Proton GPDs from lattice QCD

Martha Constantinou

Temple University

FONDAZIONE BRUNO KESSLER

ECT*
EUROPEAN CENTRE FOR THEORETICAL STUDIES IN NUCLEAR PHYSICS AND RELATED AREAS

From first-principles QCD to experiments

May 23, 2023

Collaborators

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Twist-2 PDFs and GPDs

- **C. Alexandrou**
  Univ. of Cyprus/Cyprus Institute
- **K. Cichy**
  Adam Mickiewicz University
- **K. Hadjiyiannakou**
  Cyprus Institute
- **K. Jansen**
  DESY, Zeuthen
- **A. Scapellato**
  Temple University
- **F. Steffens**
  University of Bonn

ETMC Meeting 2008

PHYSICAL REVIEW LETTERS **125,** 262001 (2020)

Unpolarized and Helicity Generalized Parton Distributions of the Proton within Lattice QCD

Constantia Alexandrou,[1,2] Krzysztof Cichy,[3] Martha Constantinou,[4] Kyriakos Hadjiyiannakou,[1] Karl Jansen,[5] Aurora Scapellato,[3] and Fernanda Steffens[6]

PHYSICAL REVIEW D **105,** 034501 (2022)

Transversity GPDs of the proton from lattice QCD

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Extended Twisted Mass Collaboration

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Twist-3 PDFs and GPDs

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- **K. Cichy**
  Adam Mickiewicz University
- **J. Dodson**
  Temple University
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TMD Collaboration

TMD Meeting 2016

PHYSICAL REVIEW D **102**, 111501(R) (2020)

Rapid Communications | Editors' Suggestion

Insights on proton structure from lattice QCD: The twist-3 parton distribution function $g_T(x)$

Shohini Bhattacharya,[1] Krzysztof Cichy,[2] Martha Constantinou,[1] Andreas Metz,[1] Aurora Scapellato,[2] and Fernanda Steffens[3]

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Parton distribution functions beyond leading twist from lattice QCD: The $h_L(x)$ case

Shohini Bhattacharya,[1] Krzysztof Cichy,[2] Martha Constantinou,[1] Andreas Metz,[1] Aurora Scapellato,[1] and Fernanda Steffens[3]

PoS PROCEEDINGS OF SCIENCE

PHYSICAL REVIEW D **106**, 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

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PROCEEDINGS OF SCIENCE

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Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

Novel approach on GPDs

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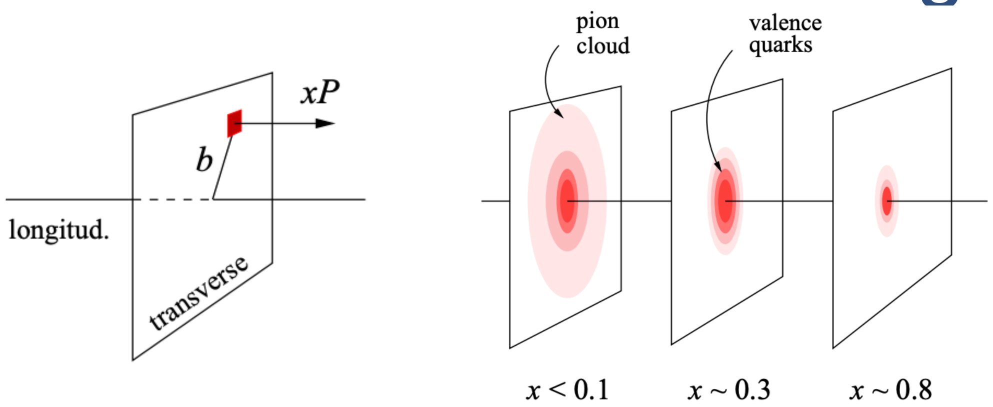
Motivation for GPDs studies

★ Crucial in understanding hadron tomography



[H. Abramowicz et al., whitepaper for NSAC LRP, 2007]

$1_{\text{mom}} + 2_{\text{coord}}$ tomographic images of quark distribution in nucleon at fixed longitudinal momentum

3-D image from FT of the longitudinal mom. transfer

★ Provide information on spatial distribution of partons inside the hadron, and its mechanical properties (OAM, pressure, etc.)

[M. Burkardt, PRD62 071503 (2000), hep-ph/0005108] [M. V. Polyakov, PLB555 (2003) 57, hep-ph/0210165]

★ GPDs are not well-constrained experimentally:

- x-dependence extraction is not direct. DVCS amplitude:

$$\mathscr{H} = \int_{-1}^{+1} \frac{H(x,\xi,t)}{x-\xi+i\epsilon} dx$$

(SDHEP [J. Qiu et al, arXiv:2205.07846] gives access to x)

- independent measurements to disentangle GPDs
- GPDs phenomenology more complicated than PDFs (multi-dimensionality)
- and more challenges …

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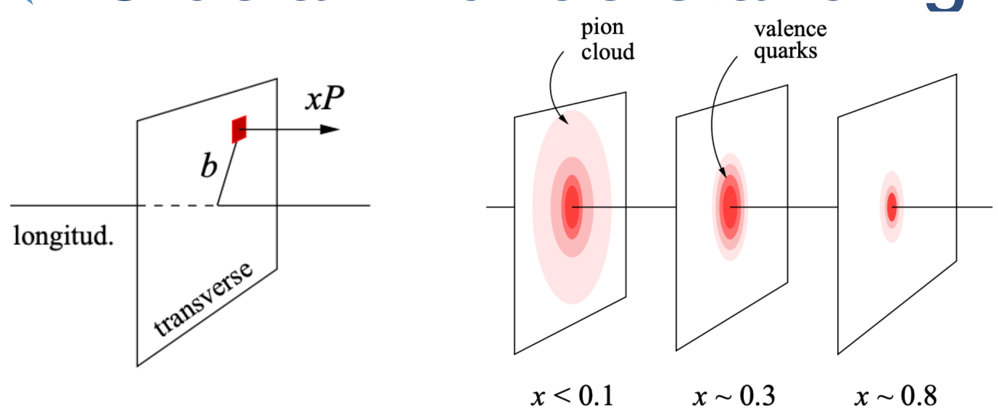

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Essential to complement the knowledge on GPD from lattice QCD

Twist-classification of PDFs, GPDs, TMDs

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

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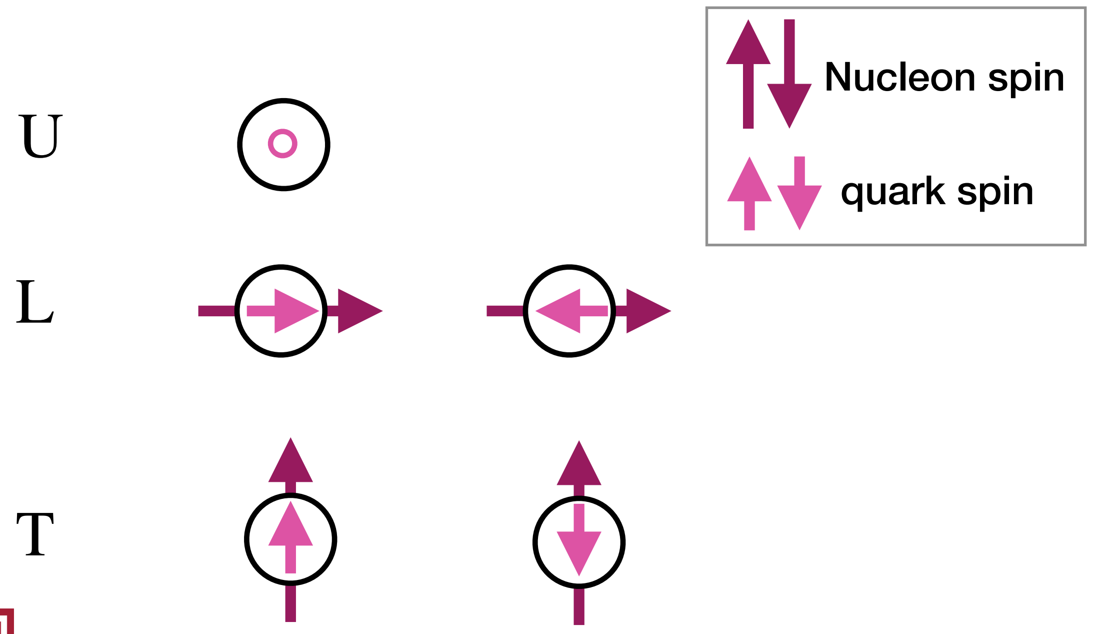
Twist-2 $(f_i^{(0)})$

Quark / Nucleon	U (γ^+)	L $(\gamma^+\gamma^5)$	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity

Probabilistic interpretation



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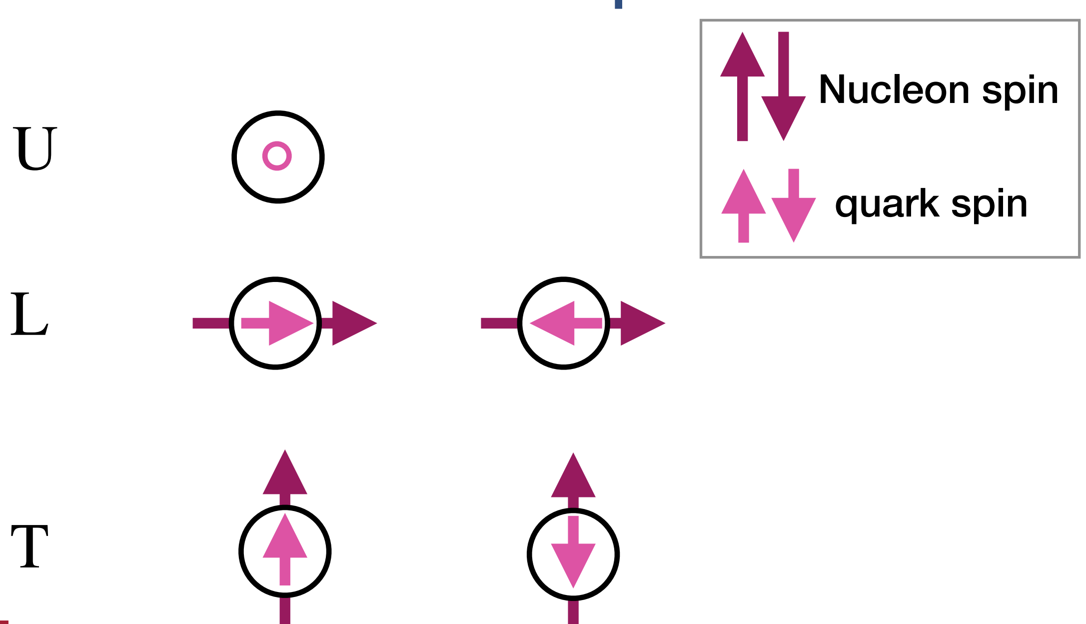
Twist-3 $(f_i^{(1)})$

Nucleon \ $\mathcal{O}$	γ^j	$\gamma^j\gamma^5$	σ^{jk}
U	G_1, G_2 G_3, G_4		
L		$\widetilde{G}_1, \widetilde{G}_2$ $\widetilde{G}_3, \widetilde{G}_4$	
T			$H_2'(x,\xi,t)$ $E_2'(x,\xi,t)$

Selected

Probabilistic interpretation



- ★ Lack density interpretation, but can be sizable
- ★ Kinematically suppressed
  Difficult to isolate experimentally
- ★ Theoretically: contain $\delta(x)$ singularities
- ★ Contain info on quark-gluon-quark correlators

Accessing information on GPDs

★ **Mellin moments** (local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z,\tfrac{1}{2}z]\,q(\tfrac{1}{2}z) = \sum_{n=0}^{\infty}\frac{1}{n!}\,z_{\alpha_1}\ldots z_{\alpha_n}\left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_n} q\right]$$

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1}\left\{\gamma^{\{\mu}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}A_{n,i}(t) - i\frac{\Delta_\alpha\sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}B_{n,i}(t)\right\} + \frac{\Delta^\mu\Delta^{\mu_1}\ldots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\ \text{even}}\Big)\Big\}$$

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local operators

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★ **Matrix elements of non-local operators**
(quasi-GPDs, pseudo-GPDs, …)

$$\langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

$$\langle N(P')|O_V^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_A^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu\gamma_5\widetilde{H}(x,\xi,t) + \frac{\gamma_5\Delta^\mu}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

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Local vs nonlocal operators

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Form Factors & Generalizations

★ **Ultra-local operators (FFS)**

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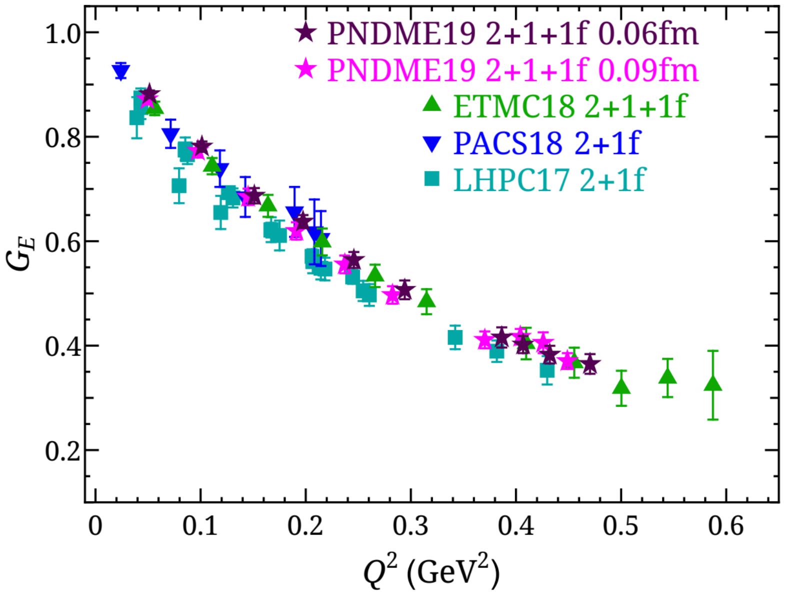

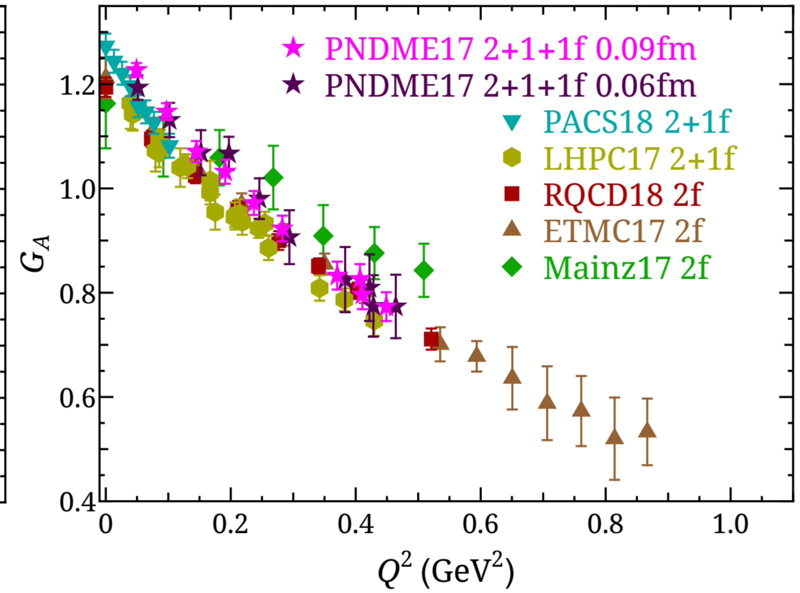

[M. Constantinou et al. (2020 PDFLattice Report), Prog.Part.Nucl.Phys. 121 (2021) 103908, arXiv:2006.08636]

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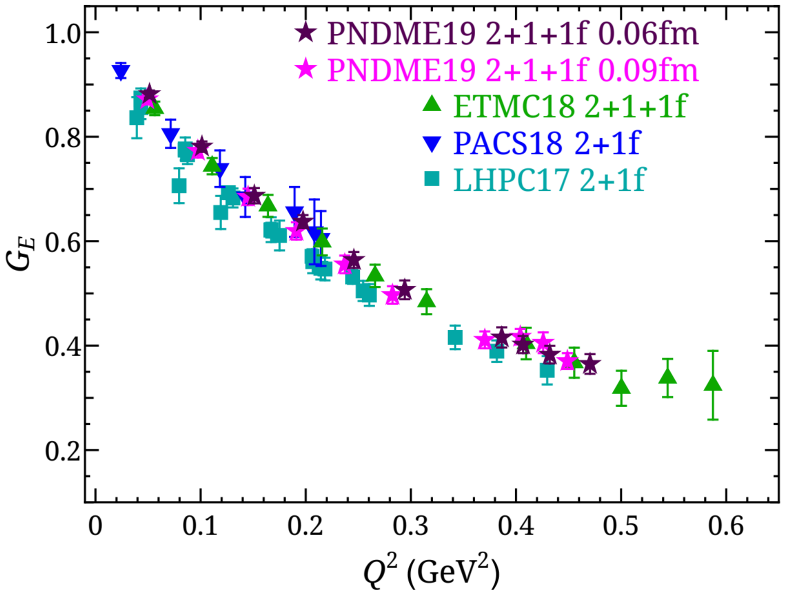

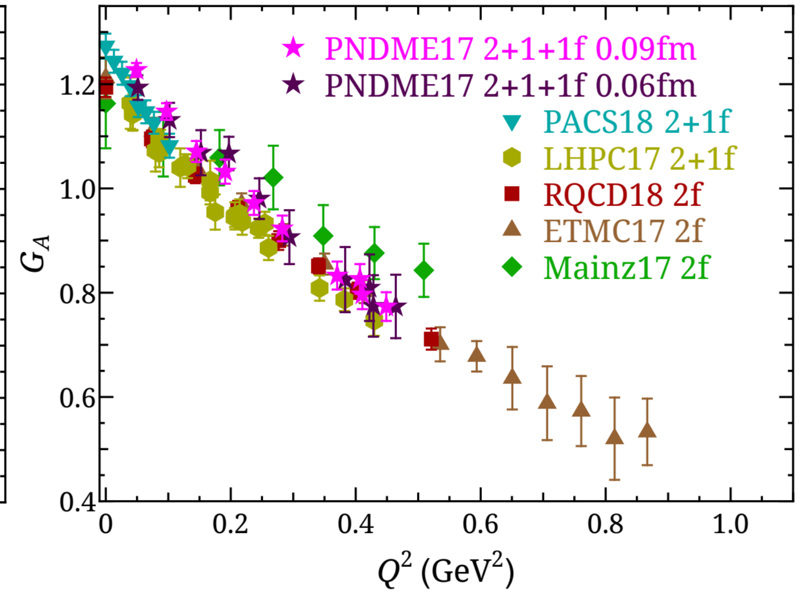

- Simulations at physical point available by multiple groups
- Precision data era
- Towards control of systematic uncertainties

[M. Constantinou et al. (2020 PDFLattice Report), Prog.Part.Nucl.Phys. 121 (2021) 103908, arXiv:2006.08636]

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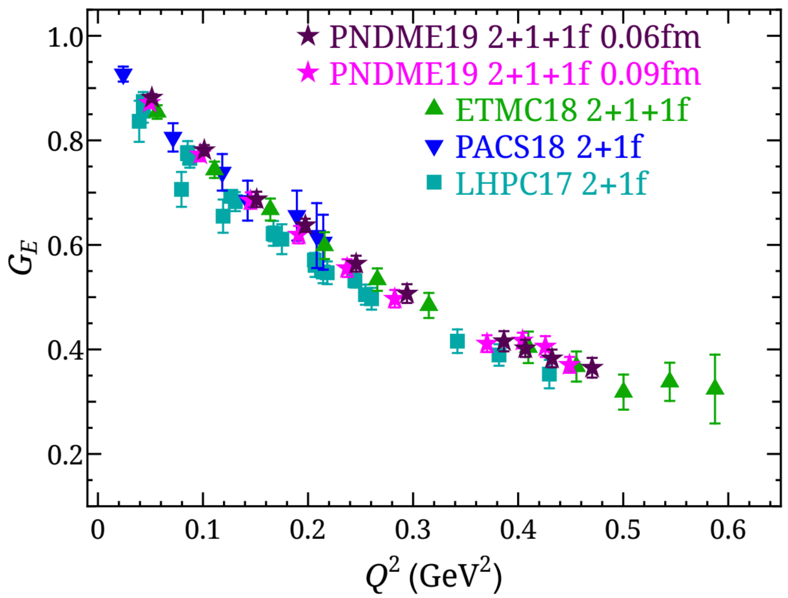
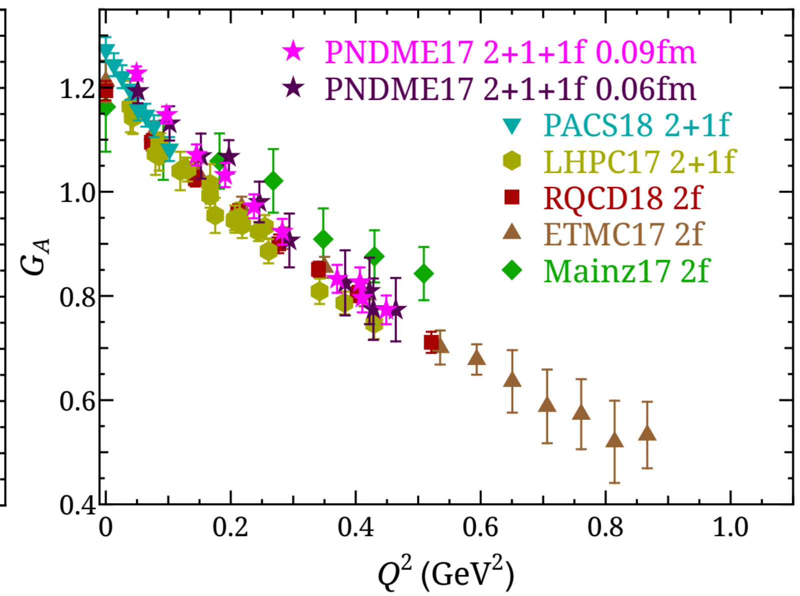


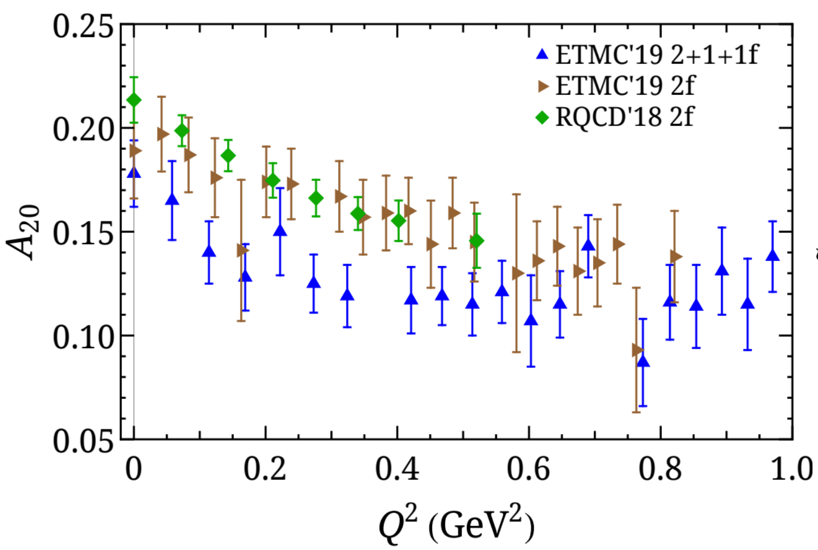


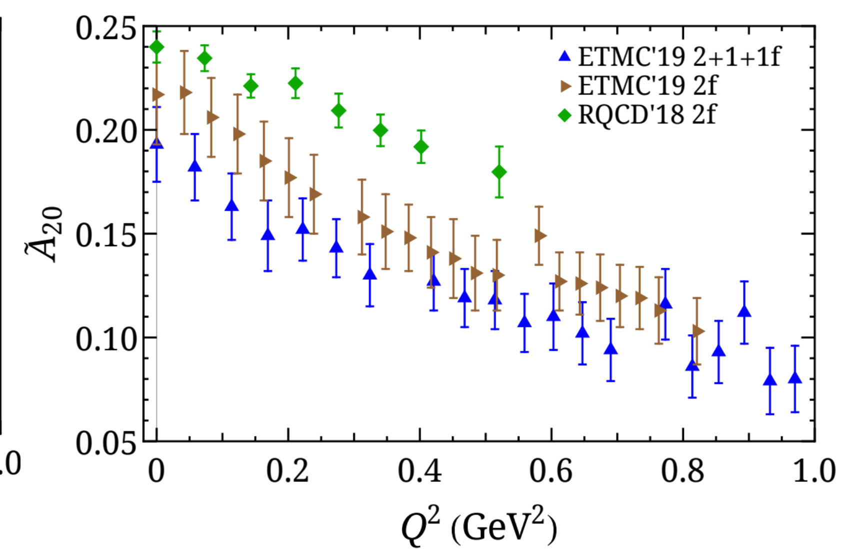
- Simulations at physical point available by multiple groups
- Precision data era
- Towards control of systematic uncertainties

★ 1-derivative operators (GFFs)

$$\langle N(p',s')|\mathcal{O}_V^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\frac{1}{2}\Big[A_{20}(q^2)\,\gamma^{\{\mu}P^{\nu\}} + B_{20}(q^2)\,\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m_N} + C_{20}(q^2)\,\frac{1}{m_N}q^{\{\mu}q^{\nu\}}\Big]u_N(p,s)$$

$$\langle N(p',s')|\mathcal{O}_A^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\frac{i}{2}\Big[\tilde{A}_{20}(q^2)\,\gamma^{\{\mu}P^{\nu\}}\gamma^5 + \tilde{B}_{20}(q^2)\,\frac{q^{\{\mu}P^{\nu\}}}{2m_N}\gamma^5\Big]u_N(p,s),$$





[M. Constantinou et al. (2020 PDFLattice Report), Prog.Part.Nucl.Phys. 121 (2021) 103908, arXiv:2006.08636]

Form Factors & Generalizations

★ Ultra-local operators (FFS)

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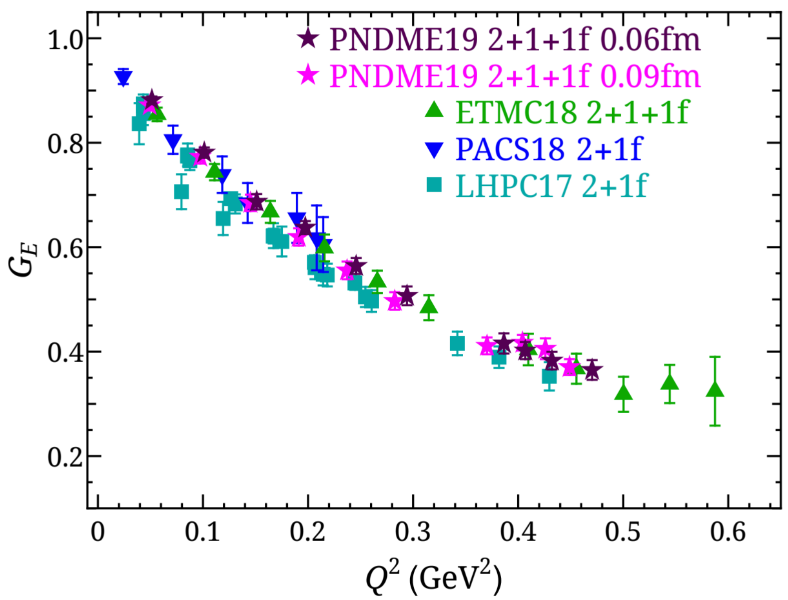

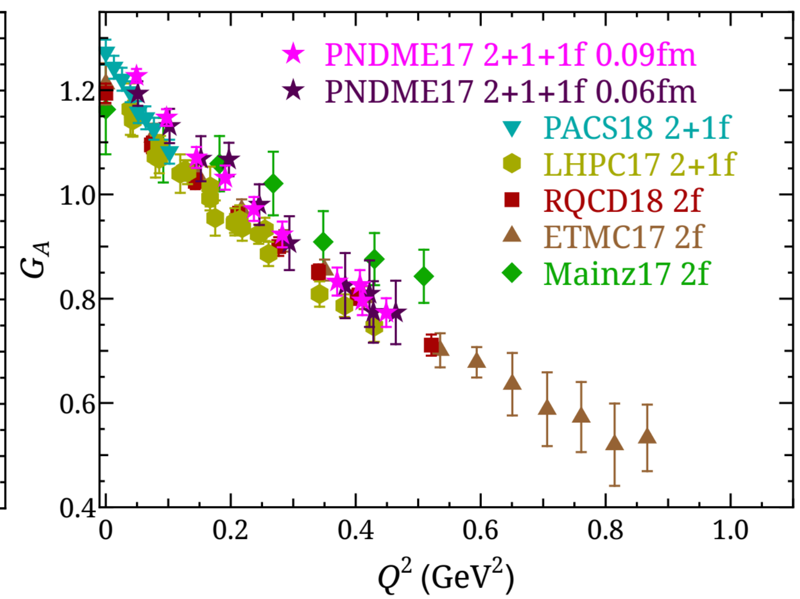

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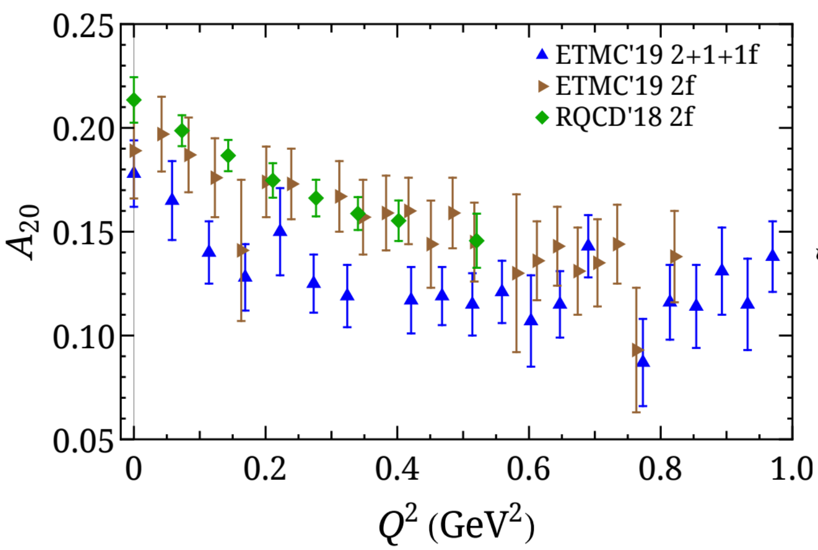

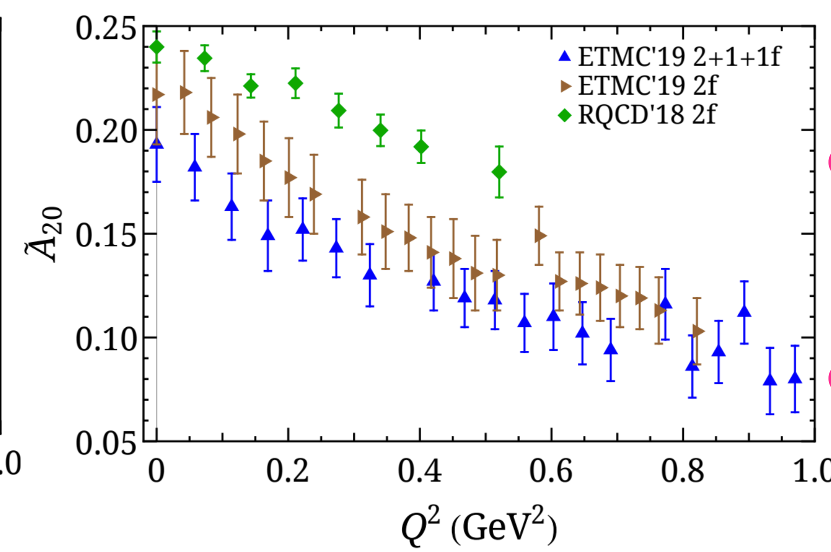

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[M. Constantinou et al. (2020 PDFLattice Report), Prog.Part.Nucl.Phys. 121 (2021) 103908, arXiv:2006.08636]

Form Factors & Generalizations

Wide -t range that comes at the cost of 1

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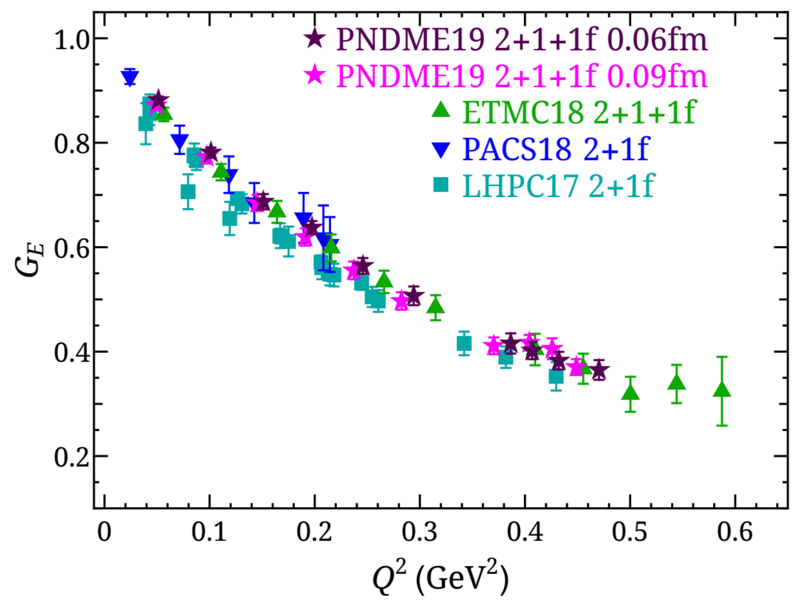

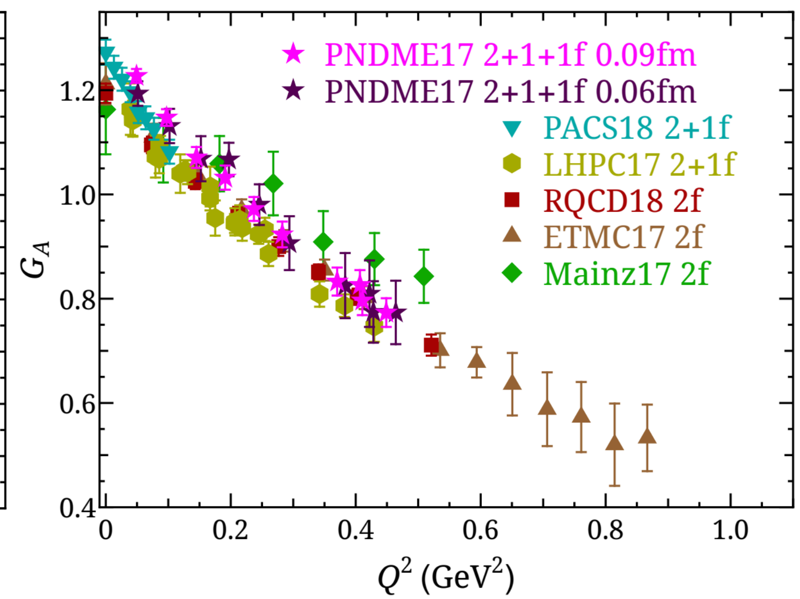

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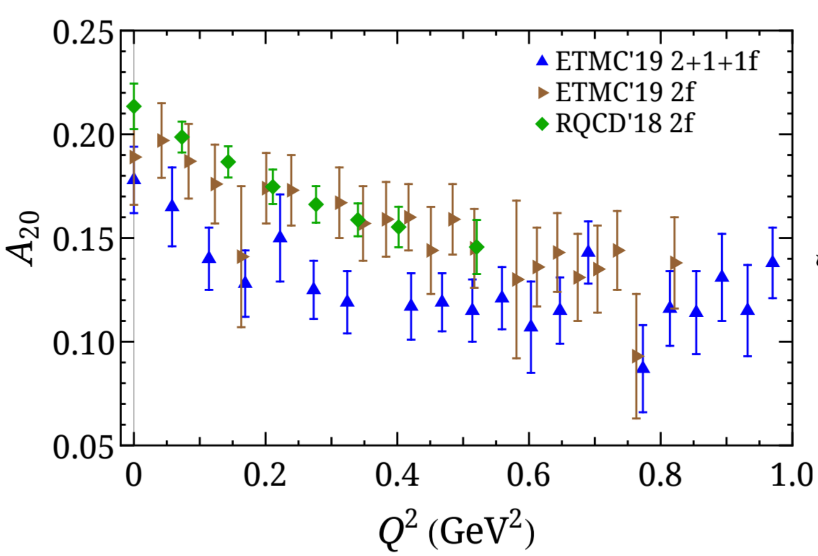

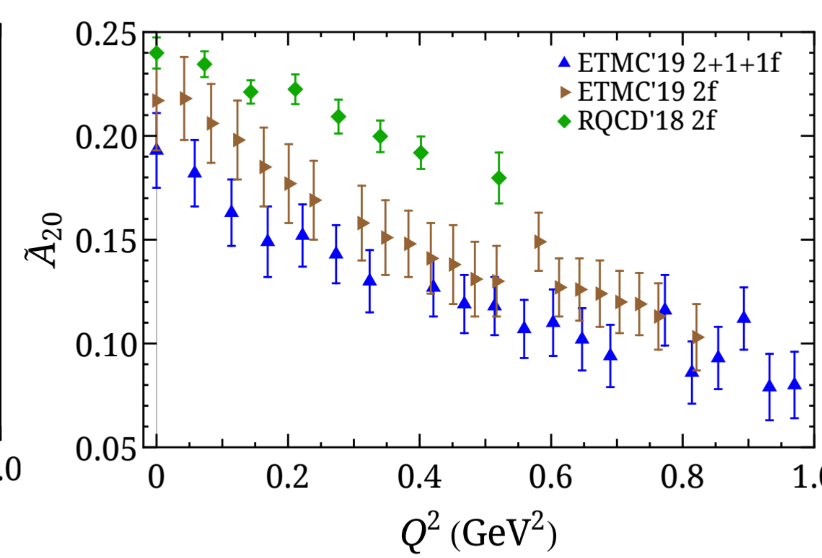

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GPDs

Through non-local matrix elements

of fast-moving hadrons

Access of GPDs on a Euclidean Lattice

[X. Ji, Phys. Rev. Lett. 110 (2013) 262002]

Matrix elements of nonlocal (equal-time) operators with fast moving hadrons

$$\tilde{q}_{\Gamma}^{\mathrm{GPD}}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \quad \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_{\mu}$$

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -\,Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$

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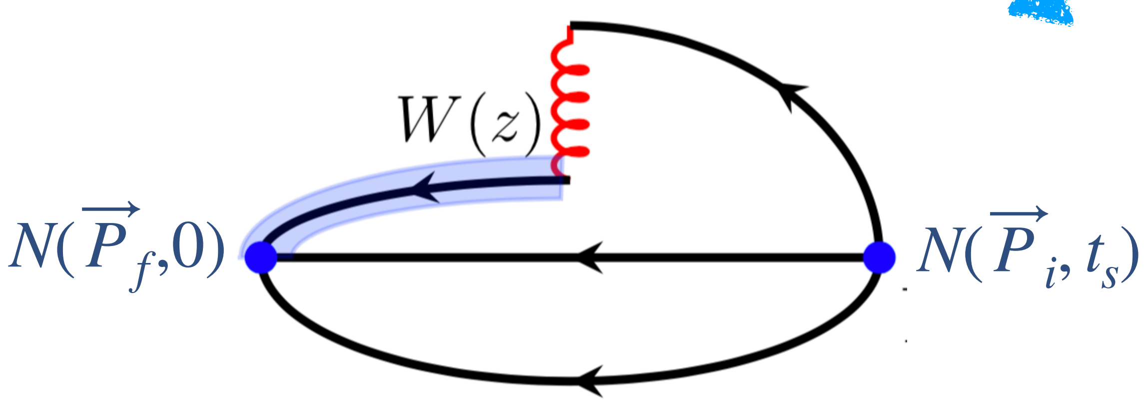

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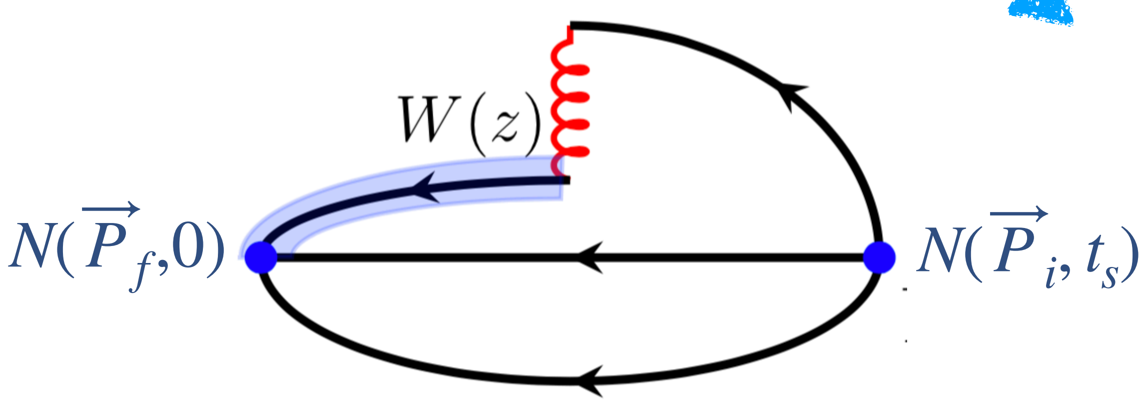

$$\Delta = P_f - P_i$$

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Variables of the calculation:

- length of the Wilson line (z)
- nucleon momentum boost (P_3)
- momentum transfer (t)
- skewness (ξ)

GPDs on the lattice

★ GPDs: off-forward matrix elements of non-local light-cone operators

$$F^{[\gamma^+]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p';\lambda' | \bar{\psi}(-\tfrac{z}{2})\,\gamma^+ \mathscr{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2}) | p;\lambda\rangle \Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

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★ Off-forward correlators with nonlocal (equal-time) operators [Ji, PRL 110 (2013) 262002]

$$\tilde{q}^{\rm GPD}_\mu(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \langle N(P_f) | \bar\Psi(z)\gamma^\mu \mathscr{W}(z,0)\Psi(0) | N(P_i)\rangle_\mu$$

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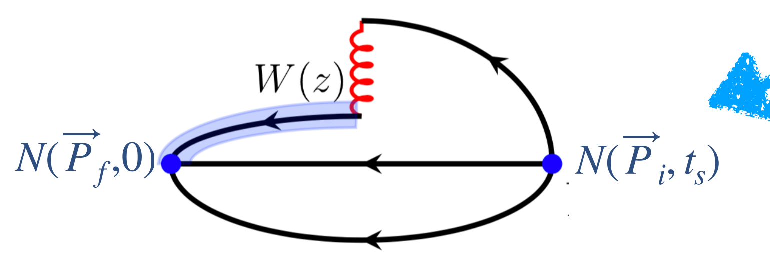

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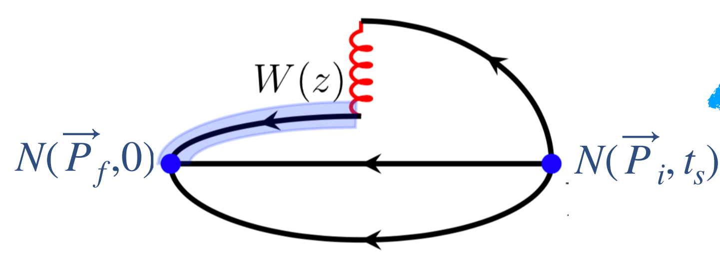

★ Potential parametrization (γ^+ inspired)

$$F^{[\gamma^0]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^0 H_{Q(0)}(x,\xi,t;P^3) + \frac{i\sigma^{0\mu}\Delta_\mu}{2M}E_{Q(0)}(x,\xi,t;P^3)\right]u(p,\lambda)$$

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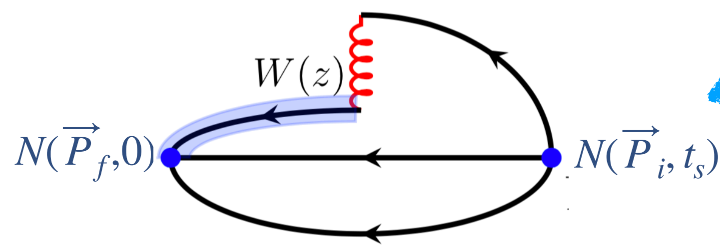

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→ reduction of power corrections in fwd limit [Radyushkin, PLB 767, 314, 2017]

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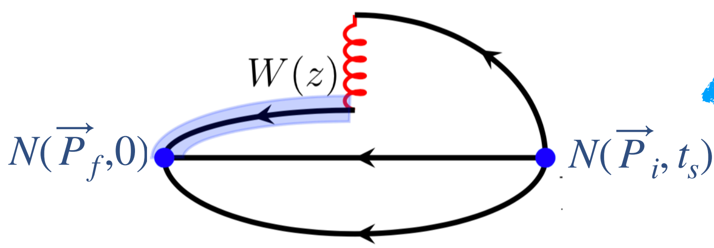

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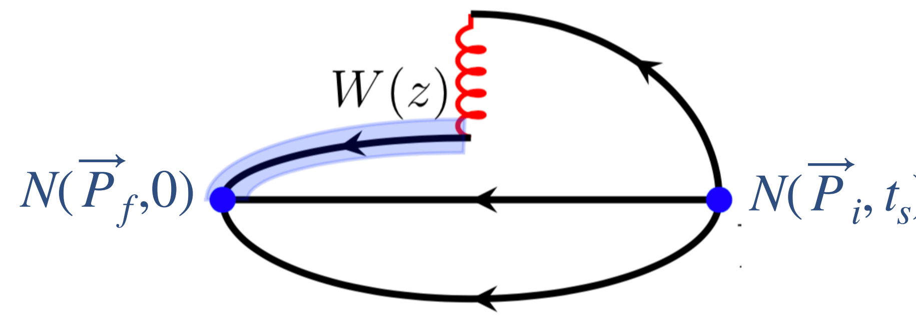
★ Symmetric frame ($\vec{p}^{\,s}_f = \vec{P} + \vec{Q}/2\,, \vec{p}^{\,s}_i = \vec{P} - \vec{Q}/2$): separate calculations at each t

Parameters of calculation

★ Nf=2+1+1 twisted mass (TM) fermions & clover term

Pion mass:	260 MeV
Lattice spacing:	0.093 fm
Volume:	32^3 x 64
Spatial extent:	3 fm



★ Proton Momentum:

P_3 [GeV]	$\vec{Q} \times \frac{L}{2\pi}$	$-t$ [GeV^2]	ξ	$N_{\rm confs}$	$N_{\rm meas}$
0.83	(0,2,0)	0.69	0	519	4152
1.25	(0,2,0)	0.69	0	1315	42080
1.67	(0,2,0)	0.69	0	1753	112192
1.25	(0,2,2)	1.39	1/3	417	40032
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★ Excited states: $T_{\rm sink}$=1, 1.12 fm

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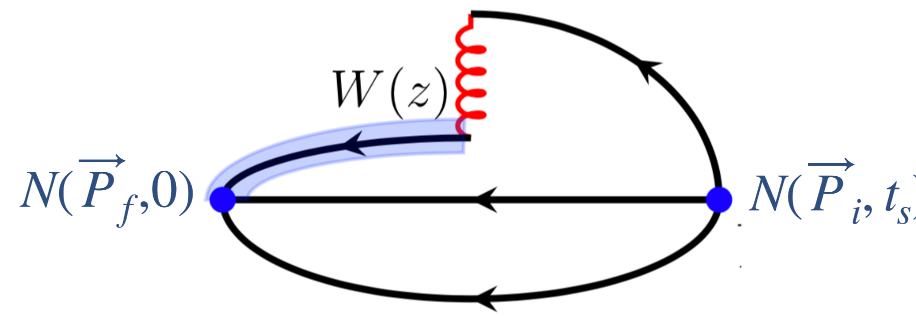

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zero skewness → (first three rows)

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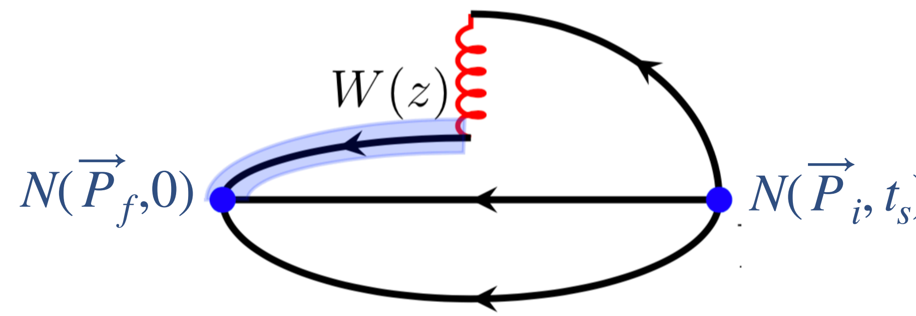

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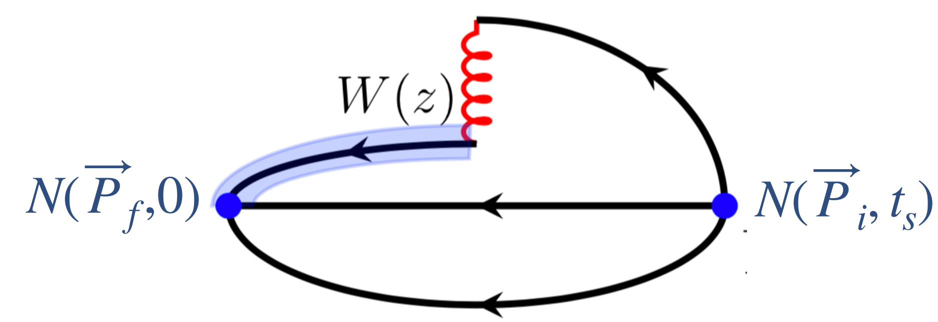

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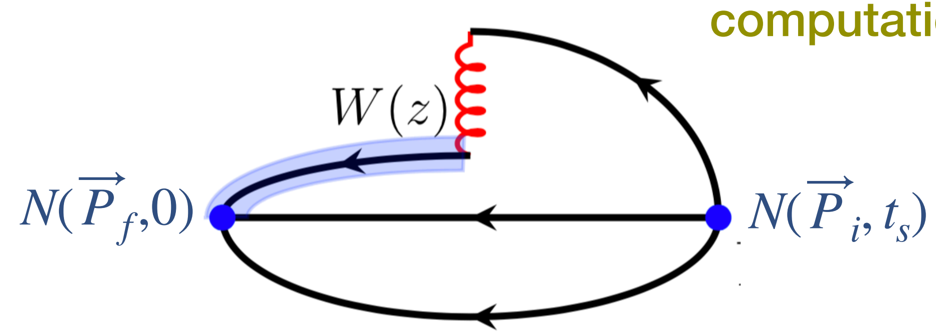
Symmetric frame very expensive computationally



★ Proton Momentum:

	P_3 [GeV]	$\vec{Q}\times\frac{L}{2\pi}$	$-t$ [GeV^2]	ξ	$N_{\rm confs}$	$N_{\rm meas}$
zero skewness	0.83	(0,2,0)	0.69	0	519	4152
	1.25	(0,2,0)	0.69	0	1315	42080
	1.67	(0,2,0)	0.69	0	1753	112192
nonzero skewness	1.25	(0,2,2)	1.39	1/3	417	40032
	1.25	(0,2,-2)	1.39	-1/3	417	40032

★ Excited states: $T_{\rm sink}$=1, 1.12 fm

First lattice calculation of x-dependent GPDs

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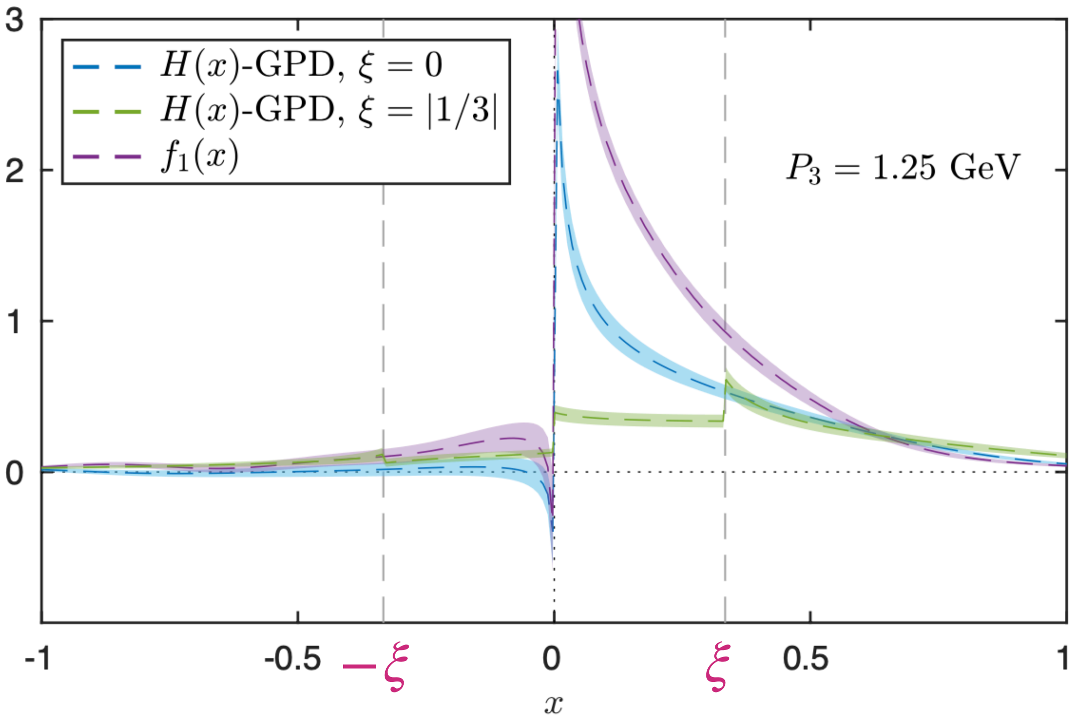

[C. Alexandrou et al., PRL 125, 262001 (2020)]

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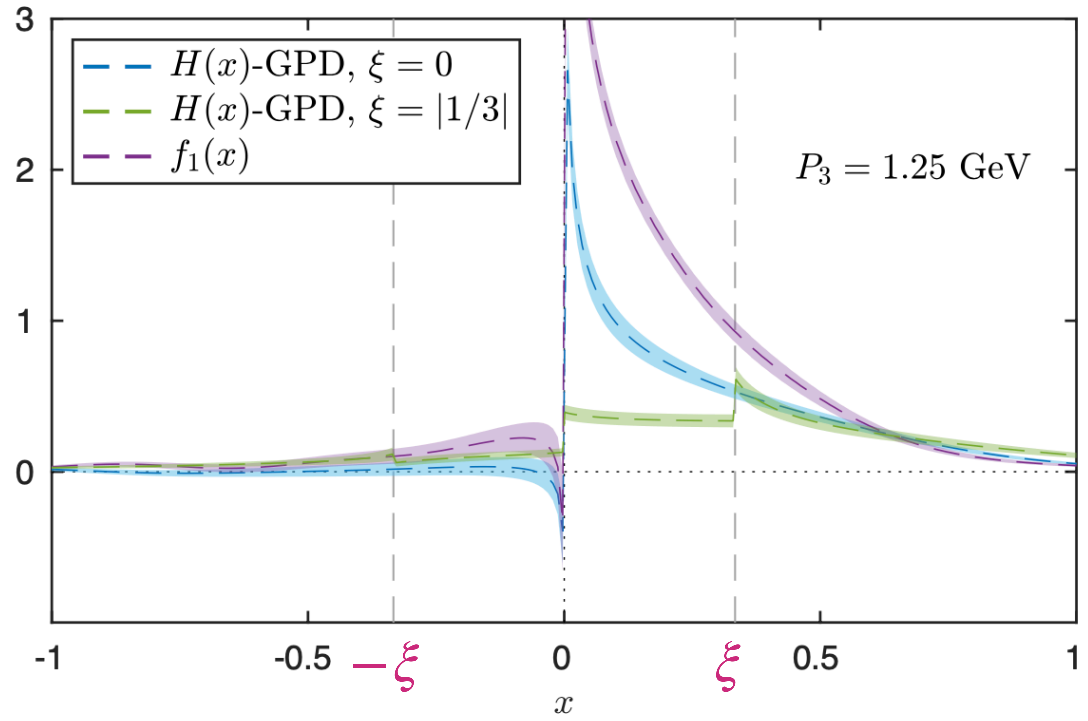

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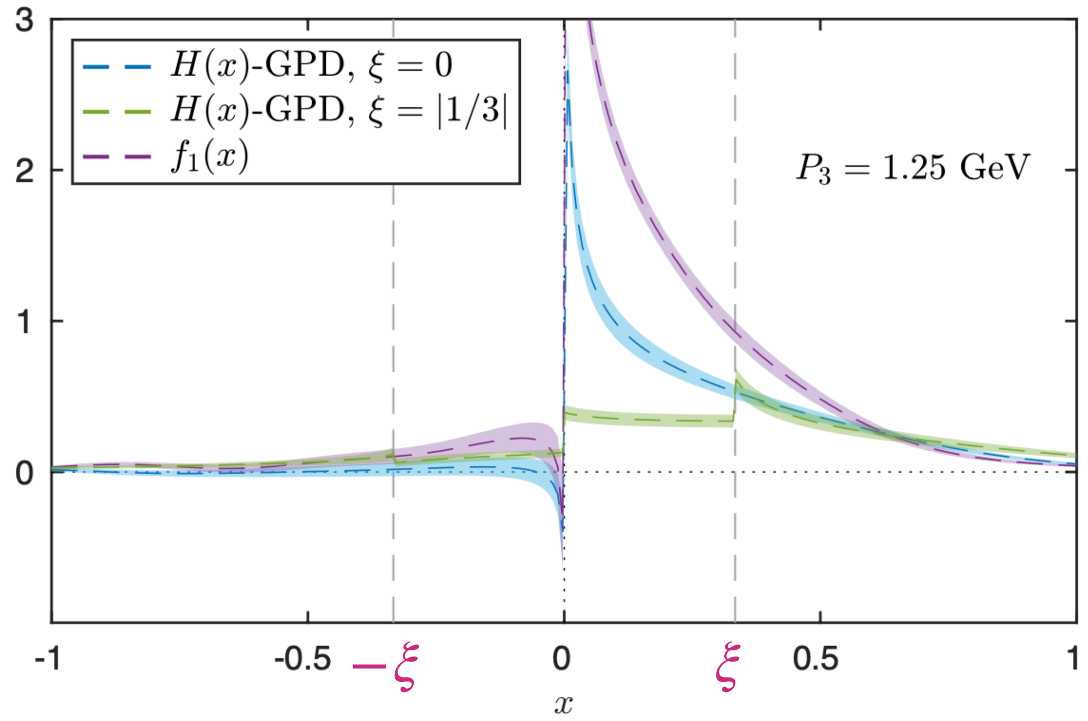

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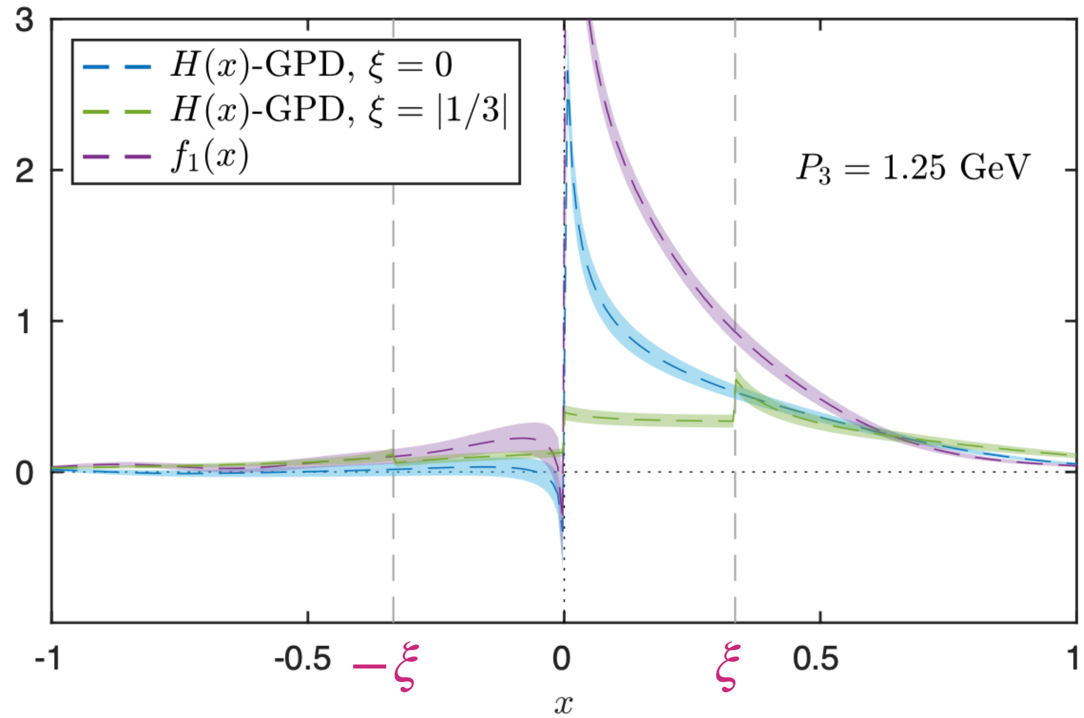

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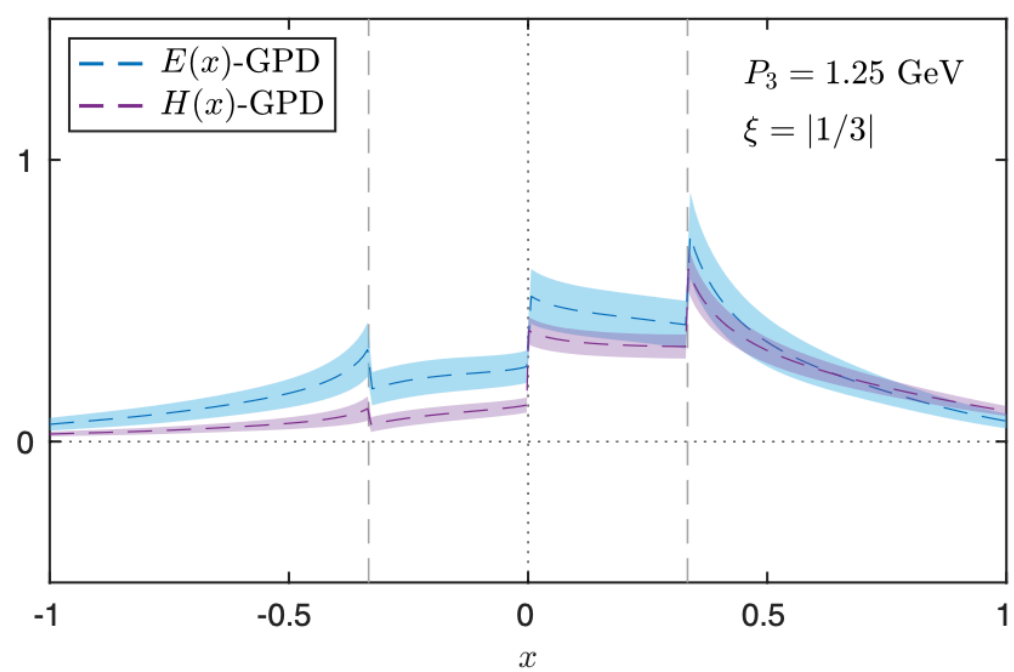

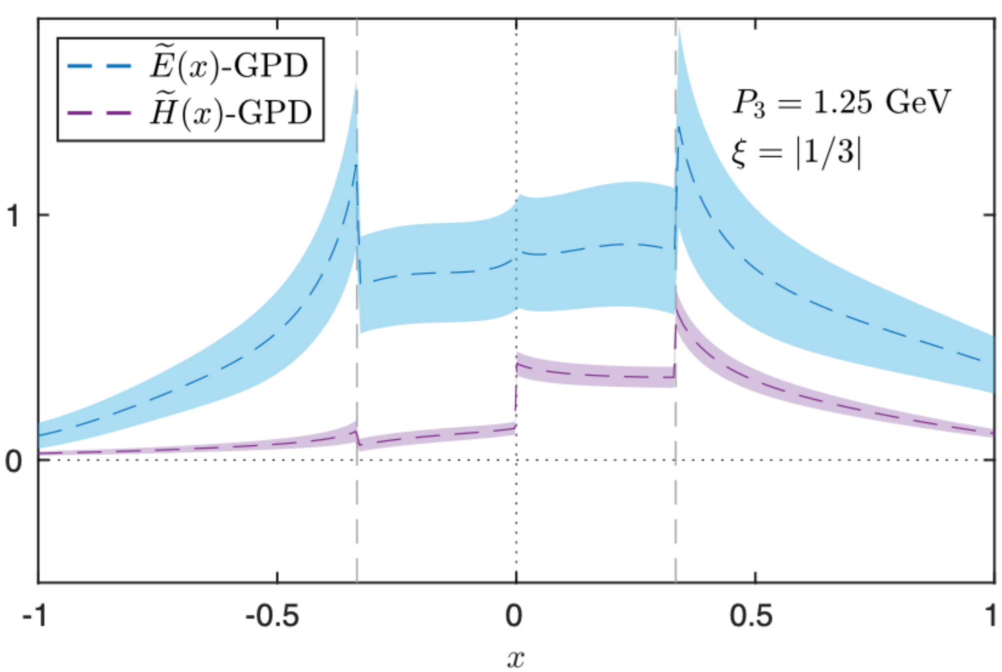

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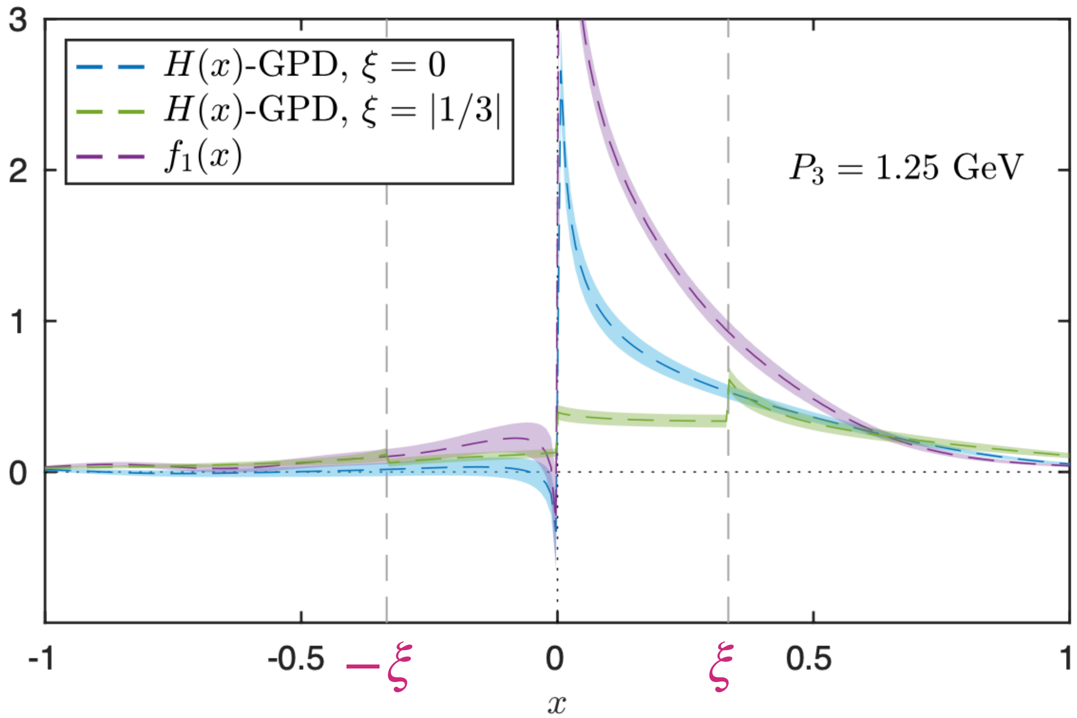

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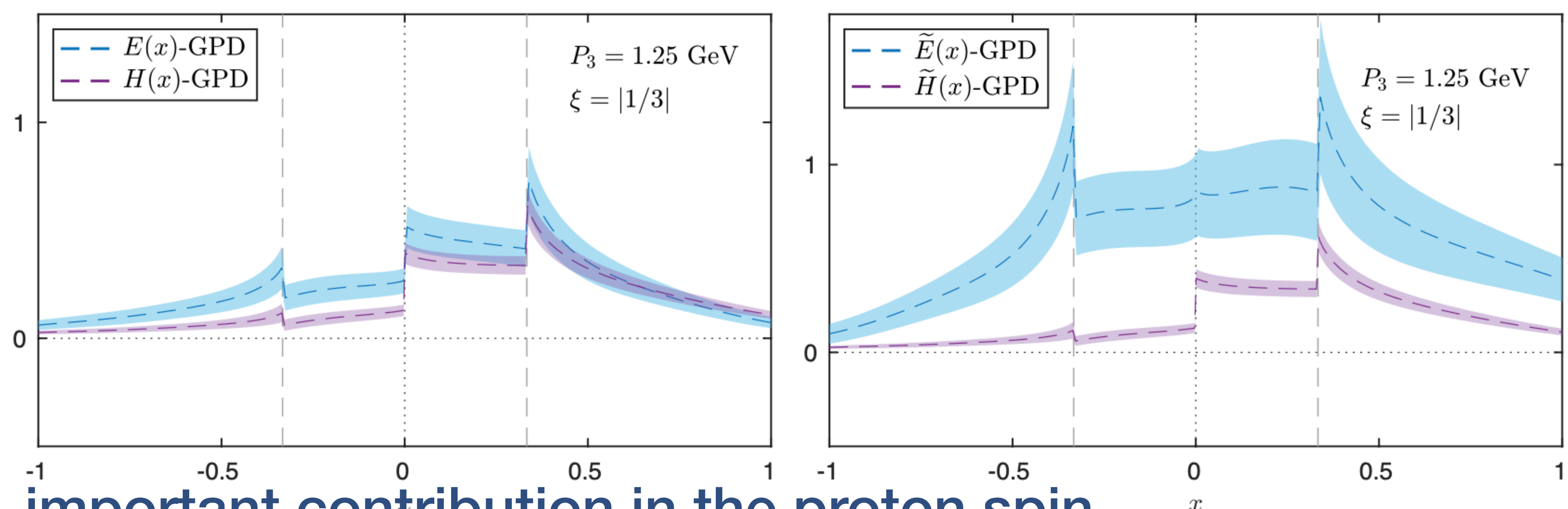

★ important contribution in the proton spin

$$\int_{-1}^{+1} dx x^2 H^q(x,\xi,t) = A_{20}^q(t) + 4\xi^2 C_{20}^q(t)\,, \qquad \int_{-1}^{+1} dx x^2 E^q(x,\xi,t) = B_{20}^q(t) - 4\xi^2 C_{20}^q(t)$$

What can we currently check using lattice results?

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★ Understanding of systematic effects through sum rules

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★ Lattice data on transversity GPDs

$$\int_{-2}^{2} dx H_{Tq}(x,0,-0.69\,\mathrm{GeV}^2,P_3) = \{0.65(4),\ 0.64(6),\ 0.81(10)\}\,, \quad \int_{-2}^{2} dx H_{Tq}(x,\tfrac{1}{3},-1.02\,\mathrm{GeV}^2,1.25\,\mathrm{GeV}) = 0.49(5)\,,$$

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$$A_{T10}(-0.69\,\mathrm{GeV}^2) = \{0.65(4)\,,0.65(6)\,,0.82(10)\}\,, \qquad A_{T10}(-1.02\,\mathrm{GeV}^2) = 0.49(5)$$

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Sum rules not imposed in calculation

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GPDs on the lattice

★ γ^+ inspired parametrization is prohibitively expensive

$$F^{[\gamma^0]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^0 H_{\mathrm{Q(0)}}(x,\xi,t;P^3) + \frac{i\sigma^{0\mu}\Delta_\mu}{2M}E_{\mathrm{Q(0)}}(x,\xi,t;P^3)\right]u(p,\lambda)$$

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Light-cone GPDs using lattice correlators in non-symmetric frames

Theoretical setup

★ Parametrization of matrix elements in Lorentz invariant amplitudes

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Proof-of-concept calculation (zero quasi-skewness):

- symmetric frame: $\vec{p}^{\,s}_f = \vec{P} + \frac{\vec{Q}}{2}$, $\vec{p}^{\,s}_i = \vec{P} - \frac{\vec{Q}}{2}$, $t^s = -\vec{Q}^2$

- asymmetric frame: $\vec{p}^{\,a}_f = \vec{P}$, $\vec{p}^{\,a}_i = \vec{P} - \vec{Q}$, $t^a = -\vec{Q}^2 + (E_f - E_i)^2$

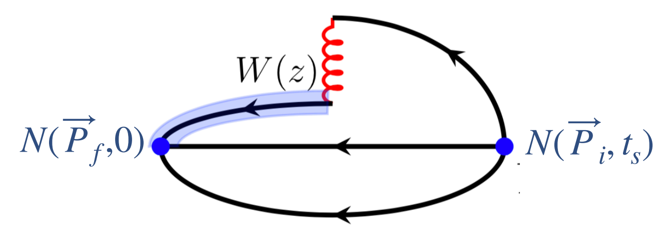
Parameters of calculation

★ Nf=2+1+1 twisted mass (TM) fermions & clover improvement

Pion mass:	260 MeV
Lattice spacing:	0.093 fm
Volume:	32^3 x 64
Spatial extent:	3 fm

★ Calculation:

- isovector combination
- zero skewness
- T_{sink}=1 fm



frame	P_3 [GeV]	$\mathbf{Q}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
symm	1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
non-symm	1.25	(±2,0,0), (0,±2,0)	0.64	0	8	269	8	17216

★ Computational cost:

- symmetric frame 4 times more expensive than asymmetric frame for same set of $\vec{Q}$ (requires separate calculations at each t)

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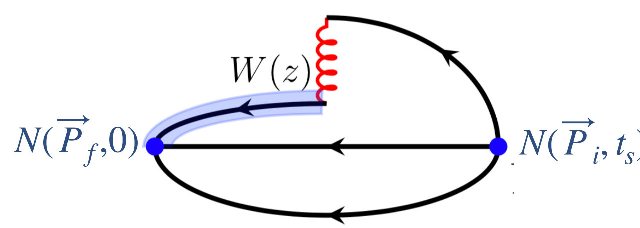

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Small difference: $t^s = -\vec{Q}^2$ $t^a = -\vec{Q}^2 + (E_f - E_i)^2$

$$A(-0.64\text{GeV}^2) \sim A(-0.69\text{GeV}^2)$$

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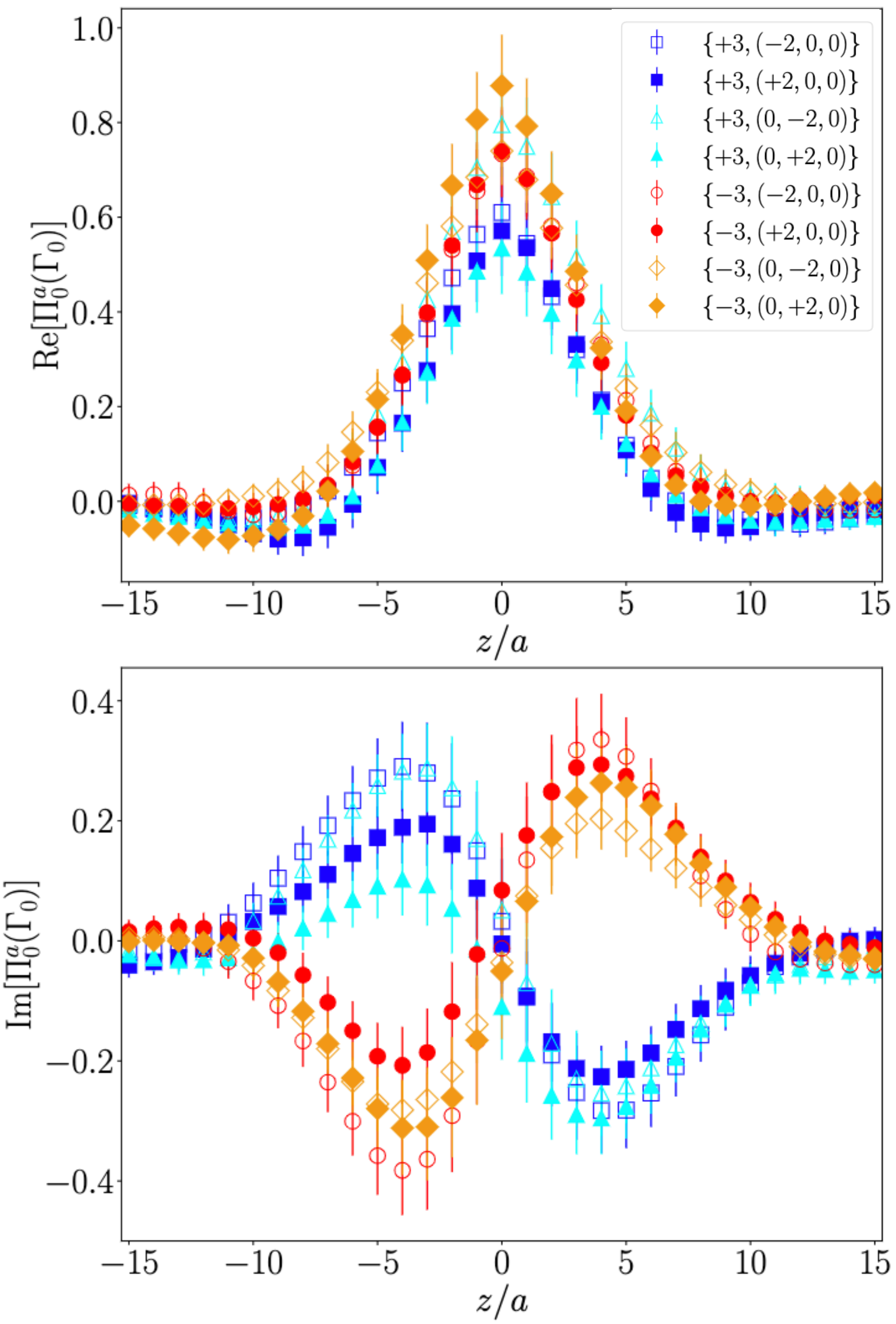

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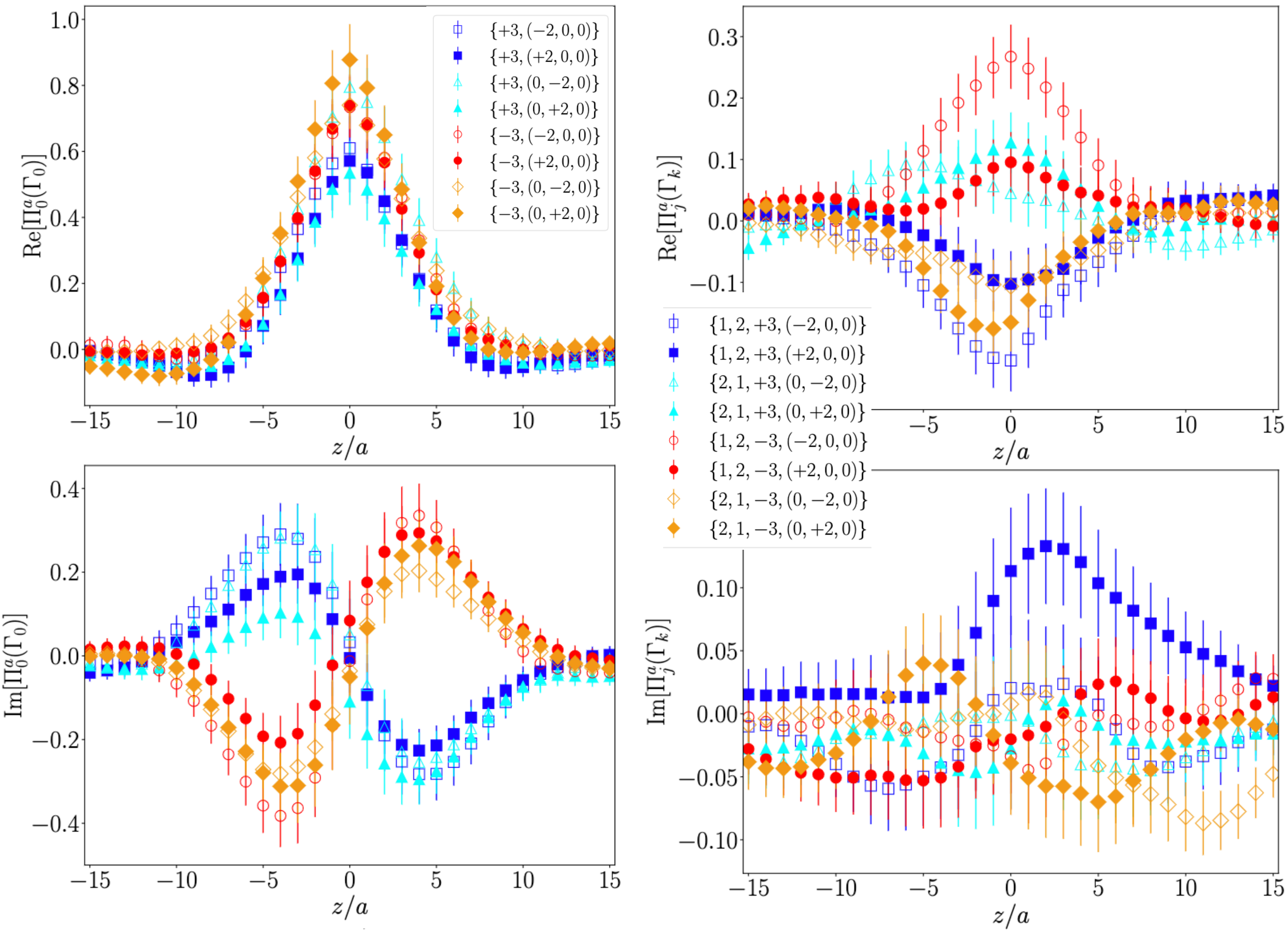

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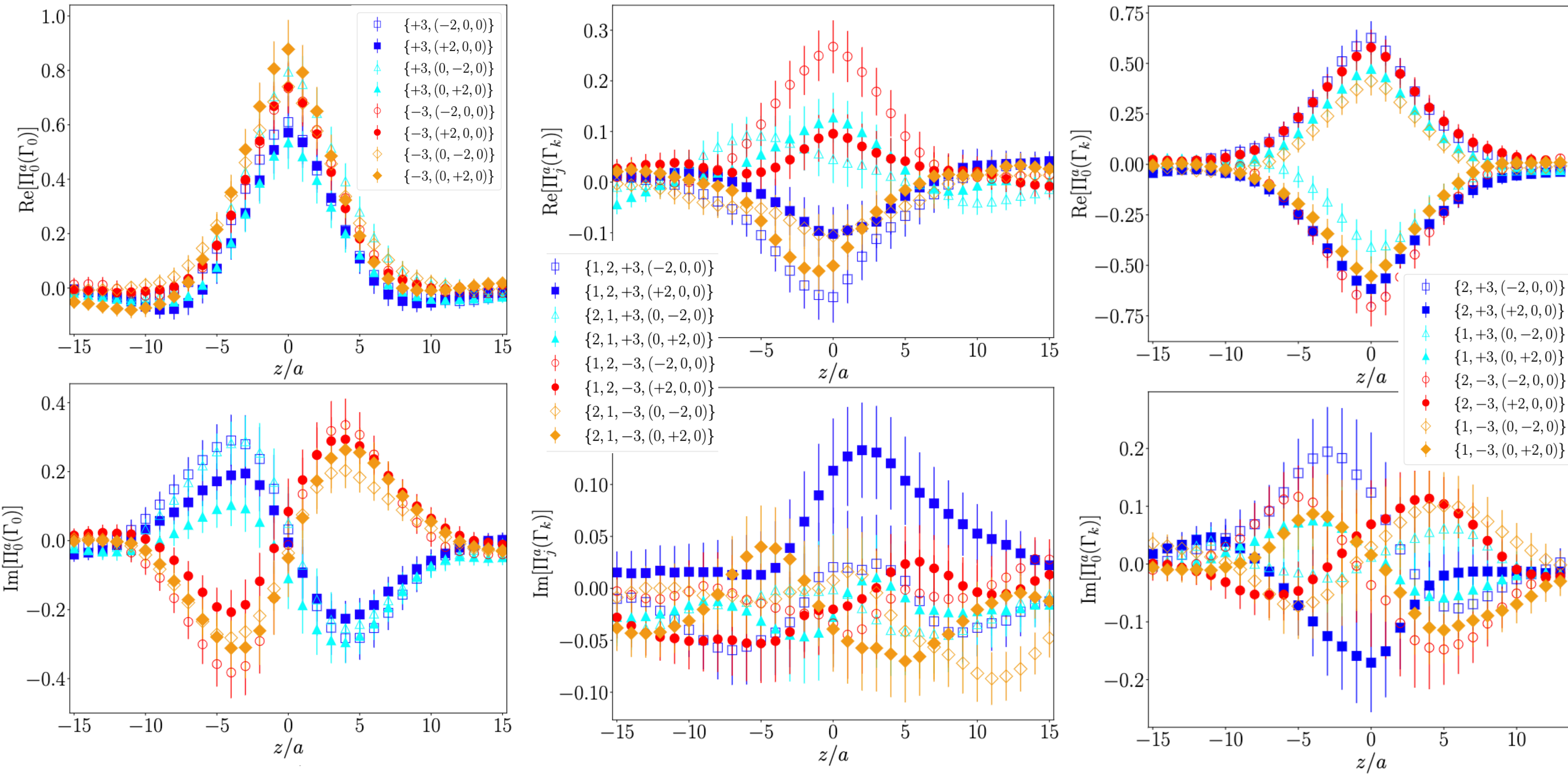

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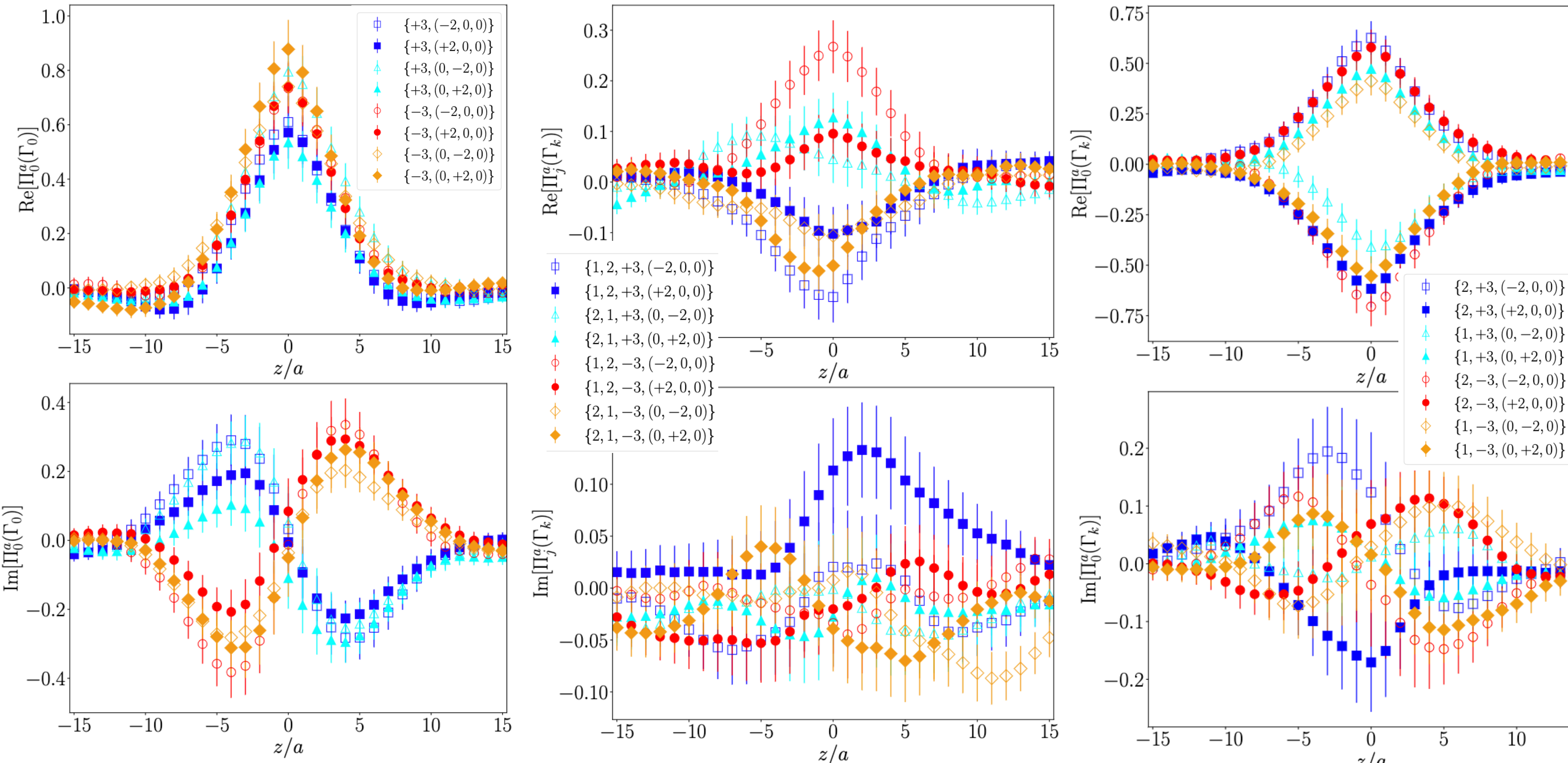

★ Asymmetric frame: ME do not have definite symmetries in $\pm P_3, \pm Q, \pm z$

★ Noisy ME lead to challenges in extracting A_i of sub-leading magnitude

How do the A_i compare between frames?

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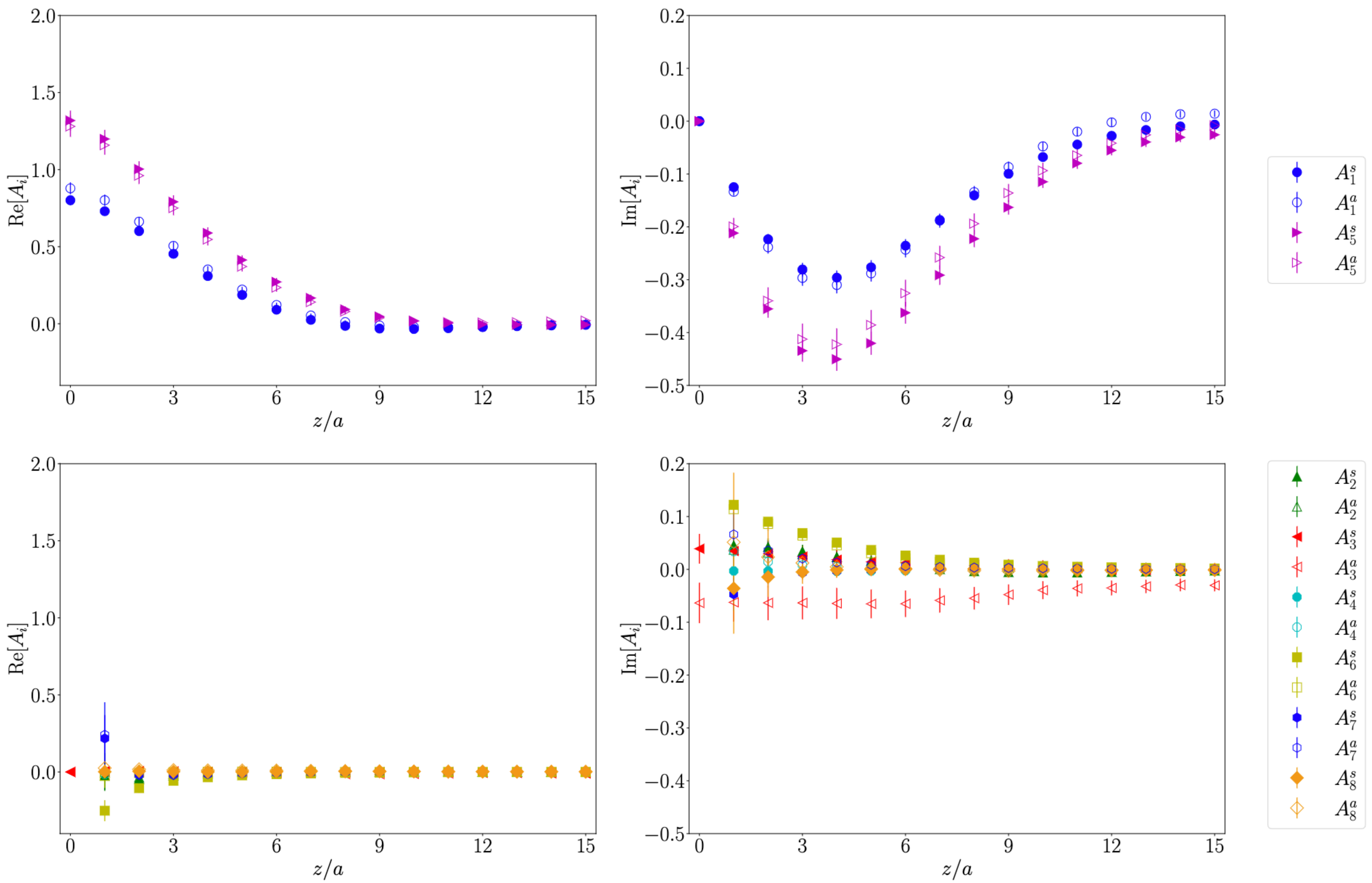

- ★ A_1, A_5 dominant contributions
- ★ Full agreement in two frames for both Re and Im parts of A_1, A_5
- ★ Remaining A_i suppressed (at least for this kinematic setup and $\xi = 0$)

quasi-GPDs in terms of A_i

- ★ The mapping of A_i to the quasi-GPDs is not unique
- ★ Construction of a Lorentz invariant definition may be beneficial

$$(\xi = 0) \qquad \Pi_H^{\text{impr}} = A_1$$

$$\Pi_E^{\text{impr}} = -A_1 + 2A_5 + 2zP_3A_6$$

- ★ All quasi-GPDs definitions converge to the same light-cone GPDs
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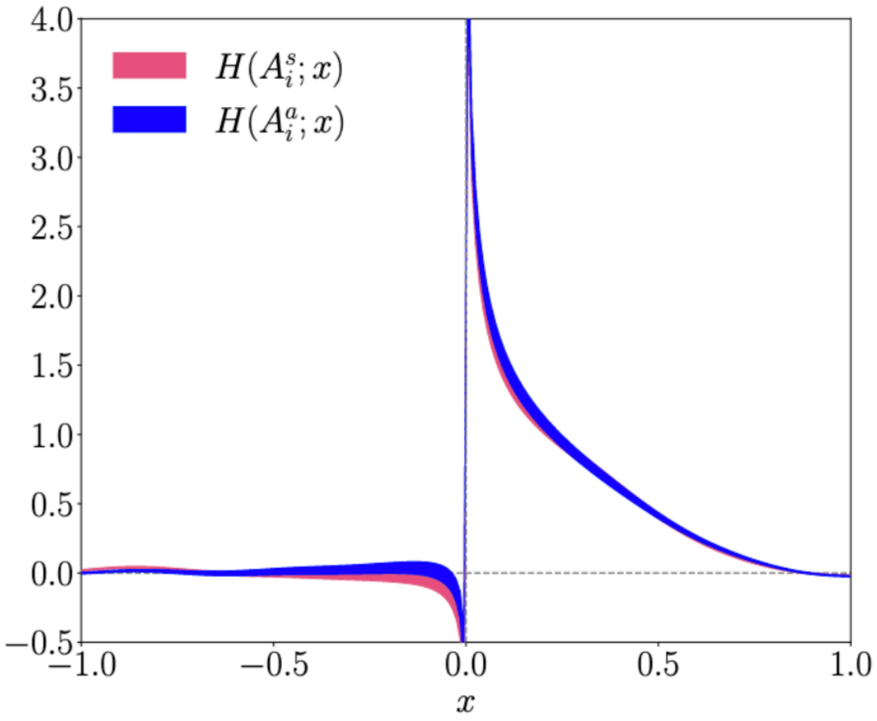
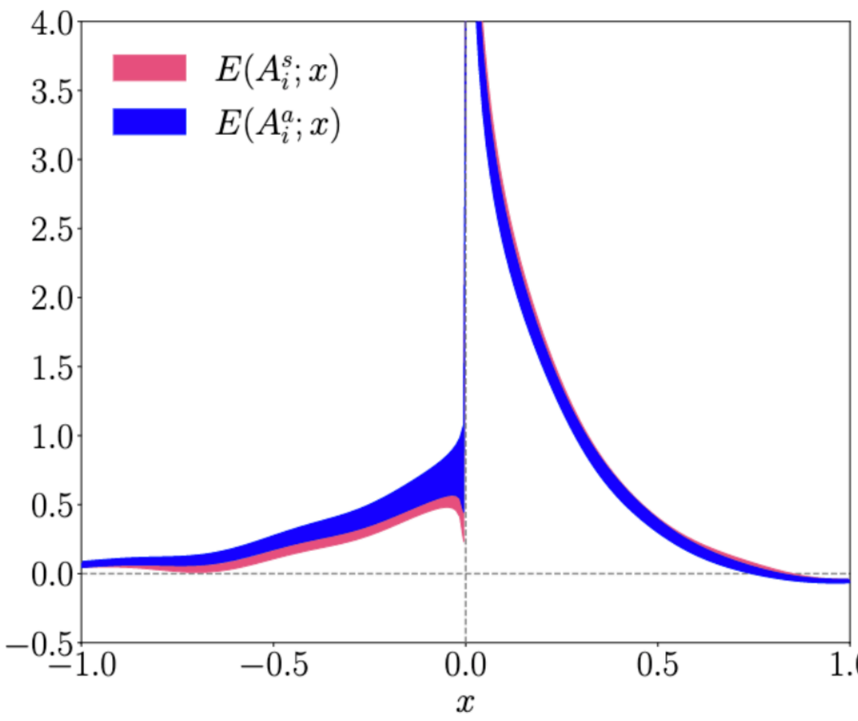




Agreement between frames for both quasi-GPDs (by definition)

Beyond exploration

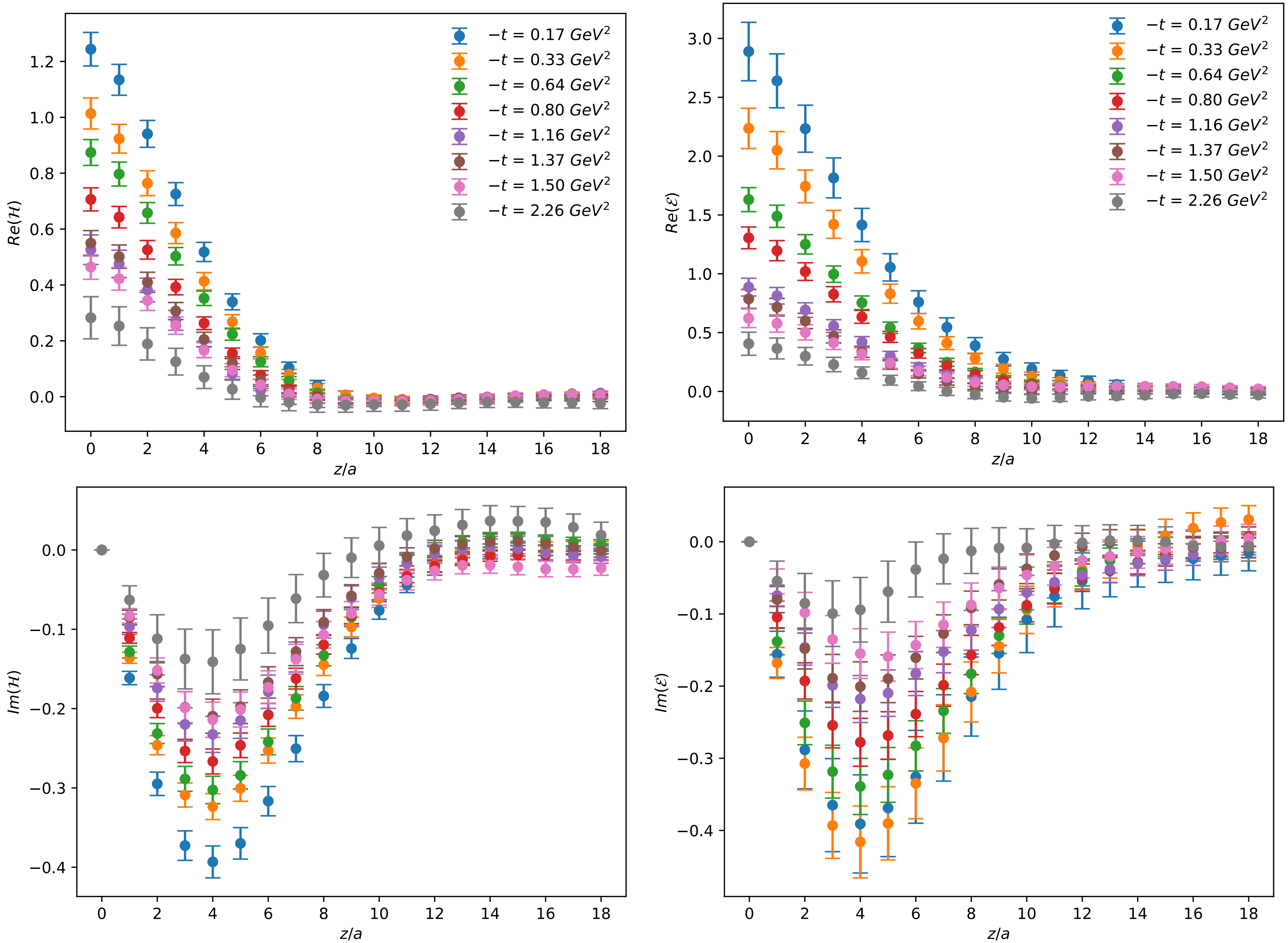
- 11 values of $-t$ (3 in symm. frame and 8 in asymm. frame)
- Separate calculation for each $-t$ value in symmetric frame
- Two groups of $-t$ value in asymmetric frame: $\vec{Q} = (Q_x,0,0), (Q_x, Q_y,0)$

frame	P_3 [GeV]	$\mathbf{\Delta}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	731	16	23392
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	(±2, ±2, 0)	1.39	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.76	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	429	8	27456
asymm	±1.25	(±1, ±1, 0)	0.33	0	16	194	8	12416
asymm	±1.25	(±2,0,0), (0,±2,0)	0.64	0	8	429	8	27456
asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.80	0	16	194	8	12416
asymm	±1.25	(±2,±2,0)	1.16	0	16	194	8	24832
asymm	±1.25	(±3,0,0), (0,±3,0)	1.37	0	8	429	8	27456
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.50	0	16	194	8	12416
asymm	±1.25	(±4,0,0), (0,±4,0)	2.26	0	8	429	8	27456

- Momentum transfer range is very optimistic
  (some values have enhanced systematic uncertainties)

Unpolarized quasi-GPDs

asymmetric frame

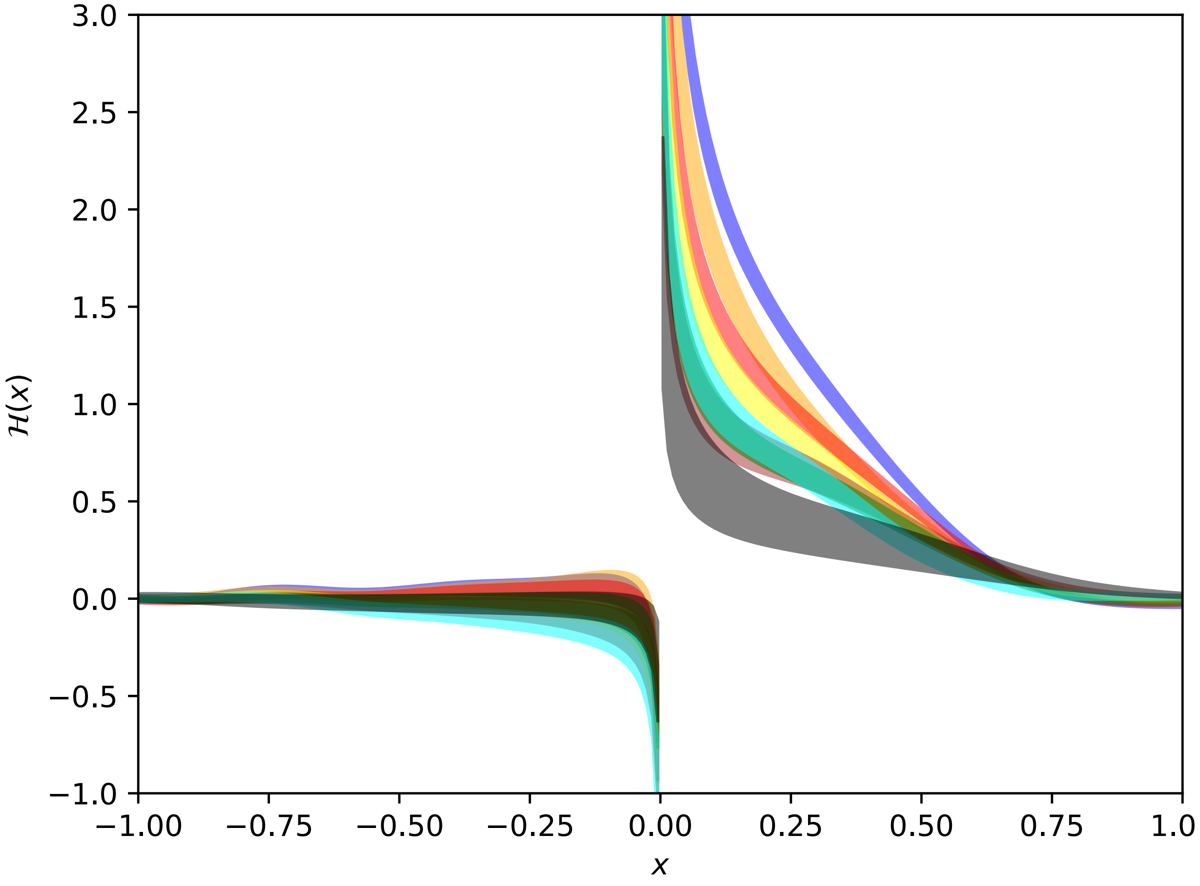


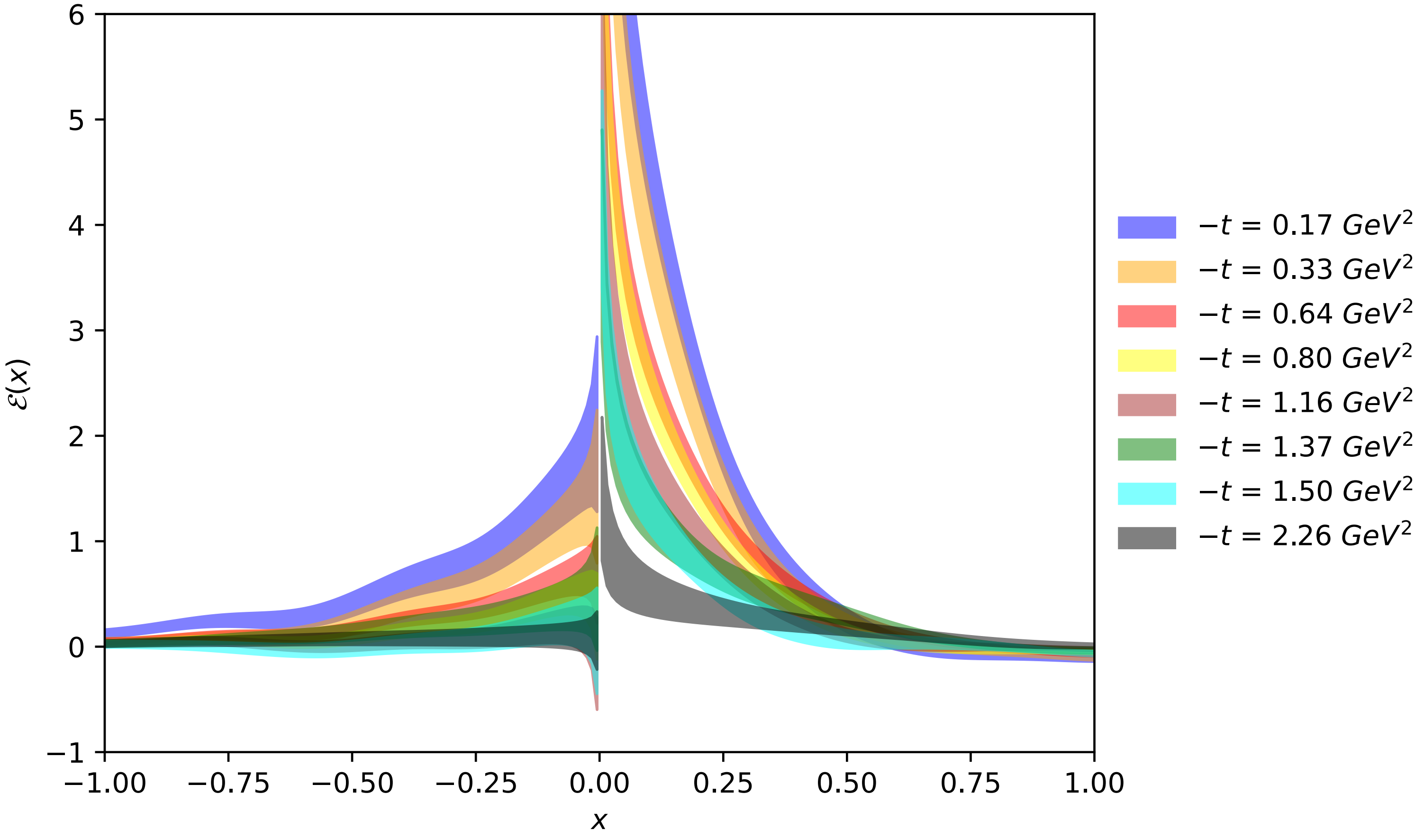
★ Impressive quality of signal quality

★ Behavior with increasing $-t$ as “expected” qualitatively

Unpolarized light-cone GPDs

- ★ quasi-GPDs transformed to momentum space
- ★ Matching formalism to 1 loop accuracy level
- ★ +/-x correspond to quark and anti-quark region
- ★ Anti-quark region susceptible to systematic uncertainties.





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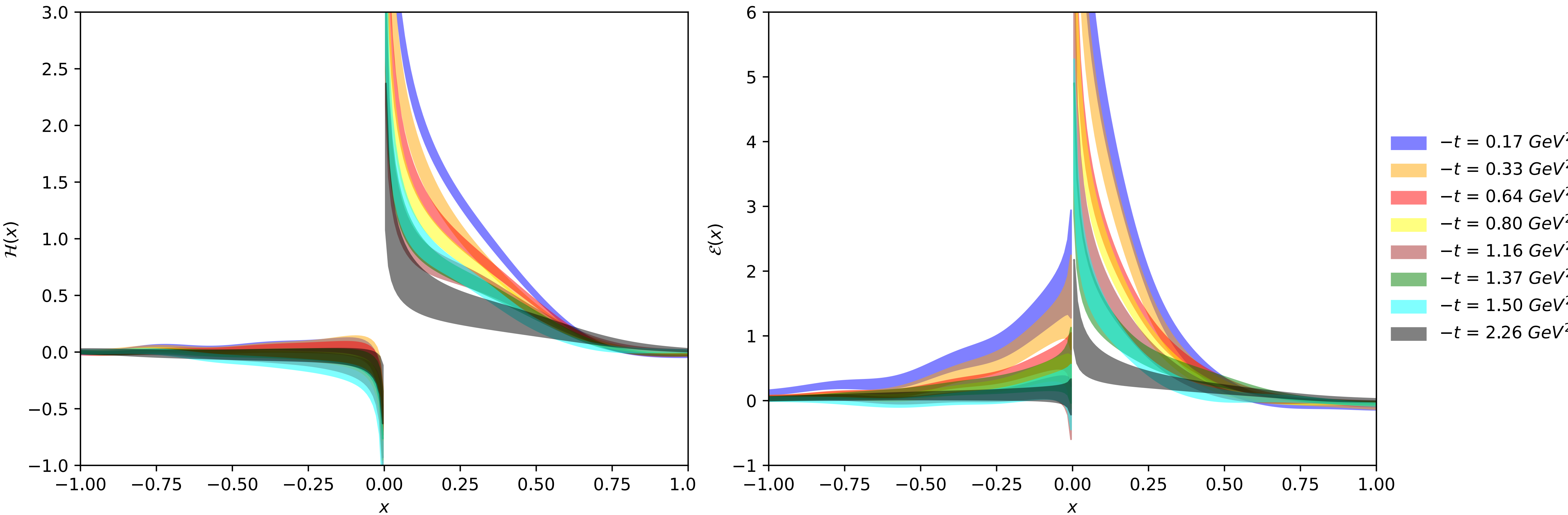
Several values of -t accessible at once



Mellin moments from non-local operators

arXiv:2305.11117

★ Leading-twist factorization formula

$$\mathscr{M}(z,P,\Delta) \equiv \frac{\mathscr{F}(z,P,\Delta)}{\mathscr{F}(z,P=0,\Delta=0)} = \sum_{n=0} \frac{(-izP)^n}{n!} \frac{C_n^{\overline{\mathrm{MS}}}(\mu^2 z^2)}{C_0^{\overline{\mathrm{MS}}}(\mu^2 z^2)} \langle x^n \rangle + \mathscr{O}(\Lambda_{\mathrm{QCD}}^2 z^2)$$

★ Avoid power-divergent mixing of multi-derivative operators

★ Wilson coefficients known to NLO (or NNLO)

★ Both isovector and isoscalar (ignores disconnected; found to be tiny)

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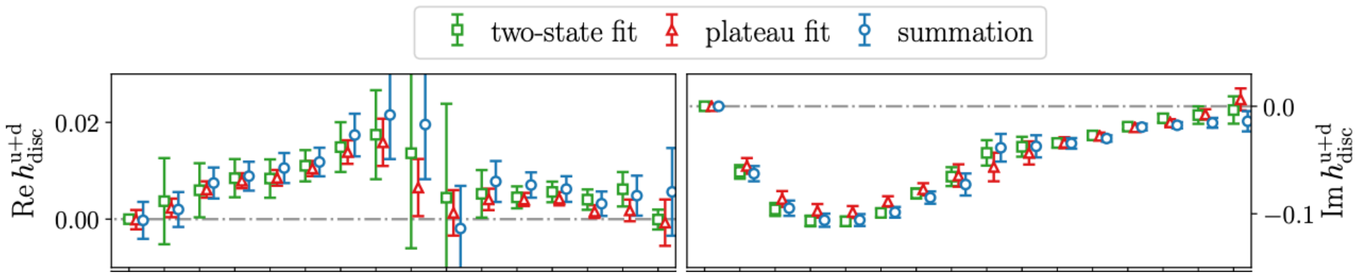

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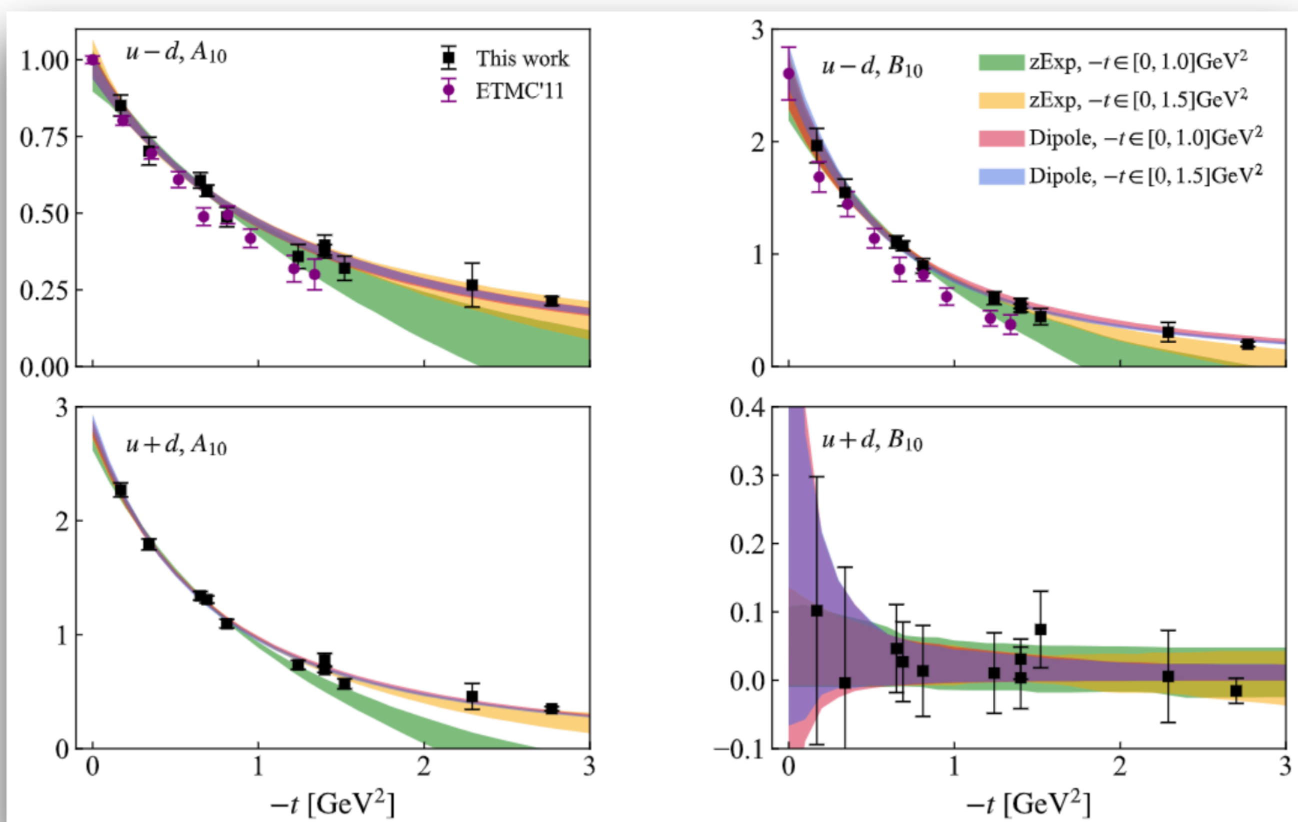

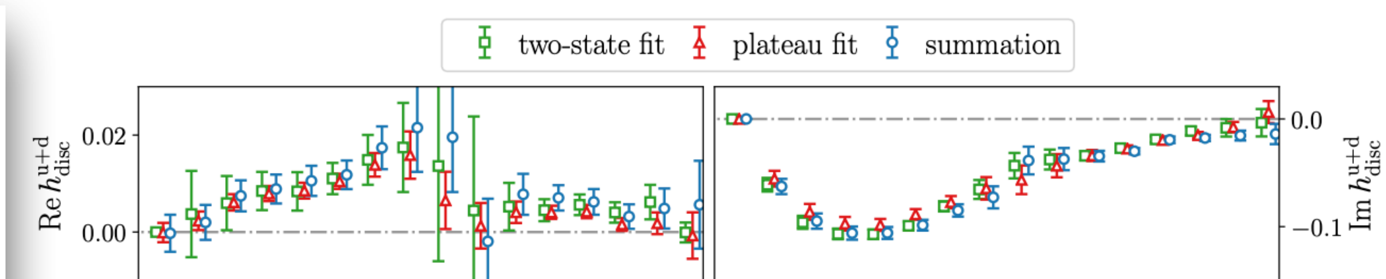

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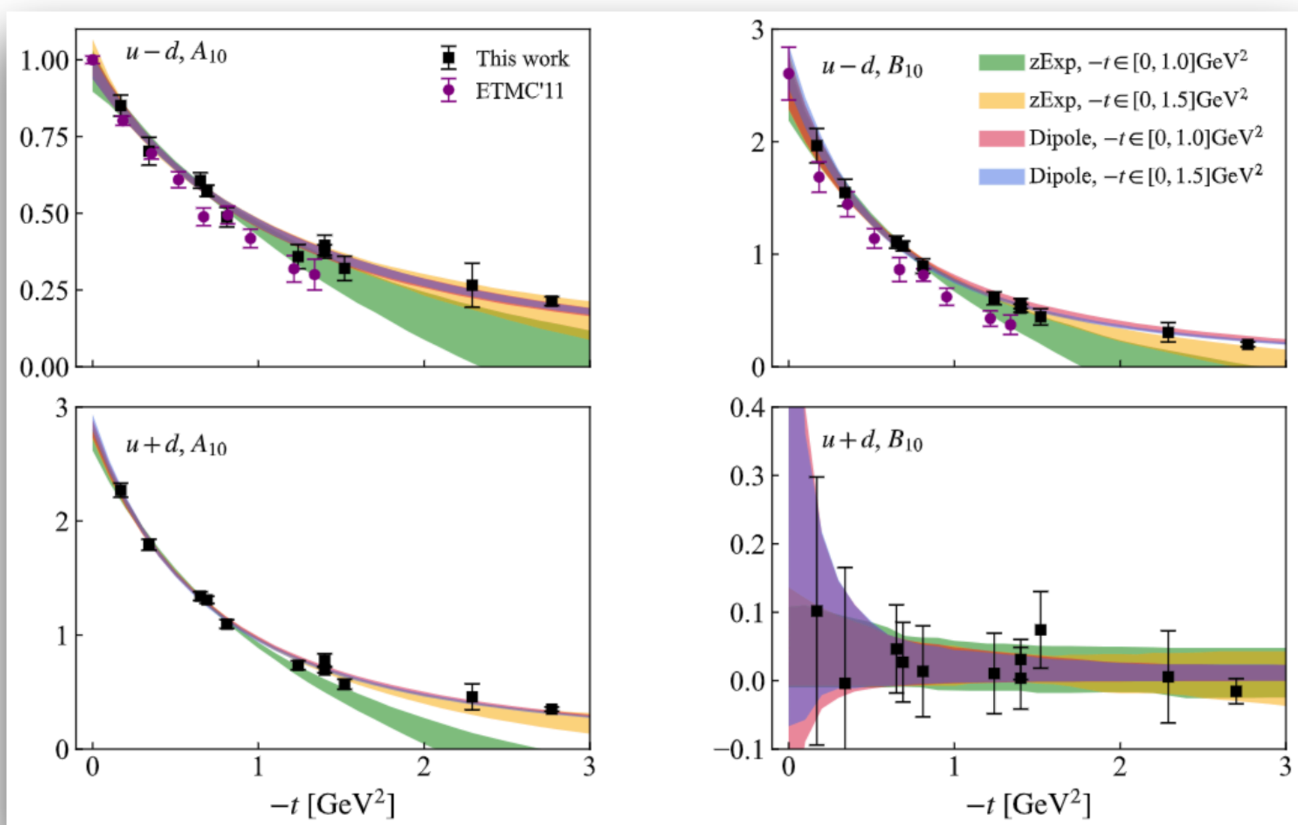

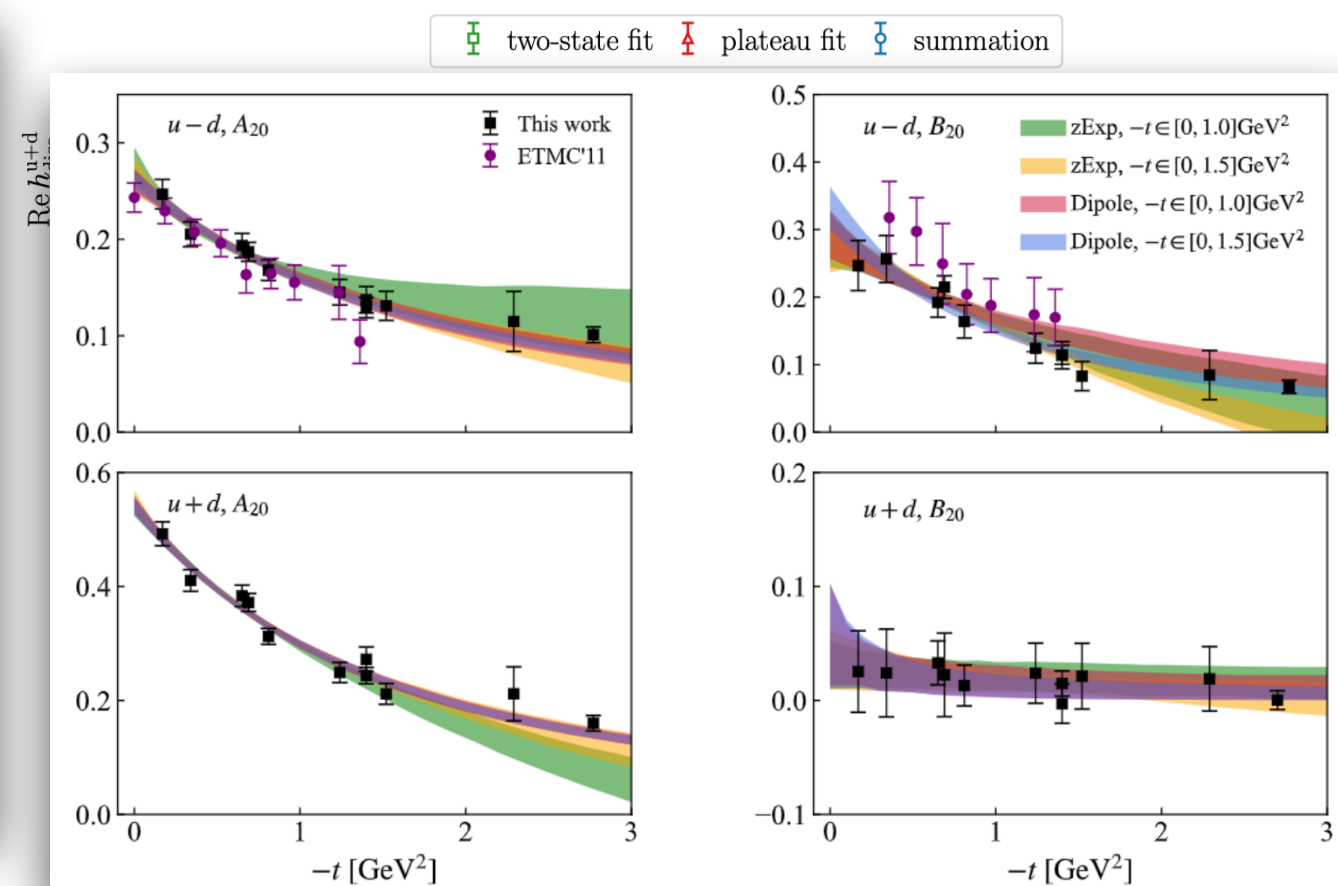

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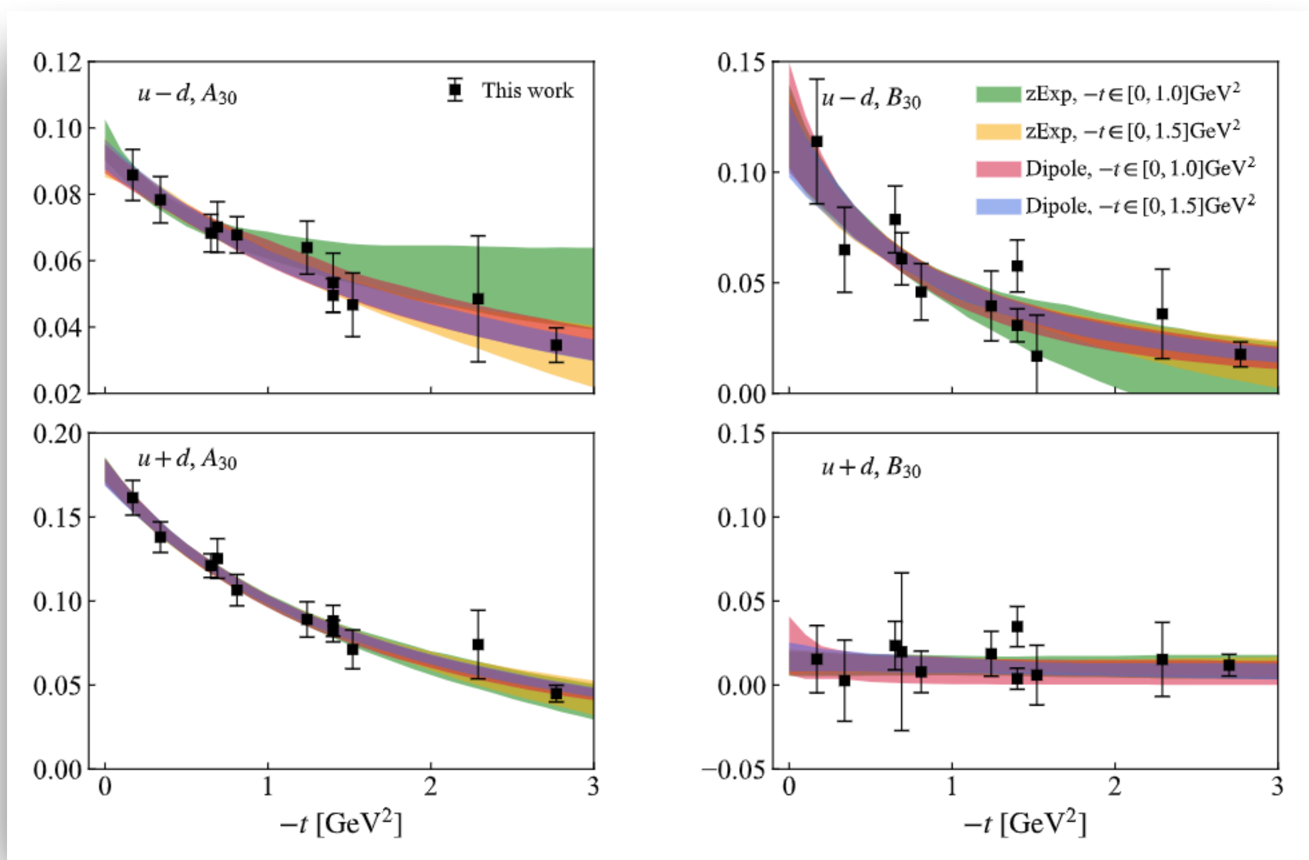

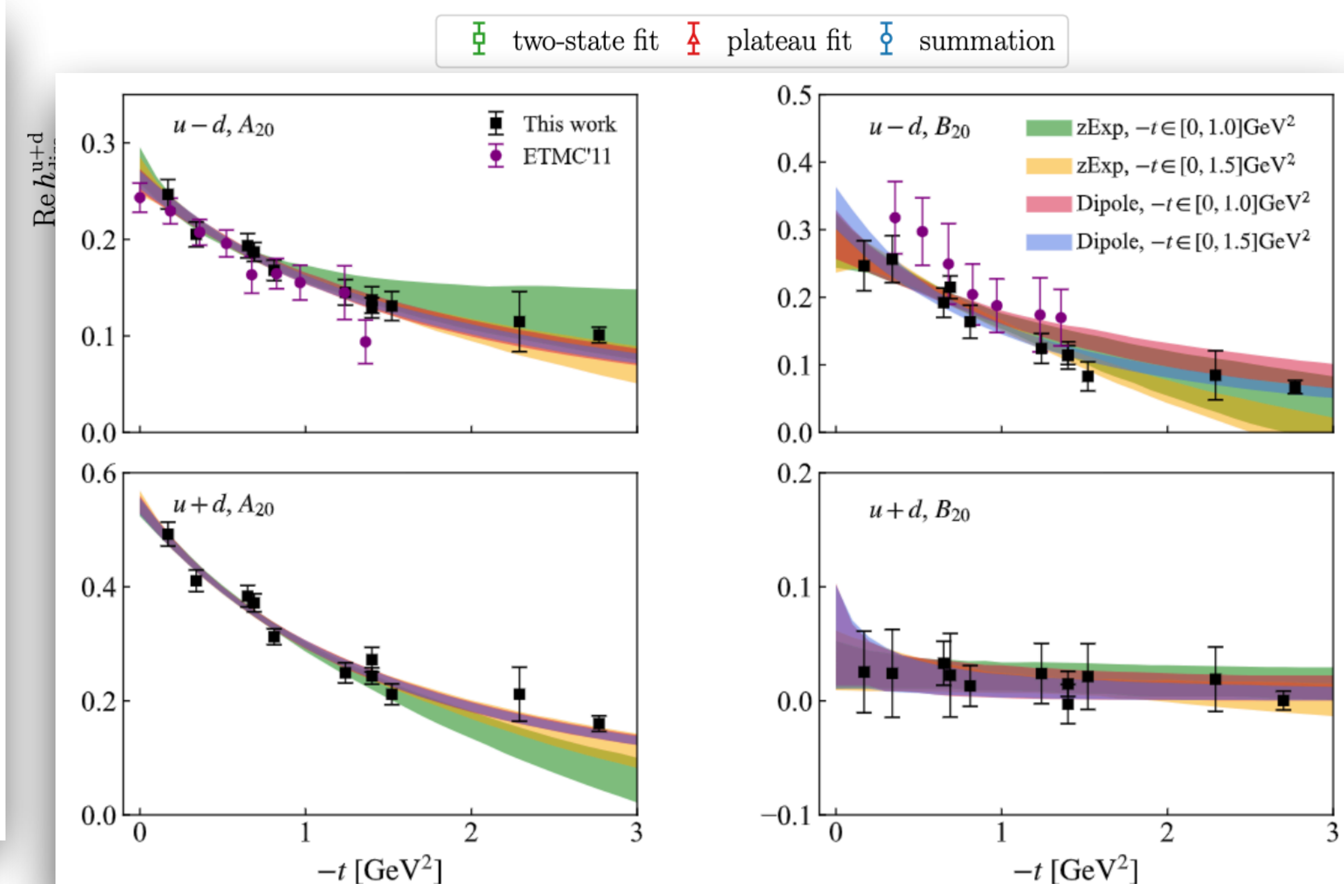

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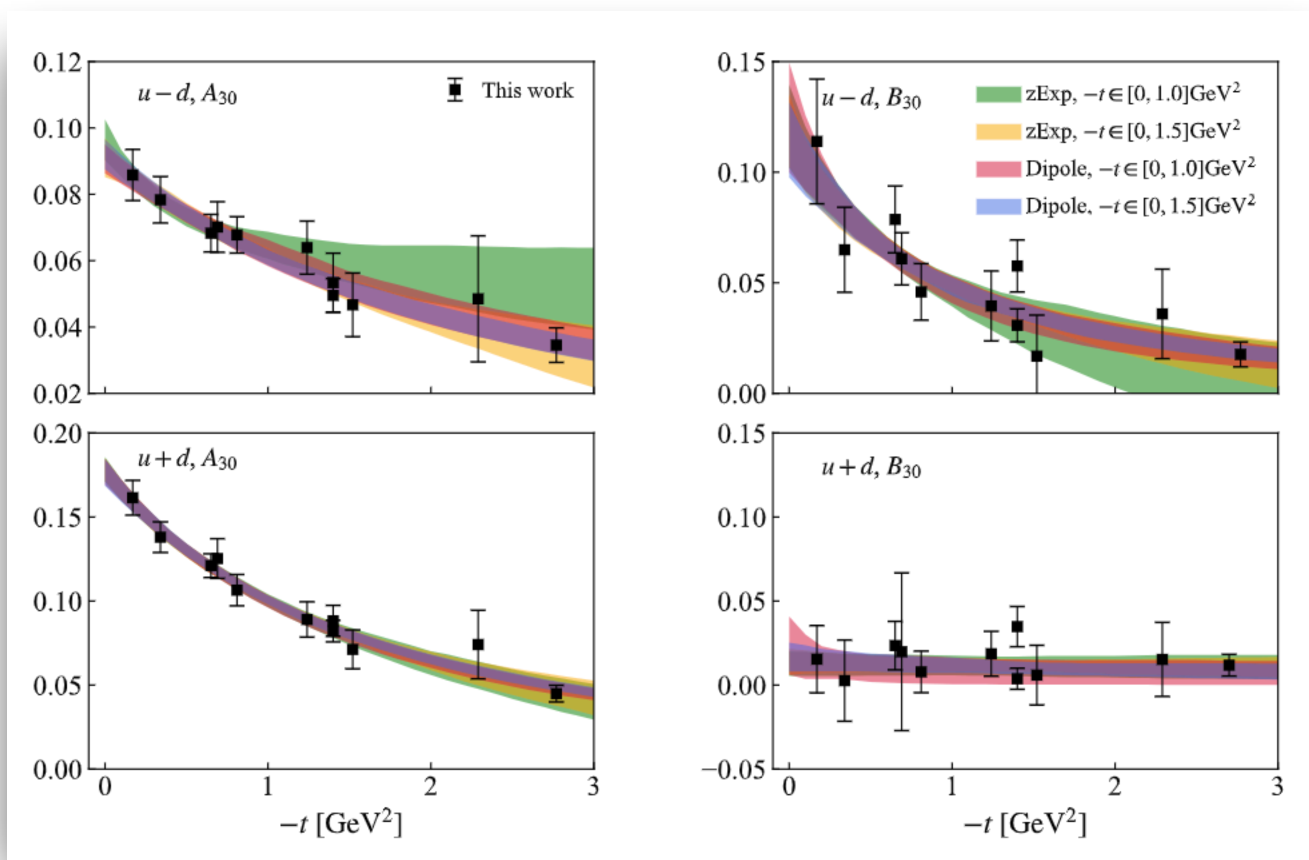

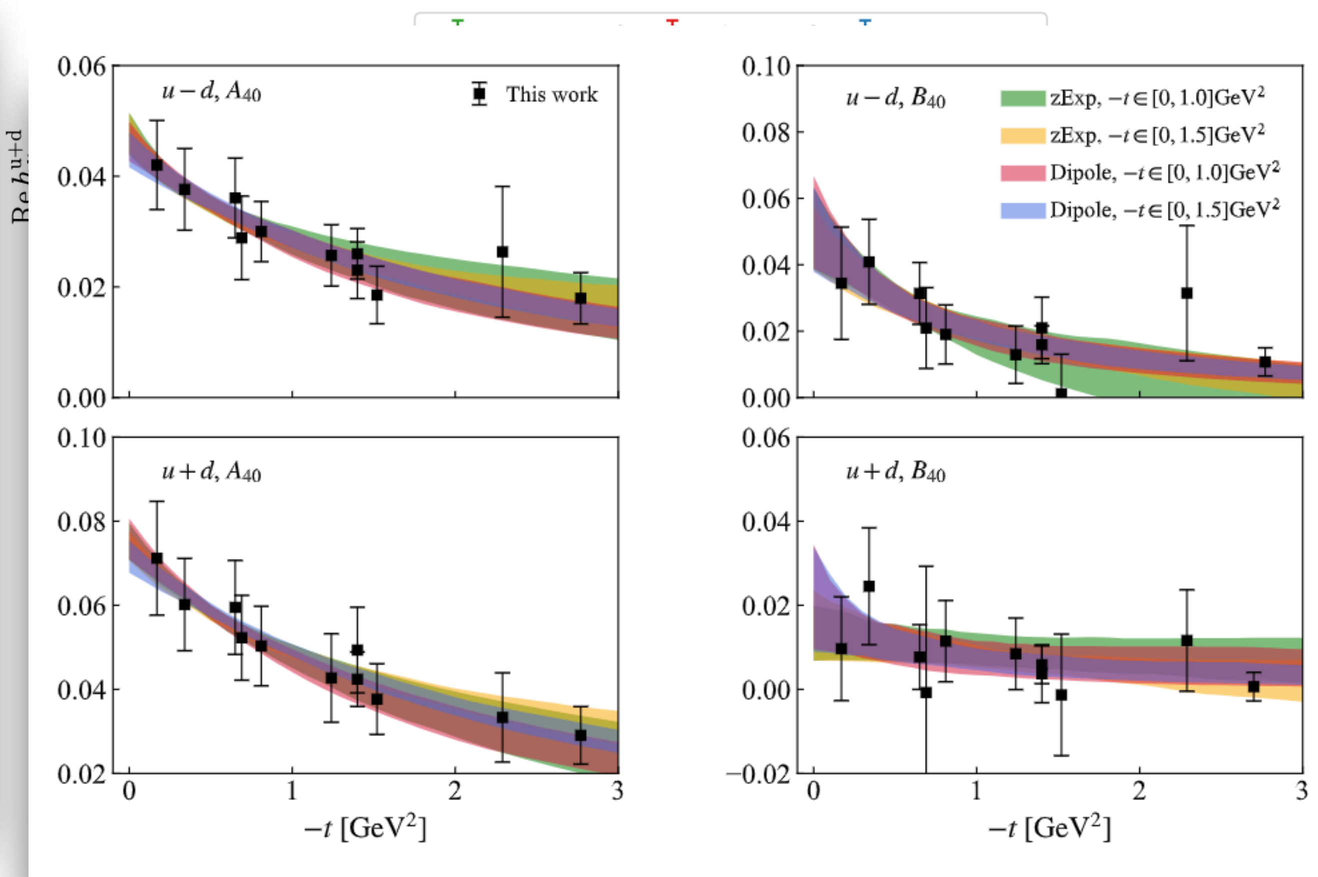

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Access to Mellin moments beyond local operators

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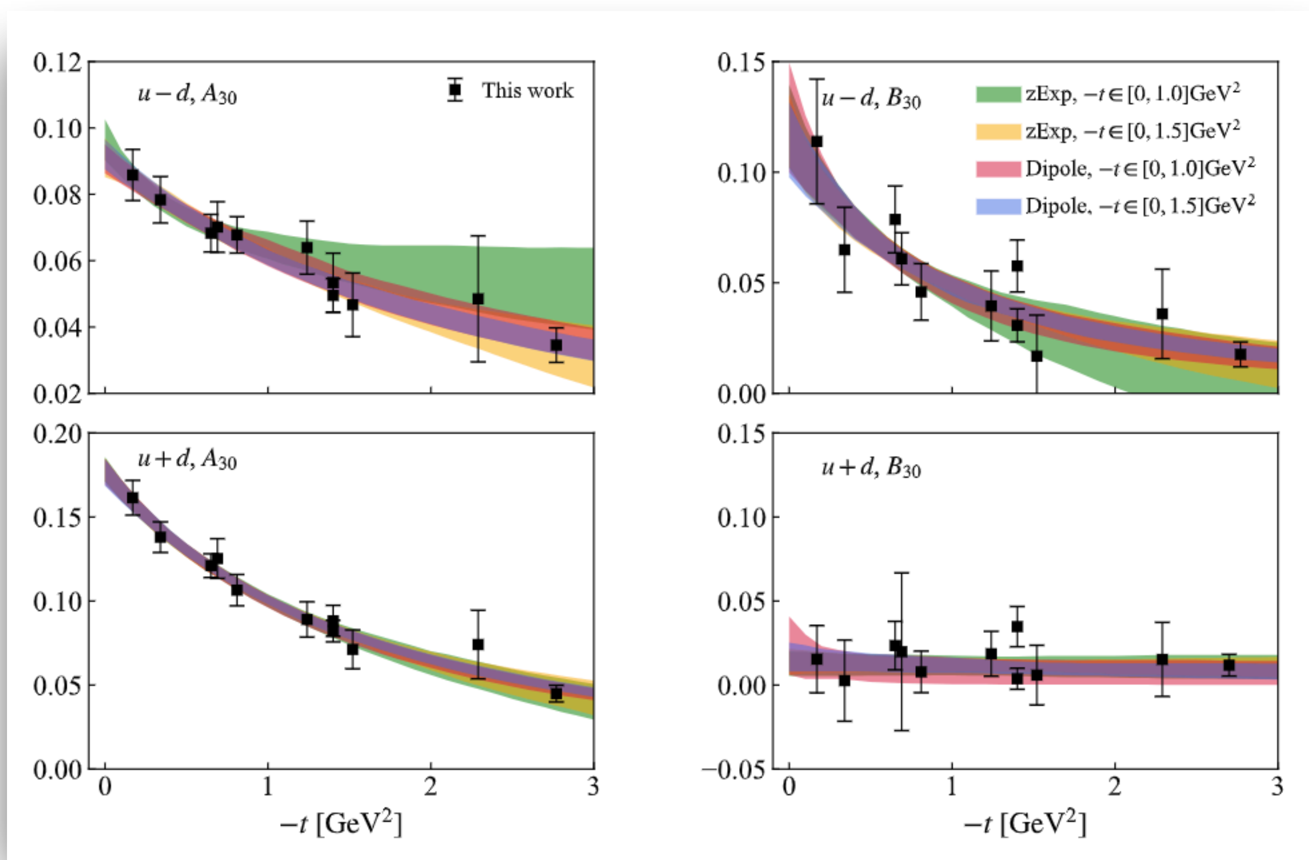

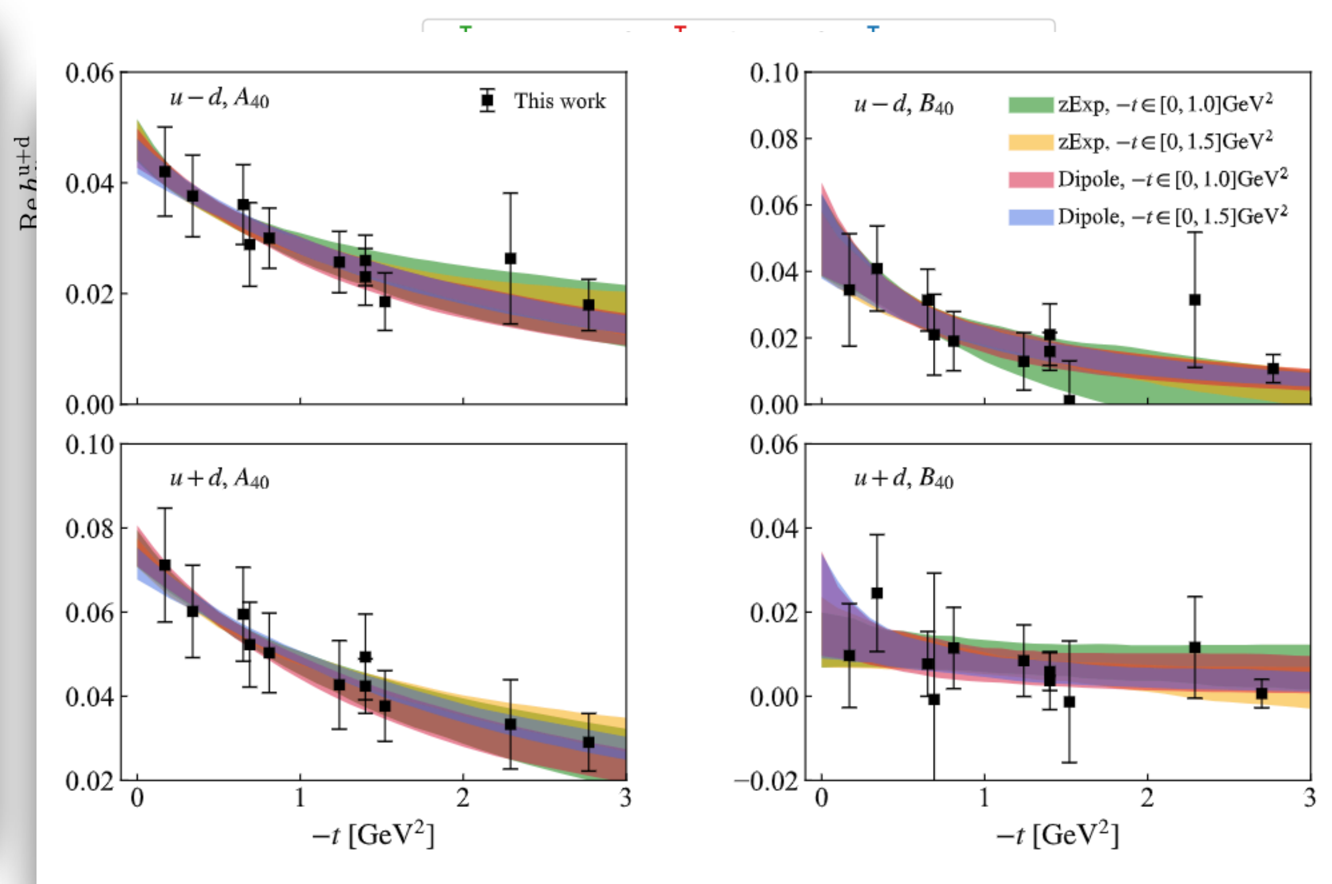

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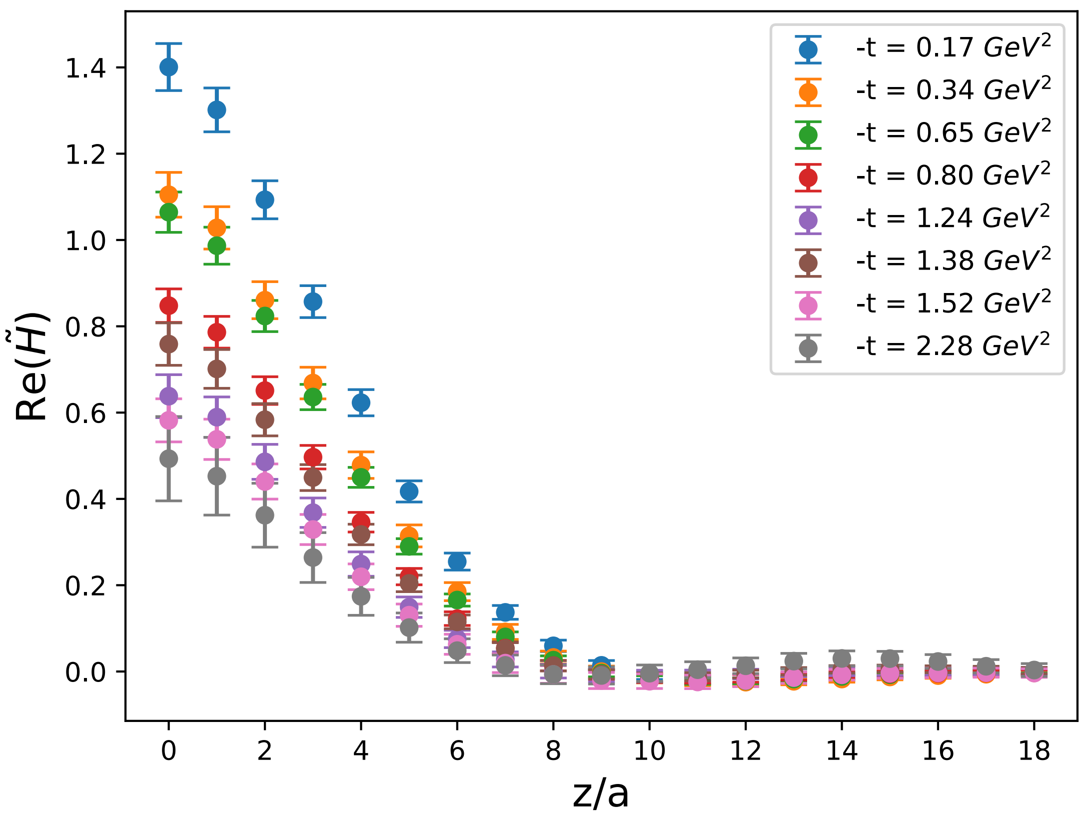

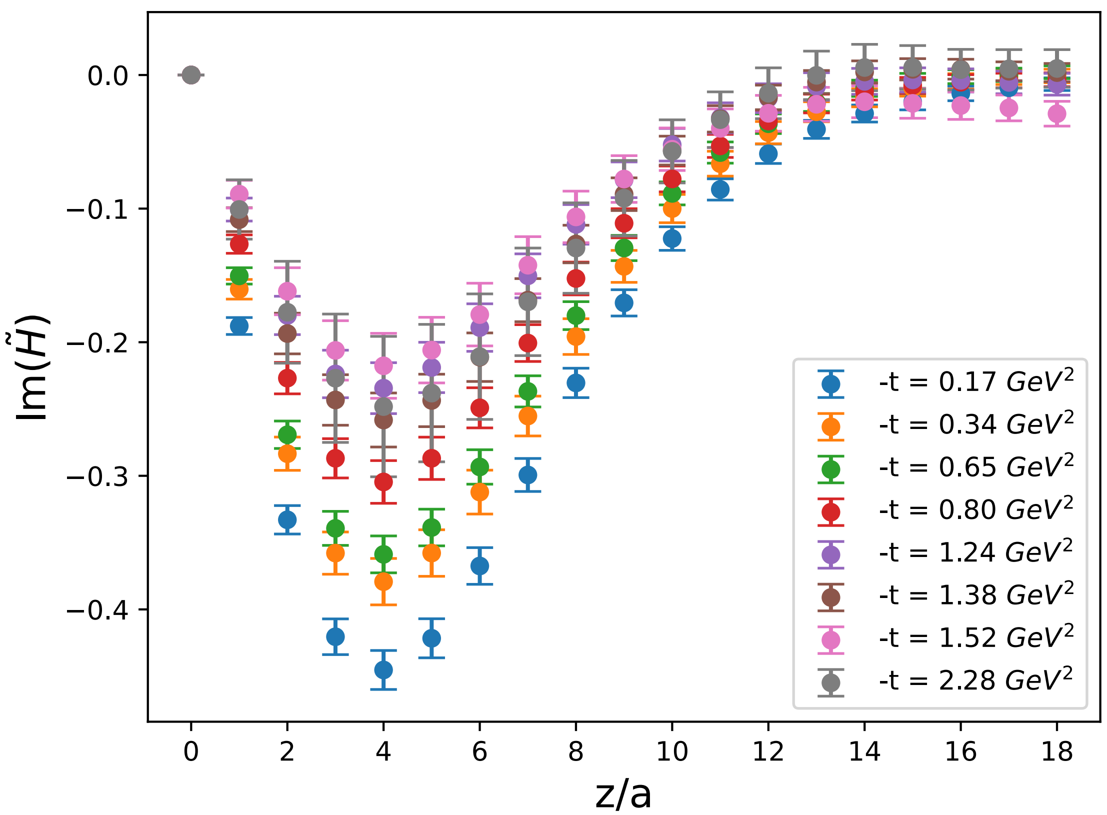

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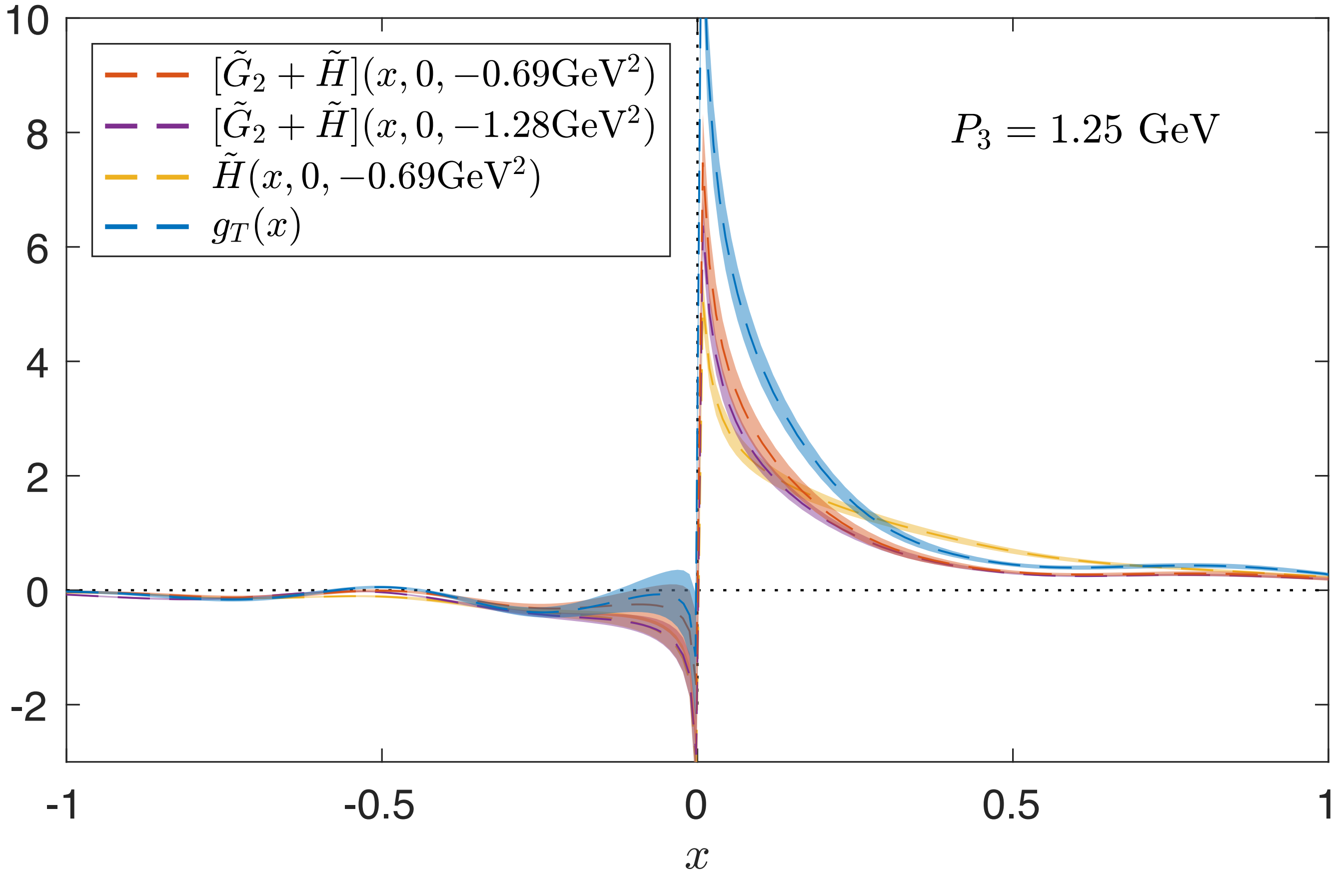
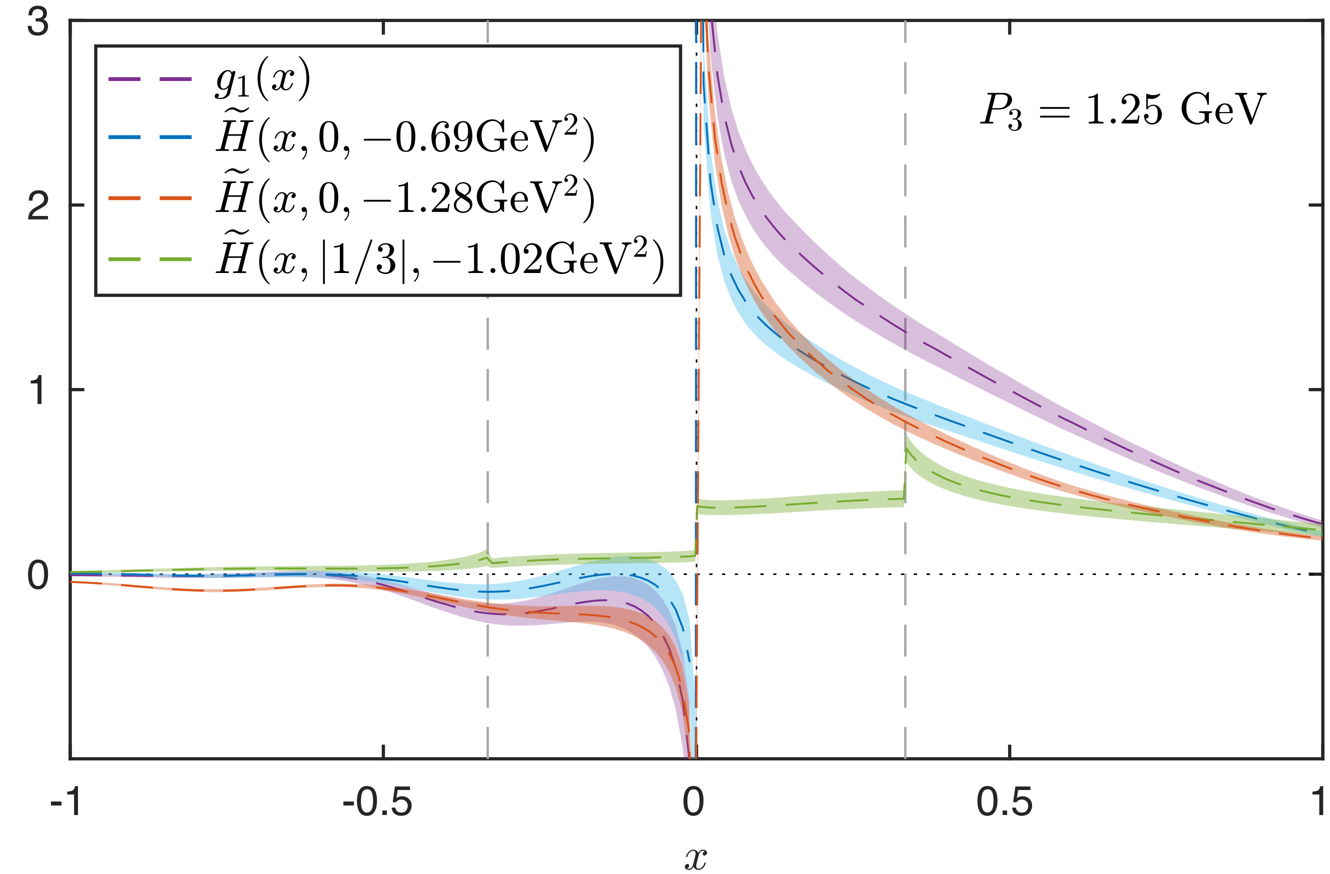
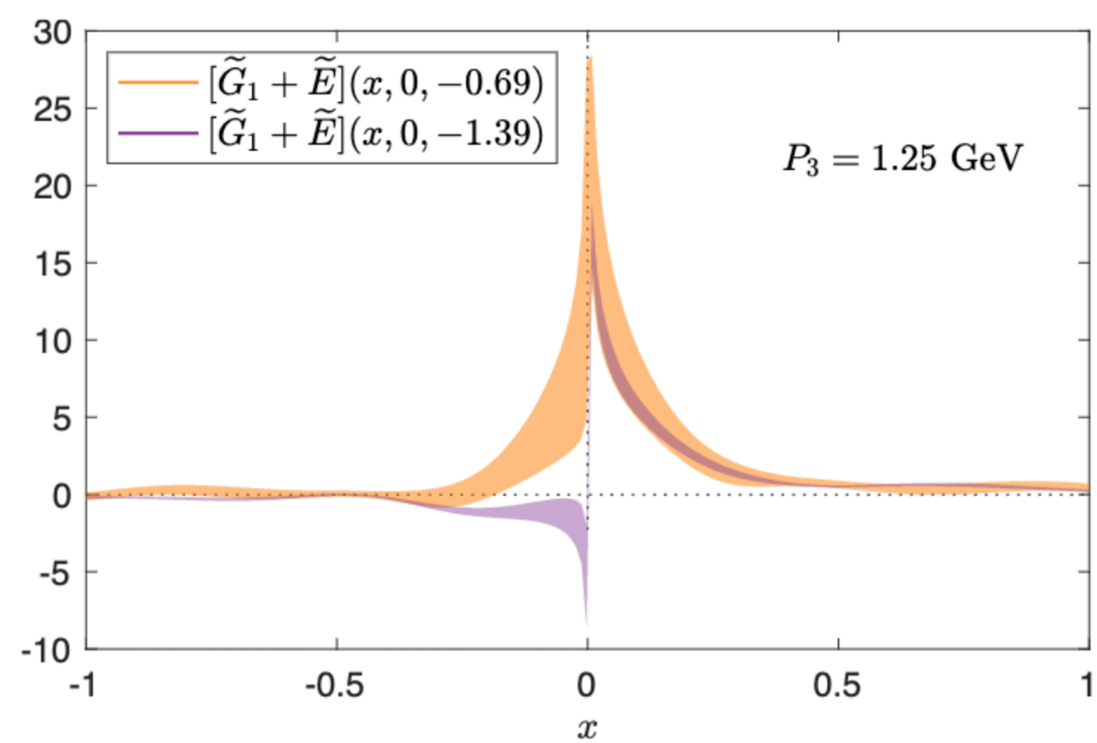
★ Twist-3 GPDs

PRELIMINARY





[S. Bhattacharya et al., PoS LATTICE2021 (2022) 054 arXiv:2112.05538]



- ★ $g_T(x)$: dominant distribution
- ★ $\widetilde{H} + \widetilde{G}_2$ similar in magnitude to $\widetilde{H}$
- ★ $\widetilde{G}_2$ is expected to be small

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PHENOMENOLOGY
LATTICE QCD
THEORY

QUARK-GLUON TOMOGRAPHY COLLABORATION

U.S. DEPARTMENT OF ENERGY | Office of Science

Award Number: DE-SC0023646

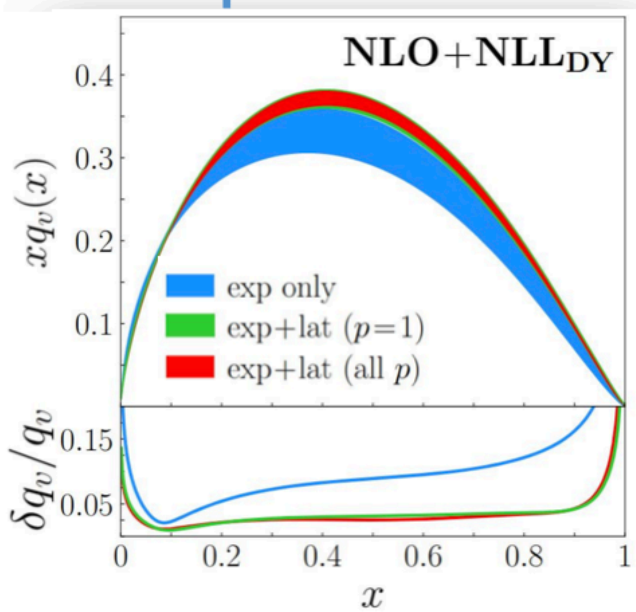
1. Theoretical studies of high-momentum transfer processes using perturbative QCD methods and study of GPDs properties

2. Lattice QCD calculations of GPDs and related structures

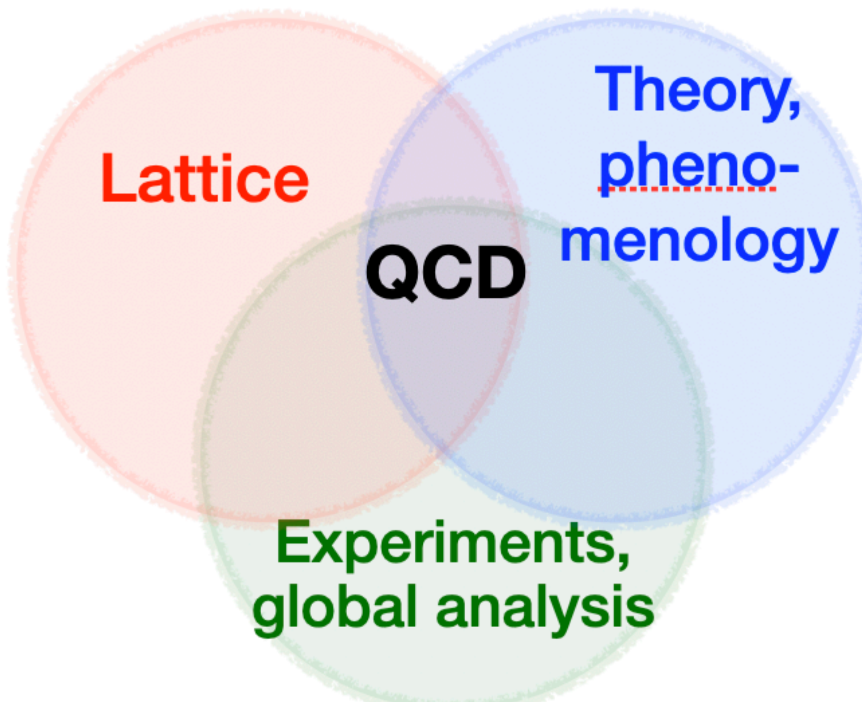
3. Global analysis of GPDs based on experimental data using modern data analysis techniques for inference and uncertainty quantification

Synergies: constraints & predictive power of lattice QCD

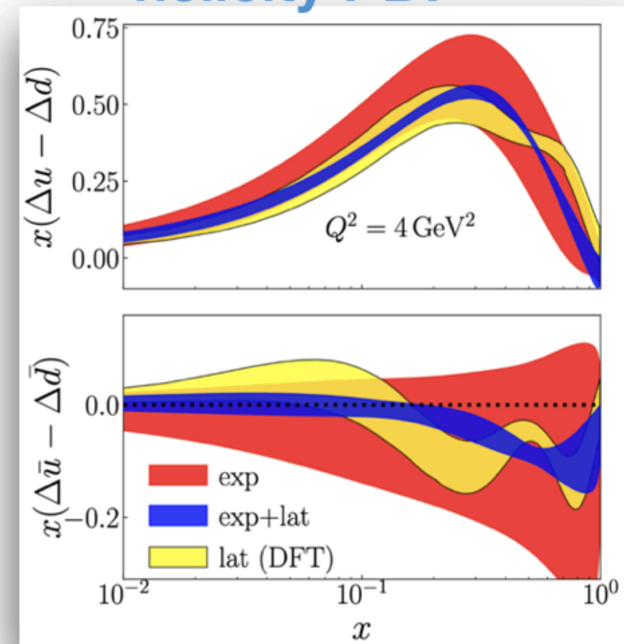
pion PDF



[JAM/HadStruc, PRD105 (2022) 114051]

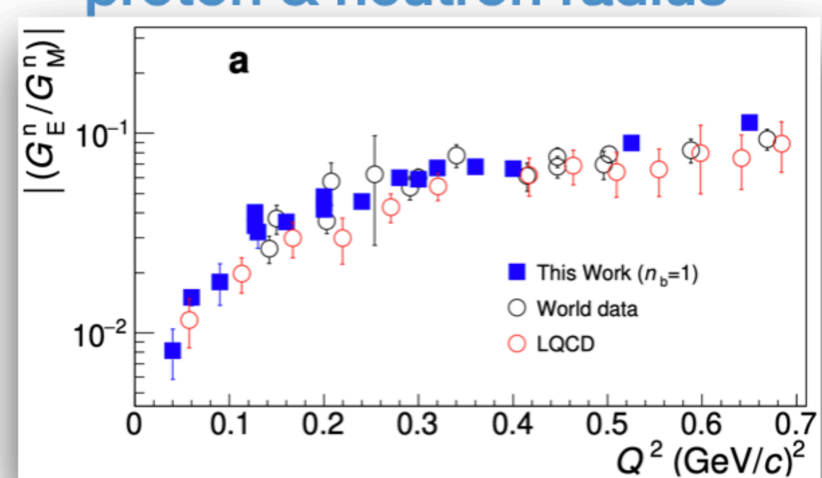


helicity PDF



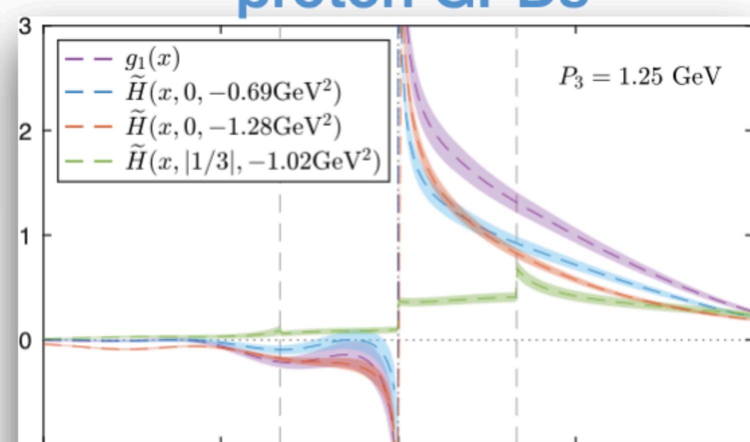
[JAM & ETMC, PRD 103 (2021) 016003]

proton & neutron radius



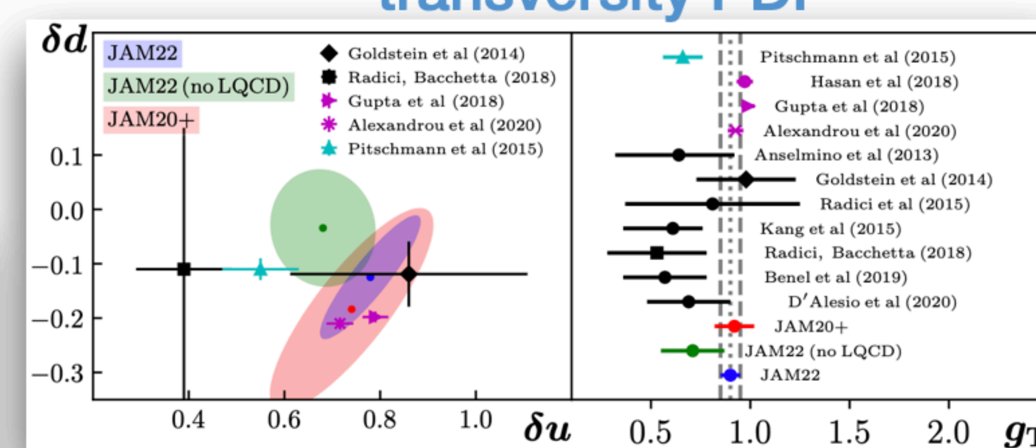
[Atac et al., Nature Comm. 12, 1759 (2021)]

proton GPDs



[ETMC, PRL 125 (2020) 262001]

transversity PDF



[JAM, PRD 106 (2022) 3, 034014]

And many more!

Summary

★ Lattice QCD data on GPDs will play an important role in the pre-EIC era and can complement experimental efforts of JLab@12GeV

★ New proposal for Lorentz invariant decomposition has great advantages:
- significant reduction of computational cost
- access to a broad range of t and ξ

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★ Mellin moments can be extracted utilizing quasi-GPDs data

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Thank you

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DOE Early Career Award (NP)
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