Color superconductivity beyond Mean Field Approximation

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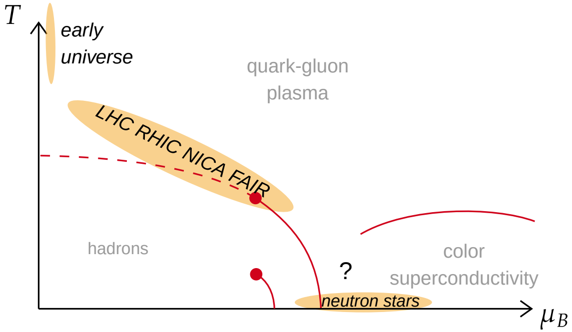
Outline

1. Color Superconductivity and Diquarks
2. Functional Renormalization Group Study
3. Phase Structure & Astrophysical Applications

Color Superconductivity and Diquarks

Why study color superconductivity?



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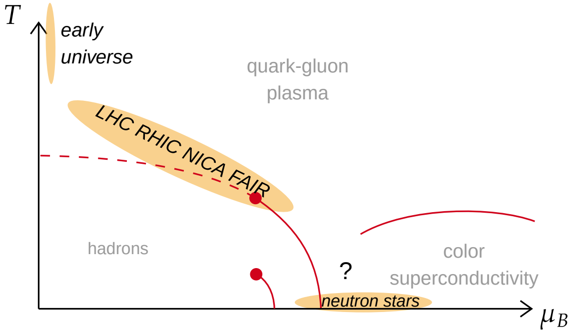

- Natural in QCD: attractive gluon exchange between quarks leads to superconductivity at high densities.
- Relevant for astrophysical observations.
- Hard to go beyond mean field.

Why study color superconductivity?

- $\langle \bar{q}q \rangle$ condensate: spontaneous chiral symmetry breaking, mesons, ...
- $\langle qq \rangle$ condensate: color superconductivity, diquarks, ...

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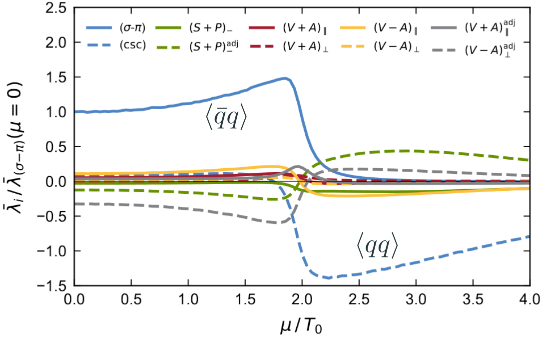

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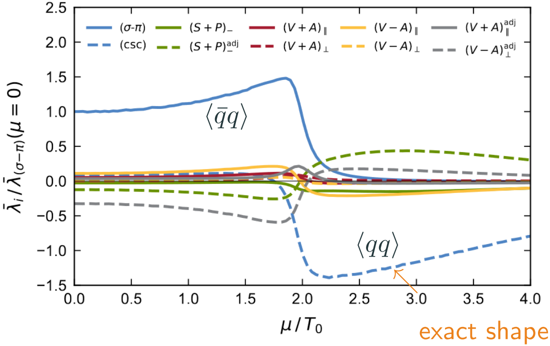


exact shape of the $\langle qq \rangle$ condensate?

2SC Phase

2SC phase: condensates of the shape

$$\Delta_A = \langle q^T C \gamma_5 \tau_2 \lambda_A q \rangle \ , \quad A = 2, 5, 7$$

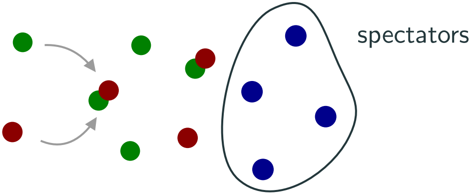
↖ some matrix structure in flavor, color and Dirac space

Main characteristics:

- Chiral symmetry is not broken.
- Color symmetry is "broken" (Higgs mechanism), but not fully:

$$SU_c(3) \rightarrow SU_c(2) \ .$$

- Only two color participate in superconductivity.



Quark-meson-diquark Model

Quark-meson-diquark model action:

$$
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S_{\text{QMD}} = \int_x \Big\{ & \bar{q} \left(\not{\partial} - \mu\gamma_0 + g_\phi \left(\sigma + i\gamma_5 \vec{\pi} \cdot \vec{\tau} \right) \right) q \\
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quark-meson
scalar-pseudoscalar
channel
$\swarrow$

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diquark kinetic term, couples to μ

meson and diquark interactions, arbitrary potential constrained by symmetries

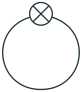
explicit chiral symmetry breaking term

Functional Renormalization Group Study

Functional Renormalization Group

- Define an effective action Γ_k such that $\Gamma_{k=\Lambda} = S_{\mathrm{QMD}}$ and $\Gamma_{k=0} = \Gamma$ by means of regulator R_k.
- The effective action follows an exact differential equation:

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \left[\left(\Gamma_k^{(2)} + R_k \right)^{-1} \partial_t R_k \right] = \frac{1}{2}$$



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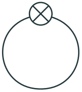

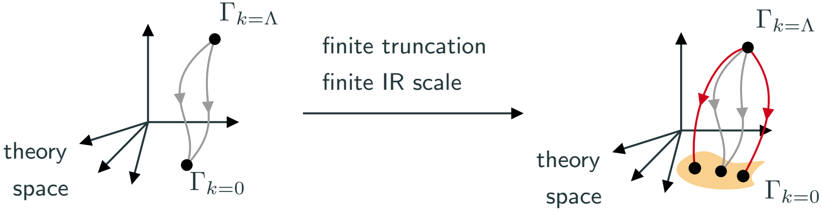

Truncation and Regulator Choice

Some FRG details used here:

- Local potential approximation (LPA): only scale dependent quantity is the potential $U_k(\sigma, \Delta)$.
  - σ: chiral condensate.
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- 3d Litim regulator

$$R_k = (k^2 - \vec{p}^2)\theta(k^2 - \vec{p}^2) \ .$$

Expect negative entropy at high μ and low T from studies of the quark-meson model $\to$ more on that later.

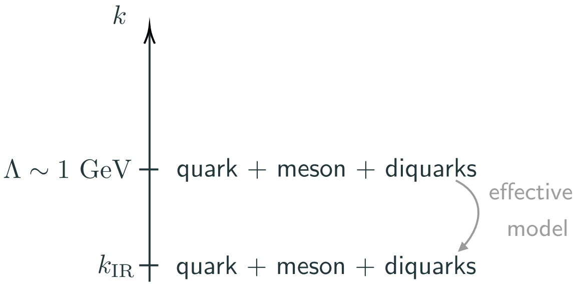
Parameter Fixing

Initial condition for the flow:

$$U_{k=\Lambda} = a_1\sigma^2 + a_2\sigma^4 + b_1\Delta^2 + b_2\Delta^4$$

with a_1, a_2, b_1 and b_2 free parameters.

- Mesonic parameters: fixed by matching vacuum observables.



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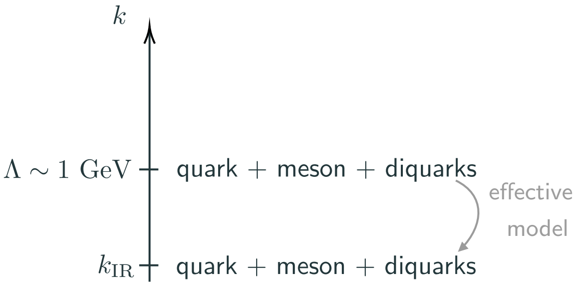

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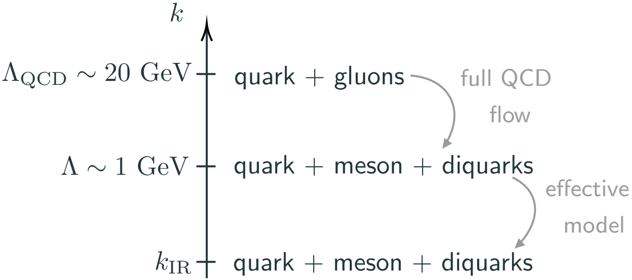
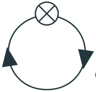
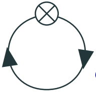
  - Use full QCD flow to initialize. [Braun, Schallmo; 2106.04198]



Flow Equation

$\partial_t U_k(\sigma, \Delta) = -$  q_r, q_g $-$  q_b $+ \frac{1}{2}$  Δ_2, σ

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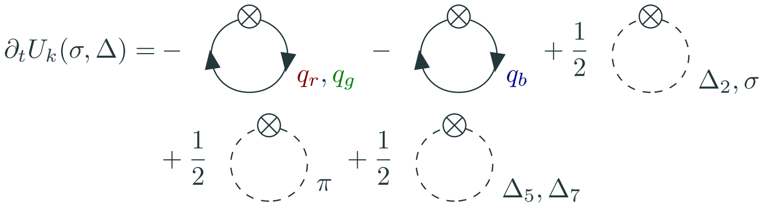
depends on $E_\Delta = \sqrt{(\epsilon_k \pm \mu)^2 + \Delta^2}$

with $\epsilon_k = \sqrt{k^2 + \sigma^2}$

$$\downarrow$$



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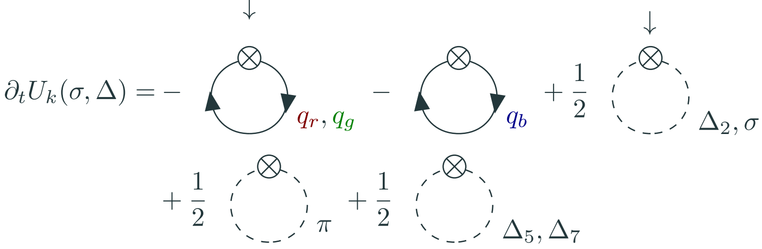
coupling between one diquark and the sigma meson



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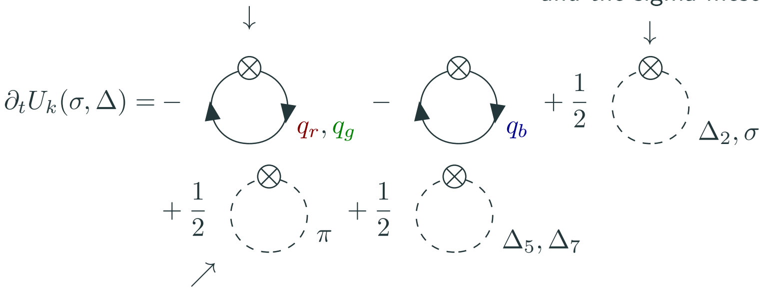

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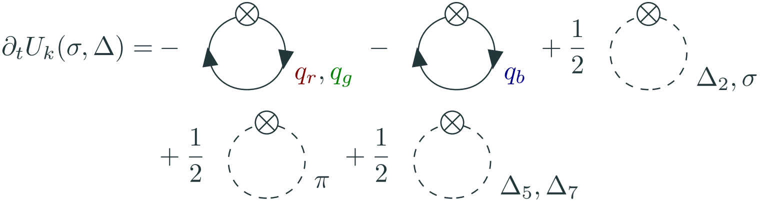

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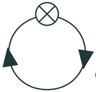
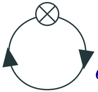
$\partial_t U_k(\sigma, \Delta) = -$  q_r, q_g $-$  q_b $+ \frac{1}{2}$  Δ_2, σ

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problem with this part

First Trouble: Bosonic Diquark Loops

Pion loops

$$\partial_t U_k = \frac{1}{\epsilon_\pi} \coth \frac{\epsilon_\pi}{2T} + \dots$$

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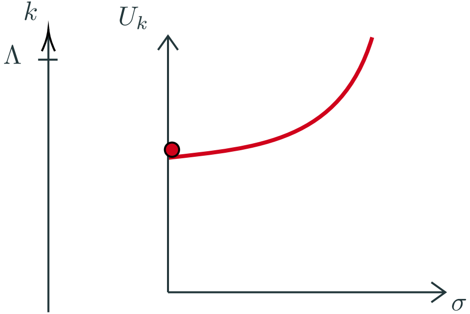

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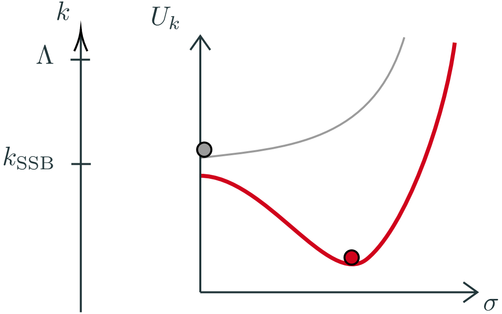

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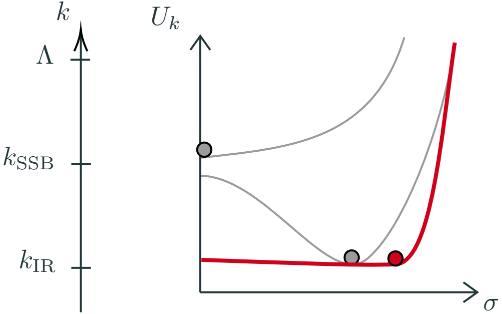

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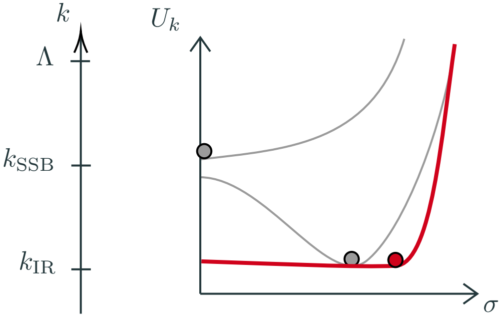

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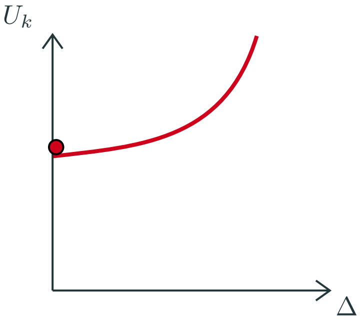

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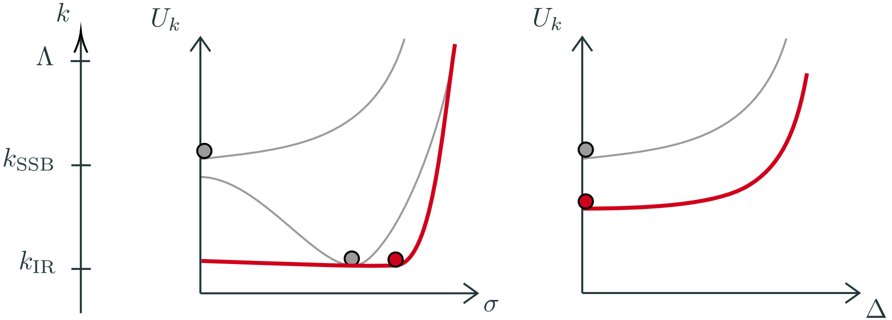

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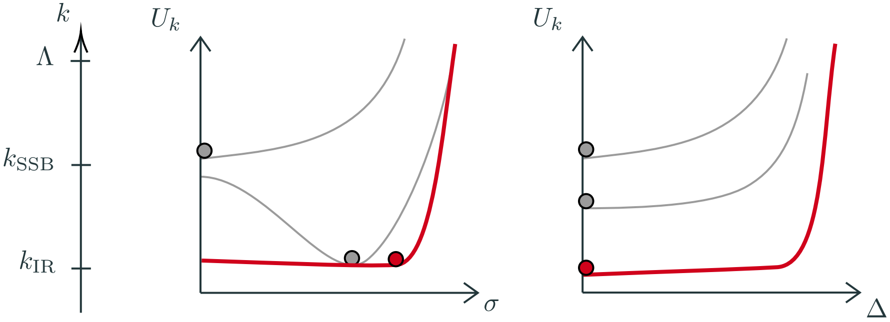
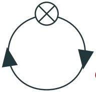


Second Trouble: Gapped Quark Loop

Gapped quark loop:

diverges at the Fermi-surface $\downarrow$

$-$  q_r, q_g

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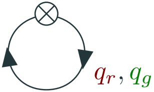
$-$ 

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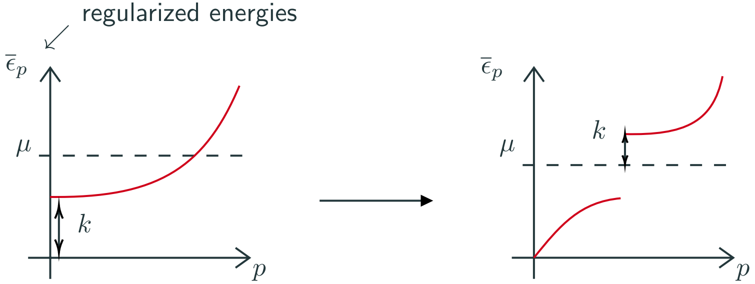
Possible solution: flow around the Fermi-surface [Braun, Dörnfeld, Schallmo, Töpfel; 2008.05978]



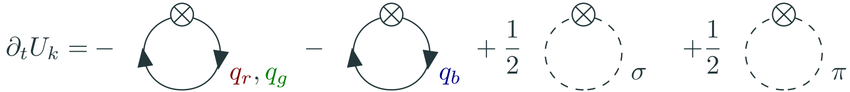
usual momentum regulator

Fermi-surface regulator

Phase Structure & Astrophysical Applications

Quark-meson-diquark Model without Diquark Loops

Diquarks only at the mean field level: none of the previously mentioned problems are present.



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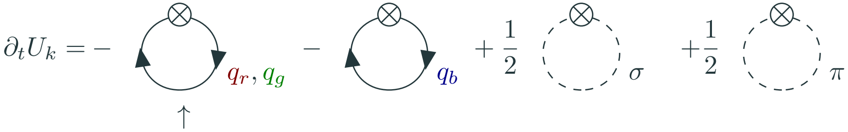

diquarks still present here

$E_{\Delta}^{\pm} = \sqrt{(\epsilon_k \pm \mu)^2 + \Delta^2}$

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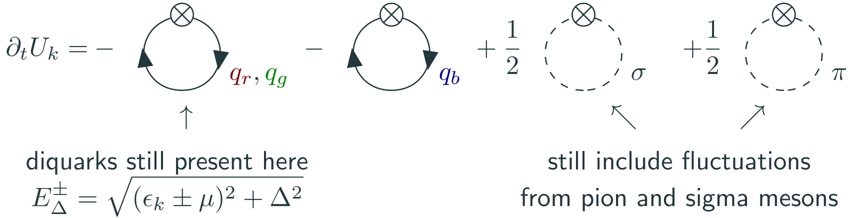

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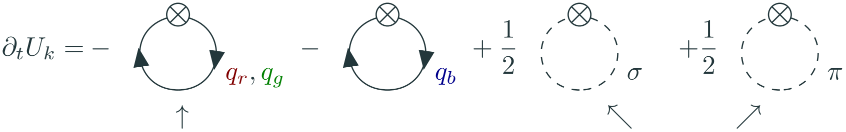


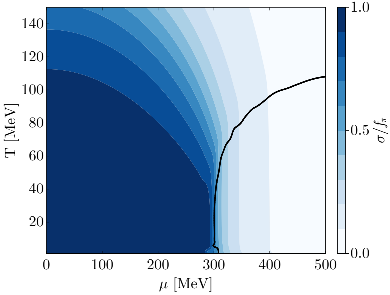
diquarks still present here

$$E_\Delta^\pm = \sqrt{(\epsilon_k \pm \mu)^2 + \Delta^2}$$

still include fluctuations from pion and sigma mesons

- Look at phase structure.
- First astrophysical applications.

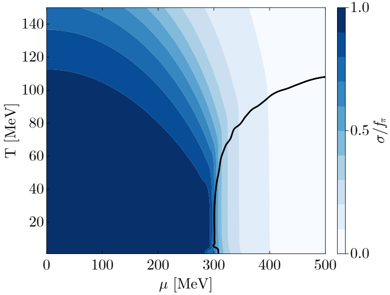
Phase Diagram



phase diagram

- Expected phase structure.

Phase Diagram



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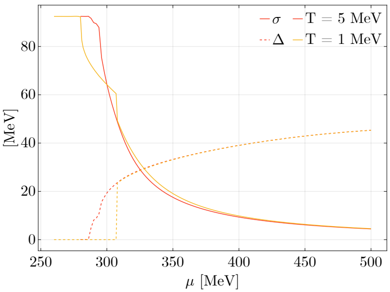
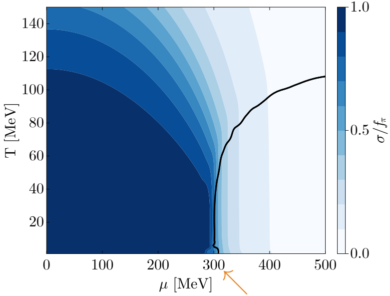
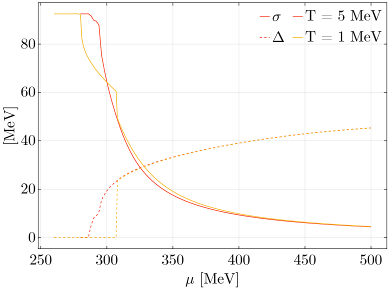


shape of the condensates

- Expected phase structure.
- Strange behavior at low T close to chiral transition.

Phase Diagram





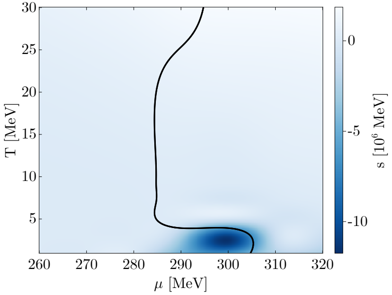
What's going on here?

phase diagram

shape of the condensates

- Expected phase structure.
- Strange behavior at low T close to chiral transition.

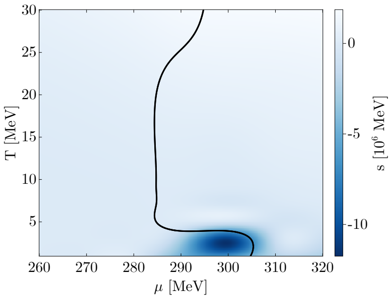
Entropy Density



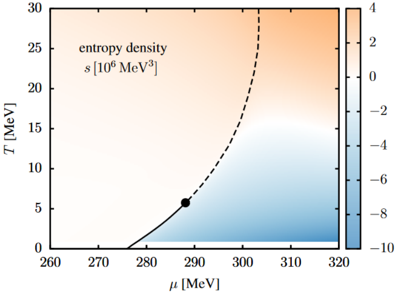
quark-meson-diquark model

Entropy Density

[Otto, Busch, Schaefer; 2206.13067]



quark-meson-diquark model

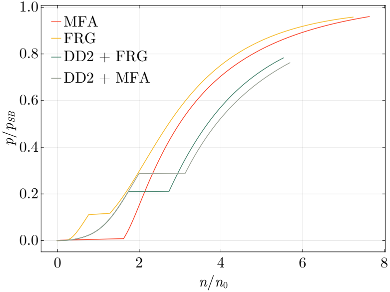


quark-meson model

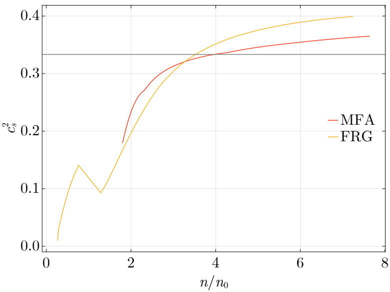
- Negative entropy is better with diquarks.
- What happens when diquark fluctuations are included?

EoS and Speed of Sounds

All at $\mathrm{T} = 1\ \mathrm{MeV}$



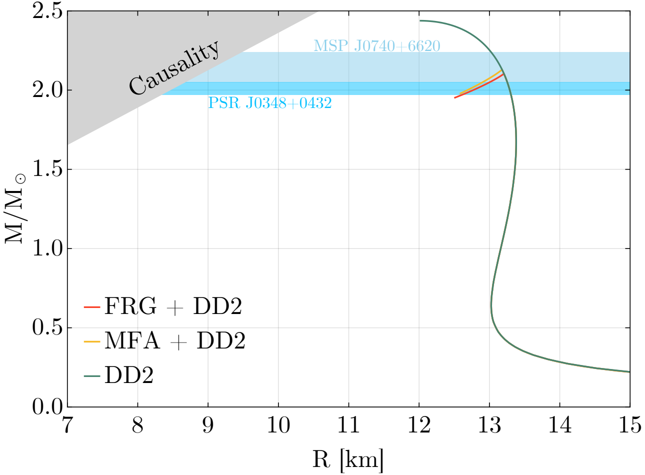
Equation of state $p(n)$.



Speed of sound c_s^2.

- DD2: hadronic EoS.
- $c_s^2 > 1/3$: found in other diquark studies.

Mass-radius Relationship



- Superconducting core → unstable with current diquark parameters.

Comment on Neutrality

More realistic EoS: impose neutrality conditions.

- Charge neutrality:

$$\frac{2}{3}n_u - \frac{1}{3}n_d - n_e = 0 \ .$$

- β-equilibrium:

$$u \leftrightarrow d + e^+ + \nu_e \ .$$

- Color neutrality:

$$n_r = n_g = n_b \ .$$

Mixing between color and flavor indices in 2SC phase: hard to impose neutrality.

Possibility of a 2SC phase in neutral matter?

Summary and Outlook

- Quark-meson-diquark model: model chiral transition and 2SC color superconducting phase.
- FRG resolution faces two problems:
  - Diquarks couple to μ: cannot flow from symmetry restored phase to symmetry broken phase.
  - Divergence at the Fermi-surface: possible resolution with Fermi-surface regulator.
- Regularization artifact (negative entropy): better with diquarks at mean-field level.
- Possibility of 2SC phase under neutrality conditions?

Backup Slides

Full Flow Equation

$$\partial_t U_k = \frac{k^5}{12\pi^2}\Bigg\{ \frac{3}{\epsilon_\pi}\coth\frac{\epsilon_\pi}{2T} + \frac{2}{\epsilon_{\Delta,0}}\left[\coth\frac{\epsilon_{\Delta,0}-2\mu}{2T} + \coth\frac{\epsilon_{\Delta,0}+2\mu}{2T}\right]$$

$$+ \sum_{i=1}^{3}\frac{\alpha_2 z_i^4 - \alpha_1 z_i^2 + \alpha_0}{(z_i^2 - z_{i+1}^2)(z_i^2 - z_{i+2}^2)}\frac{1}{z_i}\coth\frac{z_i}{2T}\Bigg\}$$

$$- \frac{k^5}{3\pi^2}\Bigg\{\frac{2}{\epsilon_k}\left[\frac{E_k^+}{E_\Delta^+}\tanh\frac{E_\Delta^+}{2T} + \frac{E_k^-}{E_\Delta^-}\tanh\frac{E_\Delta^-}{2T}\right]$$

$$+ \frac{1}{\epsilon_k}\left[\tanh\frac{\epsilon_k^+}{2T} + \tanh\frac{\epsilon_k^-}{2T}\right]\Bigg\}$$

with

$$\epsilon_k^\pm = \sqrt{k^2 + g_\phi^2\rho^2} \pm \mu = \epsilon_k \pm \mu \qquad E_\pi = \sqrt{k^2 + 2U_{k,\rho}}$$

$$E_\Delta^\pm = \sqrt{\left(\epsilon_k \pm \mu\right)^2 + g_\Delta^2 d^2} \qquad \epsilon_{\Delta,0} = \sqrt{k^2 + 2U_{k,d}}$$