Thermal photon production rate from Transverse-Longitudinal (T-L) mesonic correlator on the lattice

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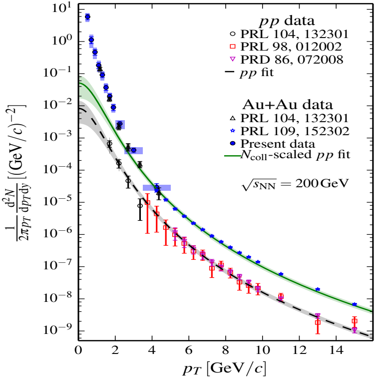
HotQCD Colloboration

Outline

- Photon and Di-lepton produced from QGP is an important probe to study Quark-Gluon-Plasma.
  - Direct photon measuremnts at RHIC and LHC clearly shows exceess of photon yield at low p_T region.
  - In addition it shows a large azimuthal anisotropy (Direct-photon puzzel).
  - Theoretical understanding requires space time intergarion of photon production rate from each stage of the plasma evoluation.

$$\frac{dN}{d^3k} \propto \int d^4X \frac{d\Gamma}{d^3k}(k, T(X), ..)$$



A. Adare et al, PRC 91,064904

J. Paquet et al, PRC 93, 044906

- The photon production rate (R_γ) and di-lepton production rate ($R_{l^+l^-}$) from a thermalized QGP can be calculated in-terms of spectral function
  L.D. McLerran and T. Toimela, Phys. Rev. D 31 (1985) 545.

$$\frac{d\Gamma_\gamma}{d^3\vec{k}} = \frac{\alpha_{em} n_b(\omega)}{2\pi^2 k} g^{\mu\nu} \rho_{\mu\nu}(\omega = |\vec{k}|, \vec{k})$$

$$\frac{d\Gamma_{l^+l^-}}{d\omega d^3\vec{k}} = \frac{\alpha_{em}^2 n_b(\omega)}{3\pi^2(\omega^2 - k^2)} g^{\mu\nu} \rho_{\mu\nu}(\omega, \vec{k})$$

- $\rho_{\mu\nu}$ is defined in terms of Electromagnetic current $J_\mu(X) = \bar{\psi}(X)\gamma_\mu\psi(X)$.

$$\rho_{\mu\nu}(K = (\omega, \vec{k})) = \int d^4X \exp\{iK.X\} \langle [J_\mu(X), J_\nu(0)] \rangle_T$$

- On the lattice we calculate the correlation function in Euclidean time.

$$G^E_{\mu\nu}(\tau,\vec{k}) = \int d^3\vec{x}\exp\left(i\vec{k}.\vec{x}\right)\langle J_\mu(\vec{x},\tau)J_\nu(\vec{0},0)\rangle$$

- Relation with spectral function,

$$G^E_{\mu\nu}(\tau,\vec{k}) = \int_0^\infty \frac{d\omega}{\pi}\rho_{\mu\nu}(\omega,\vec{k})\frac{\cosh[\omega(\tau-\frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

- Numerically unstable problem.
  1) Difference in the number of degrees of freedom.
  2) Small error in G^E become very large error in ρ.

- $\rho_{\mu\nu}$ can be decomposed,

$$\rho_{\mu\nu}(\omega, \vec{k}) = P^{T}_{\mu\nu}\rho_T(\omega, \vec{k}) + P^{L}_{\mu\nu}\rho_L(\omega, \vec{k})$$

$$\rho_V(\omega, \vec{k}) = \rho^{\mu}_{\mu}(\omega, \vec{k}) = 2\rho_T(\omega, \vec{k}) + \rho_L(\omega, \vec{k})$$

- At the photon point $\rho_L(|\vec{k}|, \vec{k}) = 0$.

$$\frac{d\Gamma_\gamma}{d^3\vec{k}} \propto 2\rho_T(|\vec{k}|, \vec{k})$$

$$\frac{d\Gamma_\gamma}{d^3\vec{k}} \propto 2\left(\rho_T(|\vec{k}|, \vec{k}) - \rho_L(|\vec{k}|, \vec{k})\right)$$

Properties of $T - L$ correlator

- At $T = 0$, $\rho_{\mu\nu} = (k_\mu \, k_\nu - g_{\mu\nu} k^2)\rho(k^2)$
  Possible when $\rho_T = \rho_L$ at $T = 0$
- $\rho_H = 2\,(\rho_T - \rho_L)$ displays pure thermal contribution.
- ρ_H is UV suppressed,

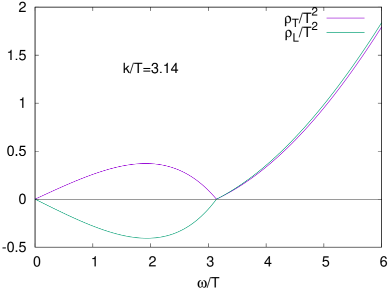
$$\rho_H \sim \frac{k^2\, O_4}{\omega^4}$$

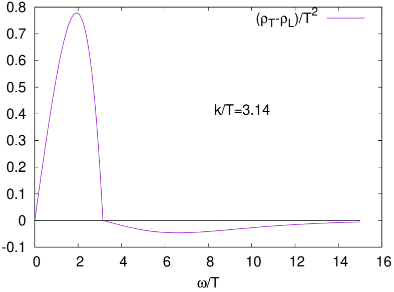
- Sum rule,

$$\int_0^\infty d\omega\, \omega\, \rho_H(\omega) = 0$$

M. Ce, T. Harris, H. B. Meyer, A. Steinberg, and A. Toniato , PRD 102, 091501

- Free result for ρ_T and ρ_L, G. Aarts and J.M. Resco,Nucl. Phys. B 726 (2005) 93



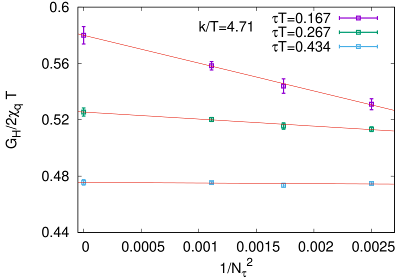


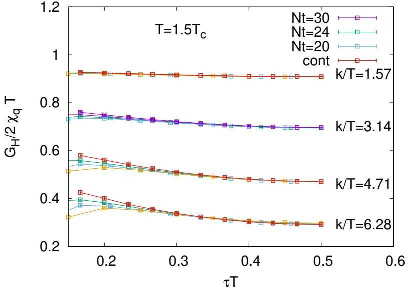
- $\rho_V = 2\rho_T + \rho_L$ has large UV part. G_V^E has large UV contribution.
- $\rho_H = 2(\rho_T - \rho_L)$ has small UV part. G_H^E has less UV contribution.

- We calculated $T - L$ correlator in pure gluonic theory at $T = 470 MeV$ and in $N_f = 2 + 1$ flavor QCD ($m_\pi = 320 MeV$) at $T = 220 MeV$.
- Lattice size:
  Gluonic theory: $120^3 \times 30, 96^3 \times 24$ and $80^3 \times 20$
  Full QCD: $96^3 \times 32$
- The available momentum for gluonic theory $\frac{k}{T} = \frac{\pi n}{2}$ and for full QCD $\frac{k}{T} = \frac{2\pi n}{3}$.
- We use clover improved Wilson fermion for the calculation of these correlation function.
- The quark masses $<<$ T.

$$G_H^E(\tau, \vec{k}) = \int_0^\infty \frac{d\omega}{\pi} 2(\rho_T(\omega, \vec{k}) - \rho_L(\omega, \vec{k})) \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

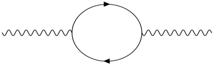
- Lattice data has cut-off effects and need continuum extrapolation.
- $$\frac{G_H}{2\chi_q T}(a) = \frac{G_H}{2\chi_q T}(a=0) + \frac{b}{N_\tau^2}$$





- Smaller cut-off dependence for G_H.
- Dominant contribution to G_H comes from the infrared part.

- $\rho_{\mu\nu}(\omega, \vec{k}) = \mathrm{Im}[\pi_{\mu\nu}(\omega, \vec{k})]$
- $M^2 \sim T^2$
  LO



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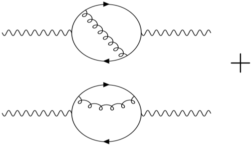
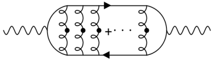
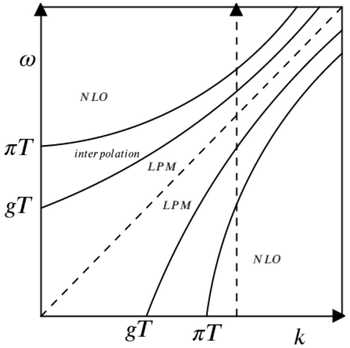
NLO (Compton Scattering +Pair Anhilation)



- $M^2 \ll T^2$
  LPM



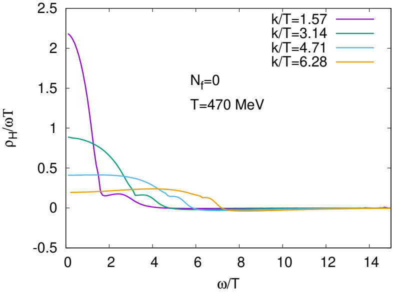


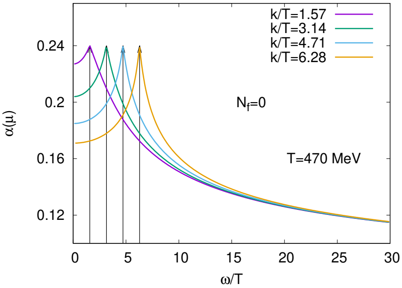
$\rho = \rho_{NLO} + \rho_{LPM} -$ common terms

G. Jackson and M. Laine, JHEP 11, (2019) 144

G. Jackson, PRD 100, 116019

- For LPM resummation LO light cone potential has been used.
- Renormalization scale $\mu = \sqrt{|\omega^2 - k^2| + (2\pi T\zeta)^2}$



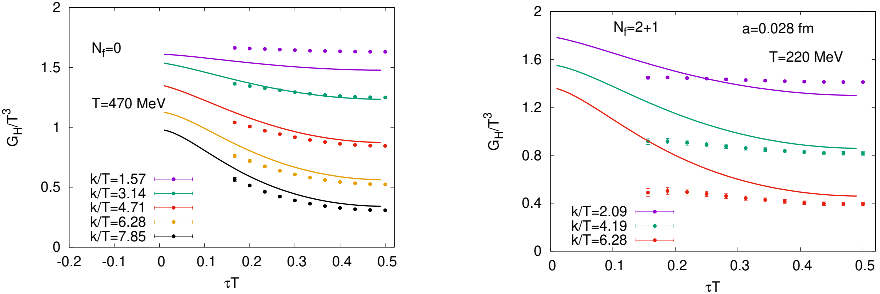


Coupling is maximum at the light cone.
S. Caron-Huot, PRD 79, 065039

- Scale setting:

$T_c/\Lambda_{\overline{MS}} = 1.24$ for $N_f = 0$
$T_c/\Lambda_{\overline{MS}} = 0.521$ for $N_f = 3$

- $\chi_q = 0.897\,T^2$ for $N_f = 0$ (using non-perturbative parametrization)
  H-T Ding, O. Kaczmarek, and F. Meyer, PRD 94, 034504
  $\chi_q = 0.872\,T^2$ for $N_f = 3$ (from $g^6 log(g)$)
  A. Vuorinen, PRD 67, 074032



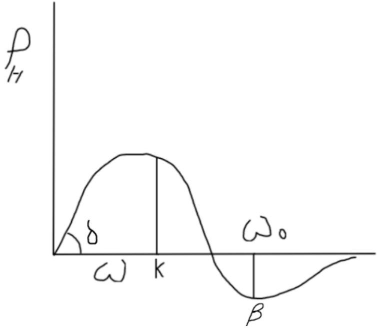
- Non-pertubative effects are important.

- For $\omega \leq \omega_0$

$$\rho_H(\omega) = \frac{\beta\omega^3}{2\omega_0^3}\left(5 - 3\frac{\omega^2}{\omega_0^2}\right) - \frac{\gamma\omega^3}{2\omega_0^2}\left(1 - \frac{\omega^2}{\omega_0^2}\right) + \delta_0\left(\frac{\omega}{\omega_0}\right)\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2$$

J. Ghiglieri, O. Kaczmarek, M. Laine, and F. Meyer, Phys. Rev. D 94, 016005.

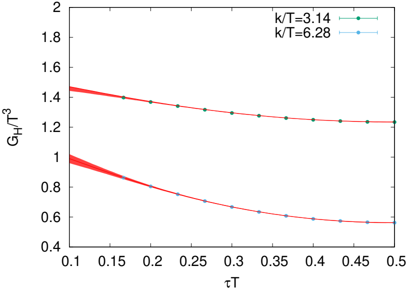
- $\beta = \rho_H(\omega_0)$ and $\gamma = \rho'_H(\omega_0)$
- For $\omega > \omega_0$ $\rho(\omega) = \sum_{i=0} \frac{A_i}{\omega^{4+2i}}$
- $\int_0^\infty d\omega\, \omega\, \rho_H(\omega, \beta, \gamma, \delta, A_i, \omega_0) = 0$
- $\omega_0 = \sqrt{k^2 + \nu(\pi T)^2}$

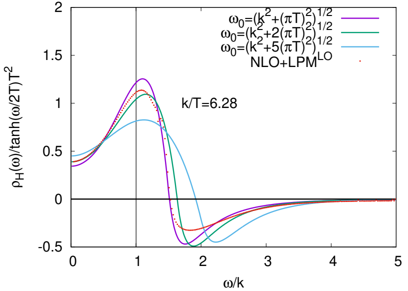


- Constrained fit with $\delta_0 \geq 0$,$\rho_H(k, \vec{k}) \geq 0$ and $\frac{\partial G_H}{\partial \tau} \leq 0$

Fitting of Mock Data

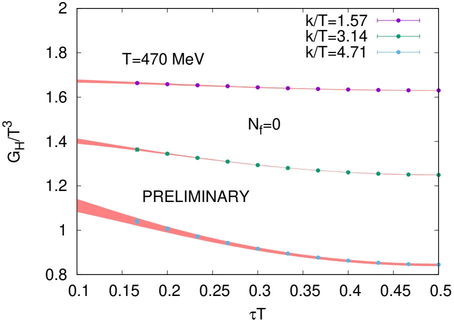
- Ten perturbative data points between 0.1875 to 0.5 in τT.
- Artificial error introduced to the order $\delta G/G = 0.001$ and trying to reconstruct the spectral function.

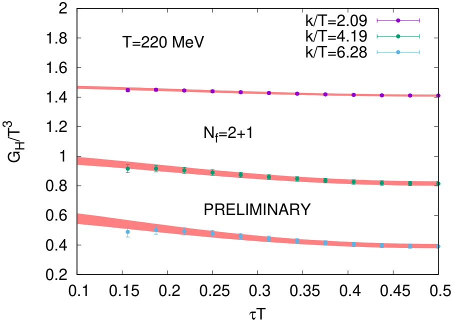




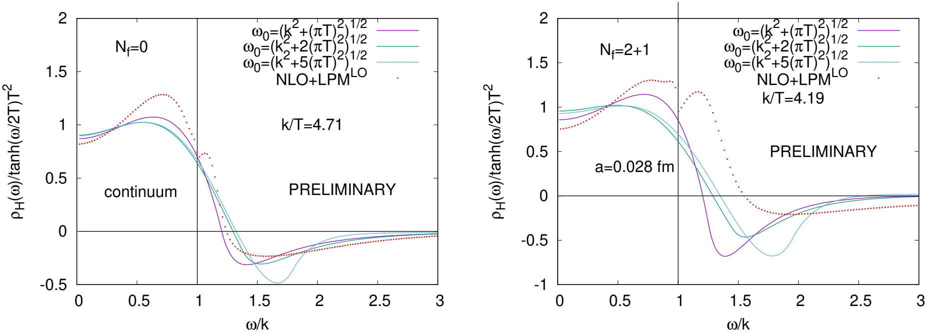
- The exact spectral function can be approximately caputured by the systematic uncertainty between $\omega_0 = \sqrt{k^2 + \pi^2 T^2}$ and $\omega_0 = \sqrt{k^2 + 5\pi^2 T^2}$

Fitting of Lattice Data





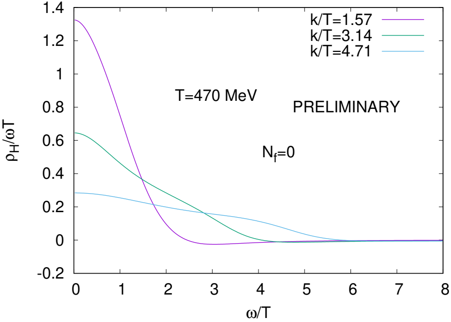
Polynomial estimated spectral function

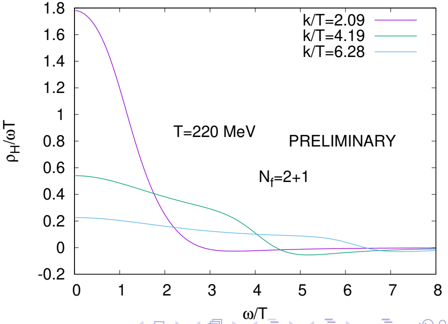


Pade Ansatz

- $$\rho_H^{PADE}(\omega,\vec{k}) = A\frac{\tanh(\omega/2T)\ (1+B\omega^2)}{(a^2+\omega^2)((\omega-\omega_0)^2+b^2)((\omega+\omega_0)^2+b^2)}$$

  M. Ce, T. Harris, H. B. Meyer, A. Steinberg, and A. Toniato , Phys. Rev. D 102, 091501(R)
- The sum rule relates B with a, ω_0 and b.
- The fit has been performed on A, a, ω_0 and b.





- Backus Gilbert estimate of the spectral function,

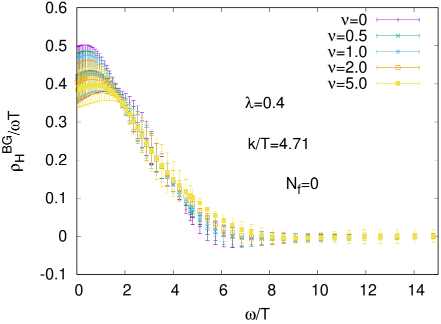
  G. Backus, F. Gilbert, Geophysical Journal of the Royal Astronomical Society 16, 169 (1968)

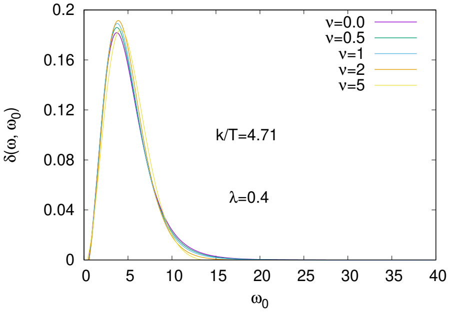
- $G_H(\tau) = \int_0^\infty \frac{d\omega}{\pi} \frac{\rho_H(\omega)}{f(\omega)} f(\omega) \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$
- $$\frac{\rho^{BG}(\omega)}{f(\omega)} = \sum_i q_i(\omega) G(\tau_i) = \int_0^\infty d\bar{\omega} \delta(\omega, \bar{\omega}) \frac{\rho(\bar{\omega})}{f(\bar{\omega})}.$$

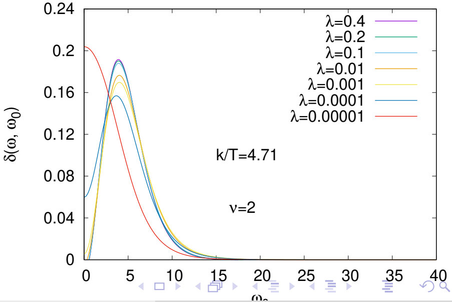
  $\delta(\omega, \bar{\omega}) = \sum_i q_i(\omega) K(\bar{\omega}, \tau_i) f(\bar{\omega})$.
- Minimize $F(\omega) = \lambda\, Width[\delta(\omega, \bar{\omega})] + (1 - \lambda)\, var[\rho_{BG}(\omega)]$

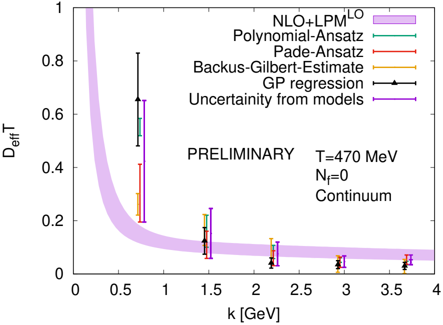
$$f(\omega) = \frac{\tanh(\omega/\omega_0)}{(\omega/\omega_0)^4}$$

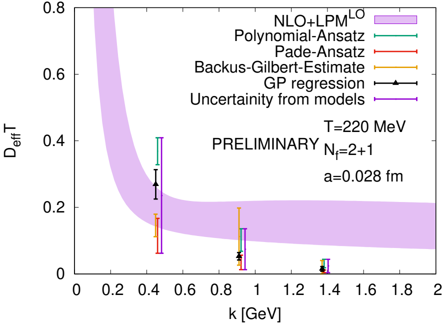
where, $\omega_0 = \sqrt{k^2 + \nu \pi^2 T^2}$.


ν=0
ν=0.5
ν=1.0
ν=2.0
ν=5.0
λ=0.4
k/T=4.71
N_f=0
ρ_H^BG/ωT
ω/T


ν=0.0
ν=0.5
ν=1
ν=2
ν=5
k/T=4.71
λ=0.4
δ(ω, ω_0)
ω_0


λ=0.4
λ=0.2
λ=0.1
λ=0.01
λ=0.001
λ=0.0001
λ=0.00001
k/T=4.71
ν=2
δ(ω, ω_0)





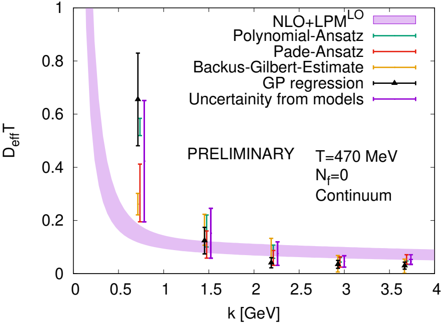
- Photon production rate,

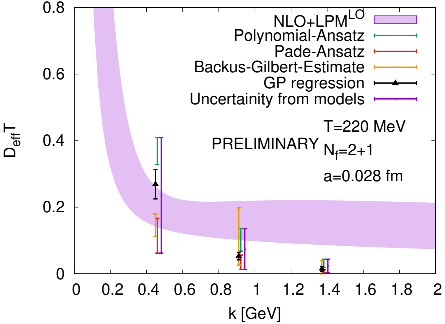
$$\frac{d\Gamma_\gamma}{d^3\vec{k}} = \frac{\alpha_{em} n_b(\omega)\chi_q}{\pi^2} Q_i^2 D_{eff}(k)$$

- Effective diffusion coefficient,

$$D_{eff}(k) = \frac{\rho_H(|\vec{k}|,\vec{k})}{2\chi_q|\vec{k}|}$$

$$\lim_{k\to 0} D_{eff}(k) = D$$





- We calculated T-L correlator in Quenched and Full QCD.
- We obtained photon production rate using 4-different methods.
- We use OPE information, at large ω and sum rules to constrain the spectral reconstruction.