CORRELATIONS IN A MOAT REGIME

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Fabian Rennecke

CRC-TR 211 Strong-interaction matter under extreme conditions

HFHF

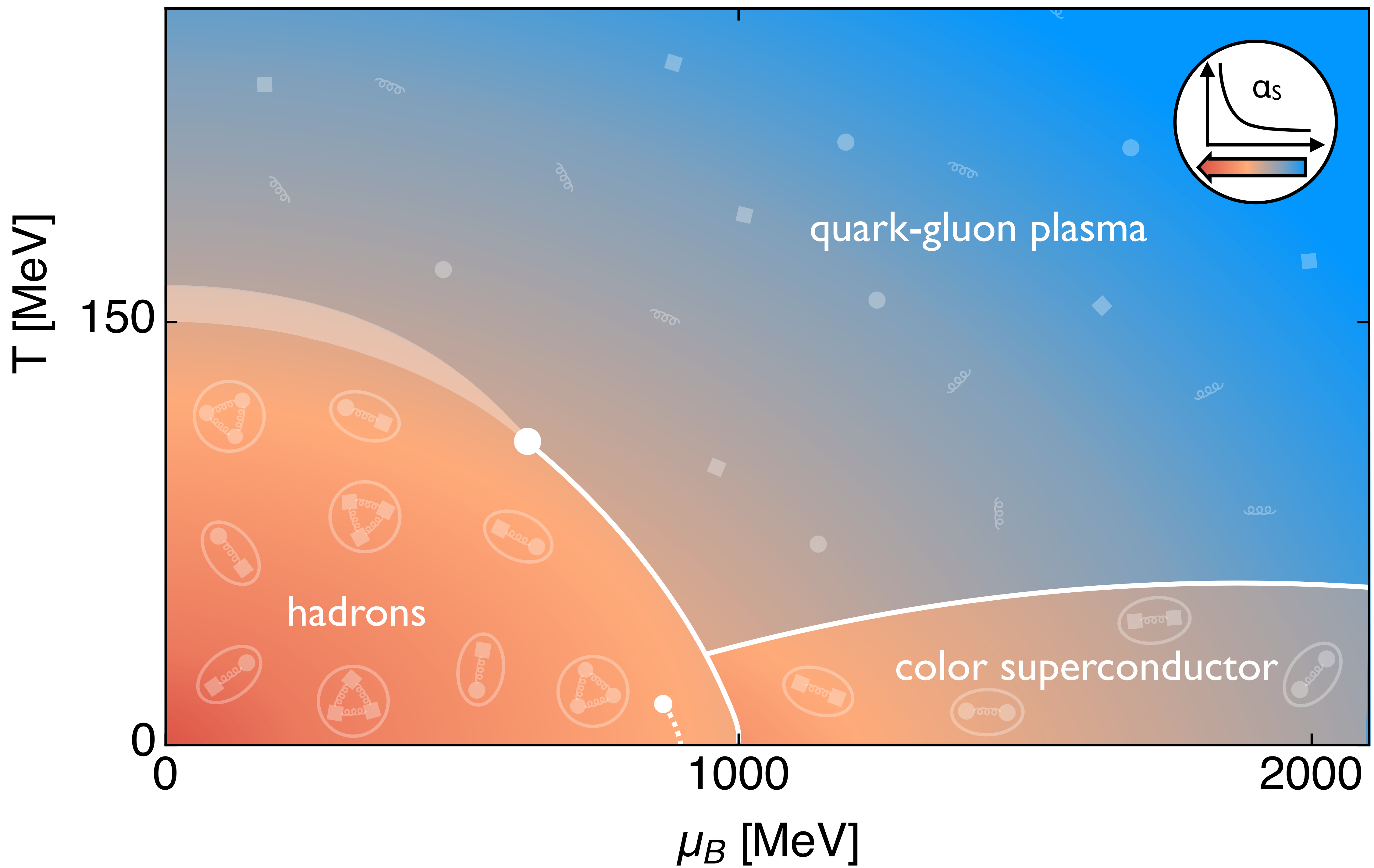
[FR, Pisarski, Rischke, arXiv:2301.11484 (2023)]

FROM FIRST-PRINCIPLES QCD TO EXPERIMENTS

ECT* TRENTO - 22/05/2023

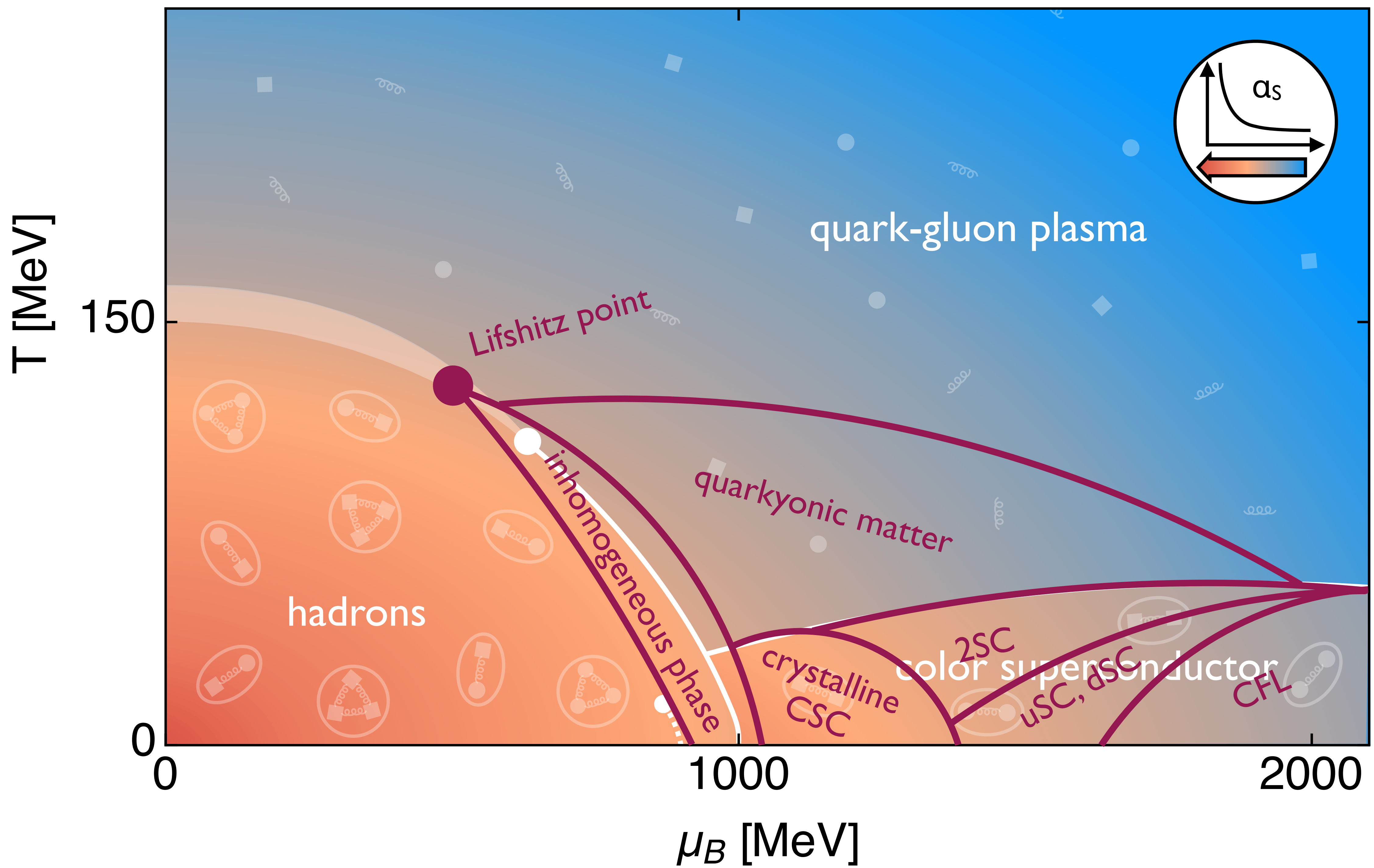
QCD PHASE DIAGRAM

in theory:



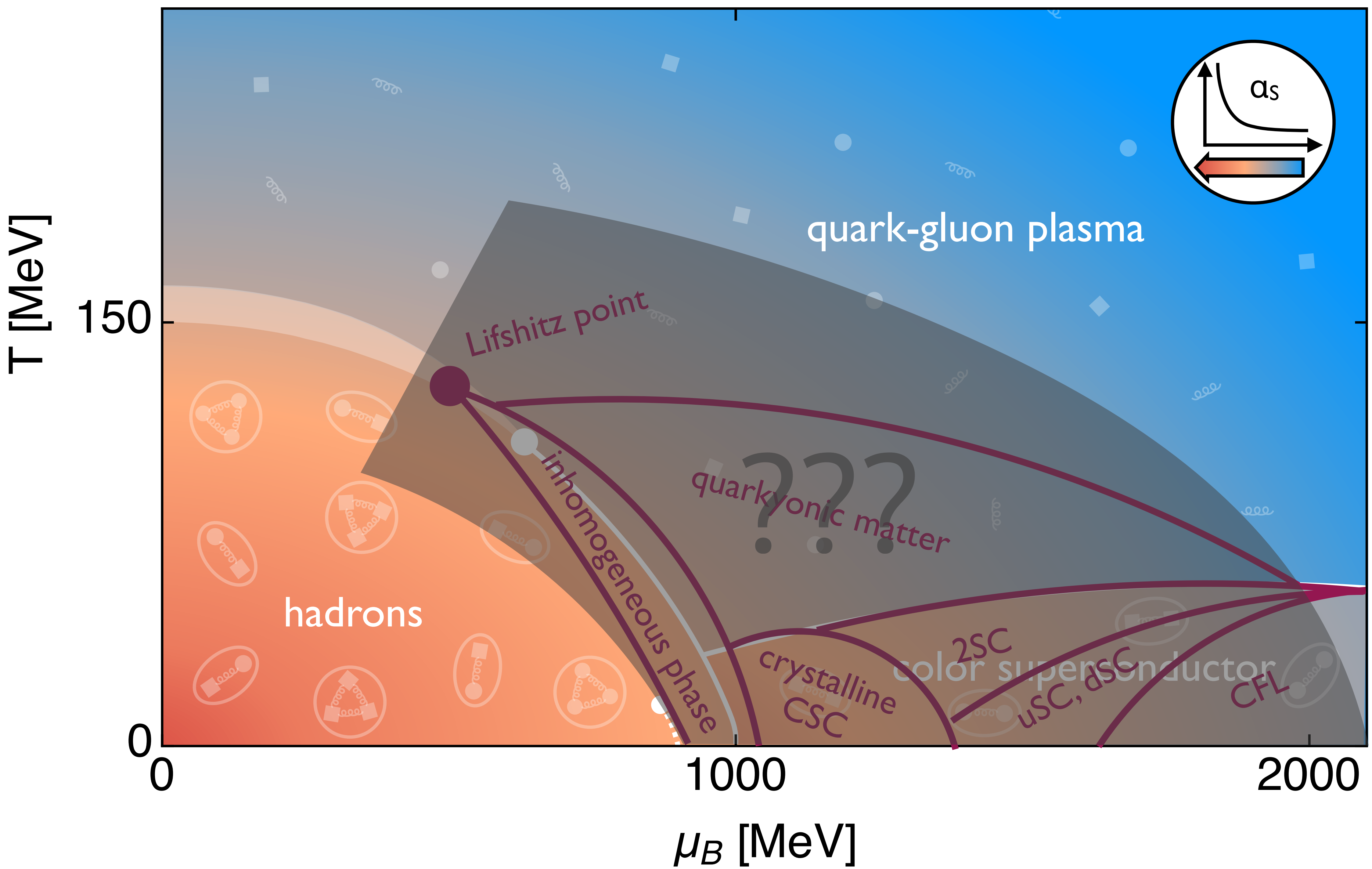
QCD PHASE DIAGRAM

in theory:



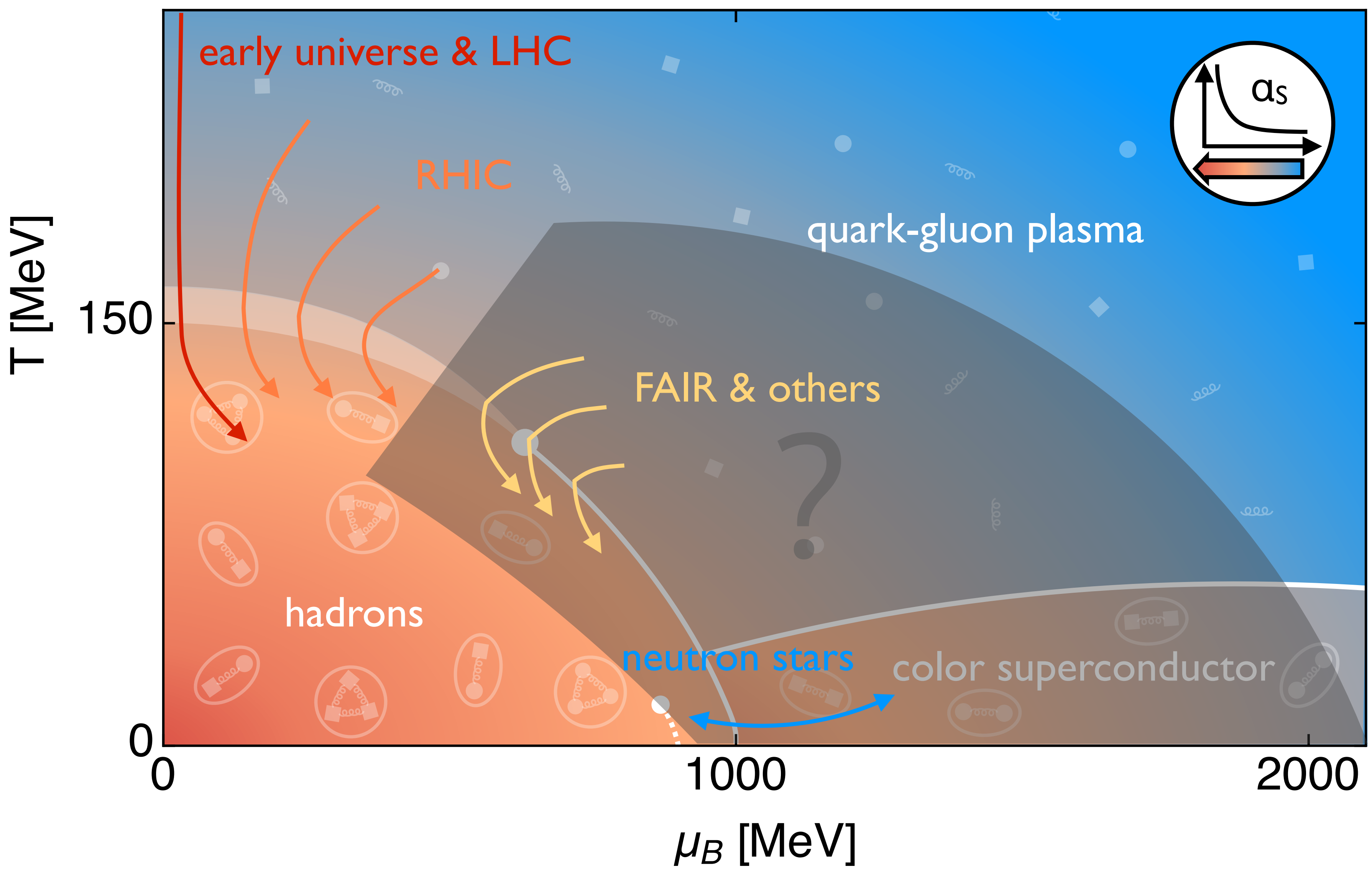
QCD PHASE DIAGRAM

in theory:



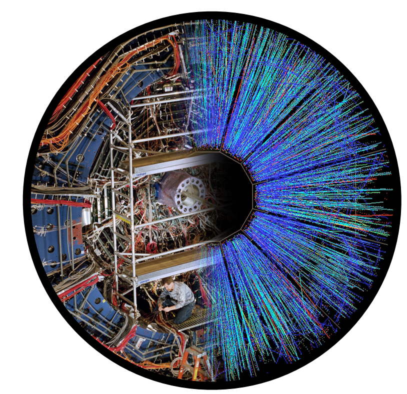
QCD PHASE DIAGRAM

in nature/experiment:

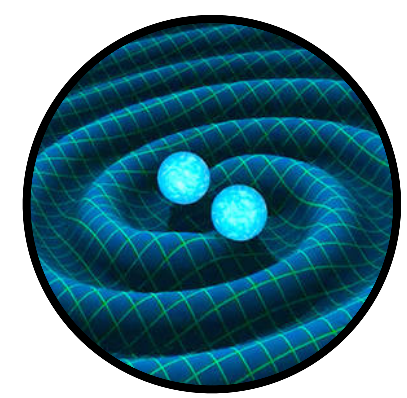


Experiments:

heavy-ion collisions

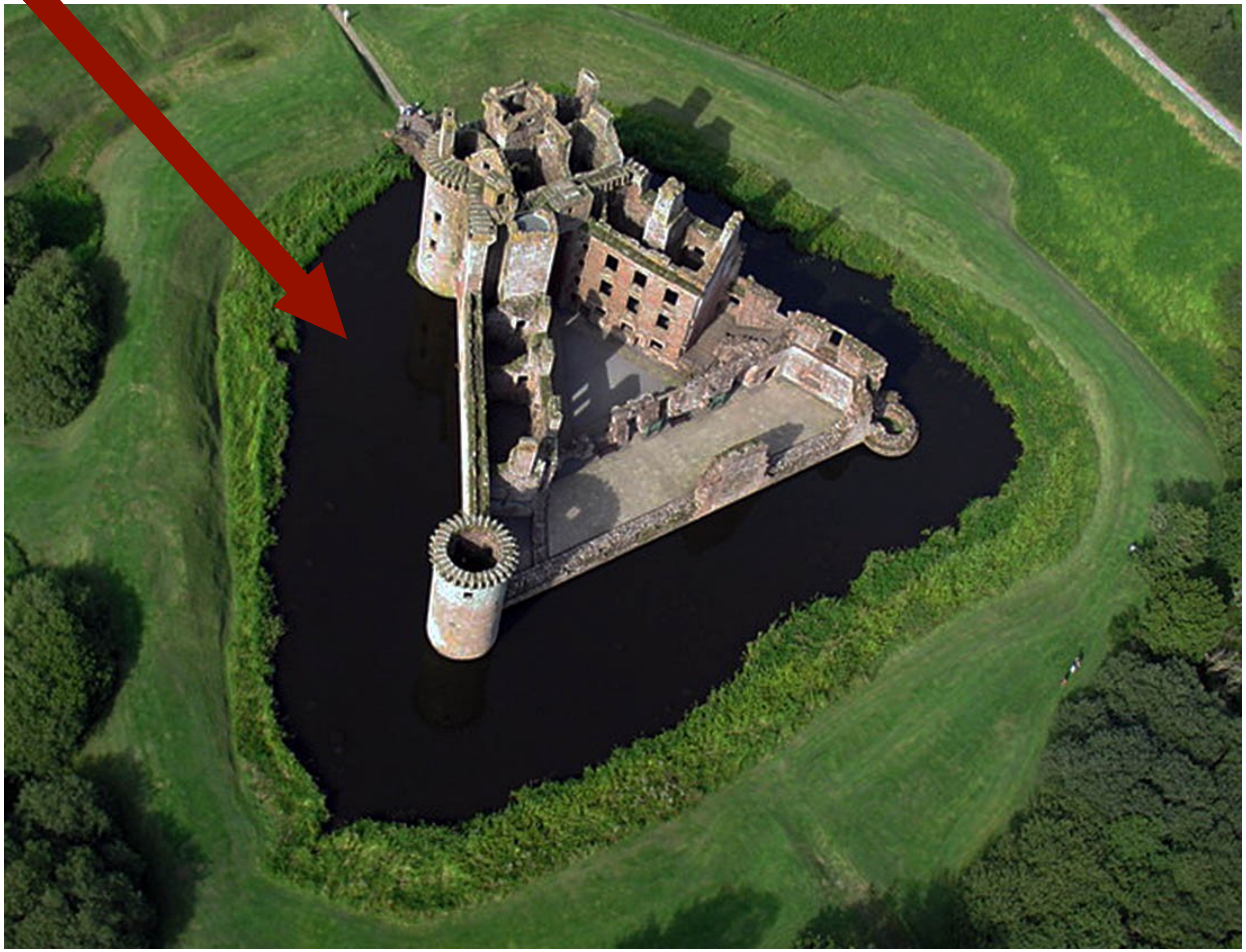


e.g. gravitational waves



MOAT REGIMES

A MOAT

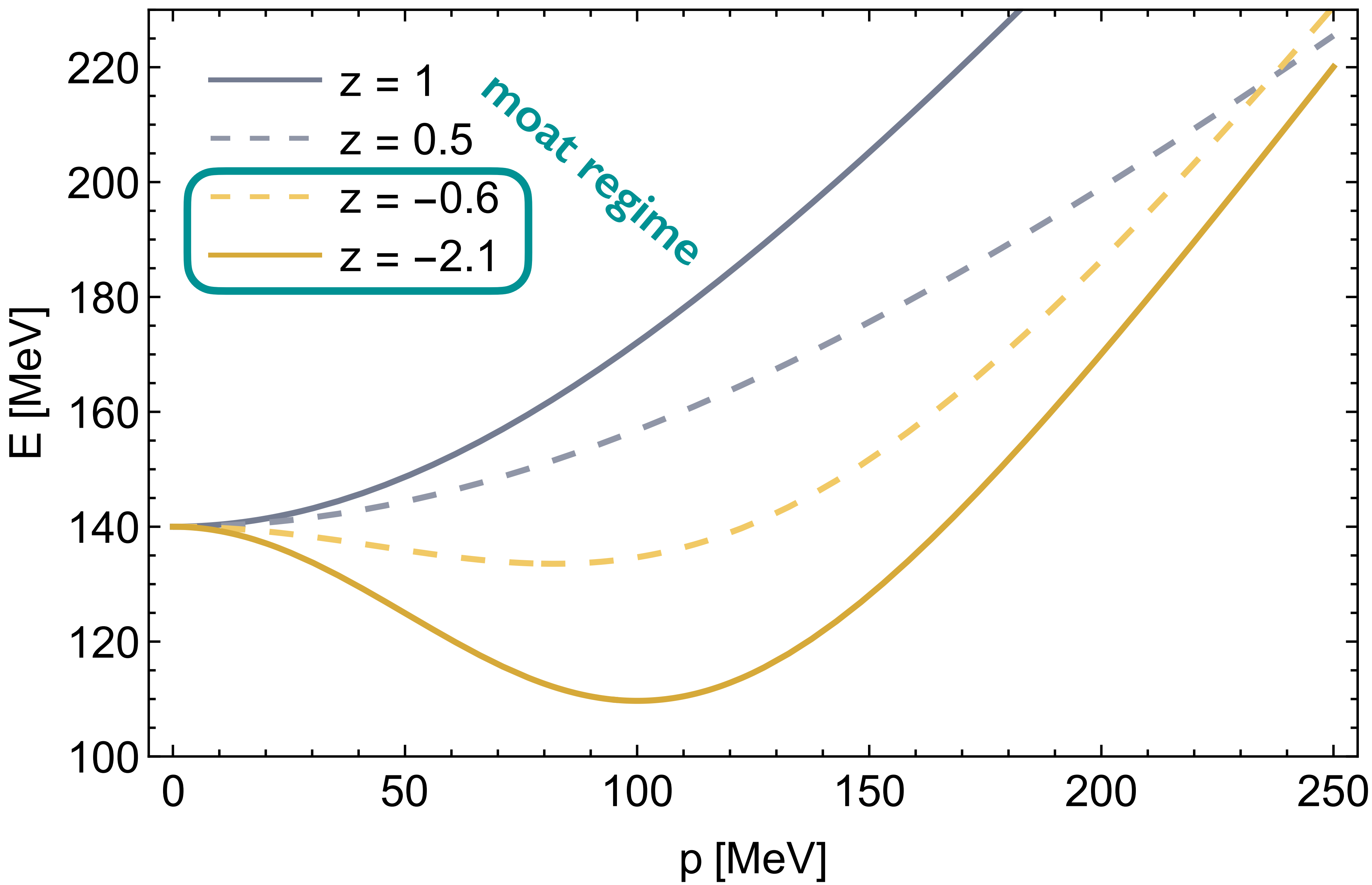


[Caerlaverock Castle, Scotland (source: Wikipedia)]

A MOAT

energy dispersion of particle ϕ:

$$E(\mathbf{p}^2) = \sqrt{Z(\mathbf{p}^2)\,\mathbf{p}^2 + m^2} = \sqrt{z\,\mathbf{p}^2 + w\mathbf{p}^4 + \mathcal{O}(\mathbf{p}^6) + m^2}$$



→ particles are favored to have nonzero momentum

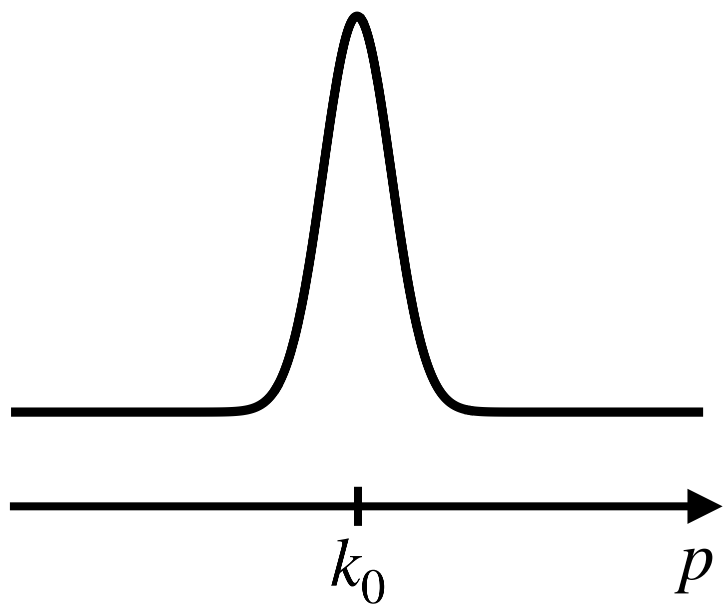
"gain energy by going faster"

WHAT DOES THE MOAT MEAN?

heuristic picture:

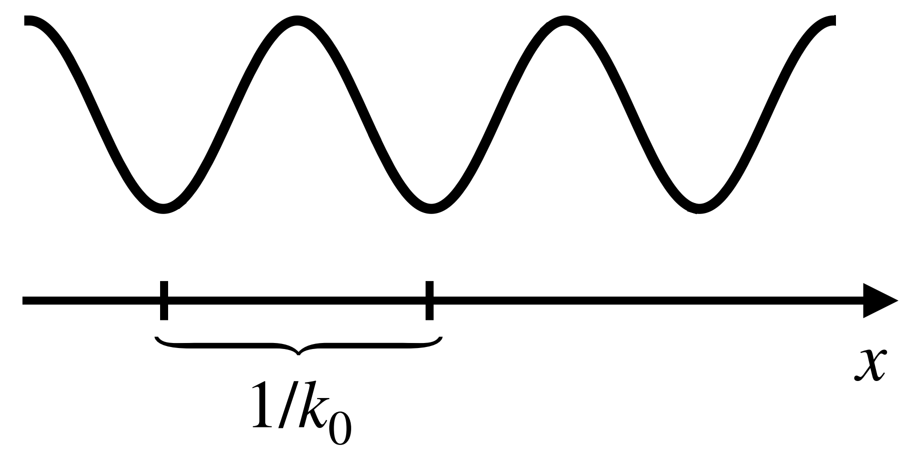
particle distribution for a deep moat:

$$n_B\big(E(\mathbf{p}^2)\big) \sim \delta(p - k_0)$$



spatial oscillation in position space

$$\mathrm{FT}\Big[n_B\big(E(\mathbf{p}^2)\big)\Big] \sim \sin(2\pi k_0 x)$$



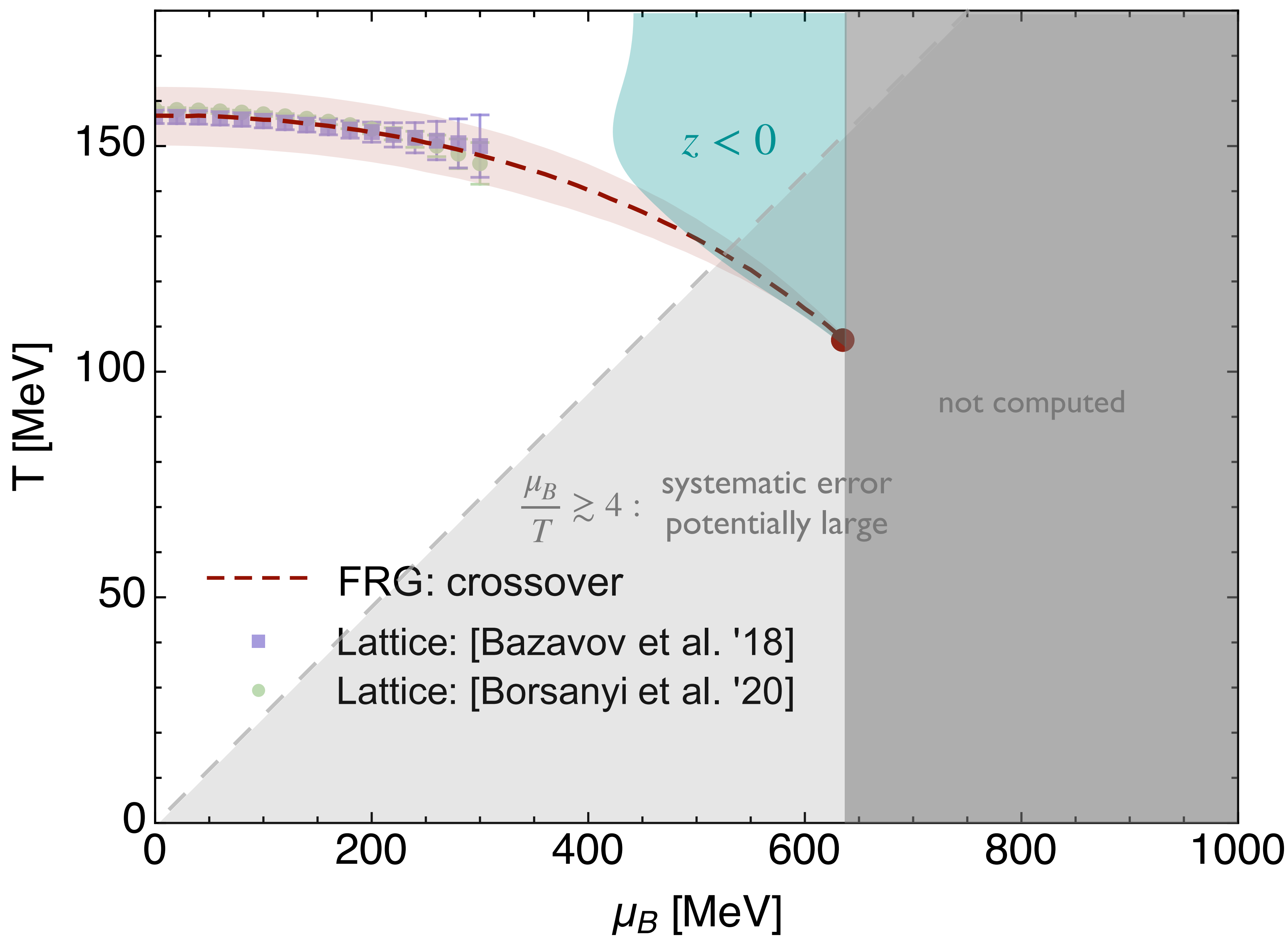
moat energy dispersion (minimal energy at k_0) $\longleftrightarrow$ spatial modulations (with wavenumber k_0)

- typical for inhomogeneous/crystalline phases or a quantum pion liquid (QπL)

WHERE CAN MOAT REGIMES APPEAR?

- many examples in low-energy models at large μ
- first indications also in QCD:

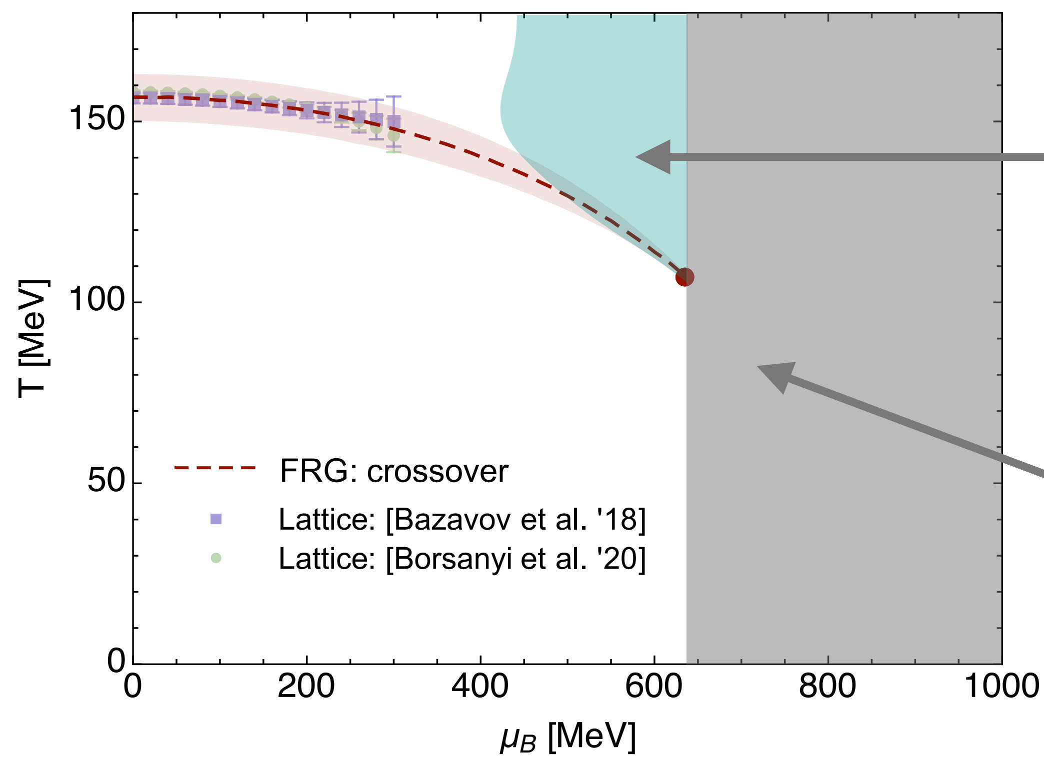
[Fu, Pawlowski, FR, PRD 101 (2020)]

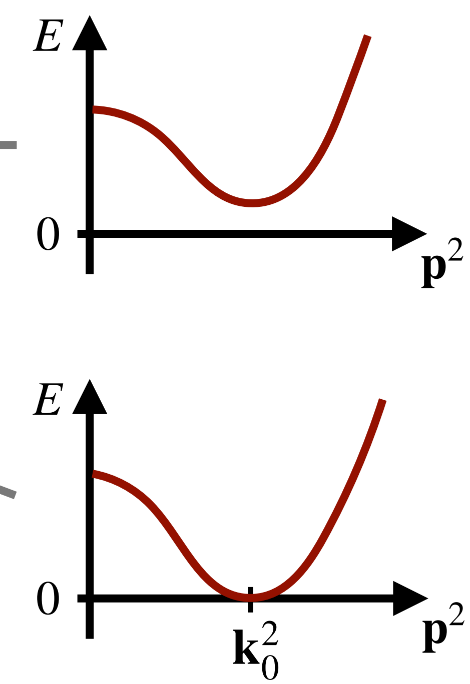


→ indication for extended region with $z < 0$ in QCD: **moat regime**

IMPLICATIONS OF THE MOAT

The energy gap might close at lower T and larger μ_B :





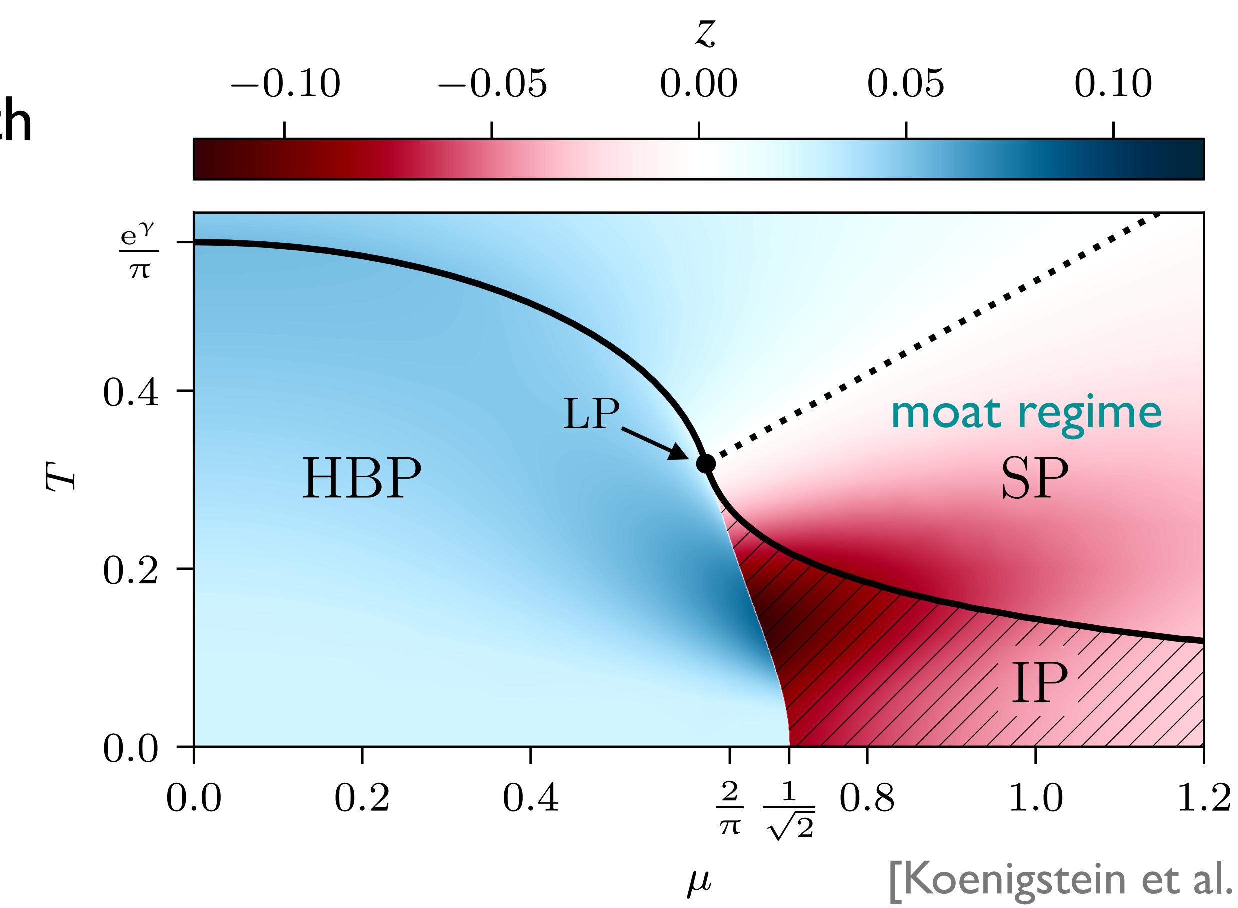
$E > 0$ for all $\mathbf{p}^2$

$E = 0$ at $\mathbf{p}^2 > 0$:

Zero energy cost to "condense" particles with nonzero momentum k_0

→ **instability towards formation of an inhomogeneous condensate**

- Example: Gross-Neveu Model in 1+1 dim. at large N_f



[Koenigstein et al. (2021)]

IMPLICATIONS OF THE MOAT

BUT: formation of inhomogeneous phases depends on dynamics of soft (massless) modes.

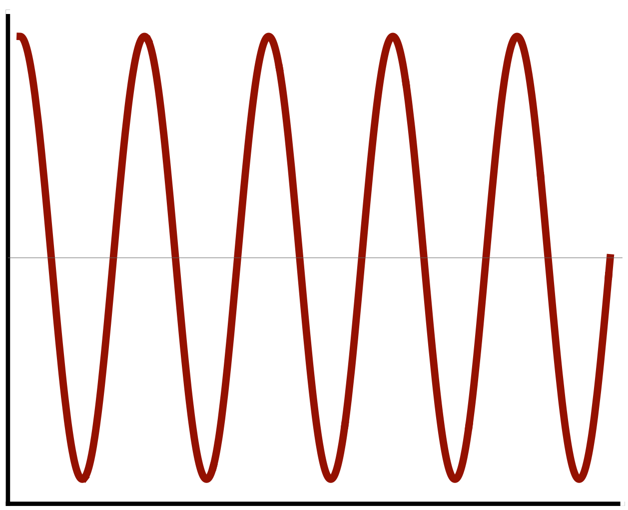
→ fluctuation-induced instabilities of inhomogeneous phases

→ other types of phases possible (possibly without long-range order!)

inhom. phase

no instability
(typical in mean-field)

$$\langle \phi(x)\phi(0)\rangle \sim \sin(k_0\, x)$$

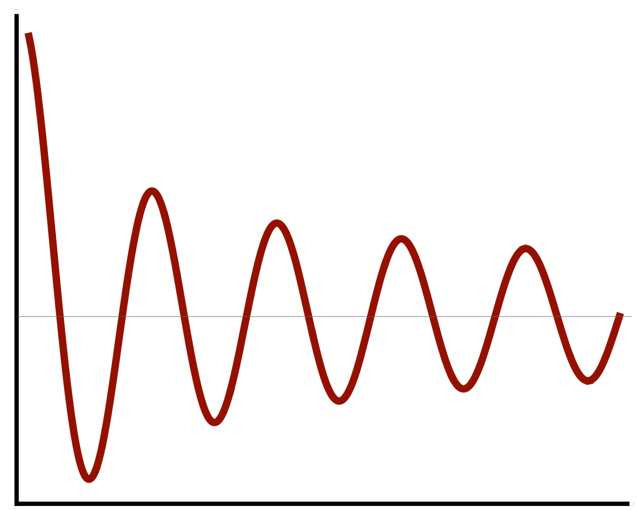


[Fukushima, Hatsuda, RPP 74 (2010)]
[Buballa, Carignano, PPNP 81 (2014)]

liquid crystal

Landau-Peierls instability
(Goldstones from spatial SB)

$$\langle \phi(x)\phi(0)\rangle \sim \sin(k_0 x)\, x^{-\alpha}$$

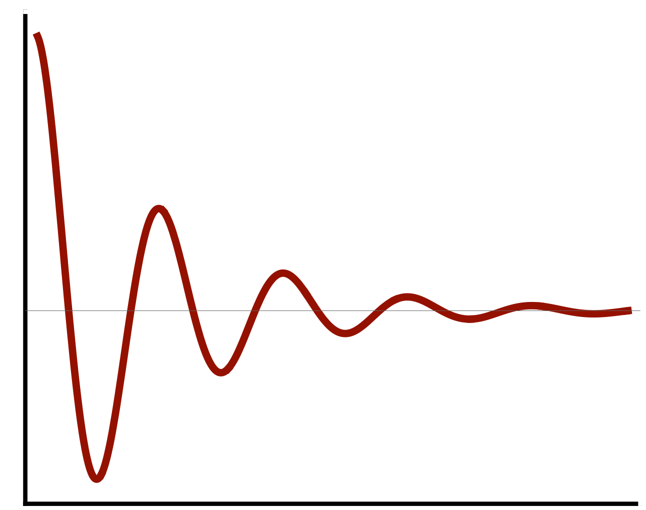


[Landau, Lifshitz, Stat. Phys. I, §137]
[Lee et al., PRD 92 (2015)]
[Hidaka et al., PRD 92 (2015)]

quantum pion liquid

PTV instability
(Goldstones from flavor SB)

$$\langle \phi(x)\phi(0)\rangle \sim \sin(k_0 x)\, e^{-mx}$$



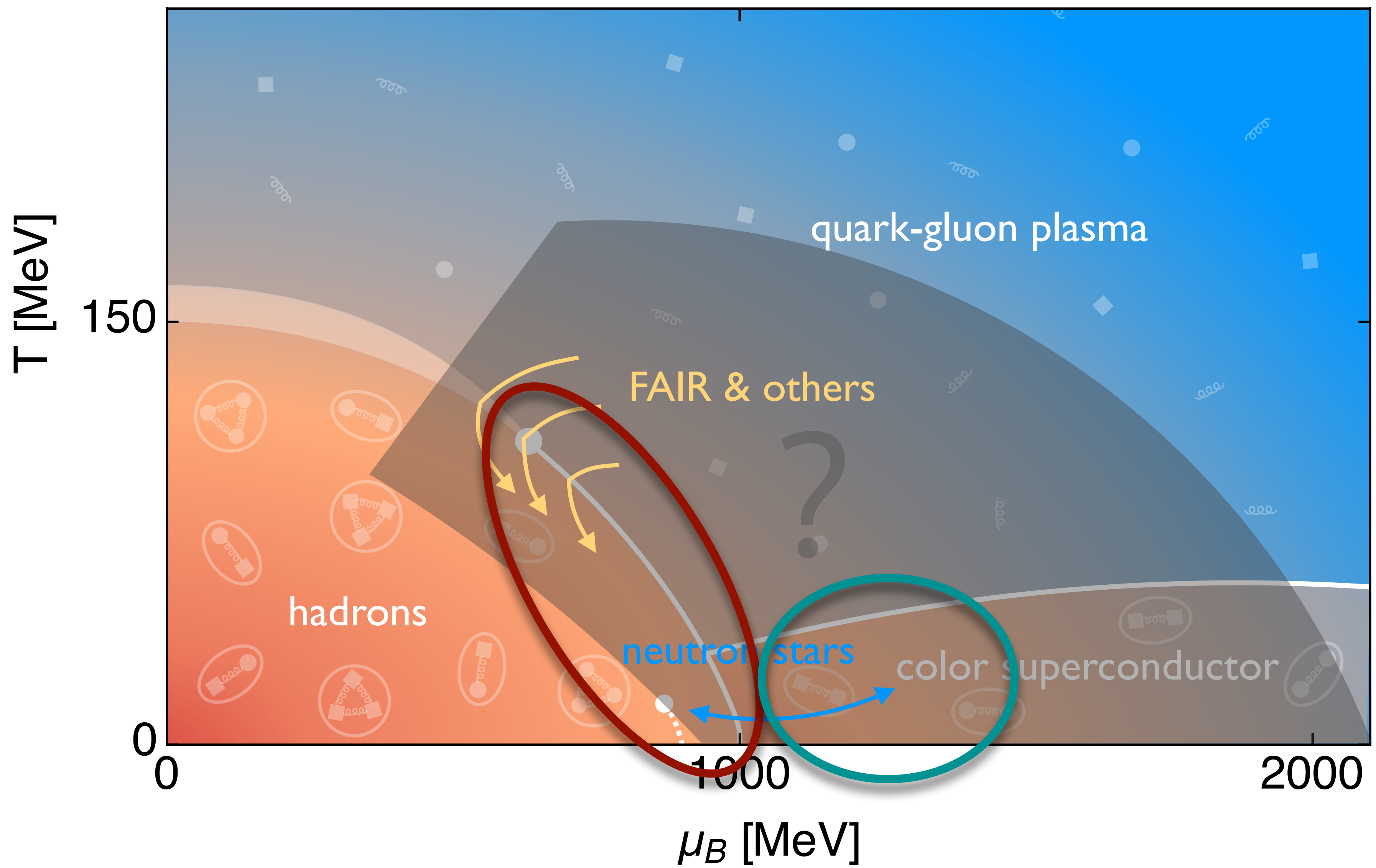
[Pisarski, Tsvelik, Valgushev, PRD 102 (2020)]
[Pisarski, PRD 103 (2021)]
[Schindler, Schindler, Ogilvie (2021)]

either way...

the moat is a common feature of regimes with spatial modulations

THE MOAT REGIME

These phases are expected in the "unknown" region of the phase diagram



CBM at FAIR will cover this region

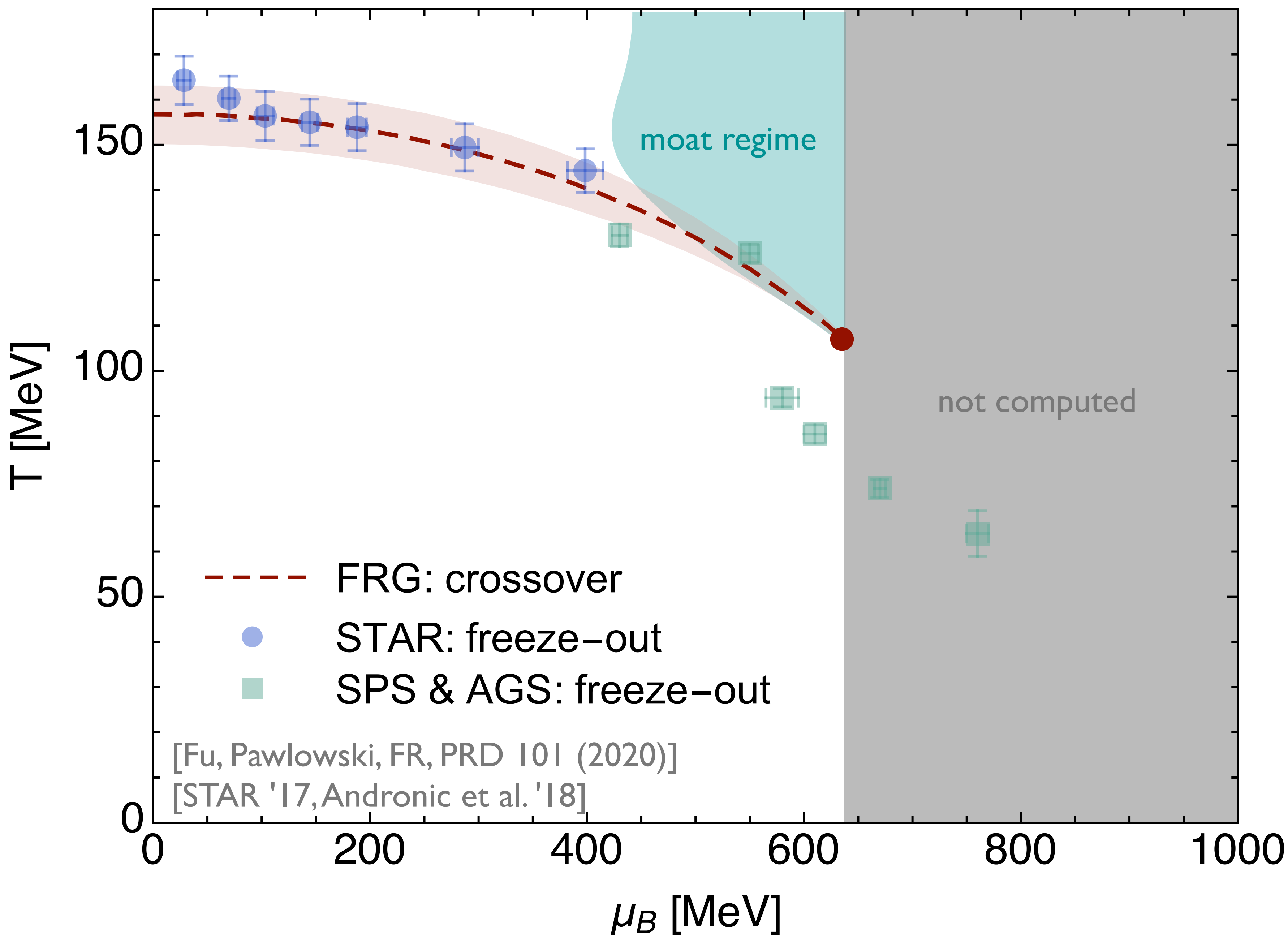
search for moats in heavy-ion collisions!

SIGNATURES OF MOATS IN HEAVY-ION COLLISIONS

[FR, Pisarski, Rischke, arXiv:2301.11484 (2023)]

PROBING THE PHASE DIAGRAM

Vary the beam energy to study the phase diagram different densities (smaller energy $\leftrightarrow$ lager μ)



STAR @ RHIC

$$\sqrt{s} = 7.7 - 200\,\mathrm{GeV}$$

$$\mu_B \approx 400 - 30\,\mathrm{MeV}$$

HADES @ GSI

$$\sqrt{s} \approx 2.4\,\mathrm{GeV}$$

$$\mu_B \approx 770\,\mathrm{MeV}$$

future experiments, e.g.,

CBM @ FAIR

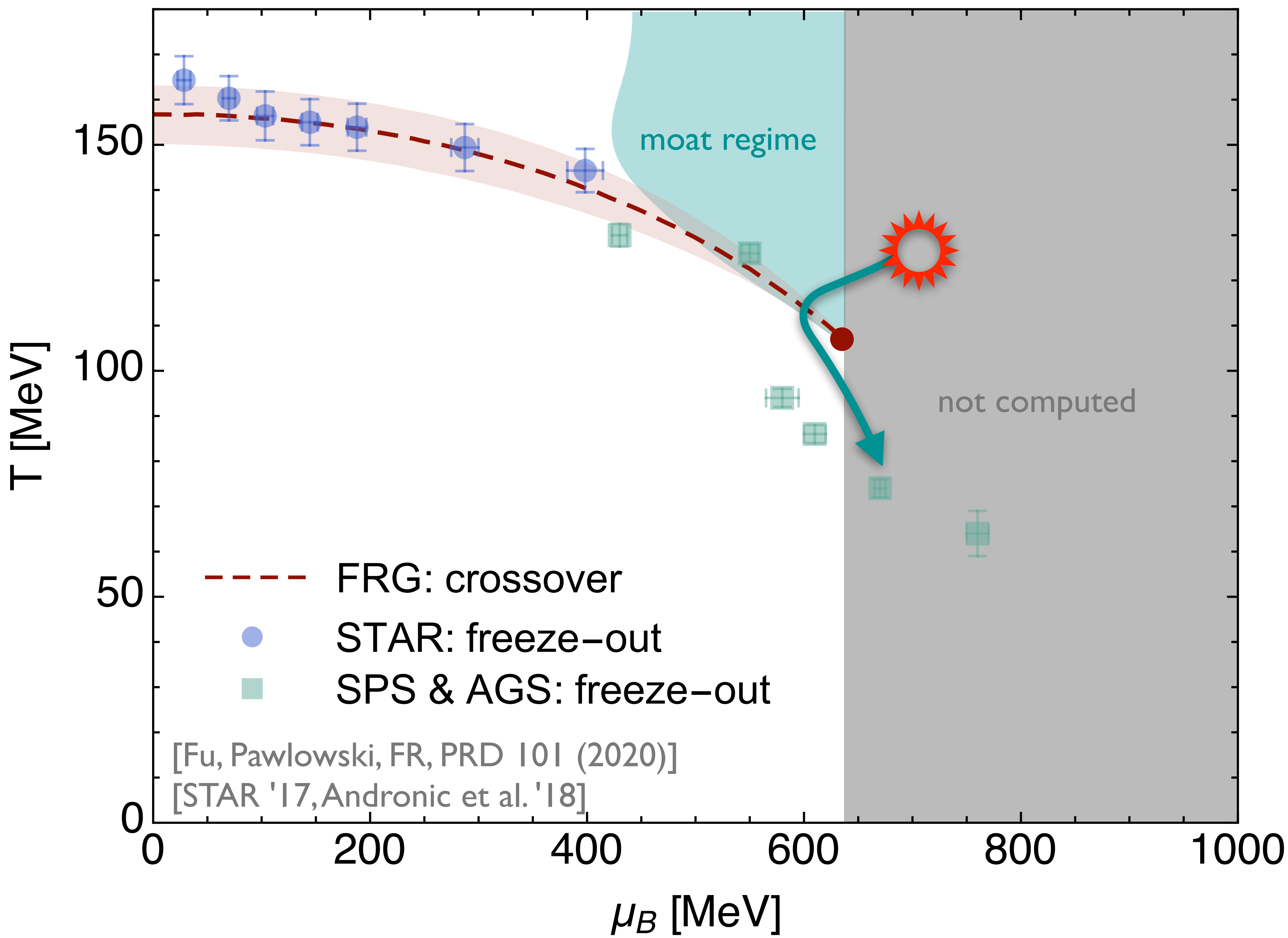
$$\sqrt{s} = 2.7 - 4.9\,\mathrm{GeV}$$

$$\mu_B \approx 730 - 540\,\mathrm{MeV}$$

also: J-PARC, NICA, HIAF

PROBING THE PHASE DIAGRAM

Vary the beam energy to study the phase diagram different densities (smaller energy ↔ lager μ)



STAR @ RHIC

$\sqrt{s} = 7.7 - 200\,\mathrm{GeV}$

$\mu_B \approx 400 - 30\,\mathrm{MeV}$

HADES @ GSI

$\sqrt{s} \approx 2.4\,\mathrm{GeV}$

$\mu_B \approx 770\,\mathrm{MeV}$

future experiments, e.g.,

CBM @ FAIR

$\sqrt{s} = 2.7 - 4.9\,\mathrm{GeV}$

$\mu_B \approx 730 - 540\,\mathrm{MeV}$

also: J-PARC, NICA, HIAF

What are the signatures of the the moat regime in heavy-ion collisions?

SEARCH FOR MOAT REGIMES

intuitive idea:

Moats arise in regimes with spatial modulations

Characteristic feature: minimal energy at nonzero momentum

$\Rightarrow$ **enhanced particle production at nonzero momentum**

→ look for signatures in the **momentum dependence** of particle correlations

(first proposed in [Pisraski, FR, PRL 127 (2021)])

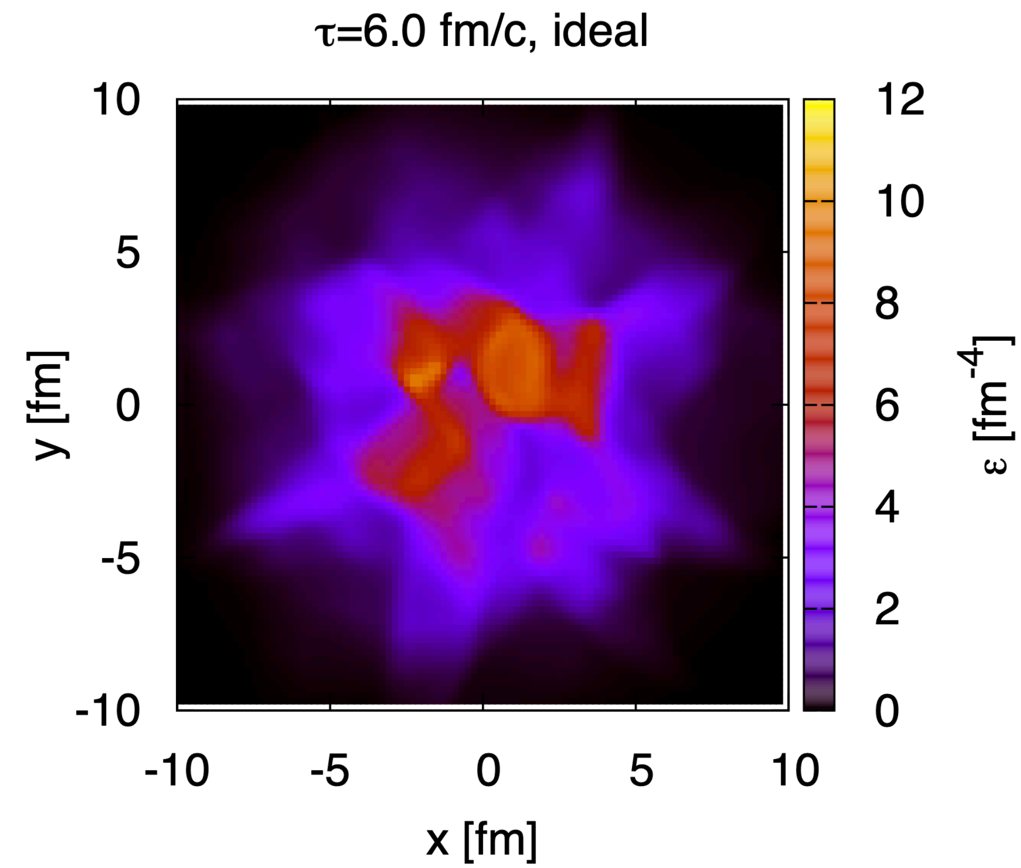
To do:

- develop new formalism to study particle correlations in moat regime
- consider two-particle correlations: **interferometry**

HYPERSURFACES IN HEAVY-ION COLLISIONS

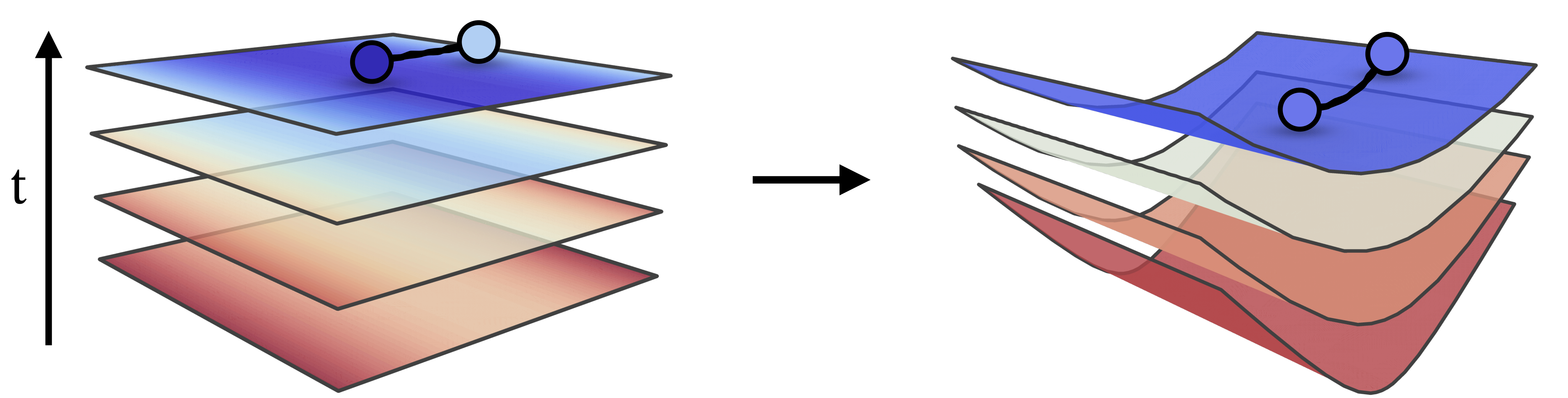
- fixed thermodynamic conditions on 3d hypersurfaces $\Sigma \neq \mathbb{R}^3$
- freeze-out typically on fixed T (or ϵ) hypersurface

→ HIC: evolution of nontrivial hypersurfaces



[Schenke, Jeon, Gale, PRL 106 (2010)]

instead of correlations on $\mathbb{R}^3$ → consider appropriate **foliation** of spacetime



A HYPERSURFACE

- hypersurface Σ defined through parametric equations:

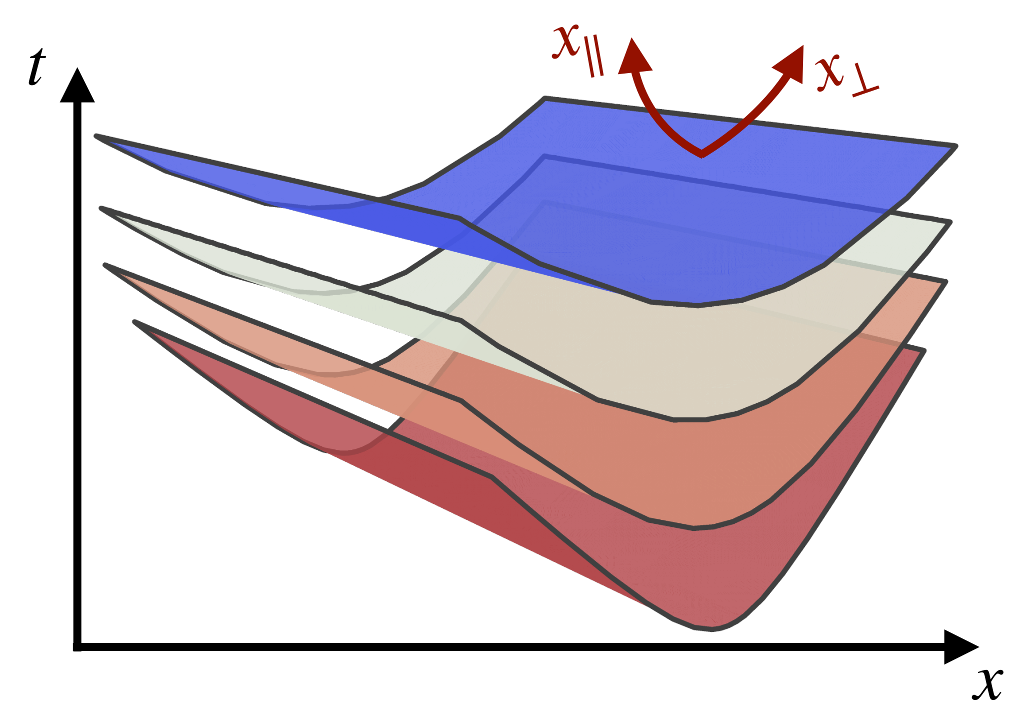
$$x^\mu = x^\mu(w^i)$$

coordinates of ambient spacetime

intrinsic coordinates of Σ $(i = 1, 2, 3)$
e.g., angles φ, ϑ on a 3-sphere

- define tangent and normal vectors of Σ:

$$e_i^\mu = \frac{\partial x^\mu}{\partial w^i} \,, \qquad \hat{v}^\mu \sim \bar{\epsilon}^{\mu\alpha\beta\gamma} e_{1\alpha} e_{2\beta} e_{3\gamma}$$



- decompose spacetime metric as

$$g^{\mu\nu} = \hat{v}^\mu \hat{v}^\nu - G^{ij} e_i^\mu e_j^\nu$$

induced metric on Σ: $G_{ij} = -\, g_{\mu\nu}\, e_i^\mu e_j^\nu$

- define '**time**' and '**space**' : $x_\parallel = \hat{v}^\mu x_\mu$ and $\mathbf{x}_\perp = \mathbf{e}^\mu x_\mu$

→ **foliation** of spacetime: $\{x_\parallel\} \times \Sigma$ instead of $\{t\} \times \mathbb{R}^3$

SPECTRA ON A HYPERSURFACE

[FR, Pisarski, Rischke, arXiv:2301.11484 (2023)]

experiments count particles $\longrightarrow$ particle number correlations

- compute **particle spectra**, e.g.,

$$n_1(\mathbf{p}_\perp) = \omega_{\mathbf{p}_\perp} \big\langle \hat{N}_1 \big\rangle = \omega_{\mathbf{p}_\perp} \big\langle a^\dagger_{\mathbf{p}_\perp} a_{\mathbf{p}_\perp} \big\rangle$$

$$n_2(\mathbf{p}_\perp, \mathbf{q}_\perp) = \omega_{\mathbf{p}_\perp} \omega_{\mathbf{q}_\perp} \big\langle \hat{N}_1 \hat{N}_2 \big\rangle = \omega_{\mathbf{p}_\perp} \omega_{\mathbf{q}_\perp} \big\langle a^\dagger_{\mathbf{p}_\perp} a_{\mathbf{p}_\perp} a^\dagger_{\mathbf{q}_\perp} a_{\mathbf{q}_\perp} \big\rangle$$

- use ladder operators in foliated spacetime (canonical quantization)

$$a_{\mathbf{p}_\perp} = i \int d\Sigma^\mu \, e^{i\bar{p}\cdot x} \frac{1}{\sqrt{2\omega_{\mathbf{p}_\perp}}} \big(\partial_\mu - i\bar{p}_\mu\big) \phi(x)$$

$d\Sigma^\mu = \sqrt{|\det G|}\; d^3 w \; \hat{v}^\mu$ on-shell momentum $\bar{p}_\| = \omega_{\mathbf{p}_\perp}$

- energy of an on-shell particle:

$$\omega_{\mathbf{p}_\perp} = \sqrt{Z(\mathbf{p}_\perp^2)\, \mathbf{p}_\perp^2 + m^2}$$

Insert expressions for ladder operators in terms of fields:

$$\big\langle a^\dagger_{\mathbf{p}_\perp} a_{\mathbf{p}_\perp} a^\dagger_{\mathbf{q}_\perp} a_{\mathbf{q}_\perp} \big\rangle \longrightarrow \big\langle \phi(x_1)\phi(y_1)\phi(x_2)\phi(y_2) \big\rangle$$

$\longrightarrow$ express n-particle spectra in terms of real-time correlations of $2n$ fields

Similar to LSZ reduction, but on Σ at (potentially) any time $x_\|$ and for general dispersion $\omega_{\mathbf{p}_\perp}$

TWO-PARTICLE SPECTRUM

- interference from two-particle scattering: need $n_2(\mathbf{p}_\perp, \mathbf{q}_\perp) = \omega_{\mathbf{p}_\perp}\omega_{\mathbf{q}_\perp}\langle \hat{N}_1 \hat{N}_2 \rangle$
- Gaussian approximation encodes relevant effects:

$$\boxed{\begin{aligned} n_2(\mathbf{p}_\perp, \mathbf{q}_\perp) &\sim \langle a^\dagger_{\mathbf{p}_\perp} a_{\mathbf{p}_\perp}\rangle \langle a^\dagger_{\mathbf{q}_\perp} a_{\mathbf{q}_\perp}\rangle + \left|\langle a^\dagger_{\mathbf{p}_\perp} a_{\mathbf{q}_\perp}\rangle\right|^2 + \left|\langle a_{\mathbf{p}_\perp} a_{\mathbf{q}_\perp}\rangle\right|^2 \\ &= n_1(\mathbf{p}_\perp)\, n_1(\mathbf{q}_\perp) + \left| n_1(\mathbf{p}_\perp, \mathbf{q}_\perp)\right|^2 + \left| \bar{n}_1(\mathbf{p}_\perp, \mathbf{q}_\perp)\right|^2 \end{aligned}}$$

particle-particle interference
(Hanbury-Brown Twiss correlation)

particle-antiparticle interference
(negligible here)

- interference in local thermal equilibrium (fluctuation-dissipation relation + sufficiently isotropic system)

average and relative pair momentum

single-particle distribution,
e.g., Bose-Einstein

$$\boxed{n_1(\mathbf{P}, \mathbf{\Delta P}) = \frac{1}{2}\int d\Sigma_X\, e^{-i\overline{\Delta P}\cdot X} \int \frac{dP_\parallel}{2\pi}\left[\left(P_\parallel + \overline{P}_\parallel\right)^2 - \frac{1}{4}\overline{\Delta P}_\parallel^2\right] f\left(X; P_\parallel, \mathbf{P}_\perp\right)\, \rho\left(X; P_\parallel, \mathbf{P}_\perp\right)}$$

average position

$\longrightarrow$ in-medium effects enter through P-dependence of the spectral function $\rho(x, y) = \langle [\phi(x), \phi(y)]\rangle$

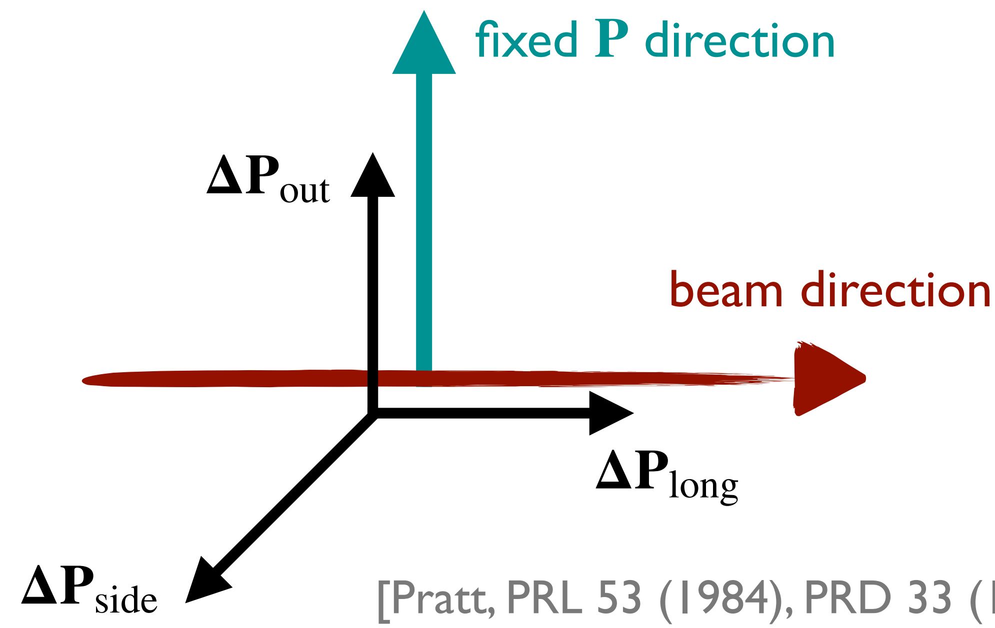
- not most general expression: involves statistical function and gradients in X
- single particle spectrum for $p = q$

[FR, Pisarski, Rischke, arXiv:2301.11484 (2023)]

INTERFEROMETRY

Compute two-particle spectrum in illustrative model

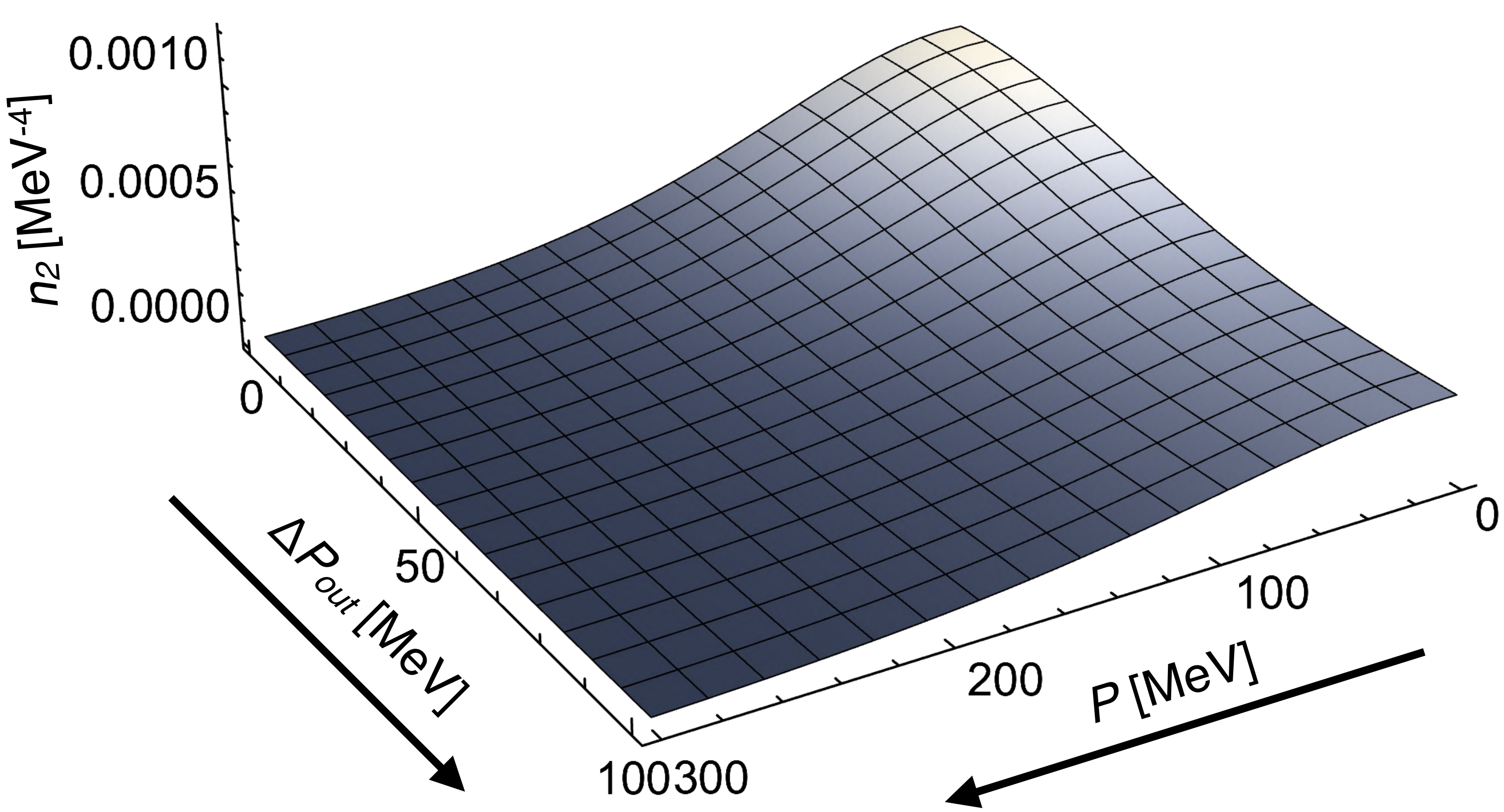
- moat quasi-particle with $k_0 = 100\,\mathrm{MeV}$
- hypersurface at fixed proper time



[Pratt, PRL 53 (1984), PRD 33 (1986)]

Remember: in-medium effects in P-dependence!

normal phase: $\omega_{\mathbf{P}_\perp} = \sqrt{\mathbf{P}^2 + m^2}$



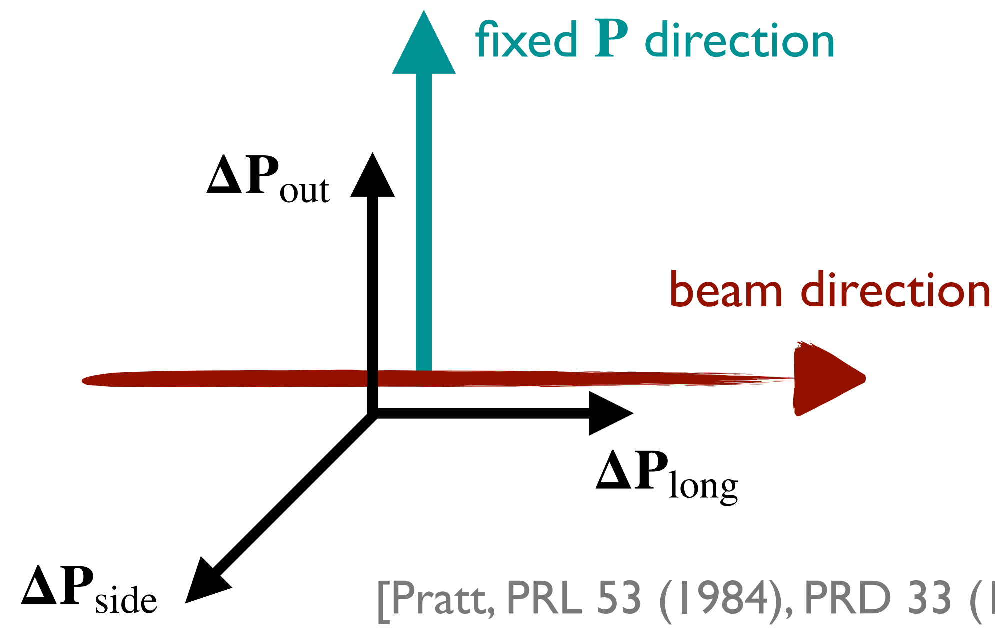
→ correlation peaks at $|\mathbf{P}| = 0$

(side- and long-correlations qualitatively the same)

INTERFEROMETRY

Compute two-particle spectrum in illustrative model

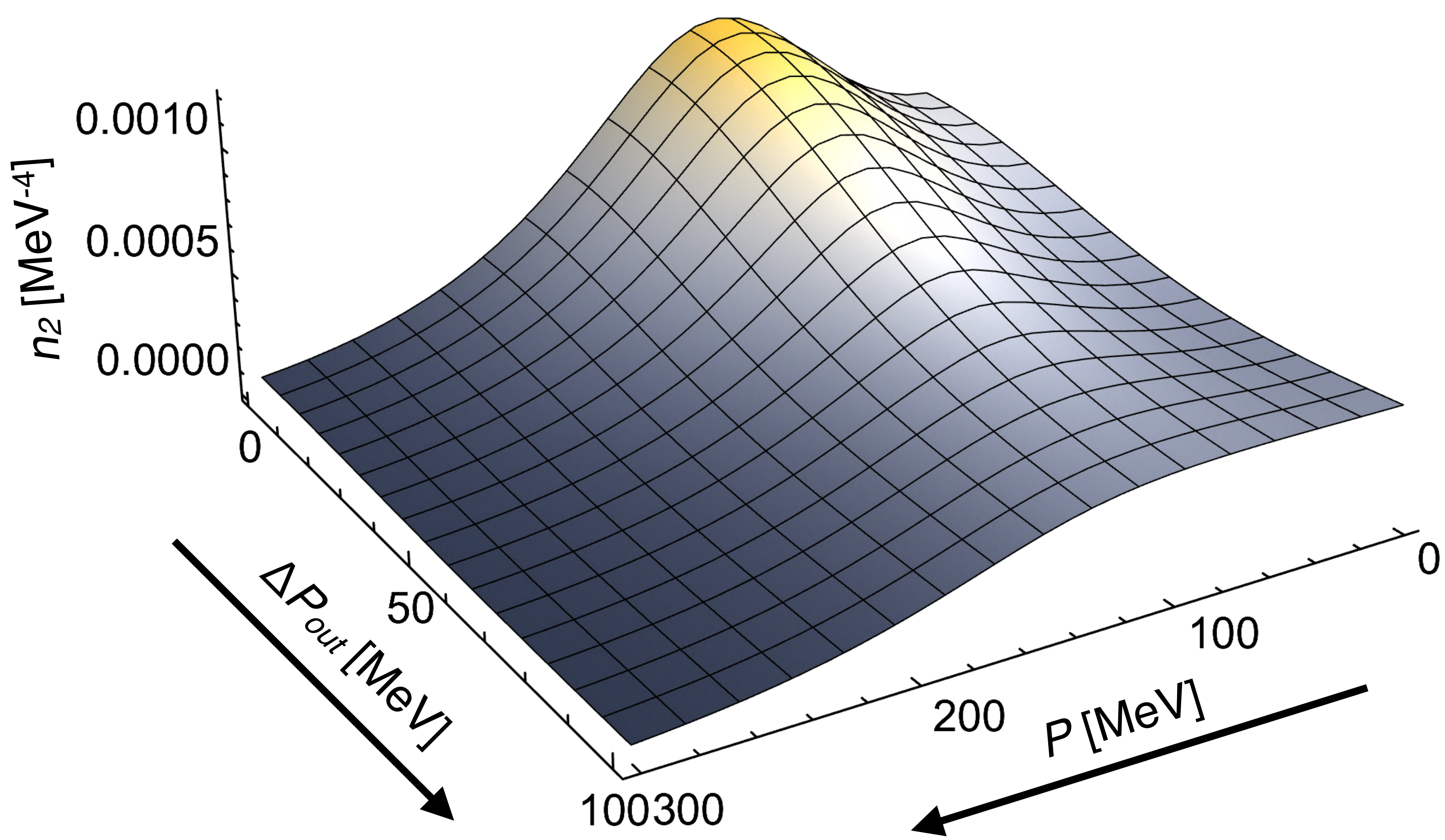
- moat quasi-particle with $k_0 = 100\,\mathrm{MeV}$
- hypersurface at fixed proper time



[Pratt, PRL 53 (1984), PRD 33 (1986)]

Remember: in-medium effects in P-dependence!

moat regime: $\omega_{\mathbf{P}_\perp} \sim \sqrt{z\,\mathbf{P}^2 + w\,\mathbf{P}^4 + m^2}\,, \quad z < 0$



→ correlation peaks at $|\mathbf{P}| = k_0 > 0$

(related to the wave number of underlying spatial modulation)

signature of a moat regime

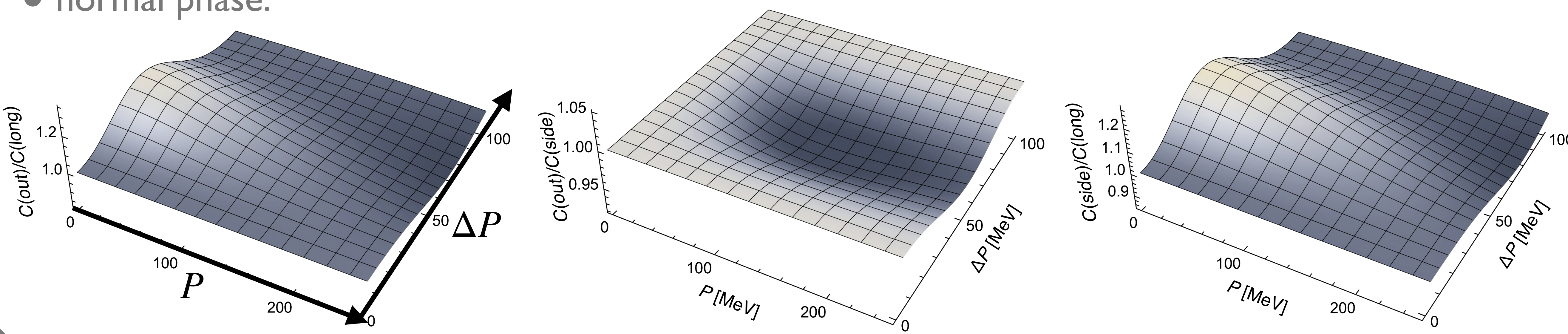
(side- and long-correlations qualitatively the same)

NORMALIZED TWO-PARTICLE CORRELATION

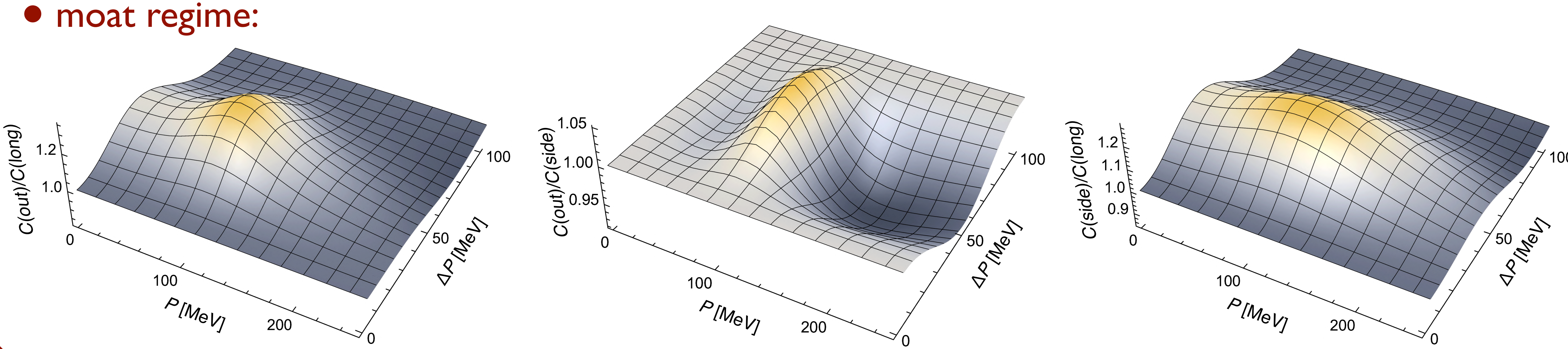
Usually measured in experiments: $$C(\mathbf{P}, \mathbf{\Delta P}) = \frac{n_2(\mathbf{P}, \mathbf{\Delta P})}{n_1\left(\mathbf{P} + \frac{1}{2}\mathbf{\Delta P}\right) n_1\left(\mathbf{P} - \frac{1}{2}\mathbf{\Delta P}\right)}$$

We propose to look at **ratios:** $C_{\text{out}}/C_{\text{long}}$, $C_{\text{out}}/C_{\text{side}}$ and $C_{\text{side}}/C_{\text{long}}$

- normal phase:



- moat regime:



HANBURY-BROWN TWISS RADII

Original idea: use intensity interferometry to measure size of astronomical objects

original experiment in Narrabri, Australia

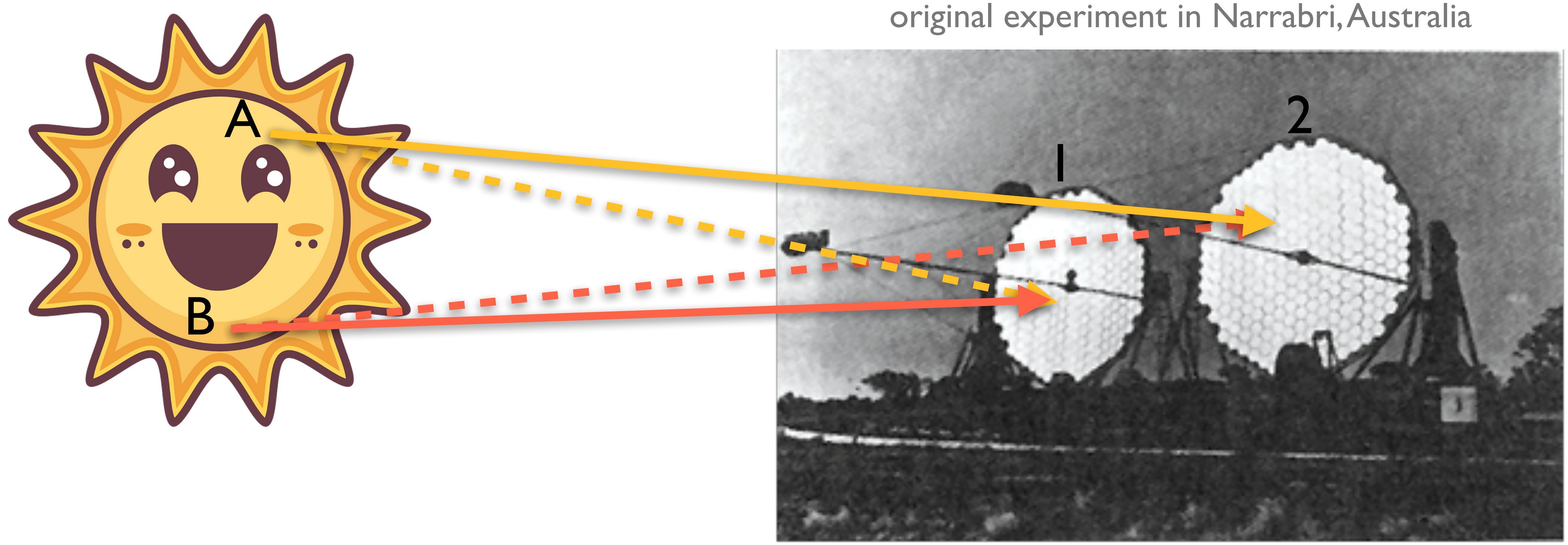


Figure 2. Picture of the two telescopes used in the HBT experiments. The figure was extracted from Ref.[1]. [Goldhaber (1991)]

- interference term (approximately) the Fourier trafo of the **emission function** $S(x, \mathbf{P}_\perp)$

$$n_1(\mathbf{P}, \mathbf{\Delta P}) \approx \int d^4x\, e^{-i\overline{\Delta P}\cdot x}\, S(x, \mathbf{P})$$

- emission function: distribution of spacetime position x and momentum $\mathbf{P}_\perp$ of particles

→ range of correlation in $\mathbf{\Delta P}$ related to inverse size of the source

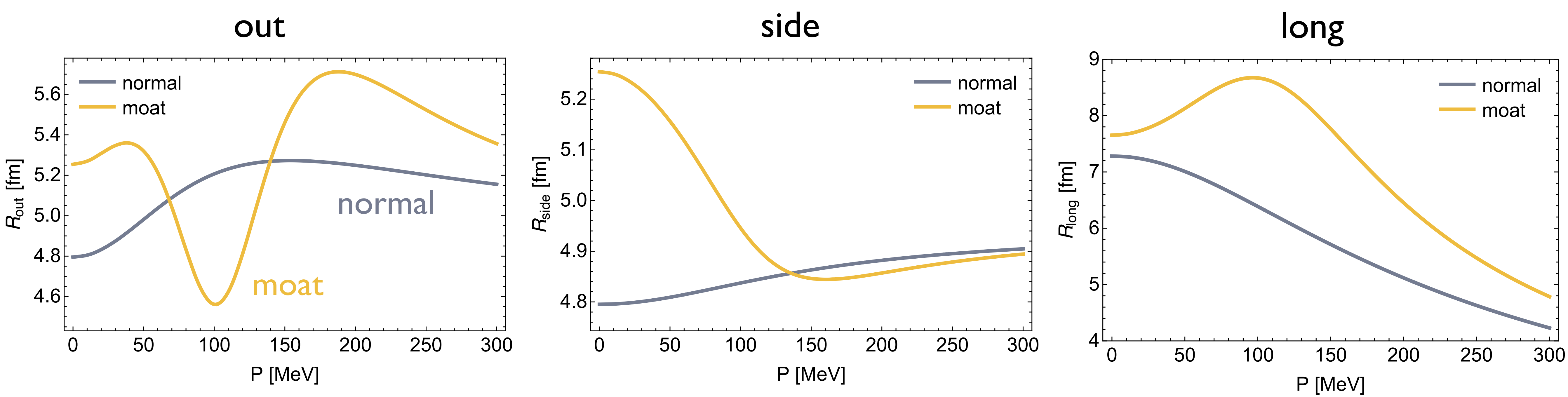
HBT RADII IN A MOAT REGIME

- define HBT radius R through range of correlation in $\mathbf{\Delta P}$

correlation is max. at $\mathbf{\Delta P} = \mathbf{0}$

$$R = \frac{1}{|\mathbf{\Delta P}^*|}\,,\ \text{with}\ \ C(\mathbf{P}, \mathbf{\Delta P}^*) = \frac{1}{2}\, C(\mathbf{P}, \mathbf{0})$$

- yields $R(|\mathbf{P}|)$:



→ **HBT radii modified in moat regime**

SUMMARY

Moats arise in regimes with spatial modulations

- expected to occur at $\mu_B \gtrsim 400$ MeV
- precursors for inhomogeneous-, liquid-crystal-like or quantum pion liquid phases

Signatures of a moat regime in particle interferometry

- developed new formalism that relates particle spectra to real-time correlation functions
- characteristic peaks at nonzero pair momentum in two-particle correlations
- also seen if thermodynamic fluctuations are taken into account [Pisraski, FR, PRL 127 (2021)]
- propose to measure ratios of normalized correlations to detect a moat regime
- in FAIR range!

Opportunity to discover novel phases with heavy-ion collisions through measurement of particle correlations

- So far: basic description of qualitative effects at intermediate stage of collision
- To do: quantitative description of moat regimes & propagation of signal to the detector

BACKUP

INTERFERENCE IN FULL GLORY

- introduce average and relative coordinates

$$X = \frac{1}{2}(x+y)\,, \qquad \Delta X = x - y$$
$$P = \frac{1}{2}(p+q)\,, \qquad \Delta P = p - q$$

- spectral and statistical function as **Wigner transformed** two-point functions

$$\rho(X,P) = \int d\Delta X_{\parallel} \int d\Sigma_{\Delta X}\, e^{iP\cdot\Delta X} \left\langle \left[\phi\left(X + \frac{1}{2}\Delta X\right), \phi\left(X - \frac{1}{2}\Delta X\right) \right] \right\rangle$$
$$F(X,P) = \frac{1}{2} \int d\Delta X_{\parallel} \int d\Sigma_{\Delta X}\, e^{iP\cdot\Delta X} \left\langle \left\{ \phi\left(X + \frac{1}{2}\Delta X\right), \phi\left(X - \frac{1}{2}\Delta X\right) \right\} \right\rangle$$

The particle-particle interference term then is general:

$$n_1(\mathbf{p}_\perp, \mathbf{q}_\perp) = \frac{1}{2} \int d\Sigma_X\, e^{-i\overline{\Delta P}\cdot X} \int \frac{dP_\parallel}{2\pi} \left[\frac{1}{4}\partial^2_{X_\parallel} + \frac{i}{2}\overline{\Delta P}_\parallel \partial_{X_\parallel} + \left(P_\parallel + \bar{P}_\parallel\right)^2 - \frac{1}{4}\overline{\Delta P}^2_\parallel \right] \left[F(X,P) - \frac{1}{2}\rho(X,P) \right]$$

AN ILLUSTRATIVE MODEL I

highlight qualitative effects

Particle in a moat regime:

- bosonic quasi-particle:

$$\rho(P) = 2\,\mathrm{Im}D_R(P) = \frac{\pi}{\omega_{\mathbf{P}_\perp}}\left[\delta(P_\parallel - \omega_{\mathbf{P}_\perp}) - \delta(P_\parallel + \omega_{\mathbf{P}_\perp})\right] \quad \text{with} \quad \omega_{\mathbf{P}_\perp} = \sqrt{Z(\mathbf{P}_\perp^2)\,\mathbf{P}_\perp^2 + m^2}$$

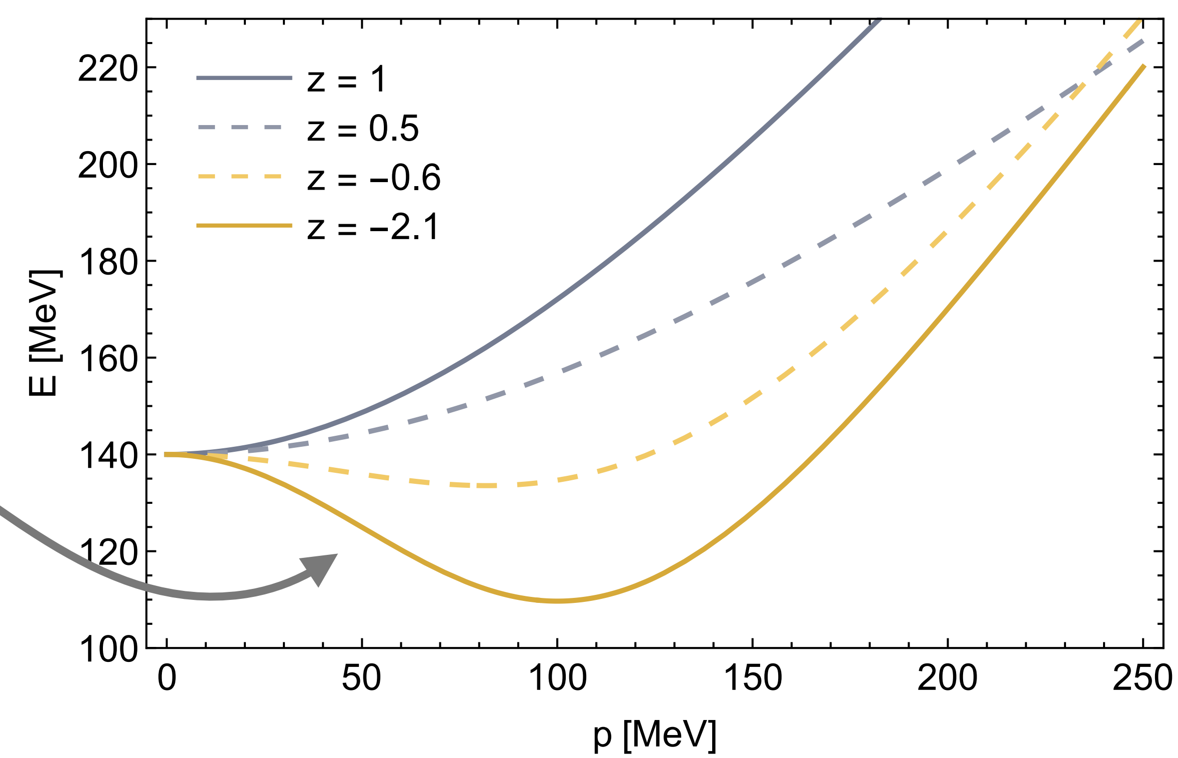
$\longrightarrow$ puts the average pair momentum on-shell

- single-particle distribution: $f\big(X; P_\parallel, \mathbf{P}_\perp\big) = n_B\big(P_\parallel\big) = \dfrac{1}{e^{P_\parallel/T} - 1}$

Wave function renormalization:

- moat spectrum, but well-defined large momentum limit (free relativistic dispersion at large $\mathbf{p}^2$)

$$Z(\mathbf{P}^2) = 1 - \frac{\lambda^2}{\mathbf{P}^2 + M^2}$$
$$\approx \underbrace{1 - \frac{\lambda^2}{M^2}}_{\mathbf{p}^2\text{-coefficient } z \text{ in dispersion}} + \frac{\lambda^2}{M^4}\mathbf{P}^2 + \mathcal{O}(\mathbf{P}^4)$$



AN ILLUSTRATIVE MODEL 2

highlight qualitative effects

Parameters:

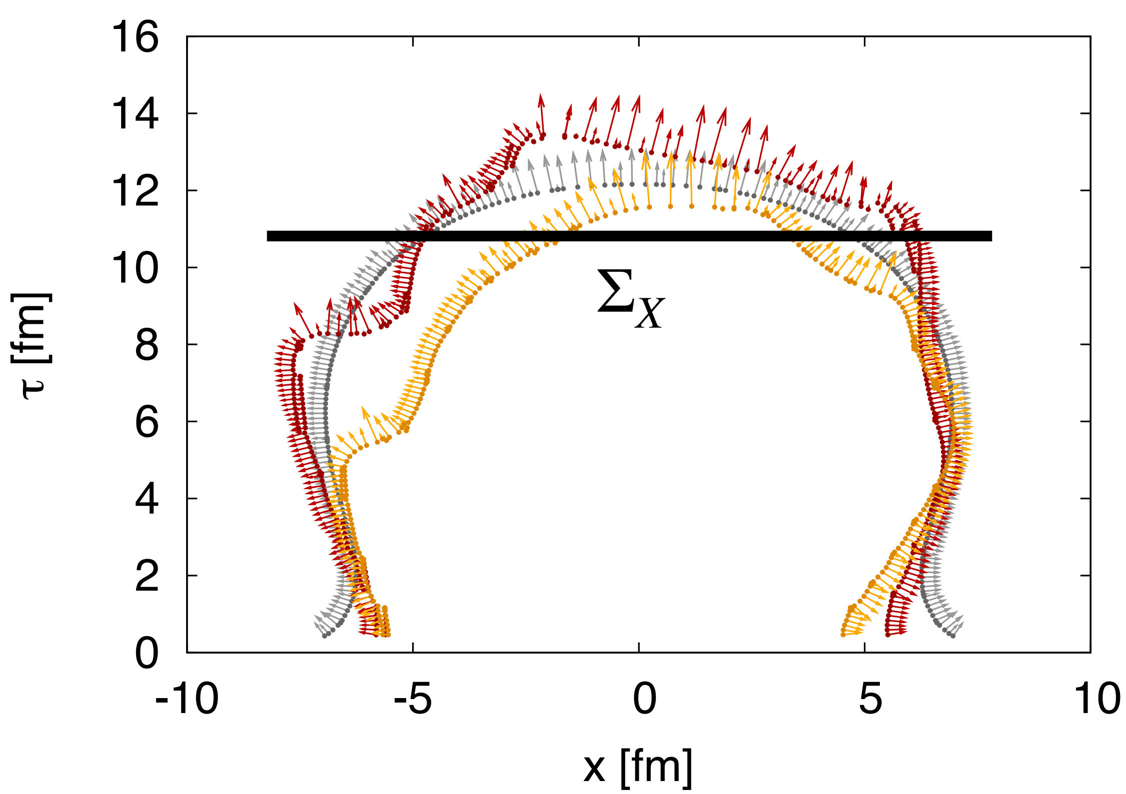
- interferometry measurements typically use pions: $m = m_\pi = 140\,\mathrm{MeV}$
- pions show indications for a moat dispersion in QCD for $\mu_B \gtrsim 450\,\mathrm{MeV}$

  [Fu, Pawlowski, FR, PRD 101 (2020)]
- choose wavenumber (min. of the energy) $\mathcal{O}(m_\pi)$: $|\mathbf{P}_{\mathrm{min}}| = 100\,\mathrm{MeV}$

Hypersurface:

- fixed T hypersurfaces in high-energy HICs approx. at fixed proper time $\tau = \sqrt{X_0^2 - X_3^2}$ (beam direction: X_3)

$\longrightarrow$ very successful in describing transverse momentum spectra



fixes temporal and spatial coordinates on Σ_X

$$X_\parallel = \tau\,, \qquad \mathbf{X}_\perp = \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix}$$

and the metric $\qquad r = \sqrt{X_1^2 + X_2^2}$

$$G^{ij} = \begin{pmatrix} \tau^{-2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & r^{-2} \end{pmatrix}$$

THERMODYNAMIC FLUCTUATIONS

n-particle correlation:

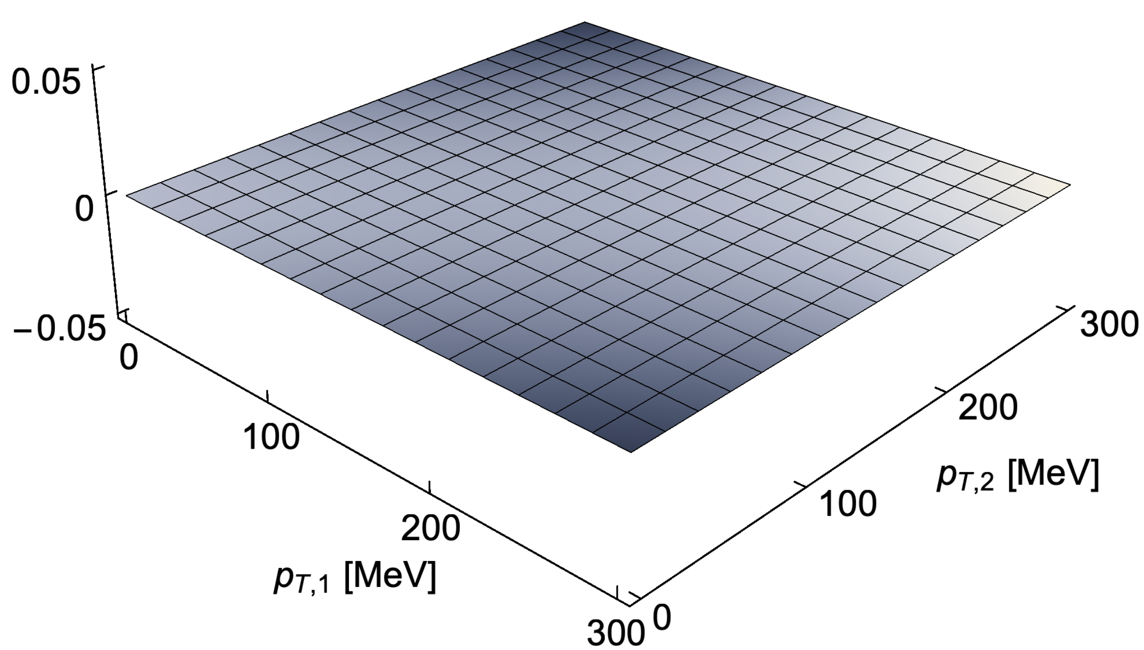
$$\left\langle \prod_{i=1}^{n} n_1(\mathbf{p}_i) \right\rangle \sim \left[\prod_{i=1}^{n} \int d\Sigma_i^{\mu} \int \frac{dp_i^0}{2\pi} (p_i)_\mu \Theta(\check{p}_i^0) \right] \left\langle \prod_{i=1}^{n} f(\check{p}_i)\, \rho(x, \check{p}_i) \right\rangle$$

thermodynamic average

[Pisraski, FR, PRL 127 (2021)]

- fluctuations, e.g., of thermodynamic quantities lead to fluctuations of F_ϕ
- consider small fluctuations in T, μ_B, u
- normalized two-particle correlation (without interference):

normal phase



moat regime