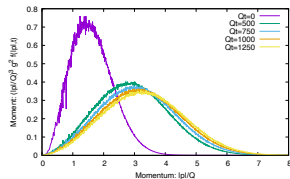
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- We consider the following initial phase-space distribution of the gluons,

$$g^2 f_g(p) = n_0 \frac{Q}{p} e^{-\frac{p^2}{2Q^2}}$$

where  $n_0/g^2 \gg 1$ .

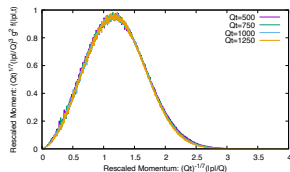


# Technical Details

- We have performed our simulations on large volume  $N_s^3 = 256^3$  lattice, with lattice spacing  $Qa_s = 0.25$ , with  $N_c = 2$  and  $N_f = 1$ .

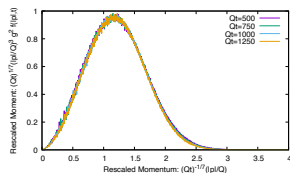
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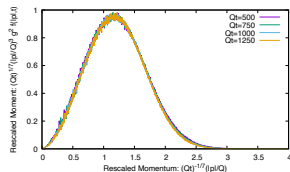
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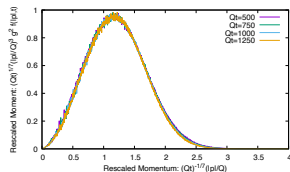
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- We have chosen a wide set of quark masses,  $m/Q = 0.0012, 0.012, 0.12, 0.24, 0.4, 1.2, 2.4, 3.6, 4.8$ . For  $Q \sim 1\text{GeV}$ , the choice of  $m/Q = 1.2$  represents a charm quark.

# Calculating Quark Spectral Functions

- Quark spectral function, in terms of quark fields, is defined as

$$\rho^{\alpha\beta}(x, y) = \left\langle \left\{ \hat{\psi}^{\alpha}(x), \hat{\bar{\psi}}^{\beta}(y) \right\} \right\rangle$$



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- In general, it can be decomposed into the following parts

$$\rho = \rho_S + i\gamma_5 \rho_P + \gamma_\mu \rho_V^\mu + \gamma_\mu \gamma_5 \rho_A^\mu + \frac{1}{2} \sigma_{\mu\nu} \rho_T^{\mu\nu}$$

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- Here, for example, the vector component of the spectral function  $\rho_V$  can be extracted as

$$\rho_V^0 = \frac{1}{4} \text{Tr}(\rho \gamma^0), \quad \rho_V^j = -\frac{E_p \rho^j}{4p^2} \text{Tr}(\rho \gamma^j)$$

# Calculating Quark Spectral Functions

- Fermionic field, in terms of creation (**b**), annihilation operators (**d**) and fermionic wavefunctions ( $\phi$ 's)

$$\Psi(t', \mathbf{x}) = \frac{1}{\sqrt{N^3}} \sum_{\lambda, \mathbf{p}} \left( \phi_{\lambda, \mathbf{p}}^u(t', \mathbf{x}) b_{\lambda}(t' = 0, \mathbf{p}) + \phi_{\lambda, \mathbf{p}}^v(t', \mathbf{x}) d_{\lambda}^{\dagger}(t' = 0, \mathbf{p}) \right)$$

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- Using the above mode function expansion, we obtain the spectral function as in

$$\begin{aligned} \rho^{\alpha\beta}(x, y) = & \sum_{\lambda, \mathbf{p}} \left( \phi_{\lambda, \mathbf{p}}^{u, \alpha}(x^0, \mathbf{x}) (\phi_{\lambda, \mathbf{p}}^{u, \gamma}(y^0, \mathbf{y}))^* \right. \\ & \left. + \phi_{\lambda, \mathbf{p}}^{v, \alpha}(x^0, \mathbf{x}) (\phi_{\lambda, \mathbf{p}}^{v, \gamma}(y^0, \mathbf{y}))^* \right) \gamma_0^{\gamma\beta} \end{aligned}$$

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- Taking the Fourier transform, we obtain the momentum space spectral function

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- Choosing  $y^0$  to be the **reference time**,  $\phi$ 's at  $y^0$  are simply given in terms of **u and v spinors (plane-wave solutions)**

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- Hence, we need to only evolve  $\phi$ 's for a specific momentum mode and plug it in the above expression to obtain spectral functions.

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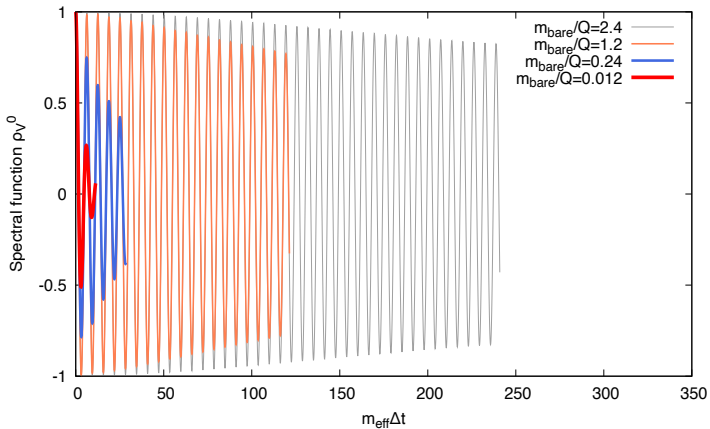
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- For light quarks with zero momenta ( $p = 0$ ), expression for the HTL spectral function simplifies to

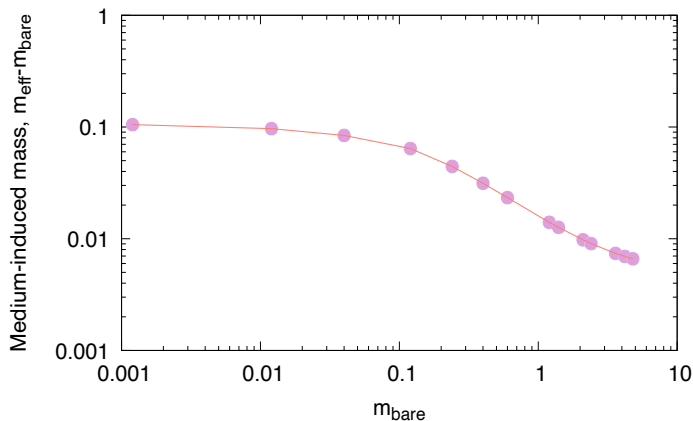
$$\text{Re}\rho_V^0(t, p = 0) \approx e^{-\gamma(m_{eff}, p=0)t} \cos[\omega(m_{eff}, 0)t]$$

# Lattice Results: quark spectral function across a mass range



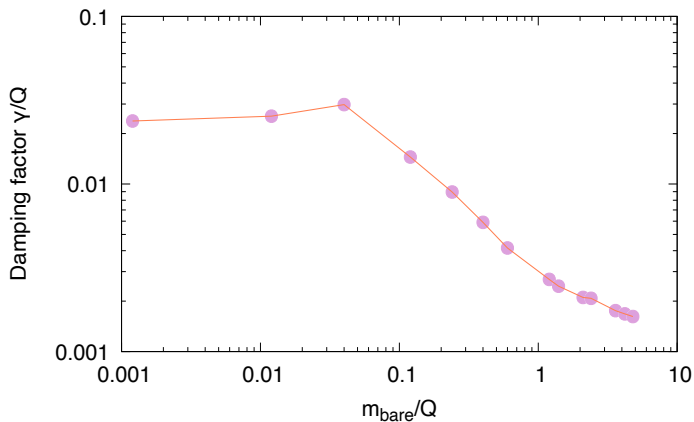
# In-Medium effects on the mass of quark quasi-particles

- We obtain effective mass ( $m_{\text{Eff}}$ ) via fit to spectral function and subtract the bare mass to obtain medium-induced mass  $\rightarrow$  medium modification decreases from  $> 99\%$  to  $< 0.1\%$  from light to heavy



# Quark Spectral Functions: Variation of $\gamma$ with $m_{\text{bare}}$

- Lifetime of quasi-particles increases  $> 10\times$  going from light to heavy



# Observations from the Spectral Function Analysis

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- In this regime, the validity of Boltzmann equation can be thus justified.
- Intermediate  $m_{bare}/Q \approx 1.2$ , i.e. **close to charm mass** lies in the transient region  $\implies$  may have enough time to diffuse but has to be evolved relativistically to capture the correct dynamics.

# Momentum Broadening: How do we calculate it?

- The fermion field at a time  $t$  in terms of time-evolved creation and annihilation operators can be defined as,

$$\Psi(t, \mathbf{x}) = \frac{1}{\sqrt{N^3}} \sum_{\lambda, \mathbf{p}} \left( u_{\lambda}(\mathbf{p}) b_{\lambda}(t, \mathbf{p}) e^{-i\mathbf{p}\mathbf{x}} + v_{\lambda}(\mathbf{p}) d_{\lambda}^{\dagger}(t, \mathbf{p}) e^{+i\mathbf{p}\mathbf{x}} \right)$$

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- Inverting the above expression using the orthonormality of Dirac spinors, we get

$$\begin{aligned} b_{\lambda'}(t, \mathbf{q}) &= \sum_{\mathbf{x}} u_{\lambda'}^{\dagger}(\mathbf{q}) \Psi(t, \mathbf{x}) e^{+i\mathbf{q}\mathbf{x}} \\ &= u_{\lambda'}^{\dagger}(\mathbf{q}) \tilde{\Psi}(t, \mathbf{q}) \end{aligned}$$

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- We start with a **single quark in a fixed momentum (P) and spin polarization (s) mode** and let it evolve in the background of gauge fields. The quark field after a time  $t'$  is

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$$\begin{aligned} \langle b_{\lambda}^{\dagger}(t = 0, \mathbf{p}) b_{\lambda'}(t = 0, \mathbf{p}') \rangle &= \delta_{\lambda \lambda'} \delta(\mathbf{p} - \mathbf{p}') \delta_{\lambda s} \delta(\mathbf{p} - \mathbf{P}) \\ \langle d_{\lambda}^{\dagger}(t = 0, \mathbf{p}) d_{\lambda'}(t = 0, \mathbf{p}') \rangle &= 0 \end{aligned}$$

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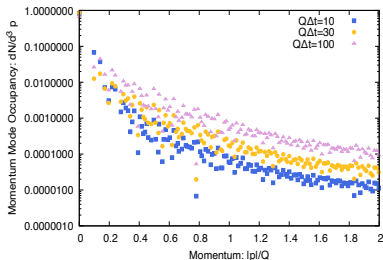
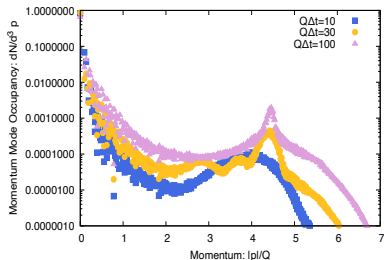
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- Momentum mode occupancy is then calculated as,

$$\frac{dN}{d^3q} = \sum_{\lambda'} \langle b_{\lambda'}^{\dagger}(t', \mathbf{q}) b_{\lambda'}(t', \mathbf{q}) \rangle = \sum_{\lambda'} |u_{\lambda'}^{\dagger}(\mathbf{q}) \tilde{\phi}_s^u(t', \mathbf{P})|^2$$

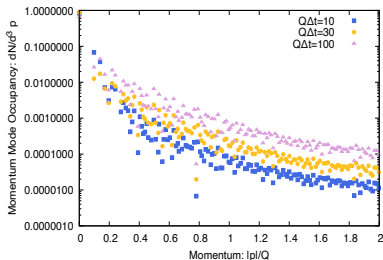
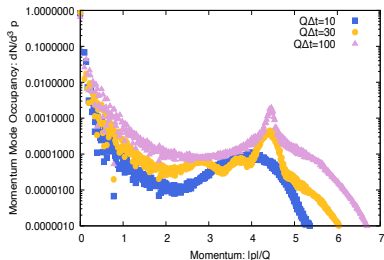
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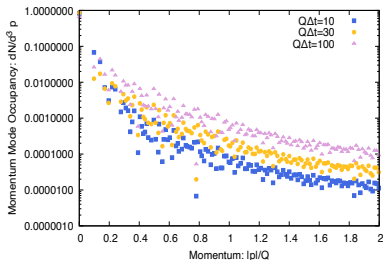
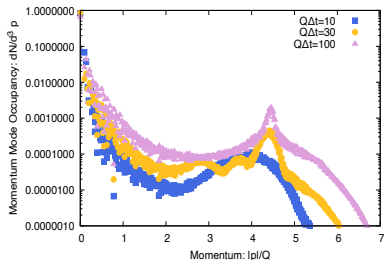


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# Momentum Broadening: Lattice Results

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- Momentum broadening of a quark with zero initial momenta can be seen quite clearly.
- But more analysis is needed to get a better understanding and reliable estimates for the momentum diffusion coefficient (**Work going on**).

## Summary and Outlook

- Heavy quarks in the presence of highly occupied gauge fields provide an excellent paradigm for studying the properties of pre-equilibrium stage of heavy-ion collisions, reheating in the early universe etc.

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- We have also set up the formalism to study heavy quark momentum broadening and extraction of heavy quark momentum diffusion coefficient.
- The immediate goal is to provide reliable estimates of heavy quark momentum diffusion coefficient in the pre-equilibrium stage (especially charm, in the context of HICs) and possibly relate it to experimental observations of charm collectivity.

Thanks :)

# Brief HTL calculation for heavy quarks with soft momenta

$$\rho_+^{HTL+\gamma}(\omega, p) = 2\pi\beta_+(\omega/p, p) + 2 \left[ \frac{Z_+(p)\gamma_+(p)}{(\omega - \omega_+(p))^2 + \gamma_+^2(p)} + \frac{Z_-(p)\gamma_-(p)}{(\omega + \omega_-(p))^2 + \gamma_-^2(p)} \right]$$

where

$$\beta_+(x, p) = \frac{m_f^2}{2p}(1-x)\theta(1-x^2) \left[ \left( p(1-x) + \frac{m_f^2}{2p} \left[ (1-x) \ln \left| \frac{x+1}{x-1} \right| + 2 \right] \right)^2 + \frac{\pi^2 m_f^4}{4p^2} (1-x)^2 \right]^{-1} \text{ with } x = \omega/p$$

# Brief HTL calculation for heavy quarks with soft momenta

For the limit  $m \gg p \implies \omega \gg p$ ,

$$\beta_+(x, p) \simeq \frac{p^2}{\pi^2 m_f^3} \rightarrow 0$$

$$Z_{\pm}(p) \simeq \frac{1}{2} \pm \frac{p}{3m_f} \rightarrow \frac{1}{2}$$

$$\omega_{\pm} \simeq m_f \pm \frac{p}{3} \rightarrow m_f$$

$$\rho_+^{HTL+\gamma}(\omega, p) = \frac{1}{2} \left[ \frac{\gamma_+(0)}{(\omega - m_f)^2 + \gamma_+^2(0)} + \frac{\gamma_-(0)}{(\omega + m_f)^2 + \gamma_-^2(0)} \right]$$

Taking Fourier transform to go from frequency domain to real-time domain, we obtain

$$\rho_+^{HTL+\gamma}(t, p) = \int d\omega \rho_+^{HTL+\gamma}(\omega, p) \approx e^{-\gamma(0)t} \cos(m_f t)$$

# Spectral Function Plots Zoomed

