Phenomenology from the three-gluon vertex in general kinematics.

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Outline

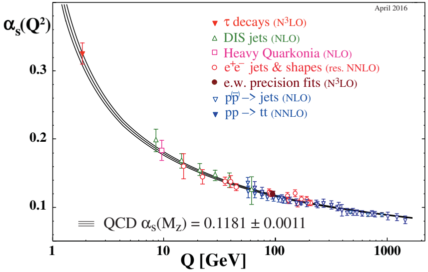
Quantum Chromo-Dynamics

QCD Lagrangian depends on a few parameters: one coupling, α_s, and quark masses (m_u, m_d, m_s, m_c, m_b and m_t).

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \sum_{f=u,\cdots,t} \bar{\psi}_f \left(i\not{D} - m_f \right) \psi_f$$

α_s acquires a renormalization scheme dependent running with the momentum.

The running of $\alpha_s(\mu^2) = \frac{g^2(\mu^2)}{4\pi}$ is controlled by its RGE, $\frac{d\alpha_s}{d\ln\mu^2} = \beta(\alpha_s)$



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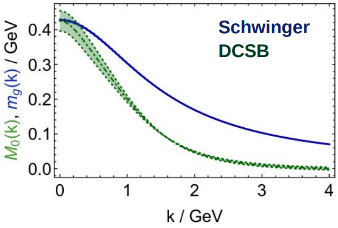
Emergent phenomena:

- Confinement (Hadron masses).
- Dynamically generated gluon-mass.
- Spontaneous chiral symmetry breaking.



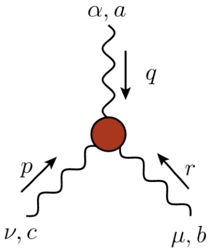
NP approaches:

- Lattice-QCD.
- FUNctional methods.
- QCD vacuum, sum rules,. . .

Gluon self-coupling

$$\mathcal{L}_{\rm YM} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a \; ; \qquad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g\, f^{abc} A_\mu^b A_\nu^c$$

- Three-gluon coupling responsible for the main differences between gluon and photon dynamics.
- It is itself a non-perturbative object which can be computed from the lattice or SDE.
- Key ingredient for sensible truncations in SDE of quark-gluon or ghost-gluon vertices, for example.



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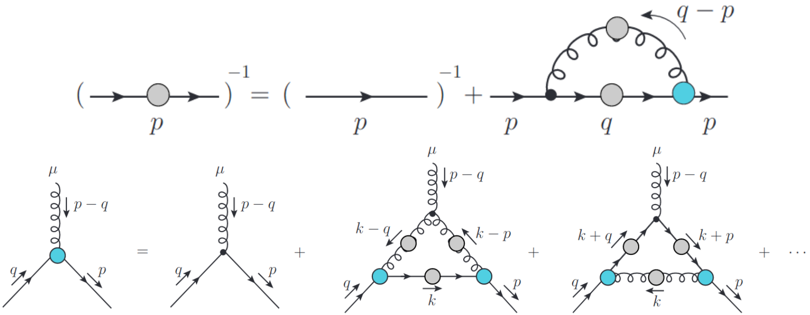

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   - QCD
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   - Evaluating 3g vertex
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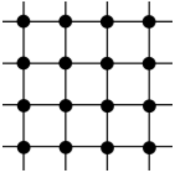
Lattice formulation

Path integral in imaginary time:

$$\langle\mathcal{O}\rangle = \frac{1}{Z}\int [dUd\psi d\bar{\psi}]\mathcal{O}(U,\psi,\bar{\psi})e^{-S(U,\psi,\bar{\psi})} \to \frac{1}{N}\sum_{i=1}^{N} O_i$$



dimensionless; lattice spacing a fixed *a posteriori*.

Pros

- **Just QCD**.
- Regularized per se ($\Lambda \sim a^{-1}$).

Cons

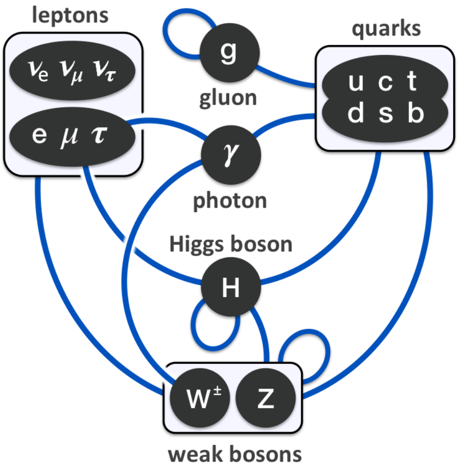
- Finite volume and discretization errors.
- Broken rotational symmetry!
- Expensive chiral fermions.

Quenched approximation

The role of fermion loops in the path integral appears as the determinant of Dirac operator D:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int [dU] \mathcal{O}(U, \psi, \bar{\psi}) e^{-S(U)} \det(D)$$

Yang-Mills theory already has a rich IR phenomenology!



Lattice setups

Exploited quenched gauge field configurations with:

β	L^4/a^4	a (fm)	confs
5.6	32^4	0.236	2000
	48^4	0.236	2000
5.7	32^4	0.182	2000
5.8	32^4	0.144	2000
	48^4	0.144	500
6.0	32^4	0.096	2000
6.2	32^4	0.070	2000
6.4	32^4	0.054	2000

- Absolute calibration for $\beta = 5.8$ taken from [S. Necco and R. Sommer, Nucl. Phys. B622, 328 (2002)].
- Relative calibrations based in gluon propagator scaling [Phys. Rev. D 98, 114515 (2018)]

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Computing three-gluon vertex in Landau gauge

Landau gauge

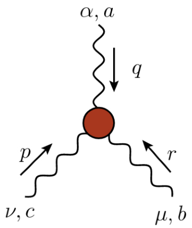
Landau gauge $\partial_\mu A^a_\mu = 0$ fixed numerically, allowing to compute gauge dependent quantities.

- Gluon propagator:

$$\Delta^{ab}_{\mu\nu}(q^2) = \langle A^a_\mu(q) A^b_\nu(-q)\rangle = \delta^{ab}\Delta(q^2) P_{\mu\nu}(q)$$

- Three-gluon vertex:

$$f^{abc}\mathcal{G}_{\alpha\mu\nu}(q,r,p) = \langle A^a_\alpha(q) A^b_\mu(r) A^c_\nu(p)\rangle \ , \quad q+r+p=0$$



Extracting the transversely projected vertex

From the lattice data, we compute the transversely projected vertex, $\overline{\Gamma}^{\alpha\mu\nu}(q,r,p)$:

$$\mathcal{G}^{\alpha\mu\nu}(q,r,p) = g\overline{\Gamma}^{\alpha\mu\nu}(q,r,p)\ \Delta(q^2)\ \Delta(r^2)\ \Delta(p^2)$$

which corresponds to the transverse projection of the 1PI vertex:

$$\overline{\Gamma}^{\alpha\mu\nu}(q,r,p) = \mathbf{\Gamma}^{\alpha'\mu'\nu'}(q,r,p) P^{\alpha}_{\alpha'}(q) P^{\mu}_{\mu'}(r) P^{\nu}_{\nu'}(p)$$

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$$\overline{\Gamma}^{\alpha\mu\nu}(q,r,p) = \mathbf{\Gamma}^{\alpha'\mu'\nu'}(q,r,p) P^{\alpha}_{\alpha'}(q) P^{\mu}_{\mu'}(r) P^{\nu}_{\nu'}(p)$$

No access to longitudinally coupled terms $V^{\alpha\mu\nu}(q,r,p) = q^{\alpha}(\cdots) + r^{\mu}(\cdots) + p^{\nu}(\cdots)$

If the 1PI vertex, $\mathbf{\Gamma}^{\alpha\mu\nu}(q,r,p)$ has longitudinally coupled term $V^{\alpha\mu\nu}(q,r,p)$:

$$\mathbf{\Gamma}^{\alpha\mu\nu}(q,r,p) = \Gamma^{\alpha\mu\nu}(q,r,p) + V^{\alpha\mu\nu}(q,r,p)$$

we will only access the transverse projection of $\Gamma^{\alpha\mu\nu}(q,r,p)$!

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Tensorial structure of $\Gamma^{\alpha\mu\nu}(q, r, p)$

The Ball-Chiu decomposition of the 1PI three-gluon vertex has 14 tensors:

$$\ell_1, \ell_2, \cdots \ell_{10}, \qquad t_1, \cdots t_4 \ ,$$

with 10 partially longitudinal and 4 transverse tensors.

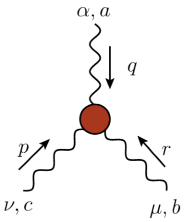
[Phys. Rev. D22 (1980) 2550]

The transversely projected tensor $\overline{\Gamma}^{\alpha\mu\nu}(q, r, p)$ will have at most the contribution of *four* independent tensors:

$$\overline{\Gamma}^{\alpha\mu\nu}(q, r, p) = \overline{\Gamma}_1 \lambda_1^{\alpha\mu\nu} + \overline{\Gamma}_2 \lambda_2^{\alpha\mu\nu} + \overline{\Gamma}_3 \lambda_3^{\alpha\mu\nu} + \overline{\Gamma}_4 \lambda_4^{\alpha\mu\nu}$$

Kinematics of the three-gluon vertex

$\overline{\Gamma}^{\alpha\mu\nu}(q,r,p)$ depends on three momenta, with $q+r+p=0$. The scalar form factors can be cast in terms of the three squared momenta.



We will write them in terms of q^2, r^2, p^2, with the angles given by:

$$\cos\theta_{qr} = \frac{p^2 - q^2 - r^2}{2\sqrt{q^2 r^2}}, \; \cdots$$

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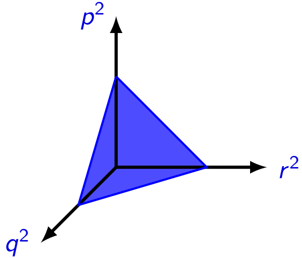

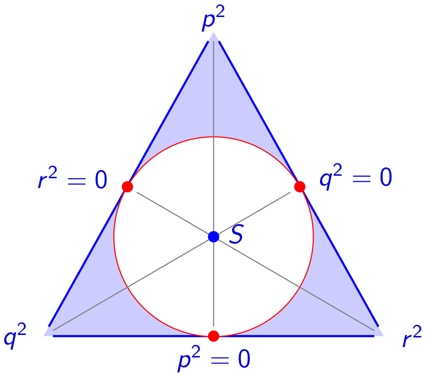

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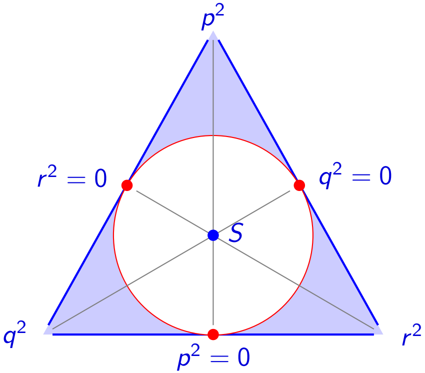

Particular cases:

Case	Def.	$\hat{qr}$	Tensors
Soft gluon	$p = 0$	π	λ_3^{sg}
Sym.	$q^2 = r^2 = p^2$	$\frac{2\pi}{3}$	$\lambda_{1,2}^{sym}$
Bisectoral	$q^2 = r^2$	$(0, \pi)$	3
General		–	4

Symmetric and soft-gluon cases already studied in [Phys.Lett.B 818 (2021) 136352]

Tensor basis

We chose the following basis:

$$\lambda_1^{\alpha\mu\nu} = \overline{\Gamma}_0^{\alpha\mu\nu} = \left(g^{\alpha'\mu'}(q-r)^{\nu'} + g^{\mu'\nu'}(r-p)^{\alpha'} + g^{\alpha'\nu'}(p-q)^{\mu'}\right) P^{\alpha}_{\alpha'}(q) P^{\mu}_{\mu'}(r) P^{\nu}_{\nu'}(p)$$

$$= \left(\ell_1^{\alpha'\mu'\nu'} + \ell_4^{\alpha'\mu'\nu'} + \ell_7^{\alpha'\mu'\nu'}\right) P^{\alpha}_{\alpha'}(q) P^{\mu}_{\mu'}(r) P^{\nu}_{\nu'}(p) \qquad \rightarrow \lambda_1^{sym}, \lambda_3^{s.g.}$$

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$$\lambda_2^{\alpha\mu\nu} = 3\frac{(r-p)^{\alpha'}(p-q)^{\mu'}(q-r)^{\nu'}}{q^2+r^2+p^2} P^{\alpha}_{\alpha'}(q)P^{\mu}_{\mu'}(r)P^{\nu}_{\nu'}(p) \qquad \to \lambda_2^{sym.}$$

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$$\lambda_3^{\alpha\mu\nu} = \frac{3}{q^2+r^2+p^2}\left(\ell_3^{\alpha'\mu'\nu'} + \ell_6^{\alpha'\mu'\nu'} + \ell_9^{\alpha'\mu'\nu'}\right) P_{\alpha'}^{\alpha}(q) P_{\mu'}^{\mu}(r) P_{\nu'}^{\nu}(p)$$

$$\lambda_4^{\alpha\mu\nu} = \left(\frac{3}{q^2+r^2+p^2}\right)^2 \left(t_1^{\alpha\mu\nu} + t_2^{\alpha\mu\nu} + t_3^{\alpha\mu\nu}\right)$$

Tensor basis

We have chosen the tensor basis *antisymmetric* under two-gluon permutation, i.e. $\{q,\alpha\} \leftrightarrow \{r,\mu\}$:

$$\lambda_i \to -\lambda_i$$

Recall

$$\langle A^a_\alpha(q) A^b_\mu(r) A^c_\nu(p)\rangle = f^{abc} g\overline{\Gamma}_{\alpha\mu\nu}(q,r,p)\ \Delta(q^2)\ \Delta(r^2)\ \Delta(p^2)$$

and

$$g\overline{\Gamma}^{\alpha\mu\nu}(q,r,p) = \sum_i \overline{\Gamma}_i(q^2,r^2,p^2)\lambda_i^{\alpha\mu\nu}(q,r,p)$$

Bose symmetry

The form-factors $\overline{\Gamma}_i(q^2,r^2,p^2)$ can only depend on symmetric combination of the momenta.

Extraction of form factors

Once we have evaluated $\overline{\Gamma}^{\alpha\mu\nu}(q,r,p)$ from the lattice, we have to solve:

$$\sum_i \overline{\Gamma}_i(q^2,r^2,p^2)\ \lambda_i^{\alpha\mu\nu}(q,r,p)\lambda_{j\,\alpha\mu\nu}(q,r,p) = \overline{\Gamma}^{\alpha\mu\nu}(q,r,p)\ \lambda_j^{\alpha\mu\nu}(q,r,p)$$

For the symmetric and soft-gluon cases we obtained a projector $\tilde{\lambda}_j(q,r,p)$ that allowed the extraction of the form factors as:

$$\overline{\Gamma}_i(q^2,r^2,p^2) = \frac{\overline{\Gamma}^{\alpha\mu\nu}(q,r,p)\tilde{\lambda}_{j\,\alpha\mu\nu}(q,r,p)}{\tilde{\lambda}_j^{\alpha\mu\nu}(q,r,p)\tilde{\lambda}_{j\,\alpha\mu\nu}(q,r,p)}$$

Renormalization

Once the bare form-factors have been obtained, we implement multiplicative renormalization for the vertex via the renormalization constant $Z_3(\mu^2)$, defined as:

$$\overline{\Gamma}_{i,R}(q^2,r^2,p^2) = Z_3(\mu)\overline{\Gamma}_i(q^2,r^2,p^2)\,.$$

We define it from the soft-gluon case by imposing:

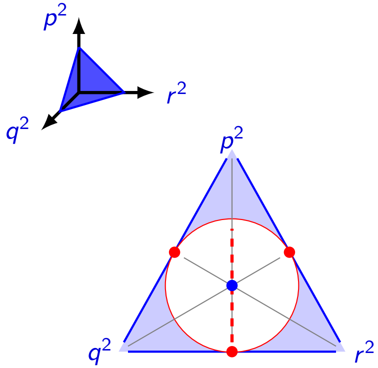
$$\overline{\Gamma}_{1,R}(\mu^2,\mu^2,0) = 1 \quad \leftrightarrow \quad Z_3(\mu) = \overline{\Gamma}_1(\mu^2,\mu^2,0)^{-1}$$

at $\mu = 4.3$ GeV. For the rest of form-factors, it implies:

$$\overline{\Gamma}_{i,R}(q^2,r^2,p^2) = \frac{\overline{\Gamma}_i(q^2,r^2,p^2)}{\overline{\Gamma}_1(\mu^2,\mu^2,0)}$$

Outline

Results for the bisectoral case $q^2 = r^2$.

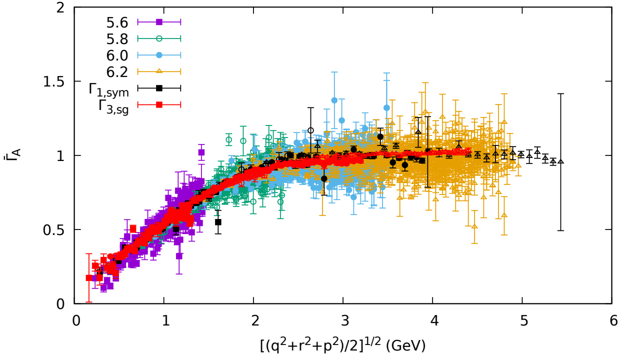


The scalar form factors can only depend on symmetric momentum variables [G. Eichmann *et al*, PRD89 (2014) 105014]:

- $s^2 = \frac{q^2+r^2+p^2}{2}$ (plane)
- $(q^2-r^2)^2+(r^2-p^2)^2+(p^2-q^2)^2$ (radius)
- $(q^2+r^2-2p^2)(r^2+p^2-2q^2)(p^2+q^2-2r^2)$ (phase)

Alternatively, we will use s and θ_{qr} for the bisectoral case.

Results for the bisectoral case $q^2 = r^2$: $\overline{\Gamma}_1$



Represented in terms of s, there is a nice overlap between the already published *symmetric* and *soft-gluon* cases, but also with the bisectoral one.

[F. Pinto-Gómez, FS, *et al* PLB838 (2023) 137737]

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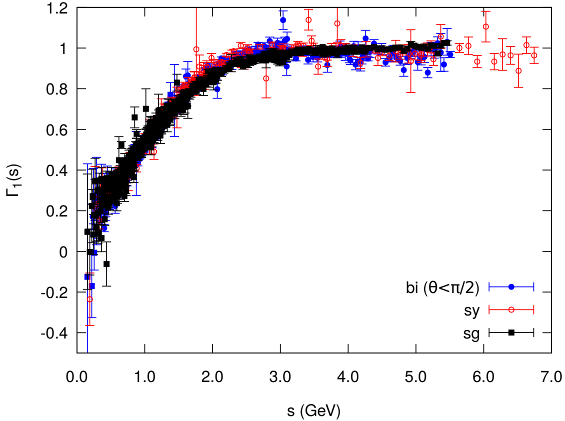

There is an excellent overlap for the deep IR (below $s \sim 1.5 - 2$ GeV).

The bisectoral case separates from the soft-gluon one at $s \sim 3$ GeV.

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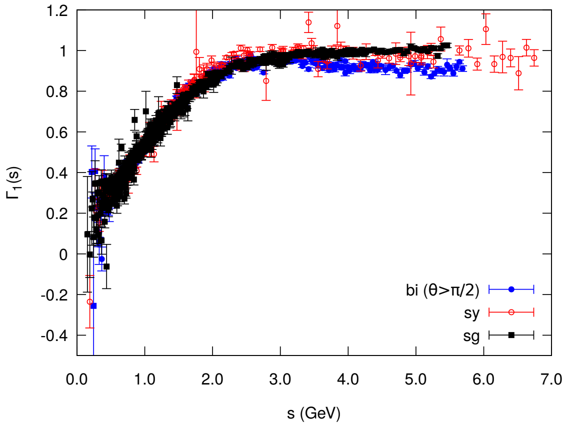

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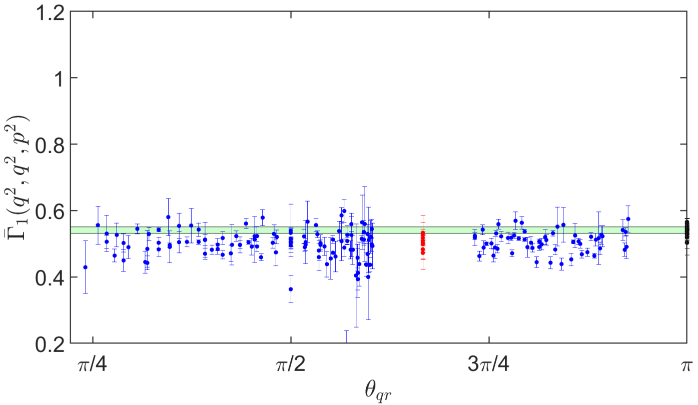
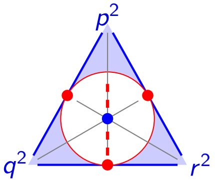
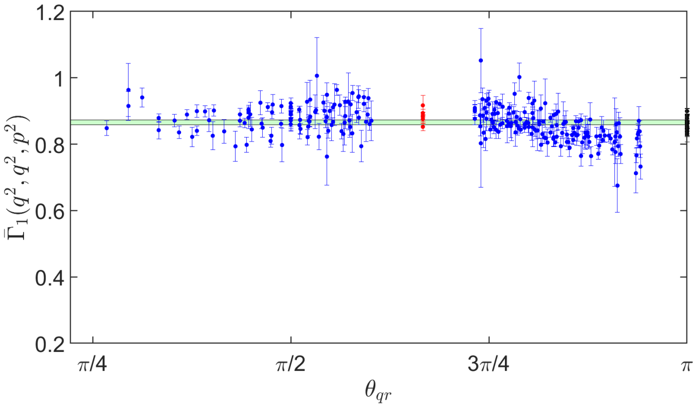


$s = 1$ GeV

For small momenta, there is a negligible effect of the angle θ_{qr}

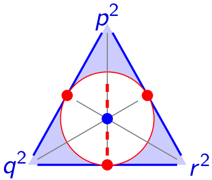


Results for the bisectoral case $q^2 = r^2$: $\overline{\Gamma}_1$

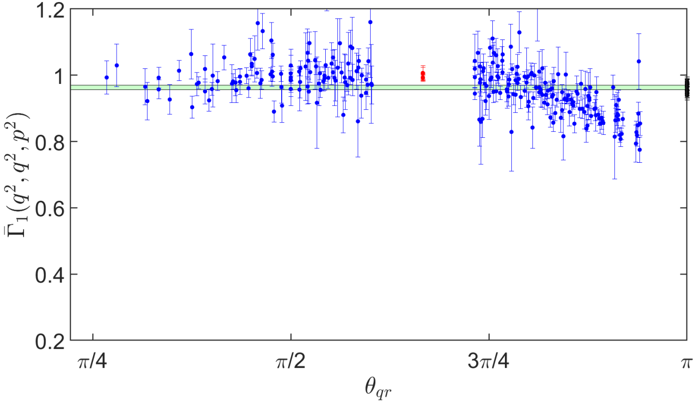


$s = 2$ GeV

For small momenta, there is a negligible effect of the angle θ_{qr}

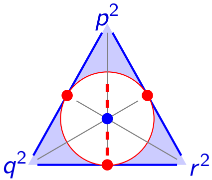


Results for the bisectoral case $q^2 = r^2$: $\overline{\Gamma}_1$

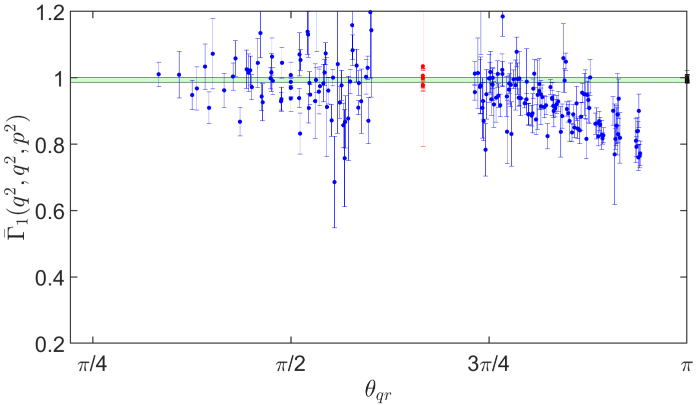


$s = 3$ GeV

For larger momenta, it gets smaller values for $\theta_{qr} \to \pi$

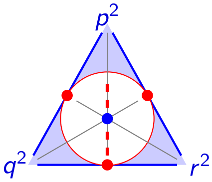


Results for the bisectoral case $q^2 = r^2$: $\overline{\Gamma}_1$

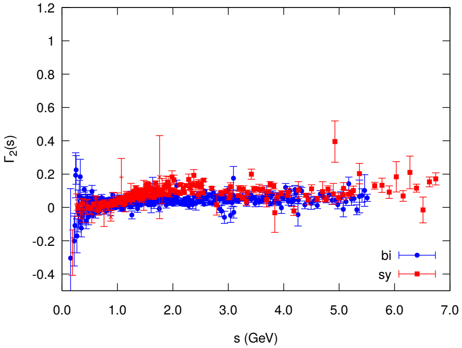


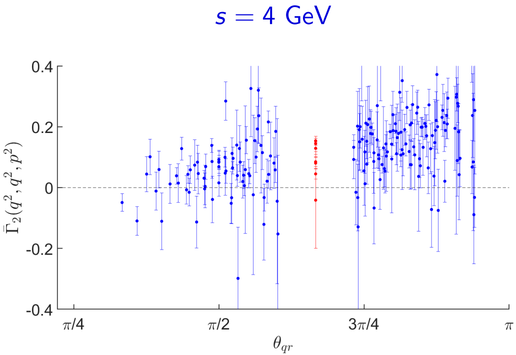
$s = 4$ GeV

For larger momenta, it gets smaller values for $\theta_{qr} \to \pi$



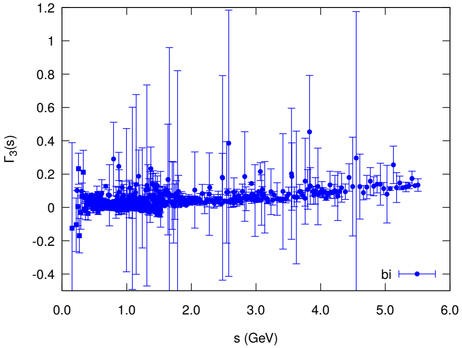
Results for the bisectoral case $q^2 = r^2$: $\overline{\Gamma}_2$

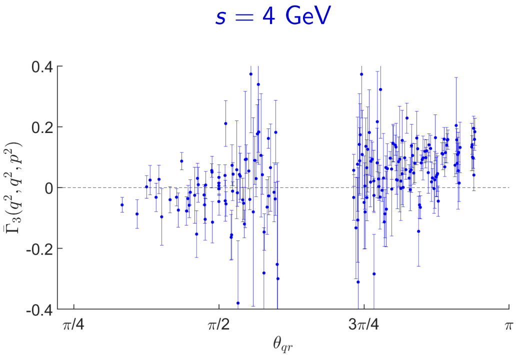




Qualitatively compatible with recent SDE results [2305.05704]

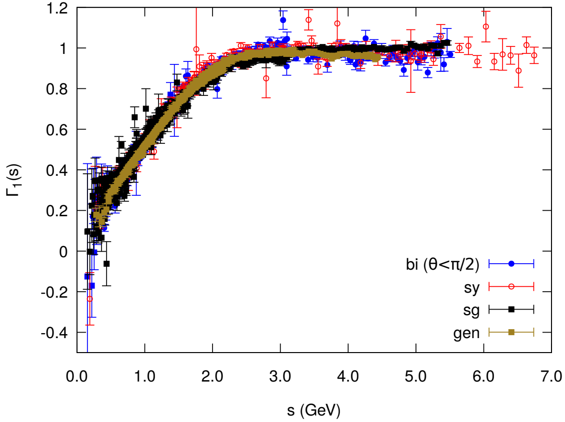
Results for the bisectoral case $q^2 = r^2$: $\overline{\Gamma}_3$





Qualitatively compatible with recent SDE results [2305.05704]

Results for the general case $q^2 \neq r^2 \neq p^2$.

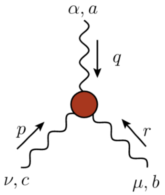


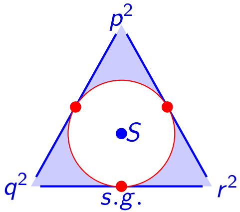
The tree-level form-factor for general kinematics, $q^2 \neq r^2 \neq p^2$, overlaps with the rest of cases for the deep IR (below $s \sim 1.5 - 2$ GeV).

The different kinematics separate from the soft-gluon one at $s \sim 3$ GeV.

Summary of results for 3g-vertex.

- $\overline{\Gamma}_1$ dominates.
- Quantitative agreement among different kinematics for $s^2 = \frac{q^2+r^2+p^2}{2} \lesssim 3$ GeV
- For $q^2 = r^2$ (bisectoral) $\overline{\Gamma}_1$ depends on θ_{qr} for large s^2.
- Preliminary data for the general case $q^2 \neq r^2 \neq p^2$ confirm the latter results.





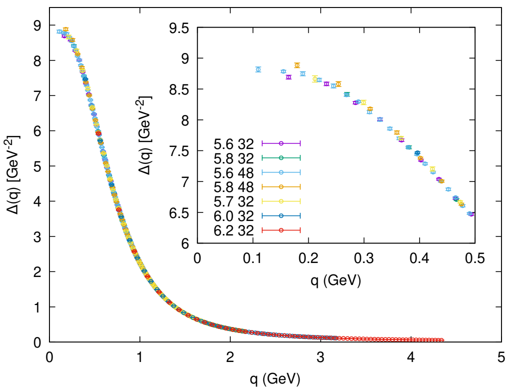
The full vertex seems to be well described by:

$$\overline{\Gamma}^{\alpha\mu\nu}(q,r,p) \approx \overline{\Gamma}^{sg}(s^2)\Big|_{s^2=\frac{q^2+r^2+p^2}{2}} \overline{\Gamma}_0^{\alpha\mu\nu}(q,r,p)$$

Outline

Schwinger mechanism

Lattice data unequivocally establish the existence of a gluon mass:



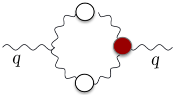
$$\Delta^{-1}(q^2) = q^2\left[1 + \Pi(q^2)\right] \xrightarrow{q\to 0} m^2_{gluon}$$

Schwinger mechanism:

If $\lim_{q\to 0} \Pi(q^2) = \frac{c}{q^2}$, a gluon mass $m^2_{gluon} = c$ emerges.

Linked to the three-gluon vertex through the gluon propagator SDE:

$$\Delta^{-1}(q^2) = \quad q^2 \quad + $$



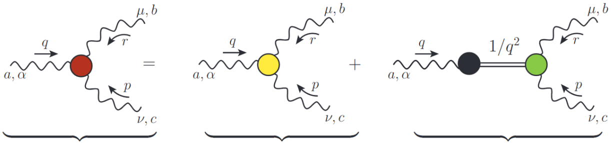
$+\ldots$

J. Papavassiliou, Chin.Phys.C 46 (2022) 11, 112001

Schwinger mechanism

Schwinger mechanism:

- mass generated through longitudinally coupled massless color excitation:



$$\mathrm{I}\Gamma_{\alpha\mu\nu}(q,r,p) = \Gamma_{\alpha\mu\nu}(q,r,p) + V_{\alpha\mu\nu}(q,r,p)$$

Schwinger mechanism

Schwinger mechanism:

- mass generated through longitudinally coupled massless color excitation:

$$\Gamma^{abc}_{\alpha\mu\nu}(q,r,p) \xrightarrow{q\to 0} \underbrace{\Gamma^{abc}_{\alpha\mu\nu}(q,r,p)}_{\text{pole-free}} + \underbrace{f^{abc}\frac{q_\alpha}{q^2} g_{\mu\nu} C_1(q,r,p) + \cdots}_{\text{longitudinally coupled}}$$

- introduces a displacement in Ward-Takahashi identity $\mathcal{C}(r^2)$:

$$\mathcal{C}(r^2) = \left.\frac{\partial C_1(q,r,p)}{\partial p^2}\right|_{q=0} = L_{sg}(r^2) - F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial \Delta^{-1}(r^2)}{\partial r^2}\right]$$

[A.C. Aguilar, FS, *et al*, PLB841 (2023) 137906]

Schwinger mechanism

All the ingredients in the displacement function can be evaluated from lattice-QCD:

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- $\Delta(r^2)$, gluon propagator.
- $L_{sg}(r^2)$, soft-gluon three-gluon vertex.
- $F(0)$ bare ghost dressing function.
- $\frac{\mathcal{W}(r^2)}{r^2} r_\rho \delta_{\mu\nu} = \left.\frac{\partial H_{\mu\nu}}{\partial q_\rho}\right|_{q=0}$

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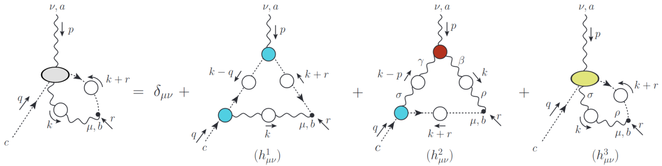
The ghost-gluon scattering kernel $H_{\mu\nu}$ can be evaluated through the solution of its SDE equation:

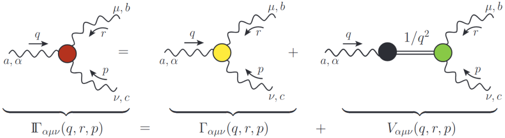


Schwinger mechanism

Lattice-evaluated Ward identity displacement function:

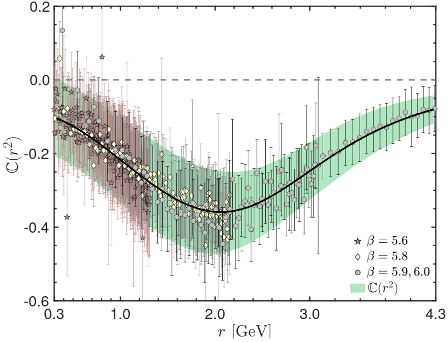
$$\mathcal{C}(r^2) = L_{sg}(r^2) - F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial\Delta^{-1}(r^2)}{\partial r^2}\right]$$

signals the presence of massless, longitudinally coupled gluon correlations.



Compatible with the solution of the gluon BSE for a massless bound state.

[A.C. Aguilar, *et al*, PRD105 (2022) 014030]



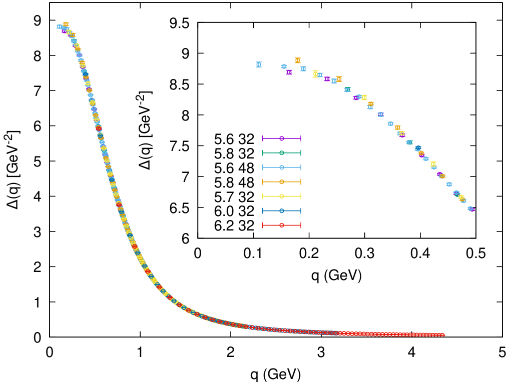
[A.C. Aguilar, FS, *et al*, PLB841 (2023) 137906]

Outline

1. Introduction
   - QCD
   - Lattice QCD
2. Three-gluon vertex
   - Evaluating 3g vertex
   - Kinematics and tensorial structure of $\Gamma^{\alpha\mu\nu}(q, r, p)$
   - Results
3. Phenomenology
   - Schwinger mechanism
   - Zero crossing
4. Conclusions

Gluon propagator

Lattice data unequivocally establish the existence of a gluon mass:



Gluon propagator:

$$\Delta^{-1}(q^2) = q^2 J(q^2) + m^2(q^2)$$

PT-BFM [D. Binosi, *et al.* PRD86 (2012) 085033]

- mass: $m_{gluon} = \lim_{q \to 0} m(q^2)$
- kinetic term presents a logarithmic divergence:

$$J(q^2)\big|_{q \to 0} \sim a \log\left(\frac{q^2}{\mu^2}\right) + b$$

related to the masslessness of the ghost

[A.C. Aguilar, *et al*, PRD89 (2014) 085008].

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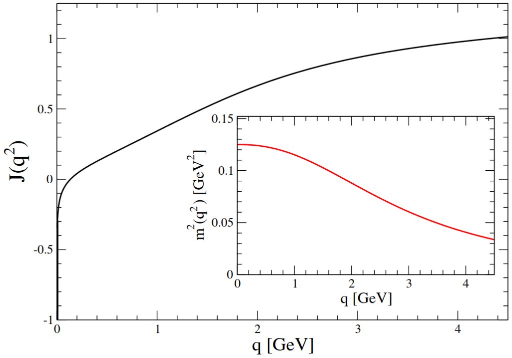

[A.C. Aguilar, FS, *et al*, PLB818 (2021) 136352]

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Zero crossing

With the three-gluon vertex written as:

$$\boldsymbol{\Gamma}^{abc}_{\alpha\mu\nu}(q,r,p) \xrightarrow{q\to 0} \underbrace{\Gamma^{abc}_{\alpha\mu\nu}(q,r,p)}_{\text{pole-free}} + \underbrace{V^{abc}_{\alpha\mu\nu}(q,r,p)}_{\text{longitudinally coupled}} \,,$$

if we assume a separation of the STI satisfied by $\boldsymbol{\Gamma}$ into two *p*artial STI's matching $\Gamma \leftrightarrow J$ and $V \leftrightarrow m^2$, then:

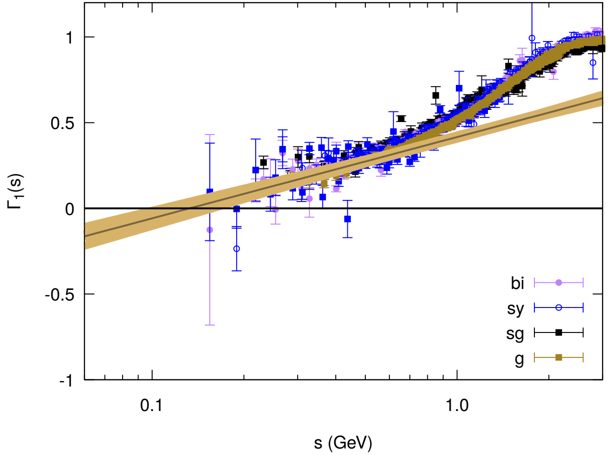
$$\overline{\Gamma}_1(s^2) \xrightarrow{s\to 0} \alpha \log(s^2/\mu^2) + \beta$$

Zero crossing

The form-factor $\overline{\Gamma}_1(s^2)$ is logarithmically divergent in the deep-IR.

[A.C. Aguilar, FS, *et al*, PLB818 (2021) 136352].

Zero crossing

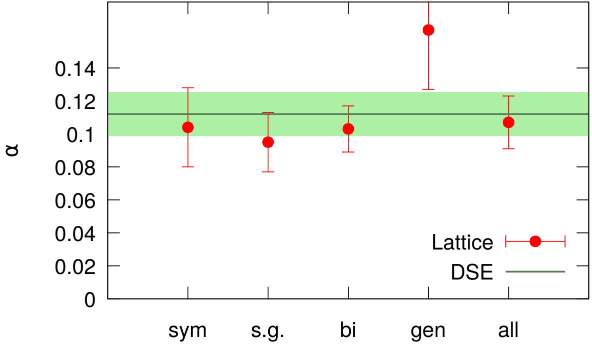


Fitting all data with $s \leq 0.5\,\text{GeV}$ to $\overline{\Gamma}_1(s^2) = \alpha \ln(s^2/\mu^2) + \beta$

The logarithmic slope obtained is $\alpha \approx 0.107(16)$, while the SDE prediction is $0.112(10)$!

A zero crossing at appears at $s \sim 130(20)$ MeV.

Zero crossing



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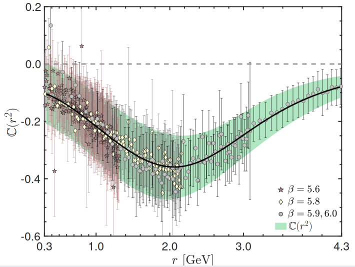
Lattice data suggest a deep-IR zero-crossing for the tree-level form factor.

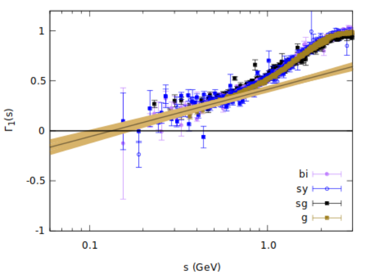
Summary

- The tree-level contribution $\overline{\Gamma}_1$ dominates the transversely projected 3g vertex.
- Planar degeneracy up to $\sim 3\,\text{GeV}$ for all kinematics.
- $\overline{\Gamma}_{\alpha\mu\nu}(q,r,p) \approx \overline{\Gamma}_1(s)\Big|_{s^2=\frac{q^2+r^2+p^2}{2}} \lambda^{\text{t.l.}}_{\alpha\mu\nu}(q,r,p)$

Scwinger mechanism



Zero-crossing



Backup slides