

Color superconductivity beyond Mean Field Approximation

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Justus-Liebig-Universität Giessen

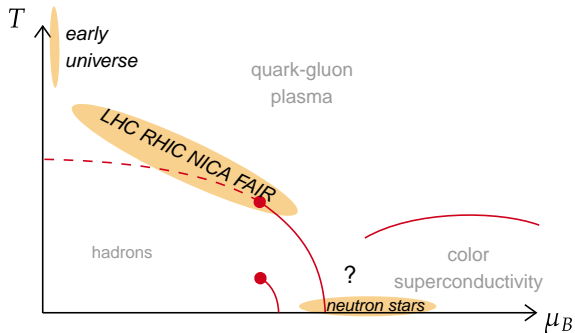


HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

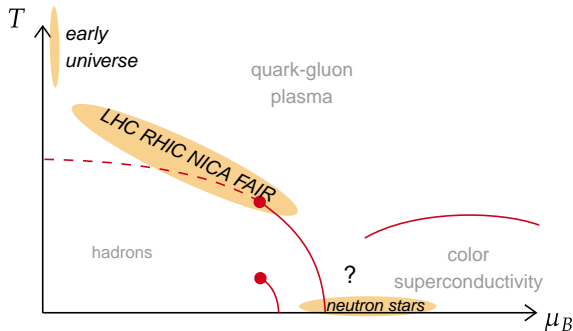
1. Color Superconductivity and Diquarks
2. Functional Renormalization Group Study
3. Phase Structure & Astrophysical Applications

Color Superconductivity and Diquarks

Why study color superconductivity?



Why study color superconductivity?



- **Natural in QCD:** attractive gluon exchange between quarks leads to superconductivity at high densities.
- Relevant for **astrophysical observations**.
- Hard to go **beyond mean field**.

Why study color superconductivity?

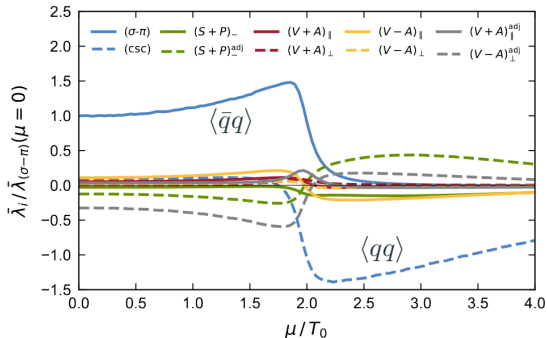
- $\langle \bar{q}q \rangle$ condensate: spontaneous chiral symmetry breaking, mesons, ...
- $\langle qq \rangle$ condensate: color superconductivity, diquarks, ...

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Full QCD flow comparing the most important channels.

[Braun, Leonhardt & Pospiech; 1909.06298].

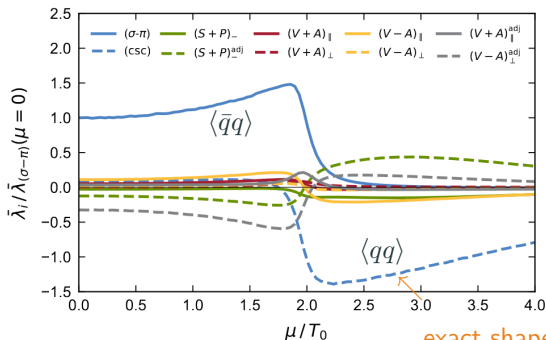


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Full QCD flow comparing the most important channels.

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exact shape of the
 $\langle qq \rangle$ condensate?

2SC Phase

2SC phase: condensates of the shape

$$\Delta_A = \langle q^T C \gamma_5 \tau_2 \lambda_A q \rangle, \quad A = 2, 5, 7$$

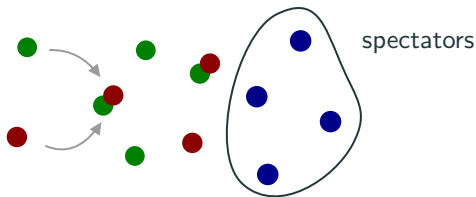
Main characteristics:

↙ some matrix structure in flavor,
color and Dirac space

- Chiral symmetry is not broken.
- Color symmetry is "broken" (Higgs mechanism), but not fully:

$$SU_c(3) \rightarrow SU_c(2).$$

- Only two color participate in superconductivity.



Quark-meson-diquark Model

Quark-meson-diquark model action:

$$\begin{aligned} S_{\text{QMD}} = \int_x \left\{ \bar{q} (\not{\partial} - \mu\gamma_0 + g_\phi (\sigma + i\gamma_5 \vec{\pi} \cdot \vec{\tau})) q \right. \\ + \frac{g\Delta}{2} (\Delta_A q^T C \gamma_5 \tau_2 \lambda_A q - \Delta_A^* \bar{q} \gamma_5 \tau_2 \lambda_A C \bar{q}^T) \\ + \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \vec{\pi})^2 \\ + ((\partial_\nu + \delta_{\nu 0} 2\mu) \Delta_a^*) (\partial_\nu - \delta_{\nu 0} 2\mu) \Delta_a \\ \left. + U(\sigma^2 + \vec{\pi}^2, \Delta_a \Delta_a^*) - c\sigma \right\} \end{aligned}$$

Quark-meson-diquark Model

Quark-meson-diquark model action:

quark-meson
scalar-pseudoscalar
channel

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meson and diquark interactions,
arbitrary potential constrained by
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quark-diquark
interaction

diquark kinetic
term, couples to μ


meson and diquark interactions,
arbitrary potential constrained by
symmetries

explicit chiral symmetry
breaking term

Functional Renormalization Group Study


Functional Renormalization Group

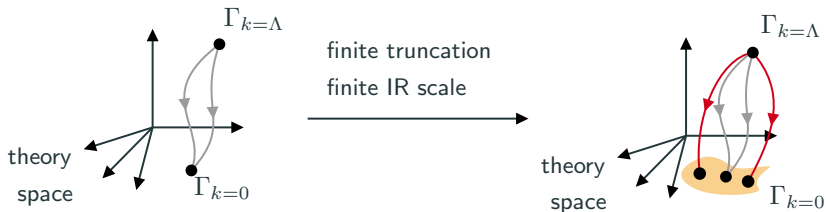
- Define an **effective action** Γ_k such that $\Gamma_{k=\Lambda} = S_{\text{QMD}}$ and $\Gamma_{k=0} = \Gamma$ by means of **regulator** R_k .
- The effective action follows an exact differential equation:

$$\partial_t \Gamma_k = \frac{1}{2} \text{Tr} \left[\left(\Gamma_k^{(2)} + R_k \right)^{-1} \partial_t R_k \right] = \frac{1}{2} \text{Tr} \left[\text{Bubble} \right]$$


Functional Renormalization Group

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Some FRG details used here:

- **Local potential approximation** (LPA): only scale dependent quantity is the potential $U_k(\sigma, \Delta)$.
 - σ : chiral condensate.
 - Δ : diquark condensate.
 - $U_k \rightarrow \infty$ number of couplings.

Truncation and Regulator Choice

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- **Local potential approximation** (LPA): only scale dependent quantity is the potential $U_k(\sigma, \Delta)$.
 - σ : chiral condensate.
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 - $U_k \rightarrow \infty$ number of couplings.
- **3d Litim regulator**

$$R_k = (k^2 - \vec{p}^2)\theta(k^2 - \vec{p}^2) .$$

Expect negative entropy at high μ and low T from studies of the quark-meson model \rightarrow more on that later.

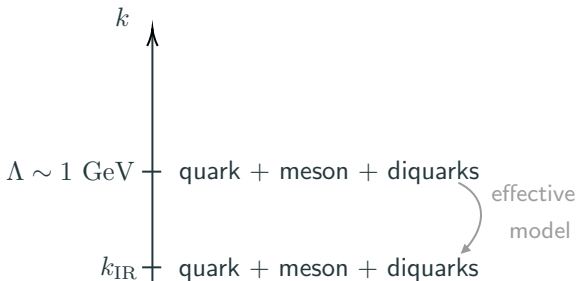
Parameter Fixing

Initial condition for the flow:

$$U_{k=\Lambda} = a_1\sigma^2 + a_2\sigma^4 + b_1\Delta^2 + b_2\Delta^4$$

with a_1 , a_2 , b_1 and b_2 free parameters.

- **Mesonic parameters:** fixed by matching vacuum observables.



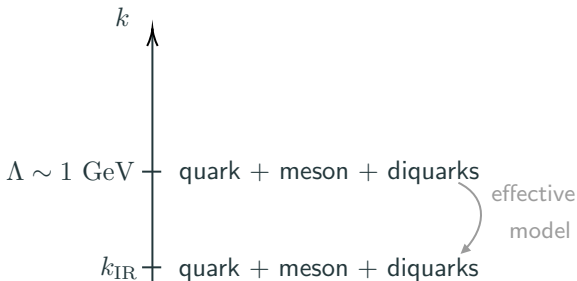
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- Harder for **diquark parameters**: no observable values.
 - Fix to pQCD.



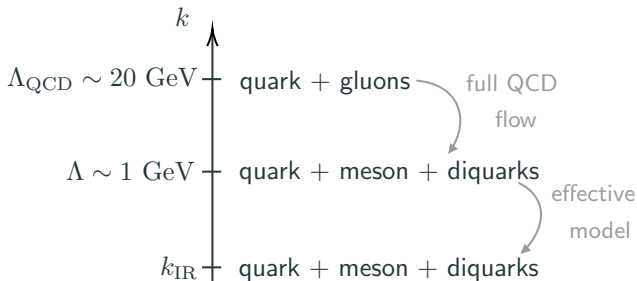
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- **Mesonic parameters**: fixed by matching vacuum observables.
- Harder for **diquark parameters**: no observable values.
 - Fix to pQCD.
 - Use full QCD flow to initialize. [Braun, Schallmo; 2106.04198]



Flow Equation

$$\partial_t U_k(\sigma, \Delta) = - \text{[Diagram 1]} - \text{[Diagram 2]} + \frac{1}{2} \text{[Diagram 3]} + \frac{1}{2} \text{[Diagram 4]} + \frac{1}{2} \text{[Diagram 5]}$$

The equation is represented by five diagrams, each featuring a circle with a cross at the top:

- Diagram 1:** A solid circle with two arrows on the right side pointing downwards. It is labeled with q_r, q_g in red and green.
- Diagram 2:** A solid circle with two arrows on the right side pointing downwards. It is labeled with q_b in blue.
- Diagram 3:** A dashed circle with no arrows. It is labeled with Δ_2, σ .
- Diagram 4:** A dashed circle with no arrows. It is labeled with π .
- Diagram 5:** A dashed circle with no arrows. It is labeled with Δ_5, Δ_7 .

Flow Equation

depends on $E_\Delta = \sqrt{(\epsilon_k \pm \mu)^2 + \Delta^2}$
with $\epsilon_k = \sqrt{k^2 + \sigma^2}$

↓

$$\partial_t U_k(\sigma, \Delta) = - \text{[diagram 1]} - \text{[diagram 2]} + \frac{1}{2} \text{[diagram 3]} + \frac{1}{2} \text{[diagram 4]} + \frac{1}{2} \text{[diagram 5]}$$

The diagrammatic equation for $\partial_t U_k(\sigma, \Delta)$ consists of five terms:

- 1. A solid circle with a crossed circle at the top and two arrows forming a clockwise loop. It is labeled q_r, q_g .
- 2. A solid circle with a crossed circle at the top and two arrows forming a clockwise loop. It is labeled q_b .
- 3. A dashed circle with a crossed circle at the top. It is labeled Δ_2, σ .
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coupling between one diquark
 and the sigma meson

$$\begin{aligned}
 \partial_t U_k(\sigma, \Delta) = & - \begin{array}{c} \downarrow \\ \text{---} \otimes \text{---} \\ \text{---} \end{array} - \begin{array}{c} \text{---} \otimes \text{---} \\ \text{---} \end{array} + \frac{1}{2} \begin{array}{c} \downarrow \\ \text{---} \otimes \text{---} \\ \text{---} \end{array} \\
 & + \frac{1}{2} \begin{array}{c} \text{---} \otimes \text{---} \\ \text{---} \end{array} + \frac{1}{2} \begin{array}{c} \text{---} \otimes \text{---} \\ \text{---} \end{array}
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q_r, q_g q_b Δ_2, σ π Δ_5, Δ_7

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 + \frac{1}{2} \begin{array}{c} \text{diagram 4} \end{array} + \frac{1}{2} \begin{array}{c} \text{diagram 5} \end{array}$$

The diagrams are:

- Diagram 1: A solid circle with a vertex (circle with an 'X') at the top. Two arrows on the circle indicate a clockwise flow. Labels q_r, q_g are at the bottom right.
- Diagram 2: A solid circle with a vertex (circle with an 'X') at the top. Two arrows on the circle indicate a clockwise flow. Label q_b is at the bottom right.
- Diagram 3: A dashed circle with a vertex (circle with an 'X') at the top. Label Δ_2, σ is at the bottom right. A downward arrow points to the vertex.
- Diagram 4: A dashed circle with a vertex (circle with an 'X') at the top. Label π is at the bottom right. An upward arrow points to the vertex.
- Diagram 5: A dashed circle with a vertex (circle with an 'X') at the top. Label Δ_5, Δ_7 is at the bottom right.

$$\sim \coth \frac{\epsilon_\pi}{2T} \text{ with}$$

$$\epsilon_\pi = \sqrt{k^2 + 2\partial_{\sigma^2} U_k}$$

Flow Equation

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$\sim \coth \frac{\epsilon_\Delta - 2\mu}{2T}$ with
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 problem with this part

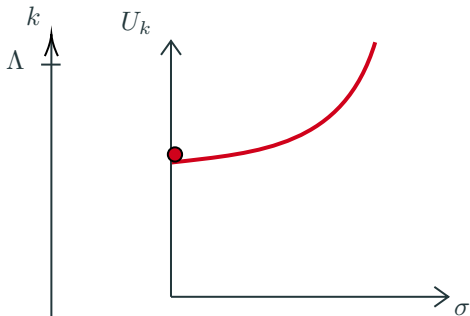
First Trouble: Bosonic Diquark Loops

Pion loops

$$\partial_t U_k = \frac{1}{\epsilon_\pi} \coth \frac{\epsilon_\pi}{2T} + \dots$$

with

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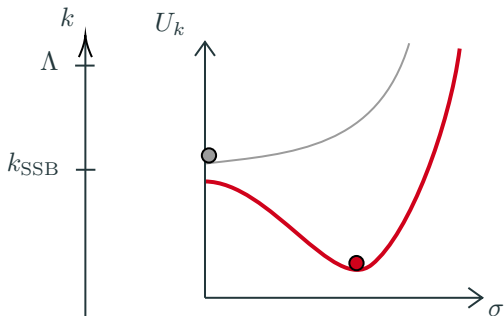
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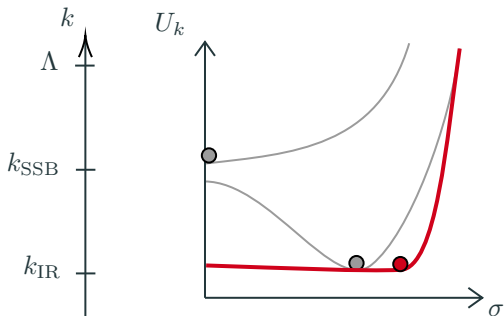
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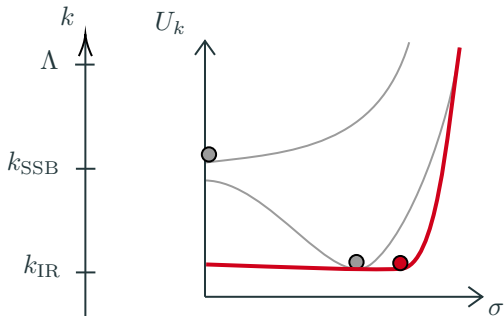
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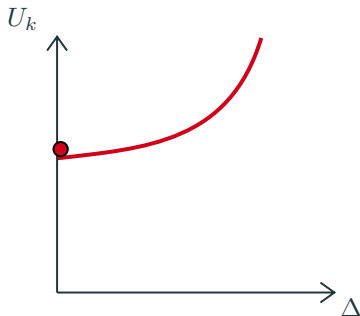


Diquark loops

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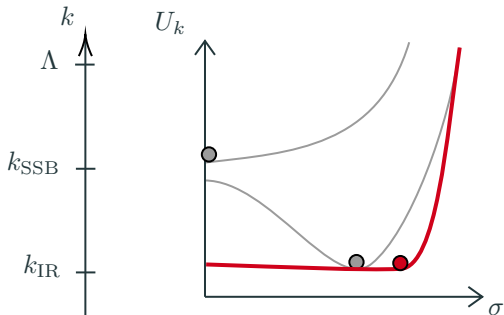
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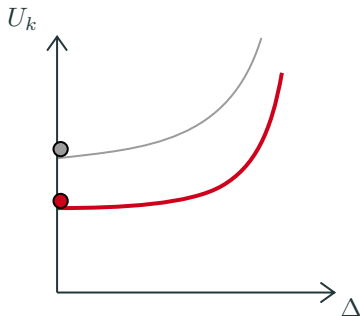


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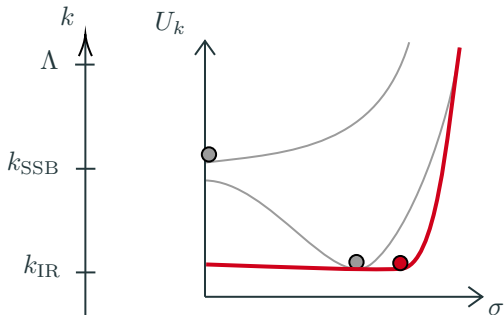
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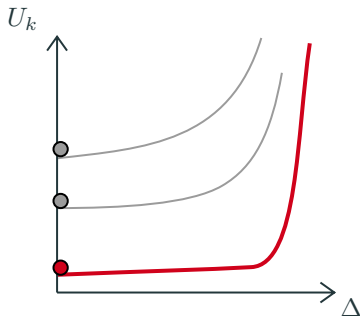


Diquark loops

$$\partial_t U_k = \frac{1}{\epsilon_\Delta} \coth \frac{\epsilon_\Delta - 2\mu}{2T} + \dots$$

with

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Second Trouble: Gapped Quark Loop

Gapped quark loop:



$$\propto \frac{\epsilon_k - \mu}{\sqrt{(\epsilon_k - \mu)^2 + \Delta^2}} \xrightarrow{\partial_{\Delta^2}} \propto \frac{\epsilon_k - \mu}{((\epsilon_k - \mu)^2 + \Delta^2)^{3/2}}$$

diverges at the Fermi-surface

↓

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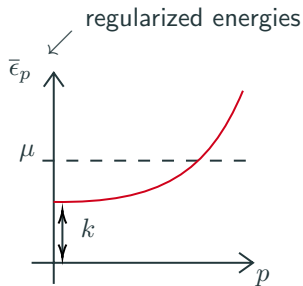
$$\propto \frac{\epsilon_k - \mu}{\sqrt{(\epsilon_k - \mu)^2 + \Delta^2}} \xrightarrow{\partial_{\Delta^2}} \propto \frac{\epsilon_k - \mu}{((\epsilon_k - \mu)^2 + \Delta^2)^{3/2}}$$

diverges at the Fermi-surface

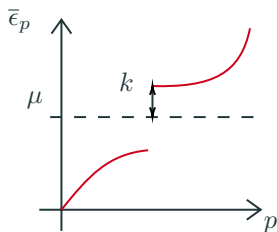


Possible solution: **flow around the Fermi-surface**

[Braun, Dörnfeld, Schallmo, Töpfel; 2008.05978]



usual momentum
regulator



Fermi-surface
regulator

Phase Structure & Astrophysical Applications

Quark-meson-diquark Model without Diquark Loops

Diquarks only at the mean field level: none of the previously mentioned problems are present.

$$\partial_t U_k = - \text{[diagram 1]} - \text{[diagram 2]} + \frac{1}{2} \text{[diagram 3]} + \frac{1}{2} \text{[diagram 4]}$$

The equation shows the time derivative of the quark loop $\partial_t U_k$ as a sum of four diagrams. Each diagram consists of a circle with a vertex at the top marked with a cross in a circle. The first two diagrams are solid lines with arrows pointing clockwise, labeled q_r, q_g and q_b respectively. The last two diagrams are dashed lines with arrows pointing clockwise, labeled σ and π respectively.

Quark-meson-diquark Model without Diquark Loops

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The diagrammatic equation shows four terms:

- Term 1: A solid circle loop with a vertex at the top (circle with an 'X'). Two arrows on the circle point clockwise. To the right of the circle are the labels q_r, q_g in red and green.
- Term 2: A solid circle loop with a vertex at the top (circle with an 'X'). Two arrows on the circle point clockwise. To the right of the circle is the label q_b in blue.
- Term 3: A dashed circle loop with a vertex at the top (circle with an 'X'). To the right of the circle is the label σ .
- Term 4: A dashed circle loop with a vertex at the top (circle with an 'X'). To the right of the circle is the label π .

diquarks still present here

$$E_{\Delta}^{\pm} = \sqrt{(\epsilon_k \pm \mu)^2 + \Delta^2}$$

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still include fluctuations
from pion and sigma mesons

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The equation shows four diagrams representing different contributions to the derivative of the potential energy $\partial_t U_k$. Each diagram consists of a circle with a cross (⊗) at the top. The first two diagrams have solid lines and arrows forming a loop, labeled q_r, q_g and q_b respectively. The last two diagrams have dashed lines and arrows forming a loop, labeled σ and π respectively. Arrows point upwards from the first and third diagrams, and upwards from the second and fourth diagrams.

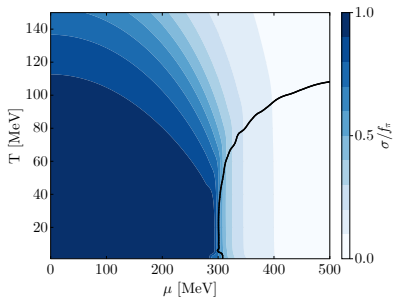
diquarks still present here

$$E_{\Delta}^{\pm} = \sqrt{(\epsilon_k \pm \mu)^2 + \Delta^2}$$

still include fluctuations
from pion and sigma mesons

- Look at phase structure.
- First astrophysical applications.

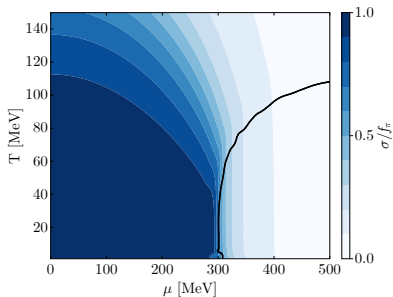
Phase Diagram



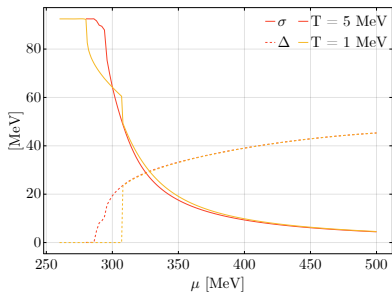
phase diagram

- Expected phase structure.

Phase Diagram



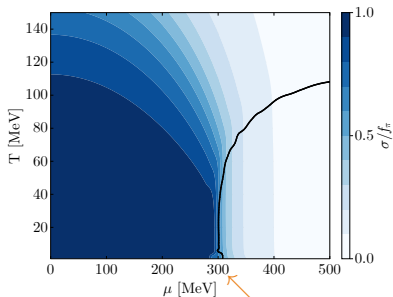
phase diagram



shape of the condensates

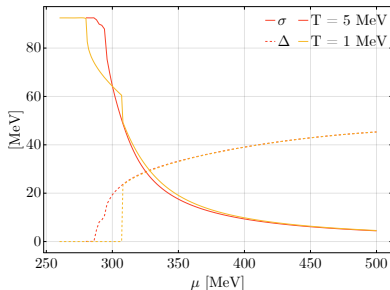
- Expected phase structure.
- Strange behavior at low T close to chiral transition.

Phase Diagram



phase diagram

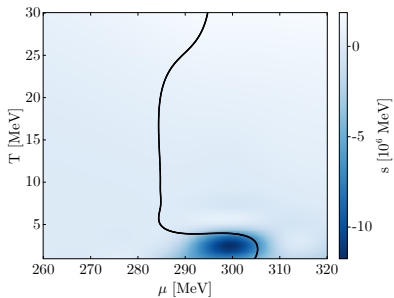
What's going on here?



shape of the condensates

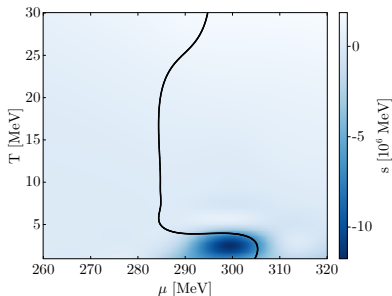
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Entropy Density

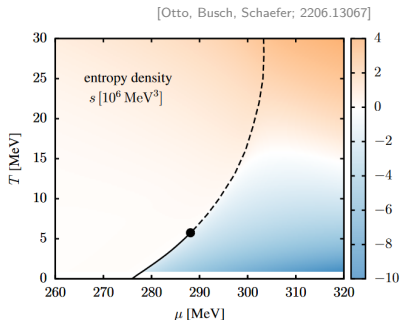


quark-meson-diquark model

Entropy Density



quark-meson-diquark model

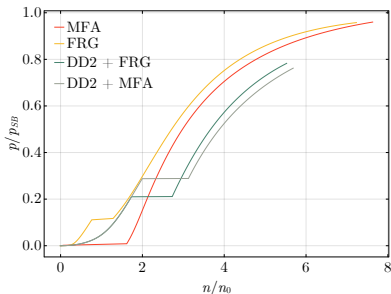


quark-meson model

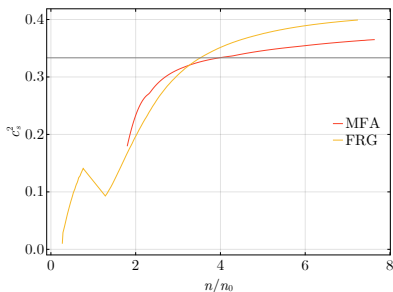
- Negative entropy is better with diquarks.
- What happens when diquark fluctuations are included?

EoS and Speed of Sounds

All at $T = 1$ MeV



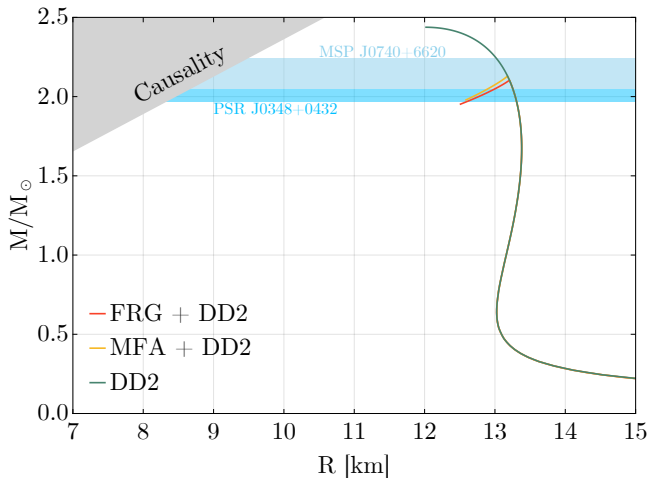
Equation of state $p(n)$.



Speed of sound c_s^2 .

- DD2: hadronic EoS.
- $c_s^2 > 1/3$: found in other diquark studies.

Mass-radius Relationship



- Superconducting core \rightarrow unstable with current diquark parameters.

Comment on Neutrality

More realistic EoS: impose **neutrality conditions**.

- Charge neutrality:

$$\frac{2}{3}n_u - \frac{1}{3}n_d - n_e = 0 .$$

- β -equilibrium:

$$u \leftrightarrow d + e^+ + \nu_e .$$

- **Color neutrality**:

$$n_r = n_g = n_b .$$

Mixing between color and flavor indices in 2SC phase: hard to impose neutrality.

Possibility of a 2SC phase in neutral matter?

Summary and Outlook

- Quark-meson-diquark model: model **chiral transition** and **2SC color superconducting phase**.
- FRG resolution faces two problems:
 - **Diquarks couple to μ** : cannot flow from symmetry restored phase to symmetry broken phase.
 - **Divergence at the Fermi-surface**: possible resolution with Fermi-surface regulator.
- **Regularization artifact** (negative entropy): better with diquarks at mean-field level.
- Possibility of 2SC phase under neutrality conditions?

Backup Slides

Full Flow Equation

$$\begin{aligned} \partial_t U_k = & \frac{k^5}{12\pi^2} \left\{ \frac{3}{\epsilon_\pi} \coth \frac{\epsilon_\pi}{2T} + \frac{2}{\epsilon_{\Delta,0}} \left[\coth \frac{\epsilon_{\Delta,0} - 2\mu}{2T} + \coth \frac{\epsilon_{\Delta,0} + 2\mu}{2T} \right] \right. \\ & + \sum_{i=1}^3 \frac{\alpha_2 z_i^4 - \alpha_1 z_i^2 + \alpha_0}{(z_i^2 - z_{i+1}^2)(z_i^2 - z_{i+2}^2)} \frac{1}{z_i} \coth \frac{z_i}{2T} \left. \right\} \\ & - \frac{k^5}{3\pi^2} \left\{ \frac{2}{\epsilon_k} \left[\frac{E_k^+}{E_\Delta^+} \tanh \frac{E_\Delta^+}{2T} + \frac{E_k^-}{E_\Delta^-} \tanh \frac{E_\Delta^-}{2T} \right] \right. \\ & \left. + \frac{1}{\epsilon_k} \left[\tanh \frac{\epsilon_k^+}{2T} + \tanh \frac{\epsilon_k^-}{2T} \right] \right\} \end{aligned}$$

with

$$\begin{aligned} \epsilon_k^\pm &= \sqrt{k^2 + g_\phi^2 \rho^2} \pm \mu = \epsilon_k \pm \mu & E_\pi &= \sqrt{k^2 + 2U_{k,\rho}} \\ E_\Delta^\pm &= \sqrt{(\epsilon_k \pm \mu)^2 + g_\Delta^2 d^2} & \epsilon_{\Delta,0} &= \sqrt{k^2 + 2U_{k,d}} \end{aligned}$$