QCD phase structure from a minimal truncated Dyson-Schwinger equations approach

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Outline

The talk is based on: **YL**, F. Gao, Y.-x. Liu and J. Pawlowski (in preparation)

Quark DSE

$$\left[S^f(p)\right]^{-1} = Z_2^f \left[S_0^f(p)\right]^{-1} + \Sigma^f(p),$$

$$\Sigma^f(p) = \frac{4}{3} Z_{1F}^f \, g^2 \sum\!\!\!\!\!\!\int_q \gamma_\mu S^f(q) \Gamma_\nu^f(k;q,p) D_{\mu\nu}(k),$$

$\sum\!\!\!\!\!\!\int_q$ stands for $\int \frac{\mathrm{d}^4 q}{(2\pi)^4}$ in the vacuum and $T \sum_n \int \frac{\mathrm{d}^3 \mathbf{q}}{(2\pi)^3}$ at finite T.

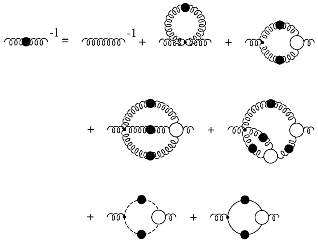
The full and bare quark propagators in the vacuum:

$$S^{-1}(p) = i\gamma \cdot p \, A(p^2) + B(p^2),$$

$$\left[S_0^f(p)\right]^{-1} = i\gamma \cdot p + m_f .$$

Gluon DSE in full QCD

Exact form (pictorial representation) [Alkofer and von Smekal 2001]:

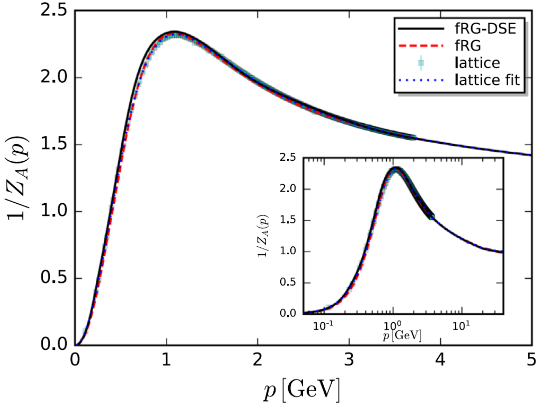


Gluon DSE from the back-coupling of quarks [Fischer 2013,2014][Isserstedt 2019,2021]:

$$[D_{\mu\nu}(p)]^{-1} = \left[D^{\rm YM}_{\mu\nu}(p)\right]^{-1} - \frac{1}{2}\sum_{f}^{N_f} Z^f_{1F}\, g^2 \sum\!\!\!\!\!\!\int_q \mathrm{Tr}\left[\gamma_\mu S^f(q)\Gamma^f_\nu(p;q,k)S^f(k)\right],$$

and also the baryonic/mesonic back-coupling [Eichmann 2016][Gunkel 2020].

Our starting point: the full gluon propagator in the vacuum: consistent results obtained from lattice and functional approaches. $N_f = 2 + 1$ [Blum 2016][Zafeiropoulos 2019][Fu 2019][Gao 2020,2021].
Parameterized formula of the $2 + 1$ flavor gluon [Gao 2021]:



Subtracted gluon DSE [Gao and Pawlowski 2020]:
finite $(T, \boldsymbol{\mu} = \{\mu_u, \mu_d, \mu_s\})$ - vacuum:

$$D^{-1}_{\mu\nu}(p)|_{T,\boldsymbol{\mu}} - D^{-1}_{\mu\nu}(p)|_{0,\mathbf{0}} = \Delta\Pi^{\mathrm{gauge}}_{\mu\nu}(p) + \Delta\Pi^{\mathrm{qrk}}_{\mu\nu}(p),$$

gauge part: (approx.) evaluated from the Yang-Mills data [Eichmann 2016]:

$$\Delta\Pi^{\mathrm{gauge}}_{\mu\nu}(p) = \left[D^{\mathrm{YM}}_{\mu\nu}(p)\right]^{-1}\Big|_T - \left[D^{\mathrm{YM}}_{\mu\nu}(p)\right]^{-1}\Big|_0 .$$

quark part:

$$\Delta\Pi^{\mathrm{qrk}}_{\mu\nu}(p) = \sum_f \left[\Pi^f_{\mu\nu}(p)|_{T,\mu_f} - \Pi^f_{\mu\nu}(p)|_{0,0}\right],$$

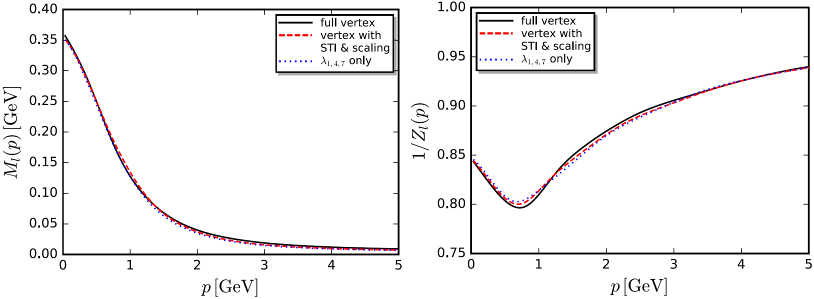
$$\Pi^f_{\mu\nu}(p) = -\frac{1}{2} Z^f_{1F}\, g^2 \sum\!\!\!\!\!\!\!\int_q \mathrm{Tr}\left[\gamma_\mu S^f(q)\Gamma^f_\nu(p;q,k)S^f(k)\right].$$

Dominant structures of the vertex

Calculation for the full vertex structures is now available [Cyrol 2018]; 3 dominant structures was found [Gao 2021]:

$$\Gamma_\mu^{\mathcal{T}_1} = \lambda_1\, \gamma_\mu^T, \qquad \Gamma_\mu^{\mathcal{T}_4} = \lambda_4\, \sigma_{\mu\nu} k_\nu, \qquad \Gamma_\mu^{\mathcal{T}_7} = \lambda_7\, \sigma_{\nu\rho} p_\nu q_\rho \gamma_\mu^T,$$

with $\gamma_\mu^T = \gamma_\mu - \frac{k_\mu \not{k}}{k^2}$.

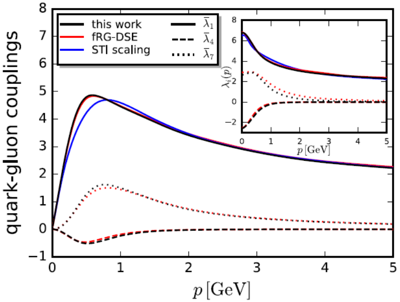


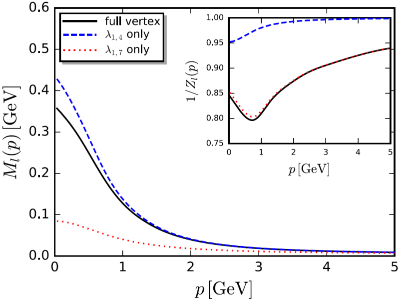
A closer look:

⋆ λ_1 relates to the Slavnov-Taylor Identity (STI);

⋆ λ_4 contributes most to the quark dynamical mass;

⋆ λ_7 contributes most to $A(p) = 1/Z_q(p)$.





Constructing the minimal vertex I: vacuum

$$\Gamma_\mu(k;q,p) = \Gamma_\mu^{\mathrm{STI}}(k;q,p) + \lambda_4(q,p)\,\sigma_{\mu\nu}k_\nu.$$

⋆ λ_1 from an STI-based part: (truncated to 2-point functions)

$$\Gamma_\mu^{\mathrm{STI}}(k;q,p) = F(k^2)\,\Gamma_\mu^{\mathrm{BC\text{-}1}}(q,p),$$
$$\Gamma_\mu^{\mathrm{BC\text{-}1}}(q,p) = \gamma_\mu \Sigma_A(q,p).$$

⋆ Reliable Ansatz for λ_4 [Skullerud 2003,2005][Chang 2011,2012][Gao 2020]:

$$\lambda_4(k;q,p) = \eta\, F(k^2)\,\Delta_B(q,p), \quad \Delta_B(\bar{p}) \sim B'(\bar{p}^2) \equiv \left.\frac{\mathrm{d}B(p^2)}{\mathrm{d}p^2}\right|_{p^2=\bar{p}^2}.$$

Derivative form for Δ_B (will be helpful at finite temperature):

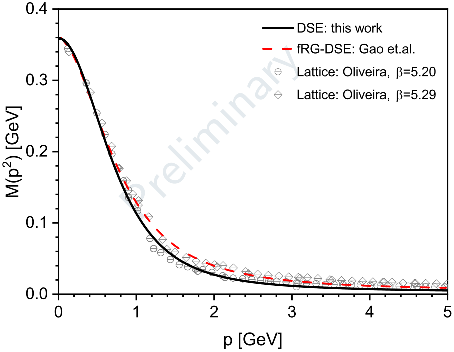
$$\Delta_B(q,p) = B'\left(\frac{q^2+p^2}{2}\right).$$

Constructing the minimal vertex I: vacuum

Ghost dressing function takes input from fRG data [Cyrol 2018];
Parameter η and the light quark mass m_l fitted from the vacuum f_π (Pagels-Stokar) and m_π (GMOR):

ζ	α_s	m_l	η
12.0 GeV	0.18	2.9 MeV	2.58

$f_\pi = 90.7\,\text{MeV},\ m_\pi = 138.8\,\text{MeV}.$
$m_s = 80\,\text{MeV}\ (\approx 27.5 m_l);$
$\Delta_{l,s} = \langle\bar{q}q\rangle_l - \frac{m_l}{m_s}\langle\bar{q}q\rangle_s = -(245.8\,\text{MeV})^3.$



Constructing the minimal vertex II: finite T

$$S^{-1}(\tilde{p}) = i\gamma_4\tilde{\omega}_p\, C(\tilde{p}) + i\gamma\cdot\mathbf{p}\, A(\tilde{p}) + B(\tilde{p}),$$
$$p^2\, D_{\mu\nu}(p) = P^E_{\mu\nu}(p)\, Z_E(p) + P^M_{\mu\nu}(p)\, Z_M(p),$$
$$\text{with } \tilde{p} = (\mathbf{p}, \tilde{\omega}_p = \omega_p + i\mu).$$

Modifications of the vertex:

$$\Gamma^{\text{BC-1}}_\mu(q,p) = \gamma_\mu\left[\delta_{\mu 4}\,\frac{C(\tilde{q}^2)+C(\tilde{p}^2)}{2} + \delta_{\mu i}\,\frac{A(\tilde{q}^2)+A(\tilde{p}^2)}{2}\right],$$

$$\lambda_4(q,p) = \eta\,\tilde{F}(k^2)\,B'\left(\frac{\tilde{q}^2+\tilde{p}^2}{2}\right),$$
$$\tilde{F}(k^2) = \left[\frac{Z(k)|_{T,0}}{Z(k)|_{T,\mu_B}}\right]^{1/2} F(k^2)\Big|_{0,0}.$$

- ⋆ Same vertex strength η as in the vacuum.
- ⋆ Mild T dependence for ghost propagator;
- ⋆ μ_B correction from RG scaling is found to be crucial; $Z = Z_E \approx Z_M$ in $\tilde{F}$.

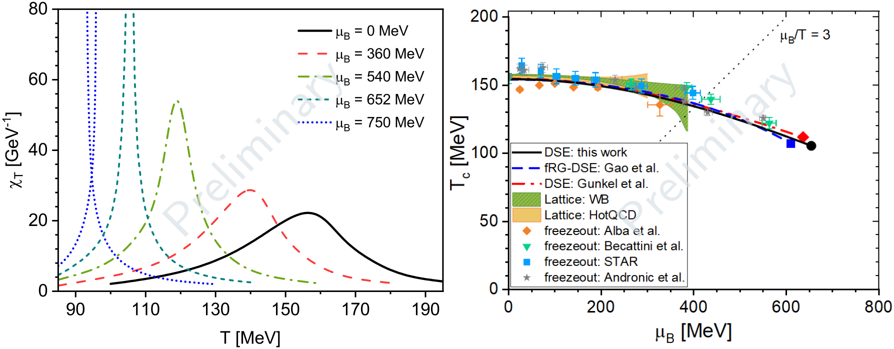
Chiral phase transition

$T_c(\mu_B)$ defined at the peak of temperature susceptibility ($\mu_{u,d} = \mu_B/3, \mu_s = 0$):

$$\chi_T(T,\mu_B) = \partial_T \left(\frac{\Delta_{l,s}(T,\mu_B)}{\Delta_{l,s}(0,0)} \right), \quad \Delta_{l,s} = \langle \bar{q}q \rangle_l - \frac{m_l}{m_s} \langle \bar{q}q \rangle_s .$$

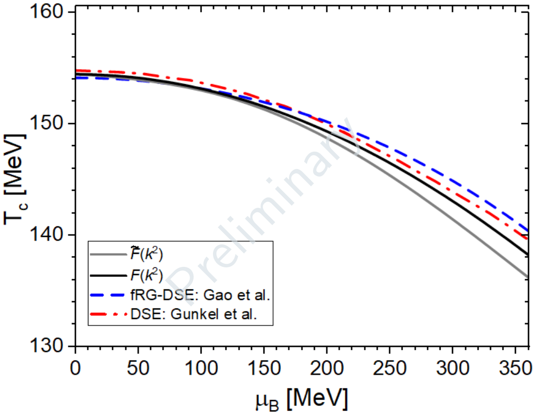
$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c(0)} \right)^2 + \lambda \left(\frac{\mu_B}{T_c(0)} \right)^4 + \cdots$$

$T_c(0) = 154.5$ MeV, $\kappa = 0.0182$; $\quad (T^E, \mu_B^E) = (105.5, 652)$ MeV.



$\tilde{F}$ vs. F

* (T, μ_B)-independent F gives $\kappa \approx 0.024$; state of the art $\kappa \approx 0.016$.



QCD equation of state within DSEs approach

From quark number density to QCD pressure:

$$n_q(T,\mu) = -N_c Z_2^f\, T \sum_n \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \mathrm{tr}_D \left[\gamma_4 S^f(p)\right],$$

$$P(T,\boldsymbol{\mu}) = P(T,\mathbf{0}) + \sum_{q=u,d,s} \int_0^{\mu_q} n_q(T,\mu)\, \mathrm{d}\mu,$$

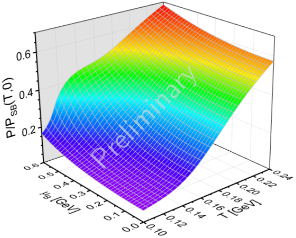
$$\boldsymbol{\mu} = (\mu_u, \mu_d, \mu_s).$$

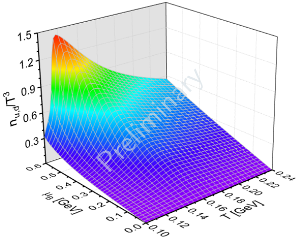
$P(T,0)$ takes input from the lattice QCD trace anomaly [Borsanyi 2010]:

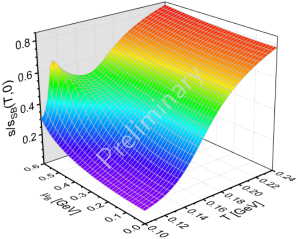
$$I(T) = (\epsilon - 3P)/T^4,$$

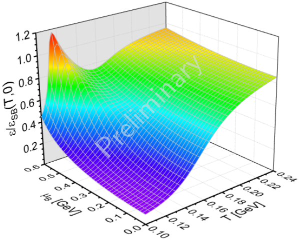
$$P(T,\mathbf{0})/T^4 = \int_0^T \mathrm{d}T'(I(T')/T').$$

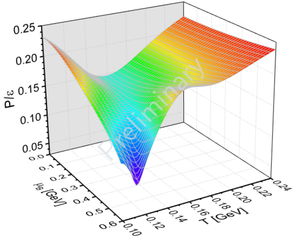
Thermodynamic functions:



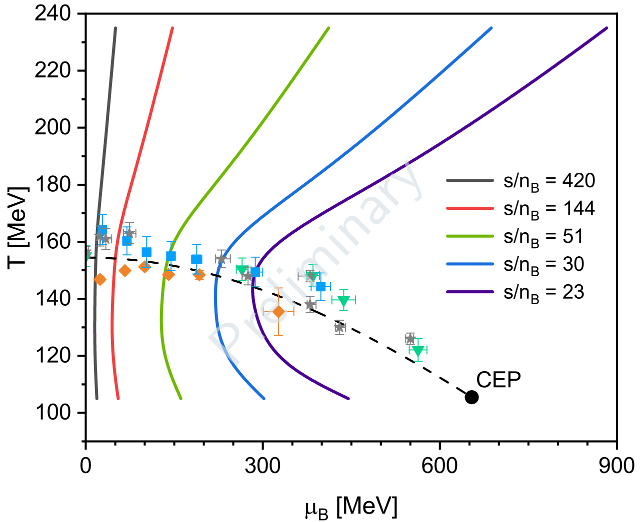








Isentropic trajectories:



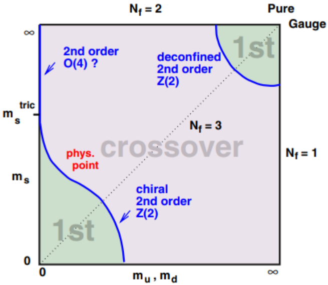
Columbia plot

For current quark masses $\mathbf{m} = \{m_u, m_d, m_s\}$:

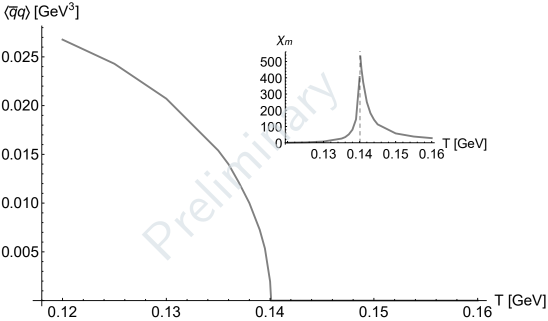
$$D^{-1}_{\mu\nu}(p)\Big|^{T,\mathbf{0},\mathbf{m}}_{0,\mathbf{0},\mathbf{m}_{\rm phy.}} = \Pi^{\rm gauge}_{\mu\nu}(p)\Big|^{T}_{0} + \Pi^{\rm qrk}_{\mu\nu}(p)\Big|^{T,\mathbf{0},\mathbf{m}}_{0,\mathbf{0},\mathbf{m}_{\rm phy.}},$$

$\mathbf{m} = \mathbf{0}$ yields $T_c = 140.1$ MeV.

verify a 2nd-order transition: $\chi_m \doteq \frac{\mathrm{d}B(\vec{0},\omega_0)}{\mathrm{d}m}$ [Qin 2011].



[de Forcrand and Philipsen, 2010]



Summary

Truncating the quark-gluon vertex

Minimal structure: STI + λ_4;
No free parameter: η is fixed in the vacuum.

Finite temperature and chemical potential

Chiral phase transition and the phase diagram.
Thermodynamic quantities and the isentropic trajectories.

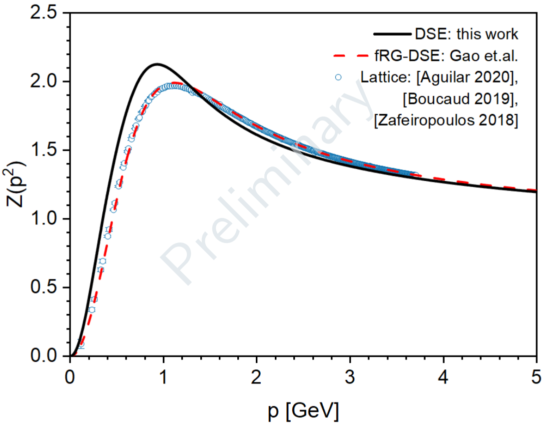
Further improvements and applications

Include λ_7 for completeness; Direct calculation on Π^{gauge}.
Polyakov loop for the hadronic phase.
Scan the Columbia plot.

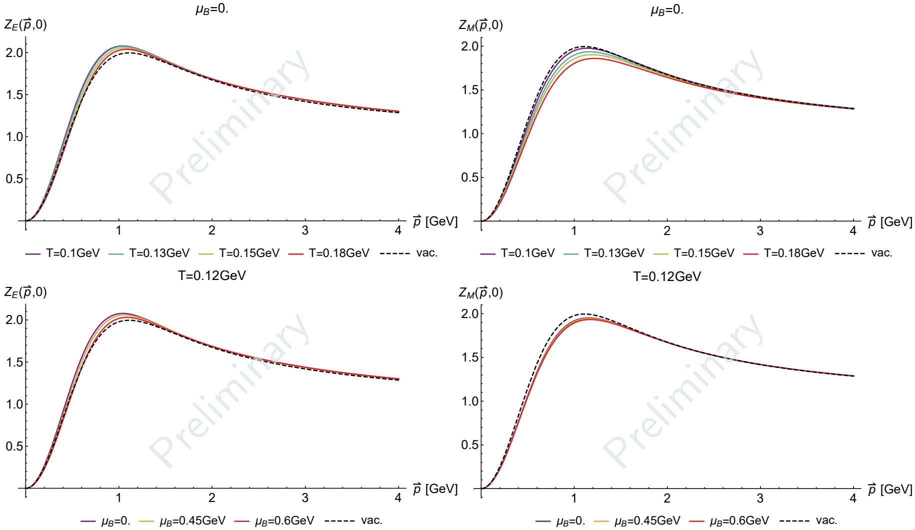
Thank you for your attention!!

Back-up

Gluon DSE from the quark back-coupling



Gluon propagator



Δ_B at finite T

$$\Delta_B = B'\left(\frac{\tilde{q}^2+\tilde{p}^2}{2}\right), \quad B'(\tilde{l}^2) \equiv \frac{\mathrm{d}B(\tilde{l}^2)}{\mathrm{d}\tilde{l}^2} = \frac{\partial B(\mathbf{l}^2, \tilde{\omega}_l^2)}{\partial \mathbf{l}^2}.$$

match of $\tilde{l}$ with the quark momenta:

$$\omega_l = \frac{\omega_q + \omega_p}{2}, \quad \mathbf{l}^2 = \frac{\mathbf{q}^2+\mathbf{p}^2}{2} + \frac{\omega_q^2+\omega_p^2}{2} - \omega_l^2.$$